

Ex. $x^3 + 3x - 1$ $a=0$ $b=1$ $\varepsilon=2^{-23}$

1st $f(a) = -1$ $f(b) = 3$ $c = \frac{0+1}{2} = .5$

iter $f(c) = .5^3 + 3(.5) - 1 = .625$

$-1 - .625 = -.625 < 0$ $b \leftarrow c$

2nd $a=0$ $b=.5$ $f(a) = -1$ $f(b) = .625$

iter $c = \frac{0+.5}{2} = .25$ $f(c) = .25^3 + 3(.25) - 1 = -.234375$

$-1 - -.234375 = .234375$ $a \leftarrow c$

3rd $a=.25$ $b=.5$ $f(a) = -.234375$ $f(b) = .625$

$c = \frac{.25+.5}{2} = .375$ $f(c) = .17734375$

$f(a) \cdot f(c) = -.04165649414$ $b \leftarrow c$

4th $a=.25$ $b=.375$ $f(a) = -.234375$ $f(b) = .17734375$

iter $c = \frac{.25+.375}{2} = .3125$ $f(c) = -.03198242188$

$f(a) \cdot f(c) = .00749588012$ $a \leftarrow c$

5th $a=.3125$ $b=.375$ $f(a) = -.03198242188$

iter $c = \frac{.3125+.375}{2} = .34375$ $f(c) = .07186889648$

$f(a) \cdot f(c) = -.00229854136$ $b \leftarrow c$

$$6\text{th} \quad a = .3125 \quad b = .34375 \quad f(a) = -.03198242188 \\ c = \frac{.3125 + .34375}{2} = .328125 \quad f(b) = .07186889648$$

$$\text{iter} \quad f(c) = .01970291138$$

$$f(a) \cdot f(c) = -.00063014682 \quad b \leftarrow c$$

$$7\text{th} \quad a = .3125 \quad b = .328125 \quad f(a) = -.03198242188 \\ \text{iter} \quad c = \frac{.3125 + .328125}{2} = .3203125 \quad f(b) = .01970291138$$

$$f(c) = -.00619840621$$

$$f(a) \cdot f(c) = .00019824004 \quad a \leftarrow c$$

$$8\text{th} \quad a = .3203125 \quad b = .328125 \quad f(a) = -.00619840621 \\ \text{iter} \quad c = \frac{.3203125 + .328125}{2} = .32421875 \quad f(b) = .01970291138$$

$$f(c) = .00673741102$$

$$f(a) \cdot f(c) = -.00004176121 \quad b \leftarrow c$$

$$9\text{th} \quad a = .3203125 \quad b = .32421875 \quad f(a) = -.00619840621 \\ \text{iter} \quad c = .322265625 \quad f(b) = .01970291138$$

$$f(a) \cdot f(c) = -.00000164762$$

$$f(c) = .00026581436 \quad b \leftarrow c$$

$$10\text{th} \quad a = .3203125 \quad b = .322265625 \quad c = .3212890625$$

$$\text{iter} \quad \text{root} \approx .3212890625$$

- 1st iter $x^3 - 2 \sin(x)$ $a = .5$ $b = 2$ $E = 2^{-23}$
 $f(a) = -.8333510772$ $f(b) = 6.181405146$
 $c = \frac{.5+2}{2} = 1.25$ $f(c) = .05515576129$
 $f(a) \cdot f(c) = -.04599169097$ $b \leftarrow c$
- 2nd iter $a = .5$ $b = 1.25$ $c = .875$ $a \leftarrow c$
 $f(c) = -.8651651295$ $f(a) \cdot f(c) = .7214188752$
- 3rd iter $a = .875$ $b = 1.25$ $c = 1.0625$ $a \leftarrow c$
 $f(c) = -.5476869797$ $f(a) \cdot f(c) = \text{pos}$
- 4th iter $a = 1.0625$ $b = 1.25$ $c = 1.15625$
 $f(c) = -.2847914008$ $f(a) \cdot f(c) = \text{pos}$ $a \leftarrow c$
- 5th $a = 1.15625$ $b = 1.25$ $c = 1.203125$
 $f(c) = \text{neg}$ $f(a) \cdot f(c) = \text{pos}$ $a \leftarrow c$
- 6th $a = 1.203125$ $b = 1.25$ $c = 1.2265625$
 $f(c) = \text{neg}$ $f(a) \cdot f(c) = \text{pos}$ $a \leftarrow c$
- 7th $a = 1.2265625$ $b = 1.25$ $c = 1.23828125$
 $f(c) = \text{pos}$ $f(a) \cdot f(c) = \text{neg}$ $b \leftarrow c$
- 8th $a = 1.2265625$ $b = 1.23828125$ $c = 1.232422$
 $f(c) = \text{neg}$ $f(a) \cdot f(c) = \text{pos}$ $a \leftarrow c$

