SOMAIYA
VIDYAVIHAR UNIVERSITY

Batch: A2 Roll No.: 16010421073
Name: Keyur Patel
Course: PSOT
Experiment / assignment / tutorial No. IA-2
Grade: Signature of the Faculty with date

IA-2 (CO5).

Name: Keyur Patel
Roll no: 16010421073
Batch: A2

(Q.1) Find the average number of customers in the system and in the queue if the system is $(M/M/1/\infty)$ and $\mu = 15, \lambda = 10$.

(Ans 1) $\rho = \frac{\lambda}{\mu} = \frac{10}{15} = \frac{2}{3}$ = service utilization factor.

L_q - average number of customers has to wait in a queue before being served $= \frac{\rho^2}{1-\rho}$

$= \frac{4/9}{1-2/3} = 1.33 \approx 1$

L_s — average number of customers in the system including waiting in queue and being served $= \frac{\rho}{1-\rho}$

$= \frac{2/3}{1-2/3} = 2.$

(Q.2) Find the service utilization factor, the average waiting time per customer in the queue and in the system for (M/M/1/∞) model if $\mu = 15$, $\lambda = 9$ per hours.
Also find the probability that
(i) a customer has to wait in the system.
(ii) there are more than 8 customers in the system.

(Ans2) $\rho = \frac{\lambda}{\mu} = \frac{9}{15} = \frac{3}{5}$ = service utilization factor.

W_q = average time an arriving customer has to wait in a queue before being served.

$= \frac{L_q}{\lambda} = \frac{1}{\lambda}\frac{\rho^2}{(1-\rho)} = \frac{9/25}{9\left(1-\frac{3}{5}\right)} = 0.1$ hrs.

W_s = average time an arriving customer spends in the system including waiting in the queue and being served

$= \frac{L_s}{\lambda} = \frac{1}{\lambda}\frac{\rho}{(1-\rho)} = \frac{3/5}{9\left(1-\frac{3}{5}\right)} = \frac{1}{6}$ hrs.

(i) P_0 - Probability that no customers in the system (idle time) $= 1-\rho$.
~~(ii)~~ (i) Probability that a ~~taueyer~~ customer has to wait in the system $= 1 - P_0 = \rho = 3/5$.

(ii) $P(n > R) = \rho^{R+1}$

$P(n > 8) = \rho^9 = \left(\frac{3}{5}\right)^9 = 0.01008$.

SOMAIYA
VIDYAVIHAR UNIVERSITY

Batch: A-2 Roll No.: 16010421073
Name: Keyur Patel
Course: PSOT
Experiment / assignment / tutorial No.
Grade: Signature of the Faculty with date

(Q.3) Find the traffic intensity of the system (M/M/1/∞) model if $\mu = 11$ per hours, $\lambda = 8$ per hours. Also find the probability that a customer has to wait for more than 20 minutes to be out of the service station.

(Ans 3): ρ = traffic intensity or service utilization factor

$$= \frac{\lambda}{\mu} = \frac{8}{11}$$

$$t = 20 \text{ min} = \frac{20}{60} \text{ hrs.}$$

A customer has to wait for more than 20 minutes to be out of the service station.

$$\Rightarrow P(W_s > t) = e^{-\mu(1-\rho)t}$$

$$\Rightarrow P(W_s > 1/3) = e^{-11(1-8/11)1/3} = 0.3679.$$

(Q.4) A customer arrives at a clinic according to a poisson process with a mean interval of 25 minutes, the doctor needs on an average 20 minutes for a patient to examine. Find

(i) the expected number of patients ~~who are not required~~ in the clinic and in the queue.

(ii) percentage of patients who are not required to wait.

(iii) on an average how much time is spent by a patient in the clinic.

(iv) the doctor will appoint another doctor if the patients time in the clinic exceeds 2 hours. How must the rate of arrivals increase so that another doctor is appointed?

