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Course: CS512

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Assignment - 0

PART - A

$$p = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \quad q = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} \quad r = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

1. $3p + 2q$:

$$3\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 12 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 3 \\ 22 \end{bmatrix}$$

2. $\hat{p}$: a unit vector in direction of p.

$$|p| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4+1+16} = \sqrt{21}$$

$$\hat{p} = \frac{p}{\|p\|} = \frac{1}{\sqrt{21}}\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}$$

3. $|P|$ and angle of P relative to positive y-axis

$$|P| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4+1+16} = \sqrt{21}$$

dot product - angle using cosine :

$$\cos(\theta) = \frac{P \cdot y}{\|P\| \|y\|}$$

$$P \cdot y = 2 \cdot 0 + (-1) \cdot (1) + 4 \cdot (0) = -1$$

magnitude of y: $|y| = 1$

$$\cos(\theta) = \frac{-1}{\sqrt{21} \cdot 1} = \frac{-1}{\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{21}}\right)$$

4. direction of cosine of P:

$$|P| = \sqrt{21}$$

$$\cos(\alpha) = \frac{2}{\sqrt{21}} = 0.43$$

$$= 0.21$$

$$\cos(\beta) = \frac{-1}{\sqrt{21}}$$

$$\cos(\gamma) = \frac{4}{\sqrt{21}} = 0.87$$

5) Angle between P and q:

$$\cos\theta = \frac{p\cdot q}{|p|\cdot|q|}$$

$p\cdot q = (2\cdot(0)) + (-1)\cdot(3) + (4)\cdot(5) = -3+20 = 17$

$|q| = \sqrt{q_1^2 + q_2^2 + q_3^2} = \sqrt{0^2+3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

Sole

$$\cos(\theta) = \frac{17}{\sqrt{21}\cdot\sqrt{34}}$$

$$\theta = \cos^{-1}\left(\frac{17}{\sqrt{21}\ \sqrt{34}}\right)$$

$= 50.49\ 02\ 877$

6) $p\cdot q$, $q\cdot p$

$$p\cdot q = q\cdot p = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} = (0) + (-3) + (20)$$

$= 17$

7) $p\cdot q$ using angle between p & q.

using the dot product formula:

$$\cos\theta = \frac{p\cdot q}{|p||q|}$$

$p \cdot q = 0 + (-3) + 20 = 17$

$$\cos\theta = \frac{17}{\sqrt{21}\,\sqrt{34}} = \frac{17}{\sqrt{714}} = \frac{17}{26.725}$$

$\cos\theta = 0.636$

$p \cdot q = |p||q| \cos\theta$

$p \cdot q = \sqrt{21} \cdot \sqrt{34} \cdot 0.636 = 17$

8. The scalar projection of q onto $\hat{q}$.

$$\text{Scalar projection} = \frac{p \cdot q}{|p|} = \frac{17}{\sqrt{21}} = 3.70$$

9. A vector that is perpendicular to p:

$$p = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$p \times s = \begin{bmatrix} i & j & k \\ 2 & -1 & 4 \\ 1 & 0 & 0 \end{bmatrix}$$

$= i(-1(0) - 4(0)) - j(2(0) - 4(1)) + k(2(0) - 1(1))$

$= 0i + 4j + k$

A vector to P:

$$p \times s = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \neq p \cdot s = 0 \therefore P \perp \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} //$$

10. $p \times q$ and $q \times p$:

$$p \times q = \begin{bmatrix} i & j & k \\ 2 & -1 & 4 \\ 0 & 3 & 5 \end{bmatrix}$$

$$= i((-1)(5) - (4)(3)) - j((2)(5) - (4)(0)) + k((2)(3) - (-1)(0))$$

$$= i(-5-12) - j(10-0) + k(6+0) = \begin{pmatrix} -17 \\ -10 \\ 6 \end{pmatrix} //$$

