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Full paper

Workspace Determination of General 6-d.o.f. Cable Manipulators

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Abstract

This paper addresses workspace determination of general 6-d.o.f. cable-driven parallel manipulators with more than seven cables. The workspace under study is called force-closure workspace, which is defined as the set of end-effector poses satisfying the force-closure condition. Having force-closure in a specific end-effector pose means that any external wrench applied to the end-effector can be balanced through a set of non-negative cable forces under any motion condition of the end-effector. In other words, the inverse dynamics problem of the manipulator always has a feasible solution at any pose in the force-closure workspace. The workspace can be determined by the Jacobian matrix and, thus, it is consistent with the usual definition of workspace in the robotics literature. A systematic method of determining whether or not a given end-effector pose is in the workspace is proposed. Based on this method, the shape, boundary, dimensions and volume of the workspace of a 6-d.o.f., eight-cable manipulator are discussed.

Keywords

Cable manipulator, cable robot, wire robot, workspace, force-closure

1. Introduction

Cable-driven parallel manipulators, also called cable manipulators for short throughout this paper, have recently attracted much interest for special applications. This type of manipulator possesses some advantages over rigid-link parallel manipulators [1]. First, their actuators can be smaller since cables take up less space when rolled around a spool. Second, for the same overall size, they can have larger workspaces because their joints can reel out a large amount of cable. Third, all of their actuators and transmission systems can always be mounted on the fixed base; thus, they have a higher payload-to-weight ratio, which makes them particularly

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attractive for high-acceleration or high-load applications. Fourth, their special designs make them light, less expensive, modular and easy to reconfigure. Finally, and also the most important characteristic for model-based controls, they have a much simpler dynamics model than their rigid-link parallel counterparts if the inertia of the cables is ignored because the mass of the cables is usually much smaller than those of the end-effector and the payload.

Cables present many advantages, but they also involve a distinct characteristic that has to be considered seriously. That is, cables are subject to unidirectional constraints (i.e., can pull but cannot push) and, thus, they can generate tension only. Therefore, a fundamental requirement for a cable manipulator to be fully controllable is that the cable forces must be able to balance any wrenches applied to the end-effector including the inertia wrench while all cables are in tension at any working configuration and speed. This all-tension condition for the cables means that all the cable forces must be nonnegative if a compression is represented by a negative force and a tension by a non-negative force. Such a force feasibility problem is indeed a force-closure or a vector-closure problem where the referred vectors are the row vectors of the Jacobian matrix of the manipulator. Precisely, the force-closure problem referred to here is indeed a wrench-closure problem because it deals with both forces and moments in three-dimensional (3-D) space. The term 'force-closure' is still employed in this paper because it has been popularly used in the robotics community. It should be pointed out that, for a spatial cable manipulator, one or more of the cables can become slack. Cable slack may lead to a violation of the force-closure condition for a cable manipulator with only seven cables. However, it may not be a problem for a cable manipulator with more than seven cables because, in this case, the force-closure condition does allow some of the individual cables to have zero tension or to be slack, as long as seven of the cables are still in tension and provide force-closure.

The unidirectional constraint of cables makes the workspace of a cable manipulator considerably different from that of a rigid-link parallel manipulator even if they have similar kinematical architectures. The workspace of a cable manipulator depends not only on the length of cables, but also on the ability of cables to generate tension. In general, a workspace of a cable manipulator is defined as the set of all the poses (i.e., both positions and orientations) that the end-effector can physically reach while the force feasibility condition (i.e., all the cables are in tension) and perhaps some additional constraints are satisfied. Those additional constraints lead to different types of workspace defined and discussed in the literature. Verhoeven and Hiller studied the controllable workspace, which is defined as the set of all the poses at which the end-effector can statically balance a specific set of external wrenches with positive cable forces [2]. Barrette and Gosselin introduced the concept of dynamic workspace for planar cable manipulators [3]. The dynamic workspace is defined as the set of all end-effector poses and accelerations at which the end-effector can be balanced with non-negative cable forces. A dynamic workspace is a workspace defined in a mixed space of both end-effector pose and acceleration. Bosscher *et al.* proposed the concept of wrench set, which is defined as the set of all external wrenches corresponding to a set of specific limits of cable forces [4]. They applied the wrench set theory to the wrench-feasible workspace defined as the set of all the poses which can balance a specified range of external wrenches. Pusey *et al.* [5] studied the static workspace of a 6–6 cable manipulator. The static workspace is defined as the set of all the poses at which the end-effector can be statically balanced with non-negative cable forces. Williams *et al.* also studied the static workspace of a six-cable manipulator for sculpture surface metrology [6]. Pham *et al.* [7] studied the force-closure workspace for both planar and spatial cable manipulators using the 'recursive dimension reduction algorithm' derived based on convex analysis presented in Ref. [8]. The algorithm is inefficient in computation. For a 6-d.o.f. n-cable manipulator, they need to decompose the R^6 space into $n!/(n-5)!$ 1-D subspaces first, then check whether all these subspaces enclose the origin of the respective subspaces. This paper provides a computationally more efficient method for workspace determination. With this method, one only needs to form and check at most C_5^n vectors in R^6 to verify whether a given end-effector pose of a 6-d.o.f. n-cable manipulator satisfies the force-closure condition. Recently, Gouttefarde and Gosselin studied the wrench-closure workspace of planar cable manipulators [9]. The wrench-closure workspace defined by them is identical to the force-closure workspace defined in this paper. However, this paper studies the workspace of spatial cable manipulators rather than planar cable manipulators. Shen *et al.* introduced the concept of an evaluation index 'set of manipulating forces' and discussed the manipulability of cable-suspended mechanisms [10]. Utsugi and Osumi proposed a parallel wire suspension mechanism with a hierarchical structure [11]. The mechanism consists of six wire-feed actuators and four moving tables in the vertical plane of the 3-D space. The characteristics of mechanical compliance in assembling process were analyzed for an incomplete geometrical constraint type realized by this mechanism, and the relationships between wire lengths and compliance characteristics were given.

