Rapidity dependent initial conditions for relativistic heavy-ion collisions

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1. Introduction
2. Useful concepts
3. TRENTo initial condition model
4. Extend TRENTo IC from 2d to 3d
5. Infer 3d-IC parameters by model-to-data comparison
6. Results

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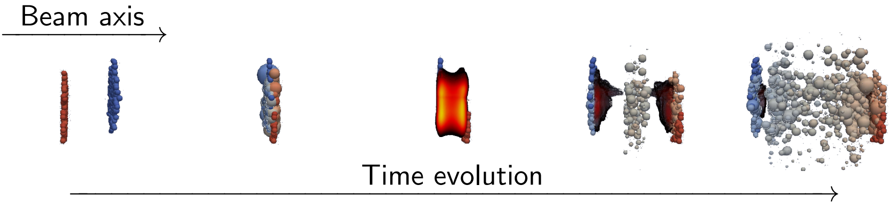
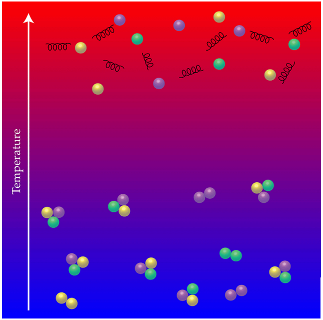
Ultra-relativistic heavy-ion collision

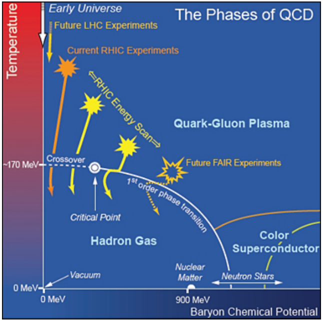


- Heavy nuclei (Au, Pb, U) are accelerated to nearly the speed of light and collide.
- Lorentz contraction in beam direction $\rightarrow$ pancake like nuclei.
- Extremely hot and dense system: $T > 300\ \mathrm{MeV} \sim 10^{12}\ \mathrm{K}$.
- Time scale $\sim 10^1\ \mathrm{fm/c}\ (10^{-22}\ \mathrm{s})$, system size $< 10\ \mathrm{fm}\ (10^{-14}\ \mathrm{m})$.

Nuclear matter in extreme conditions

- Fundamental theory: Quantum Chromodynamics (QCD), describes motion of quarks and gluons that carry color charges.
- High energy: asymptotic freedom, DoF quarks and gluons.
- Low energy: confinement, q, gs form bound states (hadrons).
- Phase transition? Hadrons $\xrightarrow{\text{high } T, \rho}$ quark-gluon plasma (QGP)





QGP at RHIC and LHC

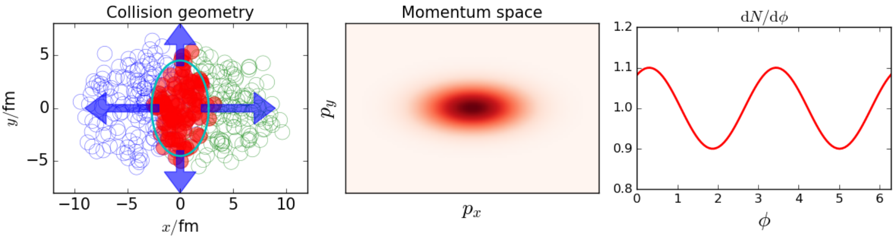
- Relativistic Heavy-ion Collider (RHIC) at Brookhaven National Lab.
  - $7.7 - 200A\ \mathrm{GeV}$.
  - Au+Au, U+U, Cu+Cu, p+Au, d+Au, ^{3}He+Au, Cu+Au ...
- Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN).
  - Increase the energy to TeV regime.
  - Pb+Pb ($2.76A, 5.02A$ TeV), p+Pb ($5.02A$ TeV).
- QGP is produced at both RHIC and LHC.

Astonishing properties of QGP: collective phenomena

- Anisotropic flow: azimuthal modulation of particle emission.

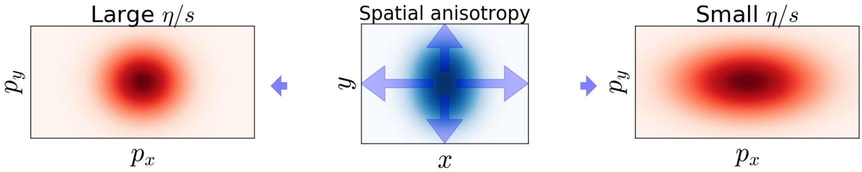
$$\frac{\mathrm{d}N}{p_T \mathrm{d}p_T \mathrm{d}\phi} \propto 1 + 2v_2 \cos(2(\phi - \Psi_2)) + 2v_3 \cos(3(\phi - \Psi_3)) + \ldots$$

- Fluid behaviour (collective flow):
  initial spatial anisotropy $\rightarrow$ final momentum anisotropy .



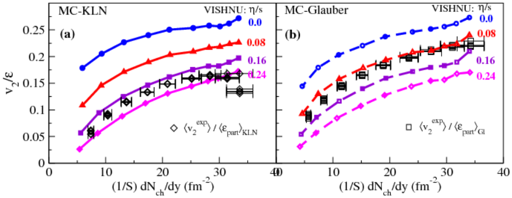
Astonishing properties of QGP: perfect liquid

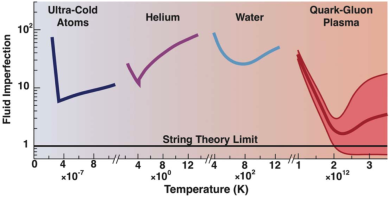
- System is well described by relativistic viscous hydrodynamics.
- Inputs: transport coefficients (shear viscosity over entropy ratio η/s), QCD equation of state.
- Viscous effect damps the development of momentum anisotropy.



