1. 换元法

$$\boxed{\int \frac{f'(x)}{f(x)}\,\mathrm{d}x = \ln|f(x)| + C.}$$

例.

$$\int \frac{x}{x^2+8}\,\mathrm{d}x$$

推断过程:

$\because \dfrac{\mathrm{d}}{\mathrm{d}x}(x^2+8) = 2x$

$\therefore$ 使用换元法, 设 $t = x^2 + 8$.

得到 $\mathrm{d}t = 2x\,\mathrm{d}x$

$$\therefore \int \frac{x}{x^2+8}\,\mathrm{d}x = \frac{1}{2}\int \frac{1}{t}\,\mathrm{d}t = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|x^2+8| + C$$

2.