2014115 ECE 423: DIGITAL COMMUNICATION SYSTEMS.

Q1 Draw and explain the operation of a pulse code modulation system.

i) Transmitter.

Source (analog/continuous time signal) → Low pass filter → Sampler → Quantizer → Encoder → PCM signal applied to channel i/p

(Quantizer and Encoder: A/D converters)

ii) transmission path.

→ regenerative repeater 1 → regenerative repeater 2 → . . . . → regenerative repeater N

iii) receiver.

→ regeneration circuit → decoder → reconstruction filter → destination

- In PCM a msg signal is rep by a Sequence of coded pulses accomplished by representing the signal in discrete form in both time and amplitude
- basic operation performed in the transmitter are sampling, quantizing and encoding. The LPF prior to sampling is included to prevent aliasing (distortion) of msg signal
- basic operations in the receiver are regeneration of impaired signals, decoding and reconstruction of the train of quantized samples.
- regeneration also occurs at intermediate points along the transmission path.

Q. What are the μ and A laws. Explain.

These are companding laws. Companding is a system in which info is first compressed, transmitted through a bandwidth limited channel and expanded at the receiving end. Ideally, compression and expansion laws are always exactly inverse so that except for the effect of compression, the expander o/p is equal to the compressor i/p. The combination of compressor and expander – compander.

μ law definition.

$$|V| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$ where, m and V are normalised i/p and o/p voltage & μ is a positive constant.

A law definition.

$$|V| = \begin{cases} \dfrac{A|m|}{1+\log A} & \text{for } 0 \le |m| \le 1/A \\ \dfrac{1+\log(A|m|)}{1+\log A} & \text{for } 1/A \le m \le 1 \end{cases}$$

Q. Discuss what you know about signal to noise ratio for PCM system

- Performance of PCM systems when used for transmitting analog signals is measured in terms of average signal to noise power ratio at the receiver o/p
- Overall SNR at the baseband o/p which is a measure of signal quality is defined as

$$\left[\frac{S}{N}\right]_0 = \frac{E\{[X_0(t)]^2\}}{E\{[n_q(t)]^2\} + E\{[n_0(t)]^2\}} \quad (1)$$

but

$$E\{[X_0(t)]^2\} = \frac{Q^2}{T_s^2} \cdot \frac{\Delta}{12} \quad (2)$$

Q is the no. of quantizer levels

average signal to quantizer noise power ratio at the o/p of the PCM system is given by

$$\frac{E\{[X_o(t)]^2\}}{E\{[n_q(t)]^2\}} = Q^2 = \frac{E\{[X_q(kT_s)]^2\}}{E\{[(X(kT_s)-X_q(kT_s)]^2\}}$$

combining eqtns above:

$$\left[\frac{S}{N_o}\right] = \frac{2^{2N}}{(1+4P_e 2^{2N})}$$ where, P_e = probability of a bit error.

For Large value of (S/N) threshold effect P_e is small

hence $1+4P_e 2^{2N} = 1$

and

$S/N = 2^{2N} = 6N\,dB$

For smaller value of (S/N) P_e is large, hence $1+4P_e 2^{2N} = 4P_e 2^{2N}$

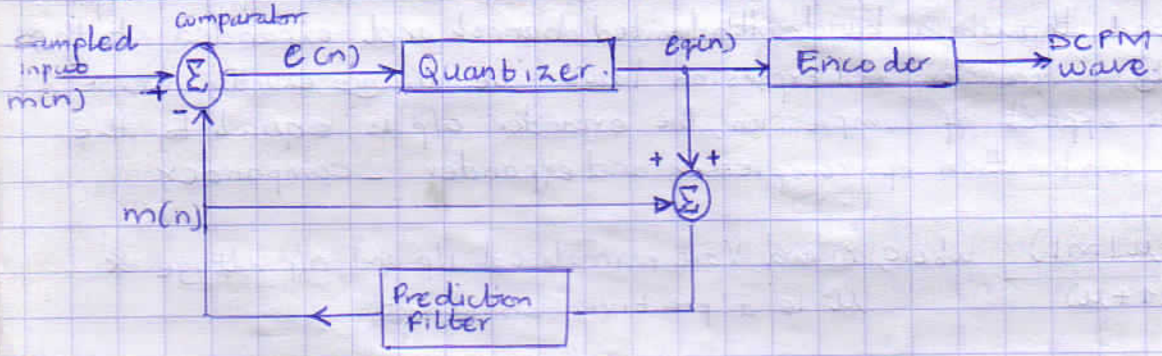
$$\left[\frac{S}{N_o}\right] = \frac{2^{2N}}{4P_e 2^{2N}} = \frac{1}{4P_e}$$

- The onset of threshold in PCM will result in a sudden increase in the o/p noise power. As input signal power is increased the o/p signal to noise power ratio reaches a value 6N dB that is independent of the signal power.
- The above threshold, increasing signal power yields no further improvement in the [S/N] the limiting value of $[S/N]_o$ depends only on the number of quantizer levels

2014/15

b) Using diagrams explain the operation of the Differential PCM systems. (Prediction filter).

Transmitter System



DPCM Receiver System

Supposing a baseband signal m(t) is sampled at the rate $f_s = 1/T_s$ to produce the sequence m(n) whose samples are T_s seconds apart, the fact that it is possible to predict future values of the signal m(t) provides motivation for the differential quantization scheme shown in transmitter system. Input signal to the quantizer is defined by $e[n] = m[n] - \bar{m}[n]$ ---- e which is the difference between unquantized input sample m[n] and prediction of it, $\bar{m}(n)$.

The predicted value is produced by using a linear prediction filter whose input consists of quantized version of input sample. The difference signal e(n) is the prediction error; amount by which the prediction filter fails to predict the input exactly.

- quantizer o/p is $e_q(n) = e(n) + q(n)$; where q - quantisation error

The quantizer output $e_q(n)$ is added to the prediction value m[n] to produce the prediction filter input.

