Quantum Machine Learning and Recent Advancements

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***Abstract*—Quantum Computing is a fastly growing area with many applications, including quantum machine learning (QML). Due to the rapid increase of computational power, machine learning models based on artificial neural networks (ANN) have become highly effective. Even though classical machine learning models have been performing well, quantum computing with machine learning enhances the performance in multiple ways. This paper studies different aspects of quantum machine learn- ing. It introduces quantum computing over classical computation, followed by the recent tools and techniques developed in the area. We look at multiple QML models like quantum kernel, quantum support vector machine (QSVM), etc. Finally, we present the literature survey to encourage researchers and academicians.**

Keywords-Classical and Quantum Computing; Quantum Machine Learning; Quantum Perceptron; Quantum Kernel; QSVM.

I. INTRODUCTION

Recent times have witnessed a slowdown in Moore's Law. An observation made by Moore in 1965 states that the number of transistors per unit area of an integrated circuit (IC) will double roughly once every two years. Moore's law has held on very well since it was stated, leading to the fantastic, powerful computers available today. The sad part is that Moore's law is reaching its saturation point now. Most experts believe that the observation made by Moore will saturate entirely by 2025 because a transistor's feature size or physical limit tends to be saturated over time [1], [2], [3], [4].

When transistors get smaller than this physical limit, quantum mechanics describe the behaviour of electrons. This may hint at looking into quantum computers and quantum information processing. Classical computers are still the best for performing addition, sorting, etc. On the contrary, quantum computers are superior in specific algorithmic tasks using quantum mechanical phenomena like superposition and en- tanglement, which are not present in classical computers. One of the earliest demonstrations where the quantum computer proved significantly faster than the best-known classical coun- terpart is Shor's algorithm for factoring numbers [2], [3]. This breakthrough brought attention to quantum computing because the approach provides an exponential speedup on the best-known classical algorithm for factoring numbers. If large enough quantum computers are built; adversaries can use them to break the RSA cryptosystem!

Rapid growth in the diverse and complex requirements of high-performance computing power has been witnessed in the last two decades. Classical computing systems provide many benefits in the presence of artificial intelligence (AI) technology, including machine learning and deep learning. However, the systems cannot provide performance requirements despite the abundance of computing resources for specific applications when applications' complexity, type, and size increase. The solution is to employ a QML technique. Therefore, it is necessary to explore this new field [2], [3], [5].

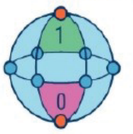
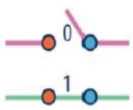
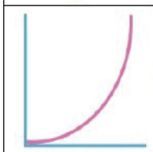
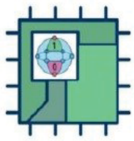
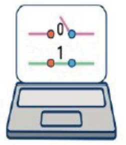
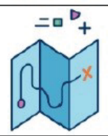
This paper presents a state-of-the-art on the different aspects of quantum machine learning. We first introduce quantum computing and architecture. Then we offer multiple QML models like quantum kernel, quantum support vector machine (QSVM), etc. We see that these methods are better than their classical counterparts in terms of accuracy and speed, even with the limited power of the present quantum computers. Finally, we present a detailed literature survey about recent advancements in the area.

The rest of the paper is organized as follows. Section II introduces classical computers. Section III briefs the fundamentals of quantum computing. Section IV expresses classical machine learning. Section V discusses QML. Section VI describes a few QC applications. Section VII provides related works. Section VIII concludes the paper.

II. CLASSICAL COMPUTING AND IT'S LIMITATIONS

In 1936 Sir Alan M Turing described the Turing machine as a straightforward machine that can capture the essence of what it means to compute. A Turing machine consists of an infinitely large tape, a head, a set of states, and some initial symbols written on the tape (input). At each state, there is a rule describing the action to be taken which is of three types, the head moves left, right, or stays and changes the alphabet on the tape. Inspired by Turing's ideas, Von- Neumann proposed an architecture that modern-day computers use and constitute a classical computing system. A typical classical computing system consists of processing, storage, and input/output units. The basic operations of the system are arithmetic and logical operations accomplished by the process- ing unit. The processing unit accesses (fetch, execution, store, etc.) both the data and instruction from the storage section. A classical computing system is for general applications and does not meet performance requirements for many complexes, large- scale applications. Subsequently, super-computing or high- performance computing systems is welcomed with quantum computing paradigms [2], [3].

TABLE I. CLASSICAL VS QUANTUM COMPUTER.

