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Expanded Optimization for Discovering Optimal Lateral Handling Bicycles

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Abstract

Previously, we introduced a method of optimizing four primary geometric parameters of a bicycle's design to minimize its Handling Quality Metric. Here we expand that method to optimize 23 geometric and inertial parameters in the benchmark parameterimization of the linear Whipple-Carvallo bicycle model. To ensure physically realizable bicycle designs we include 7 equality constraints, 21 inequality constraints, and maximal and minamal bounds on each free opmization parameter. This improves over the prior work by expanding the search space with many more parameters and the guarntee of realizability. We close by presenting five bicycle designs that have optimal later handling qualities. The bicycles are not generally self-stable and have some ununusal characteristics.

1 Introduction

Physical design features of ground vehicles can affect their lateral handling qualities. Geometry, mass, and mass distribution of the vehicle's components as well as tire characteristics are primary contributors to poor and good handling due to their important influence on the vehicle's dynamics. In past work, we have presented a theoretical and computational framework for assessing the lateral task-independent handling qualities of simplified single track vehicle designs [2, 5]. In subsequent work, we showed that minimizing our proposed handling quality metric (HQM) can produce theoretically optimal handling designs when only four geometric parameters are explored as the optimization variables [4]. The present work's goal is to expand this optimization problem to all of the geometry, mass, and inertial parameters present in the linear Whipple-Carvallo bicycle model [3]. This broadens the search space considerably but we constrain it so that only realizable optimal bicycle designs are discovered. To do so, we formulate a constrained optimization problem and use derivative-free optimization to discover optimal, yet realizable, bicycle designs.

2 Bicycle Model Parameterization

Our problem formulation relies on a new bicycle model parameterization that reflects both a reformulation of and addition to the benchmark parameterization of the linear Whipple-Carvallo bicycle model [3]. We call this the "principal parameterization" as opposed to the "benchmark parameterization" in [3]. This parameterization differs from the benchmark parameterization in three ways. Firstly, the person and rear frame are treated as separate rigid bodies each with their on inertial parameters. Secondly, we express the inertial parameters of each rigid body in terms of central principal radii of gyration to decouple the mass from the inertia terms. Lastly, we introduce two simple dimensional parameters that define the geometric extents of the person which are used to constrain the location of the person's body. Table 1 provides the parameters and the reference values which are derived from the measurements of a Batavus Browser Bicycle and the rider "Jason" presented in [5]. This parameterization can be transformed into the benchmark parameterization readily, but not vice versa.

Table 1. Full set of 47 principal parameters and their default values derived from the measurements in [5] of the Batavus Browser bicycle and rider "Jason". Note that the benchmark parameters are also used in the paper and are defined in [3].

Variable	Value	Units	Description
c	0.068581	m	Trail
w	1.1210	m	Wheelbase
λ	0.39968	rad	Steer axis tilt
g	9.81	$\mathrm{m\,s^{-2}}$	Acceleration due to gravity
v	3.0, 5.0, 7.0, 9.0	$\mathrm{m\,s^{-1}}$	Forward speed
Rear Wheel [R]			
m_R	3.11	kg	Mass
r_R	0.34096	m	Radius
x_R	0.0	m	X mass center coordinate
y_R	0.0	m	Y mass center coordinate
z_R	-0.34096	m	Z mass center coordinate
k_{Raa}	0.17050	m	Central principal radii of gyration about A_R
k_{Rbb}	0.17050	m	Central principal radii of gyration about B_R
k_{Ryy}	0.22136	m	Central principal radii of gyration about Y
Front Wheel [F]			
m_F	2.02	kg	Mass
r_F	0.34353	m	Radius
x_F	1.1210	m	X mass center coordinate
y_F	0.0	m	Y mass center coordinate
z_F	-0.34353	m	Z mass center coordinate
k_{Faa}	0.20917	m	Central principal radii of gyration about A_F
k_{Fbb}	0.20917	m	Central principal radii of gyration about B_F
k_{Fyy}	0.27179	m	Central principal radii of gyration about Y
Person [P]			
l_P	1.728	m	Body length
w_P	0.483	m	Body width
m_P	83.5	kg	Mass
x_P	0.31577	m	X mass center coordinate
y_P	0.0	m	Y mass center coordinate
z_P	-1.0990	m	Z mass center coordinate
k_{Paa}	0.36797	m	Central principal radii of gyration about A_P
k_{Pbb}	0.15276	m	Central principal radii of gyration about B_P
k_{Pyy}	0.36717	m	Central principal radii of gyration about Y
α_P	0.18618	rad	Angle about Y between A_P and X
Front Frame [H]			
m_H	3.2200	kg	Mass
x_H	0.86695	m	X mass center coordinate
y_H	0.0	m	Y mass center coordinate
z_H	-0.74824	m	Z mass center coordinate
k_{Haa}	0.29556	m	Central principal radii of gyration about A_H
k_{Hbb}	0.14493	m	Central principal radii of gyration about B_H
k_{Hyy}	0.27630	m	Central principal radii of gyration about Y
α_H	0.36995	rad	Angle about Y between A_H and X
Rear Frame [D]			
m_D	9.8600	kg	Mass
x_D	0.27595	m	X mass center coordinate
y_D	0	m	Y mass center coordinate
z_D	-0.53784	m	Z mass center coordinate
k_{Daa}	0.28587	m	Central principal radii of gyration about A_D
k_{Dbb}	0.22079	m	Central principal radii of gyration about B_D
k_{Dyy}	0.36539	m	Central principal radii of gyration about Y
α_D	1.1722	rad	Angle about Y between A_D and X

