Summary.

Assumptions

•To solve this problem, we had to restrict ourselves in scope. So we took the round hole nozzle to mean that the spray came out in a full cone shape and then devolved into a mist. Other spray patters can be found in **fig 4**.

•Following on from the previous, we also assumed that the material came out of the nozzles in a continuous manner (i.e. infinitely small droplets). For example our model is more akin to the light coming out of a lightbulb than powder coming out of an orifice. This allows us to deal with the material incident on the strip in terms of flux.

•We have assumed that all material sticks to the strip, if it is incident upon it. Not only that but also that none of the mist "floats away", so it all of the material goes where it is directed. However I feel that this could be approximated to a reasonable degree by a constant factor.

• We have neglected gravity in all these calculations, as we can assume the droplets are moving very fast and the distance to travel is small so gravity has little time to act.

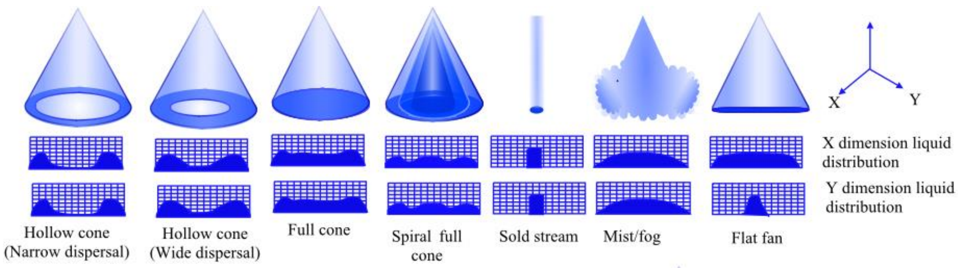


Figure 1, Different Spray Patterns

Background.

Process

In this report, for Distributions 2,3,4,5. The same method of determining the Percentage Variation is adhered to. In order to make this report clearer to the reader, this method will be outlined here. Each of these Nozzle spray distributions is based on the premise that the flux of material landing on any one point is proportional to the angle between the line connecting the point and the nozzle with the line dropped perpendicularly from the nozzle to the strip, as shown in **Figure 1**. Now this has been defined:

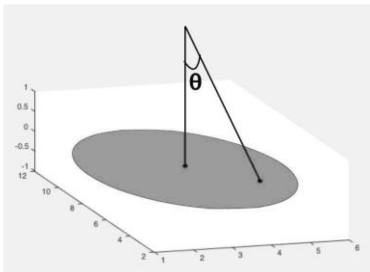


Figure 2, Nozzle is at the angle. The shaded area is the circle of incidence on the strip

1) We substitute our formula for the angle θ into the relationship $Flux\ \alpha\ f(\cos(\theta))$.

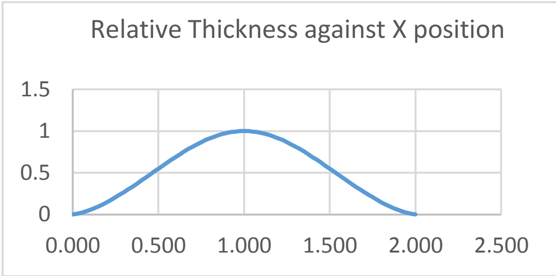


Figure 3, Cos2ϑ Dynamic pattern

2) This is then integrated with respect to y, at many different x values, changing the view from static i.e. a Nozzle spraying continuously down onto a stationary strip, to a Dynamic one. In this dynamic view the only thing we need to concern ourselves with is how the thickness of the coating varies as you move along the x axis of the strip, as in **Figure 2**

3) The values for these Integrations are then fed into a python script which simulates placing 10 nozzles in a row on a moving strip. The summed or combined distribution is calculated and the percentage variation in thickness is measured. This percentage Variation is measured for 80 different widths of gaps. Ranging from gap with = 0.025 (almost completely on top of each other) of the radius of the nozzle to 2 Radii (completely separate nozzles).

4) The Percentage Variation is then plotted against the Gap with in order to find the optimum points. Once we have these optima we can work out the wastages due to over spray at the sides, given the gap length.

Positioning of Axes

In this report we will take the axes to be as shown in the diagram, x representing Horizontal Displacement along the strip, Y representing vertical displacement also in the plane of the strip, and Z representing the Displacement up and outwards from the strip usually as the thickness of material or the magnitude of the flux. This set up is visualised in **Figure 3.**

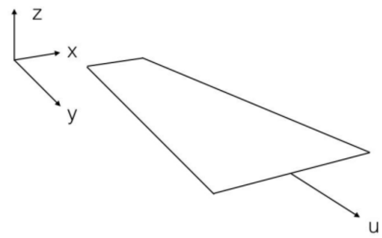


Figure 4, The Axes

Distributions:

In the project we have concentrated on 5 separate static distributions, one of them is less accurate than the others. Here we take "distribution" to mean the flux pattern produced on a stationary strip by a Nozzle positioned d metres above it. These are the 5 distributions:

-A Uniform Distribution – each point in the circle of incidence receives exactly the same amount of flux, no matter where the point is in that circle.

-Cos(2θ) Distribution - This is rather more complicated than the uniform distribution. Each point in the in the circle of incidence receives an amount of flux proportional to cos(2θ) of the angle between the line connecting the point and the Nozzle, and the line between the nozzle and the centre of the circle of incidence. Corresponds to a nozzle with total spray angle of 45°.

-Cos(3θ) Distribution - Almost identical to the previous distribution. However instead of the flux being proportional to Cos(2θ) of the angle, it is now proportional to Cos(3θ) of the angle. This Corresponds to a nozzle with total spray angle of 60°.

-Cos(5θ) Distribution – The flux at a particular point is proportional to Cos(5θ), - corresponding to a nozzle with total spray angle of 36°.

-Cos^2 (2θ) Distribution - This distribution is essentially the same as the Cos(2θ) Distribution, with the same Nozzle spray angle of 90°. However this distribution is more "concentrated" than the others. Proportionally, points with a smaller θ receive much more flux than a point with a larger θ. While this is true in the last 4 Distributions that I have described it is much more pronounced in this Cos^2 (2θ) Distribution.

Uniform Distribution:

This distribution has limited applicability in this area. It is only loose representation of the solid stream jet, intended to indicative of the method for the next Distributions. In reality the Solid Stream jet is mostly used for cutting or cleaning, due to its high power/volume flow rate.[2]

The Thickness pattern or Dynamic pattern, **Figure 5** for this distribution can be shown to be:

$$H(x) = \frac{2v\sqrt{R^2 - (x)^2}}{\pi R^2 U}$$

Where H(X) is the thickness at point x. v is the volume flow rate from the nozzle. R is the radius of the pattern. U is the velocity of the strip.

