

```

goodwill(S, B, F):
    Initialize 2-D array X with dimension (S+1)*(B+1), indexed [0...S, 0...B].
    Assign each block with initial value of 0. // base case
    for i = 1 to F do
        for s = S down to  $S_i$  do
            for b = B down to  $b_i$  do
                // recurrence expression
                if  $X[s-S_i, b-b_i] + g_i > X[s, b]$  then
                     $X[s, b] = X[s-S_i, b-b_i] + g_i$ 

```

Each time the recurrence expression is evaluated, we are trying to know if project i will provide a higher goodwill points. And the sub-problem is finding the maximum goodwill for $S - s$ students and $B - b$ buses, with projects being the subset of F without project i , recursively.

After the first for-loop ($i = 1 \dots F$) is finished, we have a table of maximum number of goodwill points given the input. To find the actual project that we have to do, we need to backtrack from $X[S, B]$

```

R <- empty set
f <- set of integers from 1 to F
s <- S
b <- B
while  $X[s, b] > 0$ :
    for i in f do
        if  $X[s-S_i, b-b_i] == X[s, b] - g_i$  then
            s =  $s-S_i$ 
            b =  $b-b_i$ 
            f.remove(i)
            R.add(i)
return R

```

R is the set of integers that represent the projects we need to do to achieve maximum goodwill units.

The total runtime of this algorithm is $O(FBS)$.

The first part of the algorithm, filling the table with nested for-loops takes $O(FBS)$ time.

Assuming each project requires at least 1 student or 1 bus, in other words, there is no “free” projects. Then each backtracking step will decrement s and/or b by at least 1, meaning that the while loop in the backtracking part will execute at most $(B+S)$ iterations. Making the time complexity of the second part to be $O(F * (B+S))$ which is smaller than $O(FBS)$ when the value of B and S is large.