$$q \times p = -(p \times q) = \begin{pmatrix} 17 \\ 10 \\ -6 \end{pmatrix}$$

11. A vector that $\perp^r$ to both p & q

$$p \times q = \begin{bmatrix} -17 \\ -10 \\ 6 \end{bmatrix}$$

$$p \cdot (p \times q) = -34 + 10 + 24 = 0$$

$$q \cdot (p \times q) = 0 - 30 + 30 = 0$$

$\begin{bmatrix} -17 \\ -10 \\ 6 \end{bmatrix}$ is $\perp$ p and q

12) The linear dependency between p, q and r: If p, q and r are linearly dependent, check determinant of the matrix formed by these vectors is zero:

$$\text{Matrix}: \begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & -2 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{\text{determinant} = 2\begin{bmatrix} 3 & -2 \\ 5 & 2 \end{bmatrix} - 0\begin{bmatrix} -1 & -2 \\ 4 & 2 \end{bmatrix} + 1\begin{bmatrix} -1 & 3 \\ 4 & 5 \end{bmatrix}}$$

$$= 2(3)(2) - (-2)(5) + 1(-1)(5) - (3)(4)$$

$$R_2 \to R_2 + \tfrac{1}{2}R_1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & -3/2 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \to R_3 - 2R_1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & -3/2 \\ 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \to \tfrac{1}{3}R_2 \text{ and } R_3 \to \tfrac{1}{5}R_3 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2C_1 + C_3 = 0$

$C_2 - \tfrac{1}{2}C_3 = 0$

$C_3 = 0$

$C_1 = C_2 = C_3 = 0$

∴ Vectors are linearly independent ~~dependent~~

13) $p^T q$ and pq^T

$$p^T q = \begin{bmatrix} 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} = 0 - 3 + 20 = 17$$

$$pq^T = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 10 \\ 0 & -3 & -5 \\ 0 & 12 & 20 \end{bmatrix}$$

PART-B

$$x = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix}, \quad y = \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & \end{bmatrix}$$

$$z = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 5 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}$$

1. $$x + 2y = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 8 & -2 & 4 \\ 6 & 0 & -6 \\ 2 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -1 & 4 \\ 5 & 3 & -2 \\ 8 & 6 & 0 \end{bmatrix}$$

2. X·Y and Y·X

$$X \cdot Y = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix} \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(4)+(1)(3)+(0)(1) \\ (-1)(4)+(3)(3)+(4)(1) \\ (3)(4)+(2)(3)+(-2)(1) \end{bmatrix}$$

$$\begin{matrix} (2)(-1)+(1)(0)+(0)(2) & (2)(2)+(1)(-3)+(0)(1) \\ (-1)(-1)+(3)(0)+(4)(2) & (-1)(2)+(3)(-3)+(4)(1) \\ (3)(-1)+(2)(0)+(-2)(2) & (3)(2)+(2)(-3)+(-2)(1) \end{matrix}$$

$$XY = \begin{bmatrix} 11 & -2 & 1 \\ 9 & 9 & -7 \\ 16 & -7 & -2 \end{bmatrix}$$

$$YX = \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(2)+(-1)(-1)+(2)(3) & (4)(1)+(-1)(3)+(2)(2) \\ (3)(2)+(0)(-1)+(-3)(3) & (3)(1)+(0)(3)+(-3)(2) \\ (1)(2)+(2)(-1)+(1)(3) & (1)(1)+(2)(3)+(1)(2) \end{bmatrix}$$

$$\begin{bmatrix} (4)(0)+(-1)(4)+(2)(-2) \\ (3)(0)+(0)(4)+(-3)(-2) \\ (1)(0)+(2)(4)+(1)(-2) \end{bmatrix}$$

$$\cancel{YX} = \begin{bmatrix} \cancel{13} & \cancel{3} & \cancel{-6} \\ \cancel{-7} & \cancel{-3} & \cancel{6} \\ \cancel{4} & \cancel{11} & \cancel{6} \end{bmatrix}$$

$$YX = \begin{bmatrix} 15 & 5 & -8 \\ -3 & -3 & 6 \\ 3 & 11 & 6 \end{bmatrix}$$

3) $(XY)^T$ and $Y^T X^T$

$$XY = \begin{bmatrix} 11 & -2 & 1 \\ 9 & 9 & 7 \\ 16 & -7 & -2 \end{bmatrix} \quad (XY)^T = \begin{bmatrix} 11 & 9 & 16 \\ -2 & 9 & -7 \\ 1 & -7 & -2 \end{bmatrix}$$