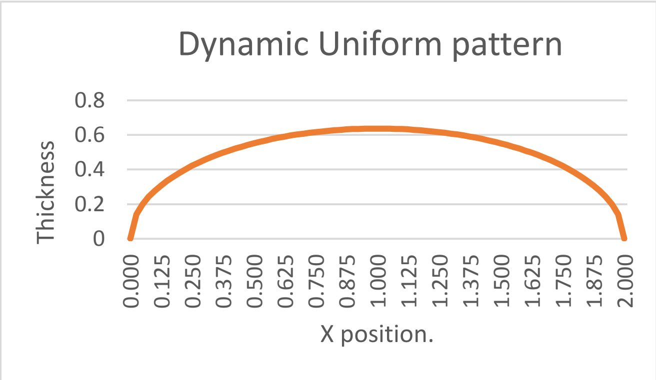


Figure 5

Now we can begin to explore the question. If we take **n** of these nozzles and space them at regular intervals of say **m** radii then we get the formula:

$$H(x) = \frac{2V}{\pi R^2 U} \sum_{m=1}^{n} \sqrt{R^2 - (x - mR)^2}$$

(Only considering the real part)

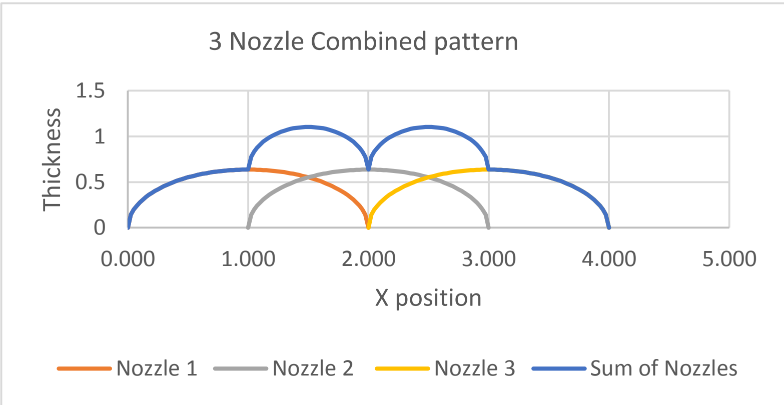


Figure 6

Now by varying the value of **m** from 0 to 2 we can change the gap in between the dynamic nozzle distributions. The aim of this report was to reduce the roughness of the shape formed as much as possible. So in order to do that, we only considered part of the graph (from centre of nozzle 1 to centre of nozzle **n**-1) to avoid the irregularity at the sides.

In order to measure the Percentage Variation of the Total Dynamic Distribution, we used the following formula:

$$Percentage\ Variation = 100 \times \frac{(\max height - \min height)}{\max height}$$

This can be thought of as how much the height of the Distribution varies as a percentage of the maximum height.

Our aim is to minimise this Percentage Variation, by making the minimum height as close to the maximum height as possible.

In this Case it occurs at Gap size = 0.450, 0.600 and 0.900, with Percentage Variation 9.22, 8.20 and 14.05 respectively.

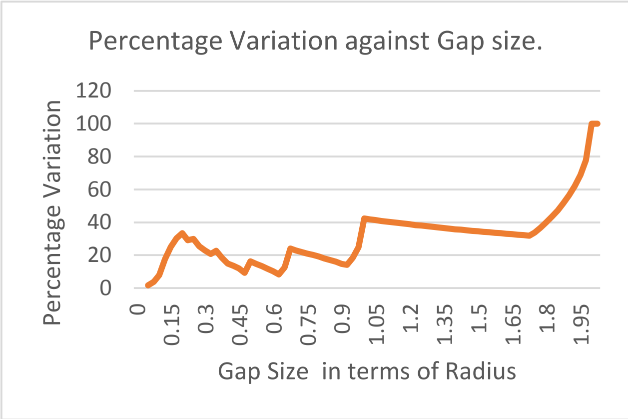


Figure 7

Cos(2θ) Distribution

This is a great deal more complicated than its Uniform cousin. In order to understand what is going on with this distribution, I have neglected velocity and volume flow from the nozzle. This sounds rather counter intuitive however it greatly simplifies the maths and allows us to think in terms of relative values. These values can be taken into account at the end of the calculations if necessary. The velocity and the volume flow only compress and stretch the pattern produced on the moving strip. This means that you can determine the Percentage Variation without considering either.

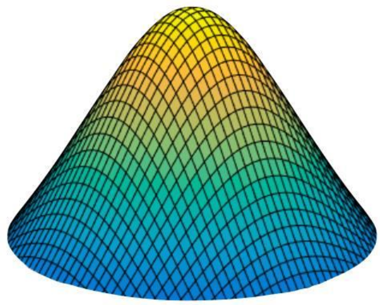


Figure 8, Cos(2theta) Static pattern

This distribution means that the flux at any one point inside the circular region of incidence, is proportional to Cos(2θ). Definition of the angle θ is given in the appendix. We define our function over the domain $\theta \in [-45, 45]$ thereby describing a nozzle with a total spray angle of 90 °. When $\theta \geq 45$ or $\theta \leq -45$ we call the value of the function 0. This in fact gives us our circle of incidence.

From part A2 of the appendix it follows:

$$Cos(\theta) = \frac{d}{\sqrt{x^2 + y^2 + d^2}}$$

Where **d** is the perpendicular height of the nozzle above the stationary strip. x and y are the position coordinates on the strip.

From this we can see that the static pattern formula is:

$$Cos(2\theta) = \frac{2d^2}{(x^2+y^2+d^2)} - 1 \quad \text{With the condition } x^2 + y^2 \leq d^2$$

This is our defining equation for this Distribution. In order to convert this from the Static case to the Dynamic case, we integrate along lines parallel to the y axis with respect to y. This gives us the product of $flux(y) \times\ y\ distance$. Giving us the total flux along the line. We do this for many different x values. (40 values from x = 0 to x=d, then use the same values in reverse for x = -d to 0 as symmetric – 80 in total). This gives us the Dynamic pattern for one nozzle, with centre x = 0, and domain [-d, d].

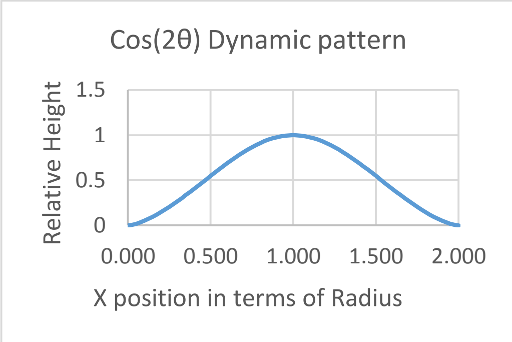
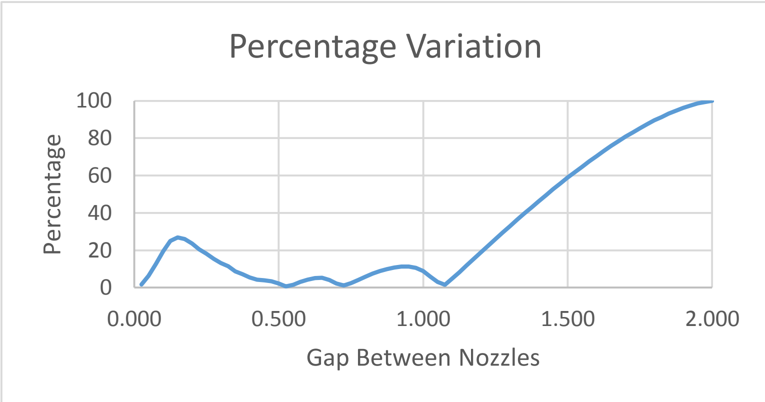


Figure 9

The question asks us to determine the "Uniformity". In order to measure this we defined instead the Percentage Variation (in part X) along the Dynamic pattern. So to start with we placed put 2 of these patterns together both with centre x = 0, and calculated the Sum value of these two nozzles at each x point. Obviously this produces just the same pattern except with twice the relative thickness of one nozzle. Then we played around with the spacing's of the centres to give us something that looked reasonably flat in the middle. Following on from this we then increased the number of nozzles to 10, and moved our interval of interest to [$Centre_2$, $Centre_9$]. In order that we might again neglect the irregularities at the edges. Fig

Figure 10, Cos3theta



The next logical step was to increase the gap between the centre of the 10 Nozzles incrementally by 0.025**d** (as we integrated the Static pattern 40 times at intervals of 0.025**d**) and measuring how flat the distribution produced was in our interval of interest, using our Percentage Variation value outlined in the Uniform Distribution section**.** This was the graph produced:

Table 1

Gap size (Radius)	Percentage Difference	Wastage (Nozzles)
0.525	0.672	3.0
0.725	1.138	3.0
1.075	1.617	3.0

The area inside these two lines is our Area of Interest. Everything outside these lines is wastage.

