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A new computation method for the force-closure workspace of cable-driven parallel manipulators

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SUMMARY
For cable-driven parallel manipulators (CDPMs), it is known that maintaining positive cable tension is critical in constraining the moving platform. Hence, the force-closure workspace of CDPMs represents a set of poses where the cable tensions can balance arbitrary external wrench applied on the moving platform, proposed by researchers. A new computation method for the force-closure workspace of CDPMs is developed in this paper, and the new method is realized by calculating the null space of the structure matrix and solving the linear matrix inequalities. The detailed calculation procedures of the force-closure workspace for the incompletely restrained, completely restrained, and redundantly restrained CDPMs are given, respectively, and the advantages of the new method are analyzed according to the time complexity. The simulation experiments of the force-closure workspace computation are implemented on a six-degree of freedom (6-DOF) CDPM with eight cables, and then the superiority of the new method over the existing algorithm is studied.

KEYWORDS: Cable-driven parallel manipulators; Force-closure workspace; Null space of matrix; Linear matrix inequalities.

1. Introduction

In order to overcome the mechanism defects of the parallel manipulator with rigid links, cables are used to replace rigid links to design the cable-driven parallel manipulator (CDPM), and expand the application fields of parallel manipulators. The CDPM is a closed-loop mechanism including multiple kinematic chains in which the moving platform and static platform are connected by cables. The main components of the CDPM are static platform, drivers, pulleys, cables, moving platform, and punctuate hinges connecting the cables with the two platforms.[1] This special mechanism structure affords the CDPM with the following advantages over the parallel manipulator with rigid links. First, the pulley can reel out a large amount of cables, and hence the CDPM has larger workspace. Second, the quality of the cable is very light, and hence the CDPM has a higher payload-to-weight radio, and is more suitable for the high-load and high-acceleration application. At last, for the special mechanism structure, the CDPM has low manufacture cost and it is easy to reconfigure. Due to these advantages, the CDPM has been applied in practice, such as in aircraft wind tunnel test,[2] large radio telescope,[3] and so on.

The CDPM has advantages in mechanical structure, but there exists an obvious specificity as well. Since cables can only work in tension, i.e., the cable tension must satisfy the unidirectional force property that the tension must maintain positive, hence it needs considering the unidirectional force property under arbitrary external wrench when the moving platform is in different poses. Then, the relation between the poses of the moving platform and the external wrench becomes a basic problem of the CDPM. Therefore, several types of workspace, such as controllable workspace, force-closure workspace, force-feasible workspace, and dexterous workspace, have been proposed.

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Here, the force-closure workspace is a set of poses of the moving platform on which the cables can balance any external wrench by all-positive cable tensions. Thus, the force-closure workspace is only dependent on poses of the moving platform, and it will not be affected by the external wrench.

In order to study the force-closure workspace, the CDPM can be classified into three kinds for an n-degree of freedom (DOF) CDPM driven by m cables: the incompletely restrained positioning mechanisms ($m < n + 1$), completely restrained positioning mechanisms ($m = n + 1$), and redundantly restrained positioning mechanisms ($m > n + 1$).[4] Moreover, the CDPM can also be classified into planar CDPM and spatial CDPM according to the spatial structure. In general, the number of cables of an n-DOF CDPM must be equal to or larger than $n + 1$ to satisfy the force-closure condition.[5] However, Verhoeven[6] proposed that an incompletely restrained positioning mechanism could satisfy the force-closure condition if the gravity was large enough to be dealt as a special cable. At the beginning of the force-closure research, Philip *et al.*[7] compared the force-closure problem of CDPM with multiple fingers grasping a frictionless rigid-body. The grasping is often solved by convex analysis,[8] thus the convex analysis can be used to calculate the force-closure workspace as well. Gouttefarde *et al.*[9] provided a particular study of the force-closure workspace for fully constrained planar CDPMs. More researchers are interested in studying the force-closure workspace of the 6-DOF spatial CDPMs with seven or more cables. Ferraresi *et al.*[10] solved the force-closure workspace of the 6-DOF CDPM with nine cables by calculating the null space of the structure matrix. Mahir *et al.*[11] applied the Dykstra's alternating projection algorithm to check the force-closure configuration. Pham *et al.*[12] proposed a recursive algorithm in which the Gaussian elimination was adopted for projection operation, but the algorithm was time-consuming. Diao *et al.*[13] developed an efficient method for the force-closure workspace of the CDPM with seven cables, in which only an inverse matrix needs to be calculated. Then, Diao *et al.*[14] extended their method to the case of more than seven cables, but the method needs to detect C_m^5 combinations for 6-DOF CDPMs. Furthermore, Lim *et al.*[15] indicated that the CDPM with more than seven cables could be transformed into the case of seven cables, and then an improved method needed to check C_m^6 combinations was proposed.