(ii) $Y^T X^T = \begin{bmatrix} 4 & 3 & 1 \\ -1 & 0 & 2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & 2 \\ 0 & 4 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 11 & 9 & 16 \\ -2 & 9 & -7 \\ 1 & -7 & -2 \end{bmatrix}$$

4) $|X|$ and $|Z|$

$$|X| = \begin{vmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{vmatrix} = 2(-14) - 1(-10) - 0 = -18$$

$$|Z| = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 1 & 2 \end{vmatrix} = 2(3) - 0 - 1(-11) = 6 + 11$$

$$= 17$$

5) $X = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & 2 \end{bmatrix}$ $Y = \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix}$

$Z = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & \cancel{2} & 2 \end{bmatrix}$

Matrix :

$R_1 \times R_2 = -2 + 3 + 0 = 1$, $R_1 \cdot R_3 = 6 + 2 + 0 = 8$

$R_2 \times R_3 = -3 + 6 - 8 = -5$

none of the products are zero, so X does not have orthogonal vectors.

Matrix Y →

$R_1 \times R_2 = 12 + 0 + (-6) = 6$

$R_1 \; R_3 = 4 + (-2) + 2 = 4$

$R_2 \; R_3 = 3 + 0(-3) = 0$

Y does not have orthogonal vectors

Matrix Z :

$R_1 R_2 = 2 + 0 + (-5) = -3$

$R_1 R_3 = 6 + 0 + (-2) = 4$

$R_2 R_3 = 3 + 4 + 10 = 17$

Z does not have orthogonal vectors

b) x^{-1} and y^{-1}

$$x^{-1} = \frac{1}{|x|} \cdot adj(x)$$

$$|x| = 2(-14) - 1(-10) = -18$$

$$adj\, x = \begin{bmatrix} A_{11}^{+} & A_{12}^{-} & A_{13}^{+} \\ A_{21}^{-} & A_{22}^{+} & A_{23}^{-} \\ A_{31}^{+} & A_{32}^{-} & A_{33}^{+} \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 14 & -11 \\ -2 & -4 & 1 \\ 4 & 8 & 7 \end{bmatrix} \quad x^{T} = \begin{bmatrix} -14 & -2 & 4 \\ 14 & -4 & 8 \\ -11 & 1 & 7 \end{bmatrix}$$

$$x^{-1} = \frac{1}{-18} \begin{bmatrix} -14 & +2 & 4 \\ 14 & -4 & -8 \\ -11 & 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7/9 & 1/9 & 2/9 \\ 7/9 & 2/9 & -8/18 \\ 11/18 & 1/18 & -1/18 \end{bmatrix}$$

$$y^{-1} = \frac{1}{|y|} \cdot adj(y) \quad |y| = 4(7)(-1) + (6)(2)(6) = 2$$

$$adj\, y = \begin{bmatrix} 6 & -6 & 6 \\ 5 & 2 & -9 \\ 3 & 18 & 3 \end{bmatrix} = y^{T} = \begin{bmatrix} 6 & 5 & 3 \\ -6 & 2 & 18 \\ 6 & -9 & 3 \end{bmatrix}$$

$$Y^{-1} = \frac{1}{22}\begin{bmatrix} 6 & 5 & 3 \\ -6 & 2 & 18 \\ 6 & -9 & 3 \end{bmatrix} = \begin{bmatrix} 3/11 & 5/22 & 3/22 \\ -3/11 & 1/11 & 9/11 \\ 3/11 & -9/22 & 3/22 \end{bmatrix}$$

