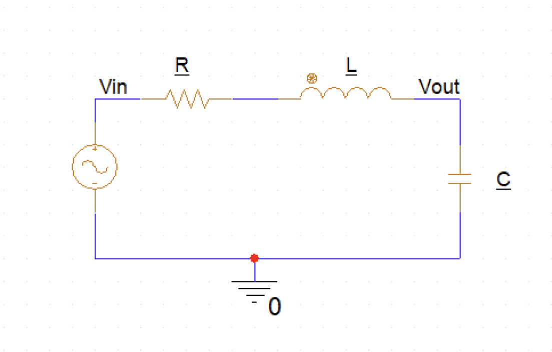
UC San Diego, ECE 100

Lab Project 1: RLC Circuit Analysis with MATLAB

1. **Low-pass RLC circuit:**



   (a) Find the transfer function, $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$, in the above low-pass RLC filter circuit.

   (b) The frequency response of the circuit can be written as $H(j\omega) = \frac{1}{1+2\zeta\frac{j\omega}{\omega_0}+(\frac{j\omega}{\omega_0})^2}$. Write ω_0 and ζ in terms of R, L, and C.

   (c) Write a MATLAB script to calculate and plot the magnitude and phase of $H(j\omega) = \frac{1}{1+2\zeta\frac{j\omega}{\omega_0}+(\frac{j\omega}{\omega_0})^2}$ for $\omega_0 = 1$ and ζ=[0.1 0.3 0.707 1.0 3.0 10.0].

      i. Create a vector named `w` to store the samples of ω between 0.01 and 100 ($0.01 \leq \omega \leq 100$). Set the step size to be 0.01.

      ii. For every ζ value on the given list,

      - Create a vector named `h` to store the the samples of $H(j\omega)$ for the given ω values.
      - Find $|H(j\omega)|$ for the given ω and store the values in a vector named `h2`.
      - Using the `plot` function, plot $|H(j\omega)|$ vs ω.
      - Use the `hold on` function to overlay the graphs.

      iii. Label your graphs: Add a proper title. Label the x-axis "frequency ω/ω_0" and add a legend.

      As you can see, it is hard to read the plot on linear axes. The standard form of plotting the frequency response is with magnitude squared in dB vs log frequency, and phase in degrees vs log frequency. The plots of the magnitude in dB and phase in degrees are called **Bode plots**.

Next, use MATLAB to draw the **Bode plots** for $H(j\omega) = \frac{1}{1+2\zeta\frac{j\omega}{\omega_0}+(\frac{j\omega}{\omega_0})^2}$ for $0.1 \le \omega \le 100$ and $\omega_0 = 1$ and ζ=[0.1 0.3 0.707 1.0 3.0 10.0].

iv. For every ζ value on the given list,

- Using the `log10` function, find $10\log_{10}|H(j\omega)|^2$ for the given ω and store the values in a vector named `hdb`.
- Using the `semilogx` function, plot $10\log_{10}|H(j\omega)|^2$ vs ω in logarithmic scale. Type `semilogx(w,hdb)`.
- Use the `hold on` function to overlay the graphs.

v. Label your graph: Add a proper title to your graph. Label the x-axis "frequency ω/ω_0". Add a legend to your graph.

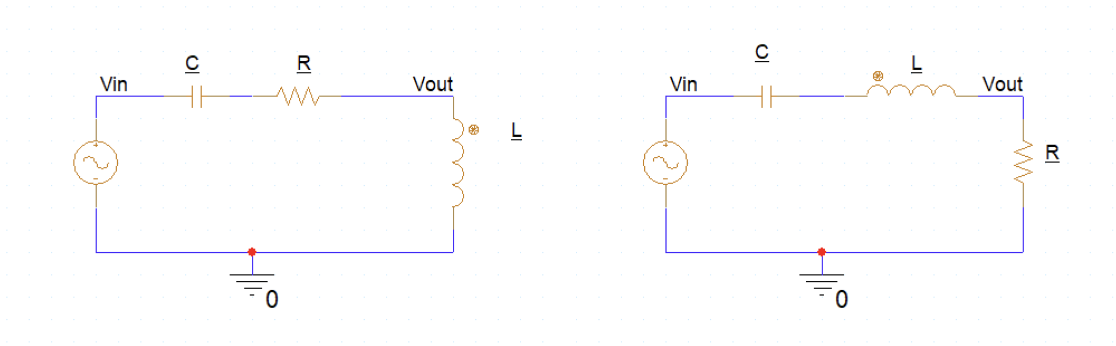
vi. For every ζ value on the given list,

- Using the `angle` function, calculate the phase of $H(j\omega)$ for the given ω values. This function will give you the phase of $H(j\omega)$ with the units of radians. Create a vector named `ph` to store the phase of $H(j\omega)$ with the units of **degrees**.
- Using the `semilogx` function, plot the phase of $H(j\omega)$ vs $\log\omega$. Type `semilogx(w,ph)`.
- Use the `hold on` function to overlay the graphs.

vii. Label your graphs: Add a proper title. Label the x-axis "frequency ω/ω_0" and add a legend.