In this paper, a new method only needing to detect C_m^7 combinations is proposed for the force-closure workspace. When $m \leq 2n + 1$, the number of the combinations in the new method is smaller than the algorithm in ref. [15]. In general, there are not too many cables in actual application, thus the condition $m \leq 2n + 1$ can be satisfied. Therefore, the new method can reduce the calculating time of the force-closure workspace, and it is applicable to the incompletely restrained, completely restrained, and redundantly restrained CDPMs. The new method is designed by finding a special external wrench applied on the moving platform. And the current pose belongs to the force-closure workspace if the external wrench can be balanced by positive cable tensions. Specifically, the new method is realized by calculating the null space of the structure matrix and solving the linear matrix inequalities. Furthermore, the computational efficiency of the new method can be improved by detecting the signal of the inner-product of the cable tension vector and the special external wrench. Finally, the proposed method is implemented on a 6-DOF CDPM with eight cables, and the results are compared with the algorithm in ref. [15]. The simulation results indicate that the computational efficiency of the force-closure workspace is improved obviously by using the new method.

The paper is organized as follows. In Section 2, the static modeling of an n-DOF CDPM is established. In Section 3, the new calculation method of force-closure workspace is developed for CDPMs, and the detailed solution procedures are given. In Section 4, simulation experiments are carried out on a 6-DOF CDPM driven by eight cables, and the experimental results are compared with an existing algorithm in ref. [15]. Finally, several remarks are concluded.

2. Static Modeling

The structural diagram of an n-DOF CDPM is shown in Fig. 1, in which the coordinate frame $Oxyz$ is the base frame, and the coordinate frame $Pxyz$ is connected to the moving platform. The cable quality is very light, thus the gravity of the cables can be neglected. Moreover, the cables can be seen as straight lines in the static balance.

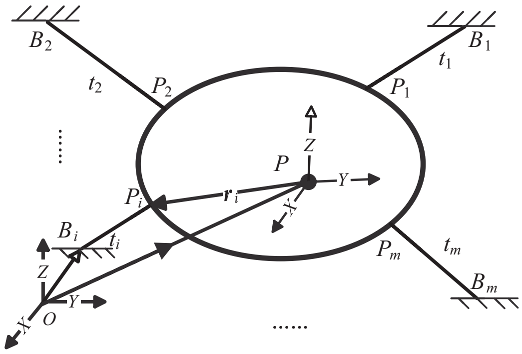

Fig. 1. Structural diagram of an n-DOF CDPM.

Based on the force equilibrium of the moving platform, the static equation can be described as

$$\begin{cases} \sum_{i=1}^{m} \mathbf{t}_i + \mathbf{f}_P = 0, \\ \sum_{i=1}^{m} \mathbf{r}_i \times \mathbf{t}_i + \mathbf{m}_P = 0, \end{cases} \tag{1}$$

where $\mathbf{t}_i$ is the cable tension, $\overrightarrow{P_iB_i}$ is the direction of $\mathbf{t}_i$, $\mathbf{f}_P$ and $\mathbf{m}_P$ are the external force and moment on the moving platform, and $\mathbf{r}_i = \overrightarrow{PP_i}$ is the force arm vector of the cable tension. These vectors are expressed in base frame $Oxyz$. Let $\mathbf{U}_i$ represent the unit vector of $\mathbf{t}_i$, here t_i represents the magnitude of the cable tension. Then, the matrix form of Eq. (1) can be written as

$$\mathbf{A} \cdot \mathbf{T} = \mathbf{W}, \tag{2}$$

where $\mathbf{T} = [\, t_1 \quad t_2 \quad \cdots \quad t_m \,]^T \in R^m$ is the cable tension vector, $\mathbf{W} = -\begin{bmatrix} \mathbf{f}_P \\ \mathbf{m}_P \end{bmatrix} \in R^n$ is the external wrench, $\mathbf{A} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 & \cdots & \mathbf{U}_m \\ \mathbf{r}_1 \times \mathbf{U}_1 & \mathbf{r}_2 \times \mathbf{U}_2 & \cdots & \mathbf{r}_m \times \mathbf{U}_m \end{bmatrix}$ is the structure matrix. Moreover,$\mathbf{U}_i$ can be formulated by $\frac{\overrightarrow{OB_i} - \overrightarrow{OP} - \overrightarrow{PP_i}}{|\overrightarrow{OB_i} - \overrightarrow{OP} - \overrightarrow{PP_i}|} = \frac{\mathbf{b}_i - \mathbf{p} - \mathbf{R} \cdot \mathbf{r}_i^P}{|\mathbf{b}_i - \mathbf{p} - \mathbf{R} \cdot \mathbf{r}_i^P|}$, here $\mathbf{b}_i = \overrightarrow{OB_i}$, $\mathbf{p} = \overrightarrow{OP}$, the superscript P denotes the vectors in coordinate frame P, $\mathbf{R}$ represents the orientation of the coordinate frame P with respect to the coordinate frame O, and $\mathbf{R}$ can be formulated as

$$\mathbf{R} = \begin{bmatrix} \cos\alpha \cdot \cos\beta & -\sin\alpha \cdot \cos\gamma + \cos\alpha \cdot \sin\beta \cdot \sin\gamma & \sin\alpha \cdot \sin\gamma + \cos\alpha \cdot \sin\beta \cdot \cos\gamma \\ \sin\alpha \cdot \cos\beta & \cos\alpha \cdot \cos\gamma + \sin\alpha \cdot \sin\beta \cdot \sin\gamma & -\cos\alpha \cdot \sin\gamma + \sin\alpha \cdot \sin\beta \cdot \cos\gamma \\ -\sin\beta & \cos\beta \cdot \sin\gamma & \cos\beta \cdot \cos\gamma \end{bmatrix},$$

where α, β, and γ are the *Z–Y–X* Euler angles.

