Incentives in CS: Exercise Set #2

Due by 11:59 PM on Sunday, February 2, 2020

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submit on Canvas. Only one person needs to submit the assignment. When submitting, please be sure to add your partner's name (if any). Be sure to add your partner to your group on Canvas.

(3) Please type your solutions. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly and clearly but not excessively.

(5) Except where otherwise noted, you may refer to anything accessible from the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are welcome to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) You have a total of 5 late days throughout the quarter.

(9) **I may add one or two more problems to this homework after lecture on January 28.**

Exercise 1 (10 points)

Consider the following game. The first number in each square denotes the left player's payoff, and the second number in each square denotes the top player's payoff. For instance, if the left player plays A, and the top player plays D, then the left player gets payoff 5 and the top player gets payoff 1.

Table 1: A 2-player 3-action game.

	D	E	F
A	(5,1)	(1,2)	(2,3)
B	(1,8)	(2,2)	(3,0)
C	(4,0)	(0,4)	(1,7)

Find a Nash equilibrium of this game. Write a brief explanation of why it is a Nash equilibrium, and state the expected payoff received by each player when this Nash equilibrium is played. (Hint: Eliminate dominated strategies first.)

Exercise 2 (10 points)

In this problem, you are asked to design a game with several properties. You will receive partial credit for designing a game that satisfies any subset of the properties **(but you may only submit one game. I.e. you cannot design 4 games, each satisfying a single property)**. Recall also the following definition:

Definition 1 (Weakly Dominate) A strategy a weakly dominates strategy b for player i if strategy a always yields at least as much payoff as b for player i, no matter what strategy the other players use, and also there exists a strategy profile for the other players such that strategy a yields strictly more payoff than strategy b against this strategy profile. A strategy is weakly dominant for player i if it weakly dominates all other strategies for player i.

Specify the payoff matrix for a game with two players and two actions each:

(i) The row player *does not* have a **weakly dominant** pure strategy.

(ii) The column player has a **weakly dominant** pure strategy. Furthermore, after deleting the column player's **weakly dominated** pure strategy, the row player now has a **weakly dominant** pure strategy. (In other words, the game is solvable by iterated deletion of weakly dominated strategies).

(iii) The game has *two* pure Nash equilibria.

(iv) The pure Nash equilibrium resulting from iterated deletion of dominated strategies is *strictly worse* for both players than the other pure Nash equilibrium (which does not result from iterated deletion of dominated strategies).

For each property, provide a brief justification of why your game satisfies it.

Exercise 3 (10 points)

Consider the following two-person zero-sum game. Both players simultaneously call out one of the numbers $\{2, 3\}$. Player I wins if the sum of the numbers called is odd and player II wins if their sum is even. The loser pays the winner the product of the two numbers called (in dollars). Find the payoff matrix, the value of the game, and an optimal strategy for each player.

Exercise 4 (10 points)

Consider the following two player game, where we refer to the row player as "Leader" and the column player as "Follower"

	Left	Right
Up	(1,1)	(3,0)
Down	(0,0)	(2,1)

a) **(2 points)** What strategy profile is a pure Nash equilibrium in this game and what payoff does Leader get in this Nash equilibrium?

b) **(2 points)** Now suppose that rather than playing the game simultaneously, the following process plays out:

- Step 1: Leader **commits** to a (possibly mixed) strategy and this commitment is guaranteed. That is, Follower can trust the Leader's commitment.
- Step 2: Follower plays a best response to the strategy Leader announced in step 1.

Concretely, suppose that in Step 1, Leader commits to playing Down. What will Leader's payoff be now?

c) **(6 points)** Show that by committing to a mixed strategy in Step 1, Leader can achieve a higher expected payoff than he can by committing to playing Down.

Exercise 5 (20 points)

This problem refers to a reputation game discussed in section 20.3 of the linked chapter (PS-Reputation-Systems.pdf). I recommend reading section 20.3 (and skimming the rest of the chapter). In that section, Repeated Prisoner's Dilemma is used as a simple model of a reputation system. For example, an airBnb guest can either have a quiet stay (Cooperate) or throw a party (Defect) and the host can either clean the apartment before the guest moves in (Cooperate) or leave it uncleaned (Defect).

Suppose now that there are $2n$ agents and that the agents are matched into pairs in a round-robin fashion. (So for example, with 6 agents, agent 1 may be matched with 2, then 3, then 4, then 5, then 6 and then 2 again and so forth.) Suppose that all actions taken are public (i.e., each agent can monitor the actions of all other agents). Suppose also that each agent begins with a *good reputation*, but will forever has a bad reputation if he ever defects against any agent with a good reputation. We call this the general reputation game.

Recall that the discounted payoff (that each player is trying to maximize) is

$$\sum_{t \geq 1} \beta^t \cdot (\text{the player's payoff in round } t).$$

In this problem, you may also want to use the fact that for $0 < \beta < 1$,

$$\sum_{t \geq j} \beta^t = \frac{\beta^j}{1 - \beta}.$$

The *reputational-grim-trigger* strategy is the following:

- Whenever you (an agent) play a round, if both you and your opponent have a good reputation, then play C.
- Otherwise, play D.

We will use the following payoff table for the stage game:

	cooperate (C)	defect (D)
cooperate (C)	$(3, 3)$	$(0, 5)$
defect (D)	$(5, 0)$	$(1, 1)$

a) **(12 points)** By writing down expressions for the discounted payoff, and adopting the single-deviation principle (that we will discuss in class on January 28), determine what must be true of the discount factor β such that it is a *subgame-perfect equilibrium* for every player to play reputational-grim-trigger. (See Section 4.3.1 of the attached chapter, PS-Extensive-Form-Games.pdf, on extensive-form games for more on the single-deviation principle.) Be sure to take into account the fact that different nodes in the game tree correspond to different numbers of agents having a bad reputation, and all types of nodes must be checked for a useful single deviation.

b) **(4 points)** *Whitewashing* is an attack in which an agent exits the system once they have a bad reputation, creates a new identity and starts afresh. (Assume that on the given platform, the user's identity is not verified, so this attack is easy to implement.) What's the highest possible discounted payoff an agent can achieve in the game using the whitewashing strategy, given that all other agents are playing the reputational-grim-trigger (in other words, re-entering every round as a new player)?

c) **(4 points)** One response to whitewashing is to introduce an *initiation fee* $f > 0$ for joining the system. Find the minimum value of f such that an agent who has defected will not choose to whitewash and then cooperate, but will instead prefer to remain in the market after defecting and keep defecting forever.