Quantum Computer		Classical Computer	
	Calculates with qubits which can represent 0 and 1 at same time		Calculates with transistors which can represent either 0 or 1
	Power increases exponentially in proportion to the number of qubits		Power increases linearly with respect to number of transistors
	Have high error rates and need to be kept ultra-cold.		Low error rate and can operate at room temperature
	Suited for problems like optimization, data analysis and simulations		Most everyday processing is best handled by classical computers

III. BUILDING BLOCKS OF A QUANTUM COMPUTER

Although quantum computers will not replace classical computers in every sense soon for tasks like text editors, scien- tific computations, etc., there is sufficient evidence to show that quantum computers can outperform classical computers [2], [3]. Table I gives fundamental differences between a quantum and classical computer. Therefore, building a quantum com- puter is an important task.

A. Qubits

Classical bits are always in the state of 0 or 1 throughout the entire process of computation. No additional information is required to represent them. Contrarily quantum bits (Qubits) can be in the superposition of the states 0 and 1. A Qubit collapses to either the 0 or 1 state when measured. These states are mutually exclusive. One way to represent a Qubit is by using two ortho-normal vectors shown in Equation 1.

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \tag{1}$$

The and are known as ket notation denoting qubit states. With the vector representation of qubits, one can describe more complex states, e.g., the vector representing the superposition state in Equation 2.

$$|q\rangle = \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} \tag{2}$$

One can express the state |q⟩ alternatively as in Equation 3 because |0⟩ and |1⟩ form an orthonormal basis,

$$|q\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \tag{3}$$

q is a state vector containing all possible Qubit informa- tion. Although q represents the superposition of 0 and 1 states, it is as well defined as the basis states.

B. Measuring Qubits

One of the most critical operations in quantum computing is the measurement of qubits. Taking a peek into what happens during a quantum computation is essential. Although Qubits are in the superposition of 0 and 1 when we measure a Qubit, it collapses to one of these states [3]. The rule for a Qubit measurement is straightforward. Equation 4 calculates the probability of measuring a state |ψ⟩ in one of the basis state |x⟩.

$$\boldsymbol{p}[|\boldsymbol{x}\rangle] = |\langle \boldsymbol{x}|\boldsymbol{\psi}\rangle|\mathbf{2} \tag{4}$$

The symbols and tell that x is a row vector and ψ is a column vector. In quantum computing, the column vectors are known as "kets", and row vectors are known as "bras". Together these vectors thus make up the bra-ket notation. For example, every ket c has a corresponding bra c , which is obtained by taking the conjugate transpose of the column vector.

In Equation 4, x can be any state to measure the probabil- ity by taking the inner product of x and the state ψ to be measured followed by squaring the result. This is because the probability of calculating a Qubit in a particular state is the absolute value of the amplitude squared. For example, consider the superposition state described in Equation 3. We calculate the probability of resulting state |0⟩ as 0.5 when we measure |q0⟩ . Computation is shown in Equation 5.

$$\begin{aligned} |q_0\rangle &= \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \\ \langle 0|q_0\rangle &= \frac{1}{\sqrt{2}}\langle 0|0\rangle + \frac{i}{\sqrt{2}}\langle 0|1\rangle \\ &= \frac{1}{\sqrt{2}}\cdot 1 + \frac{i}{\sqrt{2}}\cdot 0 \\ &= \frac{1}{\sqrt{2}} \\ |\langle 0|q_0\rangle|^2 &= \frac{1}{2} \end{aligned} \tag{5}$$

The rule (Equation 4) dictates how a piece of information is extracted from quantum states. Therefore, it is vital. A note about measurement is that when one measures a particular state, the superposition is collapsed to a single state. Any subsequent measurements of the Qubit will yield the same answer. If measuring the state of a Qubit is too early, one may lose the quantum advantage.

Consequently, the Qubit starts to act like a classical bit. Therefore, the Qubits are measured at the end of the computation. Hence the measurement operations are kept at the end of a quantum circuit [2], [3], [6].

C. Bloch Sphere Representation

The state vector corresponding to a general state of Qubit q is represented as defined in Equation 6, which is further transformed to Equation 7 and 8 taking the global phase eiφ1.

