The Cylindrical Luneburg Lens Discretization Influence on its Radiation Parameters

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Abstract—**The cylindrical Luneburg lens under analysis has a radially inhomogeneous layered structure. The influence of the number of layers on lens radiation parameters is investigated. Four types of lenses discretization are considered. For each type antenna directivity is analyzed. Radiation patterns of three-layered Luneburg lens and 3-wavelength outer radius are shown. To calculate radiation patterns and antenna gain a model based on cylindrical Green's functions for layered inhomogeneous cylindrical structures was used. A model of equivalent radial lines, in which voltages and currents are associated with the spectral components of the electromagnetic field, is used for the analysis. In addition, for the solution transfer and boundary matrices are used and boundary conditions are modeled by end loads. The optimal method of cylindrical Luneburg lens layer's permittivity and thickness choice is suggested.**

Keywords— Luneburg lens, multi-layered cylinder, lens discretization, radiation pattern, the Green's function, equivalent radial lines method, transfer and boundary matrices, boundary conditions.

I. Introduction

An important direction of modern antennas design is to provide narrow and multi-beam radiation patterns, sufficiently high gain and minimum dimensions. No less interesting are scanning antenna systems. Several types of antennas satisfy these requirements. One of them are the antenna arrays, but their gain decreases during beam steering which also requires a complex control system that is most of the cost of such an antenna. Another type is parabolic or lens antennas. In the case of a reflector antenna scanning is performed by rotating the entire antenna system mechanically, which increases the complexity, cost, weight and size of the system, or by electrical or mechanical displacement of the irradiator, but in a relatively narrow range of angles. This problem could be easily solved using the Luneburg lens with several irradiators [1, 2]. Luneburg lenses are used to build multi-beam and scanning antenna systems. Due to its design, this lens allows scanning in a wide range of angles without distortion of the radiation pattern [3].

Lens, proposed by Carl Luneburg, concentrates the radiation of the omnidirectional antenna in the desired direction due to radially inhomogeneous medium [4]. Refraction coefficient of such a lens changes from $\sqrt{2}$ in the middle to 1 at the edge, according to the next law:

$$n(r) = \sqrt{\varepsilon'(r)} = \sqrt{2-(r/a)^2}\,, \tag{1}$$

where ε' - relative permittivity of the lens material, r - radial coordinate in spherical or cylindrical coordinate system, a - the external radius of the lens.

Antennas based on the Luneburg lens are characterized by the ability of switchable scanning in a wide angle range, multi-beam radiation and high directivity.

II. Aim of the Study

It is hard enough to create a Luneburg lens of a dielectric material with a continuously changing, according to the law (1), refractive index. There are lots of researches on the different types of structures with change in the refractive index that is close to the law (1). The simplest and most commonly used method is to create a structure with a stepped change in the refractive index. This paper examines different ways of stratification with a stepped change in the refraction index. Most of the papers are devoted to the analysis of spherical Luneburg lens, but for some applications cylindrical lenses are interesting. They allow forming a multi-beam pattern with switchable steering. Linear antenna arrays can be used as irradiators for such lenses. This makes it possible to solve the problem of additional electronic scanning in the plane of the array.

This paper examines the impact of the stratification method of the lens structure by the thickness and refractive index value in terms of achieving the maximum directivity. The minimum number of layers will allow simplifying the lens design and thus making it cheaper and more manufacturable.

III. Methods Of Analysis

It should be noticed that the calculation and analysis of multilayered structures by standard methods in electromagnetic modeling software requires is very time consuming and requires a lot of computational power, especially at the stage of optimization of the structure. Research of the methods for such structures calculation that are several orders faster and give not sufficiently accurate, but acceptable results of calculations is required. This will give an opportunity to optimize the primary antenna structure of a multi-layered lens. Refined estimates in the second step can be performed using such well-known software products as Ansys HFSS, FEKO, CST Microwave Studio, etc.

The use of tensor Green's functions significantly accelerates the process of primary calculations and optimization. In [5] the effectiveness of this approach to the analysis of spherical lenses is shown. We should expect a similar effect on the analysis of multilayered cylindrical structures. Thus, the task of creating a fast algorithm to calculate and analyze a cylindrical lens antenna appeared. A method, proposed in [6], based on the use of the apparatus of tensor Green's functions, allows to solve the problem of creating compact and fast algorithms for solving the radiation problem for cylindrical lens antenna excited by different feeds.