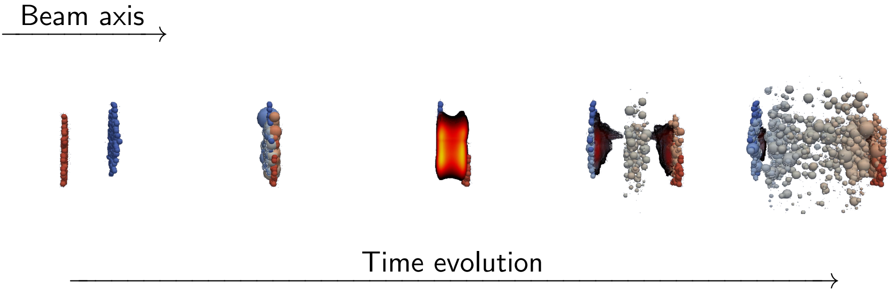
Astonishing properties of QGP: perfect liquid

- Extract η/s by comparing model calculation with experimental data.
- Extremely low $\eta/s \sim 0.08 - 0.20$, close to quantum lower bound of $\eta/s = \frac{1}{4\pi}$ $\rightarrow$ almost perfect liquid. (PRL 106, 192301 (2011))
- Large uncertainty from initial conditions.





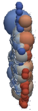
Heavy-ion collision modelling



Initial states → QGP → Hadronization → Final states interaction → Detector

- Little control of the initial condition; complication from hadronization and final state interaction.
- Need a modelling of full time evolution to extract QGP properties by model-to-data comparison.

Initial state (< 1 fm/c)



- Initially far from thermal equilibrium.
- Fast thermalization process towards local thermal equilibrium (< 1 fm/c).
- Mechanism not very well understood.

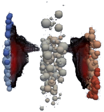
Dynamical models:
include pre-equilibrium dynamics.

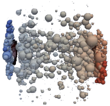
Parametric models:
phenonmenolgical models that provide entropy / energy density at later time ($\sim$ 0.6 fm/c) assuming local thermal equilibrium.

Hydrodynamics $\rightarrow$ particlization $\rightarrow$ hadronic scattering

- System is close to local thermal equilibrium.
- Solve energy momentum and charge conservation equations, $\partial_\nu T^{\mu\nu} = 0, \partial_\mu N^\mu = 0$.
- Closed by QCD equation of state $p = p(\epsilon, n)$.

- The expanding system becomes dilute and hydrodynamics ceases to apply $\rightarrow$ hadronic transport simulations.
- Particlize to hadrons on isothermal hyper-surface.





- System is still dense to allow hadronic scattering.
- Solve Boltzmann transport equation of hadrons.
- Ultra-relativistic Quantum Molecular Dynamics (UrQMD).

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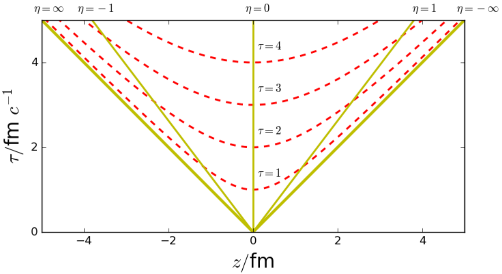
Kinematics for fast expansion system

Define proper time (τ) and space-time rapidity (η_s),

$$\tau = \sqrt{t^2 - z^2}, \eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}.$$

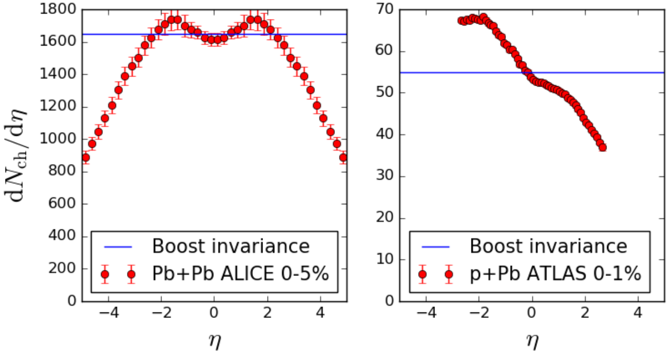
Final particle momenta, define rapidity (y) or pseudo-rapidity (η),

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}, \eta \;=\; \frac{1}{2}\ln\frac{|\mathbf{p}|+p_z}{|\mathbf{p}|-p_z} = -\ln\left(\tan\frac{\theta}{2}\right).$$



Boost-invariance approximation

- Boost-invariance approximation: system is independent of η_s.
- Reduces 3+1 d hydrodynamics to 2+1 d with 2d initial condition.
- A good approximation for large, symmetric collisions (Au+Au, Pb+Pb) within $-2 < \eta < 2$ (mid-rapidity).



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General structure of TRENTo IC

TRENTo (PRC 92 011901 (2015)) is a parametric model of initial condition assuming boost-invariance. Provide initial 2d entropy density $s(x_\perp, \eta_s = 0)$ at mid-rapidity.

TRENTo IC is developed in three layers:

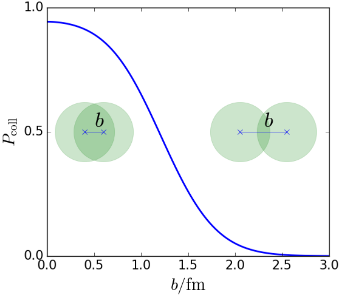
- Glauber calculation of inelastic nucleon-nucleon collision.
- Determine the densities of nuclei participant matter.
- Map participant densities to mid-rapidity entropy deposition.

Glauber calculation of nucleon-nucleon collision

- Collision probability with impact parameter b,

$$P_{\mathrm{coll}}(b) = \frac{\mathrm{d}\sigma_{NN}}{2\pi b \mathrm{d} b} = 1 - \exp\left(-\alpha T_{pp}(b)\right).$$

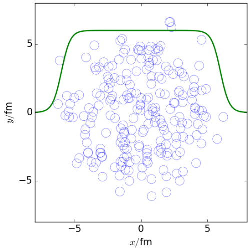
$$T_{pp} = \int \mathrm{d}x_\perp \rho\left(x_\perp + \frac{b}{2}\right) \rho\left(x_\perp - \frac{b}{2}\right).$$



- $T_{pp}(b)$: overlapping between two nucleons' density.
- Nucleon density is approximated by normal distribution.
- Parameter α is tuned to reproduce experimental cross-section $\sigma_{pp,\mathrm{inel}}(\sqrt{s})$.

