Due date: Tuesday, June 1, 9:59 AM. **Please make careful note of the due date.** You will need to submit this via GradeScope. Late problem sets are not accepted beyond a very short grace period.

When you submit to GradeScope, you will need to inform the system which page(s) of your scanned PDF contains the answer. *Do this even if your submission is a single page. Failure to do so may cost you points.*

Please review the syllabus and course reference for the expectations of assignments in this class. Remember that problem sets are not online treasure hunts. You are welcome to discuss matters with classmates, but remember the Kenny Loggins rule. Remember that you may not seek help from any source where not all respondents are subject to UC Irvine's academic honesty policy.

Problem 1 should be doable by the time this is posted; problem 2 should be doable after lecture on May 21. For problem 3, you may want to wait until the end of week nine.

1. I am teaching a class that has n teaching assistants, each of whom hold office hours outside, at JavaCity. The ith TA holds office hours starting at time S_i and ending at time E_i. Each TA holds exactly one office hour (which may last any amount of time, not necessarily one hour, and different TAs may have different interval lengths, which may overlap with one another). Attendance has been low because students, being ICS majors, are afraid of the sun, so the department announces that all TAs who do not have a student visit them this week will be fired[1]. Because all of our teaching assistants are awesome[2], we want to ensure that none get fired. However, due to student fears of the sun, we want to minimize the total number of visits made to TAs.

   For purposes of this problem, assume each TA's office hour time is one continuous interval with no breaks, and that a student visiting the sundeck at time t counts as visiting all TAs whose office hours interval contains t. Also assume that student visits are "instantaneous," in the sense that the amount of time a student stays in the sun is negligible – formally, each student visits the deck at a single point in time. This also helps to avoid exposing a student to the sun unnecessarily.

   You propose the following: first, we sort all of the intervals by end time. We send a student to attend office hours at the moment immediately prior to the end of the first ending interval. We remove from our input all TAs who overlap with this time, and if the remaining set is non-empty, we repeat.

   Prove that the **greedy algorithm** in the previous statement minimizes the number of students we need to send to visit TAs. For purposes of this homework this quarter, it is sufficient to demonstrate that for any optimal solution, there is an equally optimal choice of student visits that includes your first choice.

[1] This part is not actually true and is fiction for the problem.
[2] Worth noting: this part is true for CompSci 161 this quarter.

2. We're asked to help the captain of the UCI's tennis team to arrange a series of matches against Long Beach State's team[3]. Both teams have n players; the tennis rating (a positive number, where a higher number can be interpreted to mean a better player) of the ith member of UCI's team is a_i and the tennis rating for the kth member of LBSU's team is s_k. We would like to set up a competition in which each person plays one match against a player from the opposite school. Because we get to select who plays against whom, our goal is to make sure that in *as many matches as possible*, the UCI player has a higher tennis rating than their opponent. Give an efficient **greedy** algorithm for this problem and prove that it is correct (maximizes the number of matches in which our team has the player with the higher tennis rating).

   For CompSci 161 this quarter, for the proof, it is sufficient to demonstrate that the repair of an arbitrary inversion in any alternate ordering, compared to yours, produces an ordering that is no worse. You may use the fact from lecture that any permutation that differs from yours has a consecutive pair of elements that are inverted relative to yours, however for this particular problem, the proof may not need the fact that they are adjacent.

3. Design a strategy that minimizes the expected number of questions you will ask in the following game. You have a deck of cards that consists of one one, two twos, three threes, and so on up to nine nines for a total of 45 cards. Someone draws a card from the shuffled deck and looks at its value (hiding it from you). The goal is to determine the value of the card through asking a series of questions, each of which must be answerable with "yes" or "no" (such as "Is the card a nine?").

   To answer this question, you should express your strategy as a decision tree. You may either explicitly draw the decision tree or describe its construction in sufficient detail so that I could draw it from your description. If you draw the tree instead of describing its construction, give a brief (1-2 sentences) description of how you came up with it.

   Furthermore, briefly explain why this minimizes the expected number of questions you will ask in this game. You are not required to give a formal proof.

   Hint: The first question to ask in the optimal decision tree is "Is the card one of $\{4, 5, 9\}$?" Equivalently, the question can be "Is the card one of $\{1, 2, 3, 6, 7, 8\}$?"

Not Collected Questions

These questions will not be collected. Please do not submit your solutions for them. However, these are meant to help you to study and understand the course material better. You are encouraged to solve these as if they were normal homework problems.

This homework (approximately) covers Chapter 10 in the textbook of Goodrich and Tamassia. One of the strengths of this book is that it has a good variety and quality of practice problems.

If you need help deciding which problems to do, consider trying R-10.4, R-10.5, R-10.6, R-10.7, R-10.8, R-10.9, R-10.10, R-10.4, R-10.5, R-10.6, R-10.7, R-10.8, A-10.4, A-10.5, A-10.7

In addition, I will be releasing a set of reinforcement/practice questions, which you are encouraged to work on, both individually and in groups, and to discuss with your instructor and the TAs.

[3] Not really.