$M_q(n) = \bar{m}(n) + e_q(n)$

$m_q(n) = \bar{m}(n) + e(n) + q(n)$

but $\bar{m}(n) + e(n) = m[n]$

$\therefore m_q(n) = m(n) + q(n) \Rightarrow$ rep. a quantized version of the input sample.

The receiver is for reconstructing the quantized version of the i/p. It consists of a decoder to reconstruct the quantized error signal. the quantized version of the original input is reconstructed from the decoder o/p using same prediction filter used in the transmitter in the absence of channel noise the decoded signal at the receiver i/p is identical to the encoded signal at the transmitter o/p.

Q Explain 2014/15 Shannons law and Channel Capacity theorem,

shannon's Law - Let B denote the channel bandwidth and SNR denote the received signal to noise ratio The information capacity theorem states that ideals these two parameters are related as;

$C = B\log_2(1+SNR)$ b/s where; C is the information capacity of the channel and is defined as the max. rate at which information can be transmitted across the channel without error in bits per second

- Information capacity theorem tells us that a message signal can be transmitted through the system without error even when the channel is noisy provided that the actual signaling rate R in bits per second is less than the information capacity C.

Channel Capacity - Assuming each message to be equiprobable the info. transmitted is $\log_2\left[\frac{S+N}{N}\right]^{BT} = BT\log_2(1+S/N)$ bits

Therefore the channel capacity is given by:

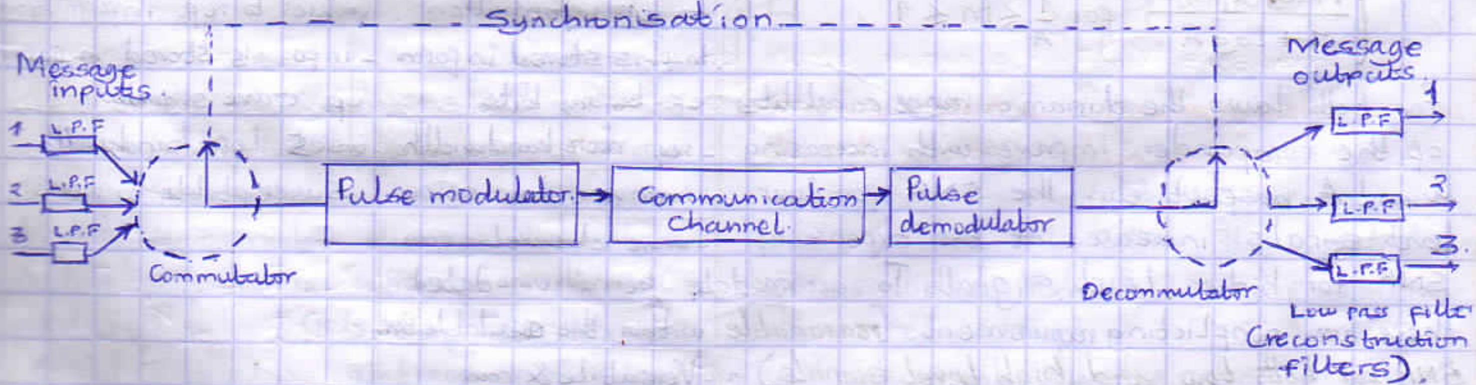
$C = B\log_2(1+S/N)$ bits/sec

If the ratio of signal energy to noise energy is identical to the corresponding power ratio, so that shannon's Law for channel Capacity is

$C = B\log_2(1+S_P/N_P)$ b/s.

- It states that, "If a channel has a bandwidth B and the mean SNR is S_P/N_P, the maximum rate at which information may be transmitted is C bits/sec

Q. With the help of a digram explain the time division multiplex system.



Each input message is first restricted in BW by a LPF and aliasing filter to remove the frequency that are unessential to an adequate signal representation.

LPF are then applied to a commutator which is usually implemented using electronic switching circuitry. The commutator takes a narrow sample of each message at rate fs.

- The multiplexed signal is applied to a pulse modulator to transform it into a form suitable for transportation over the common channel.
- At the receiving end, the receiving signal is applied to a pulse demodulator which performs reverse operation of the pulse modulator.
- The narrow samples produced at the output are distributed to appropriate Low pulse reconstruction filter by a demodulator.

Q. Explain the use companding systems in Digital communication systems.

Companding is a system in which information is first compressed, transmitted through a bandwidth limited channel and expanded at the receiving end. Ideally, compression and expansion laws are exactly inverse so that except for the effect of compression, the expander output is equal to the compression input.

→ The combination of compressor and expander - Compander.

Particular forms of compression laws used are μ-law and A-law.

μ-law

$$|V| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$; where m and V are the normalised i/p and o/p voltage.

μ is a positive consant

A-law

$$|V| = \begin{cases} \frac{A|m|}{1+\log A} & \text{for } 0 \le |m| \le 1/A \\ \frac{1+\log(A|m|)}{1+\log A} & \text{for } \frac{1}{A} \le m \le 1 \end{cases}$$

For both laws the dynamic range capability of the compander improves with increasing μ and A respectively. The SNR for low level signals increases at the expense of SNR for higher level signals. To accomodate these two conflicting requirements (reasonable SNR for both low and high level signals) a compromise is usually made in choosing the value of parameters μ and A for their respective companders, typically

μ = 255

A = 87.6.

2014/15

Q. Discuss the line coding techniques in digital communication.

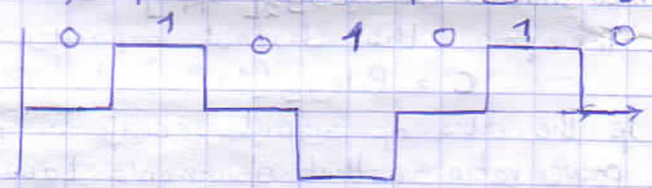
Manchester code: symbol 1 is represented by positive pulse of amplitude A followed by a negative pulse of amplitude A, with both pulses being half symbol wide. symbol 0 requires both polarities of the 2 pulses.