3. Force-Closure Workspace Analysis

Force-closure condition is an important aspect of workspace analysis for CDPMs. It is a local measure for evaluating a particular pose of the moving platform, where the cable tension can always maintain positive tension for arbitrary wrench. The mathematical description of the force-closure condition can be written as

$$\forall \mathbf{W} \in R^n, \exists \mathbf{T} > 0 : \mathbf{A} \cdot \mathbf{T} = \mathbf{W}. \tag{3}$$

As known from the definition Eq. (3), the cable tension $\mathbf{T}$ and the external wrench $\mathbf{W}$ can be infinite. The force-closure condition makes the force-closure workspace only determined by the mechanical structure of CDPMs, thus it is a basic problem in the theory analysis of the workspace. Moreover, due to the force-closure condition, the structure matrix $\mathbf{A}$ must be full rank. Otherwise, the moving platform is in a singular pose,[6] and this pose is not in the force-closure workspace. Furthermore, if the DOF is larger than the number of cables ($m < n$), the corresponding pose of the moving platform cannot satisfy the force-closure condition.

In order to design calculation algorithm of force-closure workspace for CDPMs, three force-closure conditions equivalent of Eq. (3) are given as follows:[16]

(a) The columns of $\mathbf{A}$ positively span R^n.
(b) Let structure matrix $\mathbf{A} = [\,\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_m\,]$, here $\mathbf{a}_i = [\,{}^{\mathbf{U}_i}_{\mathbf{r}_i \times \mathbf{U}_i}\,]$, $i = 1, 2, \ldots, m$, and the convex hull of $\{\mathbf{a}_i\}$ contains the neighborhood of the origin.
(c) There does not exist a vector $\mathbf{V} \in R^n$,$\mathbf{V} \neq 0$, such that $\mathbf{V} \cdot \mathbf{a}_i \geq 0$ for $i = 1, 2, \ldots, m$, where $\mathbf{V} \cdot \mathbf{a}_i$ is the inner-product of $\mathbf{V}$and $\mathbf{a}_i$.

3.1. The case of completely restrained and redundantly restrained CDPMs

According to the force-closure condition (a), one can get the following conclusion: If column vectors of $\mathbf{A}$ can positively span R^n, then $n + 1$ vectors positively spanning R^n can be found. Moreover, these $n + 1$ vectors can be obtained by the following method.[15] First, select n linearly independent columns from the structure matrix $\mathbf{A}$, and denote them as $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \ldots, \hat{\mathbf{a}}_n$. Then, let vector $\mathbf{a}_t = -(\hat{\mathbf{a}}_1 + \hat{\mathbf{a}}_2 + \cdots + \hat{\mathbf{a}}_n)$. It is not difficult to know that $n + 1$ vectors $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \ldots, \hat{\mathbf{a}}_n, \mathbf{a}_t$ can positively span R^n.[13] As $\mathbf{a}_t \in R^n$ and the column vectors of $\mathbf{A}$ can positively span R^n, $\sum_{i=1}^{m} k_i \mathbf{a}_i = \mathbf{a}_t$ is satisfied, where $k_i \geq 0$ and $\sum_{i=1}^{m} k_i \neq 0$. Thus, Theorem 1 can be obtained.

Theorem 1. Let $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \ldots, \hat{\mathbf{a}}_n$ be n linearly independent vectors from column vectors of matrix$\mathbf{A} = [\,\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_m\,]$, and $\mathbf{a}_t = -(\hat{\mathbf{a}}_1 + \hat{\mathbf{a}}_2 + \cdots + \hat{\mathbf{a}}_n)$. If the pose belongs to the force-closure workspace, then

$$\mathbf{A} \cdot \mathbf{T} = \mathbf{a}_t (\mathbf{T} > 0). \tag{4}$$

The physical meaning of Theorem 1 can be explained as follows. If cables can maintain positive tension for the external wrench -$\mathbf{a}_t$ applied on the moving platform, then the cable tension can always keep positive for arbitrary external wrench. Furthermore, there are C_m^{n+1} combinations of selecting $n + 1$ vectors from $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_m$, and it is easy to know that the space positively spanned by $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_m$ is the union of the C_m^{n+1} spaces spanned by n+1 vectors. Thus, there must exist a combination of n+1 vectors positively expressing $\mathbf{a}_t$, and this results the Theorem 2.

Theorem 2. Let $\mathbf{a}_1^i, \mathbf{a}_2^i, \ldots, \mathbf{a}_{n+1}^i$ be $n + 1$ vectors from column vectors of matrix $\mathbf{A} = [\,\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_m\,]$, and $\mathbf{A}^i = [\,\mathbf{a}_1^i \quad \mathbf{a}_2^i \quad \cdots \quad \mathbf{a}_{n+1}^i\,]$, where $i = 1, 2, \ldots, C_m^{n+1}$. If $\mathbf{A}$ satisfies the force-closure condition, then there must exist one of the C_m^{n+1} combinations satisfying

$$\mathbf{A}^j \cdot \boldsymbol{\beta} = \mathbf{a}_t, \tag{5}$$

where $\boldsymbol{\beta} = [\,\beta_1 \quad \beta_2 \quad \cdots \quad \beta_{n+1}\,]^T \geq 0$, $\mathbf{a}_t = -(\hat{\mathbf{a}}_1 + \hat{\mathbf{a}}_2 + \cdots + \hat{\mathbf{a}}_n)$, and $\mathbf{A}^j$ is full rank, here$j \in \{1, 2, \ldots, C_m^{n+1}\}$.