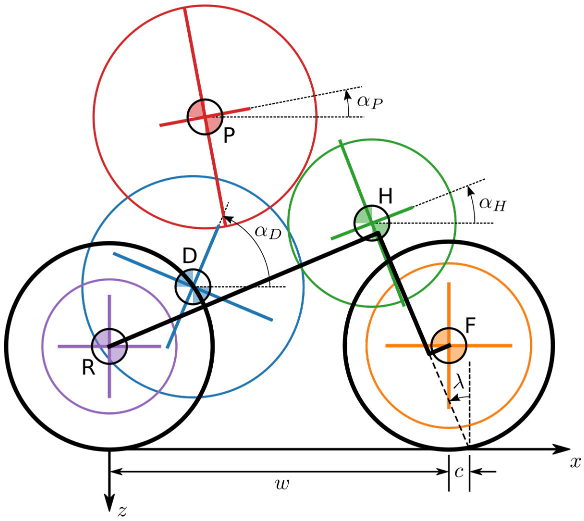

Figure 1. Depiction of the principal parameters of the Batavus Browser with rider "Jason". The solid black lines represent the essential bicycle geometry. The dotted black line represents the steer axis. The inertial properties of the five rigid bodies: front wheel (orange), rear wheel (purple), rear frame (blue), front frame (green), and person (red) are shown with the mass center and the extents of the centroidal principal radii of gyration of each rigid body. The primary principal angles, $\alpha_{D,P,H}$, are defined as the angle about the y axis from x to the maximum principal axes in the XZ plane.

3 Bounds and Constraints

The optimal principal parameters are subject to a set of constraints designed to ensure that a physically realizable bicycle is obtained from the optimization procedure. These constraints are made up of bounds on the free parameters and both equality and inequality constraints among the parameters. Below the basic constraint concepts presented and grouped by the associated rigid body or collection thereof:

Total T The combination of the five rigid bodies.

- The likely physical extents of the rigid bodies must exist above the ground plane.
- Both bicycle and rider are symmetric about the rider's sagittal plane.
- The total mass is below a reasonably human lift-able amount.
- The wheels cannot overlap.
- The bicycle cannot topple forward during hard braking or backward during hard acceleration.
- The closed-loop path tracking controlled system [2] must be stable. This is required to obtain a valid HQM value.

Rider P A single rigid body represents the rider.

- Rider mass is that of an average person.
- The rider's joint angles are fixed in a nominal configuration typical of upright bicycling and the resulting mass distribution is derived from standard body segment estimation methods.
- The rider cannot penetrate the ground.

Frames H, D Front frame (handlebar + fork) and rear frame

Table 2. Parameter lower and upper bounds.