7) Z^{-1}

$|Z| = 17$

$$\text{adj } Z = \begin{bmatrix} 3 & 13 & -11 \\ -1 & 7 & -2 \\ 4 & -11 & 8 \end{bmatrix} = Z^T = \begin{bmatrix} 3 & -1 & 4 \\ 13 & 7 & -11 \\ -11 & -2 & 8 \end{bmatrix} = \text{adj } Z$$

$$Z^{-1} = \frac{1}{|Z|}\text{adj } Z = \frac{1}{17}\begin{bmatrix} 3 & -1 & 4 \\ 13 & 7 & -4 \\ -4 & -2 & 8 \end{bmatrix} = \begin{bmatrix} 3/17 & -1/17 & 4/17 \\ 13/17 & 7/17 & -11/17 \\ -11/17 & -2/17 & 8/17 \end{bmatrix}$$

8- The product of Xs

$$Xs \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} =$$

$$Xs = \begin{bmatrix} 2 \\ 13 \\ 5 \end{bmatrix}$$

9. $X = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & 2 \end{bmatrix}$, $y = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$

$|S| = \sqrt{(-1)^2 + 4^2 + 0^2} = \sqrt{1+16+0} = \sqrt{17}$

$$\hat{s} = \frac{1}{\sqrt{17}} \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} = \begin{pmatrix} \frac{-1}{\sqrt{17}} \\ 4/\sqrt{17} \\ 0 \end{pmatrix}$$

Scalar projection $= v \cdot \hat{s}$

projection of Row 1 $\rightarrow$ $(2, 1, 0) \begin{pmatrix} \frac{-1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \\ 0 \end{pmatrix}$

$$= \frac{1}{\sqrt{17}} \left(2(-1) + 1(4) + (0)(0) \right) = \frac{1}{\sqrt{17}} (2 + 0) = \frac{2}{\sqrt{17}}$$

Row 2 $\rightarrow (-1\ 3\ 4) \begin{pmatrix} -1/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{17}} (-1(-1) + 3(4) + 0) = \frac{13}{\sqrt{17}}$

Row 3 $\rightarrow (3\ 2\ -2) \begin{bmatrix} -1/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{bmatrix} = \frac{1}{\sqrt{17}} \left(3(-1) + 2(4) + (-2)(0) \right)$

$= \frac{5}{\sqrt{17}}$

10) $|S| = \sqrt{(-1)^2 + (4)^2 + (0)^2} = \sqrt{17}$

$$\hat{s} = \frac{1}{\sqrt{17}} \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix}$$

vector projection $= \left(\frac{v \cdot \hat{s}}{\hat{s} \cdot \hat{s}} \right) \cdot \hat{s}$

$\hat{s}\cdot\hat{s} = 1 \Rightarrow v\cdot\hat{s}\cdot\hat{s}$

Row 1 $\Rightarrow (2 \quad 1 \quad 0) \quad v\cdot\hat{s} = 2/\sqrt{17}$

$$(v\cdot\hat{s})\cdot\hat{s} = \frac{2}{\sqrt{17}}\begin{pmatrix} -1/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix} = \begin{pmatrix} -2/\sqrt{17} \\ 8/\sqrt{17} \\ 0 \end{pmatrix}$$

Row 2 $\rightarrow (-1 \quad 3 \quad 4)$

$$v\cdot\hat{s} = 13/\sqrt{17}\begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -13/\sqrt{17} \\ 52/\sqrt{17} \\ 0 \end{pmatrix}$$

Row 3 $\rightarrow (-3 \quad 2 \quad -2)$

$v\cdot\hat{s} = 5/\sqrt{17}$

$$(v\cdot\hat{s})\cdot\hat{s} = \frac{5}{\sqrt{17}}\begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -5/\sqrt{17} \\ 4/\sqrt{17} \\ 0 \end{pmatrix}$$

ii) If x has columns $x_1\ x_2\ x_3$ and vector s has elements $s_1\ s_2\ s_3$ then linear combination

$$V = s_1 x_1 + s_2 x_2 + s_3 x_3$$

$s_1 = -1, \quad s_2 = 4, \quad s_3 = 0$

(i) $s_1 x_1 = -1\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$

$$s_2 x_2 = 4\begin{pmatrix}1\\3\\2\end{pmatrix} = \begin{pmatrix}4\\12\\8\end{pmatrix}$$

$$s_3 x_3 = 0\begin{pmatrix}0\\4\\2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$v = \begin{pmatrix}-2\\1\\-3\end{pmatrix} + \begin{pmatrix}4\\12\\8\end{pmatrix} + \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}2\\13\\5\end{pmatrix}$$

