Hybrid model

November 13, 2017

"Final" model

$$\frac{p}{E} \cdot \partial f = C_{\mathrm{pQCD}}^{2\leftrightarrow 2, 2\leftrightarrow 3}[f] + C_{\mathrm{Diff}}[f]$$

A "basic" pQCD model.

- "Basic": no magic tuning, no K-factor, α_s under control.
- Only uncertainty comes from the scale at which α_s is evaluated.

A diffusion component parametrizes what is missing from pQCD.

- $\hat{q}(T, E)$ can be completely parametric. Problem: $\hat{q}_c$, $\hat{q}_b$?
- Or, guided by a simple parametric model. Fit a single set of parameter for both c and b.

The pQCD component

Elastic process: intermediate propagators are all screened by Debye mass.

$$\frac{1}{s}, \frac{1}{t}, \frac{1}{u} \to \frac{1}{s + m_D^2}, \frac{1}{t - m_D^2}, \frac{1}{u + m_D^2}$$

Debye mass: simplest leading order result.

$$m_D^2 = \frac{4\pi}{3}\left(Nc + \frac{N_f}{2}\right)\alpha_s T^2$$

Inelastic process: Gunion-Bertsch matrix-element, with interference between subsequent radiation/absorption.

$$\frac{dP}{dqdk^3dt} = \frac{dP_{GB}}{dqdk^3dt}\left(1 - \cos\left(\frac{\Delta t}{\tau_k}\right)\right)$$

The pQCD component: control running coupling

Running coupling is difficult for multi-scale problem: T, Q^2, $k_\perp^2$.

- For probe-medium interaction:

$$
\begin{aligned}
|M_{2\leftrightarrow 2}|^2 &\propto \alpha_s^2(Q^2, T) \\
|M_{2\leftrightarrow 2}|^2 &\propto \alpha_s^2(Q^2, T)\alpha_s(k_\perp^2, T)
\end{aligned}
$$

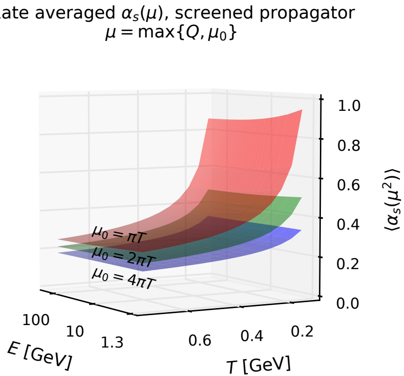
- In medium, no process slower than $t \sim 1/T$ can exist alone, since the thermal collision rate $\propto T$. So α_s scale has a lower limit of $O(T)$,

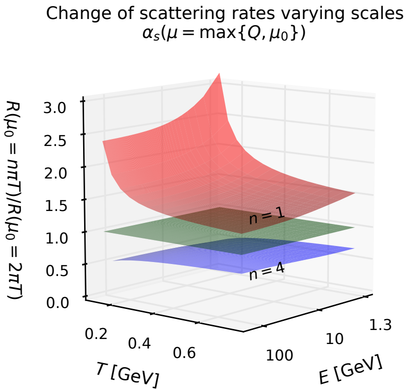
$$
\alpha_s(\mu) = \alpha_s(\mu = \max\{Q, \mu_0 T\}),
$$
$$
\alpha_s(\mu) = \alpha_s(\mu = \max\{k_\perp, \mu_0 T\}).
$$

Coupling constant must be smaller than $\alpha_s(\mu_0 T)$, μ_0 is uncertain (πT, $2\pi T$, or any number within a reasonable range).

pQCD: average $\alpha_s(\mu)$

The averaged coupling constant of elastic processes. The uncertainty of μ_0 leads to large uncertainty in scattering rates (like a K-factor but with E, T-dependence).

