[Taller 3b] Métodos iterativos

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Tabla de Contenidos

```
%load_ext autoreload
%autoreload 2
from src import eliminacion_gaussiana, gauss_jacobi, gauss_seidel
import numpy as np
```

```
[01-20 23:15:17][INFO][DELL] 2026-01-20 23:15:17.230140
[01-20 23:15:18][INFO][DELL] 2026-01-20 23:15:18.781098
```

Ejercicio 1

$x_1 + x_2 = 7$

$-2x_1 + 5x_2 = 0$

- $ X_0 = (0, 0) $
- $ X_0 = (5, 2) $

```
#definir la matriz A y el vector b
A = np.array([[1,1], [-2, 5]])
b = np.array([7, 0])

#definir n
n = len(b)
```

Probando varios valores iniciales

```
gauss_jacobi(A=A, b=b, x=[3] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A, b=b, x=[8] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:18][INFO][DELL] i= 0 x: [[3. 3.]]
[01-20 23:15:18][INFO][DELL] i= 10 x: [[5.02048 1.98976]]
[01-20 23:15:18][INFO][DELL] i= 0 x: [[8. 8.]]
[01-20 23:15:18][INFO][DELL] i= 1 x: [[-1.  -0.4]]
[01-20 23:15:18][INFO][DELL] i= 2 x: [[7.4  2.96]]
[01-20 23:15:18][INFO][DELL] i= 3 x: [[4.04  1.616]]
[01-20 23:15:18][INFO][DELL] i= 4 x: [[5.384  2.1536]]
[01-20 23:15:18][INFO][DELL] i= 5 x: [[4.8464  1.93856]]
[01-20 23:15:18][INFO][DELL] i= 6 x: [[5.06144  2.024576]]
[01-20 23:15:18][INFO][DELL] i= 7 x: [[4.975424  1.9901696]]
[01-20 23:15:18][INFO][DELL] i= 8 x: [[5.0098304  2.00393216]]
[01-20 23:15:18][INFO][DELL] i= 9 x: [[4.99606784 1.99842714]]
[01-20 23:15:18][INFO][DELL] i= 10 x: [[5.00157286 2.00062915]]
```

```
array([[5.00157286],
       [2.00062915]])
```

```
gauss_jacobi(A=A, b=b, x=[11] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A, b=b, x=[14] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:18][INFO][DELL] i= 0 x: [[11. 11.]]
[01-20 23:15:18][INFO][DELL] i= 10 x: [[4.93856 1.90784]]
[01-20 23:15:18][INFO][DELL] i= 0 x: [[14. 14.]]
[01-20 23:15:18][INFO][DELL] i= 1 x: [[-7.  -2.8]]
[01-20 23:15:18][INFO][DELL] i= 2 x: [[9.8  3.92]]
[01-20 23:15:18][INFO][DELL] i= 3 x: [[3.08  1.232]]
[01-20 23:15:18][INFO][DELL] i= 4 x: [[5.768  2.3072]]
[01-20 23:15:18][INFO][DELL] i= 5 x: [[4.6928  1.87712]]
[01-20 23:15:18][INFO][DELL] i= 6 x: [[5.12288  2.049152]]
[01-20 23:15:18][INFO][DELL] i= 7 x: [[4.950848  1.9803392]]
[01-20 23:15:18][INFO][DELL] i= 8 x: [[5.0196608  2.00786432]]
[01-20 23:15:18][INFO][DELL] i= 9 x: [[4.99213568 1.99685427]]
[01-20 23:15:18][INFO][DELL] i= 10 x: [[5.00314573 2.00125829]]
```

```
array([[5.00314573],
       [2.00125829]])
```

```
gauss_jacobi(A=A, b=b, x=[25] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A, b=b, x=[32] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:19][INFO][DELL] i= 0 x: [[25. 25.]]
[01-20 23:15:19][INFO][DELL] i= 10 x: [[4.7952  1.76448]]
[01-20 23:15:19][INFO][DELL] i= 0 x: [[32. 32.]]
[01-20 23:15:19][INFO][DELL] i= 1 x: [[-25. -10.]]
[01-20 23:15:19][INFO][DELL] i= 2 x: [[17.   6.8]]
[01-20 23:15:19][INFO][DELL] i= 3 x: [[0.2  0.08]]
[01-20 23:15:19][INFO][DELL] i= 4 x: [[6.92  2.768]]
[01-20 23:15:19][INFO][DELL] i= 5 x: [[4.232  1.6928]]
[01-20 23:15:19][INFO][DELL] i= 6 x: [[5.3072  2.12288]]
[01-20 23:15:19][INFO][DELL] i= 7 x: [[4.87712  1.950848]]
[01-20 23:15:19][INFO][DELL] i= 8 x: [[5.049152  2.0196608]]
[01-20 23:15:19][INFO][DELL] i= 9 x: [[4.9803392  1.99213568]]
[01-20 23:15:19][INFO][DELL] i= 10 x: [[5.00786432 2.00314573]]
```

```
array([[5.00786432],
       [2.00314573]])
```

