

Github link: <https://github.com/keyur2110/MLCS5710.git>

Video link: https://drive.google.com/file/d/1BJbSDpAOOGhJZdSwfImX55sDa0dlLC/view?usp=share_link

- There are 2 questions for this assignment first question is attached at the end of question 2.

Question 2:

```
In [145]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler, normalize
from sklearn.decomposition import PCA
from sklearn.cluster import KMeans
from sklearn import preprocessing
from sklearn import metrics

In [146]: dataset = pd.read_csv('CC GENERAL.csv') #importing data

In [147]: dataset.head() #first few rows of the dataset

Out[147]:

|   | CUST_ID | BALANCE     | BALANCE_FREQUENCY | PURCHASES | ONEOFF_PURCHASES | INSTALLMENTS_PURCHASES | CASH_ADVANCE | PURCHASES_FREQUENCY |
|---|---------|-------------|-------------------|-----------|------------------|------------------------|--------------|---------------------|
| 0 | C10001  | 40.900749   | 0.818182          | 95.40     | 0.00             | 95.4                   | 0.000000     | 0.166               |
| 1 | C10002  | 3202.467416 | 0.909091          | 0.00      | 0.00             | 0.0                    | 6442.945483  | 0.000               |
| 2 | C10003  | 2495.148862 | 1.000000          | 773.17    | 773.17           | 0.0                    | 0.000000     | 1.000               |
| 3 | C10004  | 1666.670542 | 0.636364          | 1499.00   | 1499.00          | 0.0                    | 205.788017   | 0.083               |
| 4 | C10005  | 817.714335  | 1.000000          | 16.00     | 16.00            | 0.0                    | 0.000000     | 0.083               |



In [148]: dataset.describe()

Out[148]:

|       | BALANCE     | BALANCE_FREQUENCY | PURCHASES   | ONEOFF_PURCHASES | INSTALLMENTS_PURCHASES | CASH_ADVANCE | PURCHASES_FREQUENCY |
|-------|-------------|-------------------|-------------|------------------|------------------------|--------------|---------------------|
| count | 8950.000000 | 8950.000000       | 8950.000000 | 8950.000000      | 8950.000000            | 8950.000000  | 8950.000000         |
| mean  | 1564.474828 | 0.877271          | 1003.204834 | 592.437371       | 411.067645             | 978.871112   | 0.490351            |
| std   | 2081.531879 | 0.236904          | 2136.634782 | 1659.887917      | 904.338115             | 2097.163877  | 0.401371            |
| min   | 0.000000    | 0.000000          | 0.000000    | 0.000000         | 0.000000               | 0.000000     | 0.000000            |


```

First, I have imported the required libraries. Then imported the dataset ‘CC GENERAL.csv’.

Dataset is also displayed using head () function and there is description of the dataset.

```
max 19043.138560      1.000000 49039.5/0000    40/61.250000    22500.000000  4/13/211/60    1.000000
```

```
In [149]: #Question 2(a)
#drooping the unwanted data
dataset = dataset.drop("CUST_ID", axis=1)
```

```
In [150]: #checking for Null values
dataset.isnull().sum()
```

```
Out[150]:
```

| BALANCE | 0 |
|----------------------------------|-----|
| BALANCE_FREQUENCY | 0 |
| PURCHASES | 0 |
| ONEOFF_PURCHASES | 0 |
| INSTALLMENTS_PURCHASES | 0 |
| CASH_ADVANCE | 0 |
| PURCHASES_FREQUENCY | 0 |
| ONEOFF_PURCHASES_FREQUENCY | 0 |
| PURCHASES_INSTALLMENTS_FREQUENCY | 0 |
| CASH_ADVANCE_FREQUENCY | 0 |
| CASH_ADVANCE_TRX | 0 |
| PURCHASES_TRX | 0 |
| CREDIT_LIMIT | 1 |
| PAYMENTS | 0 |
| MINIMUM_PAYMENTS | 313 |
| PRC_FULL_PAYMENT | 0 |
| TENURE | 0 |

```
dtype: int64
```

```
In [151]: dataset = dataset.fillna(dataset.mean()) #filling null values with mean values
```

