Lab 7&8

Problem 1

Alex, a transportation planner at a city transport company, is tasked with reviewing the city's subway network to ensure it's designed as a tree. A tree is a connected graph with no cycles and exactly N−1 edges, where N is the number of stations (vertices). Each station in the subway system is connected to other stations by specific routes (edges).

The planning department has provided Alex with the number of stations (N) and the number of routes each station connects to (degree). Alex needs to determine whether the subway network can be considered a tree based on this information.

Input:

- The number of stations, N, in the subway network.
- A list of N integers, where each integer represents the number of routes (degree) each station is connected to.

Output:

Print "Yes" if the subway system forms a tree, otherwise print "No".

Questions:

1. **Describe the algorithm approach to solve this problem.**
   (Provide a step-by-step explanation of how Alex can determine if the subway system forms a tree based on the number of stations and the degree of each station.)
2. **Consider the following inputs:**
   a. N=4, Degrees = [1, 2, 1, 2]
   b. N=5, Degrees = [2, 3, 2, 2, 3]

   Do these subway networks represent a tree? Explain your reasoning.

Problem 2

Sophie, a marine biologist, has arrived at an archipelago of N islands. Each island is connected to one or more others by bidirectional bridges. Sophie is currently at Island #1 and needs to reach Island #N to conduct her research. However, she wants to minimize the number of bridges she has to cross because the bridges are old and require a lot of effort to cross.

Sophie is wondering how many bridges she will need to cross to get to Island #N, taking the optimal route. Can you help Sophie find the minimum number of bridges she must cross to reach her destination?

You are given:

- The total number of islands N and the total number of bridges connecting them.
- A list of connections between the islands in the form of bidirectional bridges.
- The index of the starting island, which is always Island #1.

You need to determine how many bridges Sophie will cross on the shortest route, or if it's even possible to reach Island #N.

Questions:

1. Describe the approach Sophie should use to find the minimum number of bridges to cross.
2. Given the following connections between the islands:
   - `Islands: 6`
   - `Bridges: 7`
   - `Connections: 1-2, 2-3, 3-4, 1-4, 5-4, 6-5, 4-6`
   - `Sophie starts at Island 1.`

   What is the optimal route Sophie should take to reach Island #6, and how many bridges will she cross? Explain your reasoning.

Problem 3

You are working as a network engineer for a large communication company. The company's data center is designed as a network of nodes, with each node representing a server. These servers are connected by edges, which represent communication links between them. Each server in the data center is identified by a node, and there is one special node called the **head node**, which is responsible for coordinating the network.

You need to check how many servers (nodes) are unreachable from the head node. Some servers might be isolated or disconnected from the head node due to network issues. The graph representing the network may have multiple edges between the same nodes or self-loops, and it's undirected. Your task is to identify how many nodes are unreachable from the head node.

Input:

- The total number of nodes (servers) in the network, followed by the total number of communication links (edges).

- A list of M communication links between servers. Each link is represented as a pair of nodes aaa and b, where there is a communication link between node a and node b.
- The index of the head node.

Output:

- Print a single integer, denoting the number of servers (nodes) that are unreachable from the head node.

Questions:

1. Describe the algorithm approach to solve this problem.
2. Given the following input:

   ```
   10 servers in total, and 10 communication links between them.
   Connections between the servers are as follows:
   8-1, 8-3, 7-4, 7-5, 2-6, 10-7, 2-8, 10-9, 2-10, 5-10
   The head node is 2.
   ```

   How many servers are unreachable from the head node (node 2)? What are these unreachable servers? Explain your reasoning.

Problem 4

In a small village, there are n villagers, each labeled from 1 to n. A rumor is spreading that one of the villagers is secretly the village mayor. The mayor has a special role in the village and is trusted by everyone except themselves. The mayor trusts no one.

You are tasked with identifying the mayor based on a series of trust relationships. A trust relationship between two villagers is represented as a pair [a,b], meaning that villager aaa trusts villager b. If a trust relationship does not exist between two villagers, no trust relationship exists between them.

The mayor must meet two criteria:

1. The mayor trusts nobody.
2. Everyone else (except for the mayor) trusts the mayor.

If a mayor exists, return the label of the mayor; otherwise, return -1.

Example:

Input:

```
n = 3, trust = [[1, 3], [2, 3]]
```

Output:

`3`

Explanation:
Villagers 1 and 2 both trust villager 3, and villager 3 trusts no one. Therefore, villager 3 is the mayor of the village.

Questions:

1. Describe the algorithm approach to solve this problem.
2. Given the following input:
   `n = 4, trust = [[1, 4], [2, 4], [3, 4], [4, 1]]`

   Can you identify the mayor of the village? If so, who is it? If not, explain why.

Problem 5

You are tasked with evaluating a system of interconnected devices in a smart home network. Each device is represented as a node in a directed graph, and connections between devices are represented by edges. The devices communicate with each other based on these connections, and some devices are terminal devices, meaning they do not communicate with any other device.

A device is considered **safe** if every possible communication starting from that device eventually leads to a terminal device or another safe device. Your task is to find all the safe devices in the network and return them in ascending order.

The system is represented by a 0-indexed 2D array called `graph`, where each index i represents a device, and `graph[i]` contains the list of devices that device i communicates with. A device is a **terminal device** if it does not communicate with any other devices.

Input:

- An integer array `graph` of size n, where `graph[i]` is an array representing the devices that device iii communicates with.
- Each device is represented by a node labeled from 0 to n−1n-1n−1.

Output:

- Return an array containing all the safe devices, sorted in ascending order.

Example:

Input:

```
graph = [[1, 2], [2, 3], [5], [0], [5], [], []]
```

Output:

```
[2, 4, 5, 6]
```

Explanation:

- Devices 5 and 6 are terminal devices because they do not communicate with any other devices.
- Every communication starting at devices 2, 4, 5, and 6 eventually leads to either device 5 or 6, which are terminal devices.
- Therefore, the safe devices are 2, 4, 5, and 6.

Questions:

1. **Describe the approach to solve the problem of identifying safe devices.** (Explain how you can determine which devices are safe based on the communication paths and terminal devices in the network.)
2. **Consider the following graph:** `graph = [[1, 2, 3, 4], [1, 2], [3, 4], [0, 4], []]`

   Which devices are safe in this network? Explain why.