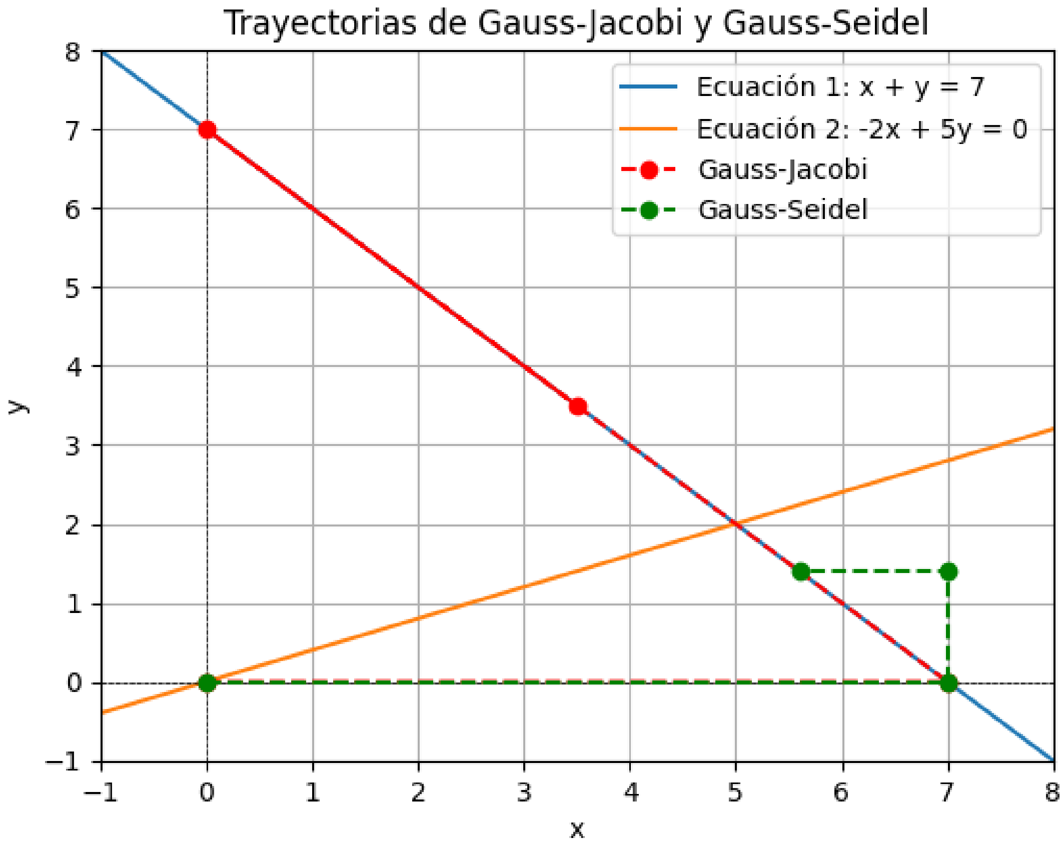
Continuacion normal del taller

```
gauss_jacobi(A=A, b=b, x=[0] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A, b=b, x=[0] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:19][INFO][DELL] i= 0 x: [[0. 0.]]
[01-20 23:15:19][INFO][DELL] i= 10 x: [[5.0512  2.02048]]
[01-20 23:15:19][INFO][DELL] i= 0 x: [[0. 0.]]
[01-20 23:15:19][INFO][DELL] i= 1 x: [[7.  2.8]]
[01-20 23:15:19][INFO][DELL] i= 2 x: [[4.2  1.68]]
[01-20 23:15:19][INFO][DELL] i= 3 x: [[5.32  2.128]]
[01-20 23:15:19][INFO][DELL] i= 4 x: [[4.872  1.9488]]
[01-20 23:15:19][INFO][DELL] i= 5 x: [[5.0512  2.02048]]
[01-20 23:15:19][INFO][DELL] i= 6 x: [[4.97952  1.991808]]
[01-20 23:15:19][INFO][DELL] i= 7 x: [[5.008192  2.0032768]]
[01-20 23:15:19][INFO][DELL] i= 8 x: [[4.9967232  1.99868928]]
[01-20 23:15:19][INFO][DELL] i= 9 x: [[5.00131072 2.00052429]]
[01-20 23:15:19][INFO][DELL] i= 10 x: [[4.99947571 1.99979028]]
```

```
array([[4.99947571],
       [1.99979028]])
```

```
# graficar trayectorias
import matplotlib.pyplot as plt
x_vals = np.linspace(-1, 8, 100)
y1 = 7 - x_vals
y2 = (2/5) * x_vals
plt.plot(x_vals, y1, label='Ecuación 1: x + y = 7')
plt.plot(x_vals, y2, label='Ecuación 2: -2x + 5y = 0')
plt.xlim(-1, 8)
plt.ylim(-1, 8)
plt.xlabel('x')
plt.ylabel('y')
plt.axhline(0, color='black',linewidth=0.5, ls='--')
plt.axvline(0, color='black',linewidth=0.5, ls='--')
plt.grid()
plt.legend()
plt.title('Trayectorias de Gauss-Jacobi y Gauss-Seidel')
# Puntos de iteración (ejemplo)
jacobi_points = [(0, 0), (7, 0), (0, 7), (3.5, 3.5)]
seidel_points = [(0, 0), (7, 0), (7, 1.4), (5.6, 1.4)]
jacobi_x, jacobi_y = zip(*jacobi_points)
seidel_x, seidel_y = zip(*seidel_points)
plt.plot(jacobi_x, jacobi_y, 'ro--', label='Gauss-Jacobi')
plt.plot(seidel_x, seidel_y, 'go--', label='Gauss-Seidel')
plt.legend()
plt.show()
```