Unipolar NRZ - Symbol 1 rep. by transmitting a pulse of amplitude A for the duration of the symbol and symbol 0 is rep. by switching off the pulse. It's an on and off switching.

polar NRZ - 1 and 0 are rep by transmitting pulses of amplitude +A and A respectively. Power spectrum of the signal is large near zero freq.

Unipolar RZ - symbol 1 is rep. by a rectangular pulse of amplitude A with half symbol width and symbol 0 is rep. by transmitting no pulse.

Bipolar RZ - positive and negative pulses of equal amplitude (+A and -A) are used alternately for symbol 1 with each having a half symbol width, no pulse is used for symbol 0.



2014/15

Q. Compare and contrast the digital communication system and the analog communication system.

Digital	analog.
Uses descrete values to rep. information	Uses continuous range of values to rep. information
Info is stored in form of binary bits	info. is stored in form of wave signals
req. more bandwidth	uses less bandwidth
immune to noise	Susceptible to noise
more channels can be accomodated within the available BW	
Versatile & can accomodate both TV and radio signals	
Denoted by square waves	Denoted by sine waves

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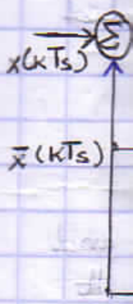

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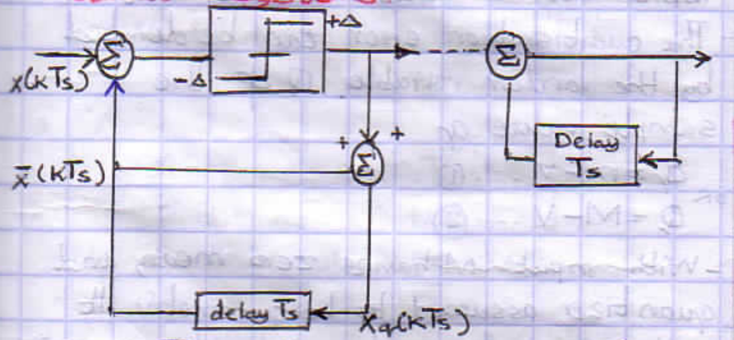
Q. Discuss the transmission noise

- Introduced everywhere in the channel between the transmitter output and the receiver input. Always present once the equipment is switched on and it causes matched filter detector to make an ocassional error in decoding whether a 0 or 1 was transmitted.
- To calculate the effect of bit error introduced by channel noise, consider a PCM system using N-bit code words $(Q = 2^N)$.
- assigning 00……00 to most negative level, 00…01 to the next level and 11….11 to the most positive level. An error that occurs in the lsb of the codeword corresponds to an error ~~in the next~~ in the ~~significant bit~~ quantized value of the sampled signal by amount Δ, an error in the next significant bit causes an error 2Δ, and an error in the i^{th} position causes an error $(2^{i-1})\Delta$.
- referring to the error as ΔQ the variance of the error is

$$E[Q_\Delta^2] = \frac{1}{N}\left(\sum_{i=1}^{N}(\Delta 2^{i-1})^2\right)$$

$$= \frac{2^{2(N-1)}}{3N} = \frac{2^{2N}}{3N}\Delta^2 \text{ for } N \gg 2.$$

Explain the operation of the delta modulation systems



- At the transmitter the sampled values $X(KT_s)$ of $x(t)$ is compared with a predicted value $\bar{x}(KT_s)$ and the difference $X(KT_s) - \bar{x}(KT_s)$ is quantized into either $+\Delta$ or $-\Delta$.
- The output of the quantizer is encoded using one binary digit per sample and sent to the receiver.
- At the receiver, the decoded value of the difference signal is added to the immediately preceeding value of the receiver output.

Operation.

$\bar{X}(KT_s') = \bar{X}(K-1)T_s'$

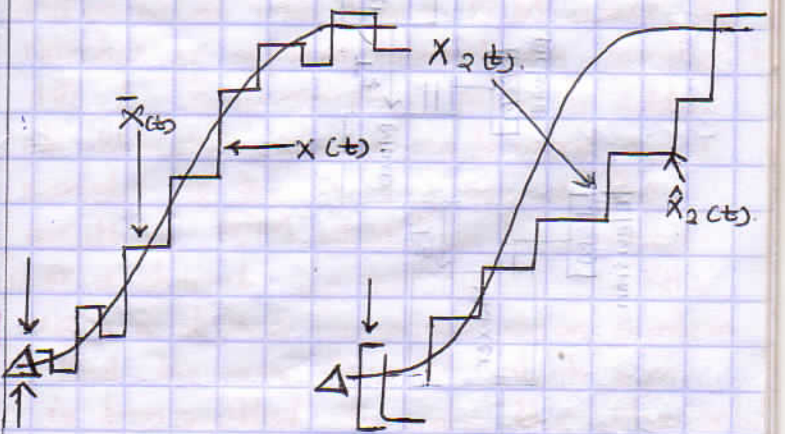
where; $\bar{x}(K-1)$ is the receiver output at $t(K-1)T_s$ and $\bar{X}(KT_s) = \bar{X}(KT_s') + [X(KT_s') - \bar{X}(KT_s)]$

$= \bar{X}(K-1)T_s \pm \Delta$.

2014/15

Q. Explain the slope overload in a digital communication system.

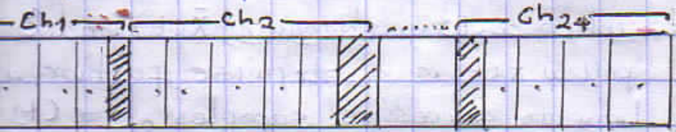
- Serious problem in delta modulation schemes due to the rate of rise of overloading.
- when $x(t)$ is changing, $\bar{x}(t)$ and $\hat{x}(t)$ follow $x(t)$ in a stepwise fashion as long as successive samples of $x(t)$ do not differ by an amount greater than the step size Δ.
- When the difference is greater than Δ, $\bar{x}(t)$ and $\hat{x}(t)$ can no longer follow $X(t)$.
- Its not determined by the amplitude of the message signal $x(t)$ but rather by its slope.