Min		Parameter		Max
$-\infty$	$\leq$	w	$\leq$	∞
$-\infty$	$\leq$	c	$\leq$	∞
$-\pi/2$	$\leq$	λ	$\leq$	$\pi/2$
1.0kg	$\leq$	m_D	$\leq$	∞
$-\infty$	$\leq$	x_D	$\leq$	∞
$-\infty$	$\leq$	z_D	$\leq$	0.0
0.0	$\leq$	k_{Daa}	$\leq$	∞
0.0	$\leq$	k_{Dbb}	$\leq$	∞
$-\pi/2$	$\leq$	α_D	$\leq$	$\pi/2$
$-\infty$	$\leq$	x_P	$\leq$	∞
$-\infty$	$\leq$	z_P	$\leq$	0.0
$-\pi/2$	$\leq$	α_P	$\leq$	$\pi/2$
0.25	$\leq$	m_H	$\leq$	∞
$-\infty$	$\leq$	x_H	$\leq$	∞
$-\infty$	$\leq$	z_H	$\leq$	0.0
0.0	$\leq$	k_{Haa}	$\leq$	∞
0.0	$\leq$	k_{Hbb}	$\leq$	∞
0.0	$\leq$	k_{Hyy}	$\leq$	∞
$-\pi/2$	$\leq$	α_H	$\leq$	$\pi/2$
0.127m	$\leq$	r_R	$\leq$	∞
1.0kg	$\leq$	m_R	$\leq$	∞
0.127m	$\leq$	r_F	$\leq$	∞
1.0kg	$\leq$	m_F	$\leq$	∞

Table 3. Equality constraints.

Constraint	Equation	Description
g_1	$I_{Dyy} = \sqrt{I_{Dxx}^2 + I_{Dzz}^2}$	Rear frame is planar.
g_2	$k_{Ryy} = r_R$	Rear wheel is a ring
g_3	$k_{Raa} = k_{Ryy}/2$	Rear wheel is a ring
g_4	$k_{Rbb} = k_{Ryy}/2$	Rear wheel is a ring
g_5	$k_{Fyy} = r_F$	Front wheel is a ring
g_6	$k_{Faa} = k_{Fyy}/2$	Front wheel is a ring
g_7	$k_{Fbb} = k_{Fyy}/2$	Front wheel is a ring

- The rear frame is planar in nature and the front frame's moments of inertia are consistently dependent.
- The mass and inertia of the frames are positive and large enough to be constructed from a steel space frame.

Wheels F, R Both front and rear wheels have identical constraints.

- Wheel radius and mass must be positive and be greater than a minimum value.
- Wheels are inertially wheel-like, i.e. symmetric about each plane and most of the mass is at the rim.

These bounds, equality, and inequality constraints are presented mathematically in tables 2, 3, 4 respectively and explained in more detail in the following sections.

Table 4. Inequality constraints.

Constraint	Equation	Description
c_1	$\sqrt{I_{Hxx}^2 + I_{Hzz}^2} \geq I_{Hyy}$	Consistent moments of inertia.
c_2	$0 \geq z_P + l_P/2 \cos\alpha_P$	Person cannot penetrate ground.
c_3	$0 \geq z_P + w_P/2 \sin\alpha_P$	Person cannot penetrate ground.
c_4	$0 \geq z_P - l_P/2 \cos\alpha_P$	Person cannot penetrate ground.
c_5	$0 \geq z_P - w_P/2 \sin\alpha_P$	Person cannot penetrate ground.
c_6	$x_T \geq \lvert z_T\rvert/4$	Maximum acceleration of $1/4g$.
c_7	$w - x_T \geq 3/4\lvert z_T\rvert$	Maximum deceleration of $3/4g$.
c_8	$2k_{Hyy} \geq \sqrt{(x_H - w)^2 + (z_H + r_F)^2}$	Minimal inertial spread.
c_9	$2k_{Dyy} \geq \sqrt{(x_D - 0)^2 + (z_D + r_R)^2}$	Minimal inertial spread.
c_{10}	$w \geq r_F + r_R$	Non-overlapping wheels.
c_{11}	$25\text{kg} \geq m_D + m_H + m_R + m_F$	Maximum bicycle mass.
c_{12}	$-z_D \geq 1.4k_{Dyy}$	Rear frame cannot penetrate ground.
c_{13}	$-z_H \geq 1.4k_{Hyy}$	Front frame cannot penetrate ground.
$c_{14,\ldots,21}$	$0 \geq s_1, \ldots, s_8$	Closed loop stability.

3.1 Person [P]

We assume that the person's joint configurations are such that they are in a nominal configuration for pedaling, i.e. an average normal everyday riding position on a typical bicycle. We retain the same configuration as they were seated on the Batavus Browser bicycle. The person is assumed to be symmetric about the XZ plane. We allow the rider to be rotated about the Y axis and positioned anywhere within the plane of symmetry above the ground.

