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Tutorial: Practice - Tut 3

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(Q.1) The following table shows the joint probability distribution of two random variables X and Y with respective values x_i and y_i.

Calculate $H(X)$, $H(Y)$, $H(X,Y)$, $H(X/Y)$ & $H(Y/X)$.

	y_1	y_2
x_1	$\frac{1}{2}$	$\frac{1}{8}$
x_2	$\frac{1}{8}$	$\frac{1}{8}$

(Q.2) Consider a DMS with source probabilities

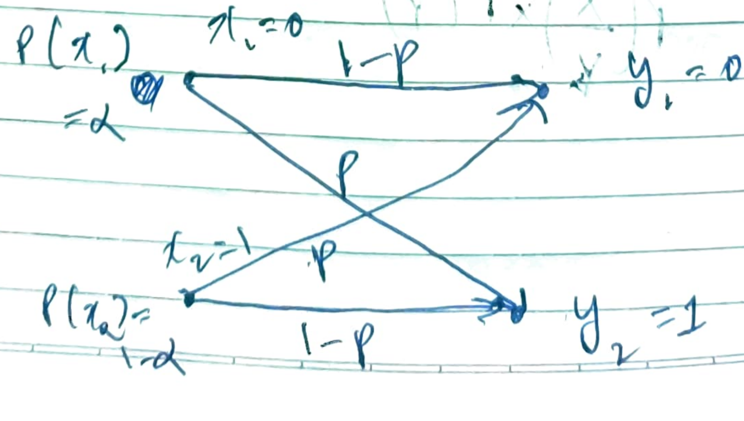
$\{0.20, 0.20, 0.15, 0.15, 0.10, 0.10, 0.05, 0.05\}$.

(i) Determine an efficient fixed length code for the source.
(ii) Determine Huffman code for this source.
(iii) Compare two codes and comment.

(Q.3) Encode the text message using LZW coding

"itty_bitty_bin".

(Q.4) Consider a BSC with $P(x_1) = \alpha$
Compute $I(X;Y)$ for $\alpha = 0.5$ & $P = 0.2$



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(Ans 1)

x\y	y_1	y_2	
x_1	1/2	1/8	$= \frac{5}{8}$
x_2	1/8	1/8	$= \frac{1}{4}$
x_3	1/8	0	$= \frac{1}{8}$
	$= \frac{3}{4}$	$= \frac{1}{4}$	

$$H(x) = \sum_{i=1}^{3} P(x_i) \log_2 \left(\frac{1}{P(x_i)}\right)$$

$$= P(x_1) \log_2 \left(\frac{1}{P(x_1)}\right) + P(x_2) \log_2 \left(\frac{1}{P(x_2)}\right) + P(x_3) \log_2 \left(\frac{1}{P(x_3)}\right)$$

$$= \frac{5}{8} \log_2 \left(\frac{8}{5}\right) + \frac{1}{4} \log_2 4 + \frac{1}{8} \log_2 8$$

$= 1.298$ bits/symbol.

$$H(Y) = \sum_{i=1}^{2} P(y_i) \log_2 \left(\frac{1}{P(y_i)}\right)$$

$$= P(y_1) \log_2 \left(\frac{1}{P(y_1)}\right) + P(y_2) \log_2 \left(\frac{1}{P(y_2)}\right)$$

$$= \frac{3}{4} \log_2 \left(\frac{4}{3}\right) + \frac{1}{4} \log_2 4$$

$= 0.8112$ bits/symbol

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$$\cancel{H(X,Y) = H(X/}$$

$$H(X/Y) = \sum_{i=1}^{2} P(Y=i)\, H(X/Y=i)$$

$$= \cancel{\frac{1}{4}\ \frac{3}{4} H\left(\frac{5}{8}, \frac{1}{4}, \frac{1}{8}\right) + \frac{1}{4} H}$$

$$= \frac{3}{4} H\left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right) + \frac{1}{4} H\left(\frac{1}{6}, \frac{1}{6}, 0\right)$$

$$= \frac{3}{4}\left[\frac{2}{3}\log_2\left(\frac{3}{2}\right) + \left\{\frac{1}{6}\log_2 6\right\} \times 2\right]$$

$$+ \frac{1}{4}\left[\left\{\frac{1}{6}\log_2 6\right\} \times 2 + 0\right]$$

$$\boxed{H(X/Y) = 1.1541 \text{ bits/symbol.}}$$

$$H(Y/X) = \sum_{i=1}^{3} P(X=i)\, H(Y/X)$$

$$= \frac{5}{8} H\left(\frac{4}{5}, \frac{1}{5}\right) + \frac{1}{4} H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{1}{8} H(1,$$

$$= \frac{5}{8}\left[\frac{4}{5}\log_2\left(\frac{5}{4}\right) + \frac{1}{5}\log_2(5)\right]$$

$$+ \frac{1}{4}\left[\left\{\frac{1}{2}\log_2 2\right\} \times 2\right]$$

$$+ \frac{1}{8}\left[\cancel{1\log_2 1}\right] \rightarrow 0$$

$$\boxed{H(Y/X) = 0.7012}$$

bits/symbol.