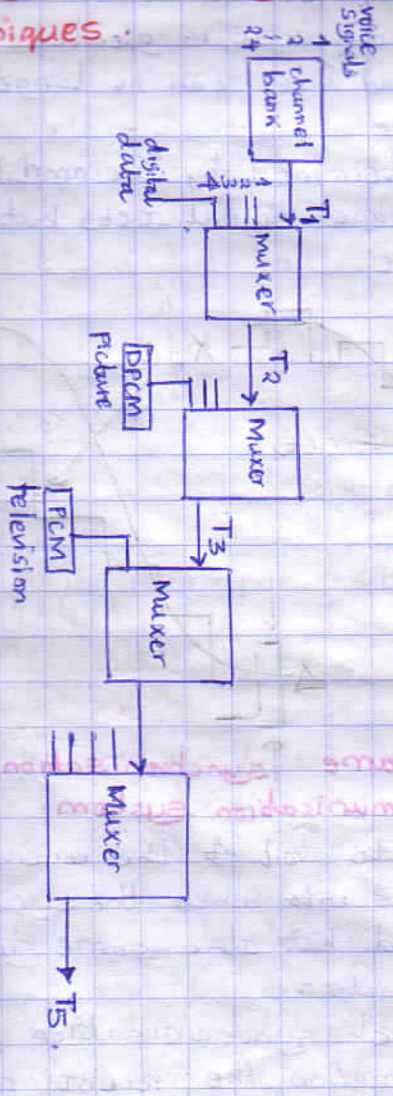
Q. Explain frame synchronisation in a digital communication system.

- necessary to avail at the receiver not only the bits into which the signal have been encoded but also some synchronisation information.
- Without such synchronisation i.e timing information, the receiver can't know which bits correspond to which of the original signal.
- An extra signal bit is made available immediately preceeding the 192 bits that

...g the encoded signal.
- 192 slots assigned to the encoded ...nal together with one extra timing signal ... adding up to a total of 193 bits ...ke up the frame.
- ...ch sample provided by the commutator ... encoded into 8 bits (7 information ...s + 1 signal bit) corresponding to $2^8 = 256$ quantization levels.
- ...e Digital signals generated during the course of one complete swap of the commutator is therefore $(24 \times 8) = 192$ bits.
- ...mmutator sweep continuously from ...1 upto S24.



...scuss the PCM signal multiplexing techniques.



Q. Discuss what you know about Quantization noise in digital communication systems 2014/15

This is an error introduced by quantization and is defined as the difference between the i/p signal, m and the o/p signal V.

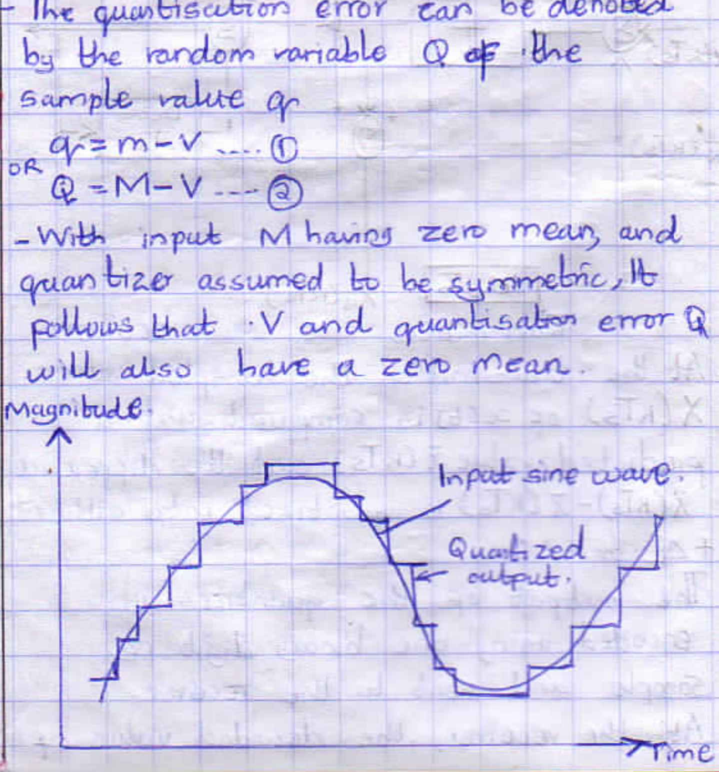
- Let the input m be the sample value of a zero mean random variable M. A quantizer g(.) maps the input random variable m of a continuous amplitude into a discrete random variable V and their respective sample values m and V are related as
- $V = g(m)$ where m - is continuous sample value and V - discrete sample value.
- The quantisation error can be denoted by the random variable Q of the sample value q

$$q = m - V \quad \text{....} \; (1)$$

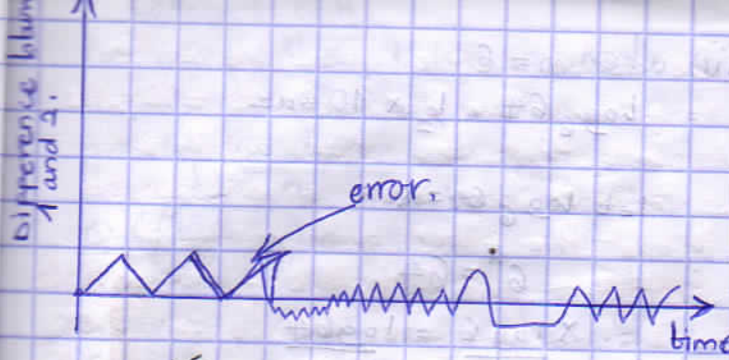
OR

$$Q = M - V \quad \text{---} \; (2)$$

- With input M having zero mean, and quantizer assumed to be symmetric, It follows that V and quantisation error Q will also have a zero mean.



difference btwn 1 and 2.



