Tractor Trailer: Tracking and Backing

Group 8
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Abstract

The objective of the problem is to help our client design port facilities by giving him or her a quick means of identifying how much road space needs to be available to handle and unload large objects. This involved an investigation into the motion of an articulated vehicle such as a tractor-trailer system going forwards or backwards. For this reason, the analysis is divided into two: the investigation of the tracking or forward movement of the vehicle, and the investigation of the reversing or backwards movement of the vehicle. To model the motion of the vehicle we use Matlab. This paper defined the three points that would be used to understand the motion of the tractor-trailer system as the front point at the centre of the tractor's steering wheels axle, the pivot point that matched with the hitch connecting the tractor and the trailer, and the back point at the centre of the trailer's rear wheels axle. Other variables were defined, as they were necessary to model the system accurately, for example, the width of the vehicle or the length of the trailer. Initially a model was designed that showed the forward motion of a tractor-trailer making an one hundred and eighty degree turn while only moving forward as this keeps the system completely stable. Later a second model was created to help understand the backwards motion of the tractor-trailer. Since this motion will be unstable, we looked for the relation between the steering and trailer angles that would allow the stability of the rig. Using these two models, we were able to design a program that shows the motion of a tractor-trailer, given known dimensions, which can be used to design a road space that is wide enough for the tractor-trailer to turn around in.

Contents

1 Introduction

The objective of the problem is to help our client design port facilities by giving him or her a quick means of identifying how much road space needs to be available to handle and unload large objects. This involved an investigation into the motion of an articulated vehicle such as a tractor-trailer system going forwards or backwards. Nowadays tractor-trailer rigs can be used in various ways. The main purpose of using it is for transport, for example to tow away cars or to transport wind turbine rotor blades on long articulated transporters. It is very difficult to drive these rigs. The trailer follows the vehicle, so the motion is stable while it moves forward. However, the motion is unstable while it moves backward since a small turning change from the tractor causes a big change from the trailer; this motion is unstable and can even lead to jack-knifing. For this reason, the analysis has been divided into two: the investigation of the tracking or forward movement of the vehicle, and the investigation of the backing or backwards movement of the vehicle.

In order to develop our models, we assumed that the wheels of the vehicle rolled without slipping. We also defined the three points that would help us understand the motion of the tractor-trailer system: the front point at the centre of the tractor's steering wheels axle, the pivot point that matched with the hitch connecting the tractor and the trailer, and the back point at the centre of the trailer's rear wheels axle. The variables describing the rig and trailer are necessary to make the models work, they are the width of the vehicle, the length of the front and the back of the tractor, the distance between the pivot and the back points and the steering or trailer angle depending on the model.

In the first part, we have attempted to create a model that, for the listed variables describing the vehicle, would give a graph showing the movement of a tractor-trailer system making a turn of one hundred and eighty degrees and then continuing its forward motion in a straight line. Since the initial conditions are so that the rig starts its turn in the origin of the x-y plane, the road space needed would be the maximum value of y plus some extra space for safety and to allow behaviour that is more realistic.

In the second part, we attempted to understand the backwards movement of a tractor-trailer system. Since its motion will be unstable in this case, we looked for the relation between the steering and trailer angles that would allow the stability of the rig. In the case that the trailer has a stabilizer, it would be useful to know what trailer angle would allow stability for a given steering angle. In the case that the driver has a means of knowing the trailer angle, then he or she could change the steering angle so that the system is stable and moves in a concentric circle. In both cases, the turning radius, the distance between the pivot point and the instant centre of rotation, for the stable vehicle is given so that it is easier to predict the next position of the tractor-trailer system.

2 Aim

The aim is to design a model such that it describes the motion of a long articulated transporter as it makes a turn of up to one hundred and eighty degrees, in doing so it must then find the area swept of roadway while the transporter is in forward motion. A second model is also needed to describe the motion of the articulated transporter reversing in the case of provision of extra roadways taking up too much space.

3 Model

3.1 Assumptions

In order to develop this model, we have assumed that the wheels roll without slipping and that all the systems are planar. A main assumption made was assuming that the angle between the front wheels and the horizontal axis stayed constant throughout the one hundred and eighty degree turning period. In reality, this would not be possible due to human error and even slight deviations in this angle would change the outcomes of the model. Another assumption made was that the pivot point or the back end did not affect the front of the vehicle. This meant that calculations were made easier for modelling the pivot point and the back of the trailer. The entire model was based on assuming that the front of the tractor, the pivot point and the back of the trailer were point particles. Making these assumptions made MATLAB calculations much easier but in turn made the model less accurate than it could have been. If there was more time to model this problem, the outcomes could have been more accurate.

3.2 Development of Model

3.2.1 Tracking

To begin modelling the problem, it was decided to treat the problem as a dynamical system instead of a geometric problem. This is because we need to keep track of key points in the system such as the pivot point for the trailer, the back of the trailer and the front of the tractor with respect to the independent variable time.

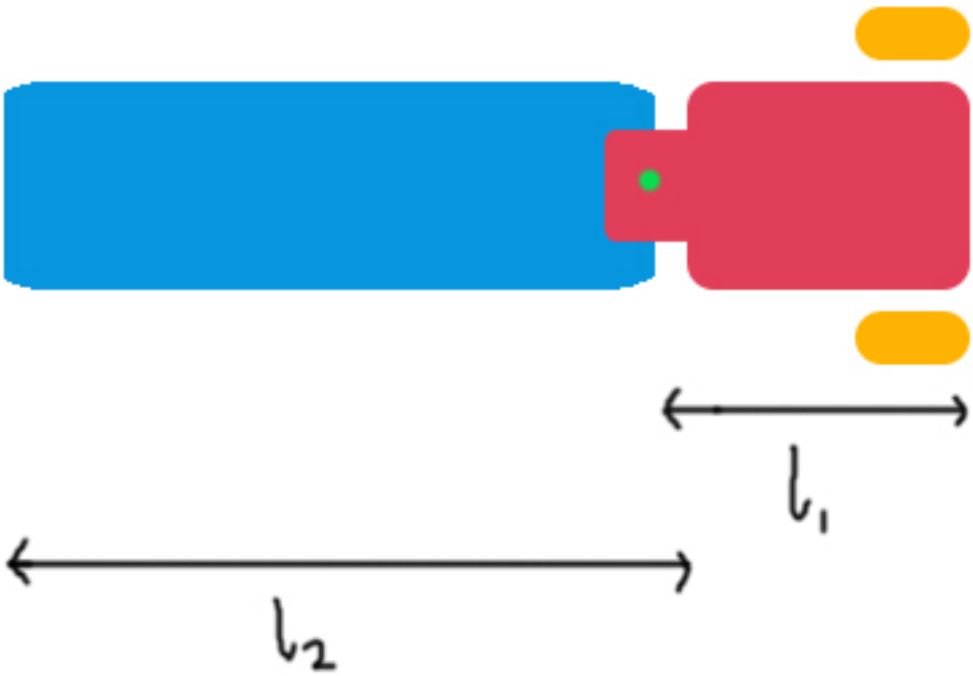

Figure 1 : Diagram of an articulated transporter

Figure 1 shows how the dimensions are measured, l_1 is the length of the tractor and l_2 is the length of the cab.