For Question 2(a), I have deleted the first column which is 'CUST_ID'. I have checked for the null values in the dataset there are 2 attributes with the null values. I have used the mean values to fill the null values of those two attributes.

```
In [152]: #question 2(b)
#Doing feature scaling for dataset
scaler = StandardScaler()
X_Scale = scaler.fit_transform(dataset)
print(X_Scale)

[[-0.73198937 -0.24943448 -0.42489974 ... -0.31096755 -0.52555097
 0.36067954]
 [ 0.78696085  0.13432467 -0.46955188 ...  0.08931021  0.2342269
 0.36067954]
 [ 0.44713513  0.51808382 -0.10766823 ... -0.10166318 -0.52555097
 0.36067954]
...
[-0.7403981 -0.18547673 -0.40196519 ... -0.33546549  0.32919999
-4.12276757]
[-0.74517423 -0.18547673 -0.46955188 ... -0.34690648  0.32919999
-4.12276757]
[-0.57257511 -0.88903307  0.04214581 ... -0.33294642 -0.52555097
-4.12276757]]
```

```
In [153]: #Normalized the scaled data
x_norm = preprocessing.normalize(X_Scale)
```

```
In [154]: print(x_norm)
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | \ |
|------|-----------|-----------|-----------|------------|-----------|-----------|-----------|---|
| 0 | -0.311938 | -0.106297 | -0.181072 | -0.152108 | -0.148760 | -0.198921 | -0.343687 | |
| 1 | 0.219925 | 0.037539 | -0.131222 | -0.099749 | -0.127037 | 0.728166 | -0.341434 | |
| 2 | 0.126682 | 0.146783 | -0.030504 | 0.030850 | -0.128790 | -0.132249 | 0.359771 | |
| 3 | 0.020589 | -0.426439 | 0.097309 | 0.229034 | -0.190618 | -0.154587 | -0.425253 | |
| 4 | -0.151595 | 0.218909 | -0.195238 | -0.146744 | -0.192075 | -0.197234 | -0.428504 | |
| ... | ... | ... | ... | ... | ... | ... | ... | |
| 8945 | -0.146893 | 0.103128 | -0.066344 | -0.0701050 | -0.026403 | -0.092916 | 0.252770 | |
| 8946 | -0.151521 | 0.105735 | -0.067173 | -0.072846 | -0.025067 | -0.095266 | 0.259162 | |
| 8947 | -0.156974 | -0.039324 | -0.085222 | -0.075675 | -0.062521 | -0.098965 | 0.181181 | |
| 8948 | -0.154320 | -0.038411 | -0.097240 | 0.073918 | -0.094139 | -0.093057 | -0.253016 | |
| 8949 | -0.115207 | -0.178881 | 0.008480 | 0.060711 | -0.091465 | -0.081732 | 0.088393 | |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | \ | |
| 0 | -0.289212 | -0.301422 | -0.287801 | -0.202878 | -0.217905 | -0.409290 | -0.225425 | |
| 1 | -0.189660 | -0.256265 | 0.160401 | 0.030761 | -0.165384 | 0.192448 | 0.228779 | |
| 2 | 0.757440 | -0.259802 | -0.191339 | -0.134880 | -0.030888 | 0.234039 | -0.108739 | |
| 3 | -0.167447 | -0.384524 | -0.108570 | -0.138184 | -0.231288 | 0.346393 | -0.251048 | |
| 4 | -0.168727 | -0.387463 | -0.285359 | -0.201157 | -0.233056 | -0.382591 | -0.153959 | |
| ... | ... | ... | ... | ... | ... | ... | ... | |
| 8945 | -0.135091 | 0.234852 | -0.134432 | -0.094764 | -0.069751 | -0.191180 | -0.096784 | |
| 8946 | -0.138507 | 0.240791 | -0.137832 | -0.097161 | -0.071515 | -0.196014 | -0.102738 | |
| 8947 | -0.143885 | 0.161230 | -0.143183 | -0.100933 | -0.082821 | -0.203625 | -0.120978 | |
| 8948 | -0.140545 | -0.189902 | 0.032623 | -0.037897 | -0.122556 | -0.227357 | -0.120224 | |

For question 2(b), first I have applied the standard scaler. And then I have normalized the data using normalize () function. In above screenshot I have displayed the dataset after the standard scaler and after normalizing to see how dataset changes.