References [12–14] classified cable manipulators into fully constrained and under-constrained types, based on the extent to which the end-effector is constrained by the cables. This paper is concerned with cable manipulators of the fully constrained type. The workspace discussed in this paper is called force-closure workspace, which is defined as the set of end-effector poses satisfying the force-closure condition. In other words, the force-closure workspace is the set of end-effector poses within which the inverse dynamics problem of the manipulator always has a feasible solution. This paper will present a systematic method to verify whether or not a given end-effector pose is in the force-closure workspace for 6-d.o.f. cable manipulators with more than seven cables. With no assumptions on the design and the architecture of cable manipulators imposed in the modeling and analysis, the method is generally applicable to any 6-d.o.f. n-cable manipulator as long as its Jacobian matrix has a full rank. Since the force-closure workspace is defined without any constraint on the external wrench applied to the end-effector and the motion (velocity and acceleration) of the end-effector, it is the most desirable workspace for applications where the associated dynamic load and acceleration of the end-effector change drastically such as a cable manipulator-based contact-dynamics testbed [15]. The workspaces studied in Refs [7, 9, 16] also fall in this category. It should be pointed out that in practice cable forces are limited by the bound of cable tensions or the stall torques of actuators of the manipulator, which will, in turn, limit the allowed external wrenches. The effect of cable-force limits on the dynamics of a cable manipulator were discussed recently in Ref. [17].

The remainder of the paper is organized as follows. The modeling of cable manipulators is presented in Section 2, which is followed by the definition of the force-closure workspace in Section 3. The systematic method of determining the force-closure workspace of a general 6-d.o.f. cable manipulators is addressed in Section 4. A numerical example of determining the force-closure workspace using the systematic method is presented and discussed in Section 5. The paper is concluded in Section 6.

2. Modeling of Cable Manipulators

The kinematics and dynamics models of a general 6-d.o.f. n-cable ($n > 7$) manipulator are derived based on the architecture shown in Figs 1 and 2, respectively. The end-effector (moving platform) is assumed to be controlled by more than seven cables with their driving actuators mounted to the fixed base.

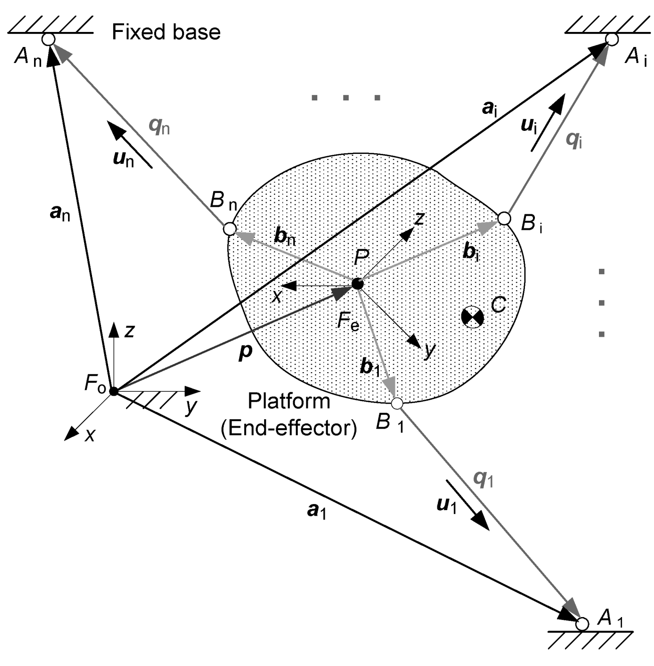

Figure 1. Kinematics notation of a general 6-d.o.f. cable manipulator.

In Fig. 1, $\mathbf{q}_i \in R^3$ $(i = 1, 2, \ldots, n)$ is the vector along the ith cable and has the same length as the cable. The length of the ith cable is represented by scalar q_i which is also considered as the manipulator's joint variable. $\mathbf{u}_i$ is the unit vector along the ith cable. A_i and B_i are the two attaching points of the ith cable on the base and the end-effector, respectively. The positions of the two attaching points are represented by vectors $\mathbf{a}_i$ and $\mathbf{b}_i$, respectively. Obviously, $\mathbf{a}_i$ is a constant vector in the base frame F_o and $\mathbf{b}_i$ is a constant vector in the end-effector frame F_e. The origin of frame F_e is fixed at a reference point P of the end-effector, which is used to define the position of the end-effector. Based on the kinematics notation, the position of the end-effector can be described as:

$$\mathbf{p} = \mathbf{a}_i - \mathbf{q}_i - \mathbf{b}_i \quad \text{for } i = 1, 2, \ldots, n, \tag{1}$$

from which one has:

$$q_i^2 = [\mathbf{a}_i - \mathbf{p} - \mathbf{b}_i]^{\mathrm{T}}[\mathbf{a}_i - \mathbf{p} - \mathbf{b}_i] \quad \text{for } i = 1, 2, \ldots, n. \tag{2}$$

Differentiating (2) with respect to time and then organizing the n resulting equations into a matrix form, one obtains:

$$\dot{\mathbf{q}} = \mathbf{J}\mathbf{t}, \tag{3}$$

where:

$$\dot{\mathbf{q}} \equiv [\,\dot{q}_1 \quad \dot{q}_2 \quad \cdots \quad \dot{q}_n\,]^{\mathrm{T}} \tag{4}$$

$$\mathbf{J} \equiv -\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_n \\ \mathbf{b}_1 \times \mathbf{u}_1 & \mathbf{b}_2 \times \mathbf{u}_2 & \cdots & \mathbf{b}_n \times \mathbf{u}_n \end{bmatrix}^{\mathrm{T}} \tag{5}$$

$$\mathbf{t} \equiv \begin{bmatrix} \dot{\mathbf{p}} \\ \boldsymbol{\omega} \end{bmatrix} \equiv [\,\dot{x} \quad \dot{y} \quad \dot{z} \quad \omega_x \quad \omega_y \quad \omega_z\,]^{\mathrm{T}}. \tag{6}$$

