Week 5

Thursday, September 23, 2021 3:43 PM

Practice Problem

$y'' + 6y' + 8y = 0$

$y(0) = 2, \quad y'(0) = 3$

$y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$r^2 e^{rt} + 6re^{rt} + 8e^{rt} = 0$

$e^{rt}(r^2 + 6r + 8) = 0$

$(r+2)(r+4) = 0$

$r = -2, -4$

e^{-2t} and e^{-4t} are solutions.

$y(t) = C_1 e^{-2t} + C_2 e^{-4t}$

$y'(t) = -2C_1 e^{-2t} - 4C_2 e^{-4t}$

$y(0) = 2, \quad y'(0) = 3$

$2 = y(0) = C_1 + C_2$

$3 = y'(0) = -2C_1 - 4C_2$

$C_1 + C_2 = 2 \quad \rightarrow C_2 = 2 - C_1$

$2C_1 + 4C_2 = -3$

$2C_1 + 4(2 - C_1) = -3$

$2C_1 + 8 - 4C_1 = -3$

$8 - 2C_1 = -3$

$-2C_1 = -11$

$C_1 = \frac{11}{2}$

$C_2 = 2 - \frac{11}{2}$

$C_2 = \frac{4}{2} - \frac{11}{2}$

$C_2 = -\frac{7}{2}$

$\therefore$ Solution to IVP is

$$y(t) = \frac{11}{2}e^{-2t} - \frac{7}{2}e^{-4t}$$