[7] and [8] show the results of using the proposed mathematical model for the study of radiation characteristics of a cylindrical Luneburg lens. Using this mathematical model the effect of the discretization method of cylindrical Luneburg lenses on its radiation characteristics was studied. As a result, we managed to choose optimal discretization method, which allows obtaining the minimum number of layers while maintaining high directivity of the antenna.

IV. Results and Analysis

The considered structure of the lens is shown in Fig. 1. The number of layers and their electrodynamic parameters are arbitrary. The primary radiation source is located at a distance r_A from the center of the lens.

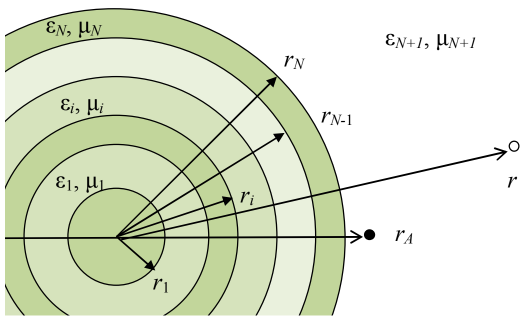

Fig. 1. Luneburg cylindrical multilayer lens

In accordance with the proposed approach, the spectral densities of the longitudinal field components are aligned with the voltages and currents in equivalent transmission lines simulating the radial structure of the lens. The remaining components of the field are calculated through longitudinal ones. The field is defined at all points of space. However, in the solution of antenna radiation problems, the field in the far zone is of greatest interest. Its calculation is simplified by using both the asymptotic of cylindrical functions and the saddle-point method, which makes it possible to significantly reduce the calculation time.

Fig. 2 shows the stratification methods, considered in this paper. Fig. 2a and 2c show discretization with uniform increments of permittivity. In the first case the layer dielectric constant was selected to correspond to the equation (1) at the boundary of the layer (Fig. 2a). In the second case constant was chosen to satisfy the formulae (1) in the middle of the layer (Fig. 2c). Fig. 2b and 2d show discretization with a uniform step along the radius of the lens. The permittivity was chosen to satisfy the equation (1) on the layers boundaries (Fig. 2b) and center (Fig. 2d).

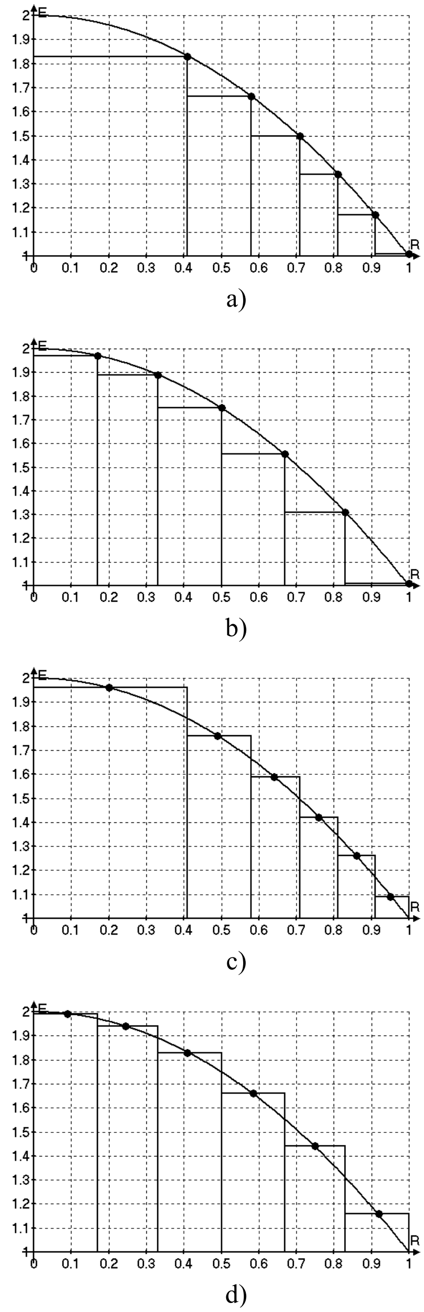

Fig. 2. Stratification curves for Luneburg lens (a) and c) with the uniform step of the dielectric constant, b) and d) with the uniform increment in thickness)

Fig. 3 shows the dependence of the antenna gain on the number of layers for the above stratification methods for cylindrical Luneburg lens. Huygens element is used as the feed.

