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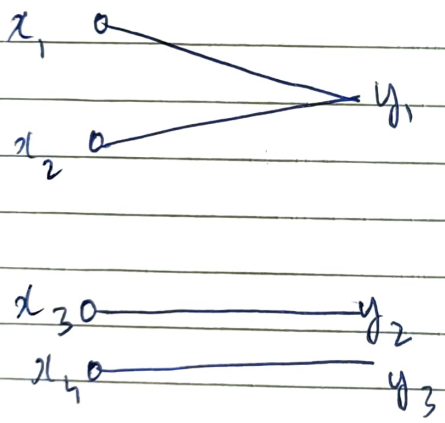
ITC - Tutorial-4

(Q.1) For the following chanel transition matrix

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

(a) Draw the chanel diagram for the following chanel transition matrix.

(b) Write the prob transition matrix for the following chanel diagram and write comment about it.



(Q.2) A binary symmetric chanel has the following noise matrix with the source probabilities.

$$P(x_1) = 2/3 \quad , P(x_2) = 1/3 \quad , P(Y/X) = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

(a) Determine $H(x)$
(b) Determine $P(Y)$
(c) determine joint probability matrix $[P(X,Y)]$. $H(Y/X)$

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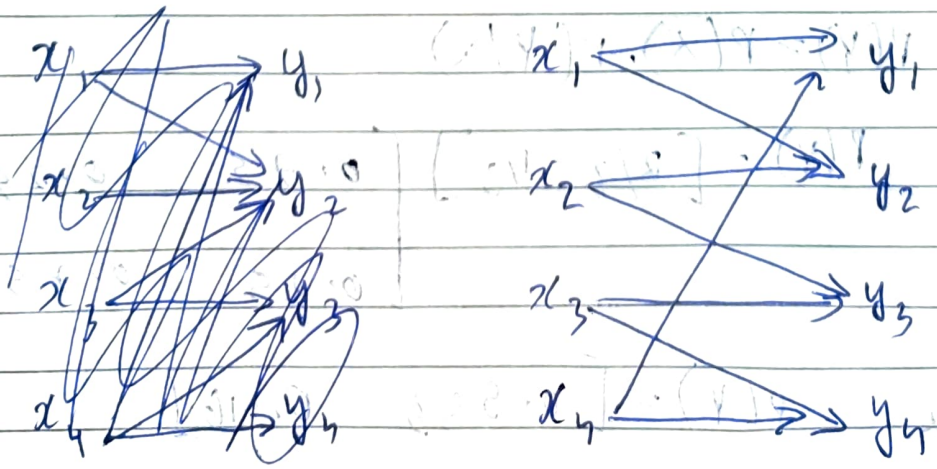
(Q.3) Match the following (X source input, Y output)

A	B	C
1) $H(X,Y)$	M.I. (Mutual Information)	$\sum_{j=1}^{m} P(x_j) \log_2 (x_j)$
2) $H(X)$	How well one can recover transmitted symbol from received symbol	$\sum_{j=1}^{m} \sum_{k=1}^{n} P(x_j, y_k) \log_2 (x_j, y_k)$
3) $H(X/Y)$	Priority Entropy	$H(X) - H(X/Y)$
4) $I(X,Y)$	Joint Entropy of Y & X	$-\sum_{j=1}^{m} \sum_{k=1}^{n} P(x_j, y_k) \log P(x_j/y_k)$

(Ans 1)
$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

a)



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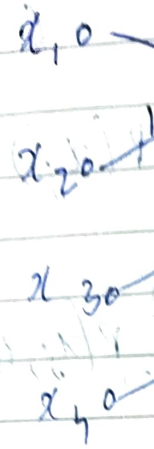
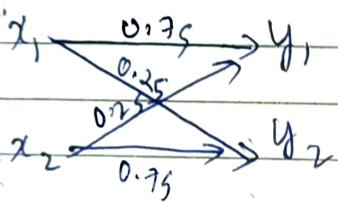
(b)



$$[P(Y/X)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(Ans 2) $P(x_1) = 2/3$; $P(x_2) = 1/3$, $P(Y/X)$

$$= \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$



$P(x_1) = 2/3$, $P(x_2) = 1/3$

$P(Y) = P(X) \cdot P(Y/X)$

$$P(Y) = \begin{bmatrix} 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

b) Answers $P(Y) = [0.583 \quad 0.416]$

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(a) $H(x) = -\sum_{j=1}^{m} p(x_j) \log_2 p(x_j)$

$= -\sum_{j=1}^{2} p(x_j) \log_2 p(x_j)$

$= -\left(\frac{2}{3} \log\left(\frac{3}{2}\right) + \frac{1}{3} \log 3\right)$

$= 0.389 + 0.528$

$= 0.917$ unit

(c) $[P(X,Y)] = [P(X)]_d [P(Y/X)]$

$[P(X,Y)] = \begin{bmatrix} 2/3 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$

$[P(X,Y)] = \begin{bmatrix} 1/2 & 1/6 \\ 1/12 & 1/4 \end{bmatrix}$

$H(Y/X) = -\sum_{j=1}^{m} \sum_{K=1}^{n} P(y_K, x_j) \log_2 P(y_K/x_j)$

$P(X,Y) = \begin{bmatrix} 1/2 & 1/6 \\ 1/12 & 1/4 \end{bmatrix}$ $\quad P(Y/X) = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$

$H(Y/X) = \frac{1}{2} \log_2\left(\frac{4}{3}\right) + \frac{1}{6} \log_2(4) + \frac{1}{12} \log_2(4) + \frac{1}{4} \log_2\left(\frac{4}{3}\right)$

$= 0.7386$ bits / message.

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(Ans 3)

A	B	C
1) $H(X,Y)$	Joint Entropy of X and Y	$-\sum_{j=1}^{m}\sum_{K=1}^{n} P(x_j,y_K)\log_2 P(x_j,y_K)$
2) $H(X)$	Priority Entropy.	$-\sum_{j=1}^{m} P(x_j)\log_2 P(x_j)$
3) $H(X/Y)$	How well one can recover transmitted symbol from recieved symbol.	$-\sum_{j=1}^{m}\sum_{K=1}^{n} P(x_j,y_K)\log P(x_j/y_K)$
4) $I(X,Y)$	MI (mutual information)	$H(X)-H(X/Y)$