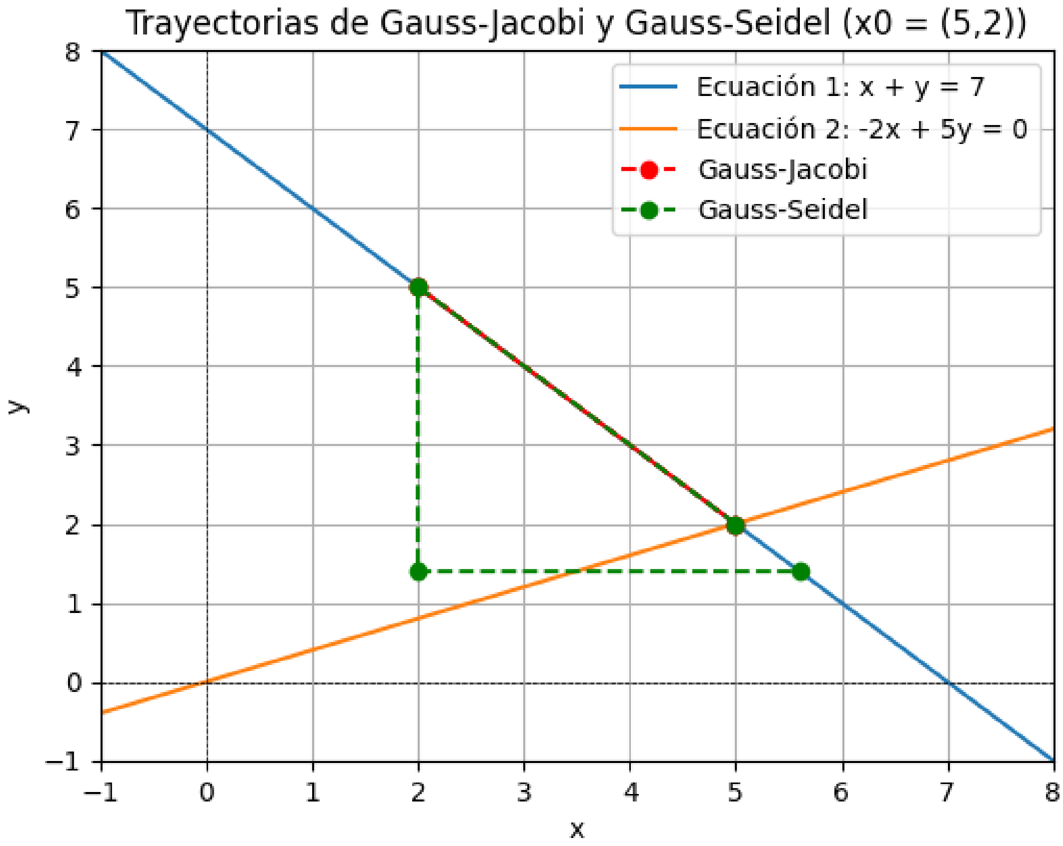
```
# x = (5, 2)

gauss_jacobi(A=A, b=b, x=[5] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A, b=b, x=[2] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:22][INFO][DELL] i= 0 x: [[5. 5.]]
[01-20 23:15:22][INFO][DELL] i= 10 x: [[5.      1.96928]]
[01-20 23:15:22][INFO][DELL] i= 0 x: [[2. 2.]]
[01-20 23:15:22][INFO][DELL] i= 1 x: [[5. 2.]]
```

```
array([[5.],
       [2.]])
```