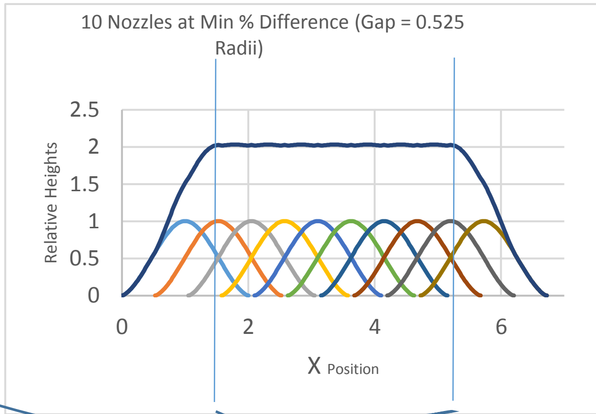


Figure 11, Cos2theta

We were justified in using only 10 Nozzles when in fact there could be more. This is because the shape produced is periodic, with period equal to 2 Radii, however this is only the case once the pattern stabilises.

We are interested in the minima of this graph. These are tabulated in Table 1. This indicates that the Nozzles should be placed at these intervals in order to achieve minimum Percentage difference. The wastage at the sides is calculated from the area in the interval [0, $Centre_2$] and [$Centre_9$, Ending $Nozzle_{10}$], decision of which nozzle arrangement is best is left to the readers own interpretation. I have given wastage to 2 decimal places, because there is considerable uncertainty in its value.

$$Wastage = \frac{Area\ outside\ of\ interest}{Area\ of\ Nozzle}$$

This definition allows us to keep things in relative terms, to stay as widely applicable as possible. This Wastage then has units m^3s^{-1}, or volume flow It also means a Value of 1 for the wastage implies that a whole nozzle's worth of flow is being wasted. See appendix for full calculation of the Wastage Values in Table 1.

Cos(3θ) Distribution

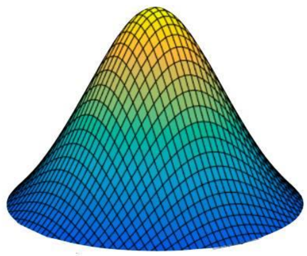


Figure 12, Cos(3theta) Static

This Distribution is essentially the same as the $Cos(2\theta)$. The difference lies rather obviously in the "3θ". Now our Distribution function is defined over the Domain $\theta \in [-30\ , 30]$ this describes a nozzle with total spray angle 60°. It must be noted that we were rather lax in our definition of **d** in the previous Section. This was because **d**, the height of the nozzle, and **R**, the radius of the circle of incidence, were equal ($Radius = tan(45)$. However in this case $Radius = dtan(30)$ and so we have to be careful to take this into account in this new distribution. All of this results in a distribution (Both Static and Dynamic), which when viewed on an absolute scale alongside the cos(2θ) counterpart, give a narrower distribution. The Cos(3θ) appears to be squashed in the x and y axis.

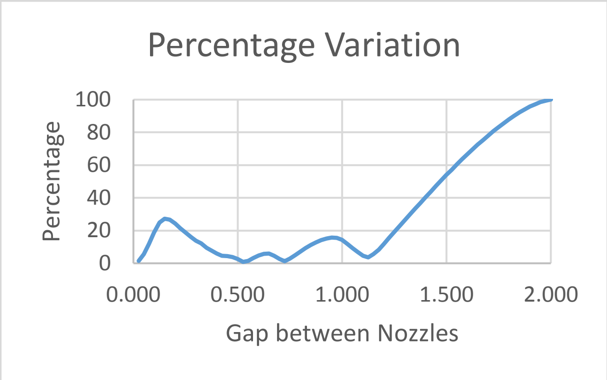
The whole process was repeated, and the resultant equation for this distribution is:

$$Cos(3\theta) = \frac{4d^3}{(x^2+y^2+d^2)^{\frac{3}{2}}} - \frac{3d}{(x^2+y^2+d^2)^{\frac{1}{2}}} \text{ With the condition } x^2 + y^2 \leq (dtan30)^2$$

Again we integrate the right hand side of this expression with respect to y at many regularly spaced x intervals to convert from static to dynamic. When the 10 nozzles are again modelled and the python script implemented the results in Table 2 are produced. The Graph of percentage Variation against Gap between nozzles is shown below, **Figure 14.**

Table 2

Gap Size (Radii)	Percentage Variation	Wastage (Nozzles)
0.500	0.846	3.0
0.700	1.273	3.0
1.100	3.69	3.0



< - Figure 14, Cos3theta

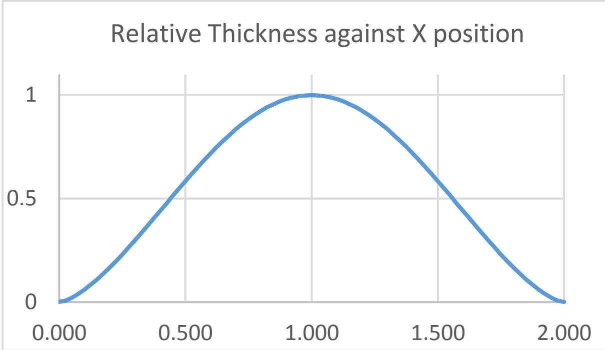


Figure 13, Cos3theta

<u>Cos(5θ) Distribution</u>

Same Process as the previous two Distributions. This time around the Nozzle has total angle of spray 36° as seen by Cos(5θ). It is only defined over the domain $\theta \in [-18\,, 18]$ and the Radius now becomes $Radius = d(tan18)$. This distribution is even narrower than the Cos(3θ) Distribution.

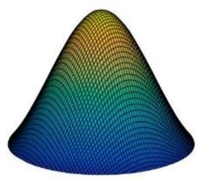


Figure 15, Cos5theta

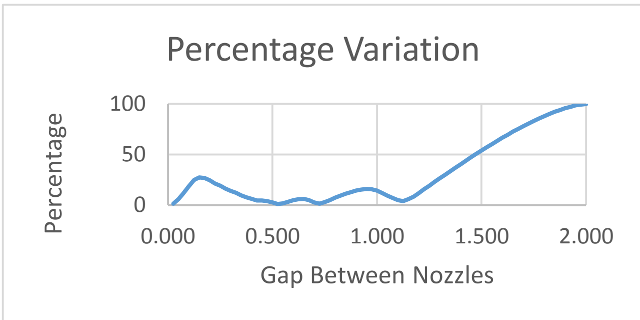
The defining equation for this Distribution is:

$Cos(5\theta) = \frac{16d^5}{(x^2+y^2+d^2)^{5/2}} - \frac{20d^3}{(x^2+y^2+d^2)^{\frac{3}{2}}} - \frac{5d}{(x^2+y^2+d^2)^{\frac{1}{2}}}$ With the condition $x^2 + y^2 \leq (dtan18)^2$

We convert it from Static to Dynamic via step 2 of the Process described in X. That done, the data is then fed into the python script to produce the Percentage Variation data. Explanation of this Python script is provided in Appendix B, B6.