In the above equations, vector $\dot{\mathbf{p}}$ represents the linear velocity of point P, vector $\boldsymbol{\omega}$ is the angular velocity of the end-effector and vector $\mathbf{t}$ represents the twist vector in R^6, which consists of both linear and angular velocities of the end-effector. Moreover, $\mathbf{J}$ is the $n \times 6$ Jacobian matrix of the cable manipulator. For a cable manipulator, the inertia of the cables is usually negligible compared to those of the end-effector and the payload. Therefore, one can ignore the inertia of the cables, which will significantly simplify the dynamics model of the manipulator. In other words, each cable can be modeled as a massless string. Based on the dynamics notation defined in Fig. 2, one can derive the equations of motion of a cable manipulator with respect to point P as follows:

$$\mathbf{J}^{\mathrm{T}}\mathbf{f} = \mathbf{w}_e + \mathbf{w}_g - \mathbf{M}\dot{\mathbf{t}} - \mathbf{N}\mathbf{t}, \tag{7}$$

which can be rewritten into a compact form as:

$$\mathbf{A}\mathbf{f} = \mathbf{w}, \tag{8}$$

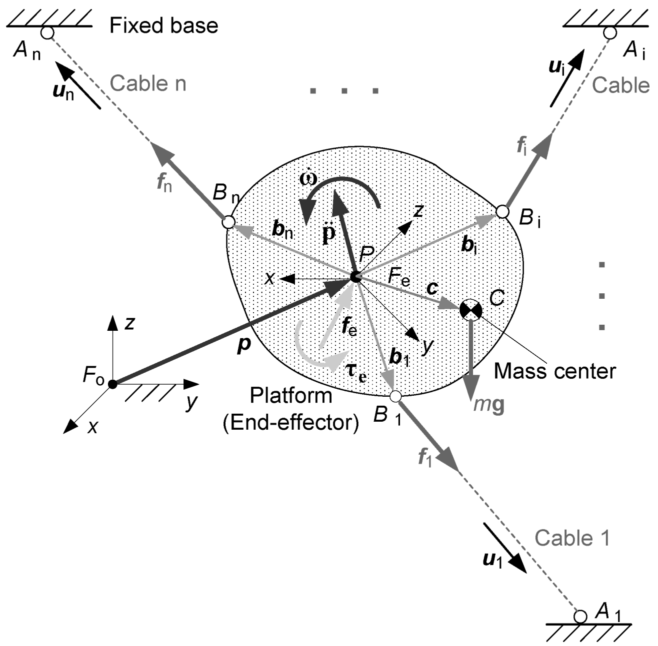

Figure 2. Dynamics notation of a general 6-d.o.f. cable manipulator.

where:

$$\mathbf{A} \equiv \mathbf{J}^{\mathrm{T}}, \quad \mathbf{f} \equiv \left[f_1 f_2 \cdots f_n \right]^{\mathrm{T}}, \quad \mathbf{w} \equiv \mathbf{w}_{\mathrm{e}} + \mathbf{w}_{\mathrm{g}} - \mathbf{M}\dot{\mathbf{t}} - \mathbf{N}\mathbf{t}$$
$$\mathbf{M} \equiv \begin{bmatrix} m\mathbf{1} & -m\mathbf{c}^{\times} \\ m\mathbf{c}^{\times} & \mathbf{I} \end{bmatrix}, \quad \mathbf{N} \equiv \begin{bmatrix} \mathbf{0} & -m(\boldsymbol{\omega} \times \mathbf{c})^{\times} \\ m(\boldsymbol{\omega} \times \mathbf{c})^{\times} & -(\mathbf{I}\boldsymbol{\omega})^{\times} \end{bmatrix} \tag{9}$$
$$\mathbf{w}_{\mathrm{e}} \equiv \begin{bmatrix} \mathbf{f}_{\mathrm{e}} \\ \boldsymbol{\tau}_{\mathrm{e}} \end{bmatrix}, \quad \mathbf{w}_{\mathrm{g}} \equiv \begin{bmatrix} m\mathbf{g} \\ \mathbf{c} \times m\mathbf{g} \end{bmatrix}.$$

In the above equations, vectors $\mathbf{w}_{\mathrm{e}} \in R^6$ and $\mathbf{w}_{\mathrm{g}} \in R^6$ are the external wrench and the gravity wrench exerted on point P, respectively, m is the mass of the end-effector including any attached payload, $\mathbf{I}$ is the 3×3 inertia tensor of the end-effector about point P, $\mathbf{g} \in R^3$ is the gravity acceleration vector, vectors $\mathbf{f}_{\mathrm{e}} \in R^3$ and $\boldsymbol{\tau}_{\mathrm{e}} \in R^3$ are external force and moment applied to point P, $\mathbf{f} \in R^n$ is a vector consisting of all individual cable forces and scalar f_i is the cable force of the ith cable. In addition, $\mathbf{0}$ and $\mathbf{1}$ are the 3×3 zero matrix and identity matrix, respectively, $\mathbf{c} \in R^3$ is the position vector of the mass center of the end-effector in frame F_{e}, and $(\cdot)^{\times}$ is a operator representing the cross product $(\cdot)\times$.

For any given motion condition (i.e., pose, velocities and accelerations of the end-effector) and external wrench, one can find a set of cable forces from (8), which is indeed the inverse dynamics problem of the manipulator. Since a cable can support a tension (i.e., a non-negative force) only, the solution of the inverse dynamics problem will not always be feasible. This force feasibility problem is indeed the basis for the determination of the force-closure workspace.

3. Workspace Definition

Before discussing the force-closure workspace, one needs to rigorously define the concept of force-closure. A cable manipulator is said to have a force-closure in a particular configuration or end-effector pose if and only if any external wrench applied to the end-effector can be sustained through a set of non-negative cable forces. In other words, the force-closure condition is satisfied if and only if the inverse dynamics problem has a feasible solution regardless of the external wrench applied to the end-effector. Such a force-closure condition can be mathematically described as:

$$\forall \mathbf{w} \in R^6, \exists \mathbf{f} \geqslant \mathbf{0}, \quad \ni \mathbf{A}\mathbf{f} = \mathbf{w}, \tag{10}$$

where $\mathbf{f} \geqslant \mathbf{0}$ means that all the components of vector $\mathbf{f}$ are greater than or equal to zero.