```
[8950 rows x 17 columns]
```

```
In [155]: X_norm = pd.DataFrame(X_norm) #converting data into pandas dataset
X_norm.head()
```

```
Out[155]:
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | |
|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|---------|
| 0 | -0.311938 | -0.106297 | -0.181072 | -0.152108 | -0.148760 | -0.198921 | -0.343687 | -0.289212 | -0.301422 | -0.287801 | -0.202878 | -0.217905 | -0.409290 | -0.225425 | -1.3251 |
| 1 | 0.219925 | 0.037539 | -0.131222 | -0.099749 | -0.127037 | 0.728166 | -0.341434 | -0.189660 | -0.256265 | 0.160401 | 0.030761 | -0.165384 | 0.192448 | 0.228779 | 2.4958 |
| 2 | 0.126682 | 0.146783 | -0.030504 | 0.030850 | -0.128790 | -0.132249 | 0.359771 | 0.757440 | -0.259802 | -0.191339 | -0.134880 | -0.030888 | 0.234039 | -0.108739 | -2.8803 |
| 3 | 0.020589 | -0.426439 | 0.097309 | 0.229034 | -0.190618 | -0.154587 | -0.425253 | -0.167447 | -0.384524 | -0.108570 | -0.138184 | -0.231288 | 0.346393 | -0.251048 | 2.0456 |
| 4 | -0.151595 | 0.218909 | -0.195238 | -0.146744 | -0.192075 | -0.197234 | -0.428504 | -0.168727 | -0.387463 | -0.285359 | -0.201157 | -0.233056 | -0.382591 | -0.153959 | -1.1230 |

After applying normalizing we get array as an output so I have converted the array into panda dataframe and displayed the dataset named 'x_norm'.

```
In [156]: #Question 2(c)
pca = PCA(2)#applying PCA and took K(components)=2
x_pca = pca.fit_transform(x_norm)
x_pca = pd.DataFrame(x_pca) #converting into pandas dataframe
x_pca.columns = ['P1','P2']
```

```
In [157]: x_pca.head() #output of the PCA after reducing it to 2 columns
```

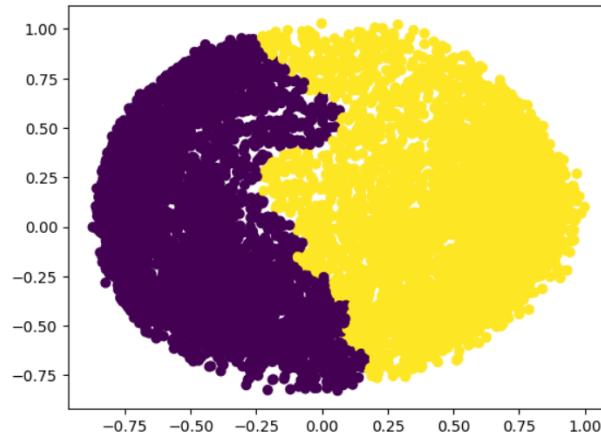
```
Out[157]:
```

| | P1 | P2 |
|---|-----------|-----------|
| 0 | -0.489825 | -0.679679 |
| 1 | -0.518791 | 0.545010 |
| 2 | 0.330885 | 0.268978 |
| 3 | -0.482374 | -0.092112 |
| 4 | -0.563289 | -0.481914 |

For Question 2(c), I have implemented PCA where I taken k = 2. So, the dataset x_norm has been transformed into array. I have again transform the array into panda dataframe which has 2 column named 'P1','P2' and the name of the dataset is x_pca. It is displayed in the screenshot.

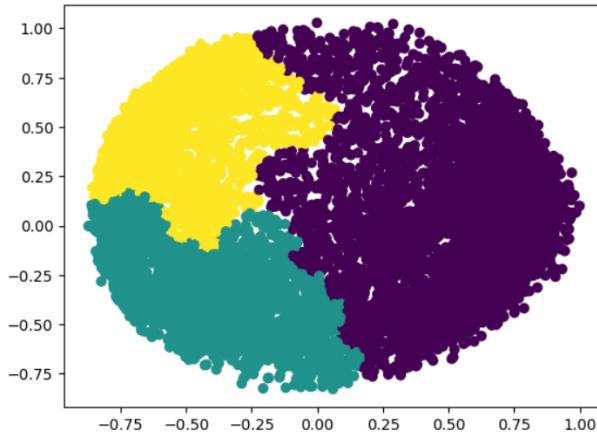
```
In [158]: #Question 2(d)
from sklearn.cluster import AgglomerativeClustering
AC2 = AgglomerativeClustering(2) #applying agglomerative clustering where k =2
a = AC2.fit_predict(x_pca)
```

```
In [159]: plt.scatter(x_pca['P1'],x_pca['P2'],c = a,cmap='viridis')
plt.show()
```