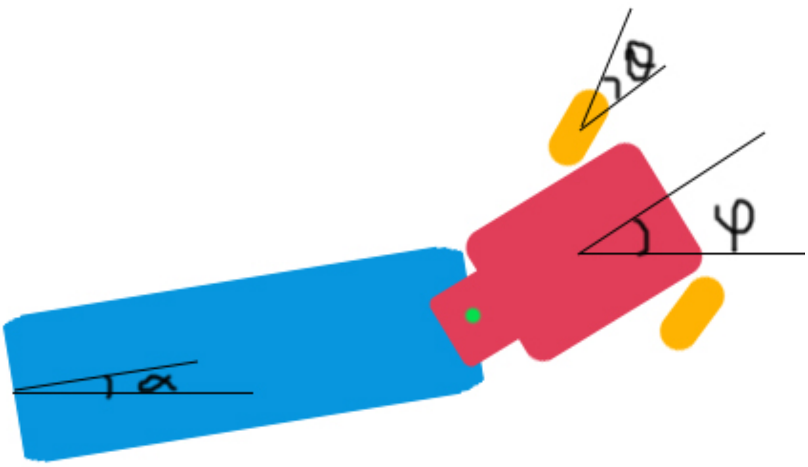

Figure 2 : Angles of an articulated lorry

Figure 2 shows how the angles of the system are measured: theta is the steering angle; phi is the cab angle measured from the horizontal and alpha is the trailer angle also measured from the horizontal.

Once naming conventions of angles and lengths were established, the next step was to find the differential equations that describe the system using V as the speed of the lorry in the direction of the steering angle. The Equations that describe the motion of the front point of the tractor (measured in line with the main axle) are:

$$\frac{dx}{dt} = vcos(\theta + \varphi)$$
$$\frac{dy}{dt} = vsin(\theta + \varphi)$$

These next equations describe the rate of change of the cab angle and the trailer angle. As the assumption was made that the steering angle would remain constant, the rate of change in theta will remain zero.

$$\frac{d\varphi}{dt} = \frac{vsin(\theta)}{l_1}$$
$$\frac{d\alpha}{dt} = \frac{vsin(\varphi - \alpha)}{l_2}$$

These four equations were used to describe every point of the articulated lorry as positions of the pivot point and the back of the trailer can be found using simple trigonometry. These equations are as follows:

The position of the pivot point:

$$xp = x - l_1 \cos(\varphi)$$
$$yp = y - l_1 \sin(\varphi)$$

The position of the back of the trailer:

$$xb = xp - l_2 \cos(\alpha)$$
$$yb = yp - l_2 \sin(\alpha)$$

These questions were then coded in MATLAB to produce the first model; the results from the first model are shown in Figure 3 for a typical articulated transporter. The graph shows the positions of the points for a large time range. While all the equations are valid it is not the solution to the problem as it shows a positions of the points if the articulated transporter is perpetually turning, hence the motion is a circle.

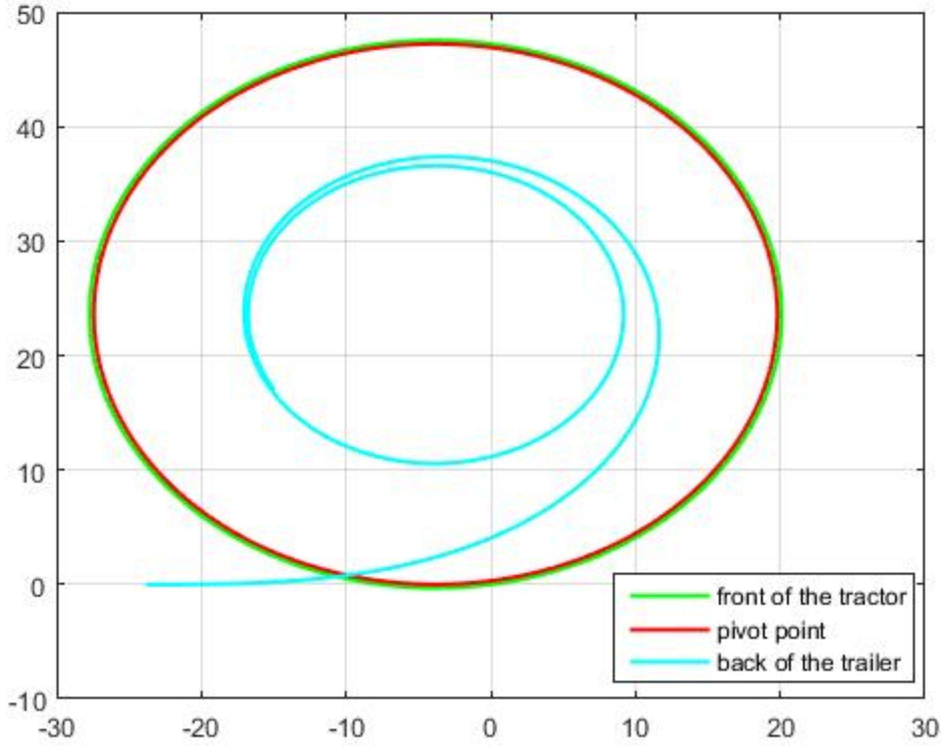

Figure 3 : First model results

To refine the model to describe the transporter making a turn of one hundred and eighty degrees instead of continually turning, the steering angle must be set to zero once the cab angle reaches one hundred and eighty degrees. This was done using the events feature in MATLAB. The results are shown in Figure 4 for a typical articulated transporter.

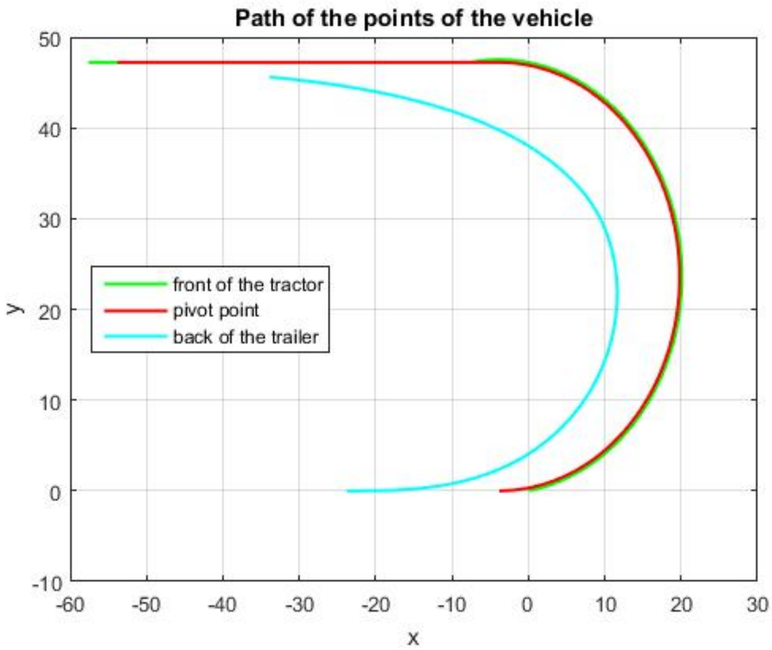

Figure 4 : Positions as a result of the second model

To finalize the model, user inputs were coded which would allow the user to input the speed of the vehicle the length of the cab, the length of the trailer and the steering angle. The model will then use these inputs to output a graph showing the path of the important points of the articulated transporter and also output the area swept by the turn.

