All quiz rules from the course syllabus are in effect for the real quiz, in addition to what follows.

If you have questions about the test, please ask *on Piazza* as a **private** post, viewable only by you and the instructors. The instructors will make an announcement via a pinned errata post if something needs to be said to the entire class. **Do not make comments on any non-private posts on Piazza during the quiz for any reason. Making such comments may subject you to a grade penalty.**

This quiz is to be individual effort. Students are permitted to use notes, electronics, and bring textbooks. The work you submit for each quiz is expected to be produced by *you, alone and solely for this assessment*. You may not reuse or repurpose anything you wrote at another time, nor may you ever use anything written by someone else during this quiz. However, despite being allowed notes and electronics, you may *not* seek out the answer to a question in any way, nor may you communicate with anyone during the exam, *for any reason*, with the exception of asking a question on Piazza *set as instructors-only for visibility*.

You will have 40 minutes for this quiz, plus an additional ten minutes to enter your answers to the answer form. Please be careful when you enter your answers, as this will very likely be graded by a computer program and not a human. Students who have timed accommodations, such as through DSC, should use the 50 minute total as a baseline and apply their accommodations from there.

Submit your answers to this Google Form:

`https://docs.google.com/forms/d/e/1FAIpQLScX8dN8PsTqh_skkUWiGZ6kuK7bjUs01CmnaX5pM1IcnL4WQw/viewform`

You may need to log in with your UCI credentials to access this. You may re-submit if you like, but your most recent submission is the one that will be graded. This means if you submit or re-submit after the end of the quiz, you will have a zero.

When 30 minutes have passed from the start of the quiz, check Piazza to see if there is a pinned thread that has clarifications to the questions. Anything on that thread by that time is something you are responsible for.

Master Theorem Questions (3 points)

Read the entire question *carefully* before working. I will provide three recurrence relations and I will ask you to use the Master Theorem to solve them, and I will ask you about your work.

Recall that the Master Theorem is as follows; we write the recurrence in the form $T(n) = aT(n/b) + f(n)$, for some $a \geq 1$, $b > 1$, and $f(n)$ is asymptotically positive.

1. If there is a small constant $\varepsilon > 0$ such that $f(n)$ is $\mathcal{O}(n^{\log_b a - \varepsilon})$, then $T(n)$ is $\Theta(n^{\log_b a})$
2. If there is a constant $k \geq 0$, such that $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$
3. If there is a small constant $\varepsilon > 0$ such that $f(n)$ is $\Omega(n^{\log_b a + \varepsilon})$, then $T(n)$ is $\Theta(f(n))$.

Consider the following three recurrence relations:

- $a(n) = 9a(n/3) + n$
- $b(n) = 9b(n/3) + n^{2.5}$
- $c(n) = 3c(n/9) + \sqrt{n}$

1. (a) Which case do you follow for $a(n)$?
   (b) If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.
2. (a) Which case do you follow for $b(n)$?
   (b) If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.
3. (a) Which case do you follow for $c(n)$?
   (b) If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.

Additional Questions (3 points)

The next two question parts deal with `QuickSort`, the relevant code of which is reproduced below for your convenience. Recall that the element at position q returned by the partition function is known as the *pivot*.

```
QuickSort(A, start, end)
  if start < end then
    q = partition(A, start, end)
    QuickSort(A, start, q − 1)
    QuickSort(A, q + 1, end)
```

We are s**o**rting a vector by QuickSort. A pivot was selected, the vector was partiti**o**ned, and the piv**o**t was placed in its correct position. Then, the algorithm was called recursively on the vect**o**r in the range start to $q - 1$. That sub-vector was partitioned, and the piv**o**t placed in its pr**o**per position. At the moment, the vector looks like this:

266	104	197	160	290	248	303	942	624	814	320	863	438	584	319	667	990

4. (a) (1 point) What element must have been the first pivot selected? *Give us the value, not the index.*

   (b) (1 point) What element must have been the second pivot selected? *Give us the value, not the index.*

5. (1 point) Consider the following vect**o**r.

   This vector is represented here with elements 1-10 in the first row, 11-20 in the second r**o**w, 21-30 in the third r**o**w, 31-40 in the fourth row, and 41-45 in the last row.

   Supp**o**se we run the deterministic selection algorithm from lecture. What is the first pivot ch**o**sen f**o**r the first partition step?

892	863	455	796	919	575	803	930	691	809
519	526	231	142	990	580	610	737	463	304
811	910	291	247	273	601	671	744	772	641
542	637	996	340	462	477	948	298	169	758
718	595	889	665	587					

Free Response Section (4 points)

This question is free response. You will need to write an answer on a piece of paper (typing a solution or using a tablet and software is okay too) and upload it to GradeScope, similar to how you do for homework. Please be sure that if you plan to produce your solution on paper, you can scan it and be prepared to upload it within the window allowed on an exam.

Supp**o**se I have a vector A of n distinct elements on which a t**o**tal **o**rder is defined. The vector also has the foll**o**wing properties:

- $A[n] < A[1]$
- $A[1 \dots j]$ is increasing, for some value j between 1 and n (exclusive). We **do not** know the value **o**f j. N**o**te that "increasing" means that $A[i] < A[i+1]$ for all pairs $i, i+1$ that fall into this subvector.
- $A[j] > A[j+1]$
- $A[j+1 \dots n]$ is increasing.

Give an algorithm to find the minimum value in this vector. You will get zero credit for this problem if your running time is $\Omega(n)$.