(v) Average time a patient has to be in queue before the doctor examines him.

(vi) Probability that the total waiting time of patient in the system is greater than 1 hour.

(vii) Percentage of patients who have to wait before they are called by the doctor for examination.

(viii) Probability that there are more than 4 patients in the queue.

(ix) it is desired that fewer than 5 patients are in the queue for 99% of the time. How fast the service rate should be?

SOMAIYA
VIDYAVIHAR UNIVERSITY

Batch: A2 Roll No.: 16010421073
Name: Keyur Patel
Course: PSOT.
Experiment / assignment / tutorial No.
Grade: Signature of the Faculty with date

(Ans 4) Given -

λ = average number of arrivals per unit time in the system $= \frac{1}{25}$ patients per minutes.

μ = average number of customers served per unit time in the system $= \frac{1}{20}$ patients per minute.

ρ = traffic intensity or service utilization factor $= \frac{\lambda}{\mu} = \frac{4}{5}$

(i) The expected number of patients in the clinic and in the queue $L_s = \frac{\rho}{1-\rho} = 4$ $L_q = \frac{\rho^2}{1-\rho} = 3.2 \simeq 3$

(ii) Percentage of patients who are not required to wait prob (no patient in the system) $= P_0$

$= 1 - \rho$

$= \frac{1}{5} = 0.2$

$\Rightarrow$ percentage of patients who are not required to wait $= 0.2 \times 100$

$= 20\%$

(iii) the average how much time is spent by a patient in the clinic.

$W_s = \frac{1}{\lambda} \frac{\rho}{(1-\rho)} = 100$ min.

(iv) the doctor will appoint another doctor if the patients time in the clinic exceeds 2 hours. How much must the rate of arrivals increase so that another doctor is appointed?

New doctor is appointed if $W_s > 2hrs$

$W_s > 2hrs = 120min \Rightarrow \frac{1}{\lambda}\frac{\rho}{(1-\rho)} > 120$

$\Rightarrow \frac{1}{\mu-\lambda} > 120$

$\Rightarrow \frac{1}{1/20-\lambda} > 120$

$\Rightarrow \lambda > 1/24.$

$\therefore \Rightarrow$ increase in arrival rate $= 1/24 - 1/25$

$= \frac{1}{600}$ per min.

(v) average time a patient has to be in queue before the doctor examines him;

$W_q = \frac{1}{\lambda}\frac{\rho^2}{(1-\rho)} = 80$ min.

(vi) Probability that the total waiting time of patient in the system is greater than 1 hours $= P(W_s > t)$

$= e^{-\mu(1-\rho)t}$, $t = 1hrs = 60min$

$P(W_s > 60) = e^{-\mu(1-\rho)t} = 0.5488$

(vii) Percentage of patients who have to wait before they are called by the doctor for examination = prob(system is busy) $= 1 - P_0$

$= P_0$

$= 0.8$

$= 80\%$

(viii) Probability that there are more than 4 patients.

SOMAIYA VIDYAVIHAR UNIVERSITY

Batch: A-2 Roll No.: 16010421073

Name: Keyur Patel

Course: PSOT

Experiment / assignment / tutorial No.

Grade: Signature of the Faculty with date

$P(n > R) = \rho^{R+1} \Rightarrow P(n > 4) = 0.3277$

(ix) It is desired that fewer than 5 patients are in the queue for 99% of the time.

$P(n < 5) \geq 99\% \Rightarrow P(n \leq 4) \geq 99\%$

$P(n > R) \Rightarrow \rho^{R+1} \Rightarrow P(n \leq R) = 1 - \rho^{R+1}$
$= 1 - \rho^5 \geq 99\%$

$\Rightarrow \left(\frac{\lambda}{\mu}\right)^5 \geq 0.01 \Rightarrow \left(\frac{1}{25\mu}\right)^5 \geq \frac{1}{100} \Rightarrow \mu \geq 0.1105$ patients per min.