Calculating participant density

- Sample nucleons from averaged nucleon distribution inside a nuclei.
- Check each pair of target/projectile nucleons for collision.
- Participants: nucleons that collide.
- Participant density of nuclei A and B,



$$T_{A,B}(x_\perp) = \sum_{i \in \mathrm{Part} A,B} \gamma_i \rho(x_\perp - x_i), \gamma_i \quad \sim \quad \Gamma(k,k)$$

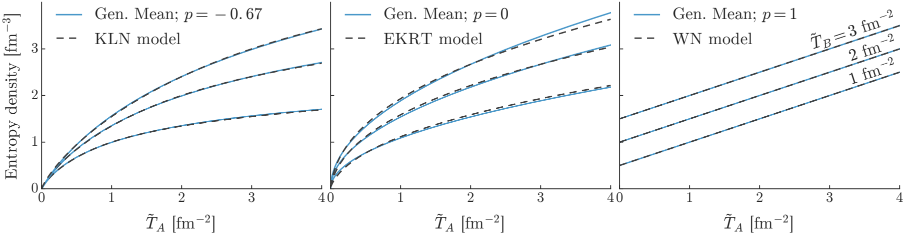
- Additional Γ fluctuation to correctly reproduce experimental pp multiplicity fluctuation.

2d entropy deposition ansatz at mid-rapidity

- Map participant densities T_A, T_B to mid-rapidity entropy deposition,

$$\mathrm{d}S/\mathrm{d}\eta_s(x_\perp, \eta_s = 0) \propto \left[\left(T_A^p(x_\perp) + T_B^p(x_\perp)\right)/2\right]^{\frac{1}{p}}.$$

- Mimic a variety of models at mid-rapidity (J. S. Moreland).
- A model-to-data comparison (J. E. Bernhard) suggests $p \approx 0$, $\rightarrow$ EKRT / IP-Glasma (successful models at mid-rapidity).



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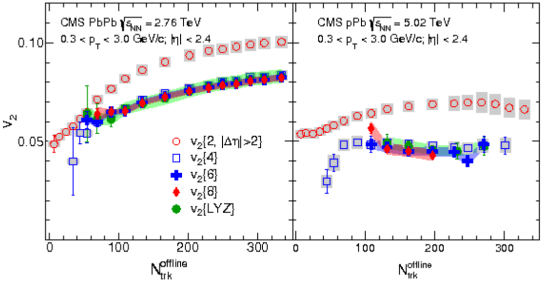
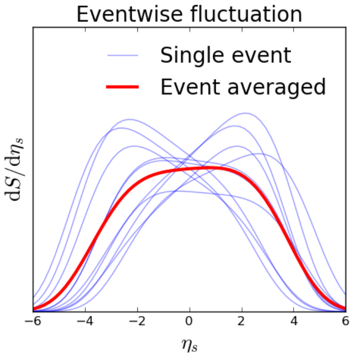
3d IC motivation: QGP in small systems?

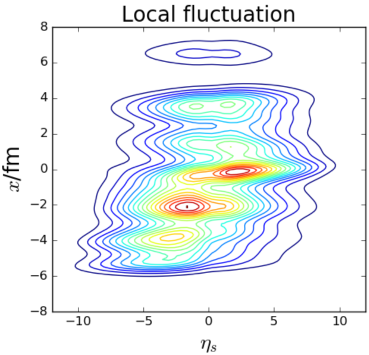
- QGP was not expected in small systems d+Au, p+Pb, ...; but exp. observed similar collective phenomena.
- QGP in small systems? Is hydrodynamics applicable?
- Boost-invariance is not a good approximation in asymmetry collision (d+Au, p+Pb). A rapidity dependent IC is demanding.



3d IC motivation: effect of longitudinal fluctuation

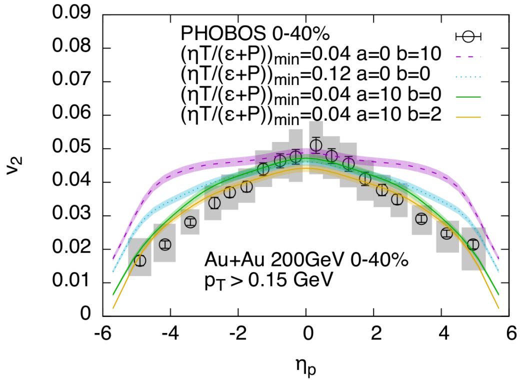
- Even in large systems, boost-invariance only works for event-averaged distributions.
- A single event contains local longitudinal fluctuations.
- Study fluctuation effects on the extraction of QGP properties.

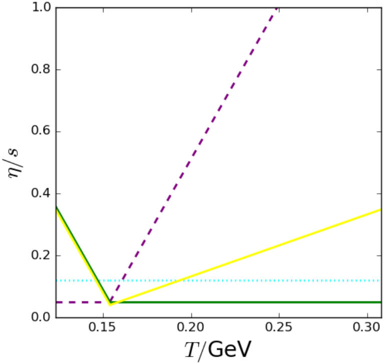




3d IC motivation: rapidity dependent observables

- η dependent observables could be sensitive to QGP properties and IC.
- $v_2(\eta)$ can be sensitive to temperature dependence of η/s (arXiv:1512.08231).





From 2d to 3d: formulae

TRENTo 3d:

- Determine 3d entropy density from participant density.

$$T_A(x_\perp), T_B(x_\perp) \quad \rightarrow \quad s(x_\perp, \eta)$$

- Simple and flexible $\rightarrow$ suitable for parametrize a variety of models.
- Preserve TRENTo 2d calculation at mid-rapidity.

From 2d to 3d: formulae

- Factorize $\perp$ and $\parallel$ to reproduce mid-rapidity TRENTo result.

$$s(x_\perp, \eta) = s(x_\perp, \eta = 0) f(x_\perp, \eta), f(x_\perp, 0) = 1.$$

- For flexibility, no use of explicit forms of f.
  $f(x_\perp, \eta)$ characterized by its first few cumulants,

mean	std	skewness	kurtosis
$\mu(x_\perp)$	$\sigma(x_\perp)$	$\gamma(x_\perp)$	$\kappa(x_\perp)$

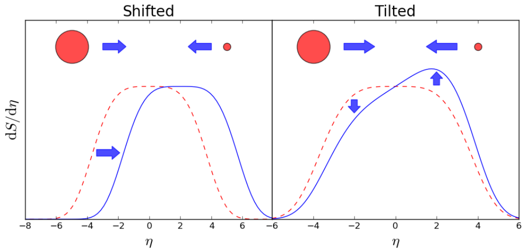
- Reconstruct $f(\eta)$ by $\mathscr{F}^{-1}$ cumulant generating function:

$$\mathscr{F}^{-1} \exp\left(i\mu k - \frac{\sigma^2}{2} k^2 + i\gamma k^3 - \kappa k^4\right)$$

