Project Portfolio Optimization in Continuous Time, Fall 2024

Frankfurt School of Finance and Management

Jan Vecer

Due Date: January 10, 2025 (midnight). Later submissions result in a grade subtraction.

Individual Project.

Output: A single file: a Jupyter notebook that runs with Python. The Jupyter notebook should be fully executable. I recommend that you try to run your code on an independent computer to make sure it works. The commands and the results should come with detailed comments. The file name should be YOURNAME.ipynb, where YOURNAME is your name. The files should be uploaded to a designated area on the campus.fs.de website.

You can use functions from my own implementation PortProject2024.ipynb.

Project Description:

Part 1: Analyze profitability of Merton's optimal portfolio for all Dow Jones stocks based on the perceived drifts for each individual stock.

1. Get data from the DJIA. Set a starting date for your analysis. I am using Jan 01, 2010. You may want to use the components of the index on that day to make sure you do not get survival bias. Compute the statistics needed for the analysis (simple return process, volatilities) and make the relevant visualizations.

2. Determine Merton's optimal portfolio. The assumption is that the simple return process $R(t)$ defined as
$$dR(t) = \frac{dX_Y(t)}{X_Y(t)}$$
is drifting with the drift μ under the physical measure, but it is driftless under the risk neutral measure. Mathematically,
$$dR(t) = \frac{dX_Y(t)}{X_Y(t)} = \mu dt + \sigma dW(t)$$
under the physical measure, but
$$dR(t) = \frac{dX_Y(t)}{X_Y(t)} = \sigma dW(t)$$
under the risk neutral measure. This corresponds to the geometric Brownian motion model of the prices. The basic assumptions are
$$\mathbb{P}^M : N(\mu t, \sigma^2 t), \qquad \mathbb{P}^Y : N(0, \sigma^2 t).$$
The optimal portfolio is just the likelihood ratio of the two probabilities:
$$M(t) = \frac{N(\mu t, \sigma^2 t)}{N(0, \sigma^2 t)} \cdot M(0) = \exp\left(\frac{\mu}{\sigma^2} R(t) - \frac{1}{2}\left(\frac{\mu}{\sigma}\right)^2 t\right) \cdot M(0)$$

and it is implemented in my file as a function `merton`. All you need to do is to choose the drift parameter μ for each stock. You can use data prior to your starting point to make a qualified choice. Keep in mind that this choice is highly subjective and highly variable depending on your time window you pick. To a large degree, this is almost an ad hoc choice.

3. Construct Markowitz mean variance portfolio for each asset using the function `w` that determines the optimal weight invested in the risky asset.

4. Compare the Merton's, Markowitz's and the portfolio invested fully in the risky asset, graph these price evolutions (in a single graph for each stock) and extract the basic performance statistics (Sharpe ratio, Maximal Relative Drawdown). Put these values in a dataframe (implicitly meaning that you get these values elegantly without for loops, etc).

5. Check if the hedging portfolios for Merton's portfolios works well. This follows from

$$dM(t) = \frac{\mu}{\sigma^2} M(t) dR(t).$$

   Illustrate on graphs.

6. Construct Bayesian Merton's portfolio that is a combination of infinitely many Merton's portfolios with the variable drift parameter μ. We assume that μ comes from a normal distribution $N(\mu_0, \sigma_0^2)$. Compare the performance of the Bayes portfolio with the plain Merton's portfolio, both using graphs and performace statistics.

Part 2: Analyze profitability of currency triplets based on the perceived mean reversion in the market for each triplet in the G10 currencies (using EUR in each existing triplet, reducing the number of triplets to 36).

1. The assumption of a mean reverting market is based on the assumption that the simple return process has a smaller final variability than implied by the risk neutral volatility. The distributional assumptions on the simple return process $R(t)$ are

$$\mathbb{P}^M : N(\mathrm{diag}(\Sigma_1) \cdot t/2, \Sigma_2 \cdot t), \qquad \mathbb{P}^Y : N(0, \Sigma_1 \cdot t)$$

   where

$$\Sigma_1 = \Sigma_Q, \qquad \Sigma_2 = \Sigma_P \cdot \frac{t}{T} + \Sigma_Q \cdot \left(1 - \frac{t}{T}\right),$$

   where Σ_Q is the objective covariance matrix and Σ_P is the subjective covariance matrix. We assume

$$\Sigma_P = \mathrm{scaling}^2 \cdot \Sigma_Q$$

   is a scaled down objective covariance matrix. This result is obtained by a highly nontrivial computation and you have it implemented in function `MY`. All what is needed is to call the function with the corresponding covariance matrix.

2. The hedging positions are implemented in function `weights`. It corresponds to positions computed as

$$w = \Sigma_2^{-1} \cdot \mathrm{diag}(\Sigma_1)/2 + (\Sigma_1^{-1} - \Sigma_2^{-1}) \cdot R(t).$$

3. Your job is to compute the optimal portfolio for each currency triplet with EUR, compute their values under all benchmarks (EUR, 2 foreign currencies, index), graph them (possibly in a single graph with 36 subgraphs), compute various performance statistics (using dataframes) and make some relevant conclusions.
4. Compute the hedging positions and the resulting replicating portfolio. Compare the replicating portfolio with the theoretical price and illustrate these results on graphs (one graph for hedges, one graph for comparison of the prices).