To prevent the rider from being positioned and oriented such that their body is not penetrating the ground we introduce two dimensions that define a cross whose apex is at the center of mass of the person and the cross axes are parallel to the principal axes in the XZ plane. $l_P/2$ is the distance along the principal axis to the tip of the toes and $w_P/2$ is the distance along the second principal axes to the tip of the hands. The constraints $c_2, \ldots, c_5$ are derived from these rules.

3.2 Front Frame [H]

The front frame is symmetric about the XZ plane so $I_{Hxy}, I_{Hyz} = 0$. We allow for any angular orientation of the XZ principal directions but limit the angle to $-\frac{\pi}{2} \leq \alpha_H \leq \frac{\pi}{2}$. We prevent the rear frame from penetrating the ground by limiting the inertial spread with respect to its mass center, c_{13}, but also set a minimum inertial spread to ensure a frame can span from the rear wheel to the mass center of the rear frame, c_8. The spread factor in c_{13} of 1.4 is based on the ratio of geometrical spread of a typical bicycle frame and its radius of gyration. The front frame is not planar but is narrow with respect to the XZ plane, which is enforced by constraint c_1.

3.3 Rear Frame [D]

Several constraints are set for the rear frame. We constrain the rear frame to be planar, g_1, and symmetric with respect to the XZ plane. We prevent the rear frame from penetrating the ground by limiting the inertial spread with respect to its mass center, c_{12}, but also set a minimum inertial spread to ensure a frame can span from the rear wheel to the mass center of the rear frame, c_9. The spread factor in c_{12} of 1.4 is based on the ratio of geometrical spread of a typical bicycle frame and its radius of gyration. Several parameters are bounded. We require the rear frame mass to be positive, the center of mass not penetrate the ground, and we allow for any angular orientation of the XZ principal directions but limit the angle to $-\frac{\pi}{2} \leq \alpha_D \leq \frac{\pi}{2}$ as angle beyond that are redundant.

3.4 Front [F] and Rear [R] Wheels

We enforce the assumption that both wheels have moments of inertia of that of a simple ring, $g_2 \ldots g_7$ and that mass and radius should be greater than a minimal size based on small purchasable spoked wheel with tire.

3.5 Total Bike [T]

The trail and wheelbase can take on any real values. The steer axis tilt is limited to ± 90 degrees. We introduce a constraint c_{10} that prevents the wheels from physically overlapping and require that the bicycle be lift-able by an average person, c_{11}. Finally, we require that the bicycle not topple forward during hard breaking or backward during hard acceleration with:

$$-\frac{3g}{4} < \text{acceleration} < \frac{g}{4}. \tag{1}$$

This translates to two constraints, c_6, c_7 that bound the total center of mass (x_T, z_T) in a triangle in the XZ plane. Lastly, we constrain the eight closed loop eigenvalues associated with the controller in [2] to be stable, i.e. have negative real parts. These are expressed in constraints $c_{14}, \ldots, c_{21}$. Closed loop stability is required for the HQM to provide a meaningful result.

4 Optimization

The above constraints leave 23 of the 47 parameters free for optimizing which we collect in the vector $\mathbf{p} \in \mathbb{R}^{23}$ and define as:

$$\begin{aligned} \mathbf{p} = [&w \quad c \quad \lambda \quad m_D \quad x_D \quad z_D \quad k_{Daa} \quad k_{Dbb} \quad \alpha_D \quad x_P \quad z_P \quad \alpha_P \\ &m_H \quad x_H \quad z_H \quad k_{Haa} \quad k_{Hbb} \quad k_{Hyy} \quad \alpha_H \quad r_R \quad m_R \quad r_F \quad m_F] \end{aligned} \tag{2}$$

Our objective in the optimization is to minimize the peak HQM value subject to the bounds, $\mathbf{p}^L, \mathbf{p}^U$, and the constraints $\mathbf{g}(\mathbf{p}), \mathbf{c}(\mathbf{p})$. Given a set of bicycle model parameter values we generate a bandwidth limited human-like controller using the methods in [5]. Once the closed loop stable controller is constructed, the HQM can be computed as per the definition in [2] and the scaler peak value returned as the objective J. This problem is presented as a non-linear programming problem in the following equation.

$$\begin{aligned} \underset{\mathbf{p}}{\text{minimize}} \quad & J(\mathbf{p}) = max(\text{HQM}(\mathbf{p})) \\ \text{subject to} \quad & \\ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0} \\ & \mathbf{c}(\mathbf{p}) = \mathbf{0} \\ & \mathbf{p}^L \leq \mathbf{p} \leq \mathbf{p}^U \end{aligned} \tag{3}$$

We make use of the derivative-free optimizer CMA-ES [1] to find solutions to this problem. The optimization supports parameter bounds and equality constraints but does not support inequality

Table 5. Peak HQM values for the reference bicycle and the optimal bicycles at each design speed.

