1811/2807/7001ICT
Programming Principles

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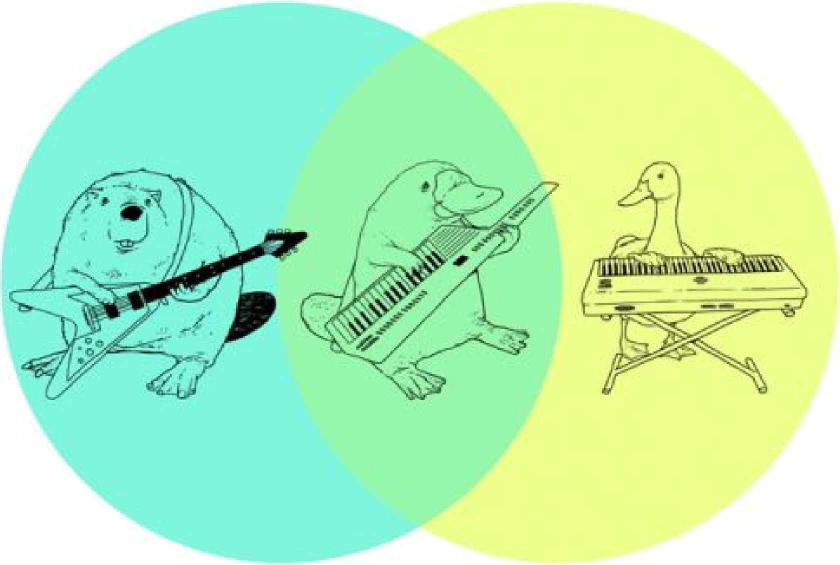
22 Sets

Sets are a mathematical construct that underpin much of modern theoretical mathematics.

They also turn out to be a very useful data structure in programs.

This section revises or introduces sets in mathematics, and the next section is about Python's `set` class.

22.1 The set concept



Definition: A *set* is a collection of distinct objects.

The “objects” could be anything.

“Distinct” means there is some way to tell them apart.

If the objects are integers, all 32s are the same. So they are not distinct, but 32 is distinct from 42.

An object can be in a set or not, but not in a set twice.

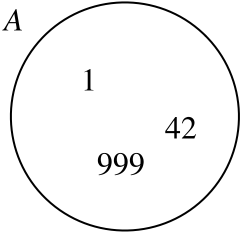
I.e. all the *elements* of a set are unique.

Sets may be represented many ways.

This set, called A, contains three integers.

$$A = \{1, 999, 42\}$$

A common way to draw sets is the Venn diagram.



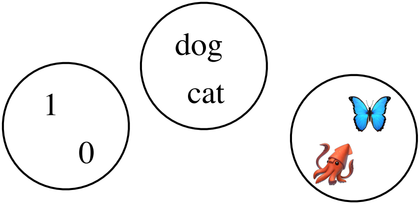
Either way, the elements are unique, and their order doesn't matter.

22.1.1 Sets in fundamental mathematics

Sets are really important in our fundamental understanding of mathematics.

> *Question:* What is 2, two, twoness?

The idea of 2 may be captured by all the examples of sets of 2 things...

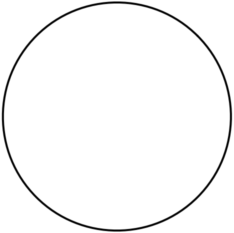


... but we are not here for esoteric math, just the practical stuff.

22.1.2 Empty sets

An empty set is written $\{\}$.

Or graphically:

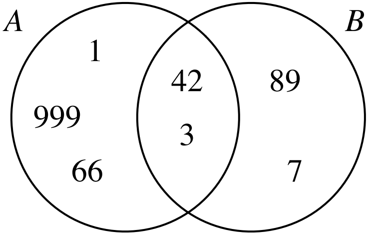


22.2 Notations and operations

Suppose we have two sets of numbers.

$$A = \{1, 3, 42, 66, 999\} \quad \textit{and} \quad B = \{3, 7, 42, 89\}$$

Numbers are unique, so these sets overlap.



22.2.1 Element, not element

The relational operator $\in$ means "in" or "is an element of". So if

$$A = \{1, 3, 42, 66, 999\} \quad and \quad B = \{3, 7, 42, 89\}$$

$1 \in A$ and $42 \in A$ and $42 \in B$ and $89 \in B$.

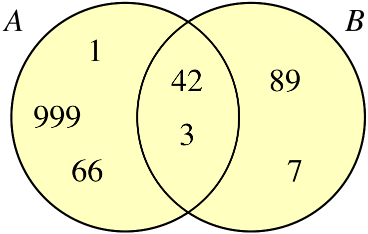
$\notin$ means "not in" or "is not an element of". So $1000 \notin A$.

22.2.2 Union

The first way we combine sets is to form the *union*, with operator $\cup$.

The union of two sets is the set that contains all the elements of both sets. So

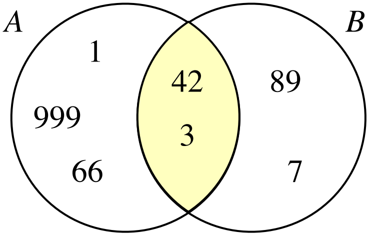
$$A \cup B = \{1, 3, 7, 42, 66, 89, 999\}$$



22.2.3 Intersection

The operator ∩ forms the *intersection* of two sets, the set of all the elements that are in both sets.

$$A \cap B = \{3, 42\}$$

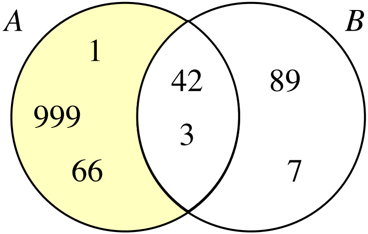


22.2.4 Difference

The operator − forms the *difference* of two sets.

$A - B$ is the set of all the elements of A that are not in B.

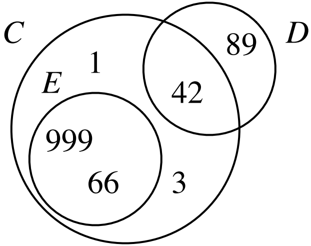
$$A - B = \{1, 66, 99\}$$



22.2.5 Subsets

Let's consider some new sets.

$$C = \{1, 42, 66, 999\}, \quad D = \{42, 89\}, and \;\; E = \{66, 999\}$$



We can define these relations by example:

- $E \subset C$, E is a *proper subset* of C as all the elements of E are also in C, but $E \neq C$.
- $E \subseteq C$, E is a *subset* of C as all the elements of E are also in C. $C \subseteq C$ too.
- $C \supset E$, C is a *proper superset* of E.
- $C \supseteq E$, C is a *superset* of E, and $C \supseteq C$ too.
- $D \not\subset C$, D is not a proper subset of C.

Section summary

This section covered:

- the concept of a set;
- set combinators, union, intersection, and difference; and
- relational operators, is/not an element, is/not a (proper) subset.