(d) When the damping factor ζ is less than 0.707, a peak occurs in $|H(j\omega)|$. Analytically, find expressions for the peak frequency, ω_{peak}, and $|H(\omega_{peak})|$ in term of ω_0 and ζ. Assuming $\omega_0 = 1$, calculate the values of ω_{peak} for $\zeta = 0.1$ and 0.3. Also find $|H(\omega_{peak})|$ at these two ω_{peak} values. Do these values agree with the peaks that you read off in your MATLAB figures for $\zeta = 0.1$ and 0.3? You can use the Data Tips (cursor) in MATLAB to answer this question.

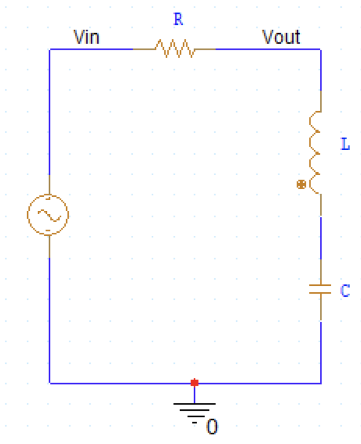
2. **High-pass and Band-pass RLC circuits:**



(a) Write the transfer functions $H_{HP}(s)$ and $H_{BP}(s)$ in terms of R, L, and C. Put them in the generalized form so the denominator is the same as the one for $H_{LP}(s)$ and the numerator is written in terms of s, ω_0 and ζ. The ω_0 and ζ are the same (in terms of R, L, and C) as for the low-pass circuit because the loop current is unchanged by changing the order of the elements. All that happens is you read the voltage across a different element. This changes the numerator.

(b) Modify your script to make the Bode plots of $H(j\omega)$ for the band-pass and high-pass filters.

3. **Mystery Filter:** What happens if you take the output across both C and L together, rather than just one of them? Write the transfer function H(s) and plot the Bode plots for $H(j\omega)$ as you did for the band-pass and high-pass filters. What could you use this circuit for?



4. **Linear Systems Tools:** MATLAB can also do symbolic algebra, including Laplace transforms. It has a number of tools that are very useful for analyzing linear systems. For example, the code below will produce Bode plots and step response plots for the circuits you analyzed above:

```
s = tf('s');                            %define s as a transfer function variable
w0 = 2*pi*15.9e3; zeta = 1/sqrt(2); %define parameters
h = 1/(1 + 2*zeta*s/w0 + (s/w0)^2); %define transfer function
bode(h);                                %create a bode plot with nice scales
figure                                  %new fig so step does not overwrite bode
step(h);                                %create scaled a step response plot
stepinfo(h)                             %compute rise time, overshoot, etc.
```

(a) Use these tools to create a single plot with all the step responses for the transfer functions in Part 1c.

(b) There is a closed form expression for the overshoot of the step-response of a second-order low-pass filter: $\%overshoot = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ for $0 < \zeta < 1$ and $\%overshoot = 0$ for $\zeta > 1$. Plot the theoretical overshoot values vs ζ for $0.01 < \zeta < 10$. Use a solid line for plotting the theoretical values. Next, find the overshoot values for each step response in part 1c with the help of the `stepinfo` function. Overlay the plot of these calculated overshoot values on your current plot. Show these values as a series of symbols, not a solid line.

Report:

Always include a schematic of the circuit(s) you are analyzing. Report should be concise, but with enough details that another engineer could repeat what you have done, including any errors you might have made. Please follow the lab report format posted on the website.
If in doubt about what to include, ask the instructor or the TA. Make sure to include the following in your report:

- Hand calculations for 1a, 1b and 1d including the derivations
- MATLAB script and plots for part 1c
- Analytical solutions (hand calculations) for part 2a
- MATLAB script and plots for the band-pass and high-pass filters
- Analytical solutions (hand calculations) and MATLAB plot for the mystery filter
- Plot of the step responses in part 4a
- Plot of %os vs damping factor overlaid on the plot of the theoretical values