Speed [m/s]	Reference Peak HQM	Optimal Peak HQM	Percent Improvement
3	13.075	2.012	85%
5	4.521	0.012	100%
7	3.043	0.022	99%
9	2.338	0.839	64%

constraints. To get around this limitation we move the inequality constraints into the objective function and penalize the objective if the constraints are violated with the following rules:

$$J(\mathbf{p}) = \begin{cases} max(\text{HQM}(\mathbf{p})) & \text{if} \quad all(\mathbf{g}(\mathbf{p})) \leq 0 \\ 30 + ||\mathbf{g}_{+}(\mathbf{p})||/10 & \text{if} \quad any(\mathbf{g}(\mathbf{p})) > 0 \text{ and } ||\mathbf{g}_{+}(\mathbf{p})|| < 30 \\ ||\mathbf{g}_{+}(\mathbf{p})|| & \text{if} \quad any(\mathbf{g}(\mathbf{p})) > 0 \text{ and } ||\mathbf{g}_{+}(\mathbf{p})|| \geq 30 \end{cases} \tag{4}$$

where $||\mathbf{g}_{+}||$ is the norm of the vector of positive elements of $\mathbf{g}$ and any, all are "any elements of" and "all elements of", respectively.

This creates a discontinuous objective function but in practice the CMA-ES algorithm is able to move into the parameter space where all the constraints are satisfied and find a (local) minima. For our purposes, this sufficiently finds parameter values that produce optimally handling bicycle designs.

5 Results

We discover four bicycles for different design speeds (3, 5, 7, and 9 $\mathrm{m\,s^{-1}}$) that have an optimally low HQM, see Table 5, and satisfy all constraints and parameter bounds. The optimal parameter values for these four bicycles are presented in Table 6. We belive these bicycles to be reasoably physically realizable. The pictorial representation of the bicycles are presented in Figure 5.

The bicycle optimized for 3 $\mathrm{m\,s^{-1}}$ is about three times larger than the reference bicycle. The wheelbase is about 3 m and it has a relatively large postive trail (0.7 m). The person is rearward on the bike and over 3 m above the ground. The mass of the front frame is the minimum posible value and the mass of the rear frame is also small.

The bicycle optimized for 5 $\mathrm{m\,s^{-1}}$ has a similar geometric scale as the reference bicycle except that the front wheel is smaller and the rear wheel is larger. The person is located low to the ground and tipped back, like one might see on a recumbent bicycle. This significantly increases the vehicles yaw moment of inertia. The trail is miminal, but suprisingly slight negative (5 mm). The steer axis tilt is slightly shallower than the reference bicycle. The mass of the rear frame is similar to the reference bicycle but the front frame is at the bound and much lower. The minor principal axis of the front frame is almost aligned with the steer axis.

The bicycle optimized for 5 $\mathrm{m\,s^{-1}}$ is one of the three that is geometrically smaller than the reference bicycle. The wheelbase is slightly larger but the wheels are approximately half the diameter. The rider is low to the ground and tipped forward. There is effiectively no trail, the steer axis is vertical, and the (very) minimal principal axis is aligned with the steer axis making the steering inertia-less. The roll moment of inertia is larger.

The bicycle optimized for 5 $\mathrm{m\,s^{-1}}$ is much larger than the reference bicycle: wheelbase and rear wheel diameter of 6 m, front wheel diameter of 4 m. The trail is large and negative, surely making the bicycle open-loop unstable.

