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Assignment - 01

PART - A

Q1111911

$$P = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = (0) \cdot q_1 + (1) \cdot q_2 + (0) \cdot r = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

1. $3p + 2q$:
 $3 \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 12 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix}$

$$\begin{bmatrix} 6 \\ 3 \\ 22 \end{bmatrix} \left(\frac{1}{\sqrt{21}} \right)^{-200} = 0$$

2. \hat{P} : a unit vector in direction of P .

$$\|P\| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$$

$$\hat{P} = \frac{P}{\|P\|} = \frac{1}{\sqrt{21}} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}$$

3. $|P|$ and angle of P relative to positive y -axis

$$|P| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4+1+16} = \sqrt{21}$$

dot product-angle using cosine:

$$\cos(\theta) = \frac{\mathbf{P} \cdot \mathbf{y}}{\|\mathbf{P}\| \|\mathbf{y}\|}$$

$$\begin{bmatrix} \mathbf{P} \cdot \mathbf{y} \end{bmatrix} = 2 \cdot 0 + [-1] \cdot (1) + 4 \cdot (0) = \begin{bmatrix} -1 \end{bmatrix} = -1$$

magnitude of \mathbf{y} : $|\mathbf{y}| = 1$

$$\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{\omega(\theta)}{\sqrt{21} \cdot 1} = \frac{-1}{\sqrt{21}} = \frac{-1}{\sqrt{21}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\theta = \cos^{-1} \left(\frac{-1}{\sqrt{21}} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

4. direction of cosine of P_i

$$|P| = \sqrt{21} \text{ (relative direction)}$$

$$\cos(\alpha) = \frac{2}{\sqrt{21}} = 0.433$$

$$= 0.21$$

$$\cos(\beta) = \frac{-1}{\sqrt{21}}$$

$$\cos(\gamma) = \frac{4}{\sqrt{21}} = 0.877$$

5) Angle between Panel Q:

$$\cos \theta = \frac{\mathbf{P} \cdot \mathbf{Q}}{|\mathbf{P}| \cdot |\mathbf{Q}|}$$

$$\mathbf{P} \cdot \mathbf{Q} = (2 \cdot 0) + (-1) \cdot (3) + (4) \cdot (5) = -3 + 20 = 17$$

$$|\mathbf{P}| = \sqrt{q_1^2 + q_2^2 + q_3^2} = \sqrt{0^2 + 3^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$$

rule $\cos(\theta) = \frac{\mathbf{P} \cdot \mathbf{Q}}{|\mathbf{P}| \cdot |\mathbf{Q}|}$ minder orthogonal ist $\theta = 90^\circ$

$$\theta = \cos^{-1} \left(\frac{17}{\sqrt{21} \cdot \sqrt{34}} \right)$$

$$\theta = \cos^{-1} \left(\frac{17}{\sqrt{21} \cdot \sqrt{34}} \right)$$

$= 50.99^\circ$ ~~28.77~~ ~~angle in dark red~~ $\rightarrow \theta$

6) $\mathbf{P} \cdot \mathbf{Q}$, $\mathbf{Q} \cdot \mathbf{P}$

$$\mathbf{P} \cdot \mathbf{Q} - \mathbf{Q} \cdot \mathbf{P} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 \\ -3 \\ 20 \end{bmatrix} = 17$$

7) $\mathbf{P} \cdot \mathbf{Q}$ using angle between \mathbf{P} & \mathbf{Q} .
using the dot product formula: $\mathbf{P} \cdot \mathbf{Q} = |\mathbf{P}| |\mathbf{Q}| \cos \theta$

$$\cos \theta = \frac{\mathbf{P} \cdot \mathbf{Q}}{|\mathbf{P}| |\mathbf{Q}|}$$

$$\vec{P} \cdot \vec{q} = 0 + (-3) + 20 = 17//$$

$$\cos\theta = \frac{17}{\sqrt{21} \cdot \sqrt{34}} = \frac{17}{\sqrt{714}} = \frac{17}{26.725}$$

$$\cos\theta = 0.636//$$

$$\vec{P} \cdot \vec{q} = 4.17(\text{ev}) \quad \text{or} \\ \vec{P} \cdot \vec{q} = \sqrt{21} \cdot \sqrt{34} \cdot 0.636 = 17//$$

$$\vec{P} \cdot \vec{q} = \sqrt{21} \cdot \sqrt{34} \cdot 0.636 = 17//$$

8. The scalar projection of \vec{q} onto \vec{q}' (9ko)

$$\text{Scalar projection} = \frac{\vec{P} \cdot \vec{q}'}{\|\vec{P}\|} = \frac{17}{\sqrt{21}} = 3.70//$$

