Technische Universität Dresden
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Fractional Brownian Motion and its Application in Financial Mathematics

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vorgelegt von

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2 Gaussian Process and Brownian Motion

In this section we start off from the general concept of probability spaces and stochastic processes. Of this, a most important case we then discribe, is Gaussian process. It bring us to introduce the Brownian Motion as a fine example.

2.1 Definition of Probability Space and Stochastic Process

DEFINITION 2.1. Let $\mathscr{A}$ be a collection of subsets of a set Ω. $\mathscr{A}$ is then a σ- *Algebra* on Ω if it satisfies the following conditions:

(i) $\Omega \in \mathscr{A}$.

(ii) For any set $F \in \mathscr{A}$, its complement $F^c \in \mathscr{A}$.

(iii) If a serie $\{F_n\}_{n\in\mathbb{N}} \subseteq \mathscr{A}$, then $\cup_{n\in\mathbb{N}} F_n \in \mathscr{A}$.

DEFINITION 2.2. A mapping $\mathcal{P}$ is said to be a *probability measure* from $\mathscr{A}$ to $\mathscr{B}(\mathbb{R}^n)$, if $\mathcal{P}\left[\sum_{n=1}^{\infty} F_n\right] = \sum_{n=1}^{\infty} \mathcal{P}[F_n]$ for any $\{F_n\}_{n\in\mathbb{N}}$ disjoint in $\mathscr{A}$ satisfying $\sum_{n=1}^{\infty} F_n \in \mathscr{A}$.

DEFINITION 2.3. A *probability space* is defined as a triple $(\Omega, \mathscr{A}, \mathcal{P})$ of a set Ω, a σ-Algebra $\mathscr{A}$ of Ω and a measure $\mathcal{P}$ from $\mathscr{A}$ to $\mathscr{B}(\mathbb{R}^n)$.

The σ- Algebra generated of all open sets on $\mathbb{R}^n$ is called the *Borel σ- Algebra* which we denote as usual by $\mathscr{B}(\mathbb{R}^n)$. Let μ be a probability measure on $\mathbb{R}^n$. Indeed, $(\mathbb{R}^n, \mathscr{B}(\mathbb{R}^n), \mu)$ can define a probability space on $\mathbb{R}^n$. A function f mapping from $(\mathcal{D}, \mathscr{D}, \mu)$ into $(\mathcal{E}, \mathscr{E}, \nu)$ is *measurable* if its collection of the inverse image of $\mathscr{E}$ is a subset of $\mathscr{D}$. A *random variable* is a real-valued measurable function on some probability space. Let $\mathcal{P}$ represent a probability measure, recall that in probability theory, for $B \in \mathscr{B}(\mathbb{R}^n)$ we call $\mathcal{P}[\{X \in B\}]$ the *distribution* of X.

DEFINITION 2.4. Let $(\Omega, \mathscr{A}, \mathcal{P})$ be a probability space. A n-dimensional *stochastic process* (X_t) is a family of random variable such that $X_t(\omega) : \Omega \longrightarrow \mathbb{R}^n, \forall t \in T$, where T denotes the set of Index of Time.

DEFINITION 2.5. A stochastic process $(X_t)_{t\in T}$ is said to be *stationary*, if

$$\mathcal{P}[X_t] = \mathcal{P}[X_{t+s}]$$

for any $t + s \in T$.

2.2 Gaussian Process

3 Fractional Brownian Motion

4 Fractional Ornstein Uhlenbeck Process Model

5 Applications in Financial Mathematics

6 Conclusion

References

[1]