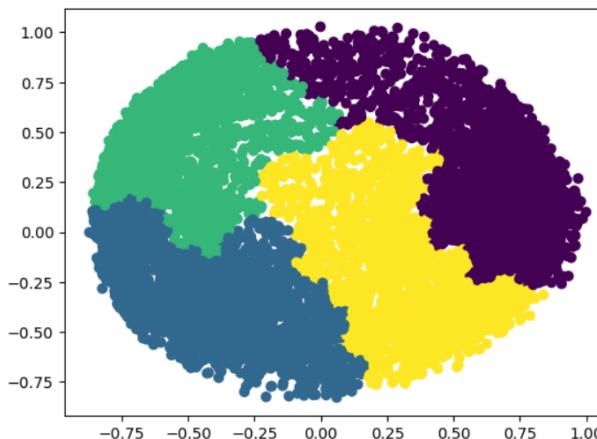
For question 2(d), I have implemented agglomerative clustering using sklearn library. Where the number of clusters is 2. Also, the output has been displayed using the scatterplot. 2 different cluster has been displayed using two different colors.

```
In [160]: AC3 = AgglomerativeClustering(3) #applying agglomerative clustering where k =3
a = AC3.fit_predict(x_pca)
plt.scatter(x_pca['P1'],x_pca['P2'],c = a,cmap='viridis')
plt.show()
```



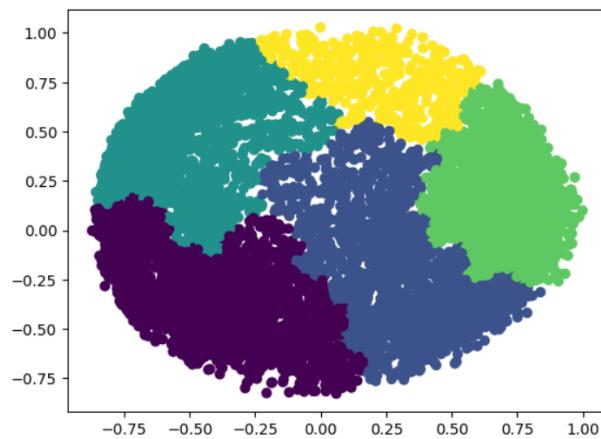
Above I have implemented the agglomerative cluster where number of clusters is 3. Three different colors represent three clusters.

```
In [161]: AC4 = AgglomerativeClustering(4) #applying agglomerative clustering where k =4
a = AC4.fit_predict(x_pca)
plt.scatter(x_pca['P1'],x_pca['P2'],c = a,cmap='viridis')
plt.show()
```



Here is the implementation of the agglomerative cluster with number of the cluster 4.

```
In [162]: AC5 = AgglomerativeClustering(5) #applying agglomerative clustering where k =5  
a = AC5.fit_predict(x_pca)  
plt.scatter(x_pca['P1'],x_pca['P2'],c = a,cmap='viridis')  
plt.show()
```



Above is the implementation of agglomerative cluster where number of cluster is 5.

Screenshot of a Jupyter Notebook interface showing code execution and a resulting bar chart.

In [163]: #Question 2(e)

```
# calculating silhouette scores of the different clusters
s2 = metrics.silhouette_score(x_pca, AC2.fit_predict(x_pca))
s3 = metrics.silhouette_score(x_pca, AC3.fit_predict(x_pca))
s4 = metrics.silhouette_score(x_pca, AC4.fit_predict(x_pca))
s5 = metrics.silhouette_score(x_pca, AC5.fit_predict(x_pca))

ss = []
ss.append (s2)
ss.append (s3)
ss.append (s4)
ss.append (s5)

print(ss)
0.43732429740165507 0.41672294024893747 0.3531061679143868 0.35055077434203463
```

In [164]:

```
# Plotting graph to compare the silhouette score
k = [2, 3, 4, 5]
plt.bar(k, ss)
plt.xlabel('Number of clusters')
plt.ylabel('Silhouette_score')
plt.show()
```

A bar chart comparing the silhouette scores for different cluster counts. The x-axis is labeled 'Number of clusters' and ranges from 1.5 to 5.5. The y-axis is labeled 'Silhouette_score' and ranges from 0.0 to 0.4. Four bars represent the scores for k=2, 3, 4, and 5. The scores decrease as the number of clusters increases.

| Number of clusters | Silhouette score |
|--------------------|---------------------|
| 2 | 0.43732429740165507 |
| 3 | 0.41672294024893747 |
| 4 | 0.3531061679143868 |
| 5 | 0.35055077434203463 |

For question 2(e), first I have calculated the silhouette score for all clusters model named “S2,S3,S4,S5” and added to the list named “ss”.