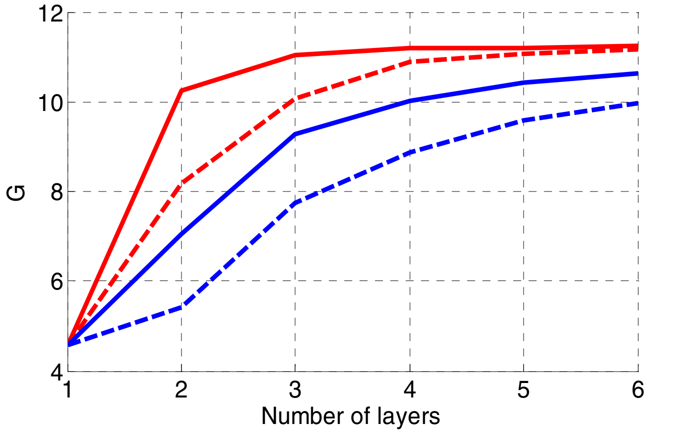

Fig. 3. Dependence of the antenna gain on the number of layers for different stratification methods (solid red - corresponds to the type (c), the red dotted line – (d), solid blue – (a) blue dotted – (b), that are shown used in Fig. 2)

Fig. 3 shows that the antenna gain increases significantly with the increase of layers number to three layers, but then it is nearly stable.

For example, the fields in the far field of a cylindrical Luneburg lens irradiated by the Huygens element, given by the densities of electric $J_z^E(r';\varphi';z')$ and magnetic $J_\varphi^M(r';\varphi';z')$ currents and located on the surface of the lens, is described in the spherical coordinate system by (2), (3):

$$E_\theta = -\frac{1}{2\pi^2}\cdot\frac{e^{-jk_0R}}{k_0R}\cdot\left(\frac{(\cos\theta)^2}{\sin\theta}+\sin\theta\right)\times \\ \times\sum_{m=0}^{\infty}\varepsilon_m j^{m+1}\cdot\cos(m\varphi)\cdot\left[\Phi_1^E+\Phi_2^E\right] \quad (2)$$

$$E_\phi = \frac{1}{2\pi^2}\cdot\frac{e^{-jk_0R}}{k_0R}\cdot\frac{Z_0}{\sin\theta}\times \\ \times\sum_{m=0}^{\infty}\varepsilon_m\cdot j^{m+1}\cdot\cos(m\varphi)\cdot\left[\Phi_1^H+\Phi_2^H\right] \quad (3)$$

where ε_m is equal to 1 for $m=0$ and 2 for $m\neq 0$, R - radial coordinate of the observation point in a spherical coordinate system.

Functions Φ_1^E, Φ_2^E, Φ_1^H, Φ_2^H have the following form:

$$\Phi_1^E(r';m;\theta)=\frac{1}{H_m^{(2)}(k_0r_6\sin\theta)}\frac{1}{\vec{Y}_{6E}^{+}}\Big[-jk_0\bar{V}_E(r_A,r_6)\times \\ \times I_0L\Delta\varphi k_0r_A\frac{\sin((k_0L\cos\theta)/2)}{(k_0L\cos\theta)/2}e^{jm\varphi_0}\,e^{jk_0z_0\cos\theta}\times \\ \times\frac{\sin(m\Delta\varphi/2)}{m\Delta\varphi/2}+\frac{Y_0}{1-(\cos\theta)^2}\frac{d\bar{V}_E(r_A,r_6)}{dr_A}I_0\Delta zLk_0r_A\times \\ \times\frac{\sin((k_0\Delta z\cos\theta)/2)}{(k_0\Delta z\cos\theta)/2}e^{jm\varphi_0}\,e^{jk_0z_0\cos\theta}\frac{\sin(mL/2)}{m\,L/2}\Big], \quad (4)$$

$$\Phi_2^E(r';m;\theta)=\frac{1}{H_m^{(2)}(k_0r_6\sin\theta)}\frac{\vec{Z}_{6EH}^{C}}{\vec{Z}_{6H}^{+}}\left[\frac{\cos\theta}{1-(\cos\theta)^2}\times\right. \\ \times\frac{jm}{r_A}\frac{H_m^{(2)}(k_0r_A\sin\theta)}{H_m^{(2)}(k_0r_6\sin\theta)}I_0\Delta zLk_0r_A\frac{\sin((k_0\Delta z\cdot\cos\theta)/2)}{(k_0\Delta z\cdot\cos\theta)/2}\times \\ \left.\times e^{jm\varphi_0}\,e^{jk_0z_0\cos\theta}\frac{\sin(mL/2)}{m\,L/2}\right], \quad (5)$$