Definition 1. The force-closure workspace is defined as the entire set of end-effector poses $\mathbf{x} = [x, y, z, \theta_y, \theta_x, \theta_z]^{\mathrm{T}}$ satisfying the force-closure condition described in (10), where x, y and z are the Cartesian coordinates of point P, and θ_y, θ_x and θ_z are a set of Euler angles (in 213 sequence) of the end-effector with respect to the base frame F_{o}.

Since the wrench $\mathbf{w}$ defined in (9) is the resultant wrench of all the inertia and external wrenches exerted on the end-effector, it is clear that a force-closure workspace is the entire set of end-effector poses which can generate any specified dynamic motion (represented by $\mathbf{t}$ and $\dot{\mathbf{t}}$) under any given external wrench (represented by $\mathbf{w}_{\mathrm{e}}$) while all the individual cable forces (represented by f_is) are nonnegative. If constraints are imposed on the acceleration or the maximum tension allowed in each cable, the workspace becomes the dynamic workspace defined by Ref. [3] and the wrench-feasible workspace defined by Ref. [4], respectively. Other than theoretical interest, the force-closure workspace is also the desirable workspace for some particular applications in industry. For example, when a cable manipulator is used for hardware-in-the-loop contact-dynamics simulation [15], the contact on the end-effector can generate very large and unpredictable acceleration and external wrench instantaneously. Therefore, the force-closure workspace is suitable for such a particular application.

4. Workspace Determination

Although cable-driven parallel manipulators have a kinematic structure similar to rigid-link parallel manipulators, formulations and analysis methods developed for workspace determination for rigid-link parallel manipulators cannot be directly applied to cable manipulators because of the unidirectional constraint of cables. The workspace determination for cable manipulators is more difficult. Several papers about workspace determination of various cable manipulators have been published in the past few years, such as those for under-constrained cable manipulators [4–6, 14], for fully constrained planar cable manipulators [3, 9, 16, 18] and for fully constrained spatial cable manipulators [7, 19, 20]. Although some of them talked about the workspace determination of fully constrained 6-d.o.f. manipulators, only Ref. [7] touched on the topic of workspace determination for fully constrained 6-d.o.f. cable manipulators with more than seven cables. However, as discussed in Section 1, the algorithm proposed in Ref. [7] for workspace determination is inefficient. In this section, a systematic and efficient method of determining the force-closure workspace of fully constrained 6-d.o.f. cable manipulators with more than seven cables is developed based on an existing force-closure theory which was originally developed for multi-finger hands. This work connects the well-developed force-closure theory with the analysis of cable manipulators.

Equation (10) indicates that the force-closure condition is satisfied, i.e., the corresponding end-effector pose is in the force-closure workspace, if and only if the column vectors of matrix $\mathbf{A}$, denoted by $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$, can positively span R^6. In other words, any vector $\mathbf{v} \in R^6$ can be expressed as $\mathbf{v} = f_1\mathbf{a}_1 + f_2\mathbf{a}_2 + \cdots + f_n\mathbf{a}_n$, where $f_i \geqslant 0$ $(i = 1, 2, \ldots, n)$. According to Ref. [21], such a force-closure condition is equivalent to the following theorem.

Theorem 1. *Equation* (10) *has a solution if and only if the non-zero projections of all the n column vectors of matrix* $\mathbf{A}$ *in every direction in R^6 do not have the same sign. In other words, the end-effector pose corresponding to matrix* $\mathbf{A}$ *is in the force-closure workspace if and only if the nonzero dot products of vector* $\mathbf{v}$ *and the column vectors of matrix* $\mathbf{A}$ *have different signs for any non-zero vector* $\mathbf{v} \in R^6$.

Note that the wording of Theorem 1 has been modified from its original form in Ref. [21] for easier understanding of its geometric interpretation. As pointed out in Ref. [21], the proof of the theorem can be found in Ref. [8]. The geometric interpretation of the theorem is: for any hyperplane passing through the origin of R^6, at least one of the vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ points to one side of the hyperplane and another one points to the other side of the hyperplane. In other words, there should not exist one hyperplane passing through the origin of R^6 and having all the vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ except those on the hyperplane point to only one side of the hyperplane.

Theorem 1 states that, if and only if the set of non-zero dot products $\mathbf{v}^{\mathrm{T}}\mathbf{a}_i$ $(i = 1, 2, \ldots, n)$ have both positive and negative numbers for every non-zero vector $\mathbf{v}$ in R^6, then matrix $\mathbf{A}$ satisfies the force-closure condition and, thus, the corresponding end-effector pose is in the force-closure workspace. This theorem, used as is, is inconvenient for verifying whether or not a given end-effector pose is in the force-closure workspace because it requires one to check the sign condition for each and every non-zero vector in R^6. A computationally more attractive and also systematic method is to check only a number of vectors in R^6 which are formed from the column vectors of matrix $\mathbf{A}$. Such a method is introduced next.

Method 1. Let $\mathbf{n} \in R^6$ be a non-zero vector perpendicular to any set of five linearly independent column vectors of matrix $\mathbf{A}$. The necessary and sufficient condition for the end-effector pose corresponding to matrix $\mathbf{A}$ to be in the force-closure workspace is that the non-zero dot products of vector $\mathbf{n}$ and the remaining column vectors of matrix $\mathbf{A}$ (i.e., those column vectors not used to form vector $\mathbf{n}$) have different signs.

Since the $\mathbf{n}$ vector is indeed the normal vector of the hyperplane (in R^6) formed by the five linearly independent column vectors used to form vector $\mathbf{n}$, it is called a normal vector for the convenience of discussions in the rest of this section. Algorithmically, Method 1 can be implemented as described in the following procedure:

(i) Select a set of five linearly independent column vectors of matrix $\mathbf{A}$ to form a normal vector $\mathbf{n}$. This is always possible because $\mathbf{A}$ has been assumed to have a full rank. For example, if $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_5$ are the five selected column vectors, then a normal vector $\mathbf{n}$ can be determined from a generalized cross-product defined as [12]:

$$\mathbf{n} = \mathbf{a}_1 \times \mathbf{a}_2 \times \cdots \times \mathbf{a}_5. \tag{11}$$

In fact, such a normal vector $\mathbf{n}$ is a candidate for vector $\mathbf{v}$ in Theorem 1. The components of normal vector $\mathbf{n}$ can be calculated as:

$$n_i = (-1)^{i+1} \det[\mathbf{a}_1^i \mathbf{a}_2^i \cdots \mathbf{a}_5^i], \quad i = 1, 2, \ldots, 6, \tag{12}$$

where $\mathbf{a}_j^i$ is equal to $\mathbf{a}_j$ with its ith component removed.