The linear combination of col of matrix using element of vectors is $v = \begin{pmatrix}2\\13\\5\end{pmatrix}$

12. $y^{+} = S$

$$\begin{pmatrix}4 & -1 & 2\\3 & 0 & -3\\1 & 2 & 1\end{pmatrix}\begin{pmatrix}t_1\\t_2\\t_3\end{pmatrix} = \begin{pmatrix}-1\\4\\0\end{pmatrix}$$

$4t_1 - t_2 + 2t_3 = -1$

$3t_1 - 3t_3$

$t_1 + 2t_2 + t_3 = 0$

(i) $4t_1 - t_2 + 2\left(t_1 - \frac{4}{3}\right) = -1$

$4t_1 - t_2 + 2t_1 - \frac{8}{3} = -1$

$6t_1 - t_2 = 5/3 \rightarrow (i)$

$t_1 + 2t_2 + t_1 - \frac{4}{3} = 0$

$2t_1 + 2t_2 = \frac{4}{3} \Rightarrow t_1 + t_2 = \frac{2}{3} \rightarrow (2)$

Add (1) and (2)

$6(t_1 - t_2) + (t_1 + t_2) = \frac{5}{3} + \frac{2}{3}$

$7t_1 = \frac{7}{3} \Rightarrow t_1 = \frac{1}{3}$

take (2) $\Rightarrow t_1 + t_2 = \frac{2}{3} \Rightarrow \frac{1}{3} + t_2 = \frac{2}{3}$

$t_2 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

take (2) $\Rightarrow t_1 + t_2 = \frac{2}{3} \Rightarrow \frac{1}{3} + t_2 = \frac{2}{3}$

$t_2 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

take $t_3 = t_1 - \frac{4}{3} \Rightarrow t_3 = \frac{1}{3} - \frac{4}{3} = \frac{-3}{3} = -1$

$\therefore t = \begin{pmatrix} 1/3 \\ 1/3 \\ -1 \end{pmatrix}$

this vector satisfies the equation $yt = s$

(13) $2t = s$

$$\begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$$

$2t_1 - t_3 = -1$

$1t_1 + 4t_2 + 5t_3 = 4$

$3t_1 + 1t_2 + 2t_3 = 0$

from i, $t_3 = 2t_1 + 1$

in second $\Rightarrow t_1 + 4t_2 + 5(2t_1 + 1) = 4$

$11t_1 + 4t_2 = -1$ — (4)

in (3) $3t_1 + 1t_2 + 2(2t_1 + 1) = 0$

$7t_1 + 1t_2 + 2 = 0 \Rightarrow 7t_1 + t_2 = -2 \rightarrow$ (5)

$t_2 = -2 - 7t_1$

this in (4)

$11t_1 + 4(-2 - 7t_1) = -1$

$11t_1 - 8 - 28t_1 = -1$

$-17t_1 = 17$

$t_1 = \frac{-7}{17}$

$t_2 = -2 - 7\left(\frac{-7}{17}\right) \Rightarrow -2 + \frac{49}{17} = \frac{15}{17}$

$t_3 = 2t_1 + 1$

$t_3 = 2\left(\frac{-7}{17}\right) + 1 = \frac{3}{17}$

$t = \begin{pmatrix} \frac{-7}{17} \\ 15/17 \\ 3/17 \end{pmatrix}$

$\therefore$ t is a unique vector when multiplied by 2, gives vector s.