I have used the bar graph to represent the silhouette score of each model. In bar graph y-axis represent the silhouette score and x-axis represent cluster models.

| | P_1 | P_2 | P_3 | P_4 | P_5 | P_6 |
|-------|--------|--------|--------|--------|--------|--------|
| P_1 | 0.0 | 0.2357 | 0.2218 | 0.3688 | 0.3421 | 0.2347 |
| P_2 | 0.2357 | 0.6 | 0.1483 | 0.2042 | 0.1388 | 0.2540 |
| P_3 | 0.2218 | 0.1483 | 0.0 | 0.1513 | 0.2843 | 0.1100 |
| P_4 | 0.3688 | 0.2042 | 0.1513 | 0.6 | 0.2932 | 0.2216 |
| P_5 | 0.3421 | 0.1388 | 0.2843 | 0.2932 | 0.0 | 0.3921 |
| P_6 | 0.2347 | 0.2540 | 0.1100 | 0.2216 | 0.3921 | 0.0 |

Ang \lim^N
 Avg Pair $[P_3, P_6] \rightarrow 0.11$

| | P_1 | P_2 | P_3, P_6 | P_4 | P_5 |
|------------|--------|--------|------------|--------|-------|
| P_1 | 0.0 | | | | |
| P_2 | 0.2357 | 0.0 | | | |
| P_3, P_6 | 0.2282 | 0.2011 | 0.0 | 0.3382 | |
| P_4 | 0.3688 | 0.2042 | 0.1864 | 0.0 | |
| P_5 | 0.3421 | 0.1388 | 0.3382 | 0.2932 | 0.0 |

$$[P_3, P_6], P_1 \rightarrow \text{Avg}(0.2218, 0.2347) \rightarrow 0.2282$$

$$[P_3, P_6], P_2 \rightarrow \text{Avg}(0.1483, 0.2540) \rightarrow 0.2011$$

$$[P_3, P_6], P_4 \rightarrow \text{Avg}(0.1513, 0.2216) \rightarrow 0.1864$$

$$P_5, [P_3, P_6] \rightarrow \text{Avg}(0.2843, 0.3921) \rightarrow 0.3382$$

$$[P_2, P_5] \rightarrow 0.1388$$

| P_1 | P_2 | P_3 | P_4 |
|------------|--------|-----------------------------|-------|
| P_1 | 0.0 | | |
| P_2, P_5 | 0.2889 | 0.0 | |
| P_3, P_6 | 0.2282 | 0.2946 0.2889 | 0.0 |
| P_4 | 0.3688 | 0.2487 | 0.0 |

$$[P_2, P_5], P_1 \rightarrow \text{Avg} = (0.2357, 0.3421) \rightarrow 0.2889$$

$$[P_3, P_6][P_2, P_5] \rightarrow \text{Avg} = (0.2011, 0.3382) \rightarrow 0.2946$$

$$\boxed{P_4}[P_5, P_2] \rightarrow \text{Avg} = (0.2042, 0.2932) \rightarrow 0.2487$$

$$[P_4, P_3, P_6] \rightarrow 0.1864$$

| P_1 | P_2 | P_3, P_4, P_6 |
|-----------------|--------|------------------|
| P_1 | 0.0 | |
| P_2, P_5 | 0.2889 | 0.0 0.2776 |
| P_3, P_4, P_6 | 0.2958 | 0.2591 0.2591 |

$$P_1 [P_3, P_4, P_6] \rightarrow \text{Avg} = (0.2282, 0.3688) = 0.2985$$

$$[P_3, P_4, P_6][P_2, P_5] \rightarrow \text{Avg} = (0.2946, 0.2487) = 0.2776$$

$$0.2776 \\ 0.2591$$

$$[P_1, P_2] [P_3, P_4, P_6] \rightarrow 0.254 + [0.2716, 0.2591]$$

| P_1 | P_2, P_3, P_4, P_5, P_6 | P_{avg} |
|---------------------------|---------------------------|-----------|
| 0.0 | 0.0 | 0.0 |
| P_2, P_3, P_4, P_5, P_6 | 0.2923 | P_{avg} |
| 0.0 | 0.0 | P_{avg} |
| 0.0 | 0.0 | P_{avg} |