(ii) Check the signs of the non-zero dot products of the normal vector $\mathbf{n}$ and the remaining column vectors of matrix $\mathbf{A}$ (i.e., $\mathbf{a}_6, \mathbf{a}_7, \ldots, \mathbf{a}_n$ in the example). If they have the same sign, one can conclude that the end-effector pose corresponding to matrix $\mathbf{A}$ is outside the force-closure workspace. Otherwise, go to Step (iii).

(iii) Select another set of five linearly independent column vectors of matrix $\mathbf{A}$ and repeat Steps (i) and (ii). There will be up to C_5^n (i.e., the number of 5-combination from the set of n column vectors of matrix $\mathbf{A}$) sets of five linearly independent column vectors of matrix $\mathbf{A}$ to form up to C_5^n normal vectors. If all of these C_5^n normal vectors have passed Step (ii), then the end-effector pose corresponding to matrix $\mathbf{A}$ is in the force-closure workspace.

The difference of Theorem 1 from Method 1 is that the latter requires one to form and check at most C_5^n normal vectors while the former requires one to check the sign condition for all non-zero vectors in R^6. Hence, Method 1 is algorithmically more convenient to use. References [16, 22] also mentioned this method in different forms and applied it to 3-d.o.f. four-cable and 3-d.o.f. six-cable planar manipulators, respectively. In this paper, Method 1 is applied to workspace determination for general 6-d.o.f. n-cable spatial manipulators. The equivalence between Method 1 and Theorem 1 was proven using convex analysis in Ref. [23].

To demonstrate that Method 1 is computationally more efficient than the algorithm presented in Ref. [7], let us take a 6-d.o.f. eight-cable manipulator, for example. The latter needs to decompose the R^6 space into $8!/(8-5)! = 6720$ 1-D subspaces first, then check whether all of these subspaces enclose the origin of the respective subspaces. The former requires at most $C_5^8 = 56$ checkings. In each checking, one needs to calculate a dot product of vectors for $8 - 5 = 3$ times.

It should be pointed out that, as a numerical method, Method 1 is computationally more expensive than the analytical methods such as those discussed in Refs [3, 14, 16]. However, those analytical methods cannot be readily applied to general 6-d.o.f. fully constrained cable manipulators. Although Ref. [14] had some discussion about the wrench-feasible workspace of general cable manipulators, only simple under-constrained cable manipulators such as the point-mass cable manipulator and the planar cable manipulator with 3-d.o.f. and three cables were discussed with examples in detail. References [3, 16], again, only studied planar cable manipulators instead of general 6-d.o.f. spatial cable manipulators.

5. Numerical Example

For illustration of the force-closure workspace, the foregoing discussed method of determining the force-closure workspace is applied to a 6-d.o.f., eight-cable manipulator in this section. The 6-d.o.f., eight-cable manipulator shown in Fig. 3 is used

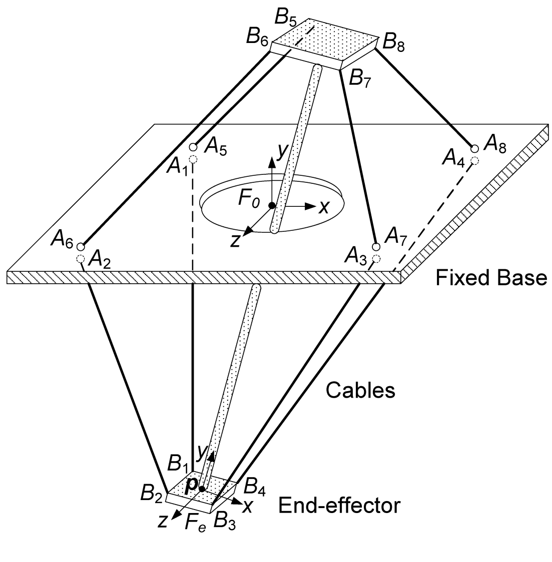

Figure 3. A fully constrained 6-d.o.f., eight-cable manipulator.

Table 1.
Dimensions (m) of the example cable manipulator

Position vector	x	y	z	Position vector	x	y	z
${}^{o}\mathbf{a}_1$	−0.5	0.00	−0.5	${}^{e}\mathbf{b}_1$	0.000	0	−0.125
${}^{o}\mathbf{a}_2$	−0.5	0.00	0.5	${}^{e}\mathbf{b}_2$	0.000	0	0.125
${}^{o}\mathbf{a}_3$	0.5	0.00	0.5	${}^{e}\mathbf{b}_3$	0.000	0	0.125
${}^{o}\mathbf{a}_4$	0.5	0.00	−0.5	${}^{e}\mathbf{b}_4$	0.000	0	−0.125
${}^{o}\mathbf{a}_5$	−0.375	0.05	0.0	${}^{e}\mathbf{b}_5$	−0.125	1	0.125
${}^{o}\mathbf{a}_6$	0.0	0.05	0.375	${}^{e}\mathbf{b}_6$	−0.125	1	0.125
${}^{o}\mathbf{a}_7$	0.375	0.05	0.0	${}^{e}\mathbf{b}_7$	0.125	1	−0.125
${}^{o}\mathbf{a}_8$	0.0	0.05	−0.375	${}^{e}\mathbf{b}_8$	0.125	1	−0.125

