Computational Physics Group Project: Ecosystem: predator and prey

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Outline

Introduction to eco-system modeling

Simulation and Implementation

Results and discussion

What are Predator Prey models and where are they used?

Systems involving competitive interaction of two "species" are some form of predator prey systems.

They deal with the general loss-win interactions and hence may have applications outside of ecosystems.

Population interaction of predator and prey in eco-system

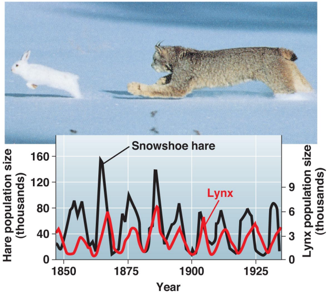

Figure:
http://www.anselm.edu/homepage/jpitocch/genbi101/ecology1intropops.html

A simplified deterministic mode: L-V equation

The dynamics of biological systems consist of one predator and one prey can be described by Lotka-Volterra (LV) equations:

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy = x(\alpha - \beta y) \\ \frac{dy}{dt} &= -\gamma y + \delta xy = -y(\gamma - \delta x) \end{aligned}$$

Where, x is the number of prey, y is the number of predator, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ represent the growth rates of two populations, and α, β, γ and δ are parameters describing the interaction of two species.

A simplified deterministic mode: L-V equation

When the biological system has reached eco-equilibrium, the number of predator and prey are supposed to be either situation below.

- 1. $x = 0, y = 0$: Both species extinct.
- 2. $x = \frac{\gamma}{\delta}, y = \frac{\alpha}{\beta}$: Predator and Prey reach a periodic stable situation. The number of animals evolve in a sinusoidal way.

Disadvantage: number of species, limit of interaction.
Advanced model: competitive L-V equation for trophic interaction; generalized L-V equation for multiple species.

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A simulation keep the essential nature of the interaction between and within the species, and predict the evolution of population step by step.

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- Predator and prey will die out if maximum age is reached or starved for enough long time
- However, simulation is a random process and change the deterministic nature of LV equation (more realistic).

Structural setup

Animal Class $\rightarrow$ Deer/ Wolf

- variables
  - presentation position
  - previous position
  - reproducing age variable
  - starving age variable
- constants
  - starvation age
  - reproduction age
- functions
  - check status: live/dead
  - check maturity: procreate/not

Structural setup of the code

Eco-system
- variables
  - a list of deer
  - a list of wolves
  - occupation matrix (0, 1, 2) $\rightarrow$ (vacant, deer, woof)
  - system time
- constants
  - Initialisation parameters: world size, starvation ages
- functions
  - initialisation
  - time evolution

Initialisation

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- Reproduction age of predators must be larger than their starvation age. (Or else wolf can sustain themselves ...)
- Starvation age of the deer is extremely large. (Always enough plants!)
- A realistic population always have some age structures, so we use a uniform initial age distribution for the animals.

Evolution of Wolves

We set up a $N \times N$ grid and simulate the eco-system with L-V equation.

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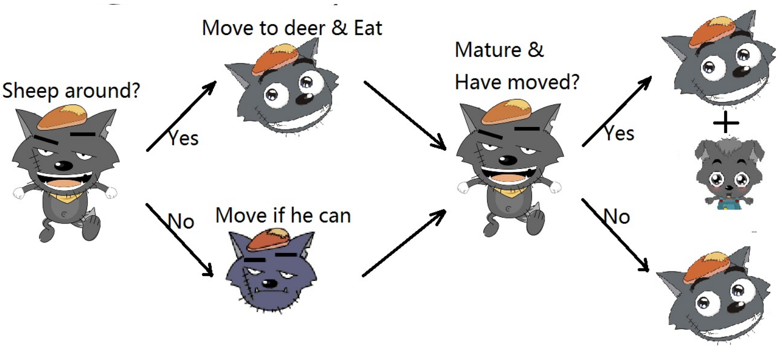
- Step 2: evolution of wolves:



Evolution of deer

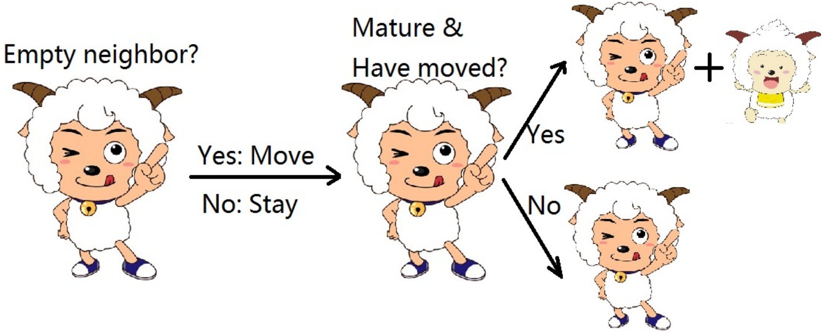
Evolution of deers:

- Step 1: Delete all unfortunate deers.

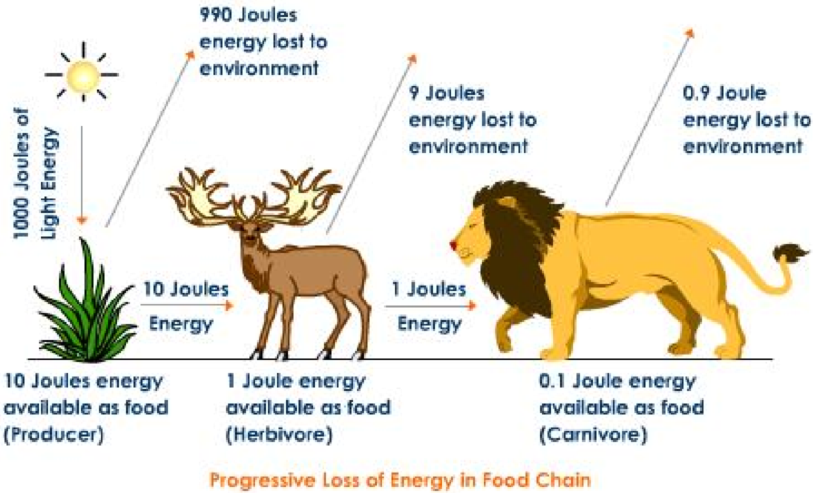
Evolution of deer

Evolution of deers:

- Step 1: Delete all unfortunate deers.
- Setp 2: Evolution of live deers.



Population interaction of predator and prey in eco-system



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Generic results

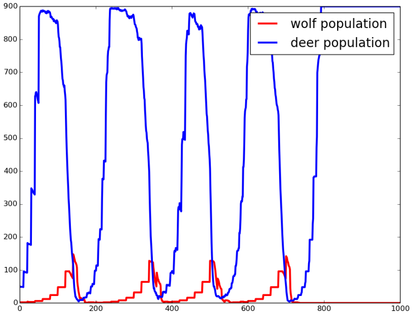


Figure: Initialised without age structure. This is an example of wolf distinction.

Generic results

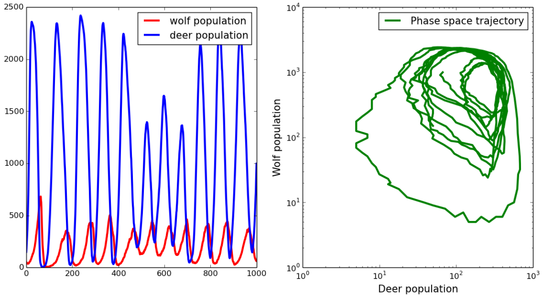


Figure: Initialised with uniform age structure. A quasi-periodic evolution is obtained.

