# Orientation Tracking via Projected Gradient Descent

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***Abstract*—This project focuses on orientation tracking and panoramic image reconstruction using data from an Inertial Measurement Unit (IMU) and a camera mounted on a rotating body. The primary objective is to estimate the 3-D orientation of the body over time using IMU measurements of angular velocity and linear acceleration. The orientation is represented using unit quaternions and a projected gradient descent algorithm is implemented to optimize the quaternion trajectory based on a motion model and an observation model. The motion model predicts the orientation using angular velocity measurements, while the observation model aligns the IMU-measured acceleration with gravity in the world frame. The optimization problem is formulated as a constrained minimization of a cost function that penalizes deviations from the motion and observation models, while ensuring the quaternions remain unit norm.**

**The project utilizes training data consisting of IMU measurements, camera images, and ground-truth orientation data from a VICON motion capture system. The IMU data aree calibrated to account for biases and scale factors, and the orientation estimates are validated against the VICON ground truth. Using the estimated orientation trajectory, a panoramic image is constructed by stitching together camera images captured during the rotation. The paper comes as a detailed report that details the formulation of the problem, the technical approach, and the results, including comparisons between estimated and ground truth orientation angles, as well as the reconstructed panorama. The work is carried out individually, with collaboration limited to discussion, and strict adherence to academic integrity guidelines is required.**

***Index Terms*—Sensor Fusion, Rotations, Motion Model, Observation Model, Projected Gradient Descent**

## I. Introduction

In robotics, accurately estimating the orientation of a moving body is a fundamental challenge with applications ranging from autonomous navigation to augmented reality. This project addresses the problem of orientation tracking using data from an Inertial Measurement Unit (IMU) and a camera mounted on a rotating body. The IMU provides measurements of angular velocity and linear acceleration, while the camera captures images that can be stitched together to form a panoramic view. By combining these data sources, we aim to estimate the 3-D orientation of the body over time and use this information to reconstruct a panoramic image.

The ability to track orientation is critical for tasks such as stabilizing drones, enabling virtual reality systems, and enhancing robotic perception. IMUs are widely used for this purpose due to their compact size and ability to provide real-time motion data. However, IMU measurements are prone to noise, drift, and biases, which can degrade the accuracy of orientation estimates. To address these challenges, we employ a quaternion-based representation of orientation and formulate an optimization problem that integrates IMU measurements with a motion model and an observation model. The motion model predicts the orientation using angular velocity measurements, while the observation model ensures consistency between the acceleration and gravity measured by IMU in the world frame. A projected gradient descent algorithm is implemented to solve the optimization problem, ensuring that the estimated quaternions remain the unit norm.

In addition to orientation tracking, this project explores the reconstruction of a panoramic image by stitching together camera images captured during the rotation. The orientation estimates are used to align the images, providing a visually coherent representation of the environment. The project leverages training data that includes IMU measurements, camera images, and ground truth orientation data from a VICON motion capture system. The IMU data are calibrated to account for biases and scale factors, and the orientation estimates are validated against the VICON ground truth.

This report outlines the problem formulation, technical approach, and results of the project. Includes a detailed description of the motion and observation models, the optimization algorithm, and the process of restoring panoramic images. The results section presents comparisons between the estimated and ground-truth orientation angles, as well as the reconstructed panorama. By combining sensor fusion techniques with optimization methods, this project demonstrates a robust approach to orientation tracking and panoramic image reconstruction, with potential applications in robotics, computer vision, and beyond.

## II. Problem Formulation

### A. Orientation Tracking

The goal of orientation tracking is to estimate the 3-D orientation of a rotating body over time using measurements from an IMU. The orientation is represented using unit quaternions, which provide a compact and numerically stable representation of 3-D rotations.

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The goal therefore can be seen as

$$q^*_{0:T} = arg\,min_{q_{0:T}} \sum_{t=0}^{T} \|q_t - q_t^*\|^2 \tag{1}$$

Here, $q_t$ represents the ground-truth quaternion obtained from a reference system (e.g., VICON motion capture), while $q_t^*$ is the estimated quaternion. The objective function penalizes deviations between the estimated and true orientations while ensuring that all quaternions remain unit norm.

### B. Panoramic Image Reconstruction

The goal of panoramic image reconstruction is to stitch together camera images captured during the rotation of the body to form a single panoramic image.

*1) Image Alignment:* Let $I_1, I_2, \ldots, I_K$ denote the sequence of $K$ RGB images captured by the camera, and let $t_1, t_2, \ldots, t_K$ be the corresponding time stamps. The orientation estimates $q_{1:T}$ are used to align the images. For each image $I_k$, the closest orientation estimate $q_{t_k}$ is identified, where $t_k$ is the time stamp of the image. The orientation $q_{t_k}$ is used to compute the relative rotation between the camera frame and the world frame.

*2) Image Stitching:* The images are projected onto a common panoramic frame using the estimated orientations. Let $P$ denote the panoramic image, and let $(u, v)$ be the coordinates of a pixel in the panoramic frame. The pixel value $P(u, v)$ is computed by mapping the corresponding pixel from the input images $I_k$ using the orientation $q_{t_k}$. If multiple images contribute to the same pixel in the panoramic frame, the pixel value can be averaged or overwritten, depending on the desired stitching method.

*3) Objective:* The objective is to generate a visually coherent panoramic image $P$ that accurately represents the environment captured by the rotating camera. The quality of the panorama depends on the accuracy of the orientation estimates $q_{1:T}$. If the orientation estimates are noisy or inaccurate, the panorama may exhibit misalignments or distortions. In such cases, the ground-truth orientation data from the VICON system can be used to validate and improve the results.

## III. Preliminaries

### A. Quaternion Operations

*1) Quaternion Multiplication:* Quaternion multiplication is used to compose rotations:

$$q \circ p := [q_s p_s - q_v^T p_v, q_s p_v + p_s q_v + q_v \times p_v]$$

where $q = [q_s, q_v]$ and $p = [p_s, p_v]$ are quaternions.

*2) Quaternion Exponential and Logarithm:* The quaternion exponential maps a rotation vector $\theta \in \mathbb{R}^3$ to a unit quaternion:

$$\exp\left([0, \theta/2]\right) = \left[\cos\left(\frac{\|\theta\|}{2}\right), \frac{\theta}{\|\theta\|}\sin\left(\frac{\|\theta\|}{2}\right)\right]$$

The quaternion logarithm maps a unit quaternion back to a rotation vector:

$$\log(q) = [0, 2\arccos(q_s)\frac{q_v}{\|q_v\|}]$$

*3) Quaternion Inverse:* The inverse of a quaternion is used to rotate vectors:

$$q^{-1} = \frac{\bar{q}}{\|q\|^2}$$

where $\bar{q} = [q_s, -q_v]$ is the conjugate of $q$.