Sufficiency: Since $\mathbf{A}^j \cdot \boldsymbol{\beta} = \mathbf{a}_t$, then Eq. (4) can be satisfied. According to Theorem 1, one knows the column vectors of $\mathbf{A}$ can positively span R^n, thus the sufficiency of Theorem 2 is proven.

Necessity: Let $S = \bigcup_{q=1,2,\ldots,Q} S_q$, where S_q is positively spanned by the vectors of the qth combination, then $\mathbf{A}^q = [\,\mathbf{a}_1^q \quad \mathbf{a}_2^q \quad \cdots \quad \mathbf{a}_{n+1}^q\,]$ is full rank for $q = 1, 2, \ldots, Q$, where Q is the total number of combinations. If the rank of $\mathbf{A}^i$ is less than n, then the space positively spanned by the columns of $\mathbf{A}^i$ will lose at least one dimension, and it belongs to a subspace $S_{q'}$where $q' \in \{1, 2, \ldots, Q\}$. Therefore, space S is equal to the space positively spanned by the column vectors of $\mathbf{A}$. Theorem 2 is proven.

The solution of Eq. (5) can be written as $\varphi = \mathbf{A}^{j+} \cdot \mathbf{a}_t + \eta \cdot \mathbf{kerA}^j$, where $\eta \in R$, $\mathbf{A}^{j+}$ is the Moore–Penrose generalized inverse of $\mathbf{A}^j$, and $\mathbf{kerA}^j$is the null space of $\mathbf{A}^j$. It is apparent that both $\mathbf{A}^{j+} \cdot \mathbf{a}_t$ and $\mathbf{kerA}^j$ are $n + 1$-dimensional vectors. Let $\mathbf{A}^{j+} \cdot \mathbf{a}_t = [\,\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_{n+1}\,]^T$ and $\mathbf{kerA}^j = [\,k_1 \quad k_2 \quad \cdots \quad k_{n+1}\,]^T$, then $\varphi = [\,\lambda_1 + \eta \cdot k_1 \quad \lambda_2 + \eta \cdot k_2 \quad \cdots \quad \lambda_{n+1} + \eta \cdot k_{n+1}\,]^T$. Thus, $\varphi \geq 0$ can be rewritten as

$$[\,\lambda_1 + \eta \cdot k_1 \quad \lambda_2 + \eta \cdot k_2 \quad \cdots \quad \lambda_{n+1} + \eta \cdot k_{n+1}\,]^T \geq 0. \tag{6}$$

According to Eq. (6), the force-closure condition can be satisfied if there exists solution of η. Next, one can solve the inequalities $\lambda_i + \eta \cdot k_i \geq 0$, $i = 1, 2, \ldots, n+1$ as follows: if $k_i = 0$, then $\lambda_i \geq 0$; if $k_i > 0$, then $\eta \geq -\lambda_i / k_i$; if $k_i < 0$, then $\eta \leq -\lambda_i / k_i$. Moreover, if there exists φ, then $\max\limits_{p=1,2,\cdots,k^+}(-\lambda_p/k_p) \leq \min\limits_{q=1,2,\cdots,k^-}(-\lambda_q/k_q)$, here k^+ and k^- are the number of the inequalities when $k_i > 0$ and $k_i < 0$, respectively.

For the force-closure analysis method in ref. [15], C_m^n combinations of vectors need to be detected, but the corresponding number is C_m^{n+1} in our new method. Since $C_m^k = m(m-1)\cdots(m-k+1)/k!$, C_m^k starts to decrease when $k > (m+1)/2$. So $C_m^{n+1} \leq C_m^n$ when $m \leq 2n+1$. Thus, the number of combinations can be decreased by using the new method when $m \leq 2n+1$ which is usually satisfied in actual application. For instance, when $n = 6, m = 10$, one knows $C_{10}^7 < C_{10}^6$. The algorithm in ref. [15] indicated that the CDPM with more than seven cables could be transformed into the case of seven cables, and then an improved method needing to check C_m^6 combinations. In our paper, the new method only needs to detect C_m^7 combinations. Therefore, when $m \leq 2n+1$, the number of the combinations in our new method is smaller than the algorithm in ref. [15]. However, if the pose does not belong to the force-closure workspace, the new method needs to check C_m^{n+1} combinations and the computational efficiency will be decreased. To solve this problem, Theorem 3 is proposed.

Theorem 3. If $\mathbf{A}^j = [\mathbf{a}_1^j \ \ \mathbf{a}_2^j \ \ \cdots \ \ \mathbf{a}_{n+1}^j]$ satisfies Eq. (5), then the signal of the inner-product of $\mathbf{a}_i^j$ and $\mathbf{a}_t$ cannot be all negative, $i = 1, 2, \ldots, n+1$.

