Ex For (7, 4) linear block code

$$[H] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

a) Find $[H]^T$ and $[G]$ matrices.
b) Find all possible code words, Hamming weight
c) If received code words are
$r_1 = [1010011]$ and $r_2 = [1111000]$
Find syndrome, corrected code word.
error pattern (single bit)
d) Find ~~standard array~~. standard array.
e) error detecting and correcting Capability of code.

Solution :- $n = 7,\ K = 4,\ n-k = 3$

a) $\because$
$$[H] = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$
$\qquad\qquad I_{3\times3} \qquad P^T$

$$\therefore [H]^T = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right], \quad [G] = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$
$\qquad\qquad I_{4\times4}, \quad P_{4\times3}$

b) To find all code words.

Encoding steps.

$$[C]_{1\times7} = [D]_{1\times4} \cdot [G]_{4\times7}$$

$$[C_1, C_2, C_3, C_4, C_5, C_6, C_7] = [D_1\ D_2\ D_3\ D_4]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= [D_1, D_2, D_3, D_4, (D_1 \oplus D_3 \oplus D_4), (D_1 \oplus D_2 \oplus D_3), (D_2 \oplus D_3 \oplus D_4)]$$

Data word				Code words								Hamming wt
D_1	D_2	D_3	D_4	C_1	C_2	C_3	C_4	C_5	C_6	C_7	~~C_8~~	$W(C)$
0	0	0	0	0	0	0	0	0	0	0		0
0	0	0	1	0	0	0	1	1	0	1		3
0	0	1	0	0	0	1	0	1	1	1		4
0	0	1	1	0	0	1	1	0	1	0		3
0	1	0	0	0	1	0	0	0	1	1		3
0	1	0	1	0	1	0	1	1	1	0		4
0	1	1	0	0	1	1	0	1	0	0		3
0	1	1	1	0	1	1	1	0	0	1		4
1	0	0	0	1	0	0	0	1	1	0		3
1	0	0	1	1	0	0	1	0	1	1		4
1	0	1	0	1	0	1	0	0	0	1		3
1	0	1	1	1	0	1	1	1	0	0		1
1	1	0	0	1	1	0	0	1	0	1		4
1	1	0	1	1	1	0	1	0	0	0		3
1	1	1	0	1	1	1	0	0	1	0		3
1	1	1	1	1	1	1	1	1	1	1		7

c] Received Codeword $r_1 = [1010011]$

To find Syndrome S

$$[S_1] = R[r_1][H^T]$$

$$= [1010011] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= [1\cdot1 \oplus 1\cdot1 \oplus 1\cdot1 \; , \; \cancel{1 \oplus 1} \; , \; \cancel{1 \oplus}\; 1\cdot1 \; , \; 1\cdot1 \oplus 1\cdot1 \oplus 1\cdot1]$$

$S_1 = \cancel{[0, 1, 0]} \quad [1, 1, 1].$

$\therefore$ Syndrome $= [111]$

$\therefore$ 6 th bit of Error pattern is set.

Syndrome = 6th row of H^T matrix.

$\therefore \quad E_1 = 0000010$

$\therefore$ Corrected Code $C_1 = r_1 \oplus E_1$

$= (1010011) \oplus (0000010)$

$$\boxed{C_1 = 1010001}$$

similarly

For $r_2 = [1111000]$

$$S_2 = r_2 \cdot H^T = [1111000] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$S_2 = [001]$

$\therefore$ 3rd bit in error

$\therefore \quad E_2 = 0010000$

$\therefore \quad C_2 = r_2 \oplus E_2 = (1111000) \oplus (0010000) = (1101000)$

e) since minimum Hamming distance for this code is $d_{min} = 3$

$\therefore$ t_{detect} = Error detection Capability

$= d_{min} - 1$

$= 3 - 1$

$= 2$ bits.

$\therefore$ 2 bits can be detected

& $t_{error\ correction} = \frac{d_{min} - 1}{2}$

$= \frac{3-1}{2}$

$= \frac{2}{2} = 1$

$\therefore$ 1 bit error can be corrected.

Single bit syndrome decoding table.

error pattern (e)	Syndrome
1 0 0 0 0 0 0	1 0 0
0 1 0 0 0 0 0	0 1 0
0 0 1 0 0 0 0	0 0 1
0 0 0 1 0 0 0	1 1 0
0 0 0 0 1 0 0	0 1 1
0 0 0 0 0 1 0	1 1 1
0 0 0 0 0 0 1	1 0 1

Syndrome $= r \cdot H^T$

$S = (c + e) H^T$

$S = c \cdot H^T + e \cdot H^T$

$S = 0 + e \cdot H^T$

S depends on error pattern.

$\therefore [S] = [e_1\ e_2\ e_3\ e_4\ e_5\ e_6\ e_7] \cdot [H^T]$.

Standard array

Syndrome	Coset leader	V_1	V_2	V_3 ---- V_{2^k}
000	0000000	0001011	0010111	→ write all code words ... 2^4
100	1000000 e_2	$e_2 \oplus V_1$	$e_2 \oplus V_2$	---→
010	0100000 e_3	$e_3 \oplus V_1$	$e_3 \oplus V_2$	---→
001	0010000	$e_4 \oplus V_1$	$e_4 \oplus V_2$	---→
110	0001000			
011	0000100			
111	0000010	0001001		
101	0000001			

$\underbrace{\quad}$ ↑ rows of $[H^T]$.

$\underbrace{\quad}$ single bit error patterns

Standard Array (7,4) LBC

Code words.

Syndrome	Coset Leader															
000	0000000	0001011	0010111	0011010	0100001	0101110	0110100	0111001	1000110	1001011	1010001	1011100	1100101	1101000	1110010	1111111
100	1000000	1001011	1010111	1011010	1100001	1101110	1110100	1111001	0000110	0001011	0010001	0011100	0100101	0101000	0110010	0111111
010	0100000	0101011	0100111	0111010	0000001	0001110	0010100	0011001	1[illegible]00110	1101011	1110001	1111100	1000101	1001000	1010010	1011111
001	0010000	0011011	0000111	0001010	0110001	0111110	0100100	0101001	1010110	1011011	1000001	1001100	1110101	1111000	1100010	1101111
110	0001000	0000011	0011111	0010010	0101001	0100110	0111100	0110001	1001110	1000011	1011001	1011000	1101101	1100000	1111010	1110111
011	0000100	0001111	0010011	0011110	0100101	0101010	0110000	0111101	1000010	1001111	1010101	1011000	1100001	1101100	1110110	1111011
1[illegible]1	0000010	0001001	0010101	0011000	0100011	0101100	0110110	011101[illegible]	1000100	1001001	1010011	1011110	1100111	$\boxed{1101010}$	1110001	111101
101	0000001	0001010	0010110	0011011	0100010	0101111	0110101	0111000	1000111	1001010	1010000	1011101	1100100	1101001	1110011	11111

Single error