PART-C

$M = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$, $N = \begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix}$, $P = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$

1. eigenvalues and corresponding eigenvalues of M $\rightarrow$

eigenvalues of M $\rightarrow$

$\det(\lambda I - A) = 0$

$\det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}\right) = 0$

let $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = 0$

let $\begin{bmatrix} \lambda-3 & -2 \\ +1 & \lambda-4 \end{bmatrix} = 0$

$= \lambda^2 - 7\lambda + 14 = 0$

by solving this we get $\lambda_1 = \frac{7+i\sqrt{7}}{2}$,

$\lambda_2 = \frac{7-i\sqrt{7}}{2}$

eigenvectors of $M \rightarrow (M - \lambda I) = 0$

for $\lambda_1 \rightarrow \begin{pmatrix} 3-\frac{7+i\sqrt{7}}{2} & 2 \\ -1 & 4-\frac{7+i\sqrt{7}}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -\frac{1+i\sqrt{7}}{2} & 2 \\ -1 & \frac{1+i\sqrt{7}}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\frac{-1+i\sqrt{7}}{2} v_1 + 2v_2 = 0$

$v_1 = \frac{-2}{\frac{-1+i\sqrt{7}}{2}}$ $\quad v_2 \Rightarrow v_1 = \frac{4}{1-i\sqrt{7}} v_2$

$V_1 = \frac{1+i\sqrt{7}}{2} V_2$

$V_1 = \text{eigenvector} = \begin{bmatrix} 1+i\sqrt{7}/2 \\ 1 \end{bmatrix}$

$\lambda_2 = \frac{7-i\sqrt{7}}{2} \Rightarrow M - \lambda_2 = \begin{bmatrix} -\frac{1-i\sqrt{7}}{2} & 2 \\ -1 & -\frac{1-i\sqrt{7}}{2} \end{bmatrix}$

$-V_1 + \frac{1-i\sqrt{7}}{2} = 0 \Rightarrow V_1 = \frac{1-i\sqrt{7}}{2} V_2$

$V_2 = \text{eigen vector} = \begin{bmatrix} \frac{1-i\sqrt{7}}{2} \\ 1 \end{bmatrix}$

2) $\begin{bmatrix} \frac{1+i\sqrt{7}}{2} & 1 \end{bmatrix} \begin{bmatrix} \frac{1-i\sqrt{7}}{2} \\ 1 \end{bmatrix} = 8+1 = 9$

$\therefore V_1 V_2 = 9$

3) The dot product between the eigenvectors of $\underline{N}$.

$\left[N - \lambda I\right] = 0$

$$\begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} = 0$$

$$= 30 - 5d - 6d + d^2 - 9 = 0$$

$$= d^2 - 11d + 21 = 0$$

$$\frac{11 \pm \sqrt{(-11)^2 - 4(1)+21}}{2} = \frac{11 \sqrt{121 - 84}}{2 \cdot 1} = \frac{11 \pm \sqrt{37}}{2}$$

$$d_1 = \frac{11+\sqrt{37}}{2} \quad d_2 = \frac{11-\sqrt{37}}{2}$$

$$M - dI = \begin{bmatrix} 5 - \frac{11+\sqrt{37}}{2} & -3 \\ -3 & 6 - \frac{11+\sqrt{37}}{2} \end{bmatrix}$$

$$= -\frac{1+\sqrt{37}}{2} x_1 - 3x_2 = 0 \Rightarrow -3x_1 + \frac{1+\sqrt{37}}{2} x_2 = 0$$

$$x_1 = \frac{1+\sqrt{37}}{6} x_2$$

$$v_1 = \begin{bmatrix} \frac{1+\sqrt{37}}{6} \\ 1 \end{bmatrix}$$

$$d_2 = \frac{11-\sqrt{37}}{2}$$

$$M - d_2 I = \begin{bmatrix} \frac{-1-\sqrt{37}}{2} & 3 \\ -3 & \frac{1-\sqrt{37}}{2} \end{bmatrix}$$

$$\frac{-1+\sqrt{37}}{}v_1 - 3v_2 = 0$$

$$-3v_1 + \frac{1-\sqrt{37}}{2}v_2 = 0$$

$$\Rightarrow v_1 = \frac{1-\sqrt{37}}{6}v_2$$

$$v_2 \begin{bmatrix} \frac{1-\sqrt{37}}{6} \\ 1 \end{bmatrix}$$

$$v_1 \cdot v_2 = \begin{bmatrix} \frac{1+\sqrt{37}}{6} & 1 \end{bmatrix} \begin{bmatrix} \frac{1-\sqrt{37}}{6} \\ 1 \end{bmatrix}$$

$$= \frac{-36}{36} + 1 = 0$$

④ eigenvectors of N are orthogonal to eachother

* All eigenvalues are real

* symmetric

⑤ $pt = 0$

Trivial Solution

$$t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(6) $\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2t_1 + 4t_2 = 0$