9. A vector that is perpendicular to \vec{P} :

$$\vec{P} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 8 \end{bmatrix} = \vec{q} \cdot \vec{P} = 0$$

$$\vec{P} \times \vec{q} = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ 3 & 2 & 1 \end{vmatrix} = 17i - 17j - 17k$$

$$= 1(-1)(0) - 1(0) - j(2)(0) - 4(1)k \\ = 2(0) - 1(1) = 0$$

$$= 0i + 0j + k = 0$$

... A vector \perp to P : perpendicular unit. with (1)
 $\Rightarrow P \times S = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$ where $P \cdot S = 0 \Leftrightarrow P \perp \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$

next, calc. binormal vector with P perpendicular
and S as the normal

10. $P \times Q$ and $Q \times P$:

$$P \times Q = \begin{bmatrix} i & j & k \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} : \text{unitary}$$

$$\begin{aligned} &= i(0) - j(5) + k(3) = -j(-5) + k(3) = -5j + 3k \\ &\Rightarrow (2) (3) = (-11(0)) \\ &(i - i(-5)(2)) + j(10 - 0) + k(6 + 0) = \begin{pmatrix} -17 \\ 10 \\ 6 \end{pmatrix} \end{aligned}$$

$$Q \times P = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - j(P \times Q) = -j \begin{pmatrix} -17 \\ 10 \\ 6 \end{pmatrix} = 17i + 10j + 6k \leftarrow \text{eq. 1}$$

11. 1) P vector [that] \perp to Q both $P \times Q = 0 \leftarrow \text{eq. 2}$

$$P \times Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{not a line. } \text{dec. eq. 1}$$

$$P \cdot (P \times Q) = 0 - 34 + 24 = 0$$

$$q \cdot (P \times V) = 0 - 30 + 30 = 0$$

$\therefore \begin{bmatrix} -17 \\ 10 \\ 6 \end{bmatrix}$ is \perp to P and Q \leftarrow eq. 3

12) The linear dependency between p_1 , p_2 and p_3

of p_1 , p_2 and p_3 are linearly dependent, check determinant of the matrix formed by these vectors is zero:

$$\text{Matrix : } \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & -2 \\ 4 & 5 & 2 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{determinant} = \begin{vmatrix} 2 & 0 & 1 \\ -1 & 1 & -2 \\ 4 & 5 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & -2 \\ 5 & 2 \end{vmatrix} - (-1) \begin{vmatrix} 0 & 1 \\ 4 & 2 \end{vmatrix} + 4 \begin{vmatrix} 0 & 0 \\ 4 & 5 \end{vmatrix}$$

$$= 2(3)(2) - (-1)(5) + 4(0) = 2(6) + 5 = 17$$

$$R_2 \rightarrow R_2 + \frac{1}{2}R_1 \Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & -\frac{3}{2} \\ 4 & 5 & 2 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_1 \Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & -\frac{3}{2} \\ 0 & 5 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \rightarrow \frac{1}{3}R_2 \text{ and } R_3 \rightarrow \frac{1}{5}R_3 \Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2C_1 + C_3 = 0 \quad 0 = 0 \quad (P+1) \cdot P$$

$$C_2 + \frac{1}{2}C_3 = 0$$

$$C_3 = 0$$

$$C_1 = C_2 = C_3 = 0$$

Vectors are linearly independent

13) $P^T q$ and $p q^T$ L.P has C.S.

$$P^T q = \begin{bmatrix} 2 & -1 & 4 \\ -1 & 0 & 1 \\ 1 & 3 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 15 \end{bmatrix} = \begin{bmatrix} 0 - 3 + 20 = 17 \\ -3 \\ 0 + 9 + 15 = 24 \end{bmatrix}$$

$$P q^T = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} (1)(0) + (-1)(3) + (4)(15) \\ (0)(0) + (1)(3) + (1)(15) \\ (1)(0) + (0)(3) + (1)(15) \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 12 \end{bmatrix} \begin{bmatrix} 10 \\ -5 \\ 20 \end{bmatrix}$$

$$(1)(0) + (-1)(3) + (4)(15) = (0)(0) + (0)(3) + (1)(15)$$

$$(1)(0) + P_A \cdot R_{ST} + T_B = (0)(0) + (0)(3) + (1)(15)$$

$$(1)(0) + (0)(-5) + (0)(15) = (0)(0) + (0)(3) + (1)(15)$$

$$x = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix}, \quad y = \begin{bmatrix} 4 \\ 3 \\ 11 \end{bmatrix} \quad P = \begin{bmatrix} 4 & 0 & -3 \\ 3 & 0 & 1 \\ 11 & 2 & 0 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$z = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 5 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$1. \quad x + 2y = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 8 & -2 & 4 \\ 6 & 0 & -6 \\ 2 & 4 & 2 \end{bmatrix}$$