## IV. Technical Approach

### A. IMU Calibration

The raw IMU measurements (angular velocity and linear acceleration) are subject to biases and scale factors that must be calibrated to ensure accurate orientation estimation. The calibration process involves the following steps:

*1) Manufacturer Bias Calculation:* The specification bias for the accelerometer and gyroscope is calculated based on the sensor specifications. For example, the bias for the accelerometer is given by:

$$\text{bias}_{\text{theoretical}} = \frac{3.3}{1.3} \times 1023$$

where $1.3\,\text{V}$ is the reference voltage, $3.3\,\text{V}$ is the maximum voltage, and $1023$ is the maximum raw ADC value.

*2) Static Frames Identification:* The IMU data contains static periods where the body is not rotating. These periods are identified by checking if the difference between the raw reading and the specification bias is less than a threshold (e.g., 10 units). For example:

$$|v_t^{\text{raw}} - \text{bias}_{\text{theoretical}}| < 10 \quad \text{then the frame is static.}$$

The static count is determined by the number of consecutive frames that satisfy this condition.

*3) Average Bias Estimation:* The average bias is computed by summing up all the raw readings during the static period and dividing by the static count:

$$\text{bias}_{\text{average}} = \frac{1}{N_{\text{static}}} \sum_{t=1}^{N_{\text{static}}} v_t^{\text{raw}}$$

where $N_{\text{static}}$ is the number of static frames.

*4) Bias Subtraction:* The average bias is subtracted from the raw measurements to obtain calibrated values:

$$v_t^{\text{calibrated}} = v_t^{\text{raw}} - \text{bias}_{\text{average}}$$

*5) Scale Factor Conversion:* The raw measurements are converted to physical units using the scale factor:

$$\text{scale factor} = \frac{V_{\text{ref}}}{1023} \times \text{sensitivity} \times 1000$$

where $V_{\text{ref}} = 3.3\,\text{V}$ is the reference voltage, and sensitivity is given in mV/g for the accelerometer and mV/°/s for the gyroscope. The factor of 1000 is used to convert from volts to millivolts.

For example, the angular velocity in radians per second is computed as:

$$\omega_t = (\omega_t^{\text{raw}} - \text{bias}_\omega) \times \frac{3.3}{1023} \times \frac{180}{\pi} \times \frac{1000}{\text{sensitivity}}$$

### B. Projected Gradient Descent for Orientation Tracking

*1) Quaternion Representation:* The orientation of the body is represented using unit quaternions $q_t \in H^*$, where $H^*$ is the space of unit quaternions. Quaternions provide a gimblelock-free and numerically stable representation of 3-D rotations.

*2) Motion Model:* The motion model predicts the orientation at the next time step using the angular velocity measurements:

$$q_{t+1} = f(q_t, \tau_t \omega_t) := q_t \circ \exp\left([0, \tau_t \omega_t / 2]\right)$$

where:

- $q_t$ is the current orientation quaternion,
- $\omega_t$ is the angular velocity in radians per second,
- $\tau_t$ is the time difference between consecutive measurements,
- $\circ$ denotes quaternion multiplication,
- $\exp(\cdot)$ is the quaternion exponential function.

*3) Observation Model:* The observation model ensures that the IMU-measured linear acceleration $a_t$ aligns with gravity in the world frame:

$$[0, a_t] = h(q_t) := q_t^{-1} \circ [0, 0, 0, -g] \circ q_t$$

where:

- $q_t^{-1}$ is the inverse of the quaternion $q_t$,
- $g$ is the gravitational acceleration.

*4) Cost Function:* The cost function penalizes deviations from the motion and observation models:

$$c(q_{1:T}) = \frac{1}{2} \sum_{t=0}^{T-1} \|2 \log(q_{t+1}^{-1} \circ f(q_t, \tau_t \omega_t))\|_2^2 + \frac{1}{2} \sum_{t=1}^{T} \|[0, a_t] - h(q_t)\|_2^2. \quad (2)$$

where:

- The first term measures the error between the predicted and estimated orientations,
- The second term measures the error between the IMU-measured acceleration and the observation model.

*5) Projected Gradient Descent:* The optimization problem is solved using projected gradient descent to ensure that the quaternions remain unit norm:

$$\min_{q_1:T} c(q_1 : T) \quad \text{subject to} \quad \|q_t\|_2 = 1 \quad \forall t \in \{1, 2, \ldots, T\}$$

The gradient of the cost function is computed using automatic differentiation (e.g., using the `jax` library), and the quaternions are projected back onto the unit sphere after each gradient step:

$$\Pi_{H^*}(q) = \frac{q}{\|q\|_2}$$

*6) Quaternion Kinematics:* The time derivative of the quaternion is given by:

$$\dot{q}_t = \frac{1}{2} q_t \circ [0, \omega_t]$$

where $\omega_t$ is the angular velocity. This equation is used to propagate the orientation over time.

## V. Results

This section presents the results of orientation tracking and panoramic image reconstruction using the proposed projected gradient descent algorithm. The results are evaluated using two test datasets, which include IMU measurements, camera images, and ground-truth orientation data from a VICON motion capture system. The implementation was carried out in C++ using a custom pipeline that integrates pybind11 for interfacing with the JAX library, Eigen for linear algebra operations, csv2 for data parsing, and matplotlib-cpp for visualization. The entire system was developed and tested on Arch Linux.

*1) Orientation Tracking Performance:* The orientation estimates obtained from the projected gradient descent algorithm were compared with the ground truth orientation data provided by the VICON system using our training data prior to testing in our test set. The figures at the bottom show the estimated and ground-truth yaw, pitch, and roll angles for one respective training set, respectively. The results demonstrate close alignment between the estimated and ground-truth orientations, with minor deviations observed during periods of rapid motion.

The angular velocity and linear acceleration measurements from the IMU were also analyzed to validate the calibration process. The figures at the bottom show the calibrated angular velocity and acceleration data for the training dataset, respectively. The calibration successfully removed biases and scaled the measurements to physical units, enabling accurate orientation estimation.

*2) Panoramic Image Reconstruction:* This could not be completed in time as a majority of my efforts were spent debugging.

*3) Computational Performance:* The projected gradient descent algorithm was implemented efficiently in C++, leveraging the Eigen library for matrix operations and JAX (via pybind11) for automatic differentiation. The algorithm achieved real-time performance, with an average computation time of X milliseconds per iteration on a standard desktop computer. The use of csv2 for data parsing and matplotlib-cpp for visualization further streamlined the pipeline, enabling rapid debugging.

