

Ex For (7,4) linear block code

$$[H] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

- Find  $[H]^T$  and  $[G]$  matrices.
- Find all possible code words, Hamming weight
- If received code words are

$$r_1 = [1010011] \text{ and } r_2 = [1111000]$$

Find syndrome, corrected code word.

error pattern (single bit)

- Find ~~standard array~~. standard array.
- error detecting and correcting Capability of code.

$$n=7, k=4, n-k=3$$

Solution :-

$$a) \therefore [H] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$I_{3 \times 3} \quad P^T$

$$\therefore [H]^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, [G] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$I_{4 \times 4} \quad P_{4 \times 3}$

- To find all code words.

Encoding steps.

$$[C]_{1 \times 7} = [D]_{1 \times 4} \cdot [G]_{4 \times 7}$$

$$[c_1, c_2, c_3, c_4, c_5, c_6, c_7] = [D_1, D_2, D_3, D_4] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= [D_1, D_2, D_3, D_4, (D_1 \oplus D_3 \oplus D_4), (D_1 \oplus D_2 \oplus D_3), (D_2 \oplus D_3 \oplus D_4)]$$

[illegible]



c] Received Code word  $r_1 = [1010011]$   
To find Syndrome  $S$

$$[S_1] = R[r_1][H^T]$$

$$= [1010011] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= [1 \cdot 1 \oplus 1 \cdot 1 \oplus 1 \cdot 1, \quad 1 \cdot 0 \oplus 1 \cdot 1 \oplus 1 \cdot 1, \quad 1 \cdot 0 \oplus 1 \cdot 1 \oplus 1 \cdot 1]$$

$$S_1 = [0, 1, 0] \quad [1, 1, 1]$$

$$\therefore \text{Syndrome} = [111]$$

$\therefore$  6th bit of Error pattern is set.  
Syndrome = 6th row of  $H^T$  matrix.

$$\therefore E_1 = 0000010$$

$$\therefore \text{Corrected Code } C_1 = r_1 \oplus E_1$$

$$= [1010011] \oplus [0000010]$$

$$C_1 = [1010001]$$

Similarly

$$\text{For } r_2 = [1111000]$$

$$S_2 = r_2 \cdot H^T = [1111000] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$S_2 = [001]$$

$\therefore$  3rd bit in error

$$\therefore E_2 = 0010000$$

$$\therefore C_2 = r_2 \oplus E_2 = [1111000] \oplus [0010000] = [1101000]$$

e) Since minimum Hamming distance for this code is  $d_{\min} = 3$

$$\begin{aligned} \therefore t_{\text{detect}} &= \text{Error detection Capability} \\ &= d_{\min} - 1 \\ &= 3 - 1 \\ &= 2 \text{ bits.} \end{aligned}$$

$\therefore 2$  bits can be detected

$$\begin{aligned} \& \text{ Error-Correction} &= \frac{d_{\min} - 1}{2} \\ &= \frac{3 - 1}{2} \\ &= \frac{2}{2} = 1 \end{aligned}$$

$\therefore 1$  bit error can be corrected.

| Single bit syndrome decoding table. |   |   |   |   |   |   |   |          |    |
|-------------------------------------|---|---|---|---|---|---|---|----------|----|
| error pattern (e)                   |   |   |   |   |   |   |   | Syndrome |    |
| 1                                   | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1        | 00 |
| 0                                   | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0        | 10 |
| 0                                   | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0        | 01 |
| 0                                   | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1        | 10 |
| 0                                   | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0        | 11 |
| 0                                   | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0        | 01 |
| 0                                   | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1        | 11 |
| 0                                   | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1        | 01 |

$$\text{Syndrome} = r \cdot H^T$$

$$S = (c + e)H^T$$

$$S = c \cdot H^T + e \cdot H^T$$

$$S = 0 + e \cdot H^T$$

S depends on error pattern.

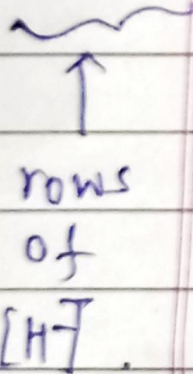
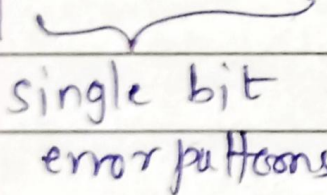
$$\therefore [S] = [e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ e_6 \ e_7] \cdot [H^T]$$



# Standard array

Date

| Syndrome | Coset leader  | $V_1$            | $V_2$            | $V_3 \dots V_2^k$                |
|----------|---------------|------------------|------------------|----------------------------------|
| 000      | 0000000       | 0001011          | 0010111          | → write all code words --- $2^4$ |
| 100      | 1000000 $e_2$ | $e_2 \oplus V_1$ | $e_2 \oplus V_2$ | →                                |
| 010      | 0100000 $e_3$ | $e_3 \oplus V_1$ | $e_3 \oplus V_2$ | →                                |
| 001      | 0010000       | $e_4 \oplus V_1$ | $e_4 \oplus V_2$ | →                                |
| 110      | 0001000       |                  |                  |                                  |
| 011      | 0000100       |                  |                  |                                  |
| 111      | 0000010       | 0001001          |                  |                                  |
| 101      | 0000001       |                  |                  |                                  |

rows of  $[H]$

single bit error patterns

Standard Array (7.4) LBC

[illegible]

| Synchrone | Casct Leader |         |         |         |         |         | Code words. |
|-----------|--------------|---------|---------|---------|---------|---------|-------------|
| 000       | 0000000      | 0001011 | 0010111 | 0011010 | 0100001 | 0101110 | 0110100     |
| 100       | 1000000      | 1001011 | 1010111 | 1011010 | 1100001 | 1101110 | 1110100     |
| 010       | 0100000      | 0101011 | 0100111 | 0111010 | 0000001 | 0001110 | 0010100     |
| 001       | 0010000      | 0011011 | 0000111 | 0001010 | 0110001 | 0111110 | 0100100     |
| 110       | 0001000      | 0000011 | 0011111 | 0010010 | 0101001 | 0100110 | 0111100     |
| 011       | 0000100      | 0001011 | 0011111 | 0011110 | 0100101 | 0101010 | 0110000     |
| 111       | 0000000      | 0001011 | 0011111 | 0011110 | 0100101 | 0101010 | 0110110     |
| 101       | 0000010      | 0001001 | 0010101 | 0011000 | 0100100 | 0101011 | 0110101     |