3.2.2 Backing

In the first part of the report, we have considered the tracking of a vehicle connected to a trailer and followed its motion while making a 180 degrees turn. Backing a trailer can be easy since the system is stable. However, this is not the case when backing a trailer since little changes in the steering angle can imply big changes in the expected outcomes.

a) Trailer angle as a function of the Steering angle

Figure 5 represents a vehicle connected to a trailer. The instant centre of rotation of the vehicle can be found by finding the intersection of the perpendiculars of the front and back wheels of the vehicle. We want this centre of rotation to be the same as the one for the trailer so that our system is stable. For that to be possible, we want the line perpendicular to the back wheels of the trailer to pass through the instant centre of rotation of the vehicle. This would imply that the centre of rotation of the vehicle is the same as the one of the vehicle-trailer system.

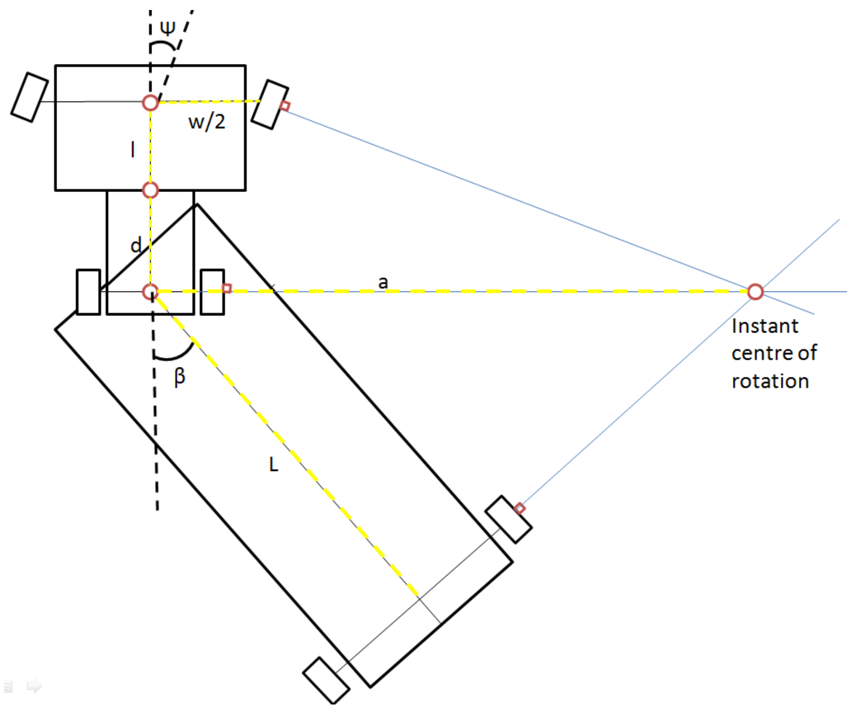

Figure 5 : Instant centre of rotation of the vehicle-trailer system

In Figure 5, we have 5 known values: the length of the front of the vehicle l, the length of the back of the vehicle d, the length between the hitch and the trailer’s rear wheels axle L, the width of the vehicle w and the steering angle Ψ. The value we are looking for is β, the trailer’s angle. In order to find this value with respect to the known values, the situation has been simplified in Figure 6 and Figure 7. These two figures help us find a, the turning radius, as a function of the steering angle Ψ and the trailer’s angle β.

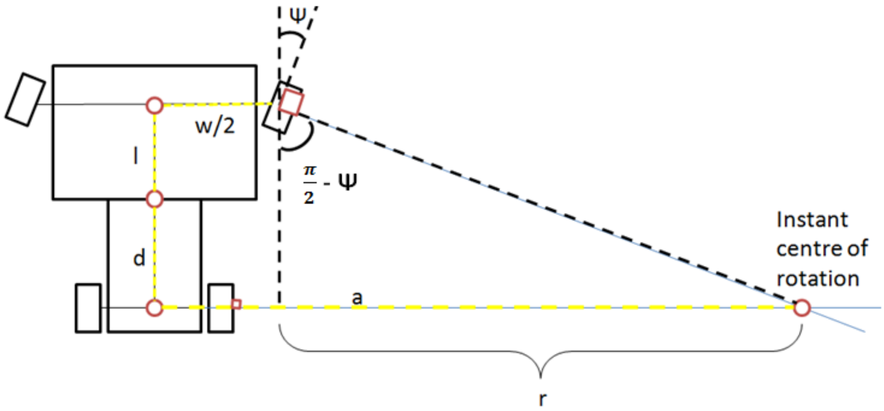

Figure 6 : Simplification 1 of Figure 5

From Figure 6, we can derive a value for r and hence for a.
From trigonometry: $\tan\left(\frac{\pi}{2} - \Psi\right) = \frac{r}{l+d}$
This gives us: $r = (l + d)\tan\left(\frac{\pi}{2} - \Psi\right)$
And therefore: $a = \frac{w}{2} + r = \frac{w}{2} + (l + d)\tan\left(\frac{\pi}{2} - \Psi\right)$ (1)

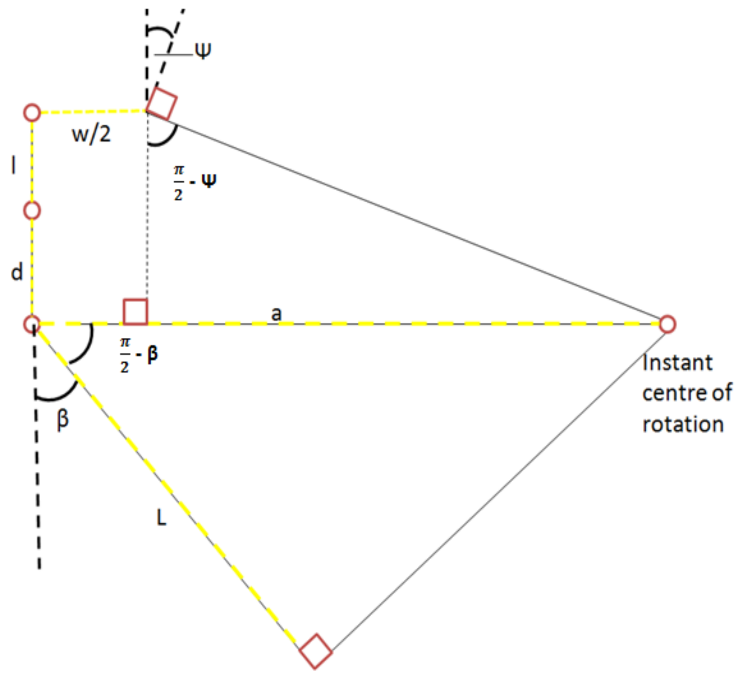

Figure 7 : Simplification 2 of Figure 5

From Figure 7, we have: $\cos\left(\frac{\pi}{2} - \beta\right) = \frac{L}{a}$
Which gives us $a = \frac{L}{\cos\left(\frac{\pi}{2} - \beta\right)}$ (2)
From equations (1) and (2):

$$\frac{w}{2} + (l + d)\tan\left(\frac{\pi}{2} - \Psi\right) = \frac{L}{\cos\left(\frac{\pi}{2} - \beta\right)}$$

$$\Rightarrow \beta = \sin^{-1}\left(\frac{L}{\frac{w}{2} + (l+d)\tan\left(\frac{\pi}{2} - \Psi\right)}\right) \qquad (3)$$

Equation (3) gives us the trailer angle suitable for the stability of the tractor-trailer system for a given steering angle. This value of β is the maximum trailer angle allowing stability.
Using equations (1) and (3), we can write a MATLAB program that will give us this trailer angle and the equivalent turning radius for different values of the steering angle. This program is shown under Appendix (1).