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle; \quad \alpha,\beta \in \mathbb{C} \tag{6}$$

$$|q\rangle = e^{i\phi_1}\alpha|0\rangle + e^{i\phi_2}\beta|1\rangle; \quad \alpha,\beta \in \mathbb{R} \tag{7}$$

$$|q\rangle = e^{i\phi_1}(\alpha|0\rangle + e^{i(\phi_2-\phi_1)}\beta|1\rangle) \tag{8}$$

Since the global phase does not contribute to the measure- ment, it cannot be measured. The Qubit is then represented as in Equation 9. The Qubit state must be normalized because α and β are not independent. The value of one determines the value of the other. This dependency allows to represent of both variables as a function of different variables, as shown in Equation 10. Considering the trigonometric identity in Equation 11, one can represent actual variables α and β in terms of a single parameter θ as shown in Equation 12. Thus |q⟩ is re-defined in Equation 13 [2], [3].

$$|q\rangle = \alpha|0\rangle + e^{i\phi}\beta|1\rangle \tag{9}$$

$$\sqrt{\alpha^2 + \beta^2} = 1 \tag{10}$$

$$\sqrt{\sin^2 x + \cos^2 x} = 1 \tag{11}$$

$$\alpha = \cos\frac{\theta}{2}, \quad \beta = \sin\frac{\theta}{2} \tag{12}$$

$$|q\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \tag{13}$$

Here θ and φ are the spherical coordinates at r = 1. Then a Qubit can be viewed as a Bloch sphere representation shown in Figure 1

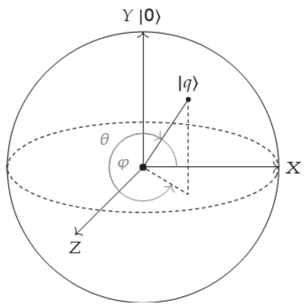


Fig. 1. Bloch sphere visualization for a Qubit

D. Quantum Gates

Classical gates are used to manipulate classical information. Quantum gates can also be utilized to manage quantum data. Quantum gates can be considered matrices that multiply a state vector into a new state vector while maintaining the vector's norm. These types of matrices are known as unitary matrices. A quantum gate is categorized as X, Y, Z, and U. The X gate, when visualized on the Bloch sphere, performs a rotation by π around the X-axis for a given input vector. In Matrix form, it can be viewed in Equation 14 and Figure 2 [2], [3].

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0| \tag{14}$$

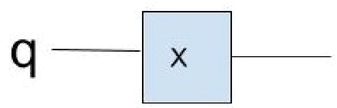


Fig. 2. Symbol for X-gate

Similar to the X gate, the Y and Z gates perform a π rotation along the Y and Z axis, respectively. Figure 3 and Equation 15 express these gates, respectively.

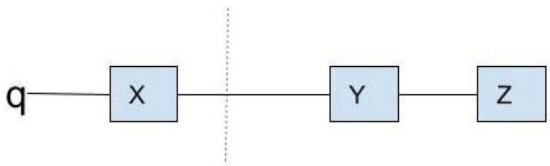


Fig. 3. Symbolic Representation of X, Y, Z gates

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{15}$$

The single Qubit state with the broadest range of applications is a U gate. This means that every other single input qubit gate can be represented as a U-gate. It makes the Bloch sphere's rotation more prevalent [7]. This is a parametrized gate of the form defined in Equation 16.

$$U(\theta, \phi, \lambda) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -e^{i\lambda}\sin(\frac{\theta}{2}) \\ e^{i\phi}\sin(\frac{\theta}{2}) & e^{i(\phi+\lambda)}\cos(\frac{\theta}{2}) \end{bmatrix} \tag{16}$$

IV. CLASSICAL MACHINE LEARNING

An artificial neuron is viewed as a mathematical function that takes an input vector where each value has a weight. Then it computes the dot product of the weight vector with the input vector. This result is given as input to a suitable activation function which returns 1 or 0 based on the input value. Here, 1 and 0 correspond to the neuron firing and not firing, respectively. An artificial neuron is the fundamental building block of an ANN architecture. The architecture is a collection of artificial neurons consisting of an input layer, hidden layer, and output layer. The number of output layer neurons depends on the number of training classes we have. Machine learning models can be broadly classified into super- vised and unsupervised models. Supervised learning models are trained with training data consisting of examples and correct classifications. On the other hand, unsupervised models try to find a hyperplane that separates the data. One of the advantages of machine learning is that it can be used to build highly accurate and reliable models for tasks such as handwriting recognition and image segmentation. On the other hand, one of the disadvantages of machine learning is that the model requires a lot of labelled data to classify objects with acceptable accuracy [2], [3], [8].

V. QUANTUM MACHINE LEARNING

Quantum computing has many potentials to reshape and improve current technology. One promising field is quantum machine learning, which covers four scenarios (Figure 4) over classical machine learning with respect to the types of algorithms and data.