Table 3

Gap Size (Radius)	Percentage Variation	Wastage (Nozzles)
0.500	1.085	2.9
0.700	1.732	3.0
1.125	5.139	3.0



Cos^2(2θ) – Distribution

Figure 16, Cos5theta

This distribution is a little more interesting than the others, however the same process applies. Squaring the term gives rise to a more concentrated distribution. As $cos(2\theta)\ \epsilon\ [-1,1]$, squaring it produces a value which is always smaller than or equal to its unsquared value. In other words, flux at the edges is much smaller, and the slopes much steeper than in comparison to the cos(2θ) Distribution. The angle of this nozzle is 45°. In this case the Radius equals the Height of the nozzle above the strip.

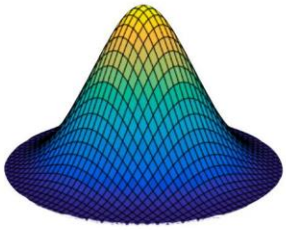


Figure 17, Cos^2(2theta) Static

The equation is:

$$Cos^2(2\theta) = \left(\frac{2d^2}{(x^2+y^2+d^2)} - 1\right)^2 \quad \text{with the condition } x^2 + y^2 \leq d^2$$

In converting from Static to Dynamic you can clearly see how the graph becomes much more concentrated than in the other distributions. This indicates the middle part of the circle of incidence receives nearly all of the coating from the nozzle.

In this Diagram X you can clearly see concentration increase in comparison to the Cos(2θ) Dynamic pattern.

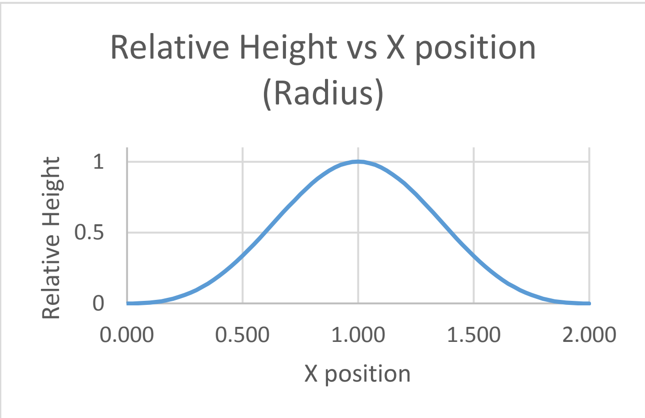


Figure 19, Cos^2(2theta)

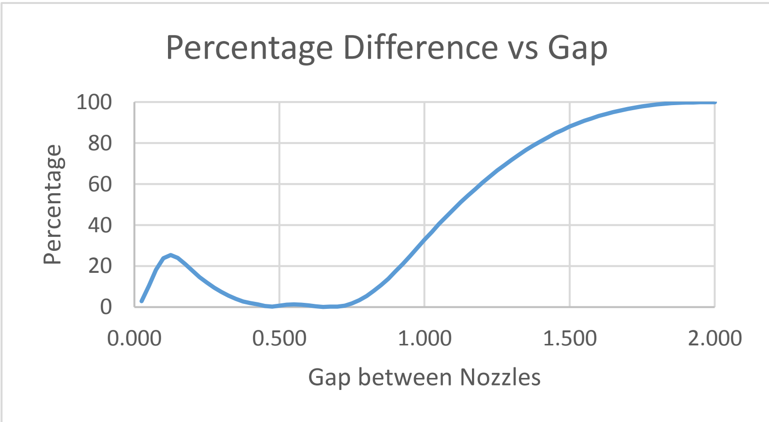


Figure 18, Cos^2(2theta)

Note also the difference in shape of this graph compared to the others. It has a much larger region of much smaller Percentage variation. The smallest Percentage Variation for this Nozzle type is 0.0878% which is a great deal smaller than the minima for the other distributions that we looked at.

Gap Size (Radius)	Percentage Variation	Wastage (Nozzles)
0.475	0.213	3.0
0.650	0.0878	3.0

Table 4

Comparing the Distributions.

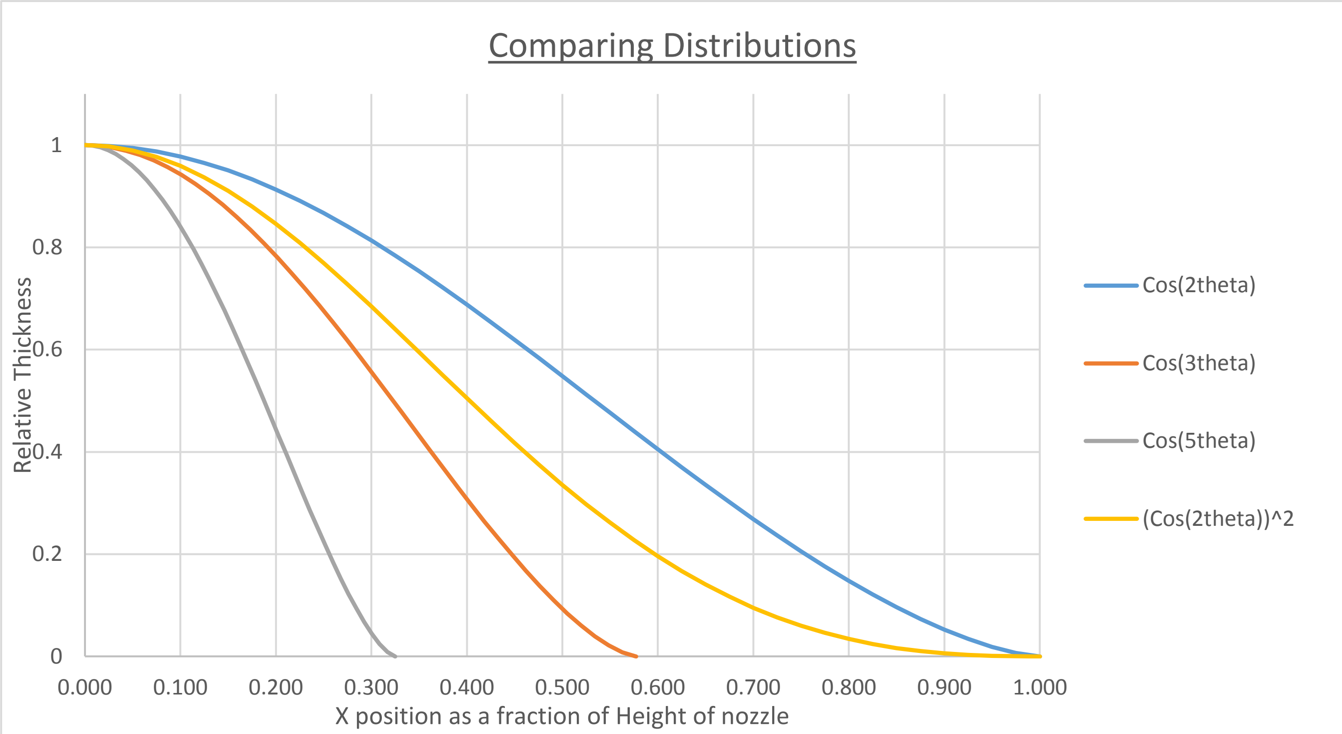


Figure 20, Comparing all the Distributions

This graph (fig) indicates the Half Dynamic patterns for each other one we looked at. Each is plotted for the same Nozzle height. However their Thicknesses have been normalised to 1 in order to compare their shapes better. (This indicates they have different nozzle volume flow rates as they have different areas.) Note how the Cos(3θ) and Cos(5θ) graphs reach 0 well before the other two. This is because the nozzle angles are smaller in these cases so you will need more than twice the Cos(5θ) nozzles in order to cover the same length as the Cos(2θ) case, if they are the same distance above the strip.