Firstly, rewrite Eq. (5) as the form$\sum_{i=1}^{n+1} \beta_i \mathbf{a}_i^j = \mathbf{a}_t$, and multiply both sides by $\mathbf{a}_t$, one gets $(\sum_{i=1}^{n+1} \beta_i \mathbf{a}_i^j) \cdot \mathbf{a}_t = \mathbf{a}_t \cdot \mathbf{a}_t$. Then, one can further have $\sum_{i=1}^{n+1} \beta_i \gamma_i = c$, where $\gamma_i = \mathbf{a}_i^j \cdot \mathbf{a}_t$ and $c = \mathbf{a}_t \cdot \mathbf{a}_t > 0$. If $\gamma_i < 0$ for $i = 1, 2, \ldots, n+1$, then $c < 0$ which is a contradiction. Theorem 3 is proven.

Based on Theorem 3, the computational time of the new method can be reduced by detecting the signal of $\mathbf{a}_i^j \cdot \mathbf{a}_t$. For example, if $\mathbf{a}_i^j \cdot \mathbf{a}_t$ is negative for $i = 1, 2, \ldots, n+1$, $C_m^{n+1} = m!/(n+1)!(m-n)!$ combinations can be reduced. If there are $m-1$ vectors $\mathbf{a}_i^j$ that yield negative inner-product of $\mathbf{a}_i^j \cdot \mathbf{a}_t$, then $C_{m-1}^{n+1} = (m-1)!/(n+1)!(m-n-1)!$ combinations can be decreased. Moreover, the more positive signals of $\mathbf{a}_i^j \cdot \mathbf{a}_t$, the higher probability of $\mathbf{A}^i$ satisfying Eq. (5). Therefore, one should check the signal of $\mathbf{a}_i^j \cdot \mathbf{a}_t$ firstly. Select k vectors from the group where $\mathbf{a}_i^j \cdot \mathbf{a}_t > 0$ and $n+1-k$ vectors from the group where $\mathbf{a}_i^j \cdot \mathbf{a}_t \leq 0$ to constitute $\mathbf{A}^i$, and k should not be less than one. Then, determine whether $\mathbf{A}^i$ meets Eq. (5) in the descending order of k.

According to the above analysis, the flowchart of the new method for the force-closure workspace is shown in Fig. 2, in which the main steps can be summarized as follows:

(1) Input the pose variables of the moving platform for CDPMs, and then formulate the structure matrix $\mathbf{A}$.
(2) Judge whether $\mathbf{A}$ is full rank. If it is not, then the pose does not belong to the force-closure workspace, and return to step 1.
(3) Find n linearly independent vectors from the column vectors of $\mathbf{A}$, and formulate a special external wrench $\mathbf{a}_t$. Then, check the signal of $\mathbf{a}_i \cdot \mathbf{a}_t$. If the inner-product signals are all the same, then the pose does not belong to the force-closure workspace, and return to step 1.
(4) Select k vectors from the group where $\mathbf{a}_i^j \cdot \mathbf{a}_t > 0$ and n+1-k vectors from the group where $\mathbf{a}_i^j \cdot \mathbf{a}_t \leq 0$ to constitute $\mathbf{A}^i$, and suppose there are p combinations. Formulate the null space $\mathbf{kerA}^i$ and the Moore-Penrose generalized inverse $\mathbf{A}^{i+}$ in descending order of k.
(5) Check the solution existence of inequalities $\mathbf{A}^{i+} \cdot \mathbf{a}_t + \eta \cdot \mathbf{kerA}^i \geq 0$. If the solution exists, then the pose belongs to the force-closure workspace. If it does not exist, one has to check the next combination. If all the combinations have been checked and there is no solution, then the pose does not belong to the force-closure workspace.

3.2. The case of incompletely restrained CDPMs

The gravity is considered as an external force in the force-closure workspace analysis in the previous Section 3.1 where the gravity is small. However, if the gravity of the moving platform is large, the gravity can be used to reduce the energy consumption and increase the workspace. In this section, we assume that there exists a special virtual cable in the direction of gravity, and the direction of

