1. 三角函数的极限:

$$\boxed{\lim_{x\to 0}\frac{\sin(x)}{x}=1}$$

$$\boxed{\lim_{x\to 0}\frac{\tan(x)}{x}=1}$$

实例.

$$\begin{aligned}
\lim_{x\to 0}\frac{\sin^3(2x)\cos(5x^{19})}{x\tan(5x^2)} &= \lim_{x\to 0}\frac{[\frac{(\sin(2x))^3}{(2x)^3}\times(2x)^3]\cos(5x^{19})}{x[\frac{\tan(5x^2)}{5x^2}\times(5x^2)]} \\
&= \lim_{x\to 0}\frac{\frac{(\sin(2x))^3}{(2x)^3}\cdot\cos(5x^{19})}{\frac{\tan(5x^2)}{5x^2}}\times\frac{(2x)^3}{x(5x^2)} \\
&= \lim_{x\to 0}\frac{(\frac{\sin(2x)}{2x})^3\cos(5x^{19})}{\frac{\tan(5x^2)}{5x^2}}\times\frac{8x^3}{5x^3}=\frac{8}{5}
\end{aligned}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos(x)}{x}=0}$$

证明:

$$\begin{aligned}
\lim_{x\to 0}\frac{1-\cos(x)}{x} &= \lim_{x\to 0}\frac{1-\cos(x)}{x}\times\frac{1+\cos(x)}{1+\cos(x)} \\
&= \lim_{x\to 0}\frac{1-\cos^2(x)}{x}\times\frac{1}{1+\cos(x)} \\
&= \lim_{x\to 0}\frac{\sin^2(x)}{x}\times\frac{1}{1+\cos(x)} \\
&= \lim_{x\to 0}\sin(x)\times\frac{\sin(x)}{x}\times\frac{1}{1+\cos(x)} \\
&= 0\times 1\times\frac{1}{1+1}=0
\end{aligned}$$

对于任意的 x, $\boxed{-1\leqslant\sin(x)\leqslant 1}$ 和 $\boxed{-1\leqslant\cos(x)\leqslant 1}$

面对 $x\to a$ 的极限, 而 $a\neq 0$ 时, 有一个很好的一般原则, 那就是用 $t=x-a$ 作替换, 将问题转化为 $t\to 0$

2. 三角函数的导数

$$\boxed{\frac{d}{dx}\sin(x) = \cos(x)}$$

$$\boxed{\frac{d}{dx}\cos(x) = -\sin(x)}$$

$$\boxed{\frac{d}{dx}\tan(x) = \sec^2(x)}$$

$$\boxed{\frac{d}{dx}\cot(x) = -\csc^2(x)}$$

$$\boxed{\frac{d}{dx}\sec(x) = \sec(x)\tan(x)}$$

$$\boxed{\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)}$$