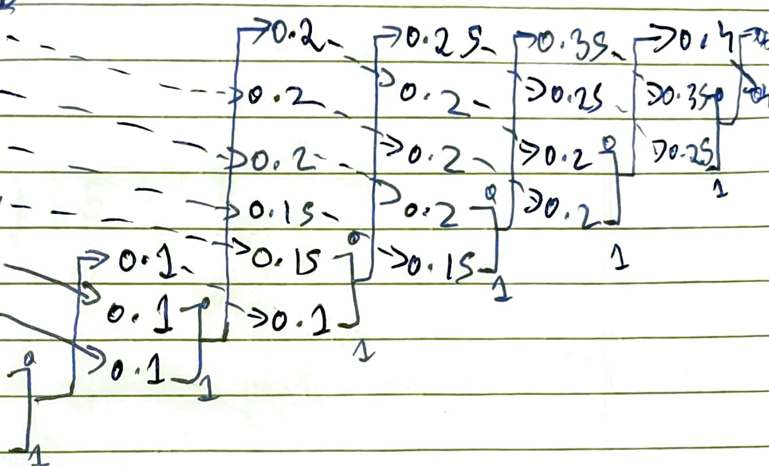
$$\Rightarrow H(X,Y) = H(X) + H(Y/X)$$

$$= 1.298 + 0.7012$$

$$\boxed{H(X,Y) = 1.9992 \text{ bits/symbol}}$$

(Ans 2) DMS (Huffman coding).

Symbol	Prob
S_1	0.2
S_2	0.2
S_3	0.15
S_4	0.15
S_5	0.1
S_6	0.1
S_7	0.05
S_8	0.05



	Code word	code length (n)
S_1	11	2
S_2	000	3
S_3	001	3
S_4	010	3
S_5	100	3
S_6	101	3
S_7	0110	4
S_8	0111	4

$$\bar{L} = \sum p(x_i) n_i$$
$$= 0.2\times2 + 0.2\times3 + (0.15\times3)\times2 + (0.1\times3)\times2 + (0.05\times4)\times2$$
$$= 1 + 0.9 + 0.6 + 0.4$$
$$= 2.9$$

$$\text{Efficiency } \eta = \frac{2.846}{2.9} \times 100\%$$
$$= 0.9815 \times 100$$
$$= 98.15\% \text{ efficiency}$$

$$H = 2\times\left(0.2 \log_2 \frac{1}{0.2}\right) + 2\times\left(0.15 \log_2 \frac{1}{0.15}\right) + 2\times\left(0.1 \log_2 \frac{1}{0.1}\right) + 2\times\left(0.05 \log_2 \frac{1}{0.05}\right)$$

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LZW Encoding

(Ans 3) Text message "itty_bitty_bin"

itty_bitty_bin

Original Dictionary

Symbol	Code
i	1
t	2
y	3
_	4
b	5
n	6

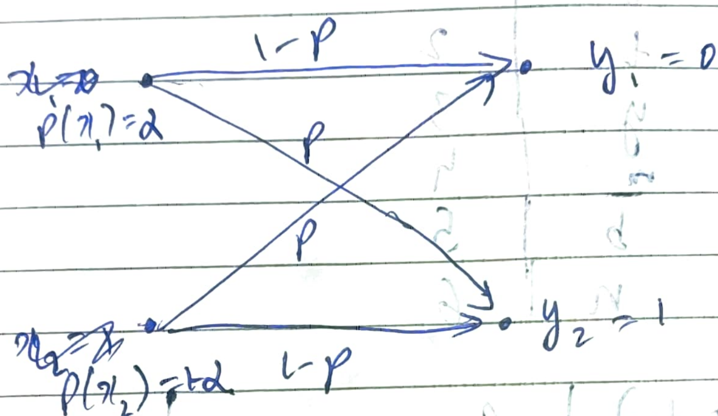
Position (Index)	Sequence (Dictionary)	Encoded (Output)
1	i	Nil
2	t	Nil
3	y	Nil
4	_	Nil
5	b	Nil
6	n	Nil
7	it	1
8	tt	2
9	ty	2
10	y_	3
11	_b	4
12	bi	5
13	itt	7
14	ty_	9
15	_bi	11
16	in	1

Encoded output is

1, 2, 3, 4, 5, 6, 1, 2, 2, 3, 4, 5, 7, 9, 11, 1

(Ans 4) BSC with $P(x_1) = \alpha$

$\alpha = 0.5$ $\qquad I(X;Y) = ?$
$p = 0.1$



$P(x_1) = 0.5$ $\qquad p = 0.1$
$P(x_2) = 1 - 0.5 = 0.5$ $\qquad 1-p = 0.9$

$x \backslash Y$	y_1	y_2
x_1	0.9	0.1
x_2	0.1	0.9

~~H(Y) =>~~

$I(X;Y) = H(Y) - H(Y/X)$ — ①

$\cancel{H(Y) = \sum_{i=1}^{2} P(y_i) \log_2 \left(\frac{1}{P(y_i)}\right)}$

$$H(x) = \sum_{i=1}^{2} P(x_i) \log_2 \left(\frac{1}{P(x_i)} \right)$$

$$P(x_1) = P(x_2) = 0.5 = \frac{1}{2}$$

$$= \cancel{0.5}\ \frac{1}{2} \log_2 (2) + \frac{1}{2} \log_2 (2)$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1 \quad \text{— } (2)$$

$$H(X/Y) = \sum_{i=1}^{2} P(Y=i)\, H(X/Y=i)$$

$$= 1 \times H(0.9, 0.1) + 1 \times H(0.1, 0.9)$$

$$= 1 \times \left[\frac{9}{10} \log_2 \left(\frac{10}{9} \right) + \frac{1}{10} \log_2 10 \right] \times 2$$

$$= 0.9379 \quad \text{— } (3)$$

From (1) & (2) & (3) we get

$$I(X;Y) = H(X) - H(X/Y)$$

$$= 1 - 0.9379$$

$$= 0.062$$