Chapter 4: *AC Machinery Fundamentals*

4-1. Develop a table showing the speed of magnetic field rotation in ac machines of 2, 4, 6, 8, 10, 12, and 14 poles operating at frequencies of 50, 60, and 400 Hz.

SOLUTION The equation relating the speed of magnetic field rotation to the number of poles and electrical frequency is

$$n_m = \frac{120 f_e}{P}$$

The resulting table is

Number of Poles	f_e **= 50 Hz**	f_e **= 60 Hz**	f_e **= 400 Hz**
2	3000 r/min	3600 r/min	24000 r/min
4	1500 r/min	1800 r/min	12000 r/min
6	1000 r/min	1200 r/min	8000 r/min
8	750 r/min	900 r/min	6000 r/min
10	600 r/min	720 r/min	4800 r/min
12	500 r/min	600 r/min	4000 r/min
14	428.6 r/min	514.3 r/min	3429 r/min

4-2. A three-phase two-pole winding is installed in six slots on a stator. There are 80 turns of wire in each slot of the windings. All coils in each phase are connected in series, and the three phases are connected in Δ. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 3600 r/min.

(a) What is the frequency of the voltage produced in this winding?

(b) What are the resulting phase and terminal voltages of this stator?

SOLUTION

(a) The frequency of the voltage produced in this winding is

$$f_e = \frac{n_m P}{120} = \frac{(3600 \text{ r/min})(2 \text{ poles})}{120} = 60 \text{ Hz}$$

(b) There are six slots on this stator, with 80 turns of wire per slot. The voltage in the coils in one phase is

$$E_A = \sqrt{2}\pi N_C \phi f = \sqrt{2}\pi (80 \text{ t})(0.060 \text{ Wb})(60 \text{ Hz}) = 1280 \text{ V}$$

Since the machine is Δ-connected, $V_L = V_\phi = 1280 \text{ V}$

4-3. A three-phase Y-connected 50-Hz two-pole synchronous machine has a stator with 2000 turns of wire per phase. What rotor flux would be required to produce a terminal (line-to-line) voltage of 6 kV?

SOLUTION The phase voltage of this machine should be $V_\phi = V_L / \sqrt{3} = 3464 \text{ V}$. The induced voltage per phase in this machine (which is equal to V_ϕ at no-load conditions) is given by the equation

$$E_A = \sqrt{2}\pi N_C \phi f$$

so

$$\phi = \frac{E_A}{\sqrt{2}\pi N_C f} = \frac{3464 \text{ V}}{\sqrt{2}\pi(2000 \text{ t})(50 \text{ Hz})} = 0.0078 \text{ Wb}$$

4-4. Modify the MATLAB in Example 4-1 by swapping the currents flowing in any two phases. What happens to the resulting net magnetic field?

SOLUTION This modification is very simple—just swap the currents supplied to two of the three phases.

```
% M-file: mag_field2.m
% M-file to calculate the net magetic field produced
% by a three-phase stator.

% Set up the basic conditions
bmax = 1;                % Normalize bmax to 1
freq = 60;               % 60 Hz
w = 2*pi*freq;           % angluar velocity (rad/s)

% First, generate the three component magnetic fields
t = 0:1/6000:1/60;
Baa = sin(w*t) .* (cos(0) + j*sin(0));
Bbb = sin(w*t+2*pi/3) .* (cos(2*pi/3) + j*sin(2*pi/3));
Bcc = sin(w*t-2*pi/3) .* (cos(-2*pi/3) + j*sin(-2*pi/3));

% Calculate Bnet
Bnet = Baa + Bbb + Bcc;

% Calculate a circle representing the expected maximum
% value of Bnet
circle = 1.5 * (cos(w*t) + j*sin(w*t));

% Plot the magnitude and direction of the resulting magnetic
% fields.  Note that Baa is black, Bbb is blue, Bcc is
% magneta, and Bnet is red.
for ii = 1:length(t)

   % Plot the reference circle
   plot(circle,'k');
   hold on;

   % Plot the four magnetic fields
   plot([0 real(Baa(ii))],[0 imag(Baa(ii))],'k','LineWidth',2);
   plot([0 real(Bbb(ii))],[0 imag(Bbb(ii))],'b','LineWidth',2);
   plot([0 real(Bcc(ii))],[0 imag(Bcc(ii))],'m','LineWidth',2);
   plot([0 real(Bnet(ii))],[0 imag(Bnet(ii))],'r','LineWidth',3);
   axis square;
   axis([-2 2 -2 2]);
   drawnow;
   hold off;

end
```

When this program executes, the net magnetic field rotates clockwise, instead of counterclockwise.