Table 6. Optimal principal parameter values for each design speed. Values with an asterisk are at a bound.

v	$3\ \mathrm{m\,s^{-1}}$	$5\ \mathrm{m\,s^{-1}}$	$7\ \mathrm{m\,s^{-1}}$	$9\ \mathrm{m\,s^{-1}}$
c	0.688	−0.005	0.001	−0.484
w	2.866	0.847	1.157	5.557
λ	−0.213	0.271	−0.028	0.239
m_D	3.22	11.53	10.32	7.02
x_D	0.958	0.287	0.427	1.020
z_D	−2.605	−2.719	−2.976	−4.065
α_D	0.663	0.915	1.123	1.077
k_{Daa}	1.449	1.218	1.324	0.861
k_{Dbb}	0.724	1.387	1.290	2.014
k_{Dyy}	1.471	1.559	1.555	2.031
m_H	0.25*	0.25*	0.49	3.54
x_H	2.356	0.532	1.145	5.615
z_H	−1.298	−0.781	−1.340	−1.776
α_H	0.572	−1.265	1.571	0.238
k_{Haa}	0.186	0.0491	0.000*	2.846
k_{Hbb}	1.259	1.145	2.006	0.168
k_{Hyy}	0.636	0.383	0.818	0.669
m_F	1.62	4.40	2.64	5.51
r_F	0.710	0.252	0.127*	2.063
k_{Faa}	0.355	0.126	0.064	1.031
k_{Fyy}	0.710	0.252	0.127	2.063
x_P	0.765	0.276	0.526	1.079
z_P	−3.194	−0.453	−0.586	−2.662
α_P	−0.661	1.150	−0.836	1.558
m_R	12.63	8.36	6.31	2.25
r_R	0.958	0.506	0.146	3.027
k_{Raa}	0.479	0.253	0.073	1.514
k_{Ryy}	0.958	0.506	0.146	3.027