a_{th} a=1.232422 b=1.2382815 c=1.23535325

f(c)=neg f(a)·f(c)=pos a< c

10th a=1.23535325 b=1.2382815 c=

root ≈ 1.236817375

1st $x + 10 - x \cos\left(\frac{50}{x}\right)$ a=120 b=130 ε=2⁻²³

iter f(a)=neg f(b)=pos c=125

f(c)=neg f(a)·f(c)=pos a< c

2nd a=125 b=130 c=127.5 f(a)·f(c)=neg

f(a)=neg f(b)=pos f(c)=pos b< c

a=125 b=127.5 c=126.25 f(a)·f(c)=pos

3rd f(a)=neg f(b)=pos f(c)=neg a< c

4th a=126.25 b=127.5 c=126.875

f(a)=neg f(c)=pos f(a)·f(c)=neg b< c

5th a=126.25 b=126.875 c=126.5625

f(a)=neg f(c)=neg f(a)·f(c)=pos a< c

a=126.5625 b=126.875 c=126.71875

f(a)=neg f(c)=pos f(a)·f(c)=neg b< c

7th $a = 126.5625$ $b = 126.71875$ $c = 126.640625$
 $f(a) = \text{neg}$ $f(c) = \text{pos}$ $f(a) \cdot f(c) = \text{neg}$ $b \leftarrow c$

8th $a = 126.5625$ $b = 126.640625$ $c = 126.6015625$
 $f(a) = \text{neg}$ $f(c) = \text{neg}$ $f(a) \cdot f(c) = \text{pos}$ $a \leftarrow c$

9th $a = 126.6015625$ $b = 126.640625$ $c = 126.6210938$
 $f(a) = \text{neg}$ $f(c) = \text{neg}$ $f(a) \cdot f(c) = \text{pos}$ $a \leftarrow c$

10th $a = 126.6210938$ $b = 126.640625$

root ≈ 126.6308594

1st Ex2. $x_0 = 2$ $\epsilon = 2^{-23}$ $f(x) = x^3 + 2x^2 + 10x - 20$

iter $f'(x) = 3x^2 + 4x + 10$ $f(2) = 16$ $f'(2) = 30$

$$x_1 = 2 - \left(\frac{16}{30}\right) = \frac{22}{15}$$

2nd $f\left(\frac{22}{15}\right) = \frac{7168}{3375}$ $f'\left(\frac{22}{15}\right) = \frac{558}{25}$ $x_2 = \frac{22}{15} - \left(\frac{\frac{7168}{3375}}{\frac{558}{25}}\right)$
 $= 1.371512014$

$f(1.371512014) = .05708664601$

$f'(1.371512014) = 21.12918367$ $x_3 = 1.368816223$

$$\left| \frac{f(1.371512014)}{f'(1.371512014)} \right| = .00270179136 > \epsilon$$

$$f'(x) = 3x^2 + 4x + 10 \quad f(x) = x^3 + 2x^2 + 10x - 20$$

4th $f(x_3) = .00004462213 \quad x_4 = 1.368808108$

$$f'(x_3) = 21.09616517$$

5th $f(x_4) = .00000000376$

$$f'(x_4) = 21.09613934$$

1st Ex3. $x_0 = 2 \quad x_1 = 1 \quad \varepsilon = 2^{-23} \quad f(1) = -7$
it's $f(2) = 16$

$$x_2 = 1 - \frac{(1-2)f(1)}{f(1)-f(2)} = \frac{30}{23}$$

2nd $x_3 = \frac{30}{23} - \frac{\left(\frac{30}{23}-1\right)f\left(\frac{30}{23}\right)}{f\left(\frac{30}{23}\right)-f(1)} = 1.37605362 \quad f\left(\frac{30}{23}\right) = -1.334757952$

3rd $x_4 = x_3 - \frac{(x_3-x_2)f(x_3)}{f(x_3)-f(x_2)} = 1.368671954 \quad f(x_3) = 1531732866$

4th $x_5 = x_4 - \frac{(x_4-x_3)f(x_4)}{f(x_4)-f(x_3)} = 1.368808108 \quad f(x_4) = -0.00287120679$

5th $x_6 = x_5 - \frac{(x_5-x_4)f(x_5)}{f(x_5)-f(x_4)} = 1.368808108 \quad f(x_5) = -0.00000601862$

root ≈ 1.368808108