b) Steering angle as a function of the Trailer angle

From equations (1) and (2):

$$\frac{w}{2} + (l+d)\tan\left(\frac{\pi}{2} - \Psi\right) = \frac{L}{\sin\beta}$$

$$\Rightarrow \ \Psi = \frac{\pi}{2} - \tan^{-1}\left(\frac{\frac{L}{\sin\beta} - \frac{w}{2}}{l+d}\right) \qquad (4)$$

Equation (4) gives us the steering angle Ψ that would allow the stability of the tractor-trailer system for a given trailer angle β. This value of Ψ is the maximum steering angle allowing stability. If the steering angle is smaller than the steering angle given by Equation (3-2-2-4), the rig will still be stable but the maximum trailer angle for stability will decrease too. If the system is stable and we change the steering angle to an angle of 0 degrees, we will expect the trailer angle to decrease until reaching an angle of 0 degrees.
Using equations (2) and (4), we can write a MATLAB program that will give us this steering angle and the equivalent turning radius (distance between the hitch and the instant centre of rotation) for the value of the trailer angle that the driver would be dealing with. This program is shown under Appendix (2).

c) Motion of the tractor-trailer assuming stability

Just like the situation where the tractor was moving forwards, the stable motion of the vehicle going backwards for constant steering and trailer angles can be simulated with a MATLAB program.

In this case we have considered three angles: the trailer angle β, the maximum steering angle for which the system is still stable Ψ and the angle between the cab axle and the x-axis γ.
In this case, the equations describing the rate of change of γ and the x-y position of the front point of the tractor will be:

$$\frac{d\gamma}{dt} = \frac{v\sin(\Psi)}{\mathrm{l+d}}$$

$$\frac{dx}{dt} = v\cos(\gamma)$$

$$\frac{dy}{dt} = v sin(\gamma)$$

These three equations can be used to describe the rest of the points of the articulated lorry. The position of the pivot point is:

$$xp = x + (l+d)\cos(\gamma)$$

$$yp = y + (l+d)\sin(\gamma)$$

The position of the back point of the trailer is:

$$xb = xp + L\cos(\beta)$$

$$yb = yp + L\sin(\beta)$$

The program can be seen under Appendix (4).

3.3 Evaluation of Model

3.3.1 Tracking

To evaluate the model going forwards, the MATLAB in-built function *ode45* was used to solve the differential equations that were found. The four equations consisted of the position vectors of the tractor, the angle Phi between tractor and trailer, and the angle Alpha between the back of the trailer and the horizontal x axis. After calculating these equations, they were used to calculate the position vectors of the pivot point, and the back end of the trailer. To assess the motion of the system, the positions of each component with respect to the x axis was plotted against the positions of each component with respect to the y axis. This gave a representation of the position of each component of the system. Tractors/trailers come in various sizes of length, which affect the radius of curvature and the area swept by the vehicle in turning through one hundred and eighty degrees as shown in Figure 8 below.

Table of Various Tractor/Trailer Combinations

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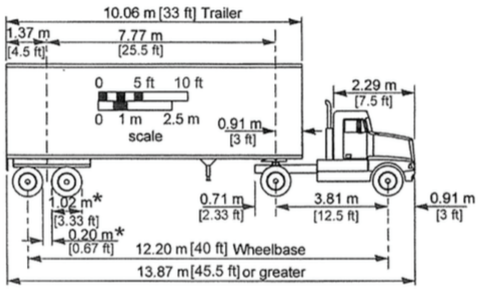
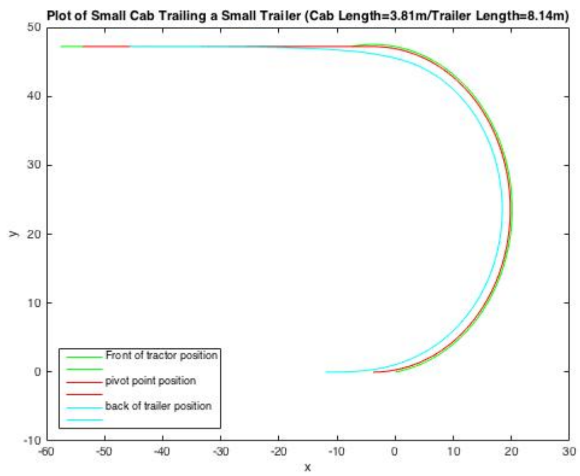
<td>$Cab\ length\ =\ 3.81m$
$Trailer\ length\ =\ 9.14m$
$v_0 = 1ms^{-1}, \theta = 0.16\ radians$
</td>
<td></td>
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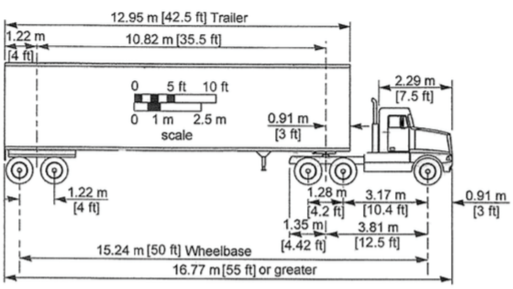
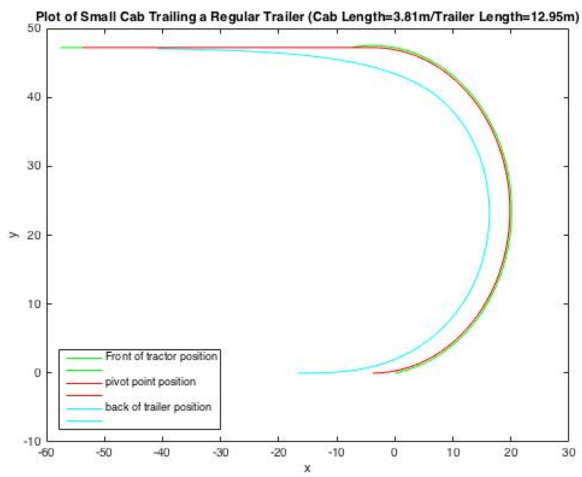
</td>
<td></td>
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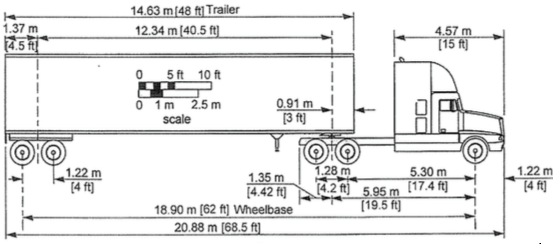
$$Trailer\ length\ =\ 13.71m$$
$$v_0 = 1ms^{-1}, \theta = 0.16\ radians$$



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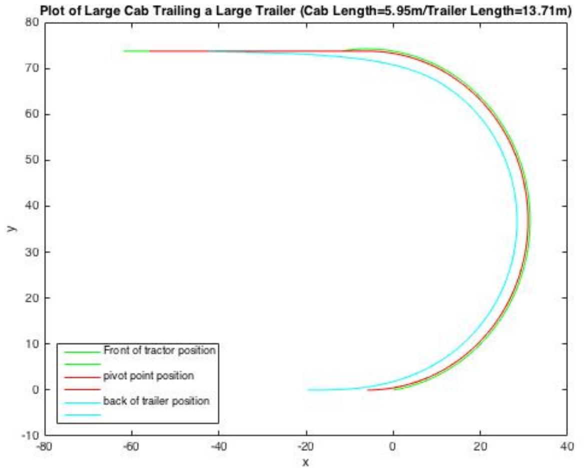