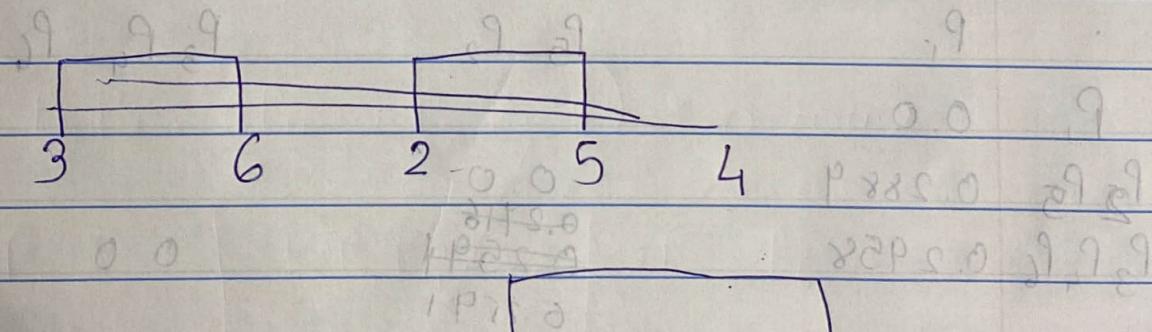
$$P_1 [P_2, P_3, P_4, P_5, P_6] \rightarrow \text{Avg}(0.2889, 0.2958) \rightarrow 0.2923$$

$$P_{avg} \leftarrow (188.0, 288.0) = pVA \leftarrow [0.9, 0.9]$$

$$P_{avg} \leftarrow (188.0, 1108.0) = pVA \leftarrow [0.9, 0.9]$$

$$P_{avg} \leftarrow (588.0, 888.0) = pVA \leftarrow [0.9, 0.9]$$

$$P_{avg} \leftarrow [0.9, 0.9]$$



$$P_{avg} = (188.0, 288.0) = pVA \leftarrow [0.9, 0.9]$$

$$P_{avg} = (188.0, 1108.0) = pVA \leftarrow [0.9, 0.9]$$

$$P_{avg} = (588.0, 888.0) = pVA \leftarrow [0.9, 0.9]$$

$$P_{avg} = [0.9, 0.9]$$

MAX - Complete

| | P ₁ | P ₂ | P ₃ | P ₄ | P ₅ | P ₆ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| P ₁ | 0.0 | | | | | |
| P ₂ | 0.2357 | 0.0 | | | | |
| P ₃ | 0.2218 | 0.1483 | 0.0 | | | |
| P ₄ | 0.3688 | 0.2042 | 0.1513 | 0.0 | | |
| P ₅ | 0.3421 | 0.1388 | 0.2843 | 0.2932 | 0.0 | |
| P ₆ | 0.2347 | 0.2540 | 0.1100 | 0.2216 | 0.3921 | 0.0 |

Pair [P₃, P₆] \rightarrow 0.1100

| | P ₁ | P ₂ | P ₃ , P ₆ | P ₄ | P ₅ |
|---------------------------------|----------------|----------------|---------------------------------|----------------|----------------|
| P ₁ | 0.0 | | | | |
| P ₂ | 0.2357 | 0.0 | | | |
| P ₃ , P ₆ | 0.2347 | 0.2540 | 0.0 | | |
| P ₄ | 0.3688 | 0.2042 | 0.2216 | 0.0 | |
| P ₅ | 0.3421 | 0.1388 | 0.3921 | 0.2932 | 0.0 |

$$P_1[P_3, P_6] \rightarrow \text{MAX}(0.2218, 0.2347) = 0.2347$$

$$P_2[P_3, P_6] \rightarrow \text{MAX}(0.1483, 0.2540) = 0.2540$$

$$P_4[P_3, P_6] \rightarrow \text{MAX}(0.1513, 0.2216) = 0.2216$$

$$P_5[P_3, P_6] \rightarrow \text{MAX}(0.2843, 0.3921) = 0.3921$$

$P_{air}[P_2 P_5]$

$\rightarrow \text{XAM}$

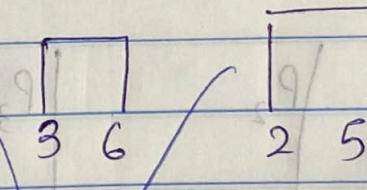
| P_1 | $P_2 P_5$ | $P_3 P_6$ | P_4 |
|-----------|-----------|-----------------------------|-----------------------------|
| P_1 | 0.0 | | |
| $P_2 P_5$ | 0.3421 | 0.0 | 0.0 |
| $P_3 P_6$ | 0.2347 | 0.3921 | 0.0 |
| P_4 | 0.3688 | 0.2042 0.2132 | 0.2216 0.2216 |

