Sudoku Solver - Optimizer Math

1 Inputs & Parameters

- $R = \text{Set of all rows of a Sudoku grid} = \{1, 2, 3, \dots, 9\}$
- $C = \text{Set of all columns a Sudoku grid} = \{1, 2, 3, \dots, 9\}$
- N= Set of all possible numbers in a Sudoku cell = $\{1,2,3,\ldots,9\}$

2 Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{if cell } (i,j) \text{ of the grid contains number } k; i \in R, j \in C, k \in N \\ 0 & \text{otherwise} \end{cases}$$

3 Objective

Min C; where C is any constant (since the purpose of the solver is to find any feasible solution)

4 Subject to Constraints

• Cell constraints: A cell can contain only a single number.

$$\sum_{k \in N} x_{ijk} = 1; \ \forall i \in R, \ j \in C$$

• Column constraints: All cells in a column must contain each number exactly once.

$$\sum_{i \in R} x_{ijk} = 1; \ \forall j \in C, \ k \in N$$

• Row constraints: All cells in a row must contain each number exactly once.

$$\sum_{i \in C} x_{ijk} = 1; \ \forall i \in R, \ k \in N$$

• Sub-grid constraints: All cells in a non-overlapping 3 by 3 grid from the top-left contains each number exactly once.

$$\sum_{i'=i}^{i+2} \sum_{j'=i}^{j+2} x_{ijk} = 1; \ \forall i \in \{1,4,7\}, \ j \in \{1,4,7\}, \ k \in \mathbb{N}$$

• Input cell constraints: These constraints correspond to cells that already have a value in the unsolved Sudoku grid.

$$x_{i,j,k=s_{ij}}=1; \ \forall i,j \ \text{which have a pre-set number } s_{ij}$$