2014/15

In a binary PSK scheme using correlation receiver, the local carrier waveform is $A\cos(\omega_c t + \phi)$ instead of $A\cos\omega_c t$ due to poor carrier synchronisation. Derive an expression for the probability of error and compute the increase in error probability when $\phi = 15°$ and $\frac{A T_b}{n} = 10$.

$$P_e = Q\left(\frac{\gamma_{max}}{2}\right)$$

$$\gamma^2_{max} = \frac{2}{n}\int_0^{T_b} \left(A\cos(\omega_c t + \phi)\right)^2 dt$$

$$= \frac{2A^2}{n}\int_0^{T_b} \cos^2(\omega_c t + \phi)\, dt$$

$$= \frac{2A^2}{n}\int_0^{T_b} \left[1 - \sin^2(\omega_c t + \phi)\right] dt$$

$$= \frac{2A^2}{n}\int_0^{T_b} \left[1 - \left(\tfrac{1}{2} - \cos 2(\omega_c t + \phi)\right)\right] dt$$

$$= \frac{A^2}{n}\left[t + \frac{\sin 2(\omega_c t + \phi)}{\omega_c}\Big|_0^{T_b}\right]$$

$$= \frac{A^2}{n}\left[T_b + \cancel{\frac{\sin 2(\omega_c T_b + \phi)}{\omega_c}}^{\,0}\right]$$

If $\phi = 15°$ and $\frac{A^2 T_b}{n} = 10$.

$$\gamma^2_{max} = \frac{A^2 T_b}{n}\left(1 + \frac{\sin 2(\omega_c t_b + \phi)}{T_b \omega_c}\right)$$

$$= 10\left(1 + \cancel{\frac{\sin 2(\omega_c T_b + 15)}{T_b \omega_c}}^{\,0}\right)$$

$$\gamma^2_{max} = 10$$

$$\gamma_{max} = \sqrt{10}$$

$$P_e = Q\left(\gamma_{max}/2\right) = Q\left(\frac{3.1623}{2}\right)$$

$$= Q(1.58115) = \underline{0.0549}$$

2014/15

Q. Compare and contrast the PSK and ASK signalling schemes.

ASK	PSK
• carrier is switched on and off	• carrier is switched between +A and −A.
. Its spectrum has an impulse at carrier freq.	. Its spectrum doesn't have an impulse at carrier freq.
. Linear modulation scheme	− non-linear modulation scheme.

on — 1
off(space) — 0

Phase carrier shifted between two values

Similarities

- Similar BW requirements.
- shape of the psd is the same

2014/15.

Q. An analog signal sampled, quantized and encoded into binary PCM wave. The number of representation levels used is 128. A synchronizing pulse is added at the end of each code word rep. a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 12 KHz using a digital communication system.

i) Find the rate (b/s) at which info. is transmitted through the channel.

$N = 128$

$BW = 12\text{ KHz}$

$$\boxed{r_b = r_s \log_2 M \text{ bit/sec}}$$

r_s – symbol rate.

T_s – bit or signalling duration.

$BW = \frac{r_s}{2}$ Hz ; $r_s = 2BW$

$r_s = 2(12{,}000) = 24000$ b/s $= 24$ kb/s

$r_b = 24000 \times \log_2 4$

$= 48$ kb/sec.

ii) Find the rate at which the analog signal is sampled. What is the max. possible value for the highest freq. component of the analog signal.

Sampling rate

$f_s = \frac{1}{T_s} = \frac{1}{(1/r_s)} = r_s = 24\,KHz$

the highest frequency component is

$f_{max} \leq f_s/2 = 24/2$

$= \underline{12\,KHz}$

2014 H5

The specifications of a PCM system include ; sampling rate = 10KHz, the no. of quantizing levels is 64. The PCM wave is transmitted over a baseband channel using discrete pulse amplitude modulation.

Determine the minimum bandwidth reqd. for transmitting the PCM wave if each pulse is allowed to take on the following no. of amplitude levels.

i) 2
ii) 4
iii) 6
iv) 8

SOLUTION

$B_{PCM} \gg r f_x$ where $r = \log_m Q$ & $m \leq Q$

m = channel symbols

Q = Quantizer levels

$f_x = 1/2\, f_s$.

i) when m = 2

$B_{PCM} \gg r f_x$

$$\left[\log_2 64\right] \times \left[\frac{1}{2} \times 10\,KHz\right]$$

$$\boxed{6 \times 5\,KHz = \underline{30\,KHz}}$$

ii) when m = 4

$$\left[\log_4 64\right] \cdot \left[\frac{1}{2} \times 10\,KHz\right]$$

$$3 \times 5\,KHz = \underline{15\,KHz}.$$

iii) when m = 6

$$\log_6 64 \cdot \frac{1}{2} \times 10\,KHz$$

let $\log_6 64 = x$

$$6^x = 64$$

$$\frac{x \log 6}{\log 6} = \frac{\log 64}{\log 6}$$

$$x = \frac{\log 64}{\log 6} = 2.321$$

$$2.321 \times 5\,KHz$$

$$= \underline{11.6\,KHz}$$

iv) when m = 8

$$\left[\log_8 64\right] \times \left[\frac{1}{2} \times 10\,KHz\right]$$

$$= 2 \times 5\,KHz$$

$$= \underline{10\,KHz}.$$

2014 H5

Compare the average power requirements of binary non-coherent ASK, coherent PSK, DPSK and non-coherent FSK signalling schemes operating at a data rate of 1200 b/s over a bandpass channel having a bandwidth of 4000 Hz, $\eta/2 = 10^{-10}$ Watts/Hz and $P_e = 10^{-4}$?

binary non-coherent ASK

$$P_e = \frac{1}{2} \exp\left(\frac{-A^2}{8N_0}\right)$$

here $N_0 = \eta B_T$ and B_T is the bandwidth of the bandpass filter.

$$P_e = \frac{1}{2} \exp\left(\frac{-A^2}{4 \times 2N_0}\right) = \frac{1}{2} \exp\left(-\frac{S_{av}}{2\eta B_T}\right)$$

where $S_{av} = A^2/4$

$$P_e = \frac{1}{2} \exp\left(-\frac{S_{av}}{2\eta B_T}\right)$$

$$\exp\left(-\frac{S_{av}}{2\eta B_T}\right) = 2P_e$$

$$-S_{av} = 2\eta B_T \ln(2P_e)$$

$$S_{av} = -2\eta B_T \ln(2P_e) = -2(2 \times 10^{-10}) 4000 \ln(2 \times 10^{-4})$$

$$8.517(2.2 \times 10^{-10})4000$$

$$= 1.363 \times 10^{-5}\ \text{Watts}.$$

$$= \underline{18.66\ dBM}.$$

Coherent PSK

$P_e = Q\left(\sqrt{\frac{A^2 T_b}{n}}\right)$ but $S_{av} = A^2/2$.