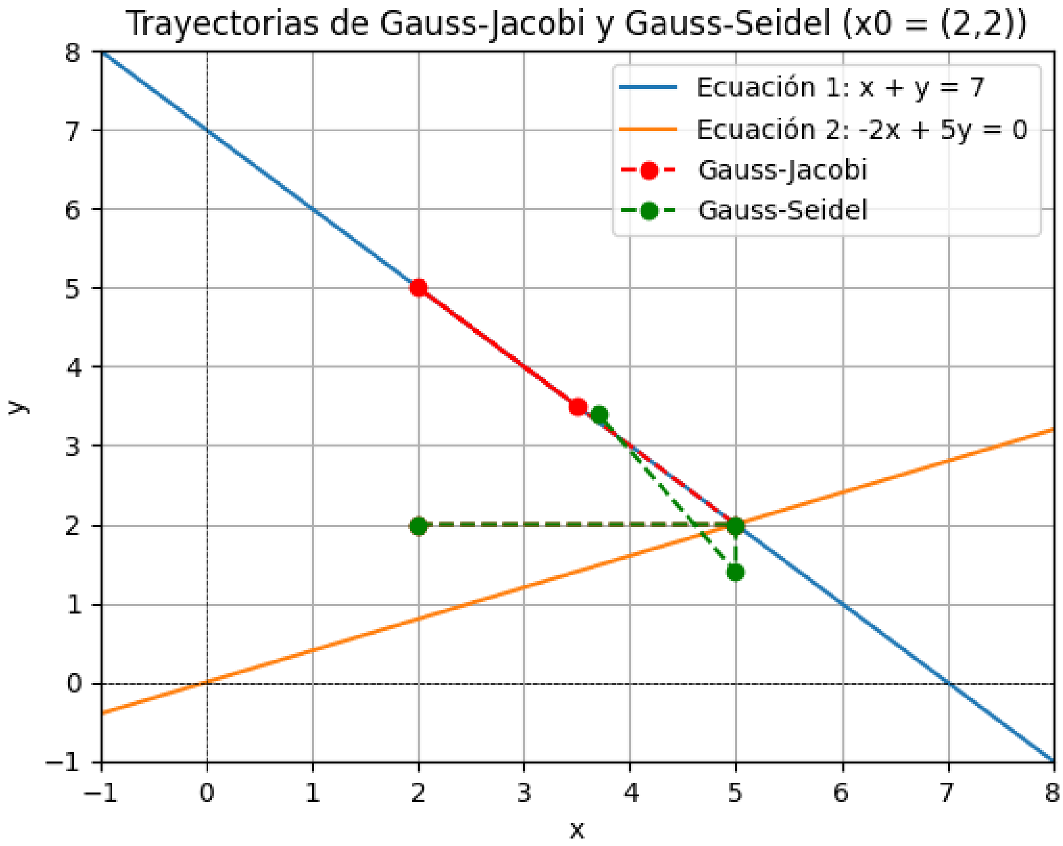
```
# graficar trayectorias
import matplotlib.pyplot as plt
x_vals = np.linspace(-1, 8, 100)
```

```
y1 = 7 - x_vals
y2 = (2/5) * x_vals
plt.plot(x_vals, y1, label='Ecuación 1: x + y = 7')
plt.plot(x_vals, y2, label='Ecuación 2: -2x + 5y = 0')
plt.xlim(-1, 8)
plt.ylim(-1, 8)
plt.xlabel('x')
plt.ylabel('y')
plt.axhline(0, color='black',linewidth=0.5, ls='--')
plt.axvline(0, color='black',linewidth=0.5, ls='--')
plt.grid()
plt.legend()
plt.title('Trayectorias de Gauss-Jacobi y Gauss-Seidel (x0 = (5,2))')
# Puntos de iteración (ejemplo)
jacobi_points = [(5, 2), (2, 5), (2,    5), (5, 2)]
seidel_points = [(5, 2), (2, 5), (2, 1.4), (5.6, 1.4)]
jacobi_x, jacobi_y = zip(*jacobi_points)
seidel_x, seidel_y = zip(*seidel_points)
plt.plot(jacobi_x, jacobi_y, 'ro--', label='Gauss-Jacobi')
plt.plot(seidel_x, seidel_y, 'go--', label='Gauss-Seidel')
plt.legend()
plt.show()
```



```
# graficar la convergencia de ambos metodos
import matplotlib.pyplot as plt
x_vals = np.linspace(-1, 8, 100)
y1 = 7 - x_vals
y2 = (2/5) * x_vals
plt.plot(x_vals, y1, label='Ecuación 1: x + y = 7')
plt.plot(x_vals, y2, label='Ecuación 2: -2x + 5y = 0')
plt.xlim(-1, 8)
plt.ylim(-1, 8)
plt.xlabel('x')
plt.ylabel('y')
plt.axhline(0, color='black',linewidth=0.5, ls='--')
plt.axvline(0, color='black',linewidth=0.5, ls='--')
plt.grid()
plt.legend()
plt.title('Trayectorias de Gauss-Jacobi y Gauss-Seidel (x0 = (2,2))')
# Puntos de iteración (ejemplo)
```

```
jacobi_points = [(2, 2), (5, 2), (2, 5), (3.5, 3.5)]
seidel_points = [(2, 2), (5, 2), (5, 1.4), (3.7, 3.4)]
jacobi_x, jacobi_y = zip(*jacobi_points)
seidel_x, seidel_y = zip(*seidel_points)
plt.plot(jacobi_x, jacobi_y, 'ro--', label='Gauss-Jacobi')
plt.plot(seidel_x, seidel_y, 'go--', label='Gauss-Seidel')
plt.legend()
plt.show()
```