$$(1)(0) + (0)(-5) + (0)(15) = (0)(0) + (0)(3) + (1)(15)$$

$$(1)(0) + (0)(-5) + (0)(15) = (0)(0) + (0)(3) + (1)(15)$$

$$(1)(0) + (0)(-5) + (0)(15) = (0)(0) + (0)(3) + (1)(15)$$

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. $X \cdot Y$ and $Y \cdot X$

$$X \cdot Y = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix} \begin{bmatrix} 0 & 4 & -1 \\ 3 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} (2)(4) + (1)(3) + (0)(1) \\ (-1)(4) + (3)(3) + (4)(1) \\ (3)(4) + (2)(3) + (-2)(1) \end{bmatrix} \\ &= \begin{bmatrix} 2(-1) + (1)(0) + (0)(2) \\ (-1)(-1) + (3)(0) + (4)(2) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &= \begin{bmatrix} 2(2) + (1)(-3) + (0)(1) \\ (-1)(2) + (3)(-3) + (4)(1) \end{bmatrix} \\ &= \begin{bmatrix} (3)(-1) + (2)(0) + (-2)(2) \\ (3)(2) + (2)(-3) + (2)(1) \end{bmatrix} \end{aligned}$$

$$X \cdot Y = \begin{bmatrix} 0 & 0 & 0 \\ 11 & 9 & 16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -6 & 2 & -2 \\ -2 & 1 & 0 \\ -7 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$Y \cdot X = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ 3 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 2 \\ 3 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(2) + (1)(-1) + (2)(3) \\ (3)(2) + (0)(-1) + (-3)(3) \\ (1)(2) + (2)(-1) + (1)(3) \end{bmatrix} = \begin{bmatrix} (4)(1) + (-1)(3) + (2)(0) \\ (3)(1) + (0)(3) + (-3)(1) \\ (1)(0) + (2)(4) + (1)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} (4)(2) + (-1)(-1) + (2)(3) \\ (3)(2) + (0)(-1) + (-3)(3) \\ (1)(2) + (2)(-1) + (1)(3) \end{bmatrix} = \begin{bmatrix} (4)(1) + (-1)(3) + (2)(0) \\ (3)(1) + (0)(3) + (-3)(1) \\ (1)(0) + (2)(4) + (1)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} (4)(2) + (1)(1) + (2)(3) \\ (3)(2) + (0)(1) + (-3)(3) \\ (1)(2) + (2)(1) + (1)(3) \end{bmatrix} = \begin{bmatrix} (4)(1) + (-1)(4) + (2)(-2) \\ (3)(4) + (0)(4) + (-3)(-2) \\ (1)(0) + (2)(4) + (1)(-2) \end{bmatrix}$$

$$yx = \begin{bmatrix} 13 & -3 & 6 \\ 7 & -3 & 6 \\ 4 & 11 & 6 \end{bmatrix}$$

$$yx = \begin{bmatrix} 15 & 5 & -8 \\ -3 & -3 & 6 \\ 3 & 11 & 6 \end{bmatrix}$$

3) $(xy)^T$ and $y^T x^T$

$$xy = \begin{bmatrix} 11 & -2 & 1 \\ 9 & 9 & 7 \\ 16 & 27 & 52 \end{bmatrix}$$

$$(xy)^T = \begin{bmatrix} 16 & 9 & 11 \\ -2 & 9 & -2 \\ 1 & 7 & 52 \end{bmatrix}$$

$$(ii) y^T x^T = \begin{bmatrix} 4 & 3 & 1 \\ -4 & 0 & 2 \\ 2 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -8 & 3 \\ 1 & 39 & 7 \\ 0 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 9 & 16 \\ -9 & 9 & -2 \\ 1 & -7 & -2 \end{bmatrix}$$

A) (x_1) and (z_1)

$$(x_1) = \begin{bmatrix} 1 & 0 \\ 2 & 3 \\ -1 & 2 \\ 3 & 2 \end{bmatrix} = 2(-4) + 1(10) - 1(10) = -18$$

$$(z_1) = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 1 & 2 \end{bmatrix} = 2(3) - 0 - 1(11) = 6 + 11 = 17$$

$$5) X = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} 4 & -18 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{Ans}$$

$$2 = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 21 & 7 \\ 8 & -8 & -7 \\ 6 & 2 & 0 \end{bmatrix} \quad \text{Ans}$$