Encoding models

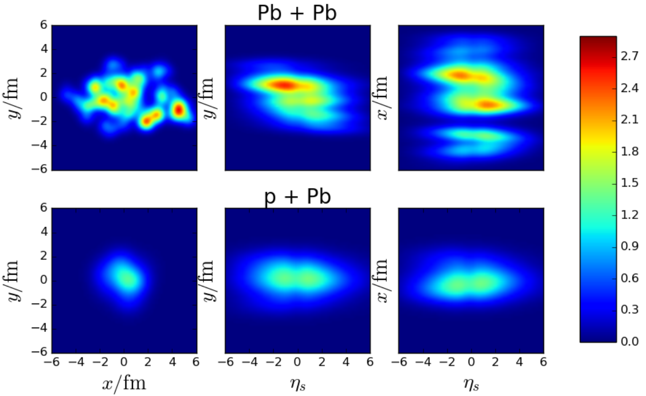
- Existing parametrizations (Piotr Bozek, arXiv:1002.4999), Shifted: $f(\eta_s) \to f(\eta_s - \mu)$; tilted: $f(\eta_s) \to f(\eta_s)\,(1 + \gamma\eta_s / y_{\text{beam}})$.
- μ and γ measures the local asymmetry of participant densities.

	μ	σ	γ	κ
shifted	$\frac{1}{2}\log(T_A/T_B)$	const.	0	const.
tilted	0	const.	$\frac{T_A - T_B}{T_A + T_B}$, $T_A - T_B, \ldots$	const.
general	$\frac{a}{2}\log(T_A/T_B)$	b	$c(T_A - T_B)$	d



Sample 3d events for Pb+Pb and p+Pb

- Fluctuations of entropy deposition in X-Y plane comes from randomized nucleon position.
- Strong local longitudinal fluctuations in Pb+Pb collisions.
- Asymmetric $\mathrm{d}S/\mathrm{d}\eta_s$ for pPb collisions.



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IC directly against experiment

- Time evolution (3+1 D hydrodynamics) is computationally expensive.
- First, compare 3d initial condition model to observables that are not sensitive to time-evolution $\rightarrow$ crude inference of IC parameters.
- Charged particle pseudo-rapidity distribution works well,

$$\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta \propto \mathrm{d}S/\mathrm{d}\eta_s$$

The validity of this approximation can be checked later.

Tools: model-to-data comparison by Bayesian analysis.

Parameter design: Latin-hypercube sampling

- Design parameter set samples from a high dimensional parameter space:

fluctuation	norm	mean	std	skewness	kurtosis	Jacobi
k	N	a	b	c	d	J
$[1, 4]$	$[8, 12]$	$[0, 2]$	$[2, 4]$	$[0, 0.5]$	$[0.2, 0.8]$	$[0.6, 0.8]$

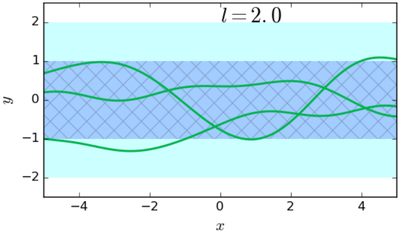
- Latin-hypercube sampling:
  Avoid over populated / sparse regions, $N_{\mathrm{samples}} \propto d$.
- Compute model output y_i on design parameter sets x_i,

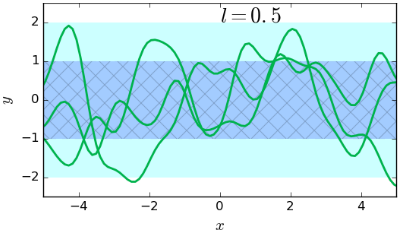
$$
\begin{array}{ccc}
x_i & \xrightarrow{\text{Model}} & y_i, \\
(k, N, a, b, c, d, J) & \xrightarrow{\text{TRENTo-3d}} & (\mathrm{d}N/\mathrm{d}\eta_1, \mathrm{d}N/\mathrm{d}\eta_2, ..).
\end{array}
$$

Gaussian process emulators: interpolate over design points

- Gaussian process (GP): a set of random variables y, labelled by x. Any finite number $y(x)$ have a joint Gaussian distribution.
- GP draws random functions by specifying mean $(= 0)$ and covariance,

$$\langle y(x_1)y(x_2)\rangle = \Sigma(x_1, x_2) = \sigma^2 \exp\left(-\frac{(x_1 - x_2)^2}{2l^2}\right)$$



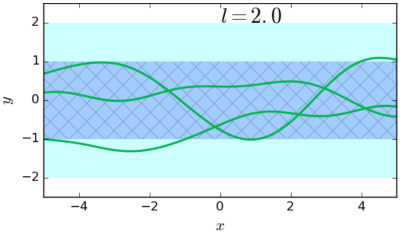


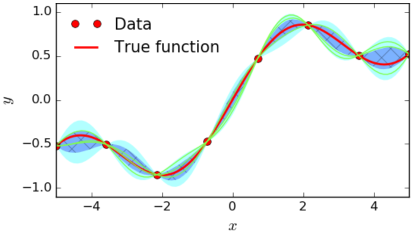
Gaussian process emulators: interpolate over design points

- Gaussian process emulator: a GP conditioned at parameter sets x_i and model outputs y_i.

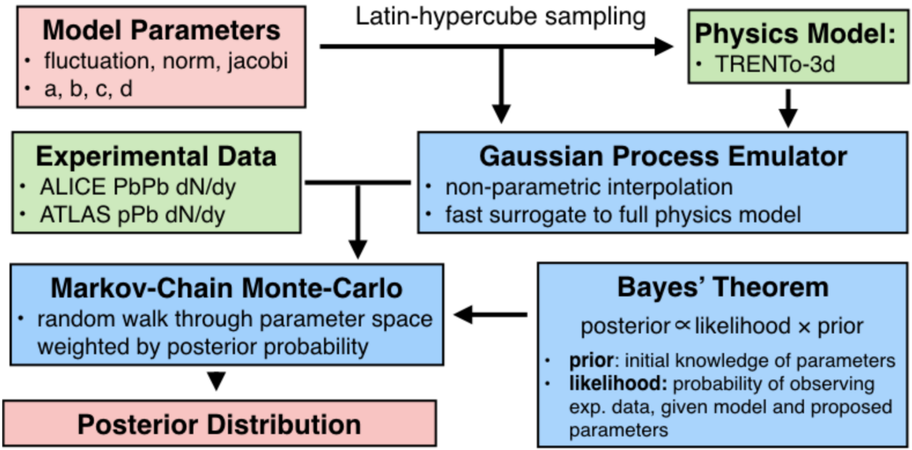
$$\Sigma'(x^*, x^*) = \Sigma(x^*, x^*) - \Sigma(x^*, x_i)\Sigma^{-1}(x_i, x_j)\Sigma(x_j, x^*)$$

- Infer model output with arbitrary parameter sets (x^*) with likelihood.
- Non-parametric interpolation over design points.