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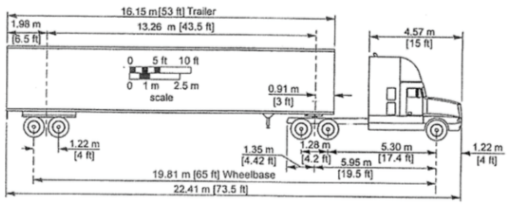
$$Cab\ length\ =\ 5.95m$$
$$Trailer\ length\ =\ 15.24m$$
$$v_0 = 1ms^{-1}, \theta = 0.16\ radians$$



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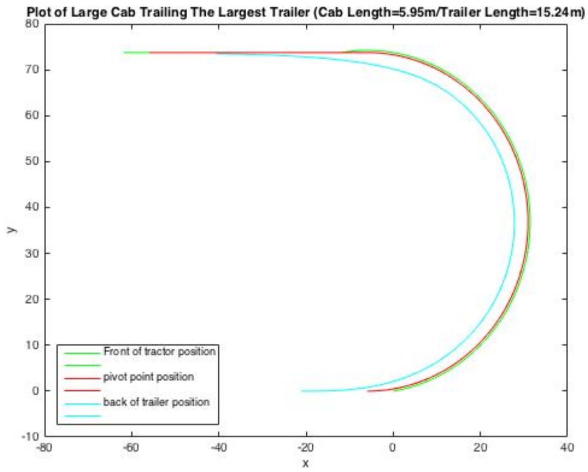

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Figure 8

The table shows various plots of different sized tractor/trailer combinations. As the trailer length increases, the radius of curvature reduces, and therefore the swept area increases. This means that a larger area of road is required for the vehicle to complete the manoeuvre. It can also be seen that the longer the trailer, the more unstable the system is. This is especially true in the case of reversing when a small change in steering angle can lead to a large change in motion.

From the plots, arbitrary values of $v_0$and θ (angle between the direction of wheels and the horizontal) were assumed. If the steering angle of the wheels is increased, this causes the back end of the vehicle to have a small radius of curvature which leads to a large swept area also. The angle θ can only be increased up to a certain critical value; after this value, the system becomes unstable. For a conventional tractor/trailer system, this value is approximately 0.35-0.4 radians. This critical value will decrease for a longer trailer.

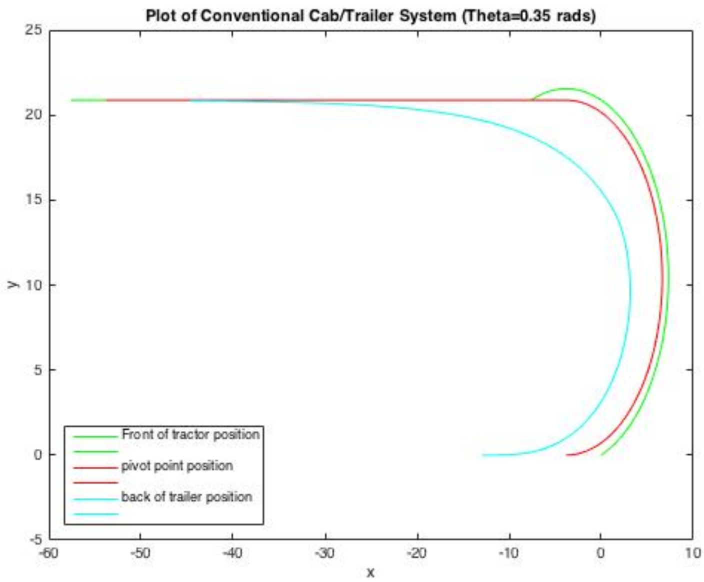

Figure 9

If the velocity at which the tractor turning the corner was to increase, the radius of curvature for each component would increase because it would take longer for the vehicle to turn. Therefore, the swept area would largely increase due to this increase in radius which is intuitive.

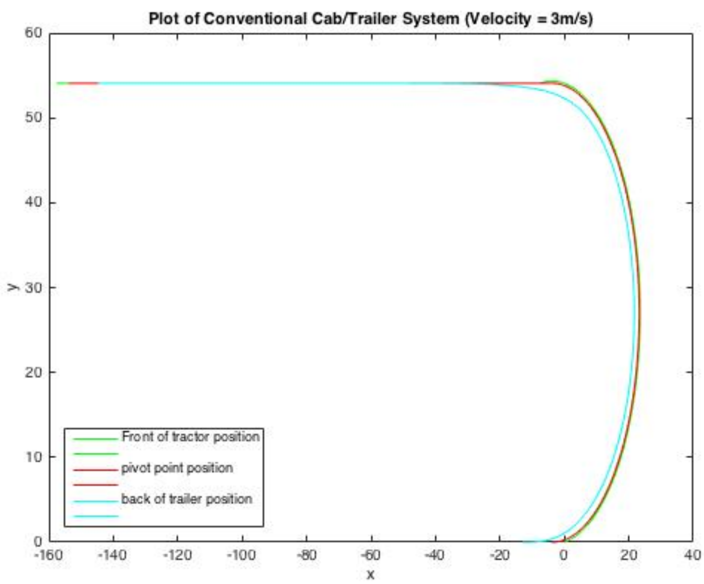

Figure 10

3.3.2 Backing

a) Trailer angle as a function of the Steering angle

We tested our model with 4 different vehicles:

Values for Vehicle 1: w = 2.44m, l = 1.38m, L = 7.77m, d = 2.43m, maxAngle = 25º
Values for Vehicle 2: w = 2.44m, l = 1.38m, L = 10.82m, d = 2.43m, maxAngle = 20º
Values for Vehicle 3: w = 2.44m, l = 3.35m, L = 12.34m, d = 2.6m, maxAngle = 30º
Values for Vehicle 4: w = 2.44m, l = 3.35m, L = 12.34m, d = 2.6m, maxAngle = 28º

The results for vehicle 1 are shown in Figure 11. The results for vehicles 2, 3 and 4 are under Appendix (3).

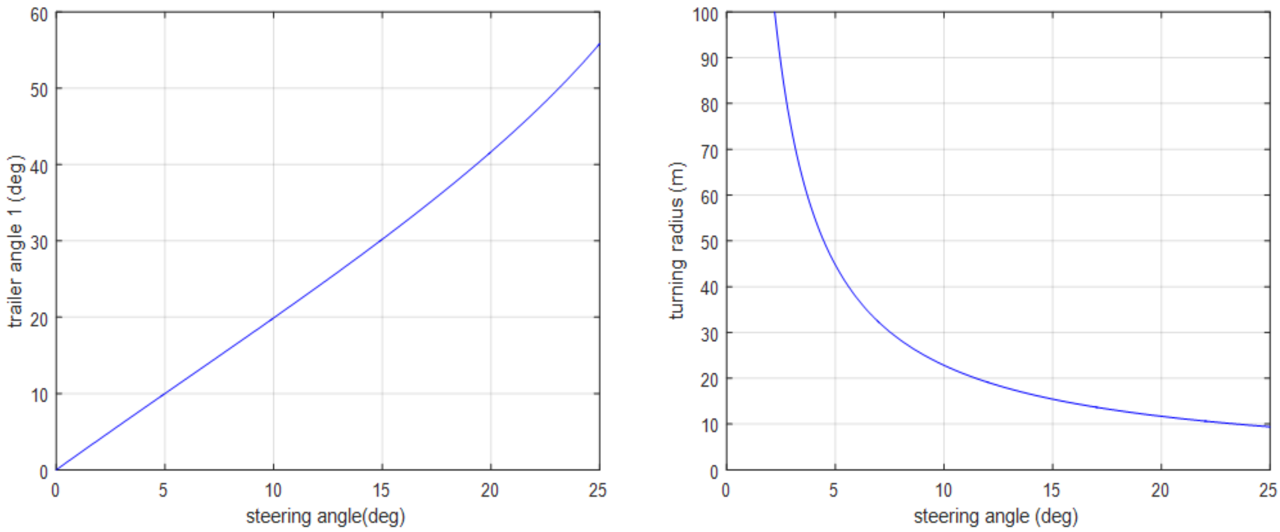

Figure 11 : Trailer angle and turning radius as a function of the steering angle for Vehicle 1

