

# STAT 400: Homework 04

*Fall 2017, UIUC*

*Due: Friday, September 29, 2:00 PM*








Please listen to [this song](#) for inspiration during this homework.

## Exercise 1

Suppose you are a [Pokémon Trainer](#) in the [Kanto region](#). You are currently walking through the tall grass of [Viridian Forest](#). When walking through tall grass, you will encounter [wild Pokémon](#). Pokémon encounters happen one-at-a-time and you may assume they are independent.

In Viridian Forest, there are five potential Pokémon that you may encounter. The following shows the probability that any one encounter is a particular Pokémon, as well as the Pokémon's [type](#).

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<b>Pokémon</b>	Caterpie	Metapod	Weedle	Kakuna	Pikachu
<b>Probability</b>	0.05	0.05	0.50	0.35	0.05
<b>Type</b>	Bug	Bug	Bug, Poison	Bug, Poison	Electric

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(a) Suppose you encounter **10** Pokémon. What is the probability that at least **1** is a Pikachu?

**Solution:**

Define  $X$  as “number of Pikachus in 10 Pokémon.” Then,  $X$  has a **binomial** distribution.

$$X \sim b(n = 10, p = 0.05)$$

$$\begin{aligned}
P[X \geq 1] &= 1 - P[X = 0] \\
&= 1 - \binom{10}{0} (0.05)^0 (0.95)^{10} \\
&= 1 - (0.95)^{10} \approx \boxed{0.40126}
\end{aligned}$$

(b) Suppose you encounter **10** Pokémon. What is the probability that more than **7** are type poison?

**Solution:**

Define  $Y$  as “number of Poison Pokémon in 10 Pokémon.” Then,  $Y$  has a **binomial** distribution.

$$Y \sim b(n = 10, p = 0.85)$$

$$\begin{aligned}
P[X > 7] &= f(8) + f(9) + f(10) \\
&= \binom{10}{8} (0.85)^8 (0.15)^2 + \binom{10}{9} (0.85)^9 (0.15)^1 + \binom{10}{10} (0.85)^{10} (0.15)^0 \\
&= 0.2758967 + 0.3474254 + 0.1968744 \approx \boxed{0.8201965}
\end{aligned}$$

(c) What is the probability that you see your **first** Pikachu after your **tenth** encounter?

**Solution:**

Define  $Z$  as “encounters until first Pikachu.” Then,  $Z$  has a **geometric** distribution.

$$Z \sim \text{Geom}(p = 0.05)$$

$$P[Z > 10] = (1 - 0.05)^{10} \approx \boxed{0.598737}$$

(d) What is the probability that you see your **third** Pikachu on your **twelfth** encounter?

**Solution:**

Define  $W$  as “encounters until third Pikachu.” Then,  $W$  has a **negative binomial** distribution.

$$W \sim NB(k = 3, p = 0.05)$$

$$P[W = 12] = \binom{11}{2} (0.05)^3 (0.95)^9 \approx \boxed{0.004333}$$

(e) Suppose you encounter **8** Pokémon. What is the probability that it is **2** Pikachus, **4** Weedles and **2** Kakunas?

**Solution:**

$$\frac{8!}{0!0!4!2!2!} (0.05)^0 (0.05)^0 (0.50)^4 (0.35)^2 (0.05)^2 \approx \boxed{0.008039062}$$

## Exercise 2

The Pokémon probabilities in Exercise 1 are from the 1998 video game [Pokémon Red](#). Also released at the same time was Pokémon Blue. The games are extremely similar. One difference, is the proportion of certain Pokémon found in certain locations. For example, in Viridian Forest, the proportion of Metapod is:

- Pokémon Red: **0.05**
- Pokémon Blue: **0.35**



Suppose it is 1998 and you are 12 years old. Assume Pokémon Red and Blue were both played equally. Your friend at school tells you they encountered 15 Pokémon in Viridian Forest, 3 of which were Metapod. What is the probability that your friend was playing Pokémon Red?

- *Hint:* What is the probability of seeing 3 Metapod out of 15 encounters in Pokémon Red?
- *Hint:* “Flip the conditional.”

**Solution:**

Define  $X$  as “number of Metapods in 15 Pokémon.” Then,  $X$  has a **binomial** distribution.

