CompSci 161
Spring 2021 Lecture 11: Divide and Conquer VI: Min and Max Concurrently

Min and Max

- We have an array of n distinct numbers.
- We want to find *both* – the min and max.
- Brute force method takes $2n - 3$ comparisons.
- Find a way that uses strictly fewer.

Total: $3\left(\frac{n-1}{2}\right)$

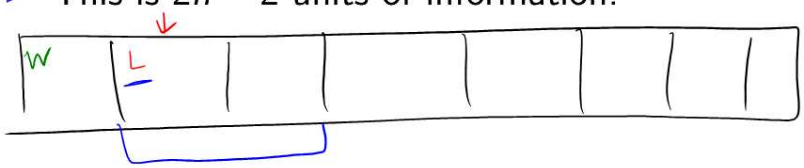
$\boxed{= \frac{3n}{2} - \frac{3}{2}}$

```
// assume n is odd
min = max = A[1]
for i=2 to n by twos      // (n-1)/2 iterations
{   if A[i] < A[i+1]:
        if A[i] < min : min = A[i]
        if A[i+1] > max : max = A[i+1]
    else // A[i+1] < A[i]
        if A[i+1] < min: min = A[i+1]
        if A[i] > max  max = A[i]
}
return min, max
```

3 comparisons per iteration

3 # Could anyone do better?

- Adversary argument:
  - All queries are made to an adversary (opponent)
  - Adversary is allowed to make up answers
  - But answers must be consistent with some input
- If we compare and find $a < b$, we say:
  - a lost the competition
  - b won the competition
- Every non-max loses at least one ← need to find $n-1$ losses
- Every non-min wins at least one
- This is $2n - 2$ units of information.



4 # What should the adversary do?

We compare a and b to gain information.

- If a, b never compared (to ANY key) before?

  User gains two units of information
- If exactly one of them compared before?

  Repeat result for repeated element

  User gains one unit regardless
- Both compared before, one won at least once?

  Let that one win again

  – ≤ 1 unit
- Both compared before, both lost before?

  ≤ 1 unit

5 ## How many comparisons can be forced?

- We need to gain $2n - 2$ units of information.
- $c_1 = \#$ comparisons that gave us one unit.
- $c_2 = \#$ comparisons that gave us two units.
- Total units of info available is at least $2n - 2$

$$\frac{3n}{2} - \frac{3}{2}$$

$$C_1 + 2C_2 \geq 2n - 2 \quad (x)$$

- At most $n/2$ comparisons give us two units

$$C_2 \leq n/2$$

$$-C_2 \geq -n/2 \quad (y)$$

$$C_1 + C_2 \geq 2n - n/2 - 2 \quad (x+y)$$

$$\underbrace{C_1 + C_2}_{\text{Total Comparisons}} \geq \frac{3}{2}n - 2$$