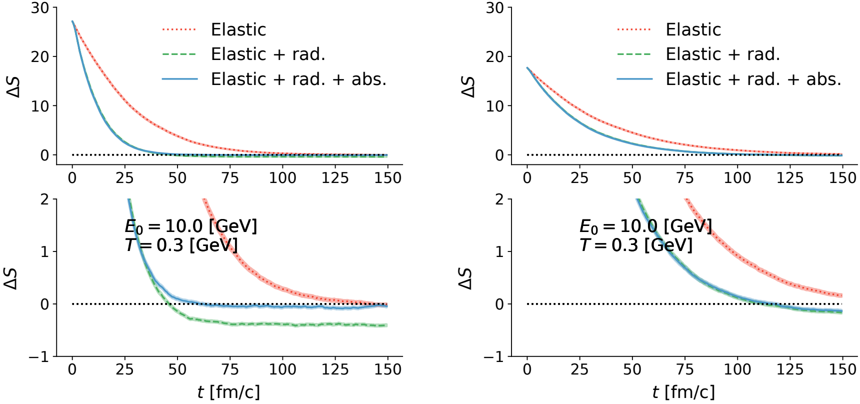




pQCD: thermalization time of charm vs bottom, $\mu = 2\pi T$

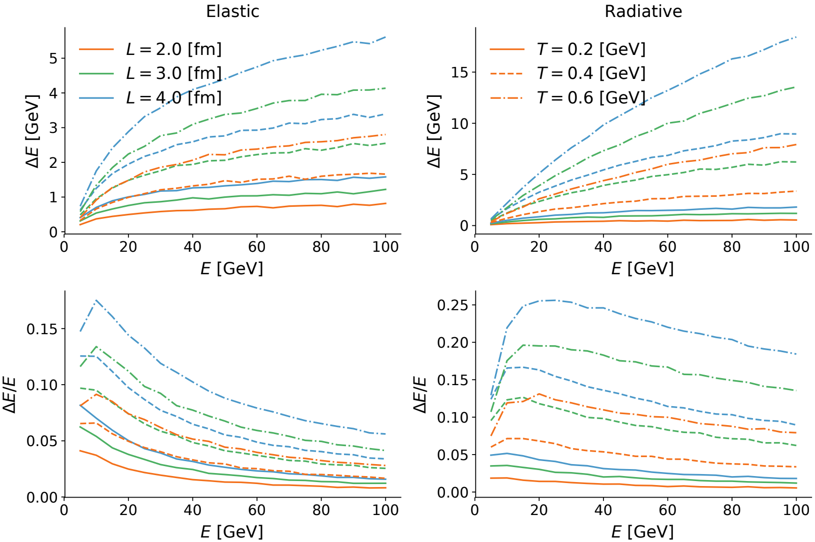
To see the approach to thermalization of an ensemble of particles $\{E_i\}$,

$$\Delta S = \langle \ln(f_0(E_i)) \rangle_i - \int f_0(E) \ln(f_0(E)) dE$$



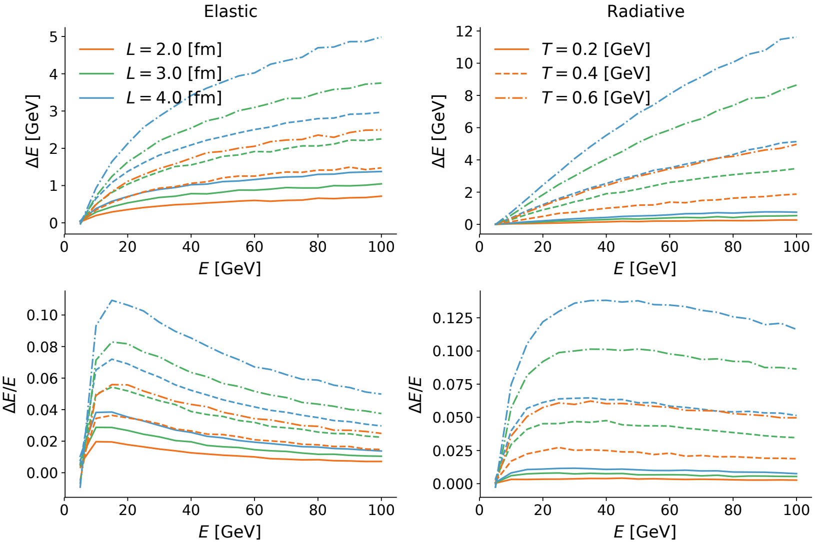
pQCD: ΔE-E in box, charm vs bottom, $\mu = 2\pi T$