Model-to-data comparison flow chart



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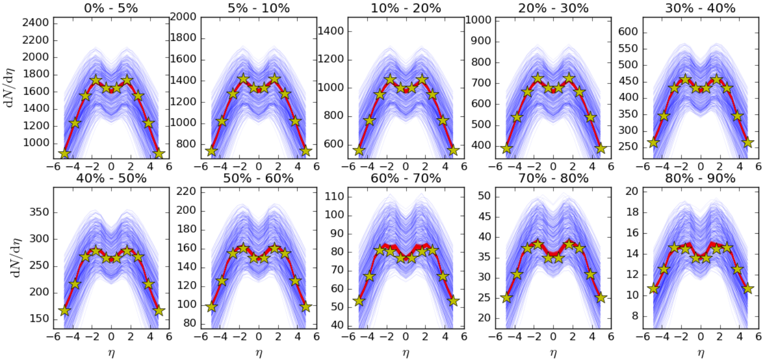
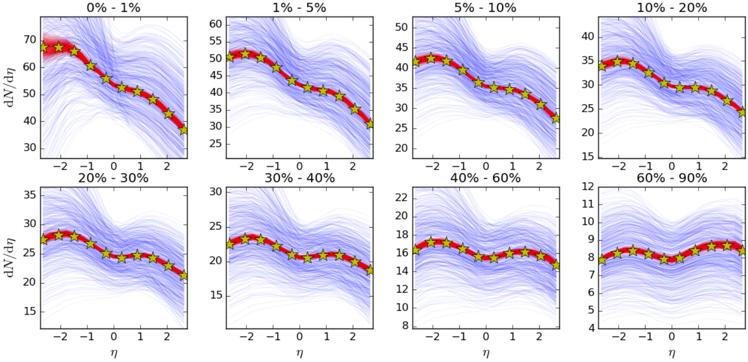
Prior / posterior of model output

- $\mathrm{d}N/\mathrm{d}\eta$ from parameters drawn from priori / posterior distributions, comparing to Pb+Pb data @ 2.76A TeV.
- The model describes Pb+Pb data of different centralities.

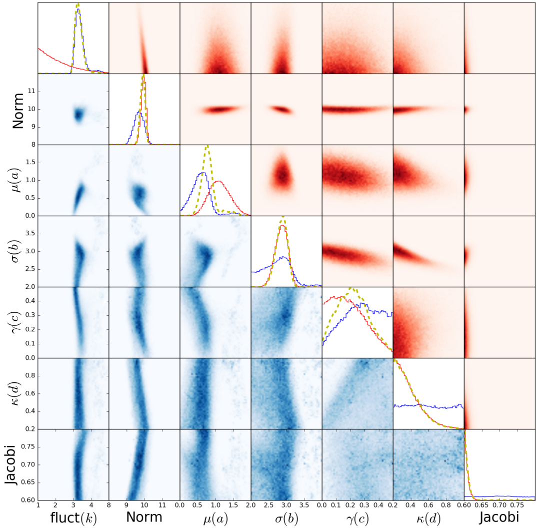


Prior / posterior of model output

- $dN/d\eta$ from parameters drawn from priori / posterior distributions, comparing to p+Pb data @ 5.02A TeV.
- The model reproduces the asymmetry distribution of different centralities in small system.



Marginalized posterior distribution of model parameters



- Red: PbPb data.
- Blue: pPb data.
- Diagonal: $P(x_i)$.
- Off-diagonal: $P(x_i, x_j)$.

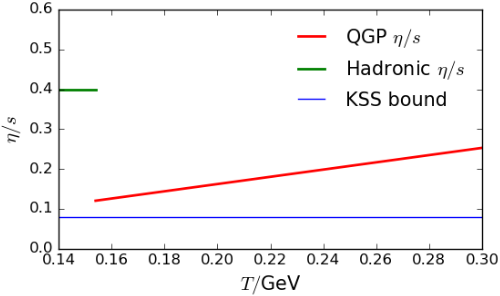
Observations:

- pPb sensitive to k.
- PbPb sensitive to σ.
- γ, μ correlation.
- Favours $a \sim 0.5 - 1$,

$$\mu \sim (0.5 - 1)\mu_{\text{com}}(x_\perp).$$

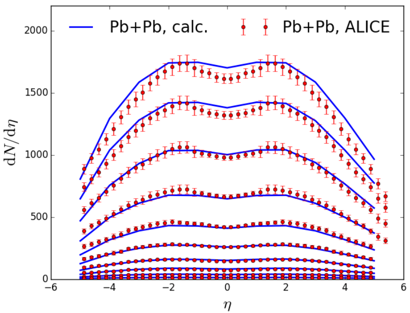
Effect of hydrodynamic evolution and hadronic scattering

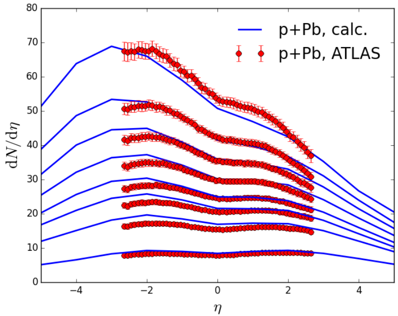
- Parameter sets around the maximum of posterior distribution $k = 3.0, a = 0.75, b = 2.8, c = 0.3, d = 0.2, J = 0.65.$
  $\rightarrow$ perform full-time evolution.
- Pseudo-critical temperature of EoS, $T_c = 0.154$ GeV.
- Switching temperature of particularization, $T_{\mathrm{sw}} = 0.148$ GeV.
- Transport coefficients $\zeta/s = 0$, $\eta/s(T)$.



Charged particle distribution

- The general agreement of $\mathrm{d}N_{ch}/\mathrm{d}\eta$ with experiment is robust against evolution.
- Validate the use of $\mathrm{d}N_{ch}/\mathrm{d}\eta$ for model-to-data comparison for initial condition.