$= Q\left(\sqrt{\frac{2 S_{av} T_b}{n}}\right)$

$Q\left(\sqrt{\frac{2 S_{av} T_b}{n}}\right) \le 10^{-4}$.

$S_{av} \ge \frac{3.7^2 \times n}{2T_b} = \frac{3.7^2 \times n}{2} \times r_b$

$S_{av} = \frac{3.7^2 (2\times10^{-10})}{2} \times 1200$

$= 1.643 \times 10^{-6}$ Watts

$= \underline{27.84 \text{ dBm}}$

Coherent DPSK

$P_e = \frac{1}{2} \exp\left(-\frac{A^2 T_b}{2n}\right)$ $S_{av} = \frac{A^2}{2}$.

$= \frac{1}{2} \exp\left(-\frac{\cancel{2} S_{av} T_b}{n}\right)$

$(10^{-4} \times 2) = \exp\left(-\frac{S_{av} T_b}{n}\right)$

$S_{av} = \frac{-n \times \ln(2\times10^{-4})}{2T_b} = \frac{-n \ln(2\times10^{-4}) r_b}{\cancel{2}}$

$= -\frac{(2\times10^{-10}) \ln(2\times10^{-4})}{\cancel{2}} \cdot 1200 = \underline{29.91 \text{ dBm}}$

non-coherent FSK

$P_e = \frac{1}{2} \exp\left(-\frac{A^2}{4N_0}\right) = \frac{1}{2} \exp\left(\frac{-A^2}{4nB_T}\right)$

$= \frac{1}{2} \exp\left(-\frac{S_{av}}{2nB_T}\right)$

$(10^{-4})2 = \exp\left(-\frac{S_{av}}{2nB_T}\right)$

$= -2(2\times10^{-10})(4000) \ln(2\times10^{-4})$

$= 1.363 \times 10^{-5}$ W

* $\underline{18.66 \text{ dBm}}$.

Q. A system puts out binary data at the rate of 60 kilobits per second. The system o/p is transmitted using a binary PAM system that is designed to have a cosine pulse. Spectrum. Determine the transmission bandwidth required for each of the following roll off factors.

i) 0.2 β = roll off factor

$B_T = r_b/2 + \beta r_b$

$= \frac{60}{2} + 0.2(60) = \underline{42 \text{ KHz}}$.

ii) 0.4

$60/2 + 0.4(60) = \underline{54 \text{ KHz}}$.

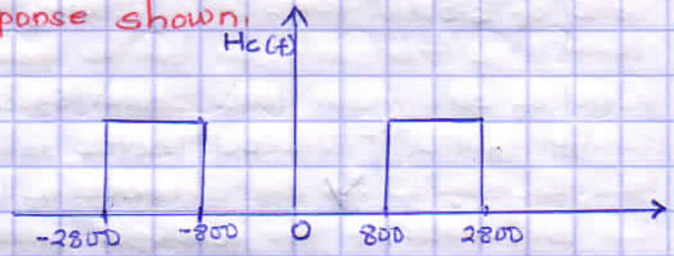
iii) 0.6

$60/2 + 0.6(60) = \underline{66 \text{ KHz}}$.

iv) 0.8

$60/2 + 0.8(60) = \underline{78 \text{ KHz}}$.

Consider a bandpass channel with the response shown.



a) A binary data is transmitted over this channel at a rate of 300 bits/sec using a non-coherent FSK signalling scheme with tone frequencies of 1070 and 1270 Hz. Calculate P_e assuming $A^2/n = 8000$.

P_e for non-coherent FSK

$P_e = \frac{1}{2} \exp\left[\frac{-A^2}{4N_0}\right]$

$N_0 = 2nr_b = 2n/T_b$; $\frac{1}{N_0} = \frac{T_b}{2n}$

$P_e = \frac{1}{2} \exp\left(-\frac{A^2 T_b}{8n}\right)$

$= \frac{1}{2} \exp\left(-\frac{A^2}{n} \cdot \frac{T_b}{8}\right)$

$= \frac{1}{2} \exp\left(-\frac{8000 T_b}{8}\right)$

But $T_b = \frac{1}{r_b} = \frac{1}{300}$

$= \frac{1}{2} \exp\left(-\frac{8000 \times 1/300}{8}\right)$

$= \frac{1}{2} \exp(-10/3)$

$= \underline{0.0178}$

b) How fast can a PSK signalling scheme operate over this scheme? Find P_e for the PSK scheme assuming coherent demodulation.

SOLUTION.

b) the PSK signalling scheme operates well over this channel because the bandwidth of the FSK signal is greater than the bandwidth of the PSK signal and that is why we say PSK signal will operate freely over this channel.

$$P_e = Q\left(\sqrt{A^2 T_b/\eta}\right) \leftarrow \text{coherent PSK}$$

$$Q\left(\sqrt{\frac{A^2}{\eta} \cdot T_b}\right)$$

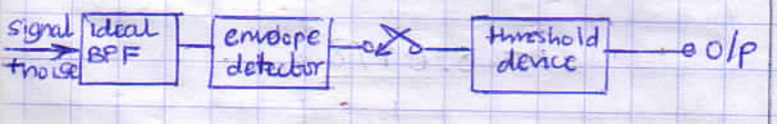
but $T_b = 1/r_b = 1/300$.

$$= Q\left(\sqrt{\left(\frac{8000}{300}\right)}\right) = Q(\sqrt{26.67})$$

$$Q(5.164) = \frac{1}{5.16 + \sqrt{2\pi}} e^{-\frac{26.67}{2}}$$

$$= 1.25 \times 10^{-7}.$$

An ASK signalling scheme uses the non-coherent demodulation scheme shown.