Charm: radiative E-loss dominates at large energy.



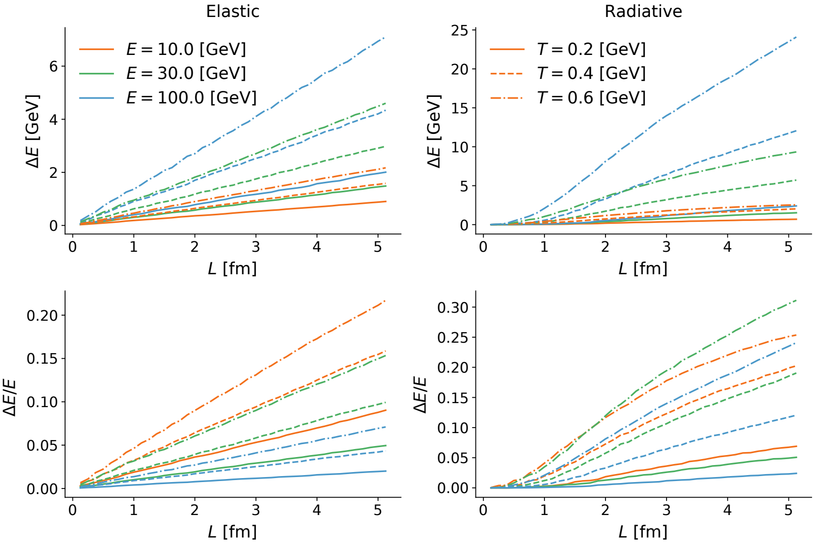
pQCD: ΔE-E in box, charm vs bottom, $\mu = 2\pi T$

Bottom: similar elastic E-loss as charm, but much less radiative E-loss.



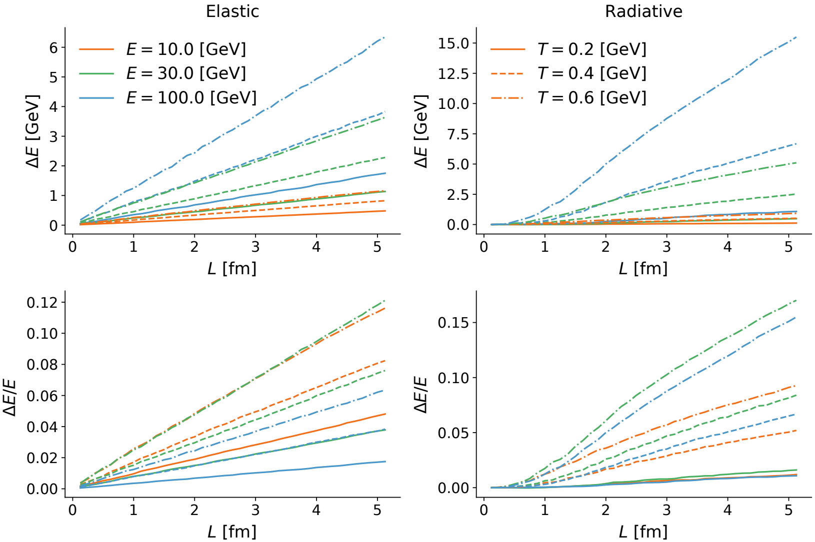
pQCD: ΔE-L in box, charm vs bottom, $\mu = 2\pi T$

Charm: non-linear $\Delta E - L$ behavior at small length.



pQCD: ΔE-L in box, charm vs bottom, $\mu = 2\pi T$

Bottom: again similar elastic E-loss, but less radiative E-loss.