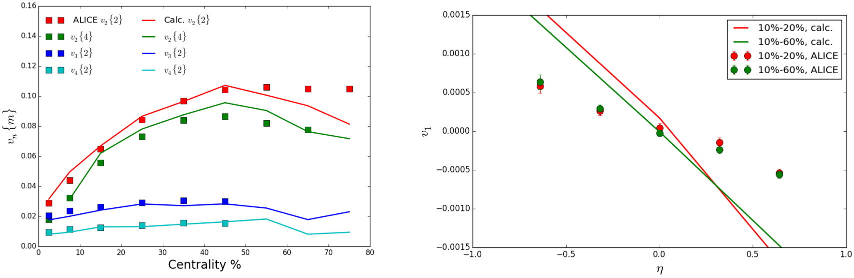




Summary and outlook

- Relativistic heavy-ion collision produces quark-gluon plasma (QGP).
- QGP properties are inferred by model-to-data comparison.
- Interest of rapidity dependent observables and small systems necessitates a rapidity dependent initial condition.
- Extend TRENTo initial condition from 2d to 3d using a general formula.
- Model-to-data comparison suggests a longitudinal distribution around the CoM rapidity.
- Enlarged data set in future model-to-data comparison $\rightarrow$ improved knowledge on both IC and QGP properties.

Back up: flow observables from 3d IC + time evolution



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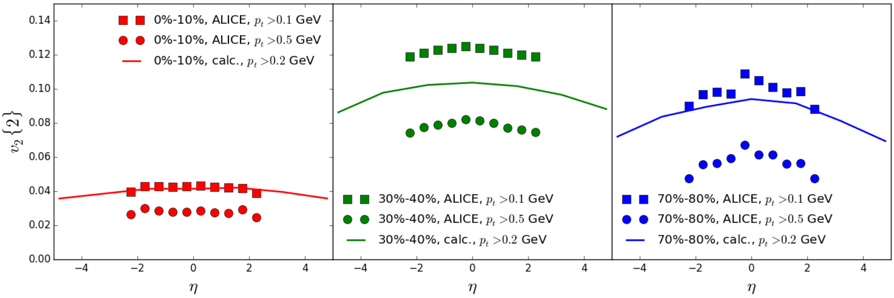
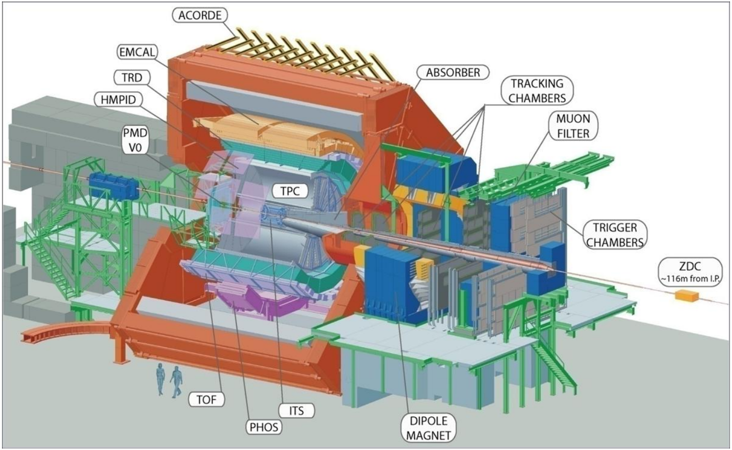


Back up: ALICE experiments

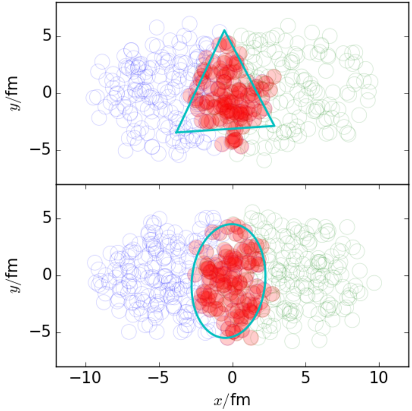


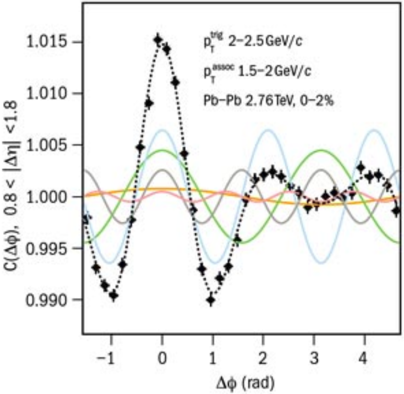
Back up: Evidence of QGP

- Elliptic flow.
- Jet quenching.
- Strangeness production.
- ...

Back up: higher order flow

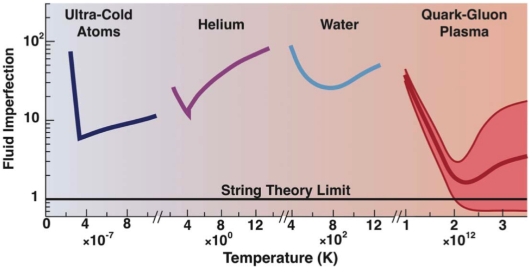
- Fluctuation of initial geometry of the collision leads to higher order anisotropic flows.





Back up: η/s for different fluid

- Extract η/s by comparing model calculation with experimental data.
- Extremely low $\eta/s \sim 0.08 - 0.20$, close to quantum lower bound of $\eta/s = \frac{1}{4\pi}$ $\rightarrow$ almost perfect liquid.
- Large uncertainty from initial conditions.