Matrix \mathbf{R}_1 :

$$R_1 \times R_2 = -2 + 3 + 0 = 1, R_1 \cdot R_3 =$$

$$R_2 \times R_3 = -3 + 6 - 8 = -5 \neq 0$$

\Rightarrow none of the products are zero, so X does not have orthogonal vectors.

$$\text{Matrix } \mathbf{R}_1 \rightarrow R_1 \times R_2 = 12 + 0 + (-6) = 6 \neq 0$$

$$R_1 \times R_3 = 14 + (-2) + 2 = 14 \neq 0$$

$$R_2 \times R_3 = 3 + 0 - 3 \neq 0 \quad \text{g does not have orthogonal vectors}$$

Matrix \mathbf{R}_2 :

$$R_1 \cdot R_2 = 2 + 0 + (-5) = -3 \neq 0$$

$$R_1 \cdot R_3 = 6 + 0 + (-2) = 4 \neq 0$$

$$R_2 \cdot R_3 = 3 + 4 + 0 = 7 \neq 0$$

\Rightarrow \mathbf{R}_2 does not have orthogonal vectors

$$b) \quad x^{-1} \text{ and } y^{-1}$$

$$x^{-1} = \frac{1}{(x)} \cdot \text{adj}(x)$$

$$(x^{-1} = 2(-14)) -1(-10) = -18 //$$

$$\text{adj}^o x = \begin{bmatrix} + & - & + \\ A_{11} & A_{12} & A_{13} \\ - & A_{22} & A_{23} \\ A_{21} & + & A_{33} \\ A_{31} & A_{32} & + \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 14 & -4 & -7 \\ -2 & -4 & 1 & 7 \\ 4 & 8 & 0 & 7 \\ -11 & -11 & -7 & 7 \end{bmatrix} \quad x^T = \begin{bmatrix} -14 & -2 & 4 \\ 14 & -4 & 8 \\ -11 & 1 & 7 \end{bmatrix}$$

$$x^{-1} = \frac{1}{-18} \begin{bmatrix} -14 & +2 & 4 \\ 14 & -4 & -8 \\ -11 & -11 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 7/9 & 1/9 & 2/9 & -8/18 \\ 7/9 & -2/9 & 1/9 & -1/18 \\ 11/18 & 1/18 & -1/18 & \end{bmatrix}$$

$$y^{-1} = \frac{1}{\text{adj}(y)} \quad (\text{adj}(y) = 4(7)(-1) + (6)(2)(6) = 22 //)$$

$$\text{adj} y = \begin{bmatrix} 6 & -6 & 6 \\ 5 & 2 & -9 \\ 3 & 18 & 3 \end{bmatrix} = y^{-2} \begin{bmatrix} 6 & 5 & 3 \\ -6 & 2 & 18 \\ 6 & -9 & 3 \end{bmatrix}$$

$$y^{-1} = \frac{1}{22} \begin{pmatrix} 6 & 5 & 3 \\ -6 & 2 & 18 \\ 6 & -9 & 3 \end{pmatrix} = \frac{1}{22} \begin{pmatrix} 3 & 5/22 & 3/22 \\ 11 & 1 & 1/22 \\ -3/11 & 9/11 & 9/11 \end{pmatrix}$$

7) Z^{-1}

$$\text{adj } Z = \begin{pmatrix} 3 & 13 & -11 \\ -1 & 7 & -2 \\ 4 & -11 & -8 \end{pmatrix} = 2^2 \begin{pmatrix} 3 & 1 & -11 \\ 13 & 7 & -2 \\ -11 & -2 & 8 \end{pmatrix} = \text{adj } Z^2$$

$$Z^{-1} = \frac{1}{17} \text{adj } Z^2 = \frac{1}{17} \begin{pmatrix} 3 & 1 & -11 \\ 13 & 7 & -2 \\ -11 & -2 & 8 \end{pmatrix} = \begin{pmatrix} 3/17 & 1/17 & -11/17 \\ 13/17 & 7/17 & -2/17 \\ -11/17 & -2/17 & 8/17 \end{pmatrix}$$

8- The product of Xs

$$X_5 = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 & 7 \\ 4 & 0 & 0 \\ 11 & 10 & 12 \end{bmatrix} = P/F$$

$$X_5 = \begin{bmatrix} 2 \\ 13 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 & 7 \\ 4 & 0 & 0 \\ 11 & 10 & 12 \end{bmatrix} = P/F$$

$$9. X = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 & 7 \\ 4 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$|\vec{s}| = \sqrt{(-1)^2 + 4^2 + 0^2} = \sqrt{1 + 16 + 0} = \sqrt{17}$$

$$\hat{s} = \frac{1}{\sqrt{17}} \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix}$$

