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(Q.1) What is the entropy of the image given where (0, 20, 50, 99) denote the grey level intensities.

(Q.2) Consider the source $S = \{S_1, S_2, S_3\}$ with probabilities $P = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$. Find

(a) Self information of each message.
(b) Entropy of source 'S'.

(Q.3) An event has four possible outcomes with probability of occurence $P_1 = \frac{1}{2}$, $P_2 = \frac{1}{2}$, $P_3 = \frac{1}{8}$ $P_4 = \frac{1}{8}$ respectively.

Determine self information in each outcome, entropy of the system also obtain the rate of information if there are 16 outcomes per second.

(Q.4) A code is composed of dots and dashes assuming that a dash (—) is 3 times as long as a dot and has a one-third probability of occurence.
Calculate
(a) The information in a dot and a dash.
(b) The entropy of dot-dash code.
(c) The average rate of information of a dot lasts for 10 ms and this same time interval is allowed between symbols.

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Ans 1. Total no of intensities → 64

Probability of $P[0] = \frac{32}{64} = \frac{1}{2}$

Probability of $P[20] = \frac{8}{64} = \frac{1}{8}$

Probability of $P[50] = \frac{16}{64} = \frac{1}{4}$

Probability of $P[99] = \frac{8}{64} = \frac{1}{8}$

Entropy of the image $= -\sum_{i=1}^{n} P_i \log_2 P_i$

$$= -\left\{ \left[\frac{1}{2}\log\left(\frac{1}{2}\right)\right] + \left[\frac{1}{8}\log\left(\frac{1}{8}\right)\right] + \left[\frac{1}{4}\log\left(\frac{1}{4}\right)\right] + \left[\frac{1}{8}\log\left(\frac{1}{8}\right)\right] \right\}$$

= ~~0.5268~~

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$$

$$= 1 + \frac{3}{4}$$

$$= 1.75$$

(Ans 2) $S = \{s_1, s_2, s_3\}$

(a) Self information of $s_1 = \log_2 2 = 1$

" " of $s_2 = \log_2 4 = 2$

" " of $s_3 = \log_2 4 = 2$

(b) Entropy of Source $S = -\sum_{i=1}^{n} P_i \log_2 P_i$

$$= -\left[\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{4}\log_2\frac{1}{4} + \frac{1}{4}\log_2\frac{1}{4}\right]$$

$$= -\left[-\frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$$

$$= \frac{3}{2}$$

(Ans 3) Self information of $P_1 = \log_2 2 = 1$

" " of $P_2 = \log_2 4 = 2$

" " of $P_3 = \log_2 8 = 3$

" " of $P_4 = \log_2 8 = 3$.

Entropy of the System $= -\sum_{i=1}^{n} P_i \log_2 P_i$

$$\Rightarrow -\left[\frac{1}{2}\log\left(\frac{1}{2}\right) + \frac{1}{4}\log\left(\frac{1}{4}\right) + \frac{1}{8}\log\left(\frac{1}{8}\right) + \frac{1}{8}\log\left(\frac{1}{8}\right)\right]$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$$

$$= 1 + \frac{3}{4} = \frac{7}{4} = 1.75 \text{ bits/symbol}$$

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Ans (a) Self information $= \log_2\left(\frac{1}{p_i}\right)$

$P_{dot} + P_{dash} = 1$

$P_{dot} = \frac{1}{3} P_{dash}$

$P_{dot} = \frac{3}{4}$ & $P_{dash} = \frac{1}{4}$.

Self information dot $= \log_2\left(\frac{4}{3}\right)$

$= 0.41$ bits

Self info dash $= \log_2(4)$

$= 2$ bits.

Entropy of code $= \sum_{i=1} P_i \log\left(\frac{1}{P_i}\right)$

$= \frac{1}{4} \log_2 4 + \frac{3}{4} \log \frac{4}{3}$.

$= \frac{2}{4} + \frac{3}{4} \times 0.41$

$= 0.5 + 0.3 = 0.87$ bits/sym

Rate $\frac{\text{message symbols}}{\text{time}}$.

$IR = rs \times H(s)$

Total time $= 10+10+10+30+10+10+10$

$= 100$ msec

message symbol $= 4$

$rs = \frac{4}{100 \times \frac{1}{1000}}$ sec.

$= 40$ = symbols / sec.

$IR = 40 \times 0.8$

$= 32$ bits / sec

∴ Information rate is 32 bits / se..