Parameter Search

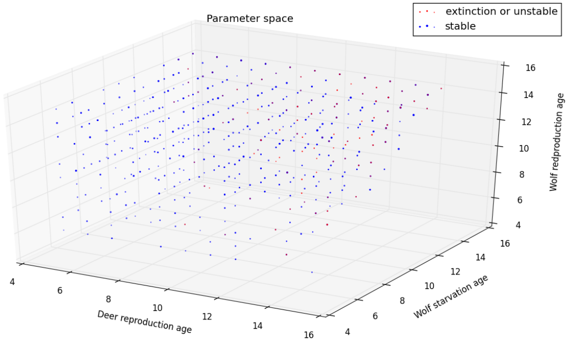
5 parameters to test (5-D parameter space)

- **Initial population of deer**
- **Initial population of wolves**
- Reproduction age of deer
- Reproduction age of wolf
- Starvation "age" of wolf

Reduce to 4 dimensions (4-D)

- **Ratio of initial populations : Size of point**
- Reproduction age of deer : x-axis
- Reproduction age of wolf : y-axis
- Starvation "age" of wolf : z-axis

Results of Full Parameter Search



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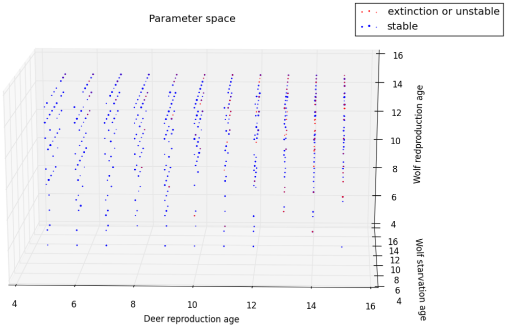

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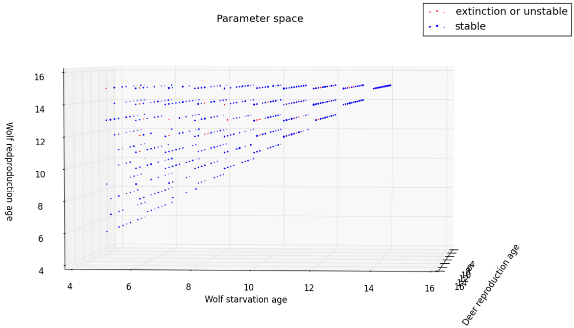

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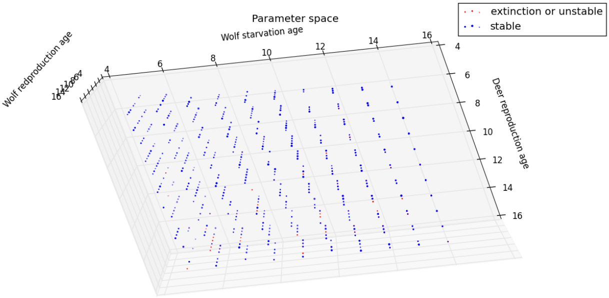

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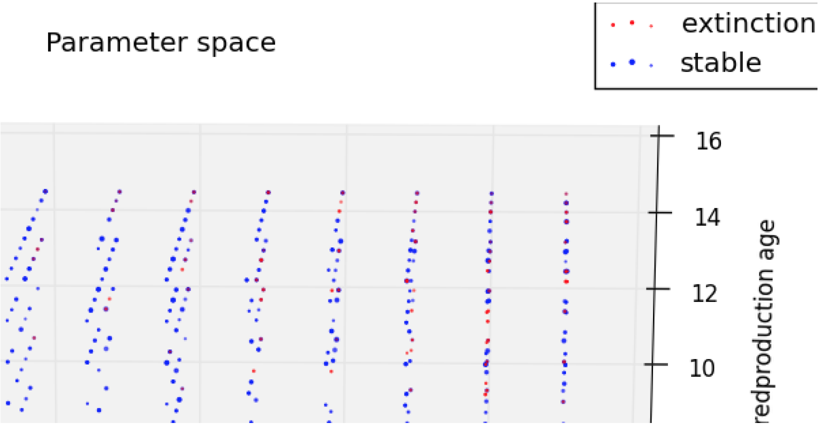