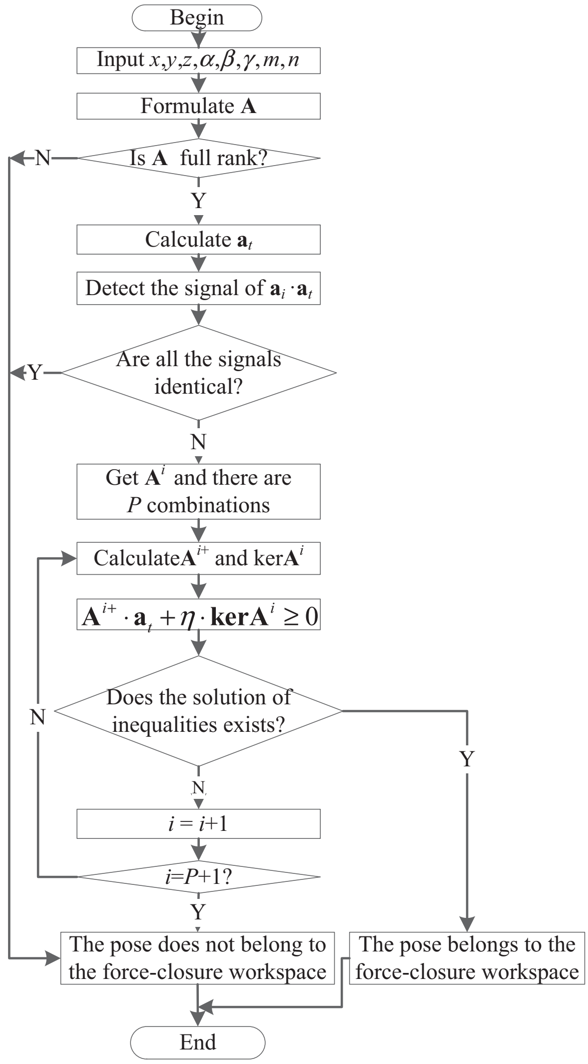

Fig. 2. The flowchart of the new method for the force-closure workspace.

the virtual cable will not change according to the movement of the moving platform.[6] Then, the new method for the force-closure workspace can be extended to the incompletely restrained CDPMs. First, the static equilibrium equation is

$$\mathbf{A} \cdot \mathbf{T} + mg \cdot \mathbf{I}_g = \mathbf{W}, \tag{7}$$

where m is the moving platform plus load total mass, g the gravity acceleration, and $\mathbf{I}_g$ the n-dimensional unit vector. Then Eq. (7) can be rewritten as

$$\begin{pmatrix} \mathbf{A} & \mathbf{I}_g \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ mg \end{pmatrix} = \mathbf{W}. \tag{8}$$

Denote $\mathbf{A}' = (\mathbf{A} \quad \mathbf{I}_g)$ and $\mathbf{T}' = (\genfrac{}{}{0pt}{}{\mathbf{T}}{mg})$, then Eq. (8) can be rewritten as

$$\mathbf{A}' \cdot \mathbf{T}' = \mathbf{W}. \tag{9}$$

Next, we take a 3-DOF CDPM with three cables as example, in which the directions of the three cables are $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. If the gravity is an external force, then $\mathbf{g} = [\,0 \quad 0 \quad -1\,]^T$, and $\mathbf{a}_1$, $\mathbf{a}_2$, and

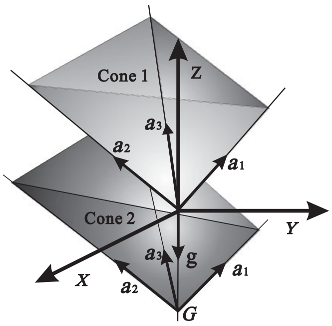

Fig. 3. Force-closure workspace analysis of a 3-DOF CDPM with three cables.

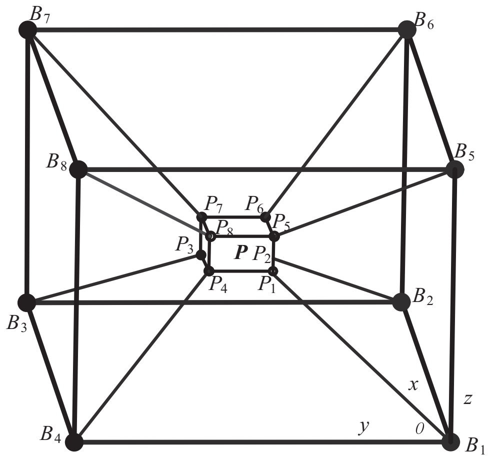

Fig. 4. Structural diagram of the 6-DOF CDPM with eight cables.

$\mathbf{a}_3$, can positively span Cone 1 in which the force-closure condition cannot be satisfied, as shown in Fig. 3. However, when the gravity is dealt as a special cable, then the Cone 1 can be shifted to point G, and a new Cone 2 is obtained. If the gravity *mg* is infinite, then Cone2 becomes the entire 3-dimensional space, i.e. $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ and $\mathbf{g}$ can positively span R^3 . As known from the force-closure condition (a), one can know that Cone2 belongs to the force-closure workspace.

4. Simulation Experiments

For the complete restrained CDPM, the computational time of the new method is shorter ($C_m^{n+1} = C_{n+1}^{n+1} = 1$). Diao *et al.*[13] developed an efficient method for the force-closure workspace of the complete restrained CDPM with seven cables. Moreover, for the incomplete restrained CDPM, the gravity can be considered as a special cable. Then the force-closure workspace of the incomplete restrained CDPM can be solved like the complete restrained CDPM. Thus the current methods can be used to analyze the two cases of incompletely restrained and completely restrained for the 6-DOF CDPM. Therefore, it is more important to study the computational time of the force-closure workspace for the redundantly restrained CDPM. Our simulation experiments are implemented on a 6-DOF CDPM driven by eight cables. The structural diagram of the 6-DOF CDPM is shown in Fig. 4, in which the drivers of the cables are, respectively, installed on the eight vertices of the cube. The size of the base is 1 m × 1 m × 1 m and the moving platform is a 0.3 m × 0.2 m × 0.1 m block. The *Z–Y–X* Euler angles, represented by α, β, and γ, respectively, are employed to define the orientation of the platform frame with respect to the base frame.