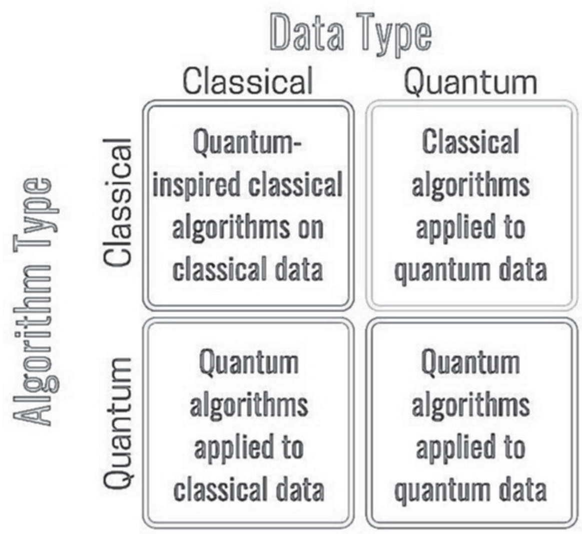


Fig. 4. Classical vs Quantum Machine Learning.

A. Quantum Perceptron

A perceptron in classical machine learning tries to find a hyper-plane that separates two classes. It is a supervised learning model that iteratively adjusts the weights after seeing each example. The QML works using a perceptron model. The main idea behind the quantum perceptron model is as follows. Instead of changing weights based on every training example, a superposition is created to represent all possible training vectors. To achieve a good speedup, Grover's search algorithm is employed. The algorithm finds the weight vector that correctly classifies the given test data [2], [3], [9], [10]. The rule for updating the weights of a quantum perceptron is as follows.

1) *Predict sign(w_i x) as the label for example x_i.*
2) *If incorrect, update $w_{i+1} = w_i + l(x_i)x_i$*

else $w_{i+1} = w_i$.

Here w_i is the weight in the ith iteration, x is the input vector, and $l(x_i)$ is the momentum.

B. Quantum SVM and Kernel Method

The SVM model is a classifier for linearly separable data. The problem with most data sets is that we don't need to have a hyperplane in the same dimension the data exists that separates the two classes. This problem is overcome by mapping the feature space to a higher dimensional space. This method is known as the kernel trick. The quantum kernel method is helpful because it is significantly harder to compute the kernel matrix classically than with a quantum computer. In addition, data is automatically cast into a higher dimensional Hilbert space when it is represented as a quantum state [3], [11], [12]. When finding a separating hyperplane in the given dimension is impossible, the feature space is cast into a higher size using the quantum kernel method. Later a quantum SVM is run to find a separating hyperplane in this dimension [13]. Figures 5 and 6 refer to quantum SVM and a typical quantum kernel method over classical machine learning.

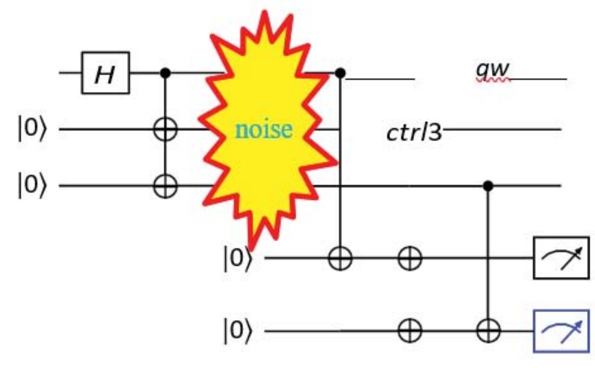


Fig. 5. Quantum Circuit for Support Vector Machine

C. Quantum Enhanced Deep Learning

The Qubit can act as a neuron. Due to some fast algorithms from quantum linear algebra, the neural network can be trained much faster than classical neural networks. This is one of the main reasons quantum machine learning models, for example, quantum kernel mapping, can enhance performance. Figure 7 describes how quantum computing can be used to speed up the process of training a deep neural network using techniques like Grover's search algorithm [3], [14].

VI. SELECTED APPLICATIONS OF QUANTUM COMPUTING

Quantum computers are ideally suited to solving complex problems that are hard for classical computers. Such an advancement creates opportunities across almost every aspect of modern life. Quantum computing can influence the world through many applications such as AI, machine learning, drug development, better Batteries, cybersecurity, cleaner fertilization, solar capture, traffic optimization, weather forecasting and climate change, financial modelling, electronic materials discovery, etc. A few are briefly described here [2], [3].