The centre frequency of the filter is f_c and the bandwidth $B = 10 r_b$ Hz. Assume that the bandwidth is such that the ASK signal passes through the filter with minimum distortion and that the filter generates no ISI. Calculate the P_e for the receiver shown above assuming that $\frac{A^2}{\eta r_b} = 200$.

$$= \frac{1}{2}\exp\left(\frac{-A^2}{8N_0}\right) = \frac{1}{2}\exp\left(\frac{-A^2}{8\eta B_T}\right)$$

$$= \frac{1}{2}\exp\left(-\frac{A^2}{8 \times 2\eta/T_b}\right)$$

$$= \frac{1}{2}\exp\left(\frac{-A^2}{8 \times 2\eta/T_b}\right)$$

$$= \frac{1}{2}\exp\left(\frac{-A^2}{16\eta r_b}\right)$$

where $r_b = 1/T_b$

$$= \frac{1}{2}\exp\left(-\frac{200}{16}\right)$$

$$= 1.863 \times 10^{-6}.$$

Q. Binary data is to be transmitted over a microwave channel at a rate of 5×10^6 bits/second. Assuming the channel noise to be white Gaussian with psd $\eta/2 = 10^{-12}$ Watt/Hz, find the power and bandwidth requirements of four phase PSK and 16-tone FSK signalling schemes to maintain an error probability of 10^{-4}.

$$T_b = 1/r_b = \frac{1}{(5 \times 10^6)} = 2 \times 10^{-7}$$

$$T_s = 2(2 \times 10^{-7}) = 4 \times 10^{-7}$$

$$(P_e)_{QPSK} = 2Q\left(\sqrt{\frac{A^2 T_s}{2\eta}}\right)$$

where $T_s = 2T_b = 0.4 \times 10^{-6}$

$\eta/2 = 10^{-12}$ Watt/Hz.

$P_e = 10^{-4}$

$$Q\left(\sqrt{\frac{A^2 T_s}{2\eta}}\right) \leq \frac{10^{-4}}{2}$$

$$\frac{A^2 T_s}{2\eta} \geq 3.9^2$$

but $S_{av} = A^2/2$

$$\frac{S_{av} T_s}{\eta} = 3.9^2$$

$$S_{av} = \frac{3.9^2 \times \eta}{T_s}$$

$$= 3.9^2 \times 2\times10^{-12} \times 0.4\times10^{6}$$

$$= 1.2168 \times 10^{-5}\ W$$

$$= -19.15\ dBM$$

QPSK requires a bandwidth of $2r_s = 5MHz$.

For a 16-tone FSK $(M=16)$

$$\frac{S_{av}}{\eta r_b} = 5$$

$$S_{av} = 5 \times \eta r_b$$

for $P_e = 10^{-4}$, bandwidth requirements

$$\geq \frac{M}{2T_s} \quad \text{where } M = 16$$

$$T_s = T_b \log_2 16 = 4T_b = 0.8\times10^{-6}$$

hence bandwidth required $\geq 10\ MHz$.

alternative formula to using Gaussian Table

$$\boxed{Q(y) = \frac{1}{y\sqrt{2\pi}} e^{-y^2/2}}$$

coherent

ASK $P_e = Q\left(\sqrt{\frac{A^2 T_b}{4\eta}}\right)$; $S_{av} = A^2/4$

PSK $P_e = Q\left(\sqrt{\frac{A^2 T_b}{\eta}}\right)$; $S_{av} = A^2/2$

FSK $P_e = Q\left(\sqrt{0.6\left[\frac{A^2 T_b}{\eta}\right]}\right)$

non-coherent

ASK $P_{e_0} = \frac{1}{2}\exp\left(\frac{-A^2}{8N_0}\right)$, $P_{e_1} = Q\left(\sqrt{\frac{A^2}{4N_0}}\right)$ For $A^2 \gg N_0$

$N_0 = \eta B_T$

PSK $P_e = \frac{1}{2}\exp\left(\frac{-A^2 T_b}{2\eta}\right)$

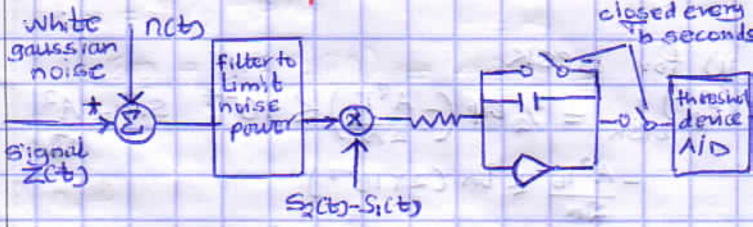
FSK $P_e = \frac{1}{2}\exp\left(\frac{-A^2}{4N_0}\right)$

QPSK $P_e = 2Q\left(\sqrt{\frac{A^2 T_s}{2\eta}}\right)$

$T_s = 2T_b$

2014/15.

Q. With the help of a diagram explain the operation of the coherent PSK.



$S_1(t) = -A\cos\omega_c t$ (0).

$S_2(t) = A\cos\omega_c t$ (1).

- the transmitted bit sequence can be recovered from the PSK signal using the intergrate and dump correlation receiver above with a local reference signal. $S_2(t) - S_1(t) = 2A\cos\omega_c t$ that is synchronized in phase and frequency with the incoming signal.

Probability of error, $P_e = Q\left(\sqrt{A^2 T_b/\eta}\right)$

the average signal power S_{av} and the signal energy bit E_{av} for the PSK scheme are;

$$S_{av} = A^2/2$$

$$E_{av} = A^2/2\, T_b$$

$$\therefore P_e = Q\left(\sqrt{\frac{2S_{av}T_b}{\eta}}\right) = Q\left(\sqrt{\frac{2E_{av}}{\eta}}\right)$$

2014/15.