Both the proposed method and the algorithm from ref. [15] are implemented in Matlab on an Intel Core i3–2120 3.3 GHz with 4 GB RAM, to calculate the force-closure workspace. Two types of movements are considered in the simulation experiments. In the first case, we change position and fix orientation of the moving platform, and position coordinates x, y, and z vary from 0 to 1 with different

Table I. The computational time for $\alpha = 0°$, $\beta = 0°$, and $\gamma = 0°$ (s).

	Sampling intervals of x, y, and z		
Methods	0.02, 0.02, 0.2	0.01, 0.01, 0.2	0.01, 0.01, 0.02
Reference[15]	23.167	93.050	803.092
Proposed	15.607	60.436	543.176
Improved efficiency	32.63 %	37.96 %	32.36%

Table II. The computational time for $\alpha = -2°$, $\beta = 0°$, and $\gamma = 2°$ (s).

	Sampling intervals of x, y, and z		
Methods	0.02, 0.02, 0.2	0.01, 0.01, 0.2	0.01, 0.01, 0.02
Reference[15]	23.694	93.160	817.568
Proposed	17.118	66.814	579.320
Improved efficiency	27.75 %	28.28 %	29.14%

Table III. The computational time for $\alpha = 2°$, $\beta = 3°$, and $\gamma = 1°$ (s).

	Sampling intervals of x, y, and z		
Methods	0.02, 0.02, 0.2	0.01, 0.01, 0.2	0.01, 0.01, 0.02
Reference[15]	25.076	98.140	867.902
Proposed	18.094	71.933	597.648
Improved efficiency	27.84 %	26.70%	31.14 %

Table IV. The computational time for $\alpha, \beta, \gamma \in [-10° \quad 10°]$ and sampling interval is $1°$(s).

	Coordinates of the moving platform (x, y, z)		
Methods	(0.3, 0.3, 0.8)	(0.4, 0.6, 0.7)	(0.8, 0.8, 0.3)
Reference[15]	34.600	32.070	34.892
Proposed	26.902	24.738	27.841
Improved efficiency	22.25%	22.86%	20.21%

sampling intervals such as (0.02, 0.02, 0.2), (0.01, 0.01, 0.2), and (0.01, 0.01, 0.02). In the second opposite case, x, y, and z are fixed, while α, β, and γ vary from -10^0 to 10^0 with sampling interval $1°$. The simulation results of the two types of movements are shown in Tables I–IV. As we change the sampling intervals (0.02, 0.02, 0.2) into (0.01, 0.01, 0.2), the number of the positions increases to four times, so the computational time increases to nearly four times.

From the simulation results in Tables I–IV, one can find the computational efficiency of the force-closure workspace can be improved by the new method. In order to describe the force-closure workspace of the CDPM intuitively, the force-closure workspace with different Euler angles are given in Figs. 5–7. From Figs. 5–7, one can see that the force-closure workspace is continuous and closed, and the whole workspace is in the cubic structure of the CDPM. Thus the force-closure workspace relates closely to the mechanical structure. That is to say the force-closure workspace can be adopted as an important performance index for designing the structure parameters of CDPMs.

In the previous simulation experiments, the gravity is considered as an external force. From the theory analysis in Section 3.2, one can assume that there exists a special virtual cable in the direction of gravity if the gravity is large enough. Under this case, the force-closure workspace for $\alpha = 2°$, $\beta = 3°$, $\gamma = 1°$ is shown in Fig. 8. The grey area represents the workspace where gravity is not dealt as an external force, and the black area is the increased space by considering the gravity as a special cable. As shown in Fig. 8, the workspace is obviously increased in the case of gravity considered as a special cable. The simulation result in Fig. 8 is consistent with the conclusion that the incompletely

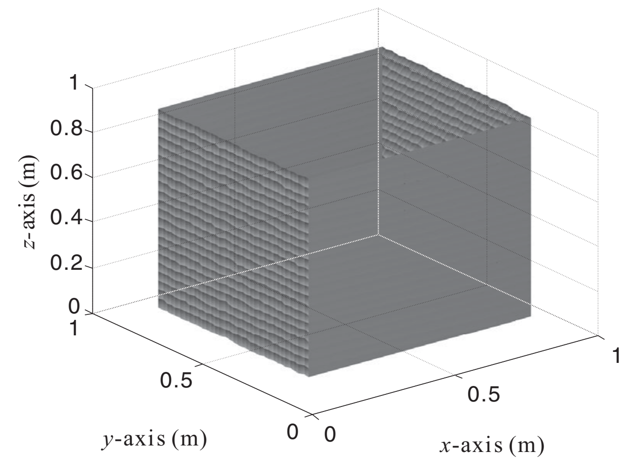

Fig. 5. The force-closure workspace for $\alpha = 0^\circ$, $\beta = 0^\circ$, $\gamma = 0^\circ$.