$$P_1 [P_2 P_5] \rightarrow \max(0.2347, 0.3421) = 0.3421$$

$$[(P_3 P_6) [P_2 P_5]] \rightarrow \max(0.2540, 0.3921) = 0.3921$$

$$P_4 [P_2 P_5] \rightarrow \max(0.2042, 0.1388) = \cancel{0.2042}$$

$00110 \leftarrow [0.2932]$

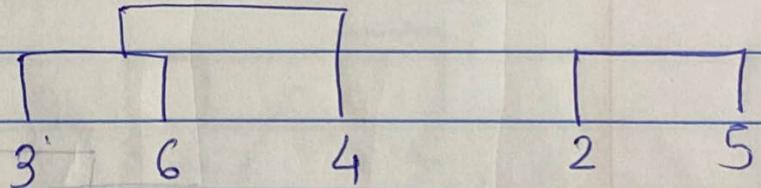


$$\text{Pair}(P_4, [P_3, P_6]) \rightarrow 0.2216$$

| | P_1 | P_2, P_5 | P_3, P_6, P_4 |
|-----------------|--------|------------|-----------------|
| P_1 | 0.0 | | |
| P_2, P_5 | 0.3421 | 0.0 | |
| P_3, P_6, P_4 | 0.3688 | 0.3921 | 0.0 |

$$P_1[P_4, P_3, P_6] \rightarrow \text{MAX}(0.3688, 0.2347) = 0.3688$$

$$[P_2, P_5][P_4, P_3, P_6] \rightarrow \text{MAX}(0.2932, 0.3921) = 0.3921$$

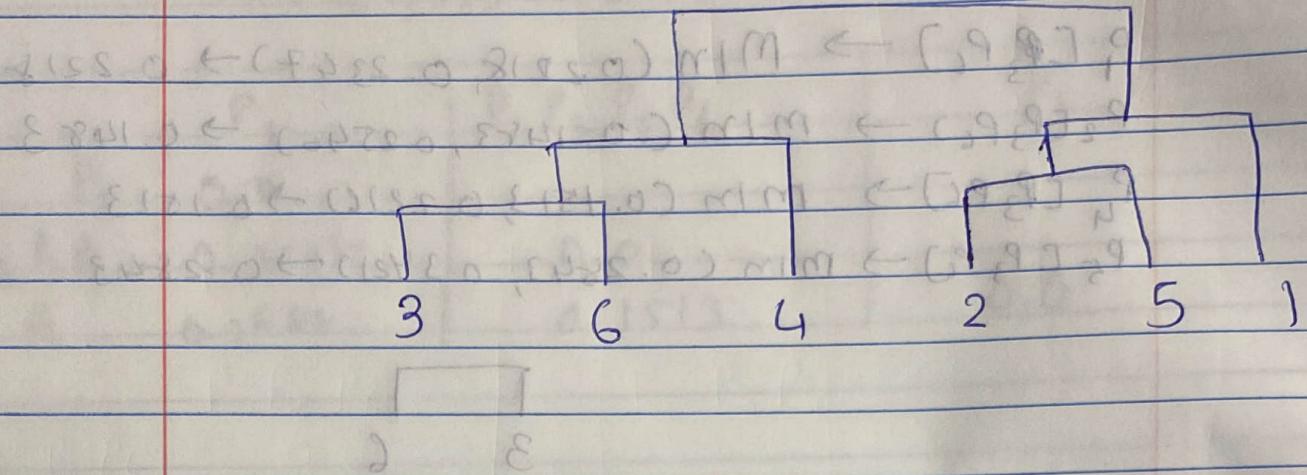


$$P_1[P_2, P_5] \rightarrow 0.3421$$

| | P_1, P_2, P_5 | P_3, P_6, P_4 |
|-----------------|-----------------|-----------------|
| P_1, P_2, P_5 | 0.0 | |
| P_3, P_6, P_4 | 0.3921 | 0.0 |

~~Q2~~

$$[P_1, P_2, P_5][P_3, P_6, P_4] \rightarrow \text{MAX}(0.3688, 0.3421) \\ = 0.3921$$