$4t_1 + 8t_2 = 0$

$t_1 = -2t_1$

for $t_2 = 1$

$t = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

for $t_2 = -1$

$t = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ //

(7)

$Mt = 0$

$M = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

$|M| = 12 + 2 = 14 \neq 0$

$\therefore$ Only solution for $t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as M is

invertible.

(1) **PART - D**

$x_1 = 0.5$ $x_2 = 0.8$ $x_3 = 0.3$

$w_1 = 0.4$ $w_2 = 0.6$ $w_3 = 0.9$

bias $= 0.2$

$y = wx + b$

$y = (0.4)(0.5) + (0.6)(0.8) + (0.9)(0.3) + (0.2)$

$= 0.2 + 0.4 + 0.27 + 0.2$

$= 1.15$

(2) Sigmoid Activation fun

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\sigma(1.15) = \frac{1}{1+e^{-(1.15)}} = 0.759$$

(3) Given values same and weights as question 1, compute the output of the neuron after applying a ReLU activation function

$ReLU(x) = \max(0, x)$

$ReLU(1.15) = \max(0, 1.15)$

$= 1.15$

(4) $W_h = \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ $b_h = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$y = W_h \cdot x + b_h$

$= \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$

$= \begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix}$

$ReLU \begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix} = \begin{bmatrix} \max(0, 1.1) \\ \max(0, 1.5) \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix}$

(5) $W_o = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}$ $b_o = [0.1]$

$y = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix} \begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix} + [0.1]$

$= \begin{bmatrix} 0.55 & -0.45 \end{bmatrix} + [0.1] = [0.2]$

$\sigma(0.2) = \frac{1}{1+e^{-0.2}} = 0.549$

(6) $x = [x_1 \quad x_2]$ $z = [z_1 \quad z_2]$

w_o b_o

w_h b_h

$L = yx + b$ $y = w_o z + b_o$

$z = W_h x + B_h$

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$$

PART-E

$f(x) = 2x^2 - 1$ $g(x) = 3x^2 + 4$

$h(x,y) = x^2 + y^2 + xy$

(1) $f'(x)$ and $f''(x)$

$$f'(x) = \frac{d}{dx}(2x^2 - 1) = 4x$$

$$f''(x) = \frac{\partial}{\partial x} 4(x) = 4$$

(2) partial derivatives

$\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$

$$\frac{\partial h}{\partial x} = \frac{\partial(x^2+y^2+xy)}{\partial x} = 2x+y$$

$$\frac{\partial h}{\partial y} = \frac{\partial(x^2+y^2+xy)}{\partial x} = 2y+x$$

(3) $\nabla h(x,y)$

$$= \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right) = (2x+y, 2y+x)$$

(4) $\frac{\partial}{\partial x} f(g(x))$. $f(x) = 2x^2+1$ $g(x) = 3x^2+4$

$$f(g(x)) = 2(3x^2+4)^2-1$$

$$= 2(9x^4+16+24x^2)-1$$

$$= 18x^4+48x^2+31$$

$$\frac{\partial}{\partial x} f(g(x)) = \frac{\partial}{\partial x} 18x^4+48x^2+31$$ without chain rule

$$= 72x^3+96x$$

$$\frac{\partial}{\partial x} + g(x) = \frac{\partial + f(x)}{d\,g(x)} \cdot \frac{d\,(g(x))}{d(x)}$$

$$= \frac{d\,2g(x)^{2-1}}{d\,g(x)} \cdot \frac{d\,(3x^2+4)}{dx}$$

$$= 4\,(g)x \cdot 6(x)$$

$$= 4\,(3x^2+4)\,(6x)$$

$$= (12x^2+16)\,6x = 72x^3 + 96x$$

with chain rule