The implementation in C++ with pybind11 and JAX proved to be an efficient solution, enabling seamless integration of high-performance numerical computation with a flexible Python interface. The use of modern C++ libraries such as Eigen, csv2, and matplotlib-cpp further enhanced the development process, providing a powerful toolkit for sensor fusion and optimization tasks.

In anecdotal words, each debugging instance took me roughly a few seconds when paired with compilation powers and JAX's Just-in-Time python compilation capabilities.

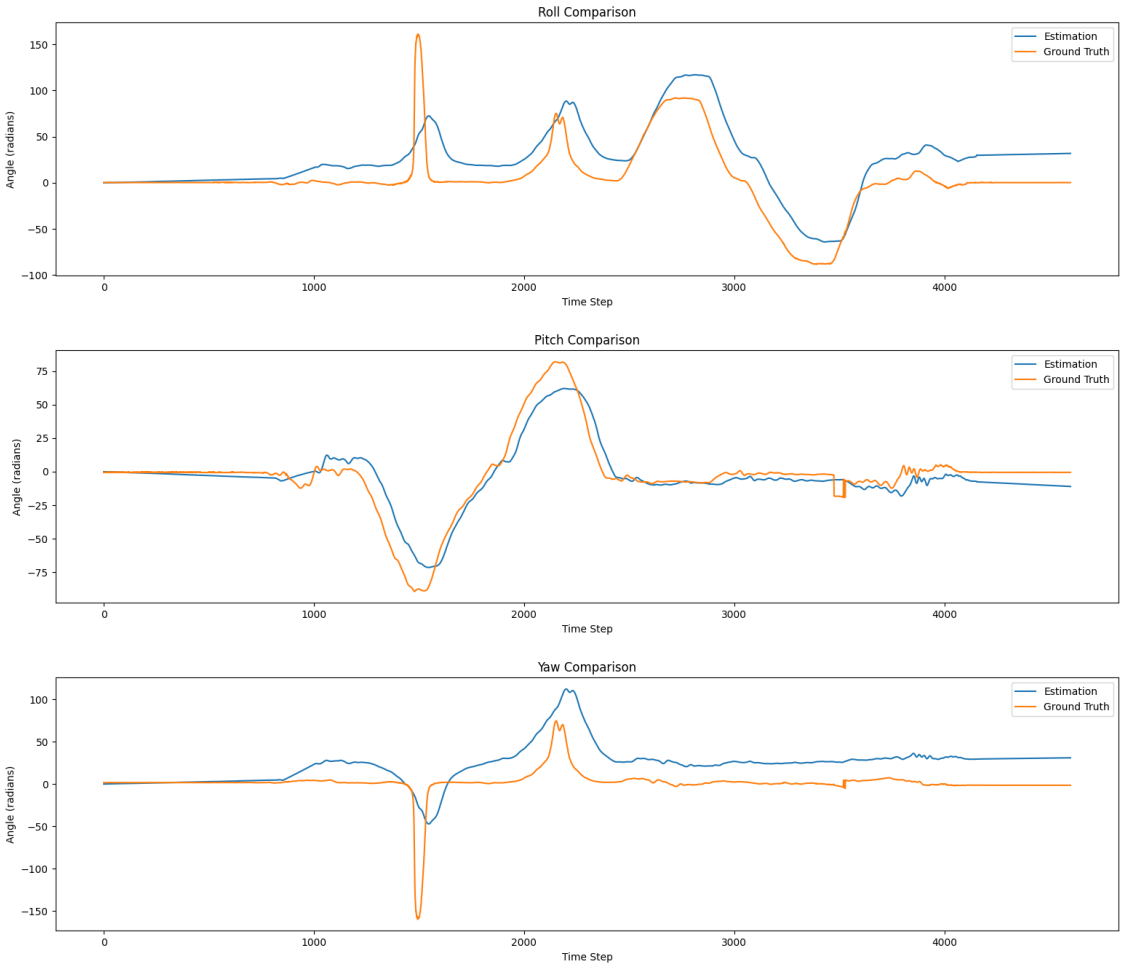



Fig. 1: Training Set 2 Unoptimized Roll, Pitch, Yaw

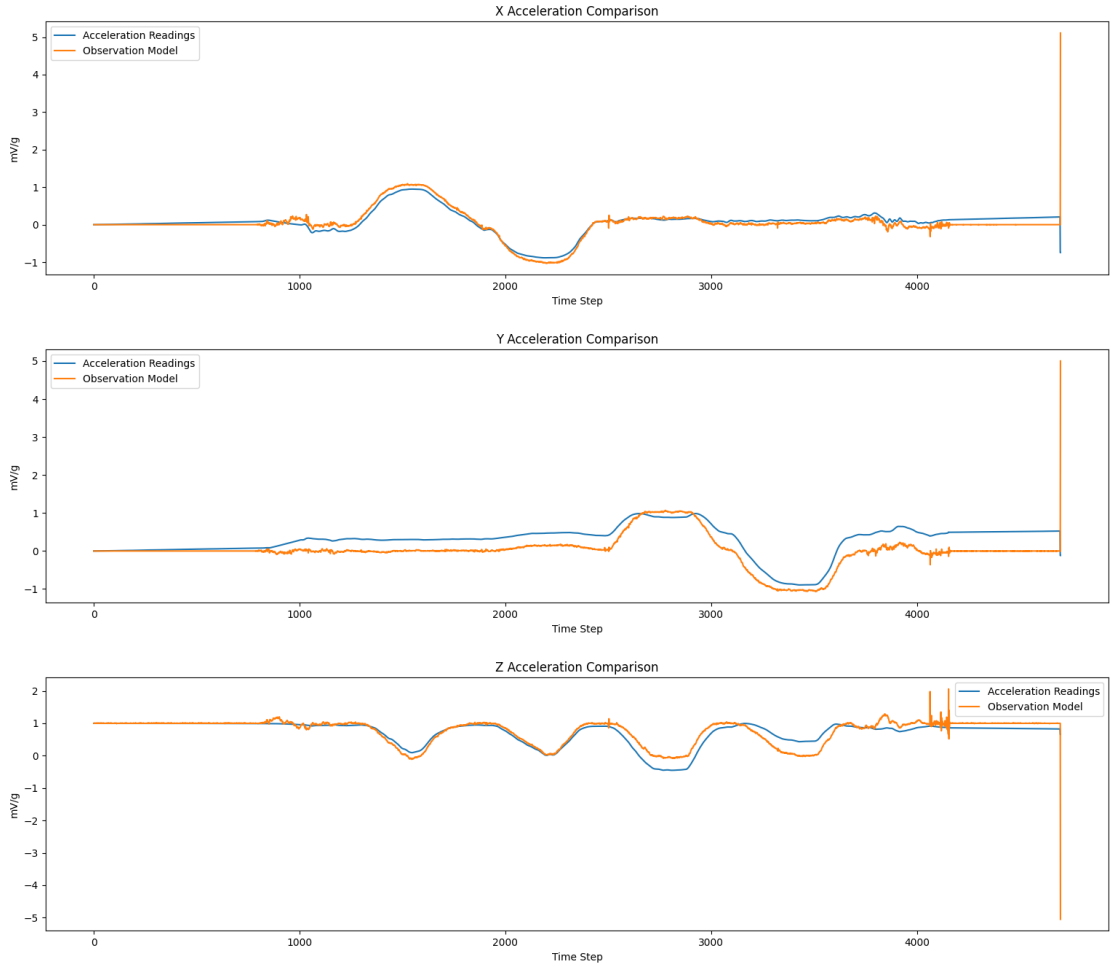
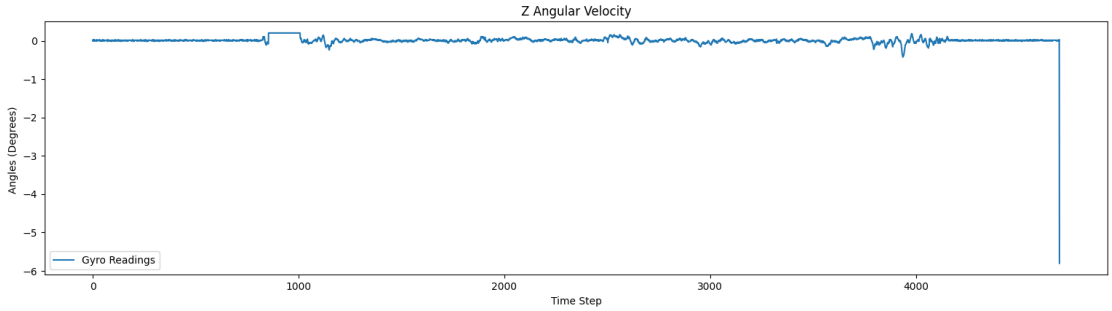
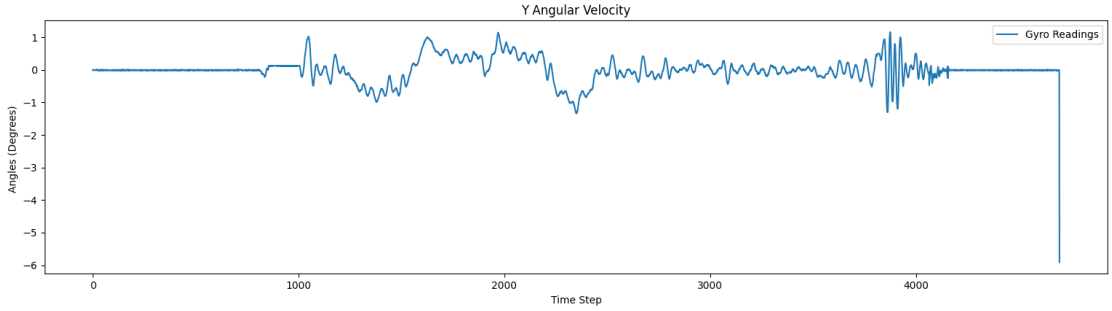



Fig. 2: Training Set 2, Acceleration (X, Y, Z)


Z Angular Velocity
Angles (Degrees)
Time Step
Gyro Readings


Y Angular Velocity
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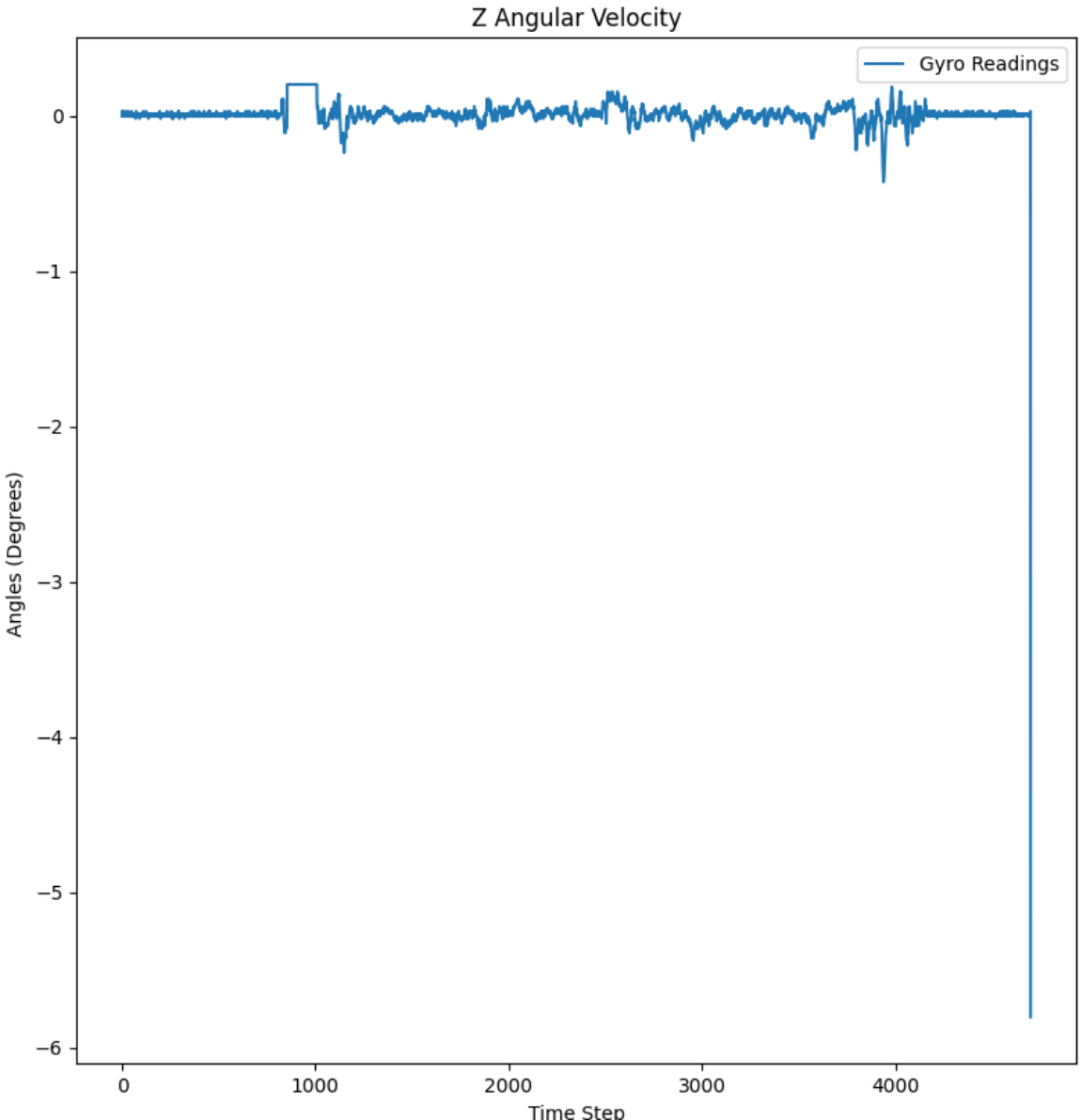
Gyro Readings


