Eco industrielle – Reinforcement learning :

Problem IV : Getting confidence in market equilibriums – an agent based perspective :

Case 1: Traditional Cournot equilibriums:

$$\mathrm{P(Q)} = 600 - Q$$

- For all cases, the cost of production c is equal to 0.
- For each firm i we can write the profit $\Pi_i = P(Q_i).Q_i - c = P(Q_i).Q_i$

Monopoly:

Let q_1: the quantity sold on the market by the monopolist: We have the following equations:

- $\mathrm{P(Q_1)} = 600 - Q_1$
- $\Pi_1 = P(Q_1).Q_1 = (600 - Q_1).Q_1$

We know that the firm wants to maximize this profit leading to :

$$\frac{\partial \Pi_1(Q_1)}{\partial Q_1} = 0$$

Hence:

$$\frac{\partial[(600 - Q_1).Q_1]}{\partial Q_1} = 0 \Leftrightarrow 600 - 2Q_1 = 0 \Leftrightarrow Q_1 = \frac{600}{2} = 300$$

For the monopoly, we have an equilibrium quantity $Q_1 = 300$.
The market price is equal to $P(Q_1) = 600 - Q_1 = 600 - 300$

$$\boldsymbol{P = 300}$$

Duopoly:

Let q_1: the quantity sold on the market by the firm 1.
Let q_2: the quantity sold on the market by the firm 2: We have the following equations:

- $\mathrm{P(Q_1}, Q_2) = 600 - (Q_1 + Q_2)$
- $\Pi_1 = P(Q_1, Q_2).Q_1 = (600 - (Q_1 + Q_2).Q_1$
- $\Pi_2 = P(Q_1, Q_2).Q_2 = (600 - (Q_1 + Q_2).Q_2$

Each firm wants to maximize its profit leading to:

$$\frac{\partial \Pi_1(Q_1, Q_2)}{\partial Q_1} = 0$$
$$\frac{\partial \Pi_2(Q_1, Q_2)}{\partial Q_2} = 0$$

We can solve the system:
For the firm 1:

$$\frac{\partial[(600 - (Q_1 + Q_2)).Q_1]}{\partial Q_1} = 0 \Leftrightarrow 600 - 2Q_1 - Q_2 = 0 \Leftrightarrow Q_1 = \frac{600 + Q_2}{2} = 300$$

For the firm 2:

$$\frac{\partial[(600 - (Q_1 + Q_2)).Q_2]}{\partial Q_2} = 0 \Leftrightarrow 600 - 2Q_2 - Q_1 = 0 \Leftrightarrow Q_2 = \frac{600 + Q_1}{2} = 300$$

We can write the system with a matrix to ease solving:

$$A.Q = B \Leftrightarrow \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} . \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} 600 \\ 600 \end{pmatrix}$$

With the invert of A, A^{-1}:

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

We have:

$$Q = A^{-1}.B$$

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.\begin{pmatrix} 600 \\ 600 \end{pmatrix}$$

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 600 \\ 600 \end{pmatrix} = \begin{pmatrix} 200 \\ 200 \end{pmatrix}$$

Finally, it gives us the following market price: $\mathrm{P}(\mathrm{Q}_1, Q_2) = 600 - (Q_1 + Q_2) = 200$

$$\boldsymbol{P = 200}$$

Triopoly:

Let q_1: the quantity sold on the market by the firm 1.
Let q_2: the quantity sold on the market by the firm 2.
Let q_3: the quantity sold on the market by the firm 3: We have the following equations:

- $\mathrm{P}(\mathrm{Q}_1, Q_2, Q_3) = 600 - (Q_1 + Q_2 + Q_3)$
- $\Pi_1 = P(\mathrm{Q}_1, Q_2, Q_3).Q_1$ = $(600 - (Q_1 + Q_2 + Q_3).Q_1$
- $\Pi_2 = P(\mathrm{Q}_1, Q_2, Q_3).Q_2$ = $(600 - (Q_1 + Q_2 + Q_3).Q_2$
- $\Pi_3 = P(\mathrm{Q}_1, Q_2, Q_3).Q_3$ = $600 - (Q_1 + Q_2 + Q_3).Q_3$

Each firm wants to maximize its profit leading to:

$$\frac{\partial \Pi_1(Q_1, Q_2, Q_3)}{\partial Q_1} = 0$$

$$\frac{\partial \Pi_2(Q_1, Q_2, Q_3)}{\partial Q_2} = 0$$

$$\frac{\partial \Pi_3(Q_1, Q_2, Q_3)}{\partial Q_3} = 0$$

We can solve the system:

For the firm 1:

$$\frac{\partial\left[\left(600 - (Q_1 + Q_2 + Q_3)\right).Q_1\right]}{\partial Q_1} = 0 \iff 600 - 2Q_1 - Q_2 - Q_3 = 0$$

$$Q_1 = \frac{600 + Q_2}{2} = 300$$

For the firm 2:

$$\frac{\partial[(600 - (Q_1 + Q_2 + Q_3)).Q_2]}{\partial Q_2} = 0 \iff 600 - 2Q_2 - Q_1 - Q_3 = 0$$

$$Q_2 = \frac{600 + Q_1 + Q_3}{2} = 300$$

For the firm 2:

$$\frac{\partial[(600 - (Q_1 + Q_2 + Q_3)).3]}{\partial Q_3} = 0 \iff 600 - 2Q_3 - Q_1 - Q_2 = 0$$

$$Q_3 = \frac{600 + Q_2 + Q_3}{2} = 300$$

We can write the system with a matrix to ease solving:

$$A.Q = B \iff \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} = \begin{pmatrix} 600 \\ 600 \\ 600 \end{pmatrix}$$

With the invert of A, A^{-1}:

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

We have:

$$Q = A^{-1}.B$$

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}.\begin{pmatrix} 600 \\ 600 \\ 600 \end{pmatrix}$$

With the invert of A, A^{-1}:

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 600 \\ 600 \\ 600 \end{pmatrix} = \begin{pmatrix} 150 \\ 150 \\ 150 \end{pmatrix}$$

Finally, it gives us the following market price: $\mathrm{P}(\mathrm{Q}_1, Q_2, Q_3) = 600 - (Q_1 + Q_2 + Q_3) = 150$

$$\boldsymbol{P = 150}$$

Synthesis:

We can summarize these results in the following table:

Market structure	**Price value [$\boldsymbol{P(Q) = 600 - Q}$]**
Monopoly	300
Duopoly	200
Triopoly	150
n-opoly	$$P = \frac{600}{n+1}$$ With $n \geq 1$.

Case 2: Roth & Erev, a Reinforcement learning process