The left hand side graphs of Figure 11 and Appendix (3) show the trailer angles we would need for the system to be stable for a given steering angle. As we could have expected, the trailer angle that allows stability for a steering angle of zero degrees, would be an angle of zero degrees itself. In this case the tractor-trailer system would move backwards in a straight line. However, the steering angle will not perfectly be a zero degree angle, which will make the trailer angle change and therefore cause the instability of the system.

The right hand side graphs of Figure 11 and Appendix (3) show the turning radius we would expect for a given steering angle assuming that the system is stable. This turning radius is the radius of the concentric circle around which the tractor-trailer system would move. Knowing the value of the turning radius can be useful to predict the position of a stable tractor-trailer system moving backwards.

Knowing the trailer angle that will allow stability for a given steering angle can be useful if the system has some kind of stabilizer or mechanism that aligns the trailer. However, this might not always be the case and it would be better to know what steering angle would allow stability for a given trailer angle.

b) Steering angle as a function of the Trailer angle

The results given by the program under Appendix (2) for the vehicles described in Appendix (3) for different steering angles Ψ are given in Figure 12.

Vehicle 1			Vehicle 2			Vehicle 3			Vehicle 4		
Trailer angle (deg)	Steering angle (deg)	Turning radius (m)	Trailer angle (deg)	Steering angle (deg)	Turning radius (m)	Trailer angle (deg)	Steering angle (deg)	Turning radius (m)	Trailer angle (deg)	Steering angle (deg)	Turning radius (m)
20	10.0500	22.7180	20	7.1400	31.6356	20	9.6861	36.0797	20	9.0040	38.7696
30	14.8990	15.5400	30	10.5688	21.6400	40	18.3128	19.1976	40	17.0435	20.6289
40	19.3192	12.0880	40	13.7138	16.8329	50	21.7832	16.1087	50	20.2945	17.3097
50	23.1218	10.1430	50	16.4490	14.1245	60	24.5449	14.2490	60	22.8917	15.3113
60	26.1734	8.9720	60	18.6727	12.4939	70	26.5420	13.1320	70	24.7763	14.1110
70	28.3925	8.2687	70	20.3097	11.5144	80	27.7473	12.5304	80	25.9165	13.4646

Figure 12 : Steering angle and turning radius of a stable tractor-trailer system for different trailer angles and vehicles

Comparing the results given in Figure 12 to the graphs in Appendix (3), we can see how we get equivalent values for the angles and turning radiuses. As we could have expected, the results for both programs show that the turning radius will be bigger for a smaller steering angle and will tend to infinity for a steering angle of zero degrees since the system would be moving in a straight line.

c) Motion of the tractor-trailer assuming stability

The results given by the program under Appendix (4) for the vehicles described in Appendix (3) for a trailer angle of 30º and velocity of 1m/s are given in Figure 13 and Appendix (5).

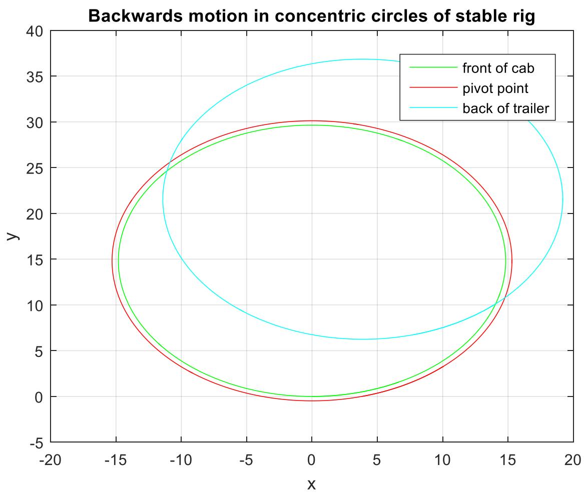

Figure 13: Backwards motion in concentric circles of stable Vehicle 1 with a trailer angle of 30 degrees and velocity of 1m/s

As we would have expected, the tractor-trailer system moves in a concentric circle for the maximum steering angle allowing stability of the rig for a given trailer angle. If we compare Figure 13 to Appendix (5) we can see that for longer vehicles the concentric circles have a longer radius and therefore a same amount of time Vehicle 1 has done at least one lap while Vehicles 2, 3 and 4 have not.

4 Discussion

4.1 Tracking

Due to the short time period given to finish the model, more time was spent on modelling the forward than the backwards motion. To simplify the model each important point on the vehicle was assumed to be a point particle. To make a more accurate model, the vehicle could have been modelled as two rigid bodies and the position of any point of both bodies could have been worked out and plotted. This method would have given much more accurate results and therefore would have created a more reliable model to work with.

4.2 Backing

In this model we have tried to understand the behaviour of a tractor-trailer system moving backwards. We have evaluated the steering and trailer angles that would allow the system to be stable and the turning radius these would imply.

Given a longer time period, modelling the reversing of such vehicles could have been looked into with more detail. We could have studied the changes in the trailer angle for different steering angles in the case of an unstable system. This would have eventually helped us find the best method to back a tractor-trailer system into a parking space.

5 Conclusion

Overall we conclude that we could get sequence of steering angles and movements that u-turn a rig or back it into proposed facilities. For tracking as the first part we have seen, we got how the key points in the system respond to changes in steering angle by computing in MATLAB. As the program letting us to know the position and the swept spaces depending on steering angles, these results are exactly what we have expected. The plotted position of key points like pivot and position of end of trailer looks working reliable. For backing as the second part we have looked, in addition, we also could get the steering angles to be stable, not to be jack-knife. Likewise, results were acted same as what we have expected. If the more steering angle is big, the more turning radius be small, and radius tend to infinity for a steering angle of zero degrees in order to move stable in straight line.

This project has merit for various sights. It not only be getting the control system design to get accurate manoeuvring of such rigs, but could apply into other situations, for example by calculating allowed roadway in facilities, increase efficiency of using the spaces and reduce wastefully taken spaces.

6 References

- Amit David Jayakaran. (2004). *Enhanced Trailer Backing.* Available: http://cimar.mae.ufl.edu/CIMAR/pages/thesis/jayakaran_a.pdf. Last accessed 18th Dec 2015.

- American Association of State Highway and Transportation Officials (2004). *Geometric Design of Highways and Streets*. USA: AASHTD. p30-34.