Ejercicio 2

$x_1 + x_2 = 6$

$-2x_1 + x_2 = 0$

```python
A2 = np.array([[1,1], [-2, 1]])
b2 = np.array([6, 0])
n = len(b2)
```

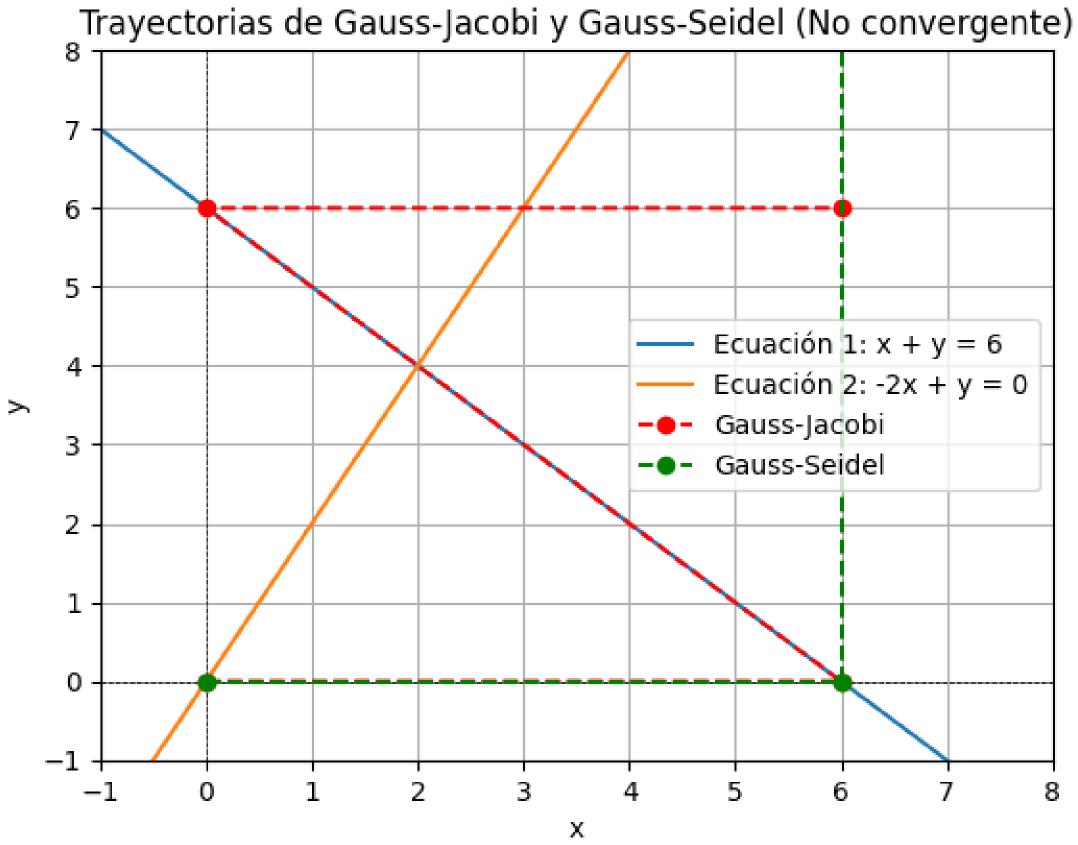
```python
gauss_jacobi(A=A2, b=b2, x=[0] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A2, b=b2, x=[0] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:23][INFO][DELL] i= 0 x: [[0. 0.]]
[01-20 23:15:23][INFO][DELL] i= 10 x: [[ 66. 132.]]
[01-20 23:15:23][INFO][DELL] i= 0 x: [[0. 0.]]
[01-20 23:15:23][INFO][DELL] i= 1 x: [[ 6. 12.]]
[01-20 23:15:23][INFO][DELL] i= 2 x: [[ -6. -12.]]
[01-20 23:15:23][INFO][DELL] i= 3 x: [[18. 36.]]
[01-20 23:15:23][INFO][DELL] i= 4 x: [[-30. -60.]]
[01-20 23:15:23][INFO][DELL] i= 5 x: [[ 66. 132.]]
[01-20 23:15:23][INFO][DELL] i= 6 x: [[-126. -252.]]
[01-20 23:15:23][INFO][DELL] i= 7 x: [[258. 516.]]
[01-20 23:15:23][INFO][DELL] i= 8 x: [[ -510. -1020.]]
[01-20 23:15:23][INFO][DELL] i= 9 x: [[1026. 2052.]]