Results of Full Parameter Search

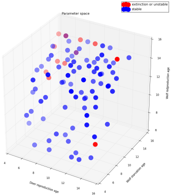
The following is an excerpt from the paper, "A. K. Dewdney, Computer Recreations" :

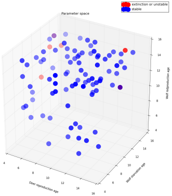
It makes perfectly good sense, however, to compile statistics on the sharks and fish of Wa-Tor, and Magi and I have done so. Our recent graphs of the shark and fish populations tend to look more like the lynx-hare charts than the Lotke-Volterra solutions do. Still, we continue to be puzzled by the long-term instability shown by certain parameter combinations. Perhaps some reader, working with his or her own WATOR program, will provide further insight. Is there some kind of general rule we might use to predict, for a given combination of parameters, whether the resulting ecology will be stable? To what extent do the cyclic fluctuations follow the Lotke-Volterra equations?

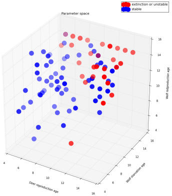
The ocean of Wa-Tor is toroidal for a very simple reason: it is much easier to

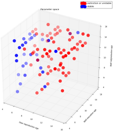
Results of Restricted Parameter Search

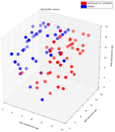
Fix initial population ratios



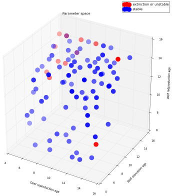




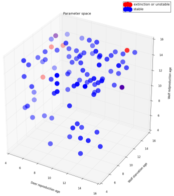




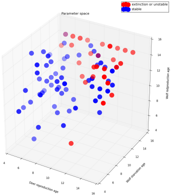
TEST



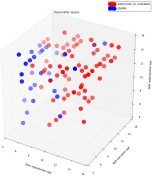
Deer: 2500; Wolves: 250



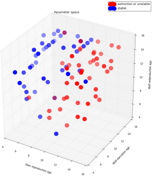
Deer: 3000; Wolves: 500



Deer: 2000; Wolves: 2000



Deer: 500; Wolves: 3000



Deer: 250; Wolves: 2500

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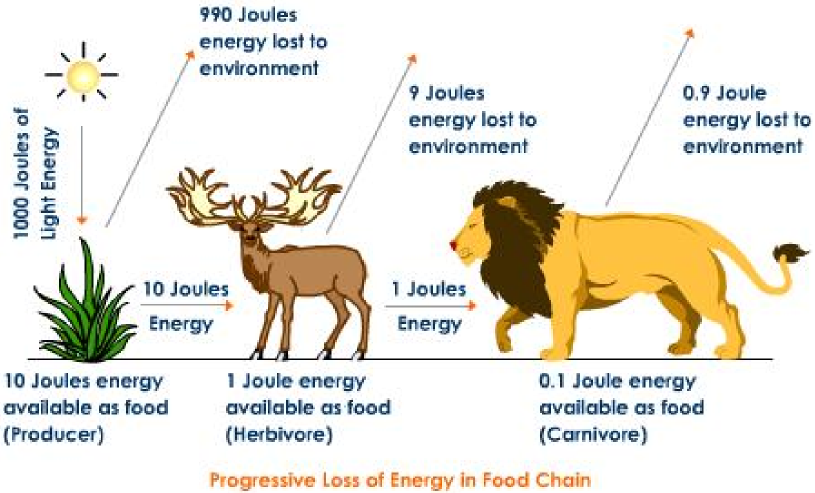

Figure: default

Ecosystem at Equilibrium

Parameters used:

- Initial number of deer: 2,500
- Initial number of wolves: 250
- Deer reproduction rate: 5
- Wolf reproduction rate: 14
- Wolf starvation rate: 11

Animation Time!