In Pokémon Red:

$$X \sim b(n = 15, p = 0.05)$$

$$P[X = 3] = \binom{15}{3} (0.05)^3 (0.95)^{12} \approx \mathbf{0.03073298}$$

In Pokémon Blue:

$$X \sim b(n = 15, p = 0.35)$$

$$P[X = 3] = \binom{15}{3} (0.35)^3 (0.65)^{12} \approx \mathbf{0.1109624}$$

We noted that

$$P(\text{Red}) = P(\text{Blue}) = 0.50.$$

Then,

$$\begin{aligned} P(\text{Red} \mid X = 3) &= \frac{P(X = 3 \mid \text{Red})P(\text{Red})}{P(X = 3 \mid \text{Red})P(\text{Red}) + P(X = 3 \mid \text{Blue})P(\text{Blue})} \\ &= \frac{0.03073298 \cdot 0.50}{0.03073298 \cdot 0.50 + 0.1109624 \cdot 0.50} \\ &\approx \boxed{0.2168947} \end{aligned}$$

### Exercise 3



As we have already noted, walking in tall grass in the Pokémon world can lead to encountering Pokémon. The mechanics of the [encounter rate](#) are actually somewhat complicated. For our sake, consider that you encounter Pokémon according to a Poisson process with an average rate of 1.5 per 10 steps in tall grass. (Assume you could have multiple encounters in one step.)

(a) Suppose you are walking in the tall grass. What is the probability that you encounter more than one Pokémon in ten steps?

**Solution:**

Define  $X$  as “number of encounters in 10 steps.” Then,  $X$  has a **Poisson** distribution.

$$X \sim \text{Pois}(\lambda = 1.5)$$

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 1 - f(0) - f(1) \\ &= 1 - \frac{1.5^0 e^{-1.5}}{0!} - \frac{1.5^1 e^{-1.5}}{1!} \\ &\approx \boxed{0.4421746} \end{aligned}$$

(b) Suppose you are walking in the tall grass. What is the probability that you encounter any Pokémon in a five steps?

**Solution:**

Define  $Y$  as “number of encounters in 5 steps.” Then,  $Y$  has a **Poisson** distribution.

$$Y \sim \text{Pois}(\lambda = \frac{1.5}{2} = 0.75)$$

$$\begin{aligned} P(Y > 0) &= 1 - P(Y = 0) \\ &= 1 - f(0) = 1 - \frac{0.75^0 e^{-0.75}}{0!} \\ &\approx \boxed{0.5276334} \end{aligned}$$

## Exercise 4

You have been a Pokémon Trainer for some time now, and your Pokédex is filling up. Suppose you have Pokémon of the following types:

- 5 Water
- 3 Flying
- 2 Electric
- 2 Fire
- 6 Psychic
- 1 Ghost

Technically Pokémon can have multiple types, but for this exercise, assume each Pokémon can be only one type.

For battling other Pokémon Trainers, a lineup consists of **six** Pokémon that you carry with you in Poké Balls.

(a) Suppose you select your lineup at random. What is the probability that it will contain 2 Water Pokémon?

**Solution:**

Define  $X$  as “number of Water Pokémon in a group of 6 selected without replacement.” Then,  $X$  has a **hypergeometric** distribution.

- From 5 Water, we want 2.
- From 14 “Not Water”, we want 4.

$$P[X = 2] = \frac{\binom{5}{2} \cdot \binom{14}{4}}{\binom{19}{6}} \approx \boxed{0.368937}$$

(b) Suppose you select your lineup at random. What is the probability that it will contain at most 2 Psychic Pokémon?

**Solution:**

Define  $Y$  as “number of Psychic Pokémon in a group of 6 selected without replacement.” Then,  $Y$  has a **hypergeometric** distribution.

$$P[Y \leq 2] = \frac{\binom{6}{0} \cdot \binom{13}{6}}{\binom{19}{6}} + \frac{\binom{6}{1} \cdot \binom{13}{5}}{\binom{19}{6}} + \frac{\binom{6}{2} \cdot \binom{13}{4}}{\binom{19}{6}} \approx \boxed{0.7431446}$$

**Gotta catch 'em all!**™