7 Appendices

Appendix (1) : MATLAB program, turning radius and trailer angle allowing a tractor-trailer system moving backwards to be stable for a range of steering angles

```
function backing1(w,l,L,d,maxAngle)
% w = width of vehicle
% l = length of front of vehicle
% L = length of trailer
% d = length of back of vehicle
% maxAngle = maximum steering angle in degrees
% psi = steering angle
% a = turning radius in m
psi = linspace(0,maxAngle,250); %*pi/180 to put in rads
%calculation and plot of instant centre of rotation/turning radius
a = w/2 + (l+d)*tan(pi/2-psi.*pi./180);
figure(1), plot(psi,a,'b')
grid on
xlabel('steering angle (deg)'); ylabel('turning radius (m)')
%calculation and plot of trailer angle
beta = asin(L./a); %theta in rad
figure (2), plot(psi,beta.*180./pi,'b-') % *180/pi to put value in degrees
grid on
xlabel('steering angle(deg)'); ylabel('trailer angle 1 (deg)');
end
```

Appendix (2) : MATLAB program, turning radius and steering angle allowing a tractor-trailer system moving backwards to be stable for a given trailer angle

```
function backing2(w,l,L,d,trailerAngle)
% w = width of vehicle
% l = distance of rear and front axels
% L = length of trailer
% d = distance of trailer hitch
% trailerAngle = trailer angle the driver is working with in deg
% psi = steering angle
% a = turning radius in m
trailerAngle = trailerAngle*pi/180; %to put in rad
% turning radius a
a = L/cos(pi/2-trailerAngle)
% gives steering angle to allow stability for a given trailer angle
psi = pi/2-atan((a-w/2)/(l+d));
psi = psi*180/pi %to give value in degrees
end
```

Appendix (3): Results of Appendix (1), Trailer angle and turning radius as a function of the steering angle for vehicles 2, 3 and 4

