Sudoku Solver - Optimizer Math

1 Inputs & Parameters

- R = Set of all rows of a Sudoku grid = $\{1, 2, 3, \ldots, 9\}$
- C = Set of all columns a Sudoku grid = $\{1, 2, 3, \ldots, 9\}$
- N = Set of all possible numbers in a Sudoku cell = $\{1, 2, 3, \ldots, 9\}$

2 Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{if cell } (i,j) \text{ of the grid contains number } k;\ i \in R,\ j \in C,\ k \in N \\ 0 & \text{otherwise} \end{cases}$$

3 Objective

Min C; where C is any constant (since the purpose of the solver is to find any feasible solution)

4 Subject to Constraints

- **Cell constraints**: A cell can contain only a single number.

$$\sum_{k \in N} x_{ijk} = 1;\ \forall i \in R,\ j \in C$$

- **Column constraints**: All cells in a column must contain each number exactly once.

$$\sum_{i \in R} x_{ijk} = 1;\ \forall j \in C,\ k \in N$$

- **Row constraints**: All cells in a row must contain each number exactly once.

$$\sum_{i \in C} x_{ijk} = 1;\ \forall i \in R,\ k \in N$$

- **Sub-grid constraints**: All cells in a non-overlapping 3 by 3 grid from the top-left contains each number exactly once.

$$\sum_{i'=i}^{i+2} \sum_{j'=j}^{j+2} x_{ijk} = 1;\ \forall i \in \{1, 4, 7\},\ j \in \{1, 4, 7\},\ k \in N$$

- **Input cell constraints**: These constraints correspond to cells that already have a value in the unsolved Sudoku grid.

$$x_{i,j,k=s_{ij}} = 1;\ \forall i, j \text{ which have a pre-set number } s_{ij}$$