Caveats.

A great deal of the data and graphs here have been produced using sequences of numerical methods. For example in creating the dynamic patterns we integrated 40 separate times in order to get 40 different values. This discretisation makes the maths and analysis a great deal easier and more manageable. However, it also means that we may be missing the picture slightly. In the Percentage Variations graphs there will most likely be gaps between nozzles which produce a percentage variation very much lower than the ones we found. There might even be a 0 point!

The reason that we did not try to do more and more integrations to try and increase the "resolution" of our data, is that there is simply very little chance a real world nozzle will fit our distribution (Cos(2θ or others) so exactly that our data will reflect it perfectly. It may have merit as a purely theoretical exercise, but as a practical piece of information it would be of very little use.

Conclusion

From the results given in this report we can conclude that there are indeed nozzle spacing's which minimise the variations in thickness when positioned in rows and incident on a moving strip. For the unsquared Distributions we looked at, they all occurred in roughly the same place. When the gap between the nozzles is 0.500 Radii. This is exactly as it should be because those distributions are essentially the same, just stretched or squashed in the x axis. With the data produced by us we were able to determine with certainty that a Nozzle with a $Cos^2(2\theta)$ Distribution of flux can have a percentage variation 0f 0.0878% and most likely smaller than that if we were to increase the resolution of our data by increasing the number Integrations in step 2.

The wastage volume calculated for all of our different distributions was disappointingly high. However the wastage as a percentage of the total volume used can be decreased by using many more nozzles than the 10 we did our calculations for. Alternatively one could explore with combinations of nozzles with varying volume flows, for example putting a smaller nozzle on at either end of the strip to increase the region of uniformity so less volume needs to be wasted.

In our opinion, this project has merit, however in order to be useful in any sort of practical context it would be required that: more arrangements need to be considered and a greater volume of practical data on different nozzle spray patterns needs to be analysed. This would enable the reader to implement the basic findings in this report with a much larger degree of confidence.

References:

[1] Picture of different spray patterns.

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Company who published it - BETE

http://www.bete.co.uk/resources/engineering-resources/guide-to-spray-properties/2--basic-spray-patterns

[2] Accessed online 17/03/15

Lechler

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Appendix A- Mathematical Derivations

A.1 Thickness Pattern for Uniform Distribution.

(1) Volume of Cylinder:

$$V = \pi R^2 H$$

(2) Volume flow from Nozzle:

$$\frac{dV}{dt} = v \quad \text{Implies } V = vt \text{ given } V(t = 0) = 0$$

(3) Time spent in flow:

$$t = \frac{2\sqrt{d^2 - x^2}}{U}$$

Substituting 2 into 1: (4)

$$H = \frac{vt}{\pi R^2}$$

Substituting 3 into 4:

$$H(x) = \frac{2v}{\pi R^2 U}\sqrt{R^2 - x^2}$$

Where:
v is the volume flow rate out of the nozzle, ***R*** is the radius of the circle of incidence, ***U*** is the speed of the strip and ***x*** is the position.

A.2 Deriving the function of θ in terms of the *x* and *y* position.

$$\boldsymbol{D} = \begin{pmatrix} 0 \\ 0 \\ d \end{pmatrix}, \quad \boldsymbol{r} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} y, \quad \boldsymbol{r} \to \boldsymbol{D} = \begin{pmatrix} -x \\ 0 \\ d \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} y$$

Taking the dot product of these two vectors:

$$(\boldsymbol{r} \to \boldsymbol{D}) \cdot \boldsymbol{D} = |r - D||D| \cos\theta = d^2$$
$$|\boldsymbol{r} \to \boldsymbol{D}| = \sqrt{x^2 + y^2 + d^2}\,, \ |\boldsymbol{D}| = d$$

Rearranging gives:

$$Cos(\theta) = \frac{d}{\sqrt{x^2+y^2+d^2}}$$
$$\therefore \theta = \cos^{-1}\left(\frac{d}{\sqrt{x^2+y^2+d^2}}\right)$$

A.3 The formulae for the distributions

A3.1 Case 1. The Cos(2θ) Distribution. This implies that the flux at any given point is proportional to Cos(2θ) of the angle joining the point to the Nozzle and from the Nozzle to the strip.

$$\cos(2\theta) = 2cos^2\theta - 1$$

-Upon substituting in the formula previously found for θ.

$$=2\left(\frac{d^2}{x^2+y^2+d^2}\right) - 1$$

A3.2. The Cos(3θ) Distribution. This implies that the flux at any given point is proportional to Cos(3θ) of the angle joining the point to the Nozzle and from the Nozzle to the strip.

$$\cos(3\theta) = \cos(2\theta + \theta)$$

$$=\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$

$$= (2cos^2\theta - 1)\cos\theta - 2\, sin^2\theta \cos\theta$$

$$= (2cos^2\theta - 1)\cos\theta - 2(1 - cos^2\theta)\cos\theta$$

$$= 2cos^3\theta - \cos\theta - 2\cos\theta + 2\, cos^3\theta$$

$$= 4\, cos^3\theta - 3\cos\theta$$

$$= 4\left(\frac{d^3}{(x^2+y^2+d^2)\sqrt{x^2+y^2+d^2}}\right) - 3\left(\frac{d}{\sqrt{x^2+y^2+d^2}}\right)$$

-Upon substituting in the formula previously found for θ.

A3.3 Case 3. The Cos(5θ) Distribution. This implies that the flux at any given point is proportional to Cos(5θ) of the angle joining the point to the Nozzle and from the Nozzle to the strip.

$$\cos(5\theta) = \cos(3\theta + 2\theta)$$

$$= \cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta$$

$$= (4\, cos^3\theta - 3\cos\theta)\,(2cos^2\theta - 1) - (*\, 4\sin\theta\, cos^2\theta - \sin\theta)(2\sin\theta\cos\theta)$$

$$= 8cos^5\theta - 4cos^3\theta - 6cos^3\theta + 3\cos\theta - 8sin^2\theta cos^3\theta + 2sin^2\theta\cos\theta$$

$$= 8cos^5\theta - 4cos^3\theta - 6cos^3\theta + 3\cos\theta - 8(1 - cos^2\theta)cos^3\theta + 2(1 - cos^2\theta)\cos\theta$$

$$= 16cos^5\theta - 20cos^3\theta + 5\cos\theta$$

$$=\frac{16d^5}{(x^2+y^2+d^2)^2\sqrt{(x^2+y^2+d^2)}} - \frac{20d^3}{(x^2+y^2+d^2)\sqrt{x^2+y^2+d^2}} + \frac{5d}{\sqrt{x^2+y^2+d^2}}$$

$$*\sin 3\theta = \sin(2\theta + \theta)$$

$$=\sin 2\theta\cos\theta - \cos 2\theta \sin\theta$$

$$=2cos^2\theta\sin\theta + 2cos^2\theta\sin\theta - \sin\theta$$

$$=4cos^2\theta\sin\theta - \sin\theta$$

A3.4 Case 4. The Cos^2(2θ) Distribution. This implies that the flux at any given point is proportional to Cos^2(2θ) of the angle joining the point to the Nozzle and from the Nozzle to the strip.