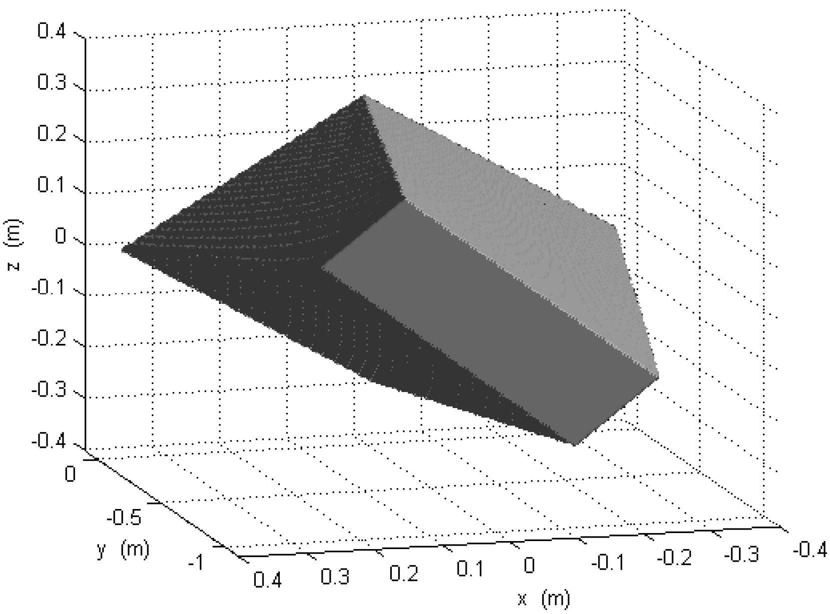

Figure 4. Translational sub-workspace when the end-effector has a fixed orientation of $(\theta_y, \theta_x, \theta_z) = (0, 0, 0)$.

as an example for force-closure workspace determination. In this example, attaching points B_1 and B_4, B_2 and B_3, B_5 and B_6 as well as B_7 and B_8 are assumed to be co-located. The end-effector (moving platform) of the manipulator is controlled by eight cables with their driving actuators mounted to the fixed base. The dimensional parameters of the manipulator are listed in Table 1, where the left-superscript ${}^{o}(\cdot)$ indicates that the vector inside $(\cdot)$ is expressed in the base frame F_o (which is also an inertial frame) and ${}^{e}(\cdot)$ indicates that it is in the end-effector frame F_e.

Since the workspace is 6-D, it cannot be entirely shown using a 3-D graphic representation. All one can present are some sub-workspaces or projections of the workspace in the visible 3-D space. Figures 4–7 show two such sub-workspaces de-

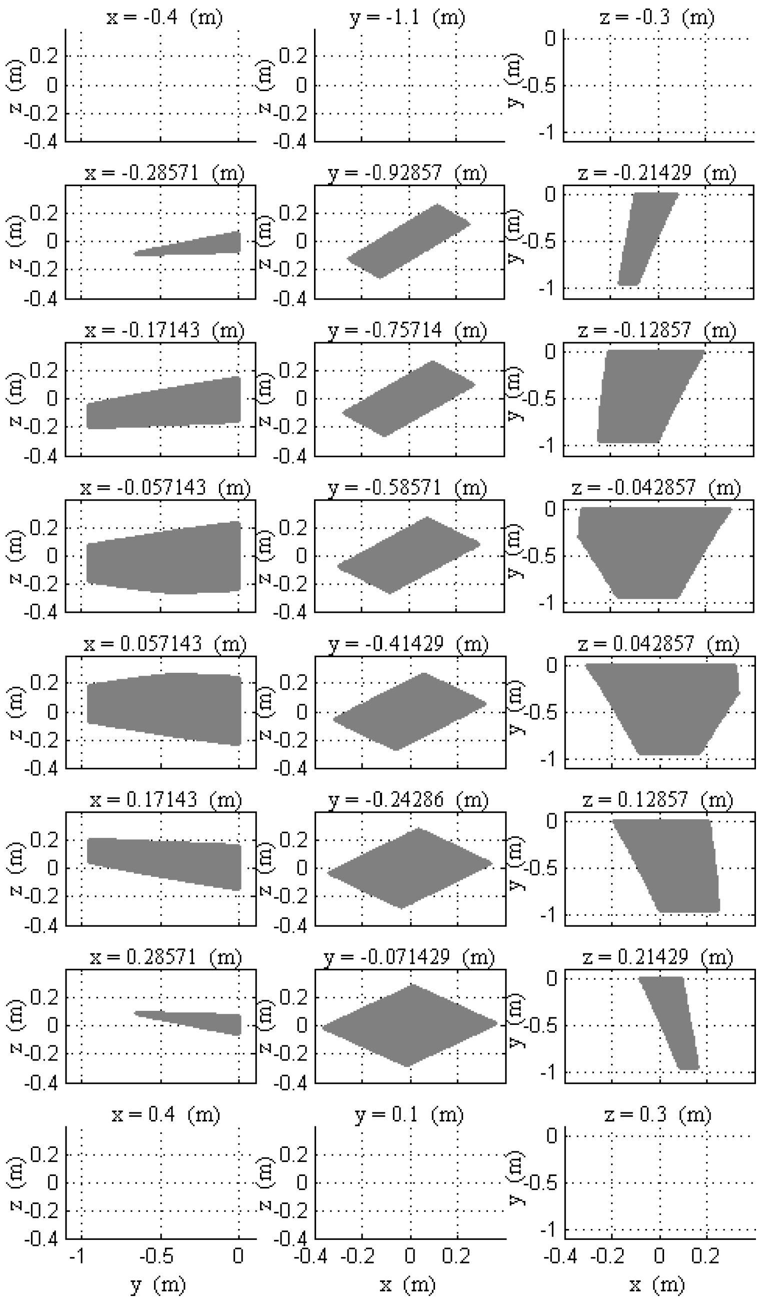

Figure 5. Cross-sections of the workspace in Fig. 4 along the x-, y- and z-axes.