3. Problem B-4 on page 658.

A three phase four pole winding of the double layer type is to be installed on a 48 slot stator. The pitch of the stator windings is 5 / 6 and there are 10 turns per coil in the windings. All coils in each phase are connected in series and the three phases are connected in Δ. The flux per pole in the machine is 0.054 Wb and the speed of rotation of the magnetic field is 1800 RPM.

a. What is the pitch factor of this winding?

$\mathrm{RPM} := \mathrm{min}^{-1}$

$\mathrm{Degrees\_Mechanical} := \mathrm{deg}$ $\quad$ $\mathrm{Degrees\_Electrical} := \mathrm{deg}$

$\mathrm{poles} := 4 \quad \mathrm{phases} := 3 \quad \mathrm{slots} := 48 \quad \mathrm{pitch} := \frac{5}{6} \quad \mathrm{turns} := 10 \quad \varphi := 0.054\cdot\mathrm{Wb} \quad \mathrm{speed} := 1800\cdot\mathrm{RPM}$

$$\rho := \mathrm{pitch}\cdot\frac{360\cdot\mathrm{deg}}{2} = 150\cdot\mathrm{deg} \qquad k_p := \sin\left(\frac{\rho}{2}\right) = 0.966$$

b. What is the distribution factor of this winding?

$$n := \frac{\mathrm{slots}}{\mathrm{poles}\cdot\mathrm{phases}} = 4$$

$$\gamma := \frac{360\cdot\mathrm{deg}}{\mathrm{slots}}\cdot\frac{\mathrm{poles}}{2} = 15\cdot\mathrm{Degrees\_Electrical}$$

$$k_d := \frac{\sin\left(\frac{n\cdot\gamma}{2}\right)}{n\cdot\sin\left(\frac{\gamma}{2}\right)} = 0.958$$

c. What is the frequency of the voltage produced in this winding?

$$f_e := \mathrm{speed}\cdot\frac{\mathrm{poles}}{2} = 60\cdot\mathrm{Hz}$$

d. What are the resulting phase and terminal voltages of this stator?

$$N_p := n\cdot\mathrm{turns} = 40$$

$$E_G := \sqrt{2}\cdot\pi\cdot N_p\cdot k_p\cdot k_d\cdot\varphi\cdot f_e = 532.63\,\mathrm{V}$$

For a four-pole machine with pole pairs in series,

$$V_T := \frac{\mathrm{poles}}{2}\cdot E_G = 1.065\cdot\mathrm{kV}$$

4. Problem B-5 on page 658.

A three phase Y connected six pole synchronous generator has six slots per pole on its stator winding. The winding itself is a chorded (fractional pitch) double layer winding with eight turns per coil. The distribution factor kd=0.956 and the pitch factor kp=0.981. The flux in the generator is 0.02 Wb per pole and the speed of rotation is 1200 RPM. What is the line voltage produced by this generator at these conditions?

$phases := 3 \quad poles := 6 \quad n := 6 \quad turns := 8 \quad k_d := 0.956 \quad k_p := 0.981 \quad \varphi := 0.02 \cdot Wb \quad speed := 1200 \cdot min^{-1}$

$slots := n \cdot poles = 36 \qquad n := \dfrac{slots}{phases} = 12 \qquad N_p := n \cdot turns = 96 \qquad f_e := speed \cdot \dfrac{poles}{2} = 60 \cdot Hz$

$E_G := \sqrt{2} \cdot \pi \cdot N_p \cdot k_p \cdot k_d \cdot \varphi \cdot f_e = 480.003\ V$

For a Y connected winding,

$V_T := \sqrt{3} \cdot E_G = 831.39\ V$

Therefore, the magnitude of the internal generated voltage E_A = 13275 V, and the torque angle is δ = 30.3°.
(c) Ignoring losses, the input power would be equal to the output power.

$$P_{in} = P_{out} = PF \times S = (0.85)(100M) = 85MW$$

And

$$n_m = \frac{120 f_e}{P} = \frac{120(50)}{2} = 3000 r/\min$$

The applied torque would be

$$\tau_{app} = \frac{P_{in}}{\omega_m} = \frac{85M}{(3000r/\min)(2\pi rad/r)(1\min/60s)} = 270563 N\cdot m$$

Problem 5-23:

A three-phase Y-connected synchronous generator is rated 120-MVA, 13.2-kV, 0.8-PF-lagging, and 60-Hz. Its synchronous reactance is 0.9-Ω, and its resistance may be ignored.
(a) What is its voltage regulation?
(b) What would the voltage and apparent power rating of this generator be if it were operated at 50 Hz with the same armature and filed losses as it had at 60 Hz?
(c) What would the voltage regulation of the generator be at 50 Hz?