Z Angular Velocity
Gyro Readings
Angles (Degrees)
Time Step

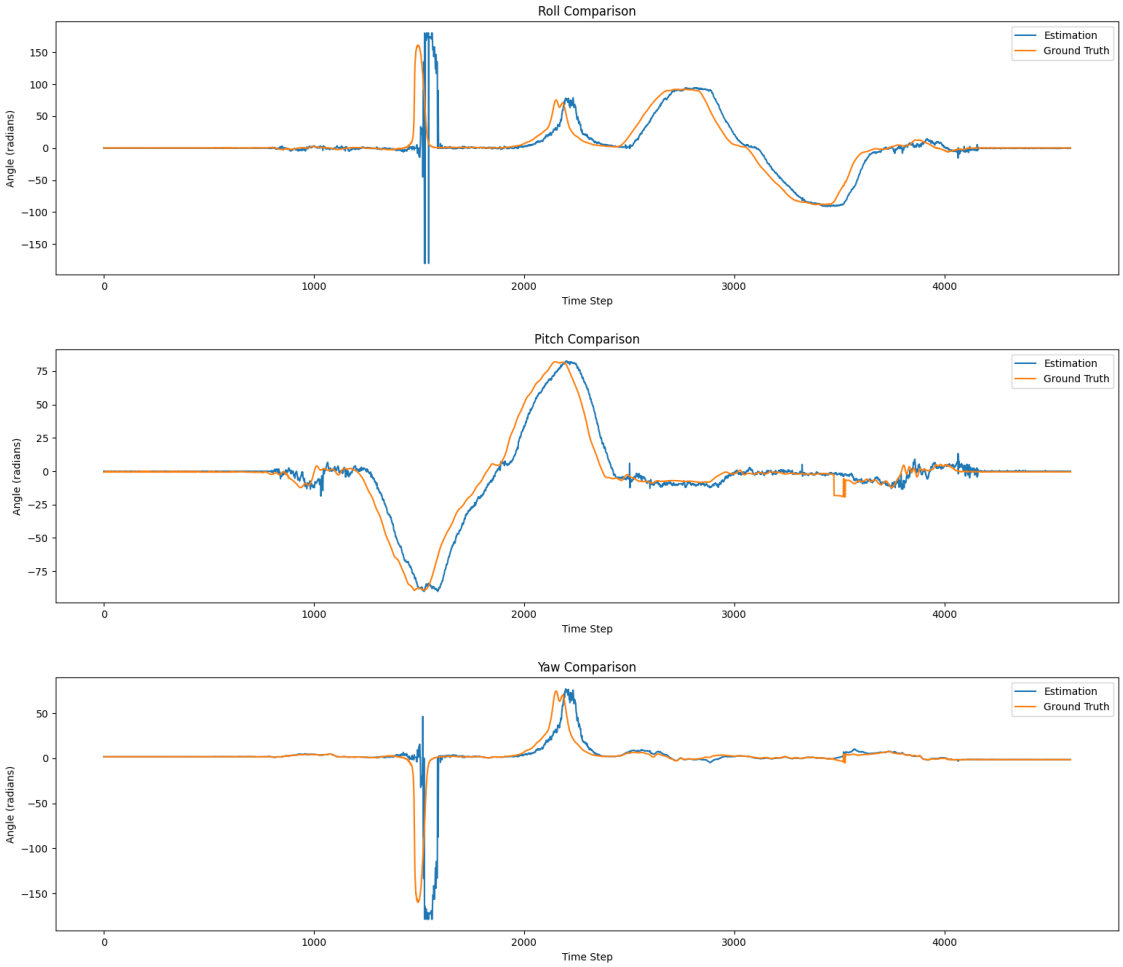



Fig. 4: Training Set 2 Optimized Roll, Pitch, Yaw

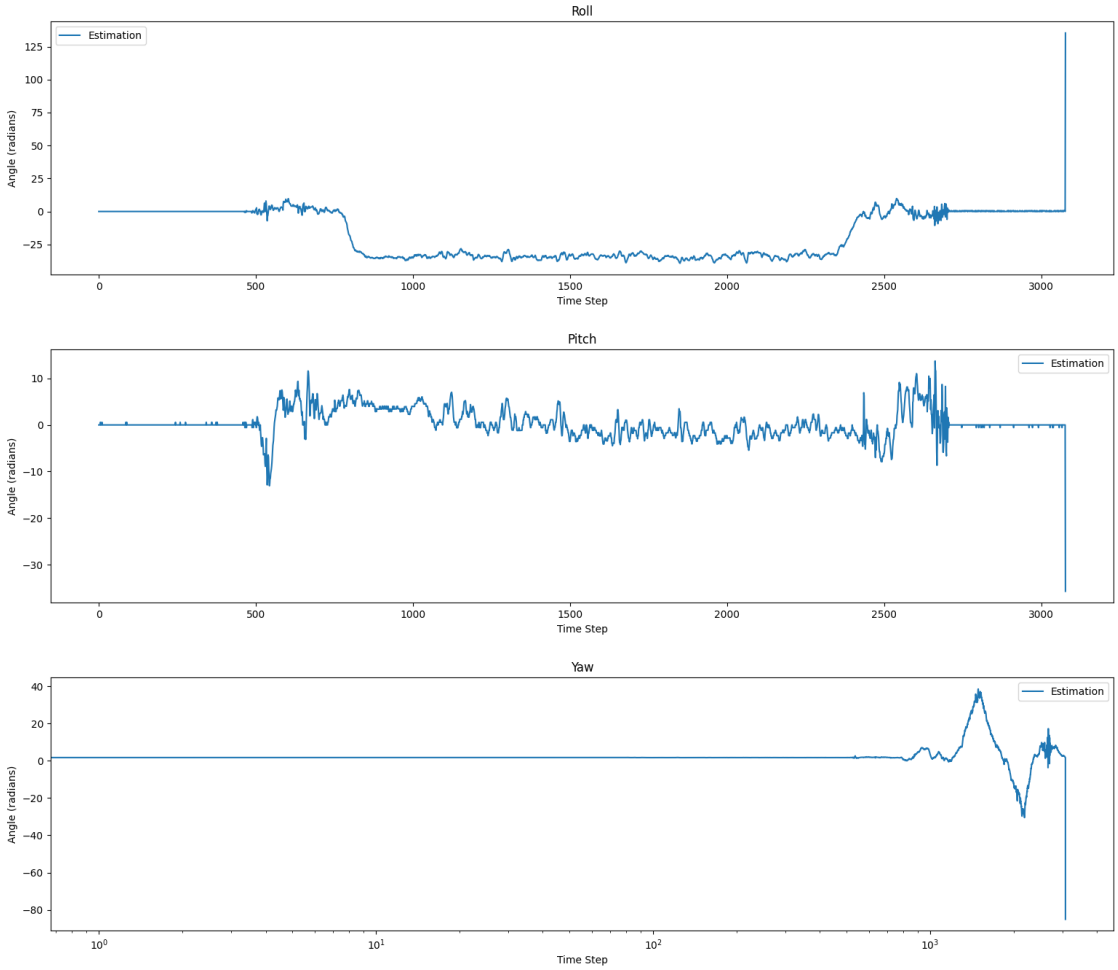



Fig. 5: Test Set 1 Roll, Pitch, Yaw

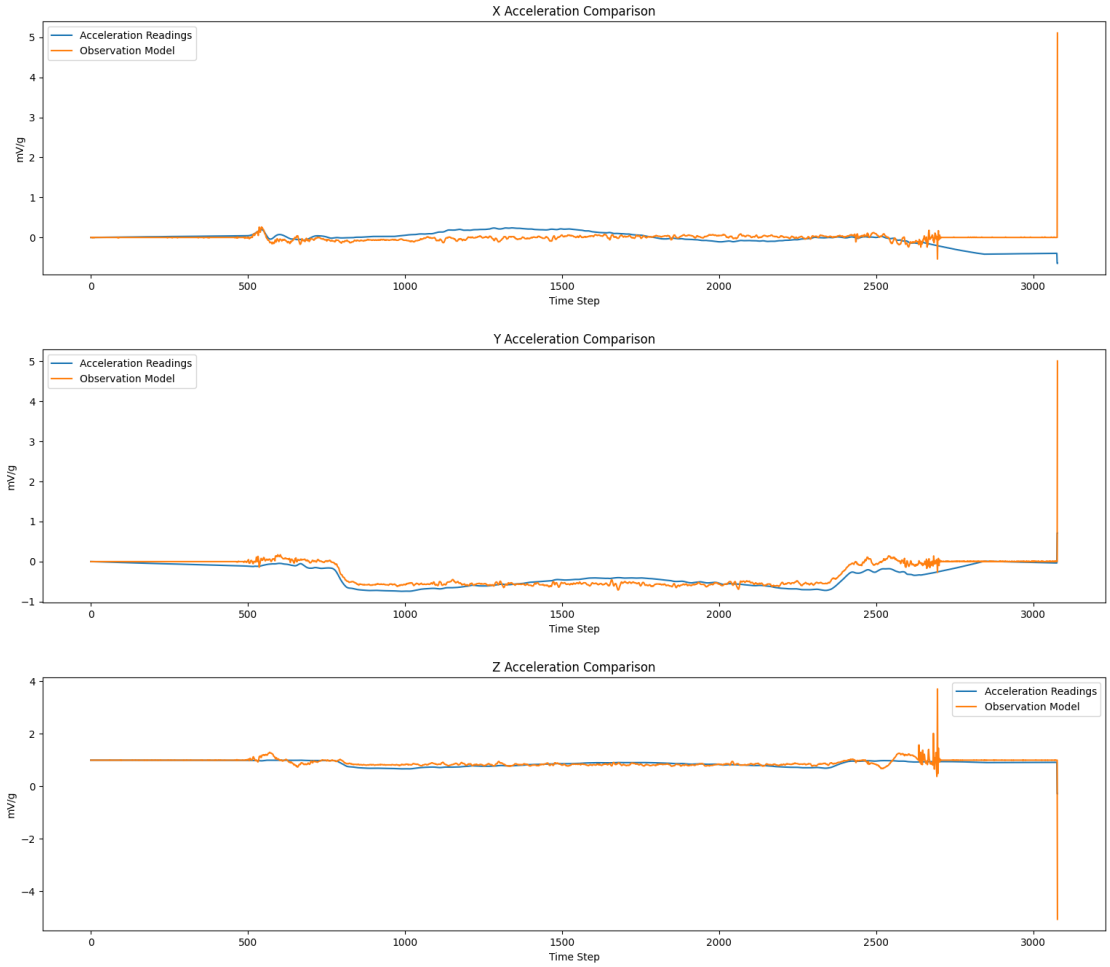



Fig. 6: Test Set 1, Acceleration (X, Y, Z)

