

Tut no: - 8

Q.

Design a Turing machine for language

$$L = \{ a^n b^{2n} \mid n \geq 0 \}$$

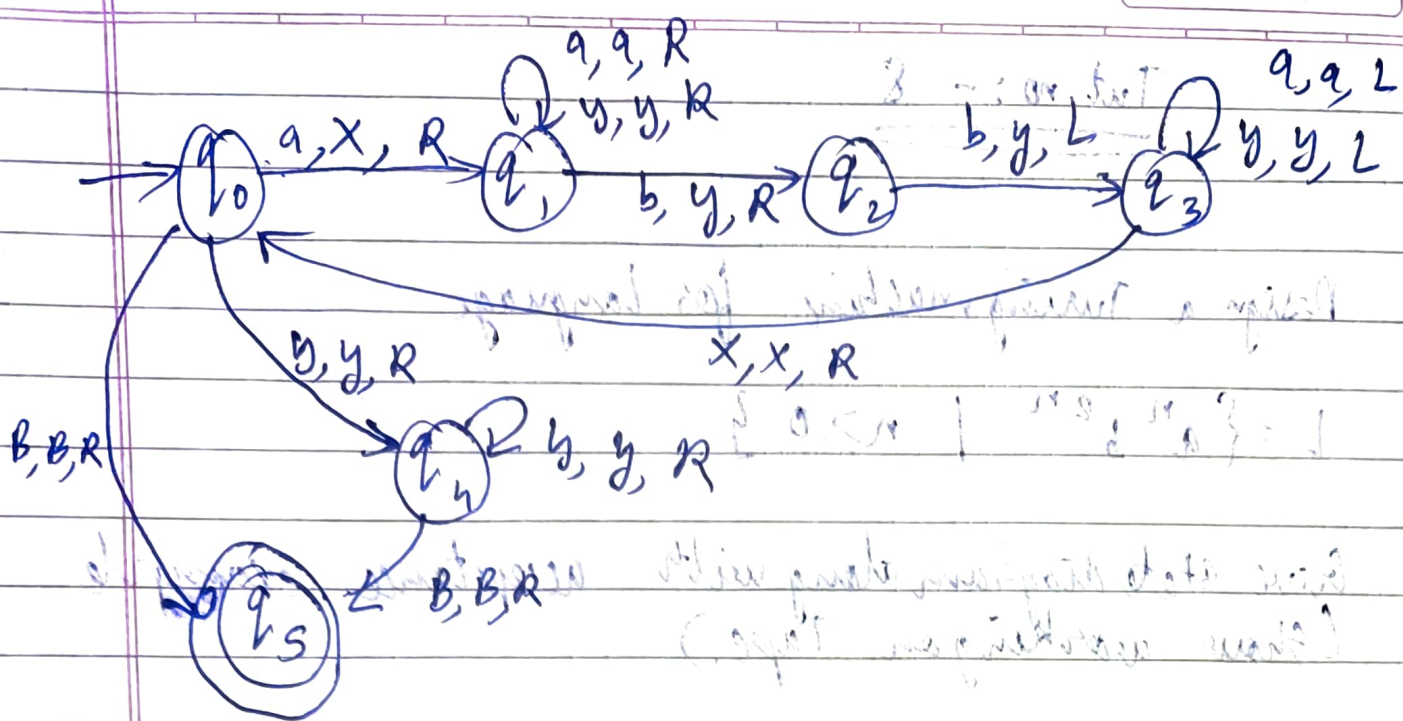
Give state diagram along with acceptance example
(Show working on Tape.)

Ans

Here $L = \{ \epsilon, ab, aabb, aaabbbb, \dots \}$
i.e. every 'a' should match 2 'b's on the tape.

Logic

- If tape doesn't read anything other than B i.e. blank then also string accepted.
- Otherwise tape read with begin with 'a' since it is $a^n b^{2n}$ i.e. there is no 'a' after 'b' or no 'b' before 'a'.
- Whenever 'a' is read its replaced by 'x' and search for 'b' where we replace 2 'b's with 'y'.
- When all 'a' have been replaced by 'x' then all 'b' should also have been converted to 'y'.
- Final state should be reached when the tape has all 'x' and 'y' or only B i.e. blank and we have reached end.
 $R \rightarrow \text{Right} \quad L \rightarrow \text{left} \quad B \rightarrow \text{Blank}$



- $q_0 \rightarrow$ for replacing a with X then move right
 $q_1 \rightarrow$ for replacing b with Y then move right
 $q_2 \rightarrow$ for replacing b with Y then move left
 $q_3 \rightarrow$ Comeback
 $q_4 \rightarrow$ check if any b is left as Blank (B) is there

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$Q = (\text{finite set of states}) = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

$$\Sigma = (\text{finite set of i/p symbols}) = \{a, b\}$$

$$\Gamma = (\text{finite set of tape symbols}) = \{B, a, b, X, Y\}$$

$$\delta = (\text{transition function}) = Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$$

$$q_0 = (\text{initial state}) = q_0$$

$$B = (\text{represents all empty cells}) = B$$

$$F = (\text{Final state}) = \{q_5\}$$

Transition table

| Q / Γ | a | b | x | y | B |
|-------------------|-------------|-------------|-------------|-------------|---------------------|
| $\rightarrow q_0$ | q_0, x, R | H | H | q_4, y, R | q_5, B, R |
| q_1 | q_0, a, R | q_2, y, R | H | q_1, y, R | H |
| q_2 | H | q_3, y, L | H | H | H |
| q_3 | q_3, a, L | H | q_0, x, R | q_3, y, L | \times H \times |
| q_4 | H | H | H | q_4, y, R | q_5, B, R |
| q_5 | q_5 | q_5 | q_5 | q_5 | q_5 |

Acceptance Example \rightarrow Testing with a a b b b b

B | a | a | b | b | b | b | B B \rightarrow blank

$\uparrow q_0$
B | x | a | b | b | b | b | B

$\uparrow q_1$
B | x | a | b | b | b | b | B \times \times

$\uparrow q_2$
B | x | a | y | b | b | b | B

$\uparrow q_3$
B | x | a | y | y | b | b | B

$\uparrow q_3$
B | x | a | y | y | b | b | B

$\uparrow q_3$
B | x | a | y | y | b | b | B

$\uparrow q_0$
B | x | a | y | y | b | b | B

$\uparrow q_1$
B | x | x | y | y | b | b | B
 $\uparrow q_2$

B | x | x | y | y | b | b | B

B | x | x | y | y | b | b | B

B | x | x | y | y | y | b | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

B | x | x | y | y | y | y | B

Since this ends at final state
It is accepted.