[01-20 23:15:23][INFO][DELL] i= 10 x: [[-2046. -4092.]]
```

```
array([[-2046.],
       [-4092.]])
```

```python
# graficaer trayectorias
x_vals = np.linspace(-1, 8, 100)
y1 = 6 - x_vals
y2 = (2) * x_vals
plt.plot(x_vals, y1, label='Ecuación 1: x + y = 6')
plt.plot(x_vals, y2, label='Ecuación 2: -2x + y = 0')
plt.xlim(-1, 8)
plt.ylim(-1, 8)
plt.xlabel('x')
plt.ylabel('y')
plt.axhline(0, color='black',linewidth=0.5, ls='--')
plt.axvline(0, color='black',linewidth=0.5, ls='--')
plt.grid()
plt.legend()
plt.title('Trayectorias de Gauss-Jacobi y Gauss-Seidel (No convergente)')
# Puntos de iteración (ejemplo)
jacobi_points = [(0, 0), (6, 0), (0, 6), (6, 6)]
```

```
seidel_points = [(0, 0), (6, 0), (6,    12), (24, 12)]
jacobi_x, jacobi_y = zip(*jacobi_points)
seidel_x, seidel_y = zip(*seidel_points)
plt.plot(jacobi_x, jacobi_y, 'ro--', label='Gauss-Jacobi')
plt.plot(seidel_x, seidel_y, 'go--', label='Gauss-Seidel')
plt.legend()
plt.show()
```