$$cos^2(2\theta) = (2cos^2\theta - 1)^2 = 4cos^4\theta - 4cos^2\theta + 1$$

$$= \frac{4d^4}{(x^2+y^2+d^2)^2} - \frac{4d^2}{(x^2+y^2+d^2)} + 1$$

Appendix B

B1 Uniform Distribution

Column Beneath is calculated from the formula:

$$H(x) = \frac{2v}{\pi R^2 U}\sqrt{R^2 - x^2}$$

X Position	Raw Thickness	Relative Thickness	Radius Gap	Percentage Variation	Radius Gap	Percentage Variation
0.000	0.636620	1.000000	0.000	//////////////////////	1.000	41.781413
0.025	0.636421	0.999687	0.025	1.611698	1.025	41.326131
0.050	0.635824	0.998749	0.050	3.888430	1.050	40.864430
0.075	0.634827	0.997184	0.075	7.841726	1.075	40.431660
0.100	0.633429	0.994987	0.100	17.609375	1.100	39.989965
0.125	0.631627	0.992157	0.125	25.106553	1.125	39.578202
0.150	0.629418	0.988686	0.150	30.206427	1.150	39.154815
0.175	0.626796	0.984568	0.175	33.466240	1.175	38.762756
0.200	0.623758	0.979796	0.200	29.182163	1.200	38.356099
0.225	0.620297	0.974359	0.225	29.898647	1.225	37.982633
0.250	0.616405	0.968246	0.250	25.320249	1.250	37.591209
0.275	0.612075	0.961444	0.275	22.973391	1.275	37.235409
0.300	0.607297	0.953939	0.300	20.695103	1.300	36.857767
0.325	0.602061	0.945714	0.325	22.673244	1.325	36.518895
0.350	0.596354	0.936750	0.350	18.432262	1.350	36.153582
0.375	0.590163	0.927025	0.375	14.877151	1.375	35.831105
0.400	0.583472	0.916515	0.400	13.633261	1.400	35.476606
0.425	0.576264	0.905193	0.425	11.916603	1.425	35.170234
0.450	0.568520	0.893029	0.450	9.225958	1.450	34.824880
0.475	0.560217	0.879986	0.475	16.178493	1.475	34.534633
0.500	0.551329	0.866025	0.500	14.700004	1.500	34.196471
0.525	0.541829	0.851102	0.525	13.271644	1.525	33.922794
0.550	0.531683	0.835165	0.550	11.743912	1.550	33.589354
0.575	0.520853	0.818153	0.575	10.193597	1.575	33.333333
0.600	0.509296	0.800000	0.600	8.197812	1.600	33.001217
0.625	0.496962	0.780625	0.625	12.577229	1.625	32.764980
0.650	0.483790	0.759934	0.650	23.960383	1.650	32.429040
0.675	0.469710	0.737818	0.675	22.972229	1.675	32.216561
0.700	0.454638	0.714143	0.700	21.998967	1.700	31.868133
0.725	0.438472	0.688749	0.725	21.034463	1.725	33.856217
0.750	0.421085	0.661438	0.750	20.071317	1.750	36.803877
0.775	0.402319	0.631961	0.775	19.100186	1.775	40.000000
0.800	0.381972	0.600000	0.800	18.108673	1.800	43.486727
0.825	0.359775	0.565133	0.825	17.079395	1.825	47.321731
0.850	0.335361	0.526783	0.850	15.986380	1.850	51.587708
0.875	0.308202	0.484123	0.875	14.787656	1.875	56.411011
0.900	0.277496	0.435890	0.900	14.046594	1.900	62.003290
0.925	0.241895	0.379967	0.925	18.485098	1.925	68.775010
0.950	0.198785	0.312250	0.950	24.860886	1.950	77.779514
0.975	0.141460	0.222205	0.975	42.264973	1.975	100.000000
1.000	0.000000	0.000000	1.000	41.781413	2.000	100.000000

B2 Cos(2θ)

These tables show integrations at regular intervals along the stationary Distribution of Cos(2θ). This converts the stationary problem into a dynamic one.

Initial Distribution is: $\int_0^{\sqrt{d^2-x^2}} \frac{2d^2}{x^2+y^2+d^2} - 1\ dy$

This integral is evaluated at 40 different fixed x intervals. With ***d*** equal to 1. (The value of ***d*** makes absolutely no difference to the relative values. In any of the Distributions)

X position (Radius)	Raw Values	Relative
0.000	1.14159	1
0.025	1.13999	0.998598446
0.050	1.13518	0.994385024
0.075	1.12721	0.987403534
0.100	1.11613	0.977697772
0.125	1.10201	0.965329059
0.150	1.08496	0.950393749
0.175	1.06509	0.932988201
0.200	1.04253	0.91322629
0.225	1.01743	0.891239412
0.250	0.989954	0.867171226
0.275	0.960277	0.841175028
0.300	0.928579	0.813408492
0.325	0.895052	0.784039804
0.350	0.859891	0.75323978
0.375	0.823297	0.721184488
0.400	0.78547	0.688049124
0.425	0.746614	0.654012386
0.450	0.706929	0.619249468
0.475	0.666614	0.583934688
0.500	0.625867	0.548241488
0.525	0.584878	0.512336303
0.550	0.543837	0.476385567
0.575	0.502924	0.440546956
0.600	0.462319	0.404978145
0.625	0.422195	0.369830675
0.650	0.382719	0.335250834
0.675	0.344056	0.301383159
0.700	0.306367	0.268368679
0.725	0.269811	0.236346674
0.750	0.234548	0.205457301
0.775	0.200738	0.175840713
0.800	0.168548	0.147643199
0.825	0.138151	0.121016302
0.850	0.10974	0.096129083
0.875	0.0835312	0.073170928
0.900	0.0597822	0.052367487
0.925	0.0388235	0.034008269
0.950	0.0211224	0.018502615
0.975	0.00746183	0.006536348
1.000	0	0

Radius Gap	Percentage Difference	Radius Gap	Percentage Difference
0.025	1.648313646	1.025	5.893612077
0.050	6.273201385	1.050	3.047948455
0.075	12.63039844	1.075	1.6168326
0.100	19.46506374	1.100	4.7228866
0.125	24.94038677	1.125	8.3067477
0.150	26.81734654	1.150	11.8906088
0.175	26.00346561	1.175	15.4474899
0.200	23.56882068	1.200	19.004371
0.225	20.57744368	1.225	22.519118
0.250	18.08930683	1.250	26.033865
0.275	15.40825824	1.275	29.4918491
0.300	13.12709943	1.300	32.9498332
0.325	11.41358349	1.325	36.3366007
0.350	8.818585274	1.350	39.7233682
0.375	7.200731907	1.375	43.0248162
0.400	5.526841641	1.400	46.3262642
0.425	4.199696086	1.425	49.5284647
0.450	3.956772297	1.450	52.7306652
0.475	3.391503624	1.475	55.8196025
0.500	2.221369132	1.500	58.9085398
0.525	0.672431743	1.525	61.8701986
0.550	1.458454424	1.550	64.8318574
0.575	3.041335716	1.575	67.6516088
0.600	4.33183566	1.600	70.4713602
0.625	5.156300679	1.625	73.1340499
0.650	5.348654582	1.650	75.7967396
0.675	4.085856638	1.675	78.2854615
0.700	2.295756124	1.700	80.7741834
0.725	1.138207391	1.725	83.0699989
0.750	2.414468876	1.750	85.3658144
0.775	4.083935091	1.775	87.4461585
0.800	5.872534909	1.800	89.5265026
0.825	7.453377156	1.825	91.3624244
0.850	8.850490333	1.850	93.1983462
0.875	9.968098675	1.875	94.7489116
0.900	10.80049067	1.900	96.299477
0.925	11.23415874	1.925	97.4961037
0.950	11.21917887	1.950	98.6927304
0.975	10.56410862	1.975	99.3463652
1.000	8.892375306	2.000	100

B3 Cos(3θ)

The table below shows integrations at regular intervals along the stationary Distribution of Cos(3θ). This converts the stationary problem into a dynamic one.We must put these ***tan30*** values in because the circle of incidence has shrunk so we need to keep the ***x*** and the ***y*** values within this circle.