Vehicle 2: w = 2.44m, l = 1.38m, L = 10.82m, d = 2.43m, maxAngle = 20º

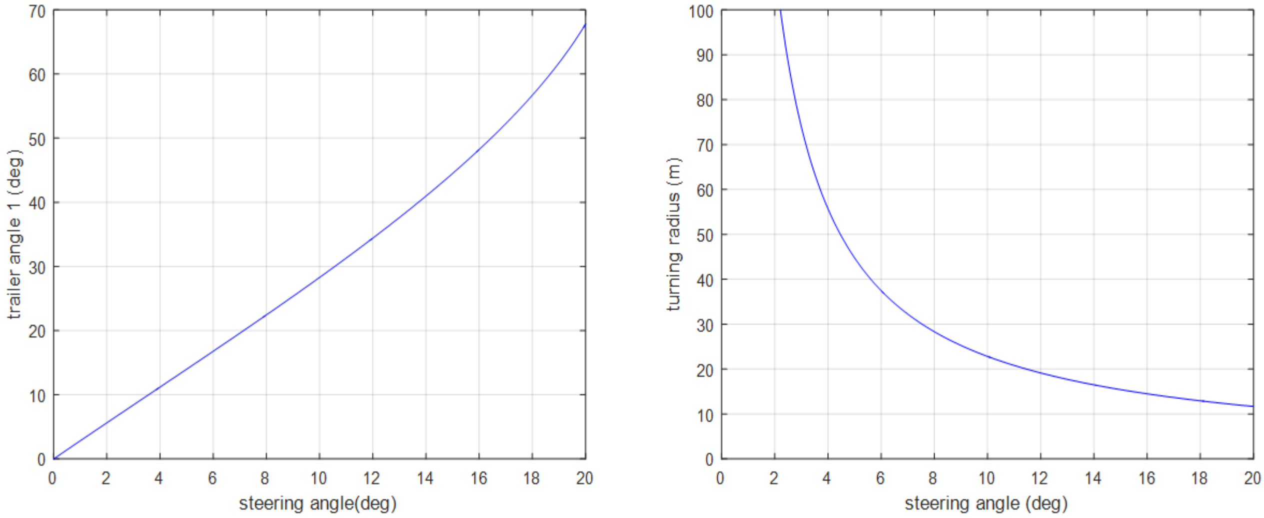

Figure 13

Vehicle 3: w = 2.44m, l = 3.35m, L = 12.34m, d = 2.6m, maxAngle = 30º

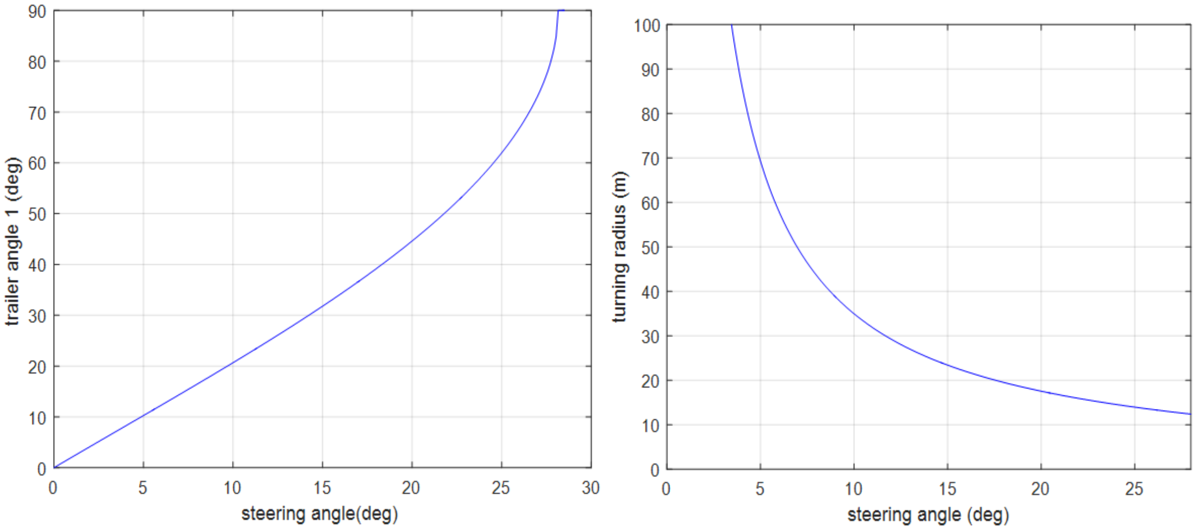

Figure 14

Vehicle 4: w = 2.44m, l = 3.35m, L = 12.34m, d = 2.6m, maxAngle = 28º

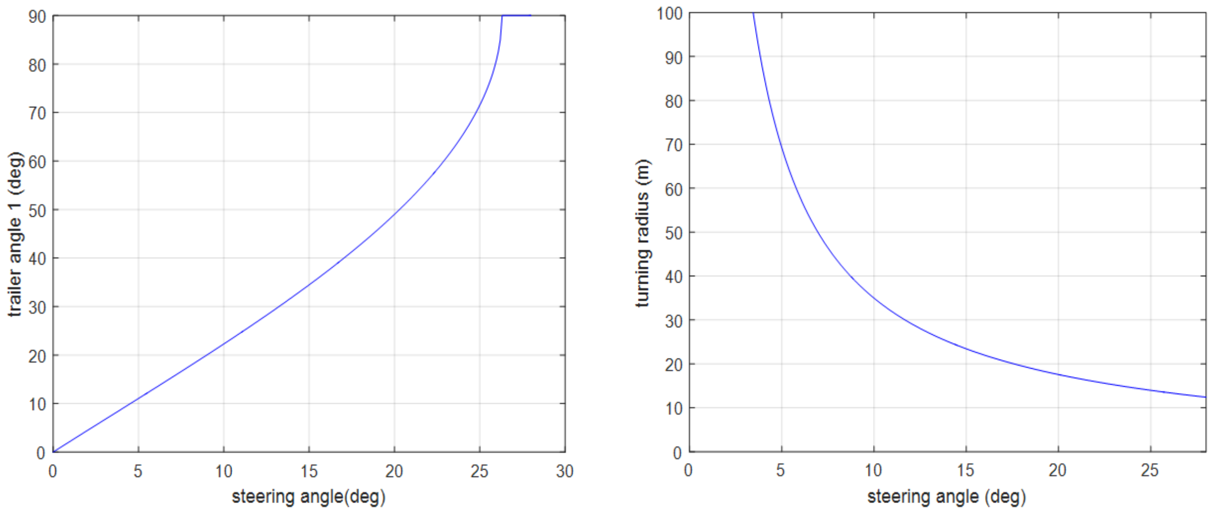

Figure 15

Appendix (4): MATLAB program simulating the backwards motion in concentric circles of a stable rig
Equations:

```
function [dY] = equationsBacking3(T,Y)

% global parameters
global Velocity steeringAngle l d;

dY = zeros(3,1); % make a column vector ready to put values in
% now set out the vector of derivatives wrt T

%Trailer Points
dY(1) = Velocity*sin(steeringAngle)/(l+d); %dgamma
dY(2) = Velocity*cos(Y(1));      %dx back
dY(3) = Velocity*sin(Y(1));      %dy back
```

ODE solver:

```
%%Clear
clc; clear; close all;
%%Define as global the parameters to be investigated

global Velocity steeringAngle l d;
Velocity = input('Enter velocity ');
l = input('Enter front vehicle length ');
d = input('Enter back vehicle length ');
L = input('Enter tail length ');
w = input('Enter width of vehicle ');
trailerAngle = input('Enter trailer angle ');

% calculation of steering angle
trailerAngle = trailerAngle*pi/180; %to put in rad
% turning radius a
a = L/cos(pi/2-trailerAngle);
% gives steering angle to allow stability for a given trailer angle
steeringAngle = pi/2-atan((a-w/2)/(l+d));

%% Now set up the model and run it.
TimeRange=linspace(0,100,250);
IC = [0;0;0];
```

```
[T,Y]=ode45(@equationsBacking3, TimeRange, IC);

gamma = Y(:,1); xf = Y(:,2); yf = Y(:,3);

xp = xf+(l+d)*cos(gamma);
yp = yf+(l+d)*sin(gamma);
xb = xp+L*sin(trailerAngle);
yb = yp+L*cos(trailerAngle);

%% Plot the results.
figure(1); plot(xf, yf, 'g-', xp, yp, 'r-',xb, yb, 'c-'), grid on, hold on
legend('front of cab','pivot point','back of trailer')

title('Backwards motion in concentric circles of stable rig');
xlabel('x'); ylabel('y');
```

Appendix (5) : Backwards motion in concentric circles of stable vehicles 2, 3 and 4 for a trailer angle of 30 degrees and velocity of 1m/s

Vehicle 2: w = 2.44m, l = 1.38m, L = 10.82m, d = 2.43m

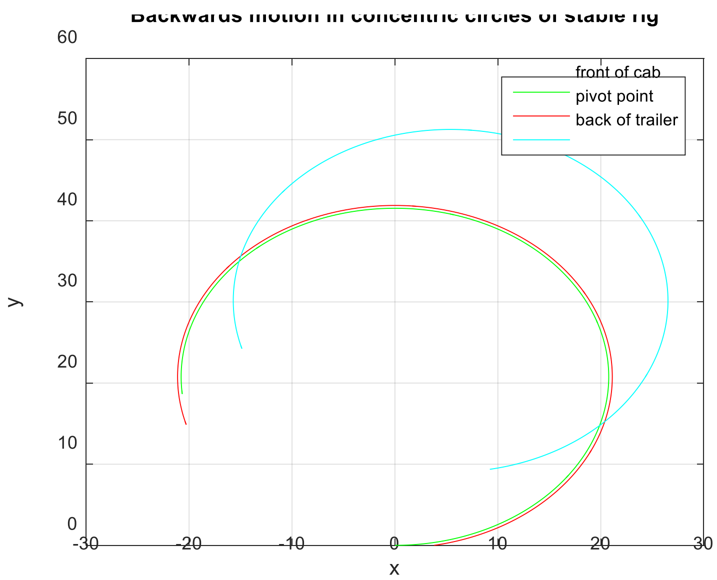

Figure 16

Vehicle 3: w = 2.44m, l = 3.35m, L = 12.34m, d = 2.6m

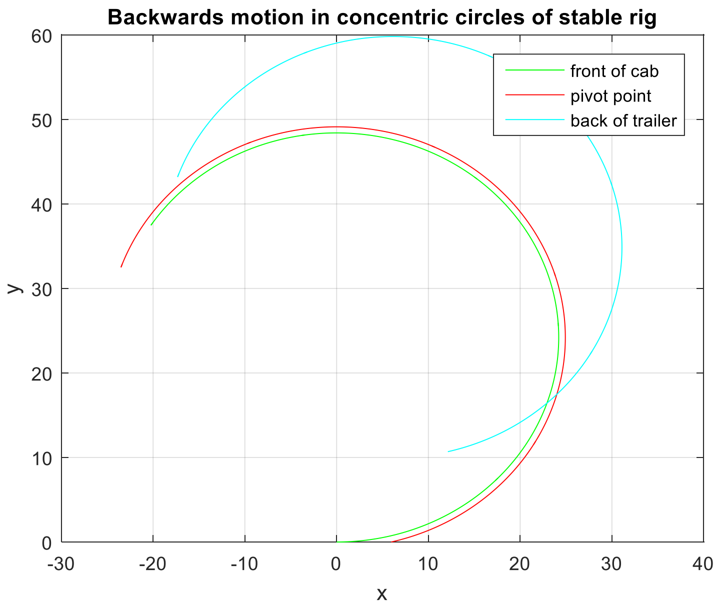

Figure 167

Vehicle 4: w = 2.44m, l = 3.35m, L = 13.26m, d = 2.6m

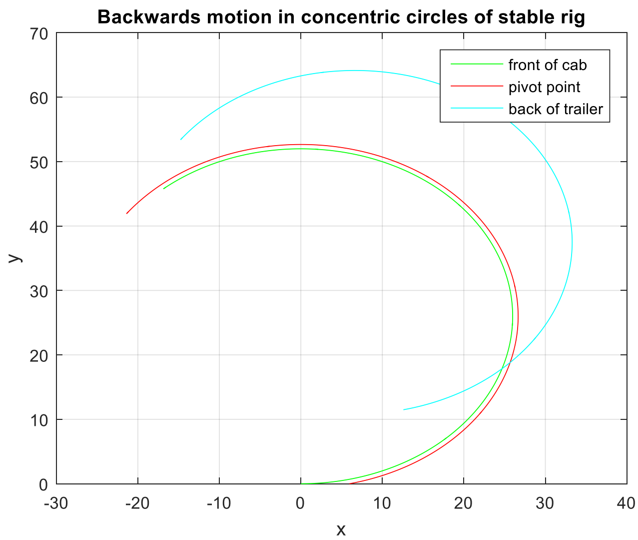

Figure 178