Ejercicio 3

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & 1 & -1 \\ 0 & 3 & -1 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 1360 \\ 1130 \\ 1350 \\ 3650 \end{bmatrix}$$

```
A3 = np.array([[1, -1, 2, 0], [-1, 1, -1, 3], [2, -1, 1, -1], [0, 3, -1, 2]])
b3 = np.array([1360, 1130, 1350, 3650])
n = len(b3)
```

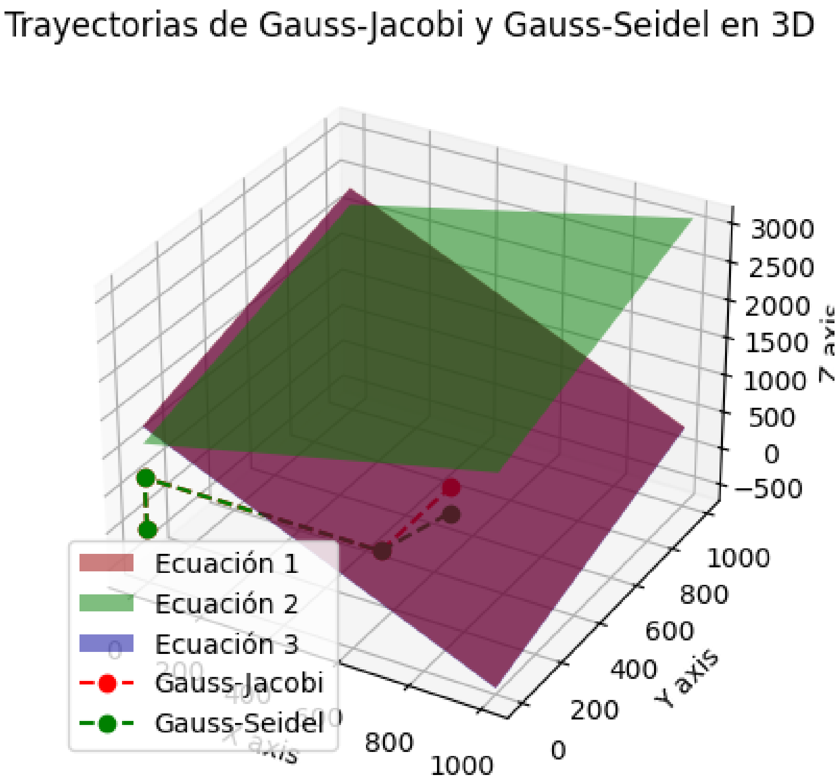
```
gauss_jacobi(A=A3, b=b3, x=[0] * n, tol=1e-5, max_iter=10)  # type: ignore
gauss_seidel(A=A3, b=b3, x=[0] * n, tol=1e-5, max_iter=10)  # type: ignore
```

```
[01-20 23:15:23][INFO][DELL] i= 0 x: [[0. 0. 0. 0.]]
[01-20 23:15:23][INFO][DELL] i= 10 x: [[  9890452.5   -16286231.25   14521376.25   -6565926.25]]
[01-20 23:15:23][INFO][DELL] i= 0 x: [[0. 0. 0. 0.]]
[01-20 23:15:23][INFO][DELL] i= 1 x: [[ 1360.   2490.   1120. -1350.]]
[01-20 23:15:23][INFO][DELL] i= 2 x: [[ 1610.   7910.   4690. -7695.]]
[01-20 23:15:23][INFO][DELL] i= 3 x: [[  -110.    28795.    22670.   -30032.5]]
[01-20 23:15:23][INFO][DELL] i= 4 x: [[-15185.     98712.5  100400.    -96043.75]]
[01-20 23:15:23][INFO][DELL] i= 5 x: [[-100727.5     288933.75    395695.     -233728.125]]
[01-20 23:15:23][INFO][DELL] i= 6 x: [[-501096.25     596913.125  1366727.5     -210180.9375]]
[01-20 23:15:23][INFO][DELL] i= 7 x: [[-2135181.875     -136781.5625    3924751.25      2169372
[01-20 23:15:23][INFO][DELL] i= 8 x: [[ -7984924.0625   -10567161.71875    7573409.375      19
[01-20 23:15:23][INFO][DELL] i= 9 x: [[-2.57126205e+07 -7.70558979e+07 -5.99003469e+06  1.125