Computer info. is transmitted over a radio link at the rate of 10^6 bits per second and the power spectral density of the noise at the receiver input is 10^{-10} Watt/hz. Find the average carrier power required to maintain an average probability error $P_e \leq 10^{-4}$ for

i) Coherent binary FSK

ii) DPSK.

i) PSK.

$$P_e = Q\left(\sqrt{\frac{A^2 T_b}{\eta}}\right) = Q\left(\sqrt{\frac{2S_{av}T_b}{\eta}}\right) ; S_{av} = \frac{A^2}{2}$$

$$Q\left(\sqrt{\frac{2S_{av}T_b}{\eta}}\right) \leq 10^{-4}$$

$$\frac{2S_{av}T_b}{\eta} \geq 3.70^2$$

$$(S_{av})_{PSK} \geq \frac{3.70^2}{2T_b}\eta = 3.70^2 \times \eta/2 \times \frac{1}{T_b}$$

but $r_b = 1/T_b$

$$= 3.70^2 \times 10^{-10} \times 10^6$$

$$= 6.845 \times 10^{-4}$$

in dBm $= 10\log(1000 \times 6.845\times10^{-4})$
$= -1.646$ dBm

ii) for DPSK

$$(P_e)_{DPSK} = \frac{1}{2}\exp\left(-\frac{A^2 T_b}{2\eta}\right) \le 10^{-4} \quad S_{av} = \frac{A^2}{2}$$

$$-\frac{A^2 T_b}{2\eta} \le \ln(2\times10^{-4})$$

$$-\frac{S_{av} T_b}{\eta} \le \ln(2\times10^{-4})$$

$$S_{av} \ge -\eta_{/T_b}(\ln 2\times10^{-4}) \; ; \frac{1}{T_b} = r_b$$

$$\ge -\frac{(10^{-10})}{10^{-6}}\ln(2\times10^{-4})$$

$$\ge 8.517\times10^{-4}\ W$$

$$= -0.697\ \text{dBM}$$

2014/15

Q. An FSK system transmitts binary at the rate of 2.5×10^6 bits/second. During the course of transmission white Gaussian noise of zero mean and power spectral density of 10^{-15} Watts/hertz is added to the signal. In the absence of noise, the amplitude of the received sinusoidal wave for digit 1 or 0 is 1mV. Determine the average probability of symbol error for the following system configurations
i) coherent binary FSK
ii) non-coherent binary FSK.

$r_b = 2.5\times10^6$ bits/sec. $T_b = \frac{1}{r_b}$

$G_n(f) = 10^{-15}$

$A = 1$ mV

i) coherent binary FSK

$$P_e = Q\left(\sqrt{\frac{0.61 A^2 T_b}{\eta}}\right)$$

$$= Q\left(\sqrt{\frac{0.61(1\times10^{-3})^2}{10^{-15}} \times \frac{1}{(2.5\times10^6)}}\right)$$

$$= Q\left(\sqrt{244}\right)$$

$$Q(15.62) = \frac{1}{15.62\sqrt{2\pi}} e^{-(15.62^2)/2}$$

$$= 2.65\times10^{-55}$$

ii) non-coherent FSK.

$$P_e = \frac{1}{2}\exp\left(-\frac{A^2}{4N_0}\right)$$

$$N_0 = \frac{2\eta}{T_b} \; ; \frac{1}{N_0} = \frac{T_b}{2\eta}$$

$$= \frac{1}{2}\exp\left(-\frac{A^2 T_b}{8\eta}\right)$$

$$= \frac{1}{2}\exp\left(-\frac{A^2}{8\eta r_b}\right) \quad T_b = \frac{1}{r_b}$$

$$= \frac{1}{2}\exp\left(\frac{-(10^{-3})^2}{8\times(2.5\times10^6)\times10^{-15}}\right)$$

$$= \frac{1}{2}e^{-50} \; ; \frac{1}{2}\times(1.9287\times10^{-22})$$

$$= 9.64375\times10^{-23}$$

2014/15

Q. A correlation receiver for a ASK system uses a carrier reference $A\sin\omega_c t$ for detecting
$S_1(t) = A\cos(\omega_c t + \Delta\phi)$
$S_2(t) = A\sin(\omega_c t + \Delta\phi)$.
Assuming that $S_1(t)$ and $S_2(t)$ are equiprobable and the noise is White and Gaussian with PSD of $\eta/2$. Find the probability of incorrect decoding.

$$P_e = Q\left(\frac{\gamma_{max}}{2}\right)$$

$$\gamma_{max}^2 = \frac{2}{\eta}\int_0^{T_b}(A\sin\omega_c t)^2 dt = \frac{2A^2}{\eta}\int_0^{T_b}\sin^2\omega_c t\, dt$$

$$= \frac{2A^2}{\eta}\int_0^{T_b}\frac{1}{2}(1-\cos 2\omega t)\,dt$$

$$= \frac{A^2}{\eta}\left(T_b - \cancel{\frac{\sin 2\omega_c T_b}{2\omega_c}}^{\,0}\right)$$

$$\gamma_{max}^2 = \frac{A^2 T_b}{\eta} \; ; \gamma_{max} = \sqrt{\frac{A^2 T_b}{\eta}}$$

$$P_e = Q\left(\frac{\gamma_{max}}{2}\right) = Q\left(\frac{1}{2}\sqrt{\frac{A^2 T_b}{\eta}}\right)$$

$$= Q\left(\sqrt{\frac{A^2 T_b}{4\eta}}\right)$$

Q. A data transmission system used a PSK signalling scheme. The low digital signals are specified as
binary 1 = $A\cos 25{,}000t$; $0 \le t \le 0.2$ ms
binary 0 = $-A\cos 25{,}000t$; $0 \le t \le 0.2$ ms
- The carrier amplitude at the receiver input is 1mV and the power spectral density of the additive white Gaussian noise at the input is $\eta/2 = 10^{-11}$ Watt/Hz. Assuming that the receiver is matched receiver, determine the minimum error probability.