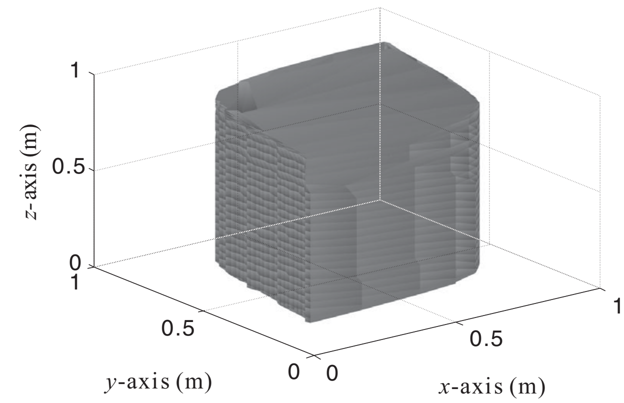

Fig. 6. The force-closure workspace for $\alpha = -2^\circ$, $\beta = 0^\circ$, $\gamma = 2^\circ$.

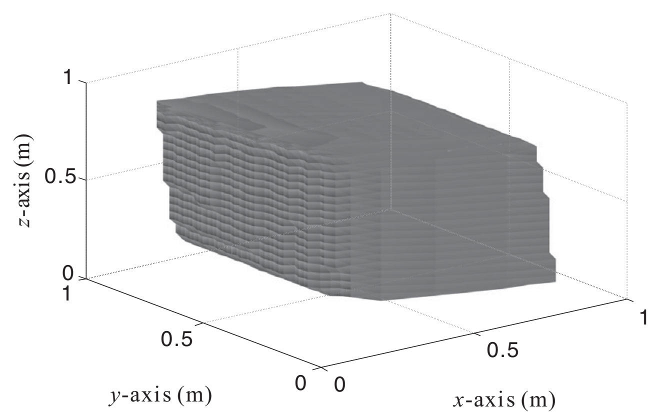

Fig. 7. The force-closure workspace for $\alpha = 2^\circ$, $\beta = 3^\circ$, $\gamma = 1^\circ$.

restrained CDPMs can also satisfy the force-closure condition if the gravity is dealt as a special cable. Although the gravity and cable tension of CDPMs cannot be infinite in actual application, one can change the workspace by counterweight in a finite set of the external force. The gravity is an important factor for the computation of the force-closure workspace. Furthermore, when the gravity is considered as a special cable, the number of cables increases. The computational time of the force-closure workspace is shown in Table V–VII. Compared with the results in Table I–III, one

Table V. The computational time when the gravity is considered as a special cable, $\alpha = 0°$, $\beta = 0°$, and $\gamma = 0°$(s).

	Sampling interval of *x*, *y*, and *z*		
Methods	0.02, 0.02, 0.2	0.01, 0.01, 0.2	0.01, 0.01, 0.02
Reference[15]	43.948	170.014	1459.20
Proposed	21.223	81.754	743.815
Improved efficiency	51.71%	51.91%	49.03%

Table VI. The computational time when the gravity is considered as a special cable, $\alpha = -2°$, $\beta = 0°$, and $\gamma = 2°$(s).

	Sampling interval of *x*, *y*, and *z*		
Methods	0.02, 0.02, 0.2	0.01, 0.01, 0.2	0.01, 0.01, 0.02
Reference[15]	47.266	169.927	1549.604
Proposed	26.565	97.896	883.379
Improved efficiency	43.80%	42.38%	42.993%

Table VII. The computational time when the gravity is considered as a special cable, $\alpha = 2°$, $\beta = 3°$, and $\gamma = 1°$(s).

	Sampling interval of *x*, *y*, and *z*		
Methods	0.02, 0.02, 0.2	0.01, 0.01, 0.2	0.01, 0.01, 0.02
Reference[15]	44.982	175.331	1644.90
Proposed	26.579	100.457	907.689
Improved efficiency	40.91%	42.70%	44.82%

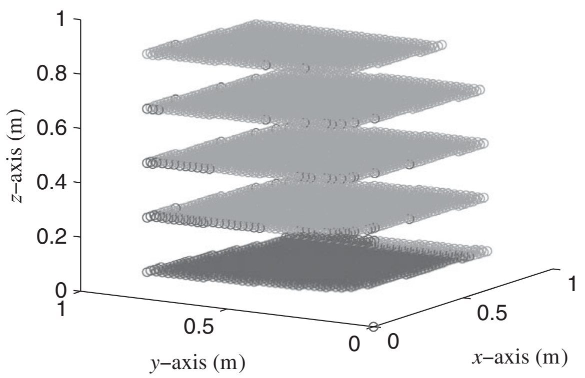

Fig. 8. The force-closure workspace when the gravity is dealt as a special cable.

can find that the improvement of the computational efficiency is more obvious when the number of cables increases, and this simulation result is consistent with the theory analysis in Section 3.1.

5. Conclusion

A new computation method for force-closure workspace of CDPMs is proposed, and this new method is applicable to the incompletely restrained, completely restrained, and redundantly restrained CDPMs. The computational efficiency of the new method can be improved obviously, compared with the existing algorithm in ref. [15]. Moreover, simulation experiments indicate that the improvement of the computational efficiency is more obvious when the cable number increases. Therefore, the new method is still suitable to the CDPMs with larger number of cables. However, physical constraints of the cable tension are omitted in the force-closure workspace, and this is not true for actual CDPMs. To the end, physical constraints of the cable tension should be considered in the future work, and this problem is the determining of the force-feasible workspace.

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