Single link

| P_1 | P_2 | P_3 | P_4 | P_5 | P_6 |
|--------------|--------|--------|--------|--------|-------|
| P_1 0.0 | | | | | |
| P_2 0.2357 | 0.0 | | | | |
| P_3 0.2218 | 0.1483 | 0.0010 | | | |
| P_4 0.3688 | 0.2042 | 0.1513 | 0.0 | | |
| P_5 0.3421 | 0.1388 | 0.2843 | 0.2932 | 0.0 | |
| P_6 0.2347 | 0.2540 | 0.1100 | 0.2216 | 0.3921 | 0.0 |

Pair (3, 6) \rightarrow 01100

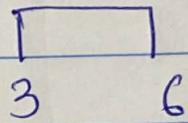
| P_1 | P_2 | P_3 | P_4 | P_5 |
|--------------|--------|--------|--------|-------|
| P_1 0.0 | | | | |
| P_2 0.2357 | 0.0 | | | |
| P_3 0.2218 | 0.1483 | 0.0 | | |
| P_4 0.3688 | 0.2042 | 0.1513 | 0.0 | |
| P_5 0.3421 | 0.1388 | 0.2843 | 0.2932 | 0.0 |

$$P_1 [P_3 P_6] \rightarrow \min(0.2218, 0.2347) \rightarrow 0.2218$$

$$P_2 [P_3 P_6] \rightarrow \min(0.1483, 0.2546) \rightarrow 0.1483$$

$$P_4 [P_3 P_6] \rightarrow \min(0.1513, 0.2216) \rightarrow 0.1513$$

$$P_5 [P_3 P_6] \rightarrow \min(0.2843, 0.3421) \rightarrow 0.2843$$



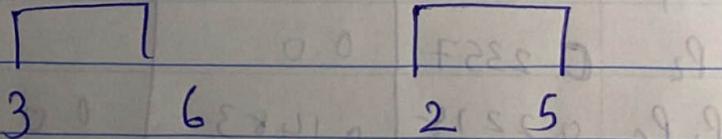
$$\textcircled{1} [P_2 P_5] \rightarrow 0.1384$$

| P_1 | $P_2 P_5$ | $P_3 P_6$ | P_4 |
|---------------------|-----------|-----------|-------|
| P_2 0.0 | | | |
| $P_2 P_5$ 0.2357 | 0.0 | 0.0 | |
| $P_3 P_6$ 0.2218 | 0.1483 | 0.0 | |
| P_4 0.3684 | 0.2042 | 0.1513 | 0.0 |

$$P_1 [P_2 P_5] \rightarrow \min(0.2357, 0.3421) \rightarrow 0.2357$$

$$[P_3 P_6] [P_2 P_5] \rightarrow \min(0.1483, 0.2843) \rightarrow 0.1483$$

$$P_4 [P_2 P_5] \rightarrow \min(0.2042, 0.2932) \rightarrow 0.2042$$

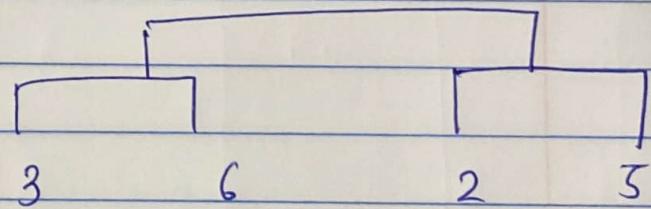


$$P_{\text{pair}} [P_3 P_6] [P_2 P_5] = 0.1483$$

| P_1 | $P_2 P_5 P_3 P_6$ | P_4 |
|-------------------|-------------------|--------|
| P_1 | 0.0 | |
| $P_2 P_5 P_3 P_6$ | 0.2218 | |
| P_4 | 0.3688 | 0.1513 |

$$P_1 [P_2 P_5 P_3 P_6] \rightarrow \min(0.2357, 0.2218) = 0.2218$$

$$P_4 [P_2 P_5 P_3 P_6] \rightarrow \min(0.2042, 0.1513) = 0.1513$$



$$P_{\text{pair}} [P_2 P_5 P_3 P_6] P_4 = 0.1513$$

| P_1 | $P_2 P_5 P_3 P_6 P_4$ |
|-----------------------|-----------------------|
| P_1 | 0.0 |
| $P_2 P_5 P_3 P_6 P_4$ | 0.2218 |

$$P_1 [P_2 P_5 P_3 P_6 P_4] = \min(0.2218, 0.3688)$$

$$= 0.2218$$