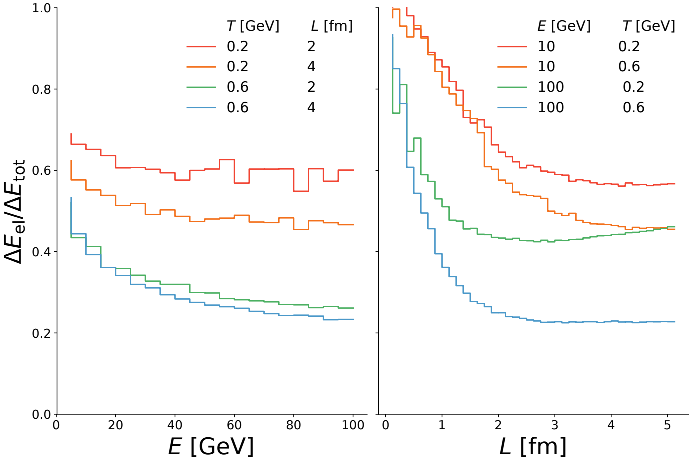


pQCD: relative importance of $\Delta E_{\rm el}$ vs $\Delta E_{\rm rad}$, $\mu = 2\pi T$

Charm:

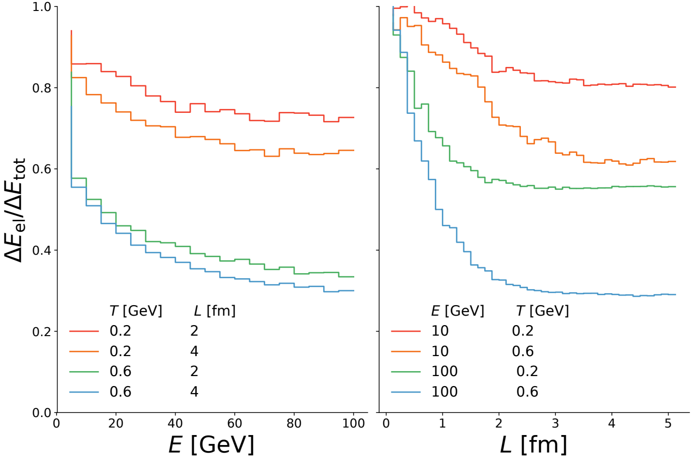
It looks $\Delta E_{\rm rad}$) always contributes $> 50\%$ for large path length.

$\Delta E_{\rm el}$ only dominates at small path length (where LPM is important).



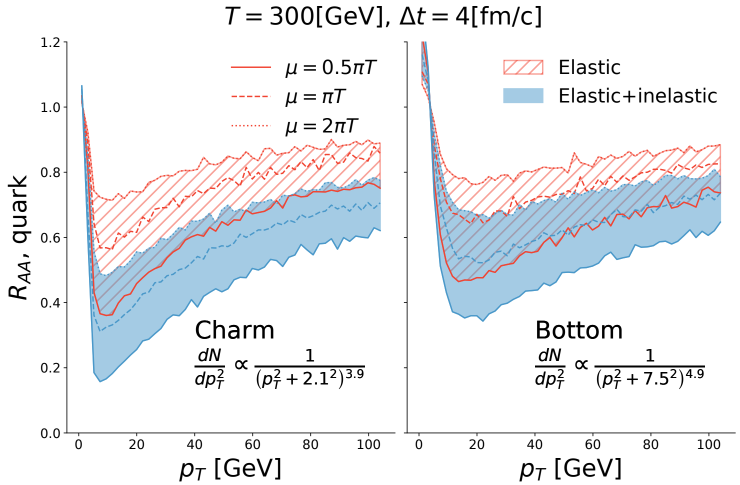
pQCD: relative importance of $\Delta E_{\rm el}$ vs $\Delta E_{\rm rad}$, $\mu = 2\pi T$