Scalar projection = $V \cdot \hat{s}$

projection of Row 1 $\rightarrow (2 \ 1 \ 0) \begin{pmatrix} -1 \\ \sqrt{17} \\ 4 \\ 0 \end{pmatrix}$

$$= \frac{1}{\sqrt{17}} (2(-1) + 1(4) + 0(0)) = \frac{1}{\sqrt{17}} (2 + 4) = \frac{6}{\sqrt{17}}$$

$$\text{Row 2} \rightarrow (-1 \ 3 \ 4) \begin{pmatrix} -1/\sqrt{17} \\ 3/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{17}} (-1(1) + 3(6) + 4(0)) = \frac{13}{\sqrt{17}}$$

$$\text{Row 3} \rightarrow (3 \ 2 \ -2) \begin{pmatrix} -1/\sqrt{17} \\ 2/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{17}} (3(-1) + 2(4) + (-2)(0))$$

= $\frac{-3 + 8}{\sqrt{17}} = \frac{5}{\sqrt{17}}$

(1)

scalar projection = $\frac{V \cdot \hat{s}}{|\vec{s}|}$

$$10) |\vec{s}| = \sqrt{(-1)^2 + (4)^2 + (0)^2} = \sqrt{17}$$

$$\hat{s} = \frac{1}{\sqrt{17}} \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix}$$

$$\text{vector projection} = \left(\frac{V \cdot \hat{s}}{|\vec{s}|} \right) \cdot \hat{s}$$

$$\hat{g} \cdot \hat{s} \hat{v} = \hat{v} \cdot \hat{s} \hat{g}$$

$$\text{Row 1} \Rightarrow \begin{pmatrix} 2 & 1 & 0 \end{pmatrix} \cdot \hat{s} = \frac{2}{\sqrt{17}} \begin{pmatrix} -2/\sqrt{17} \\ 8/\sqrt{17} \\ 0 \end{pmatrix}$$

$$(\hat{v} \cdot \hat{s}) \cdot \hat{s} = \frac{2}{\sqrt{17}} \begin{pmatrix} -2/\sqrt{17} \\ 8/\sqrt{17} \\ 0 \end{pmatrix}$$

$$\text{Row 2} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \hat{s} = \frac{13}{\sqrt{17}} \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \frac{-13}{\sqrt{17}} \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$$

$$\text{Row 3} \Rightarrow \begin{pmatrix} 3 & 0 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \hat{s} = \frac{5}{\sqrt{17}} \begin{pmatrix} -5 \\ 4 \\ 0 \end{pmatrix} = \frac{-5}{\sqrt{17}} \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$$

ii) If x has columns x_1, x_2, x_3 and vector s has elements s_1, s_2, s_3 then linear combination

$$v = s_1 x_1 + s_2 x_2 + s_3 x_3 =$$

$$s_1 = -1 \begin{pmatrix} s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, s_3 = 0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$(i) s_1 x_1 = -1 \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

$$S_2 \times 2 = 4 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ 8 \end{pmatrix}$$

$$S_3 \times 3 = 0 \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$v = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ 12 \\ 8 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 13 \\ 5 \end{pmatrix}$$

The linear combination of el of matrix
using element of vectors is $v = \begin{pmatrix} 2 \\ 13 \\ 5 \end{pmatrix}$

$$\frac{v}{\epsilon} = \frac{-2}{\epsilon} + \frac{1}{\epsilon} + \frac{13}{\epsilon} + \frac{5}{\epsilon} \in \text{el}$$

$$12. \quad y^t = S$$

$$\begin{pmatrix} 4 & 1 & 2 & 1 \\ 3 & 0 & -3 & 8 \\ 1 & 2 & 1 & 8 \end{pmatrix} \cdot \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 8 \end{pmatrix}$$

$$4t_1 - t_2 + 2t_3 = 4$$

$$3t_1 - 3t_2 + 8t_3 = 8$$

$$t_1 + 2t_2 + t_3 = 0$$

$$(i) \quad 4t_1 - t_2 + 2\left(t_1 - \frac{4}{3}\right) = -1$$

$$4t_1 - t_2 + 2t_1 - \frac{8}{3} = -1$$

$$6t_1 - t_2 = \frac{5}{3} \quad (i)$$

$$t_1 + 2t_2 + t_3 - \frac{4}{3} = 0 \quad (1)$$

$$2t_1 + 2t_2 = \frac{4}{3} = (t_1 + t_2) + \frac{t_2}{2} = \frac{2}{3} \rightarrow (2)$$