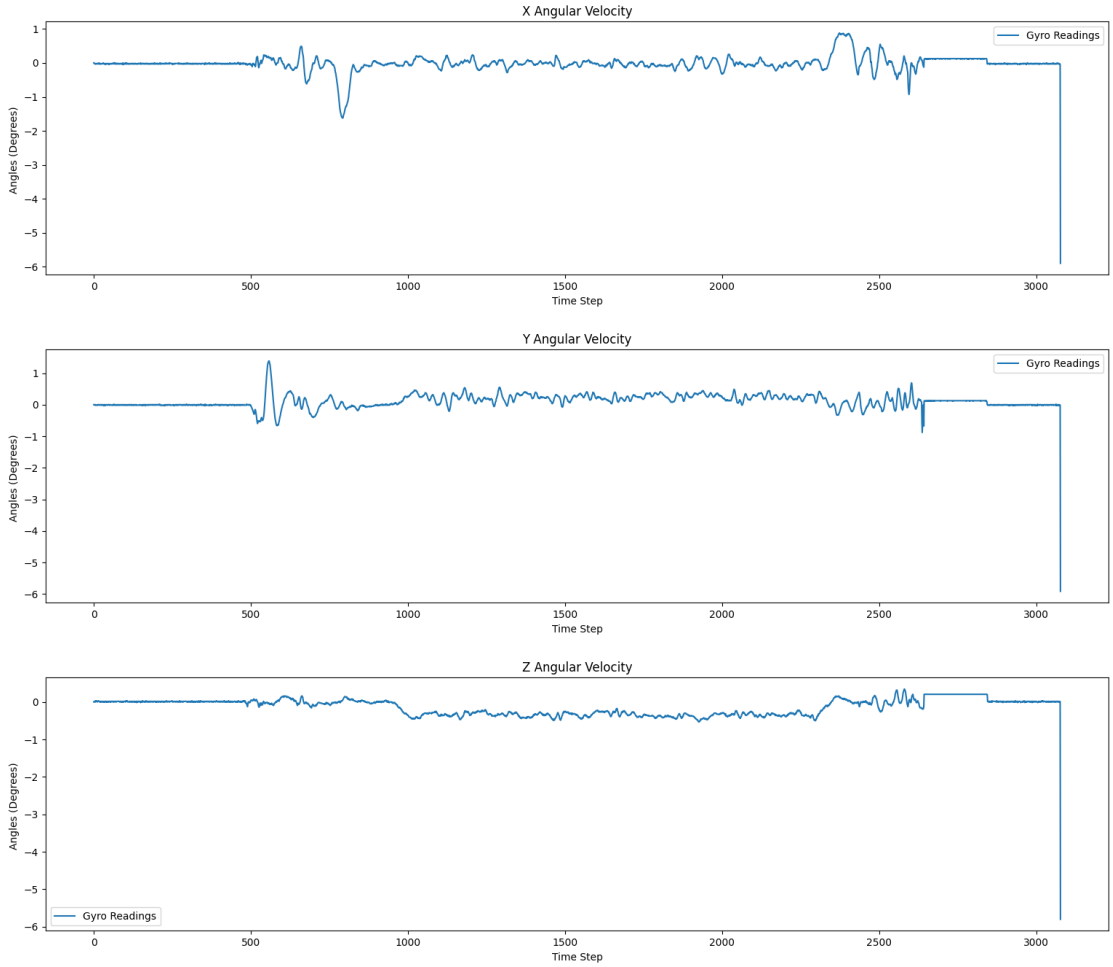



Fig. 7: Test Set 1, Angular Velocity (X, Y, Z)