Bottom: $\Delta E_{\rm rad}$ not important at very low energy ($E < 5 GeV$)



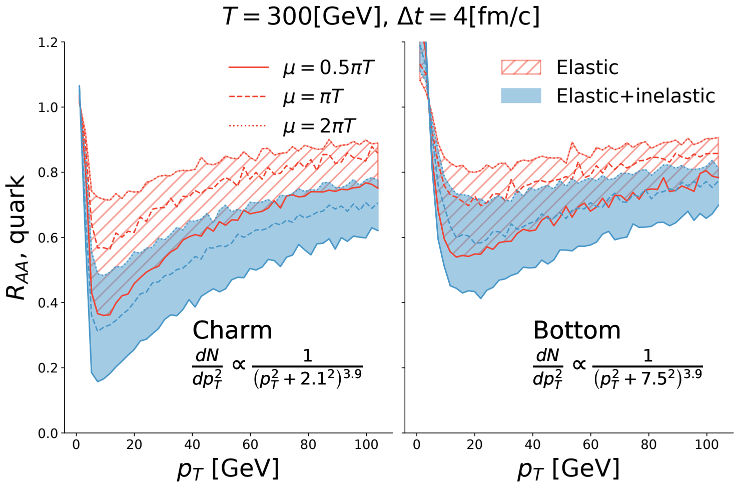
pQCD: comparison of charm / bottom R_{AA} in box.

Something like what Shanshan compares in the HQ collaboration. Heavy quark produced according to parametrized spectra and then propagates in static box for $\Delta t = 4$ fm/c.



pQCD: comparison of charm / bottom R_{AA} in box.

If we use the same power law spectra $\propto p_T^{3.9}$, it appears there is less "suppression" for bottom.



Diffusion component

Role of pQCD and diffusion,

- pQCD component: highly anisotropic scatterings (μ).
- Diffusion component: what is missing from pQCD (a, b, c...).
  (1) Non-perturbative origin.
  (2) Assume to be isotropic.
  (3) Assume it is pure diffusion, without induce extra radiation.

Choose a parametrization,

- Need mass dependence. Currently all parameters are fitted to charm meson data. But to calculate bottom quark, we probably need,
  (1) a new fit
  (2) rescale it by an *ad hoc* mass-dependence.
  (3) *start with a simple parametization with mass dependence (also based on assumptions) and fit to both charm and bottom.

Diffusion: parametrization with mass (extremely naïve)

For example, pQCD cross-section at small t looks like,

$$\frac{d\sigma}{dt} = \frac{1}{(s-M^2)^2}\frac{(s-M^2)^2}{(t-m_D^2)^2}$$

What if there are non-perturbative corrections (naïve guess),

$$\frac{d\sigma}{dt} = \frac{1}{(s-M^2)^2}\left\{\frac{(s-M^2)^2}{(t-m_D^2)^2}\left(1+A\frac{\Lambda^2}{-t}\right)+B\frac{\Lambda^2}{\left(\sqrt{s}-M^{*2}\right)^2+\Lambda^2}\right\}$$

- Blue: maybe NP effect gives heavy quark a different form factor.
- Red: maybe NP effect allows resonances.

Eventually, they just contribute to diffusion coefficient,

$$\frac{\hat{q}}{T^3} \sim a\frac{\Lambda^2}{T^2} + b\frac{\Lambda^2}{ET}$$

Next:

- Settled down the pQCD component (I think this part is completed).
- Work on the Hybrid model right now. Few problems to solve
  - A discretization scheme for the Langevin equation in the presence of scatterings.
  - Reasonable parametizations that include mass dependence.