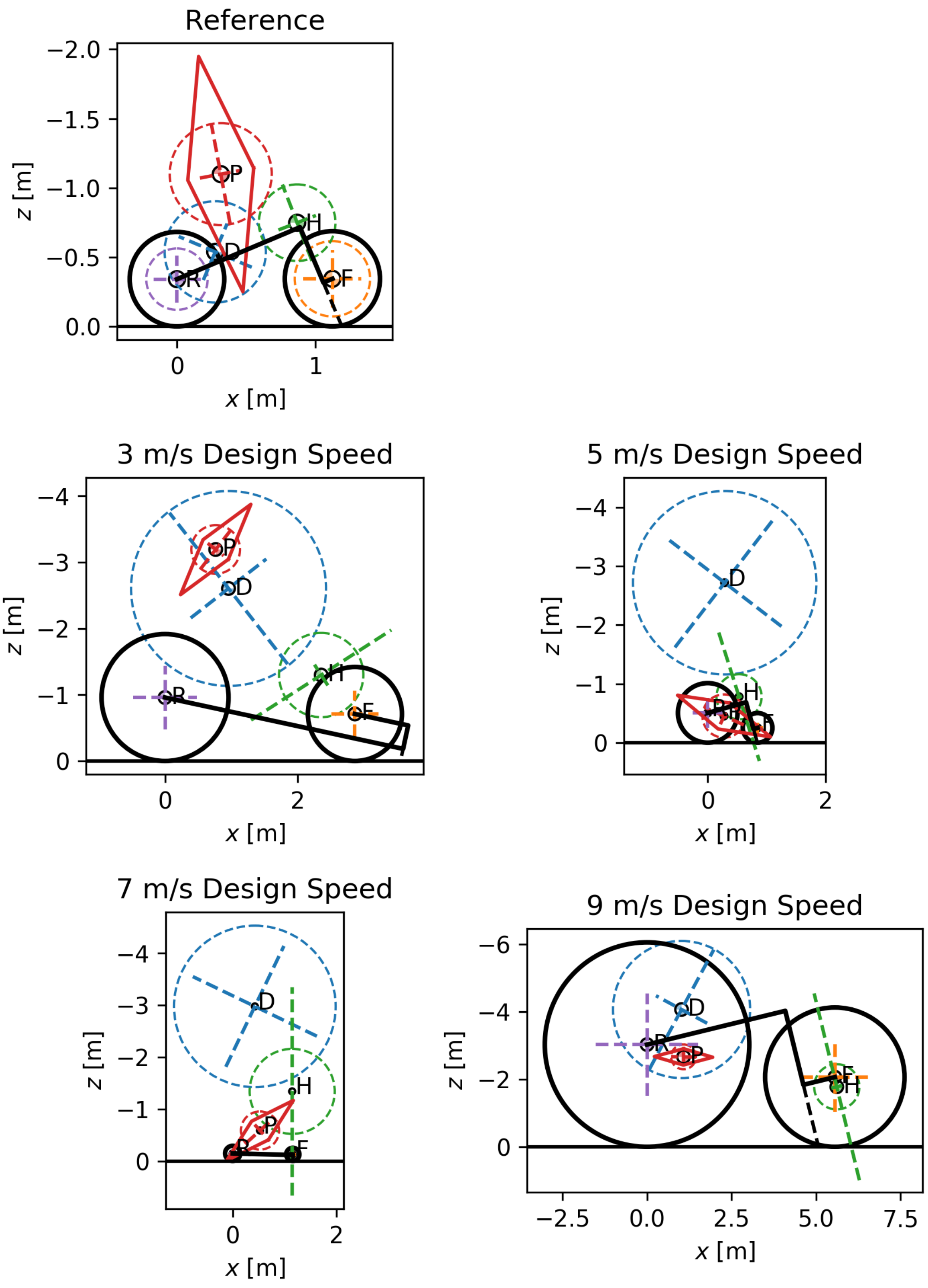

Figure 2. Depictions of the bicycle geometry and geometric representations of the inertial quantities for the reference bicycle and four optimal solutions at 3, 5, 7, and 9 m/s. Five rigid bodies are shown for each bicycle: front wheel (orange), rear wheel (purple), rear frame (blue), front frame (green), and person (red). The solid black lines represent the essential bicycle geometry. The dotted black line represents the steer axis. The solid colored curves represent the contours of solid ellipsoids with equivalent inertia as the principal inertia of the associated rigid body. The dotted colored lines represent the extents of the centroidal radii of gyration of each rigid body.

The parameters of the bicycles can be distilled into open loop eigenvalues for a clearer understanding of the dynamics, shown in Figure 5. The 3 and 9 design speed have a simlar eigenvalue pattern as well as the 5 and 7. The 7 $\mathrm{m\,s^{-1}}$ design is self-stable at speeds greater than 1.2 $\mathrm{m\,s^{-1}}$ but the other three designs are not self-stable at any travel speed from $0\,\mathrm{m\,s^{-1}}$ to $10\,\mathrm{m\,s^{-1}}$. Both the 5 and 7 designs have a weave mode that increases in frequency rapidly with speed. The weave frequency of the 3 m/s design is higher than the reference design and present at all speeds.

6 Discussion and Conclusion

We have demonstrated the ability to find optimal parameter values of the linear Whipple-Carvallo bicycle model under constraints that enforce a physically realizable bicycle with an objective of improving lateral handling qualities. We showcase four optimal bicycle designs for a range of target travel speeds. The resulting bicycles are similar to the familiar and popular bicycle design but have some oddities: very large size relative to the person, large negative trail, large positive trail, minimal steer inertia, recumbent rider orientation, and both very low and high rider mass center locations. Three of the four bicycles are not open-loop self stable for any speed up to 10 $\mathrm{m\,s^{-1}}$ and the weave model frequencies are significantly larger than the reference bicycle but all bicycles are controllable and have a minimal HQM as defined in [2].

The bicycles are reasonably physically realizable but some additional constraints could be introduced to further improve this. In the 5 $\mathrm{m\,s^{-1}}$ design the person's body overlaps the wheels, which would be almost impossible to realize. A constraint that ensures the torso cannot occupy the same space as the wheels would solve this. The masses of the front and rear frames are low with respect to the geometrical spread a steel space frame would have to occupy. The frames also need to be able to span the space between the respecitve wheel, mass center of the frame, and the steer axis and for the rear frame span to the rider's support locations. We currently only expliclty deal with spanning the space between the mass center of the frame and the respective wheel. Another more restricting appraoch would be to specify a generic geometric structure for the front and rear frames. If done, maximal stress and deflections of the frames under load could also be minimized.

Each optimizal solution requires approximately 12 hours of computation time on a high end consumer desktop machine running on a single thread. The number of iterations are typically in the hundreds of thousands for convergence. There are many avenues for speeding this up that could likely result in solutions in one or two hours on the same machine but this is still slow to be particualrly pleasant to utlizie in the process of designing a bicycle. Furthermore, the time required to translate the resulting parameters in to an actual bicycle design complete with structural destails is extremely time consuming. Never-the-less, the method shows promise for optimizing an entire vehicle for optimal dynamics. This method can be applied to a whole host of human operated vehicles opening up many new designs, but as with any optimization it only captures a very small set of the variables that a designer has to take into account for a vehicle.