Add (1) and (2)

$$6(t_1 + t_2) + (t_1 + t_2) = \frac{6}{3} + \frac{2}{3}$$

$$7t_1 + \frac{7}{3} = \frac{8}{3} \Rightarrow t_1 = \frac{1}{3}$$

$$\text{take } (2) \Rightarrow t_1 + t_2 = \frac{2}{3} \Rightarrow \frac{1}{3} + t_2 = \frac{2}{3}$$

$$t_2 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\text{take } (2) \Rightarrow t_1 + t_2 = \frac{2}{3} \Rightarrow \frac{1}{3} + t_2 = \frac{2}{3}$$

$$t_2 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\text{take } t_3 = t_1 + \frac{4}{3} = \left(\begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right) + \left(\begin{array}{c} 4 \\ 0 \\ 0 \end{array} \right) = \left(\begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right) + \left(\begin{array}{c} 4 \\ 0 \\ 0 \end{array} \right) = \left(\begin{array}{c} 5 \\ 0 \\ 0 \end{array} \right) = \frac{5}{3}$$

$$\therefore t = \left(\begin{array}{c} 1/3 \\ 1/3 \\ -1 \end{array} \right)$$

This vector satisfies the equations. Yet

$$t = \left(\begin{array}{c} 1/3 \\ 1/3 \\ -1 \end{array} \right)$$

$$1 = \frac{1}{3} + \frac{1}{3} + (-1) = 0$$

$$(B) \quad 2t = s$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}$$

$$2t_1 - t_3 = -1$$

$$1t_1 + 4t_2 + 5t_3 = 4$$

$$3t_1 + 5t_2 + 2t_3 = 0$$

from $t_1 + t_3 = 2t_1 + 1$

in second $\Rightarrow t_1 + 4t_2 + 5(2t_1 + 1) = 4$

$$11t_1 + 4t_2 = 1 - ④$$

in (3) $3t_1 + 4t_2 + 2(2t_1 + 1) = 0$

$$\begin{bmatrix} 3t_1 + 4t_2 + 2(2t_1 + 1) = 0 \\ 11t_1 + 4t_2 = 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 7t_1 + t_2 = -2 \\ 11t_1 + 4t_2 = 1 \end{bmatrix} \rightarrow ⑤$$

$$t_2 = -2 - 7t_1$$

this in (4) $11t_1 + 4(-2 - 7t_1) = -1$

$$11t_1 - 8 - 28t_1 = -1$$

$$-17t_1 = 7 \quad \text{C.M. } ⑥$$

$$t_1 = \frac{-7}{17} \cdot (A - TB) \quad \text{Ans}$$

$$t_2 = -2 - 7\left(\frac{-7}{17}\right) \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + \frac{4}{17} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{15}{17} \quad \text{Ans}$$

$$t_3 = 2 \pm 1$$

$$t_3 = 2 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + 1 = \frac{1}{17} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$t = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

t is a unique vector when multiplied by S ,
gives vector s .

PART-C

$$(A^T)^{-1} = \frac{1}{16} A + \frac{1}{16} I$$

$$\textcircled{3} M = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \quad \textcircled{4} N = \begin{bmatrix} 5 & 3 \\ -3 & 7 \end{bmatrix} \quad \textcircled{5} P = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$$

1. eigenvalues and corresponding eigenvalues
of $M \rightarrow$
eigenvalues of $M \rightarrow \lambda = 16 \pm 1$

$$\det(\lambda I - A) = 0$$

$$\det \begin{bmatrix} 1-\lambda & 0 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = 0$$

$$\text{let } \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = 0$$

$$\text{let } \begin{bmatrix} d-3 & -2 \\ -1 & d-4 \end{bmatrix} = 0$$

$$d^2 - 7d + 14 = 0$$

by solving this we get $d_1 = \frac{7+i\sqrt{7}}{2}$, $d_2 = \frac{7-i\sqrt{7}}{2}$

eigenvectors of M for d_1

$$\begin{pmatrix} 3 - \frac{7+i\sqrt{7}}{2} & 2 \\ -1 & 4 - \frac{7+i\sqrt{7}}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1+i\sqrt{7}}{2} & 2 \\ -1 & \frac{1+i\sqrt{7}}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\frac{-1+i\sqrt{7}}{2} v_1 + 2v_2 = 0$$