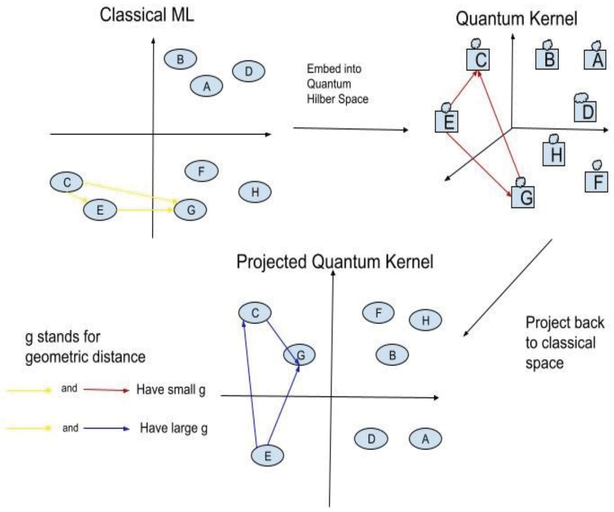


Fig. 6. Quantum kernel method

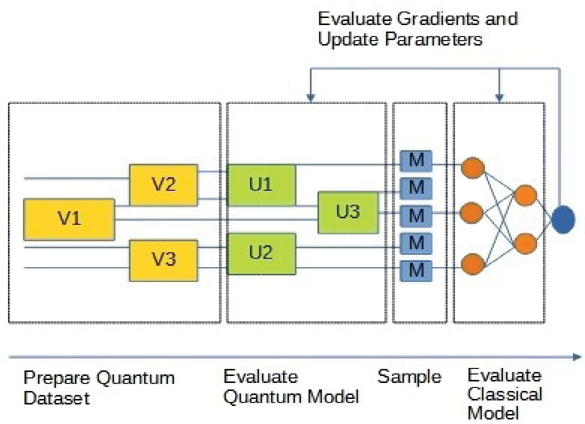


Fig. 7. Quantum Deep Learning Visualized

A. Quantum Machine Learning

Quantum machine learning is one of the most popular applications of quantum computing. The people working in the area are mainly concerned about applying the principles of quantum computing to classical machine learning. This endeavour hopes to obtain speedup and improved accuracy over its classical counterparts [15].

B. Computational Chemistry

The fundamental problem with computational chemistry is that when researchers try to understand a chemical reaction consisting of small molecules, the amount of information required to carry out the simulation is so large that even supercomputers cannot handle them. Since Quantum computers are inherently quantum mechanical, the hope is that researchers will be able to simulate and understand these reactions efficiently [16]. Primary applications include drug design, testing, and material design.

C. Cryptography and Cyber Security

Shor's algorithm poses a threat to the RSA crypto-system. It has made researchers consider developing secure crypto-systems against adversaries with a quantum computer. This field is known as post-quantum cryptography (PQC) and is currently based on quantum computers' inability to solve some problems based on Lattices quickly [17]. The other use of quantum computers includes secure communication. This area is known as quantum key distribution (QKD). Further cyber forensics recently started applying quantum machine learning to detect phishing-based attacks better [18].

VII. RELATED WORKS

QML is recently explored in diverse directions by researchers and academicians. Schuld et al. [19] showed the improved performance and generality of classical machine learning algorithms. The idea is to use quantum kernels to embed the data into higher dimensional Hilbert space to improve the data classification. It means that this model outperforms classical algorithms. Sajjan et al. [20] applied quantum machine learning to chemistry in reaction dynamics and the study of phase transition. The approach showed how quantum machine learning enhances drug design and material manufacturing. Figure 8 offers an application of machine learning in quantum chemistry. It describes how the paradigm of machine learning can be applied to chemical reaction data sets to obtain a better understanding of the data. Dborin et al. [21] proposed a method to pre-train matrix weight in parametrized quantum circuits representing quantum neural networks. This method increases the training speed of the neural network. Figure 9 denotes pre-train matrix weights in parametrized quantum circuits, i.e., the training circuit implementation.