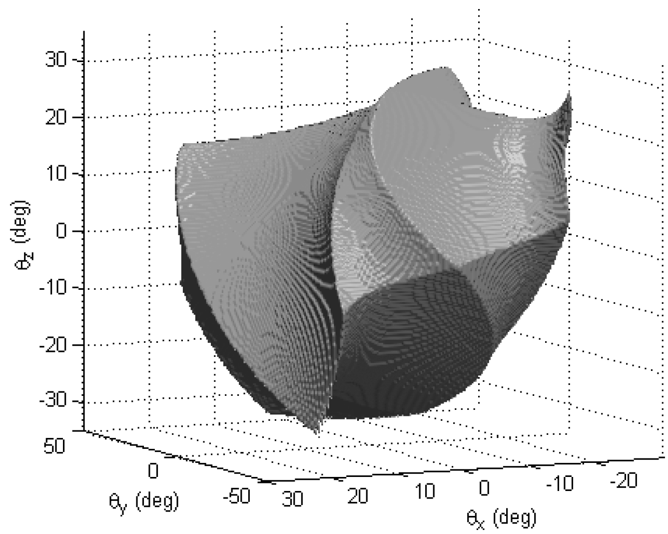

Figure 6. Rotational sub-workspace when the end-effector's position is fixed at $(x, y, z) = (0, -0.45, 0)$.

termined by examining the force-closure condition of the manipulator for each of a set of selected end-effector configurations within a 3-D box covering the workspace. Specifically, a 3-D box covering a sub-workspace is chosen first. Then the box is discretalized along each of the three axes into 200 parts, respectively; thus, one will have 8×10^6 points in the 3-D box. Each point represents a pose of the end-effector of the cable manipulator. Finally, the foregoing discussed method is used to check the force-closure status for each of these points inside the box. Such a scanning procedure is implemented using MATLAB on a desktop PC with a 3.2 GHz CPU and 2 GB memory. MATLAB code runs relatively slow and it took about 11 h to compute the entire workspace.

Figures 4 and 5 show the translational sub-workspace when the end-effector has a fixed orientation of $(\theta_y, \theta_x, \theta_z) = (0, 0, 0)$. It is a convex polyhedron. Figures 6 and 7 present the rotational sub-workspace in terms of the Euler angles in 213 sequence when the end-effector's position is fixed at $(x, y, z) = (0, -0.45, 0)$. It is bounded by concave surfaces. Figure 8 shows how the translational sub-workspace changes its volume with respect to the variation of each of the three Euler angles. Obviously, the maximum translational sub-workspace is achieved when the end-effector has an orientation of $(\theta_y, \theta_x, \theta_z) = (0, 0, 0)$, where θ_y, θ_x and θ_z are a set of Euler angles in 213 sequence. The volume of the translational sub-workspace reduces nonlinearly with the increase of the end-effector orientation. For example, the volume of the translational sub-workspace will reduce to about 50% of its maximum value if the end-effector rotates $\pm 30°$ about the x-axis and to about 65% of its maximum value if the end-effector rotates $\pm 30°$ about the y- and z-axis, respectively.

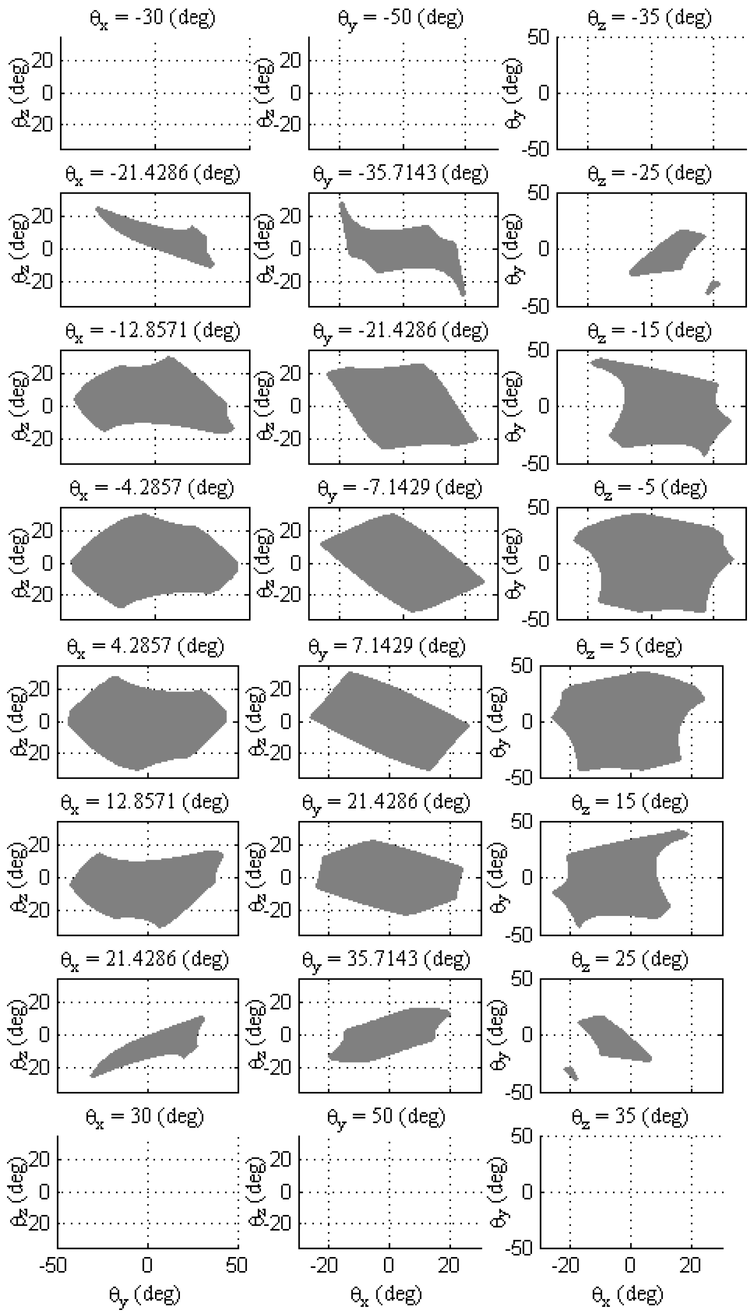

Figure 7. Cross-sections of the workspace in Fig. 6 along the θ_x, θ_y and θ_z axes.