$$v_1 = \frac{-2}{-1+i\sqrt{7}} \cdot v_2 \Rightarrow v_2 = v_1 \cdot \frac{4}{1-i\sqrt{7}} = \frac{4}{1-i\sqrt{7}} v_2$$

$$V_1 = \frac{1+i\sqrt{7}}{2} N_2$$

$V_1 = \text{eigenvector} =$

$$\begin{bmatrix} 1+i\sqrt{7}/2 \\ 1-i\sqrt{7}/2 \end{bmatrix}$$

$$d_2 = \frac{7-i\sqrt{7}}{2} \Rightarrow M - d_2 = \begin{bmatrix} -1-i\sqrt{7} & 2 \\ 0 & 1-i\sqrt{7} \end{bmatrix}$$

$\rightarrow \text{rank } M - d_2 = 1$

$$-V_1 + \frac{1-i\sqrt{7}}{2} = 0 \Rightarrow V_1 = \frac{1-i\sqrt{7}}{2}$$

$$V_2 = \text{eigenvector} =$$

$$\begin{bmatrix} -i\sqrt{7} \\ 2 \end{bmatrix}$$

$$2) \quad \begin{bmatrix} 1+i\sqrt{7} & 1 \\ 0 & 1-i\sqrt{7} \end{bmatrix} \begin{bmatrix} 1-i\sqrt{7} \\ 2 \end{bmatrix} = 8+1=9$$

$$\therefore V_1 V_2 = 9$$

3) The dot product between the eigenvectors of $M - d_1 I$

$$V_1 \cdot (M - d_1 I) V_2 = 0$$

$$\begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} = 0 \quad (\text{FEV-1})$$

$$= 30 - 5d - 6d + d^2 - 9 = 0$$

$$\therefore d^2 - 11d + 21 = 0$$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(21)}}{2(1)} = \frac{11 \pm \sqrt{121 - 84}}{2} = \frac{11 \pm \sqrt{37}}{2}$$

$$d_1 = \frac{11 + \sqrt{37}}{2}, \quad d_2 = \frac{11 - \sqrt{37}}{2}$$

$$M - d_1 I = \begin{bmatrix} 5 - \frac{11 + \sqrt{37}}{2} & 3 \\ 6 & 6 - \frac{11 + \sqrt{37}}{2} \end{bmatrix}$$

$$= -\frac{1 + \sqrt{37}}{2} x_1 - 3x_2 = 0 \Rightarrow -3x_1 + \frac{1 + \sqrt{37}}{2} x_2 = 0$$

$$x_1 = \frac{1 + \sqrt{37}}{6} x_2$$

$$v_1 = \begin{bmatrix} \frac{1 + \sqrt{37}}{6} \\ 1 \end{bmatrix}$$

$$d_2 = \frac{11 - \sqrt{37}}{2}$$

$$M - d_2 I = \begin{bmatrix} \frac{11 - \sqrt{37}}{2} & 3 \\ -3 & \frac{11 - \sqrt{37}}{2} \end{bmatrix}$$

$$-1 - \sqrt{37}v_1 - 3v_2 \left[\begin{array}{c} v_1 = 0 \\ v_2 = 0 \end{array} \right] - \left[\begin{array}{c} 0 \\ 0 \end{array} \right]$$

$$-3v_1 + \frac{1 - \sqrt{37}}{2}v_2 = 0$$

$$\Rightarrow v_1 = \frac{1 - \sqrt{37}}{6}v_2$$

$$v_2 \left[\begin{array}{c} 1 - \sqrt{37} \\ 6 \end{array} \right]$$

$$v_1 \cdot v_2 = \left[\begin{array}{c} 1 + \sqrt{37} \\ 6 \end{array} \right]$$

$$1 + \left(\frac{1 - \sqrt{37}}{6} \right)^2$$

$$= \frac{-36}{36} + 1 = 0$$

- ④ eigenvectors of N are orthogonal to each other
 * All eigenvalues are real
 * symmetric

$$⑤ pt = 0$$

Trivial Solution

$$t = \left[\begin{array}{c} 0 \\ 0 \end{array} \right]$$

$$⑥ \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2t_1 + 4t_2 = 0$$

$$4t_1 + 8t_2 = 0$$

$$t_1 = -2t_2$$

$$+ (-8)(10) + (8 \cdot 0)(10) + \text{for } t_2 = 1 (A=0) = 0$$

$$\text{for } t_2 = 1$$

$$t = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$t = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

