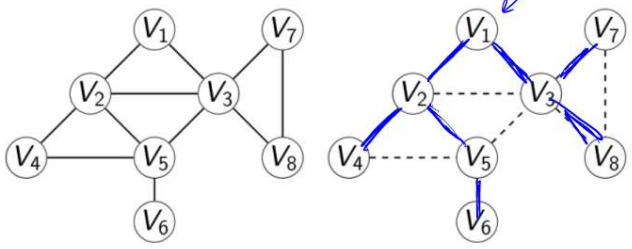
CompSci 161
Spring 2021 Lecture 5:
Graphs: BFS and Graph Coloring

Breadth-First Search



Start: layer 0

$L_0 = \{v_1\}$

$L_1 = \{v_2, v_3\}$

$L_2 = \{v_4, v_5, v_7, v_8\}$

$L_3 = \{v_6\}$

$L_4 = \emptyset$

- Did we find the shortest path(s)?
- What is the running time of BFS?

$O(n + m)$

$|V|$ $|E|$

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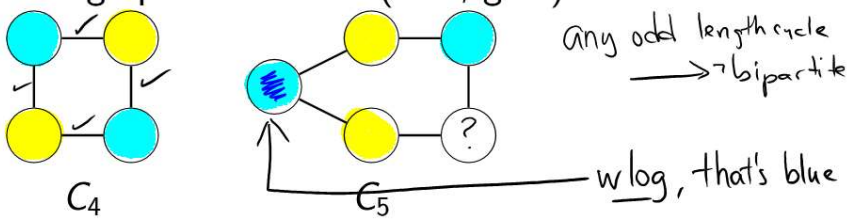
Graph Coloring

Given a (simple, undirected) graph $G = (V, E)$

- Assign each vertex a color
- Every edge must be dichromatic

Chromatic number ($\chi(G)$) minimum distinct colors

Bipartite graph: two colors (blue, gold)



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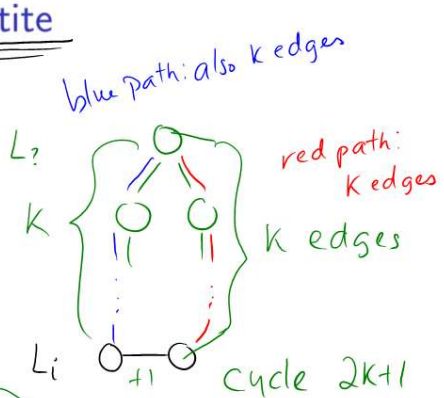
Using BFS to check bipartite

- I have a graph
- I want to know if it is bipartite
- How can I use BFS?
  - do BFS (arbitrary start)
  - even layers → blue
  - odd layers → gold
  - now check for bipartite
  - if not, we'll also have a proof



if not rejected, accept

for each $e=(u,v)$
if color(u) = color(v)
reject

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Big Extra Credit Question

- **Input**: Any simple graph G
- **Output**: Chromatic number of G
- To claim extra credit:
  - Provide an algorithm
  - Prove running time polynomial in n and m
    $\mathcal{O}(n^{100000})$ is okay.
    $\mathcal{O}(1.01^n)$ is not.
  - Prove the algorithm is correct.
- Worth an A+ in this class