Initial Distribution is: $\int_0^{\sqrt{(dtan30)^2-(xtan30)^2}} \frac{4d^3}{((xtan30))^2+y^2+d^2)^{\frac{3}{2}}} - \frac{3d}{((xtan30)^2+y^2+z^2)^{\frac{1}{2}}}\, dy$

This integral is evaluated at 40 different fixed x intervals.

X position (radius)	Raw Values	Relative Values
0.000	0.565121	1.000000
0.025	0.564436	0.998788
0.050	0.562385	0.995159
0.075	0.558976	0.989126
0.100	0.554226	0.980721
0.125	0.548157	0.969982
0.150	0.540796	0.956956
0.175	0.532177	0.941705
0.200	0.522340	0.924298
0.225	0.511330	0.904815
0.250	0.499197	0.883345
0.275	0.485995	0.859984
0.300	0.471784	0.834837
0.325	0.456628	0.808018
0.350	0.440594	0.779645
0.375	0.423753	0.749845
0.400	0.406180	0.718749
0.425	0.387953	0.686495
0.450	0.369150	0.653223
0.475	0.349854	0.619078
0.500	0.330149	0.584209
0.525	0.310121	0.548769
0.550	0.289857	0.512911
0.575	0.269446	0.476793
0.600	0.248978	0.440575
0.625	0.228543	0.404414
0.650	0.208233	0.368475
0.675	0.188143	0.332925
0.700	0.168366	0.297929
0.725	0.149001	0.263662
0.750	0.130145	0.230296
0.775	0.111904	0.198018
0.800	0.094384	0.167016
0.825	0.077702	0.137496
0.850	0.061984	0.109683
0.875	0.047374	0.083829
0.900	0.034038	0.060231
0.925	0.022188	0.039262
0.950	0.012115	0.021437
0.975	0.004294	0.007599
1.000	0.000000	0.000000

Radius Gap	Percentage Difference	Radius Gap	Percentage Difference
0.025	1.435175226	1.025	11.844066
0.050	5.619438	1.050	9.328133
0.075	11.752500	1.075	6.909878
0.100	18.831757	1.100	4.702793
0.125	24.885233	1.125	3.685830
0.150	27.200369	1.150	5.642402
0.175	26.654281	1.175	8.323350
0.200	24.310288	1.200	11.885065
0.225	21.344268	1.225	15.501105
0.250	18.906635	1.250	19.117145
0.275	16.176145	1.275	22.711065
0.300	13.867892	1.300	26.304986
0.325	12.163656	1.325	29.859977
0.350	9.442632	1.350	33.414968
0.375	7.779525	1.375	36.914572
0.400	6.026123	1.400	40.414177
0.425	4.585996	1.425	43.840877
0.450	4.395266	1.450	47.267576
0.475	3.836933	1.475	50.604207
0.500	2.642840	1.500	53.940837
0.525	0.846162	1.525	57.168642
0.550	1.547682	1.550	60.396446
0.575	3.340187	1.575	63.496649
0.600	4.838201	1.600	66.596853
0.625	5.820044	1.625	69.548787
0.650	6.086317	1.650	72.500721
0.675	4.693507	1.675	75.282019
0.700	2.648880	1.700	78.063317
0.725	1.272840	1.725	80.648764
0.750	2.795910	1.750	83.234210
0.775	4.940640	1.775	85.594041
0.800	7.197646	1.800	87.953872
0.825	9.268951	1.825	90.050786
0.850	11.189010	1.850	92.147699
0.875	12.843035	1.875	93.930132
0.900	14.235920	1.900	95.712564
0.925	15.230513	1.925	97.096401
0.950	15.771979	1.950	98.480237
0.975	15.631328	1.975	99.240119
1.000	14.414249	2.000	100.000000

B4 Cos(5θ)

Thee table directly below shows integrations at regular intervals along the stationary Distribution of Cos(5θ). This converts the stationary problem into a dynamic one. ***d*** = Nozzle Height, ***r =*** radius=(***dtan18), xtan(18)*** = X position We must put these ***tan18*** values in because the circle of incidence has shrunk so we need to keep the ***x*** and the ***y*** values within this circle.

Initial Distribution is:

$$\int_0^{\sqrt{(r)^2+(xtan18)^2}} \frac{16d^5}{((xtan18)^2+y^2+d^2)^{\frac{5}{2}}} - \frac{20\mathrm{d}^3}{((xtan18)^2+y^2+d^2)^{\frac{3}{2}}} + \frac{5d}{((xtan18)^2+y^2+d^2)^{\frac{1}{2}}}\,dy$$

This integral is evaluated at 40 different fixed x intervals.

X position (radius)	Raw Values	Relative Values
0.000	0.203845	1.000000
0.025	0.203621	0.998901
0.050	0.202950	0.995609
0.075	0.201835	0.990140
0.100	0.200279	0.982506
0.125	0.198287	0.972734
0.150	0.195867	0.960862
0.175	0.193026	0.946925
0.200	0.189776	0.930982
0.225	0.186126	0.913076
0.250	0.182090	0.893277
0.275	0.177681	0.871648
0.300	0.172916	0.848272
0.325	0.167810	0.823224
0.350	0.162383	0.796600
0.375	0.156653	0.768491
0.400	0.150641	0.738998
0.425	0.144369	0.708229
0.450	0.137860	0.676298
0.475	0.131138	0.643322
0.500	0.124228	0.609424
0.525	0.117157	0.574736
0.550	0.109953	0.539395
0.575	0.102643	0.503535
0.600	0.095258	0.467305
0.625	0.087829	0.430859
0.650	0.080387	0.394356
0.675	0.072968	0.357958
0.700	0.065606	0.321840
0.725	0.058337	0.286183
0.750	0.051201	0.251177
0.775	0.044240	0.217028
0.800	0.037498	0.183953
0.825	0.031024	0.152193
0.850	0.024872	0.122014
0.875	0.019105	0.093721
0.900	0.013796	0.067676
0.925	0.009038	0.044337
0.950	0.004960	0.024330
0.975	0.001767	0.008667
1.000	0.000000	0.000000