Back up: QCD, asymptotic freedom

- QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i \left(\gamma^\mu D_\mu \right)_{ij} - m\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}.$$

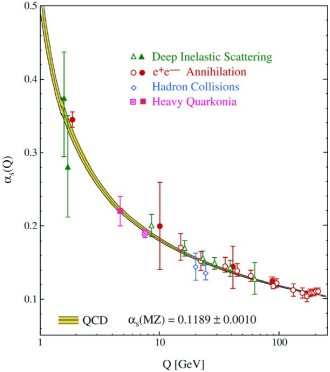
- ψ quark field.
- $G^a_{\mu\nu}$ gluon field tensor,

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu.$$

- D_μ covariant derivative,

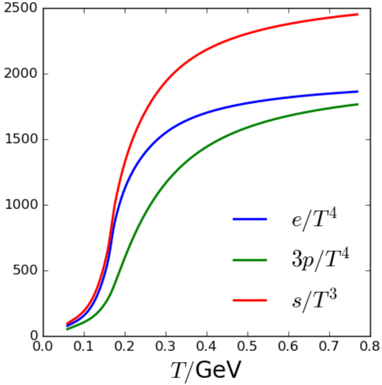
$$\left(D_\mu \right)_{ij} = \partial_\mu \delta_{ij} - i g A^a_\mu t^a_{ij}.$$

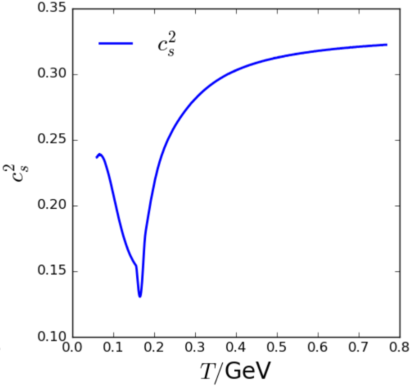
- g coupling constant.



Back up: QCD EoS

- QCD equation of state at zero n_B (HotQCD collaboration).





Back up: 3+1 D relativistic viscous hydrodynamics - 1

- Conserved currents,

$$\partial_\nu T^{\mu\nu} = 0, \partial_\nu N^\nu = 0.$$

- Define flow velocity u^μ, energy density ϵ of local-rest (Laudau) frame,

$$\epsilon u^\mu = T^\mu_\nu u^\nu.$$

- Decomposition of T^μ_ν and N^ν into equilibrium quantities (ϵ, p, n) and deviations from local equilibrium $\pi^{\mu\nu}, \Pi, V^\mu$

$$\begin{aligned} T^\mu_\nu &= \epsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \\ N^\mu &= n u^\mu + V^\mu. \end{aligned}$$

- Closed by EoS $p = p(\epsilon, n)$ from other calculation (Lattice QCD / phenomenology models).

Back up: 3+1 D relativistic viscous hydrodynamics - 2

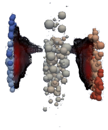
- Israel-Stewart relativistic hydrodynamics: shear stress tensor and bulk pressure are dynamical quantities (Iu. Karpenko, doi:10.1016/j.cpc.2014.07.010),

$$
\begin{aligned}
\langle u^\gamma \partial_{;\gamma} \pi^{\mu\nu} \rangle &= -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_{;\gamma} u^\gamma, \\
u^\gamma \partial_{;\gamma} \Pi &= -\frac{\Pi - \Pi_{NS}}{\tau_\Pi} - \frac{4}{3}\Pi \partial_{;\gamma} u^\gamma.
\end{aligned}
$$

- Relax to Naiver-Stokes case,

$$
\begin{aligned}
\pi^{\mu\nu}_{NS} &= \eta(\Delta^{\mu\lambda}\partial_{;\lambda} u^\nu + \Delta^{\nu\lambda}\partial_{;\lambda} u^\mu) - \frac{2}{3}\eta \Delta^{\mu\nu}\partial_{;\lambda} u^\lambda, \\
\Pi_{NS} &= -\zeta \partial_{;\lambda} u^\lambda.
\end{aligned}
$$

Back up: isothermal hyper-surface

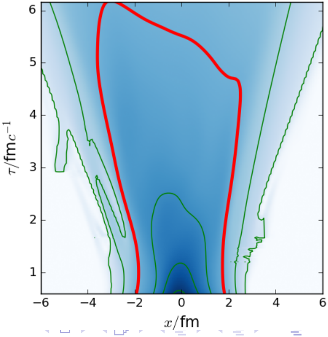


- The expanding system becomes dilute and hydrodynamics ceases to apply.
- Particlize energy density to hadrons.

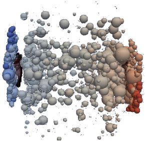
- Particles are sampled from a isothermal hyper-surface:

$$E\frac{\mathrm{d}N_i}{\mathrm{d}^3p} = \int_\Sigma f_i(x,p)p^\mu \mathrm{d}^3\sigma_\mu. \qquad (1)$$

- f_i is the distribution function of particle type "i" and $\mathrm{d}\sigma_\mu$ is the surface element.



Back up: hadronic re-scattering → calculation meets experiment

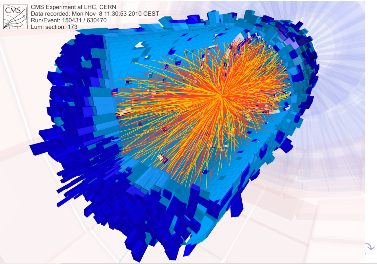


- System is still dense to allow hadronic scattering.
- Solving Boltzmann transport equation,

$$\frac{\mathrm{d}}{\mathrm{d}t} f_i(x, p) = C_i(x, p). \tag{2}$$

RHS: gain and loss of particles due to collisions.

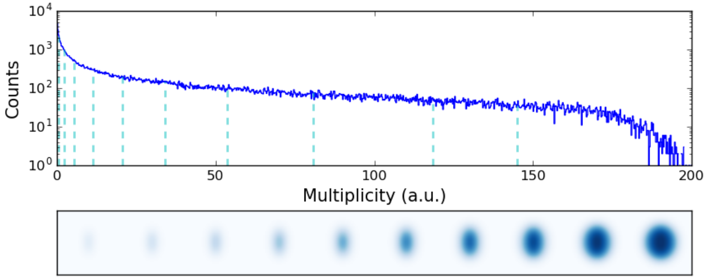
- Implementation: Ultra-relativistic Quantum Molecular Dynamics (UrQMD).
- Extract experimental observables from hadrons final states.



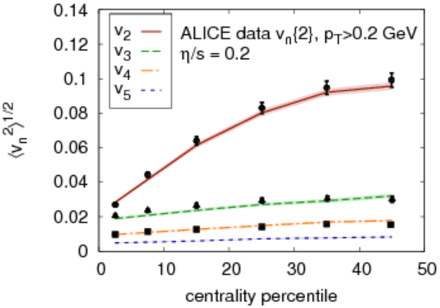
Back up: centrality class and collision geometry

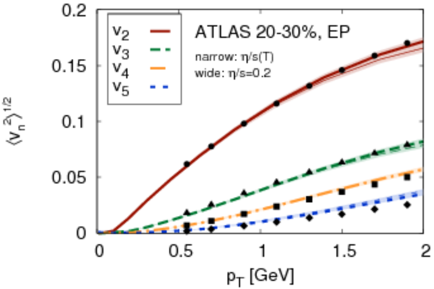
- Interested in initial spatial anisotropy (eccentricity).
- Experimental no control over initial eccentricity.