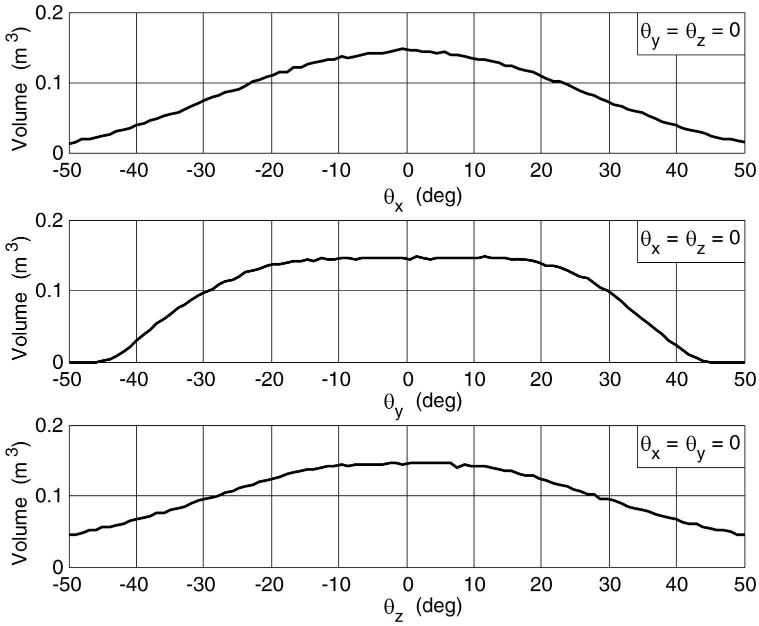

Figure 8. Volume of the translational sub-workspace *versus* end-effector orientation.

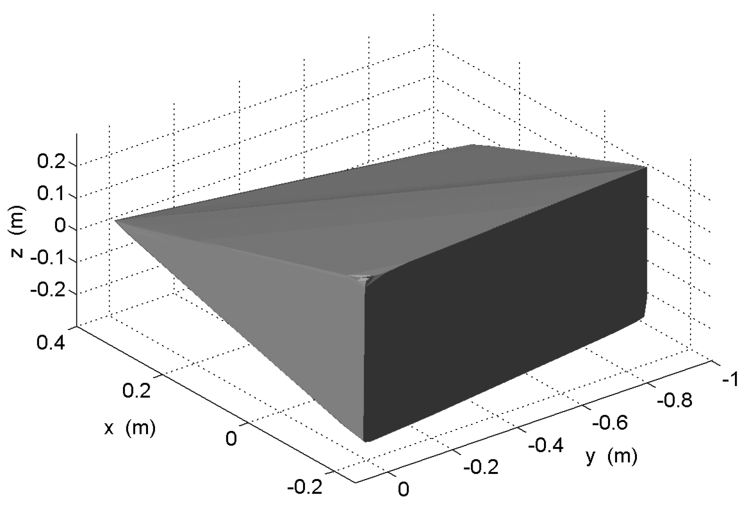

Figure 9. Translational sub-workspace of a 6-d.o.f. seven-cable manipulator [20] when the end-effector has a fixed orientation of $(\theta_y, \theta_x, \theta_z) = (0, 0, 0)$.

It is noticed that the volume of the force-closure workspace will increase with the increase of the number of cables. For example, the volume of the translational sub-workspace of the eight-cable manipulator (see Fig. 4) presented in this paper has a 23% increase compared to that of a seven-cable manipulator (see Fig. 9) of the same physical size. This is desirable for many applications of cable manipu-

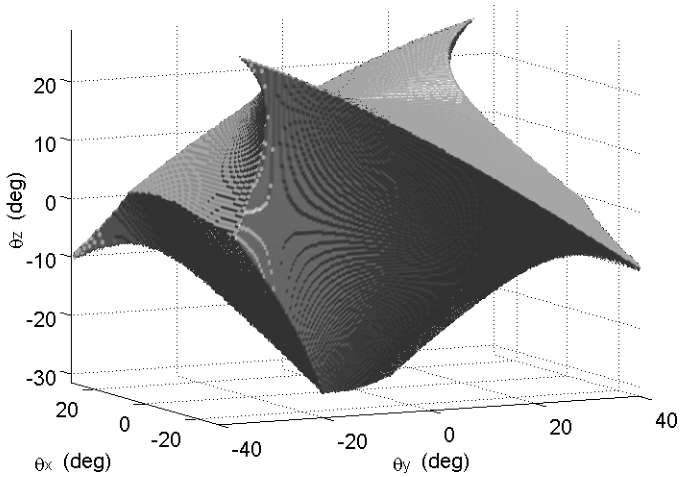

Figure 10. Rotational sub-workspace of a 6-d.o.f. seven-cable manipulator [20] when the end-effector's position is fixed at $(x, y, z) = (0, -0.45, 0)$.

lators. Of course, the probability of cable collision will increase with the increase of the number of cables, too. This problem may reduce the benefit of an increased workspace. It is also observed that the translational sub-workspace in Fig. 4 and the rotational sub-workspace in Fig. 6 of the eight-cable manipulator are more isotropic in geometry than those of the seven-cable manipulator (see Figs 9 and 10). If the translational or rotational sub-workspace is geometrically isotropic, its geometry is spherical. This is also a very desirable feature for practical applications because the end-effector motion can be more easily planned in a more isotropic workspace. It is still an open question about whether these two findings are general and what their mathematical rationales are. Further research is certainly required to reveal the quantitative relations between the characteristics of workspace and the number of cables for general n-cable manipulators.

6. Conclusions

This paper studied the force-closure workspace of a general 6-d.o.f. fully constrained cable-driven parallel manipulator with more than seven cables. The workspace is defined as the set of all the end-effector poses satisfying the force-closure condition. Having force-closure in a specific end-effector pose means that any external wrench applied to the end-effector can be balanced through a set of nonnegative cable forces under any end-effector motion condition. Since there is no limitation on the external wrench and the dynamic motion of the end-effector, such a workspace is the most desirable workspace for applications having large end-effect load variations. The workspace can be determined by the Jacobian matrix and thus, it is consistent with the usual definition of workspace in the robotics literature. An efficient and systematic method is introduced for determining whether a given end-effector configuration is in the workspace or not. Based on the proposed method, the shape, boundary, dimensions and volume of the workspace of a 6-d.o.f., eight-cable manipulator were graphically illustrated and discussed, as an application example.

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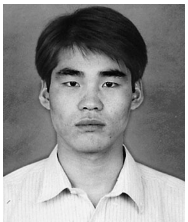

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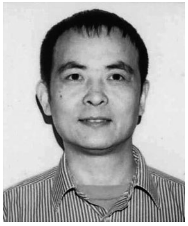

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