Radius Gap	Percentage Difference	Radius Gap	Percentage Difference
0.025	1.307005	1.025	15.644717
0.050	5.193485	1.050	13.385457
0.075	11.105536	1.075	11.128968
0.100	18.270141	1.100	9.027580
0.125	24.743047	1.125	7.037851
0.150	27.415746	1.150	5.139391
0.175	27.105273	1.175	6.259388
0.200	24.864968	1.200	8.405161
0.225	21.907130	1.225	10.967892
0.250	19.517465	1.250	13.974834
0.275	16.759796	1.275	17.478525
0.300	14.434653	1.300	21.128897
0.325	12.789569	1.325	24.768672
0.350	9.942909	1.350	28.408448
0.375	8.264211	1.375	32.020211
0.400	6.463802	1.400	35.631975
0.425	4.900225	1.425	39.197724
0.450	4.791178	1.450	42.763472
0.475	4.282737	1.475	46.264024
0.500	3.060853	1.500	49.764576
0.525	1.084777	1.525	53.179524
0.550	1.580466	1.550	56.594471
0.575	3.558176	1.575	59.901886
0.600	5.309252	1.600	63.209301
0.625	6.529238	1.625	66.385342
0.650	6.984311	1.650	69.561382
0.675	5.600021	1.675	72.579313
0.700	3.431964	1.700	75.597243
0.725	1.723453	1.725	78.426550
0.750	2.750490	1.750	81.255856
0.775	4.875884	1.775	83.860286
0.800	7.448503	1.800	86.464716
0.825	9.869971	1.825	88.798666
0.850	12.173389	1.850	91.132615
0.875	14.234894	1.875	93.133307
0.900	16.061222	1.900	95.133999
0.925	17.500978	1.925	96.700257
0.950	18.496507	1.950	98.266516
0.975	18.791609	1.975	99.133258
1.000	17.955292	2.000	100.000000

B5 $\text{Cos}^2(2\theta)$

The table below shows integrations at regular intervals along the stationary Distribution of Cos^2(2θ). This converts the stationary problem into a dynamic one.

Initial Distribution is: $\int_0^{\sqrt{d^2-x^2}} \left(\frac{4d^2}{x^2+y^2+d^2}-2\right)^2 \, dy$

This integral is evaluated at 40 different fixed x intervals.

X postion (Radius)	Raw Values	Relative Values
0.000	0.429204	1
0.025	0.428089	0.997402168
0.050	0.424762	0.989650609
0.075	0.419271	0.976857159
0.100	0.411697	0.959210539
0.125	0.402147	0.936960047
0.150	0.390758	0.91042488
0.175	0.377689	0.87997549
0.200	0.363115	0.846019608
0.225	0.347232	0.809013895
0.250	0.330241	0.76942666
0.275	0.312354	0.727751838
0.300	0.293782	0.684481039
0.325	0.274738	0.64011053
0.350	0.255426	0.595115609
0.375	0.236044	0.549957596
0.400	0.216778	0.50506985
0.425	0.197801	0.460855444
0.450	0.179271	0.417682501
0.475	0.161327	0.375874875
0.500	0.144095	0.335726135
0.525	0.127679	0.297478588
0.550	0.112168	0.261339596
0.575	0.097631	0.227469455
0.600	0.084122	0.195994679
0.625	0.071677	0.167000308
0.650	0.060319	0.140537367
0.675	0.050055	0.116623098
0.700	0.040879	0.095243754
0.725	0.032774	0.076358794
0.750	0.025710	0.059901119
0.775	0.019650	0.045781726
0.800	0.014546	0.033890877
0.825	0.010344	0.024100428
0.850	0.006981	0.016264667
0.875	0.004388	0.010222924
0.900	0.002489	0.005798292
0.925	0.001200	0.002796968
0.950	0.000431	0.001004504
0.975	0.000075	0.000175648
1.000	0.000000	0

Radius Gap	Percentage Difference	Radius Gap	Percentage Difference
0.025	2.957092766	1.025	36.6795277
0.050	10.3404885	1.050	40.5042824
0.075	18.25634633	1.075	44.1181816
0.100	23.75897367	1.100	47.7320808
0.125	25.34734812	1.125	51.1190949
0.150	23.91248361	1.150	54.506109
0.175	20.95803959	1.175	57.6535866
0.200	17.65891952	1.200	60.8010642
0.225	14.64228395	1.225	63.7005013
0.250	11.88031362	1.250	66.5999384
0.275	9.437203037	1.275	69.2462325
0.300	7.394117219	1.300	71.8925266
0.325	5.493067939	1.325	74.2839535
0.350	4.00063027	1.350	76.6753804
0.375	2.700072991	1.375	78.8133148
0.400	1.919263368	1.400	80.9512492
0.425	1.253234699	1.425	82.8397452
0.450	0.546147792	1.450	84.7282412
0.475	0.213271317	1.475	86.3740087
0.500	0.765604393	1.500	88.0197762
0.525	1.135207595	1.525	89.4317155
0.550	1.246136422	1.550	90.8436548
0.575	1.114716356	1.575	92.0327397
0.600	0.821176968	1.600	93.2218246
0.625	0.420357752	1.625	94.2008695
0.650	0.087734226	1.650	95.1799144
0.675	0.263906899	1.675	95.9634905
0.700	0.247249381	1.700	96.7470666
0.725	0.663161826	1.725	97.3512409
0.750	1.775942691	1.750	97.9554152
0.775	3.347126173	1.775	98.3978784
0.800	5.398299211	1.800	98.8403416
0.825	7.849217211	1.825	99.140474
0.850	10.73271648	1.850	99.4406064
0.875	13.90646091	1.875	99.6198528
0.900	17.42113257	1.900	99.7990992
0.925	21.08570392	1.925	99.8819848
0.950	24.97574932	1.950	99.9648704
0.975	28.86488848	1.975	99.9824352
1.000	32.854773	2.000	100

B6 Explanation of how Percentage Variation values were calculated.

Radius	Nozzle 1	Nozzle 2	Nozzle 3	Nozzle 4	Nozzle 5	Nozzle 6	Nozzle 7	Nozzle 8	Nozzle 9	Nozzle 10	SUM
0.025	Value 0										SUM 1
0.050	Value 1	Value 0									SUM 2
0.075	Value 2	Value 1	Value 0								SUM 3
0.100	Value 3	Value 2	Value 1	Value 0							SUM 4
0.125	Value 4	Value 3	Value 2	Value 1	Value 0						SUM 5
0.150	Value 5	Value 4	Value 3	Value 2	Value 1	Value 0					SUM 6
0.175	Value 6	Value 5	Value 4	Value 3	Value 2	Value 1	Value 0				SUM 7
0.200	Value 7	Value 6	Value 5	Value 4	Value 3	Value 2	Value 1	Value 0			SUM 8
0.225	Value 8	Value 7	Value 6	Value 5	Value 4	Value 3	Value 2	Value 1	Value 0		SUM 9
0.250	Value 9	Value 8	Value 7	Value 6	Value 5	Value 4	Value 3	Value 2	Value 1	Value 0	SUM 10
0.275	Value 10	Value 9	Value 8	Value 7	Value 6	Value 5	Value 4	Value 3	Value 2	Value 1	SUM 11
….	….	….	….	….	….	….	….	….	….	….	….
….	….	….	….	….	….	….	….	….	….	….	….

This represents 10 nozzles spraying in a row with a gap of 0.025 of a Radius between them. From here, if you plot "SUM" against Radius, you will get the combined Dynamic distribution of all 10 nozzles. The Percentage difference value is then calcuated from the maximum of SUM, and the minimum value of SUM (in the range [midpoint nozzle 2 : midpoint nozzle 9] so that we can ignore the edge's anaomalous values). Percentage difference is defined as (max - min)/ max.

Once we have done this we increase the gap between Nozzle 1 and Nozzle 2 to 0.5 from 0.025, and Nozzle 2 to 3 etc. We then repeat the entire process until the Gap is equal to 2 Radii and we have 10 competely separate, none-overlapping Nozzles. This gives us the Percentage Variation value at 80 different points. This is exactly what the Python Script which caluclated the Percentage Variation values does.