Univariate simulation

For fixed regression parameter β_1, non-parametric function f, variance components $\boldsymbol{\theta} = (\phi, \sigma^2, \theta_2, \theta_3)$, we generate the covariate X, the random intercept b_i, the OU process U_i and the measurement error ϵ with the assumption that all observation time points are the same for all subjects $t_i = t$. Denote Y_{ij} to be the univariate hormone response for subject i at time point j, we simulate the longitudinal response data according to the model

$$Y_{ij} = \beta_1 \text{age}_i + f(t_i) + b_i + U_i(t_{ij}) + \epsilon_{ij}.$$

where the nonparametric periodic function of time is generated from the sine function $f(t) = 5\sin(\pi t/15)$ with period length 30; the b_i are independent random intercepts following a normal distribution with mean zero and variance ϕ; the mean-zero Gaussian process U_i modelling the serial correlation are simulated from stationary OU process with covariance function $cov(U_i(s), U_i(t)) = \theta_3^2 exp\{-\theta_2|s-t|\}/2\theta_2$, and the independent measurement errors ϵ_i are from Normal distribution with mean zero and variance σ^2. The simulated response is of one cycle for now.

The variance components $\theta = (\phi, \boldsymbol{\sigma^2}, \boldsymbol{\theta_2}, \boldsymbol{\theta_3})$ and the smoothing parameter λ will be estimated using Fisher-Scoring algorithm.