7 Reproducibility

All of the source code, data, and documents needed to reproduce the presented results and this paper can be found at the repository hosted at `https://github.com/moorepants/BMD2019`.

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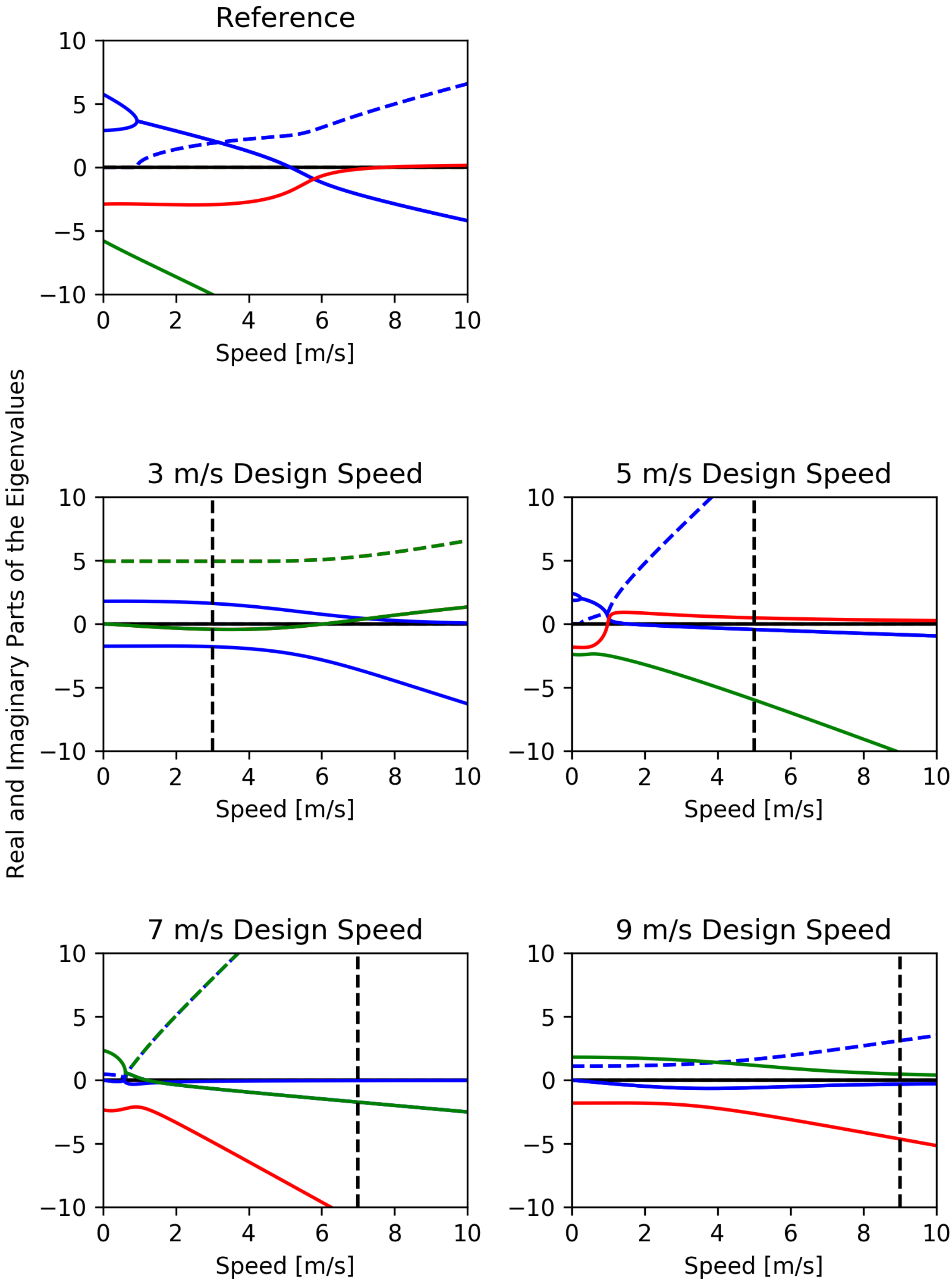

Figure 3. Real (solid) and imaginary (dashed) parts of the open-loop eigenvlues plotted versus travel speed. The vertical dashed black line indicates the design speed.

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