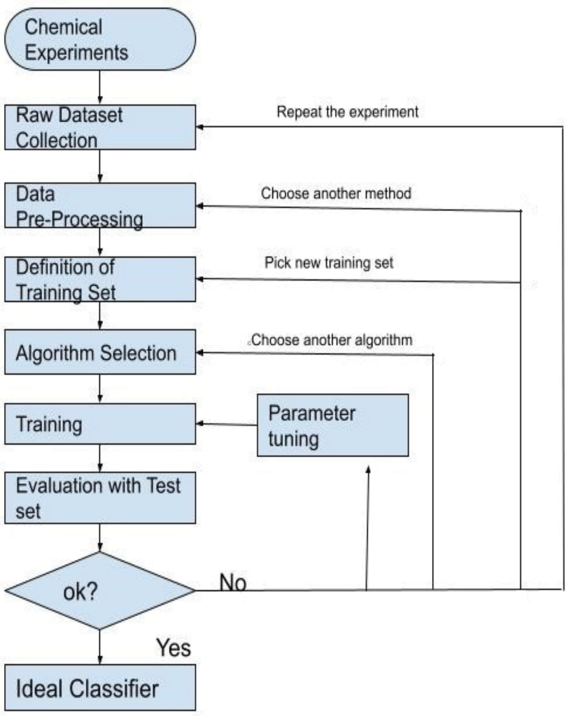


Fig. 8. Machine learning for quantum chemistry

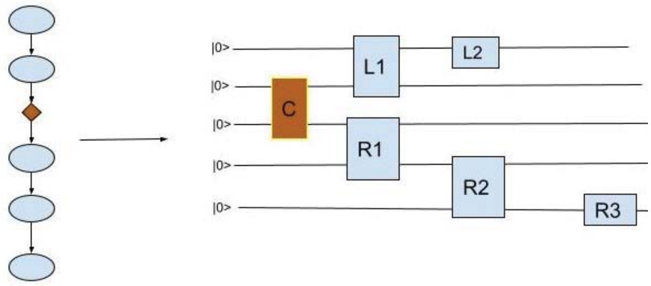


Fig. 9. Pre-Train Matrix Weights in Parametrized Quantum Circuits

Gibbs et al. [22] showed how quantum machine learning could be used in the quantum dynamical simulation. The proposed algorithms are resource efficient and can be implemented in a near-term quantum computer called a noisy intermediate scale quantum (NISQ) computer. The device is expected to provide an exponential speedup. Dilip et al. [23] provided a method for compressing the data for training a quantum machine learning model. They addressed the problem of efficiently compressing and loading data into quantum machine learning models for a competitive accuracy compared to a classical model with millions of parameters. Huang et al.

[24] provided an improved performance by a few quantum ker- nel methods applied over classical machine learning models. They also proposed ways to engineer the data set so that the quantum computer can obtain an advantage in terms of accu- racy. Even with 30 qubits, near-term quantum computers can use the quantum projected kernel method to provide higher ac- curacy. Caro et al. [25] described performance generalization of quantum machine learning. Generalization is the term used to describe how well the model performs on the training data set and the testing data set. The authors also showed that the classification of quantum states across a phase transition with a quantum convolutional neural network could be accomplished with only a minimal training data set. Besides, the QML is welcoming in many thrust research directions. Other potential applications, e.g., are learning quantum error-correcting codes and quantum dynamical simulation. The applications provide an idea to understand the performance of quantum machine learning algorithms when applied to real-world classification problems. For example, Havenstein et al. [26] performed a comparison study on two datasets- Breast Cancer and Wine dataset. Between classical SVM and quantum SVM. The latter approach significantly outperforms the former for both datasets, as visible from Tables II and III. It demonstrates the advantage of quantum machine learning and grows interest in studying the field.

TABLE II. ACCURACY COMPARISON OF BREAST-CANCER DATASET

Classifiers	Back-End	Algorithm	Wall Time	Accuracy (%)
Quantum SVM	ibmqx4	SVM RBF	6.91	80
Quantum SVM	statevector	SVM RBF	5.96	100
Classical SVM	CPU	SVM	6.9	85

TABLE III. ACCURACY COMPARISON OF WINE DATASET

Classifiers	Back-End	Algorithm	Wall Time	Accuracy (%)
Quantum SVM	ibmqx4	VAR SVM	6.44	93
Quantum SVM	statevector	VAR SVM	5.98	100
Classical SVM	CPU	SVM	6.2	90

VIII. CONCLUSION

This paper has provided a literature survey and explored some fundamental aspects of quantum computing, specifically in the quantum machine learning domain. We discussed some of the recent advances in the area of quantum machine learning. We showed that quantum machine learning is better than its classical counterpart in many current practical problems. Quantum Computing is an emergent field with a lot of potential applications. Future work includes exploring more interesting real-world machine-learning applications for enhanced performance.

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