$$\Phi_1^H(r';m;\theta)=\frac{1}{H_m^{(2)}(k_0r_6\sin\theta)}\frac{1}{\vec{Z}_{6H}^{+}}\left[\frac{\cos\theta}{1-(\cos\theta)^2}\times\right. \\ \times\frac{jm}{r_A}\bar{I}_H(r_A,r_6)I_0\Delta zLk_0r_A\frac{\sin((k_0\Delta z\cos\theta)/2)}{(k_0\Delta z\cos\theta)/2}\times \\ \left.\times e^{jm\varphi_0}\,e^{jk_0z_0\cos\theta}\frac{\sin(mL/2)}{m\,L/2}\right], \quad (6)$$

$$\Phi_2^H(r';m;\theta)=\frac{1}{H_m^{(2)}(k_0r_6\sin\theta)}\frac{\vec{Y}_{6HE}^{C}}{\vec{Y}_{6E}^{+}}\Big[-jk_0\times \\ \times\frac{H_m^{(2)}(k_0r_A\sin\theta)}{H_m^{(2)}(k_0r_6\sin\theta)}I_0L\Delta\varphi k_0r_A\frac{\sin((k_0L\cos\theta)/2)}{(k_0L\cos\theta)/2}\times \\ \times e^{jm\varphi_0}\,e^{jk_0z_0\cos\theta}\frac{\sin(m\Delta\varphi/2)}{m\Delta\varphi/2}+\frac{Y_0k_0\sin\theta}{1-(\cos\theta)^2}\times \\ \times\frac{H_m'^{(2)}(k_0r_A\sin\theta)}{H_m^{(2)}(k_0r_6\sin\theta)}I_0\Delta zLk_0r_A\frac{\sin((k_0\Delta z\cos\theta)/2)}{(k_0\Delta z\cos\theta)/2}\times \\ \times e^{jm\varphi_0}\,e^{jk_0z_0\cos\theta}\frac{\sin(mL/2)}{m\,L/2}\Big], \quad (7)$$

where for the longitudinal, relative to z axis, dipole: L - its length along the z axis, $\Delta\varphi$ - angular width along the coordinate φ; for an azimuthally located dipole: Δz - width along the z axis, L - its length along the coordinate φ; φ_A, z_A, r_A - source coordinates, $H_m^{(2)}(x)$ - Hankel function of the second kind, $\vec{Y}_{6E}^{+}=\vec{Y}_{6E}^{+}+\vec{Y}_{6E}^{+}$ - input conductivity of an equivalent electric line to the left and to the right relative to the reference cross-section r_0 outside the cylinder surface $(r'>r_6)$, $\vec{Z}_{6H}^{+}=\vec{Z}_{6H}^{+}+\vec{Z}_{6H}^{+}$ - Input resistance of an equivalent magnetic line to the left and to the right relative to the reference cross-section r_0, $\bar{Y}^C_{6HE}$, $\bar{Z}^C_{6EH}$ - conductivity and resistance of the junction formed by the electric and magnetic lines at the outer boundary of the structure, $\bar{I}_H(r_A,r_6)$, $\bar{V}_E(r_A,r_6)$ - equivalent current in the magnetic line and voltage in the electric line, respectively, $\gamma_i = \sqrt{k_i^2 - h^2}$ - wavenumber in equivalent line, k_0 - wavenumber in free space, $k_0 = k_0\sqrt{\varepsilon_i}$, $Z_0 = 120\pi$ Ohm.

The radiation patterns of four lenses having an outer radius of 3λ, three layers and different discretization methods were compared. Fig. 4 shows the radiation patterns for the stratification types (a) and c (Fig. 2), and in Fig. 5 for types (b) and (d) (see Fig. 2).

Radiation pattern of a Luneburg lens with uniform increment in permittivity and Huygens element as a radiator is shown in Fig. 4. It can be seen that the radiation pattern of the lens, with sampling shown in Fig. 2, c, is narrower and has a lower sidelobe level, than the one with the discretization shown in Fig. 2, a.