Impact parameter	Overlap	Multiplicity	Eccentricity
Large	Small	Small	Large
Small	Large	Large	Small



Back up: more on flow observables





Back up: KLN IC model

- Saturation based IC model (PRC 71 054903 (2005)).
- Entropy – Gluon multiplicity per unity rapidity:

$$\frac{\mathrm{d}N}{\mathrm{d}y} = \frac{K}{S}\frac{4\pi N_c \alpha_s}{N_c^2 - 1}\int_0^\infty \frac{\mathrm{d}p_t^2}{p_t^4} x_2 G_{A_2}(x_2, p_t^2) x_1 G_{A_1}(x_1, p_t^2).$$

- Unintegrated gluon distribution with saturation phenomenon:

$$xG(x, p_t^2) = \begin{cases} \frac{S}{\alpha_s(Q_s)} p_t^2 (1-x)^4, & p_t < Q_s \\ \frac{S}{\alpha_s(Q_s)} Q_s^2 (1-x)^4, & p_t > Q_s \end{cases}$$

- Local saturation scale,

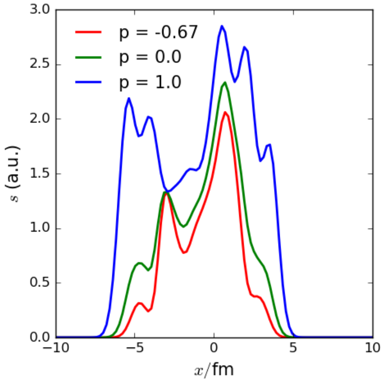
$$Q_s^2(x_\perp, y) \propto T_A(x_\perp)\exp(\pm\bar{\lambda} y)$$

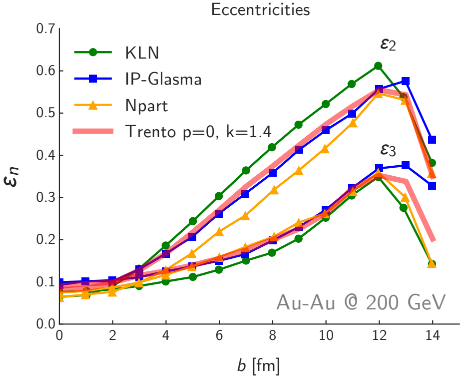
- Simplified to,

$$\frac{\mathrm{d}S}{\mathrm{d}y} \propto \frac{1}{\alpha_s(Q_s)}\left(2 + \ln\frac{Q_{\mathrm{max}}}{Q_{\mathrm{min}}}\right).$$

Back up: uncertainty in IC affects transport properties extraction

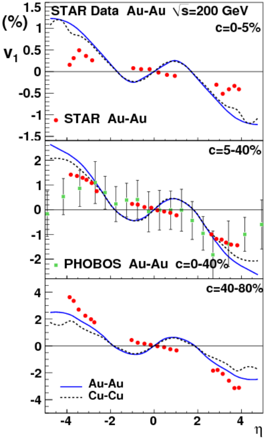
- Varying $p \to$ continuously interpolates existing models.

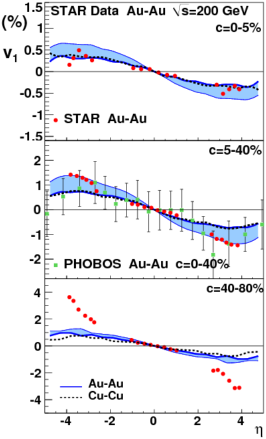




Back up: more rapidity dependent observables

- Directed flow: deflection of produced matter away from beam axis.
- Directed flow could be sensitive to EoS and IC.

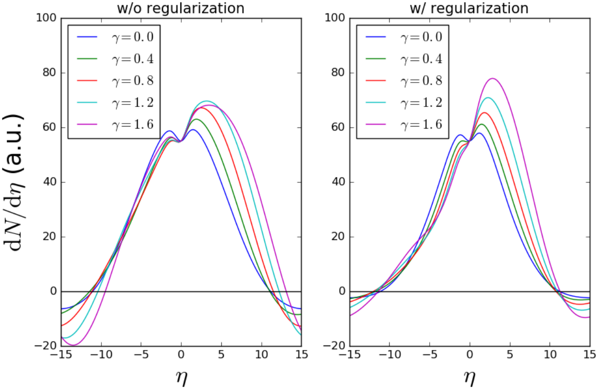




Back up: regulating cumulants generating function

- The reconstructed function may not be well-behaved.
- To regulate (1) and result in well behaved function,

$$\gamma \to \gamma \exp\left(-\sigma^2 k^2/2\right), \kappa \to \kappa \exp\left(-\sigma^2 k^2/2\right).$$



Back up: Bayes' theorem

- Posterior probability distribution of parameters by Bayes' theorem.

$$P(x^*|\{\mathbf{x}_i, y_i\}, y_{\mathrm{exp}}) \propto P(y_{\mathrm{exp}}|\{\mathbf{x}_i, y_i\}, x^*)P(x^*).$$

  Prior: $P(x^*)$, posterior $P(x^*|\{\mathbf{x}_i, y_i\}, y_{\mathrm{exp}})$.
- Study this high posterior dimensional distribution function by Markov-chain Monte-Carlo procedure.

Back up: high dimensional data reduction

- Model outputs are high dimensional vector.
- Redundant and impractical to emulate each component.
- Principal Component Analysis (PCA): reduce high dimensional output to a few principal components that covers most of the data variance.
- Model output is recovered by transforming back the truncated principal components.

Back up: model-to-data comparison details

- A 100 sets of parameters are sampled within reasonable ranges.
- 1000 events are generated for PbPb and pPb systems.
- $\mathrm{d}S/\mathrm{d}\eta_s$ calculated for each centrality case; compared to $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$
- Perform PCA on $\mathrm{d}S/\mathrm{d}\eta_s$, and take the first 5 principal components.
- Generate 10^6 samples from posterior distribution using MCMC.