⑦

$$M_1 = 0$$

$$M_2 = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \xrightarrow{-1} = (10)0$$

$$(M) = 2+2 = 4 \neq 0 \quad -(21.1)$$

\therefore Only solution for $t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as $M \neq 0$

invertible: (each row has unique non-zero entries)
if two rows have the same entries

then the matrix is not invertible

①

PART D

$$\begin{bmatrix} 2 & 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$$

$t_3 = 0.3$

$$x_1 = 0.5 \quad x_2 = 0.8$$

$$w_1 = 0.4 \quad w_2 = 0.6 \quad w_3 = 0.9$$

$$\text{bias} = 0.2$$

$$y = w_1 x_1 + b$$

$$y = (0.4)(0.5) + (0.6)(0.8) + (0.9)(0.3) + 0.2$$

$$= 0.2 + 0.48 + 0.27 + 0.2 = 1.15$$

②

Sigmoid Activation fun'

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\sigma(1.15) = \frac{1+1}{1+e^{-(1.15)}} = \frac{1+0.75}{1+0.25} = 0.75$$

Ans: 0.75

③ Given values same as question 1, compute the output of the neuron after applying a ReLU activation function

$$\text{ReLU}(m) = \max(0, m)$$

$$\text{ReLU}(1.15) = \max(0, 1.15) = 1.15$$

④ $w_n = \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$, $b_n = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$, $a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$y = w_n \cdot x + b_n$$

$$= \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix}$$

$$\text{ReLU} \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} \max(0, 1.0) \\ \max(0, 1.5) \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix}$$

⑤ $w_0 = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}$, $b_0 = \begin{bmatrix} 0.1 \end{bmatrix}$

$$y = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix} \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix} + \begin{bmatrix} 0.1 \end{bmatrix}$$

$$= [0.55 \quad -0.45] + [0.1] = [0.2]$$

$$\sigma(0.2) = \frac{1}{1+e^{-0.2}} = 0.549$$

$$⑥ \quad z = \begin{bmatrix} a_1 & a_2 \\ w_n & b_n \end{bmatrix} \xrightarrow{\text{matrix multiplication}} \begin{bmatrix} z_1 & z_2 \\ w_0 & b_0 \end{bmatrix}$$

$$L = y_n + b \quad y = w_0 z + b_0$$

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial z} = \frac{\partial L}{\partial y} \cdot \frac{\partial z}{\partial x}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

PART-B

$$f(x) = \begin{cases} 2x^2 - 1 & (x < 0) \\ 3x^2 + 4 & (x \geq 0) \end{cases}, \quad g(x) = \begin{cases} 3x^2 & (x < 0) \\ 2x & (x \geq 0) \end{cases}$$

$$h(x, y) = x^2 + y^2 + xy$$

$$① \quad f'(x) \text{ and } f''(x) \quad [x=0, x=0] = 0$$

$$f'(x) = \frac{d}{dx} (2x^2 - 1) = 4x //$$

$$f''(x) = \frac{d}{dx} 4x = 4 //$$

② partial derivatives
 $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$

$$\frac{\partial h}{\partial x} = \frac{\partial(x^2 + xy)}{\partial x} = 2x + y$$

$$\frac{\partial h}{\partial y} = \frac{\partial(x^2 + xy^2 + xy)}{\partial y} = (2y + x) + x + x = 2y + 3x$$

③ $\nabla h(x, y)$

$$= \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y} \right) = (2xy, 2y+x)$$

④ $\frac{\partial}{\partial x} f(g(x))$. $f(x) = 2x^2 + 1$ $g(x) = 3x^2 + 4$
 $f(g(x)) = 2(3x^2 + 4)^2 - 1$
 $= 2(9x^4 + 16 + 24x^2) - 1$
 $= 18x^4 + 48x^2 + 31$

$$\frac{\partial}{\partial x} f(g(x)) = \frac{\partial}{\partial x} (18x^4 + 48x^2 + 31) \quad \left. \begin{array}{l} \text{chain rule} \\ \text{constant} \end{array} \right\}$$

$$= 72x^3 + 96x$$

$$\frac{\partial}{\partial x} + g(x) = \frac{\partial + f(x)}{\frac{\partial g(x)}{dx}} \cdot \frac{\partial (g(x))}{\partial x}$$

$$= \frac{\partial (2g(x)^2 - 1)}{\partial g(x)} \left[\frac{\partial (3x^2 + 4)}{\partial x} \right]$$

$$= 4(g(x)^2 + 1) \cdot 6x$$

$$= 4(3x^2 + 4)(6x)$$

$$= (12x^2 + 16) 6x = 72x^3 + 96x$$

$$(12x^2 + 16x) = \left(\frac{N6}{C6}, \frac{N6}{C6} \right) =$$