Radiation pattern of a Luneburg lens with layers of equal thickness and Huygens element as a radiator is shown in Fig. 5. It is shown that the sampling shown in Fig. 2, d has a narrower radiation pattern and lower sidelobe level compared to the one in Fig. 2, b.

Evaluation of sampling methods according to beam width and sidelobe level allows to conclude that the two most optimal are variants in Fig. 2, c and d. This is confirmed by a sharp increase in antenna gain graphs for up to 3 layers that is presented in Fig. 3.

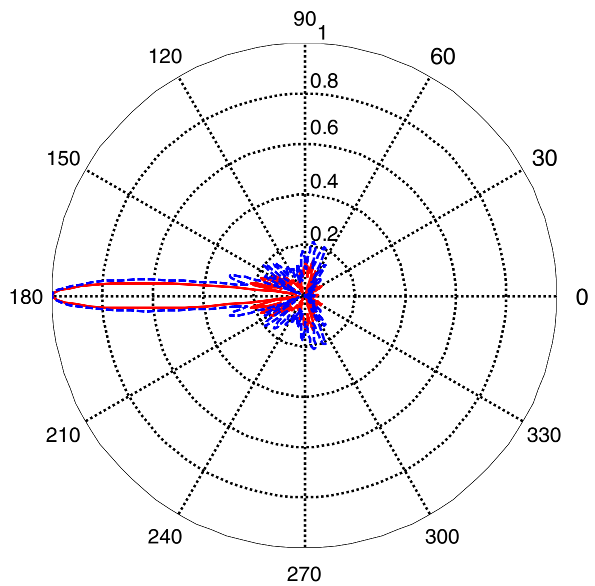

Fig. 4. Radiation pattern in the azimuth plane of a cylindrical Luneburg lens with three layers and the outer radius 3λ (Blue dotted line - sampling from Fig. 2, a; red solid line - sampling from Fig. 2, c)

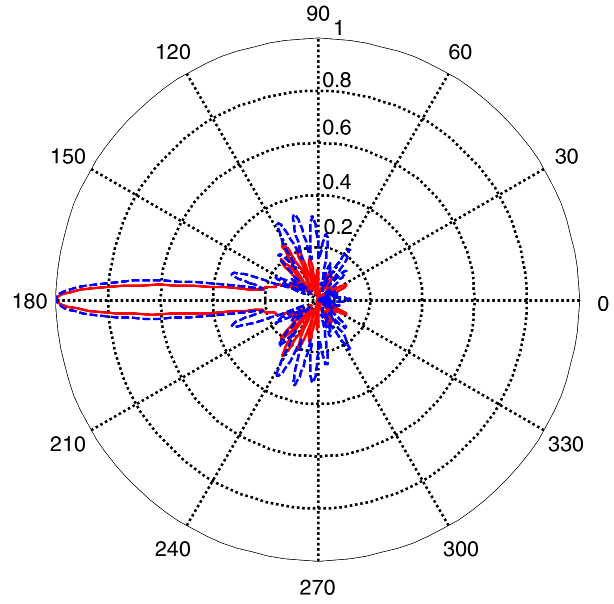

Fig. 5. Radiation pattern in the azimuth plane of a cylindrical Luneburg lens with three layers and the outer radius 3λ (Blue dotted line - sampling from Fig. 2, b; red solid line - sampling from Fig. 2, d)

V. Conclusion

The proposed approach makes it possible to perform the calculation of the scattered field for wave diffraction problems on a multilayer cylinder in a single solution. The proposed method takes into account correlations between the components of the field inside the cylinder during the theoretical stage of the solution which drastically reduces the computational time because there is no need to determine the field at all points in space. This is especially important for the optimization and synthesis of antennas based on Luneburg lens.

Lenses sampling method affects its' antenna characteristics strongly. In case of increasing the number of layers more than 3-4, a method of stratification practically doesn't affect the antenna characteristics of the Luneburg lens. Changes in the coefficient of refraction of the stratified lens must be as close to the law (1) as possible because it affects the gain and radiation pattern of the antenna system. The most optimal way to sample corresponds to the partition in Fig. 2, c. It has a maximum antenna gain, the minimum width of the main lobe of the radiation pattern and minimum level of sidelobes.

Using the sampling shown in Fig. 2, b a lens antenna with good radiation characteristics with only three layers can be developed, thereby reducing its cost and complexity of manufacturing.

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