[01-20 23:15:23][INFO][DELL] i= 10 x: [[-6.50744685e+07 -4.08835337e+08 -1.66094395e+08  5.30
```

```
array([[-6.50744685e+07],
       [-4.08835337e+08],
       [-1.66094395e+08],
       [ 5.30207632e+08]])
```

```
#graficar la trayectoria
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
x_vals = np.linspace(0, 1000, 100)
y_vals = np.linspace(0, 1000, 100)
X, Y = np.meshgrid(x_vals, y_vals)
Z1 = (1360 + Y - 2*X)
Z2 = (1130 + X + Y)/1
Z3 = (1350 - 2*X + Y)
```

```
ax.plot_surface(X, Y, Z1, alpha=0.5, rstride=100, cstride=100, color='r', label='Ecuación 1')
ax.plot_surface(X, Y, Z2, alpha=0.5, rstride=100, cstride=100, color='g', label='Ecuación 2')
ax.plot_surface(X, Y, Z3, alpha=0.5, rstride=100, cstride=100, color='b', label='Ecuación 3')
ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
plt.title('Trayectorias de Gauss-Jacobi y Gauss-Seidel en 3D')
# Puntos de iteración (ejemplo)
jacobi_points = [(0, 0, 0), (0, 0, 680), (680, 0, 680), (680, 340, 680)]
seidel_points = [(0, 0, 0), (0, 0, 680), (680, 0, 680), (680, 340, 340)]
jacobi_x, jacobi_y, jacobi_z = zip(*jacobi_points)
seidel_x, seidel_y, seidel_z = zip(*seidel_points)
ax.plot(jacobi_x, jacobi_y, jacobi_z, 'ro--', label='Gauss-Jacobi')
ax.plot(seidel_x, seidel_y, seidel_z, 'go--', label='Gauss-Seidel')
plt.legend()
plt.show()
```


Trayectorias de Gauss-Jacobi y Gauss-Seidel en 3D
Z axis
Y axis
X axis
Ecuación 1
Ecuación 2
Ecuación 3
Gauss-Jacobi
Gauss-Seidel