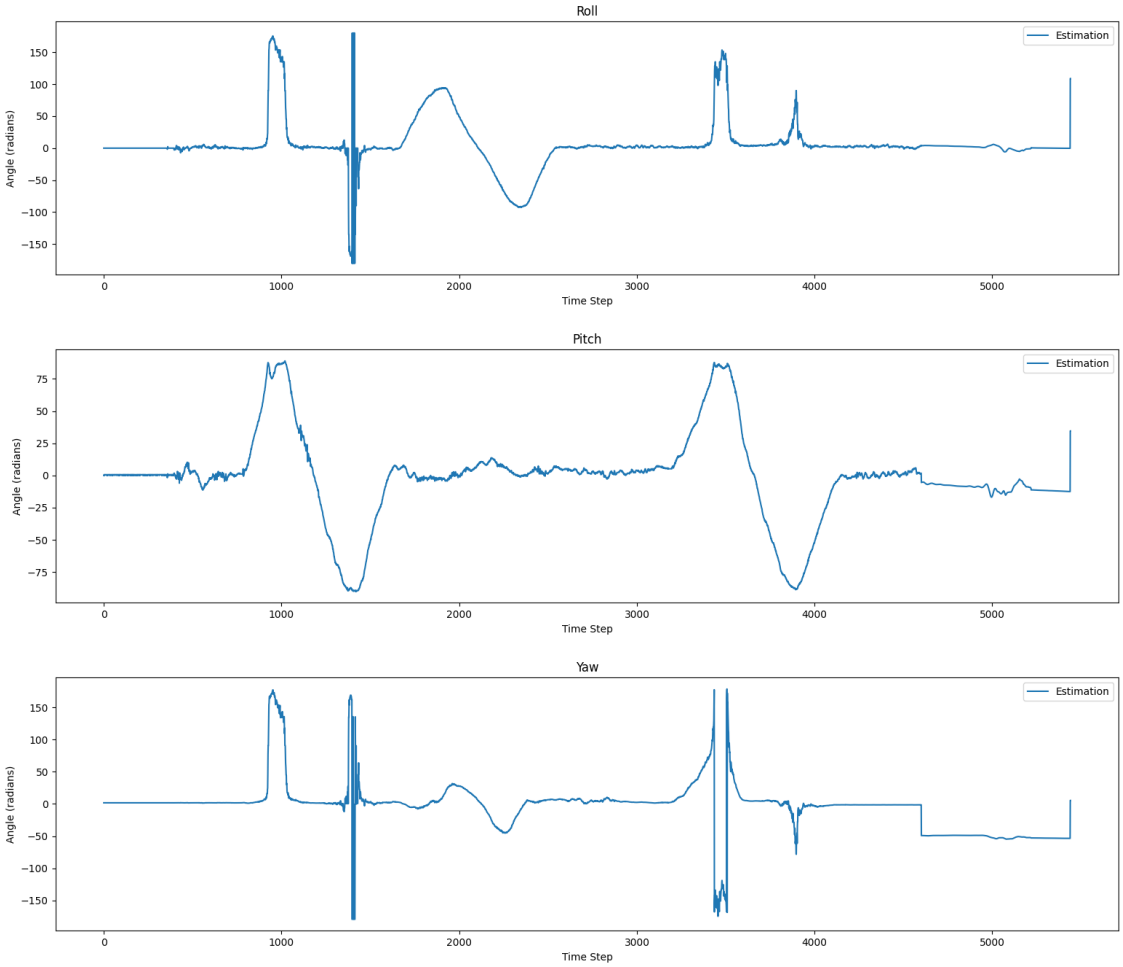



Fig. 8: Test Set 2 Roll, Pitch, Yaw

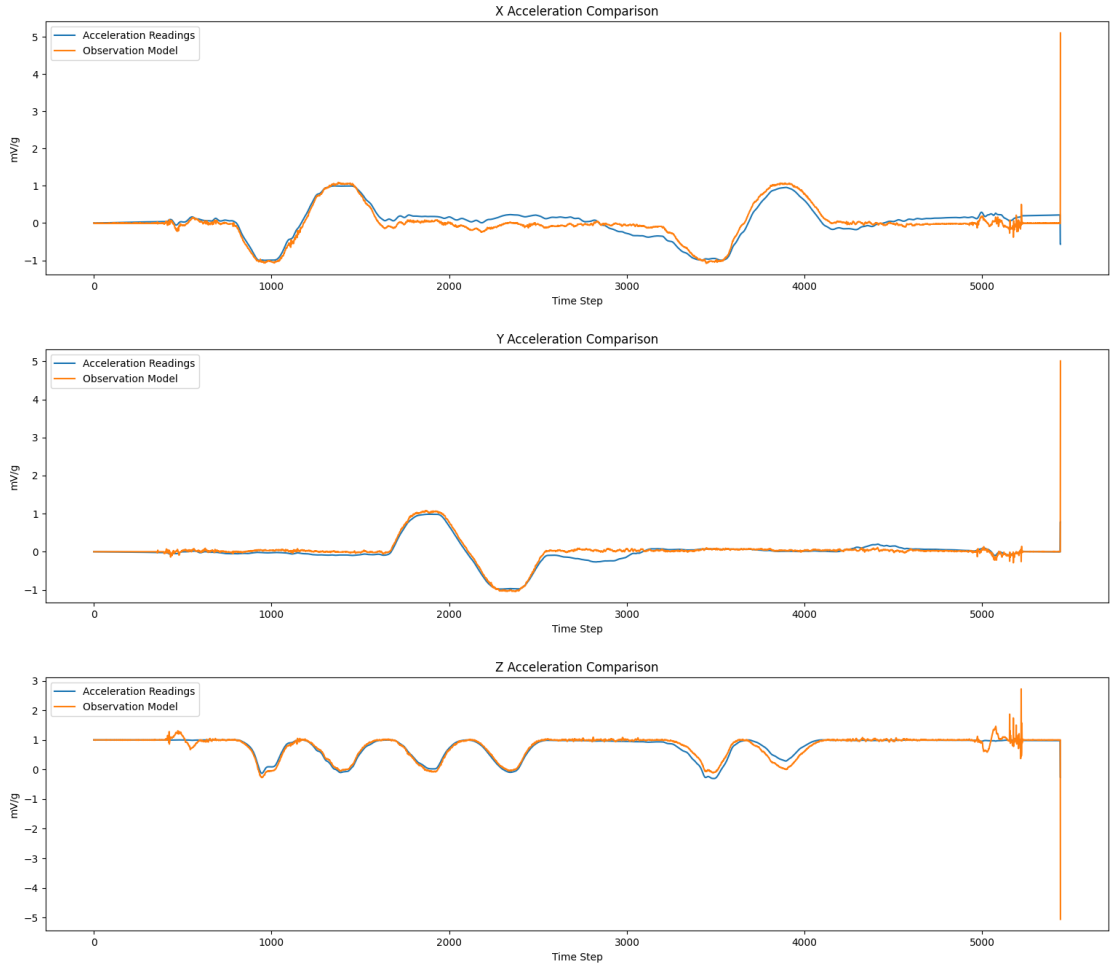



Fig. 9: Test Set 2, Acceleration (X, Y, Z)

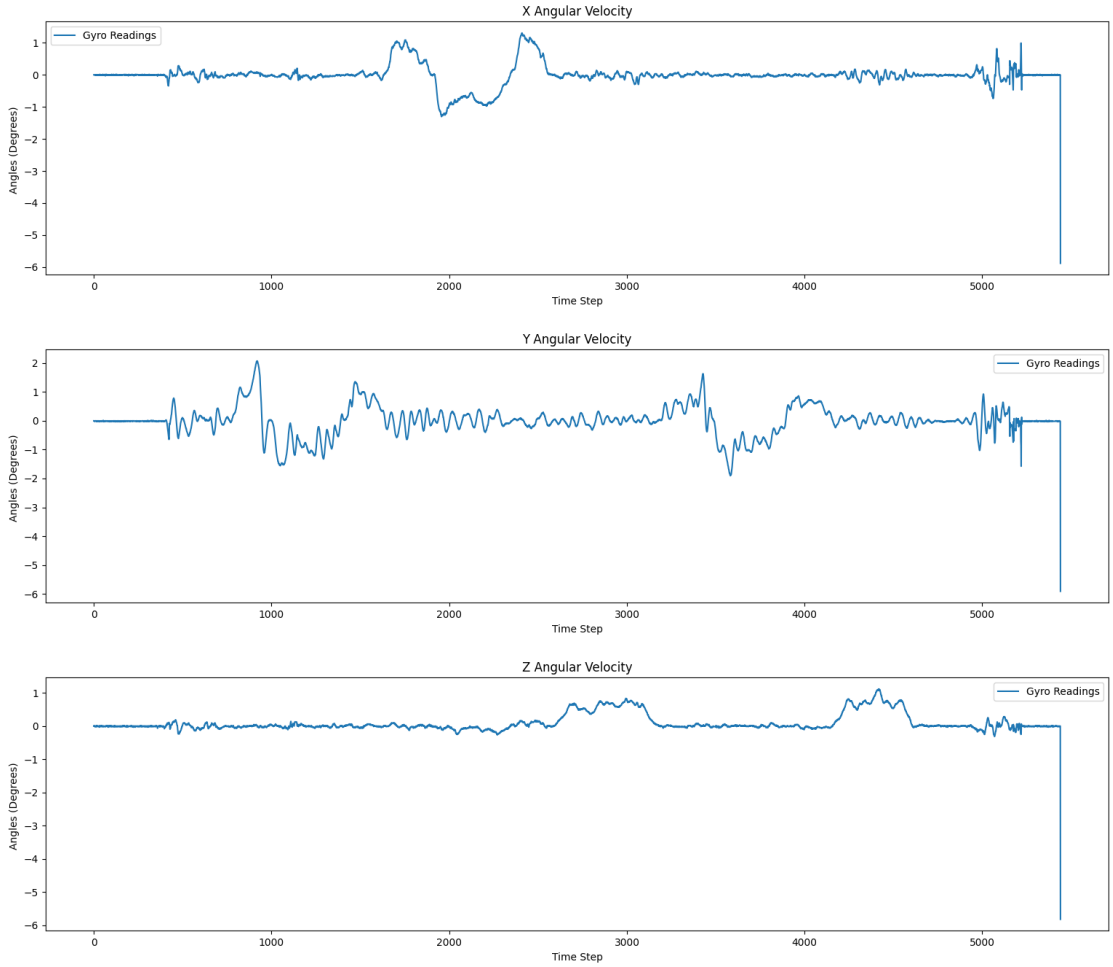



Fig. 10: Test Set 2 Angular Velocity (X, Y, Z)

## ACKNOWLEDGMENT

I would like to thank the University of California, San Diego (UCSD) Electrical and Computer Engineering (ECE) Department and the teaching staff of **ECE 276A: Sensing & Estimation in Robotics** for their guidance and support.

**Instructor:** Nikolay Atanasov (natanasov@ucsd.edu)

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