Solution:

(a) The phase voltage is

$$V_{\phi,base} = \frac{V_T}{\sqrt{3}} = \frac{13.2k}{\sqrt{3}} = 7621V$$

The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{120M}{\sqrt{3}(13.2k)} = 5249A \quad \Rightarrow \quad I_A = 5249\angle -36.87^\circ A$$

$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87^\circ$$

The internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$E_A = 7621\angle 0° + j(0.9)(5249\angle -36.87°) = 11120\angle 19.9° \text{V}$$

The resulting voltage regulation is

$$VR = \frac{E_A - V_T}{V_T} \times 100\% = \frac{11120 - 7621}{7621} \times 100\% = 45.9\%$$

(b) The voltage of the generator is directly proportional to the speed of the generator, the voltage rating and the apparent power rating of the generator will be reduced by a factor of 50/60.

$$V_{T,50} = \frac{50}{60} V_{T,60} = \frac{50}{60}(13.2k) = 11.0kV$$
$$S_{50} = \frac{50}{60} S_{60} = \frac{50}{60}(120M) = 100MVA$$

Also, the synchronous reactance will be reduced by a factor of 50/60.

$$X_{S,50} = \frac{50}{60} X_{S,60} = \frac{50}{60}(0.9) = 0.75\Omega$$

(c) At 50 Hz rated conditions, the phase voltage is

$$V_{\phi,base} = \frac{V_T}{\sqrt{3}} = \frac{11.0k}{\sqrt{3}} = 6351V$$

And the armature current would be

$$I_A = I_L = \frac{S}{\sqrt{3} V_T} = \frac{100M}{\sqrt{3}(11.0k)} = 5249A \quad \Rightarrow \quad I_A = 5249\angle -36.87° A$$
$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87°$$

The internal generated voltage is

$$E_A = V_\phi + R_A I_A + jX_S I_A$$
$$E_A = 6351\angle 0° + j(0.75)(5249\angle -36.87°) = 9264\angle 19.9° \text{V}$$

The resulting voltage regulation is

$$VR = \frac{E_A - V_T}{V_T} \times 100\% = \frac{9264 - 6351}{6351} \times 100\% = 45.9\%$$

Because voltage, apparent power, and synchronous reactance all scale linearly with frequency, the voltage regulation at 50 Hz is the same as that at 60 Hz.

Problem 5-24:

Two identical 600-kVA, 480-V synchronous generators are connected in parallel to supply a load. The prime movers of the two generators happen to have different speed droop characteristics. When the field currents of the two generators are equal, one delivers 400-A at 0.9-PF-lagging, while the other delivers 300-A at 0.72-PF-lagging.
(a) What are the real power and reactive power supplied by each generator to the load?
(b) What is the overall power factor of the load?
(c) In what direction must the field current on each generator be adjusted in order for them to operate at the same power factor?

Solution:

(a) The real and reactive powers are

$$P_1 = \sqrt{3}V_T I_L \cos\theta = \sqrt{3}(480)(400)(0.9) = 299kW$$
$$Q_1 = \sqrt{3}V_T I_L \sin\theta = \sqrt{3}(480)(400)(\sin(\cos^{-1} 0.9)) = 145kVAR$$
$$P_2 = \sqrt{3}V_T I_L \cos\theta = \sqrt{3}(480)(300)(0.72) = 180kW$$
$$Q_2 = \sqrt{3}V_T I_L \sin\theta = \sqrt{3}(480)(300)(\sin(\cos^{-1} 0.72)) = 173kVAR$$

(b) The overall power factor can be found from the total real power and reactive power supplied to the load.

$$P_{total} = P_1 + P_2 = 299k + 180k = 479kW$$
$$Q_{total} = Q_1 + Q_2 = 145k + 173k = 318kVAR$$

The overall power factor is

$$PF = \cos\left(\tan^{-1}\frac{Q_{total}}{P_{total}}\right) = \cos\left(\tan^{-1}\frac{318k}{479k}\right) = 0.83 - lagging$$

(c) The field current of generator 1 should be increased, and the field current of generator 2 should be simultaneously decreased.

Problem 5-25:

A generating station for a power system consists of four 120-MVA, 15-kV, 0.85-PF-lagging synchronous generators with identical speed droop characteristics operating in parallel. The governors on the generators' prime movers are adjusted to produce a 3-Hz