1.指数公式

$$b^0 = 1 \tag{1}$$

$$b^1 = b \tag{2}$$

$$b^x b^y = b^{x+y} \tag{3}$$

$$\frac{b^x}{b^y} = b^{x-y} \tag{4}$$

$$(b^x)^y = b^{xy} \tag{5}$$

2.对数公式

$$\log_b(1) = 0 \tag{6}$$

$$\log_b(b) = 1 \tag{7}$$

$$\log_b(xy) = \log_b(x) + \log_b(y) \tag{8}$$

$$\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y) \tag{9}$$

$$\log_b(x^y) = y \log_b(x) \tag{10}$$

换底法则 对于任意的底数$b > 1$和$c > 1$及任意的数$x > 0$,

$$\boxed{\log_b(x) = \frac{\log_c(x)}{\log_c(b)}} \tag{11}$$

3.自然对数

$e = 2.718281828459045 23\ldots$

$$e^{\ln(x)} = x \tag{12}$$

$$\ln(e^x) = x \tag{13}$$

$$\ln(1) = 0 \tag{14}$$

$$\ln(e) = 1 \tag{15}$$

$$\ln(xy) = ln(x) + ln(y) \tag{16}$$

$$\ln(\frac{x}{y}) = \ln(x) - \ln(y) \tag{17}$$

$$\ln(x^y) = y \ln(x) \tag{18}$$

4.常用对数求导

$$\lim_{n\to\infty}(1+\frac{x}{n})^n = e^x \tag{19}$$
$$\lim_{n\to\infty}(1+\frac{1}{n})^n = e \tag{20}$$
$$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)} \tag{21}$$

证明过程:

假设$g(x) = \log_b x$,则

$$\begin{aligned} g'(x) &= \lim_{\Delta x\to 0}\frac{\log_b(x+\Delta x) - \log_b x}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{1}{\Delta x}\log_b x + \Delta x x \\ &= \lim_{\Delta x\to 0}\log_b(1+\frac{\Delta x}{x})^{\frac{1}{\Delta x}} \\ &= \log_b e^{\frac{1}{x}} \\ &= \frac{1}{x}\log_b e \end{aligned}$$

由换底法则,得

$$\begin{aligned} \log_b e &= \frac{\ln e}{\ln b} \\ &= \frac{1}{\ln b} \end{aligned}$$

所以,得出结论

$$\begin{aligned} g'(x) &= \frac{1}{x}\log_b e \\ &= \frac{1}{x\ln b} \end{aligned}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \tag{22}$$
$$\frac{d}{dx}b^x = b^x\ln(b) \tag{23}$$
$$\frac{d}{dx}e^x = e^x \tag{24}$$