This is due at 9:59 AM on the morning of Friday, April 23. You will need to submit to GradeScope.

Master Theorem Questions

Read the entire question *carefully* before working. I will provide three recurrence relations and I will ask you to use the Master Theorem to solve them, and I will ask you about your work. On quiz 2, you will get a similar question, but will be asked on that to enter your responses into a Google Form. If any directions are unclear, please let me know.

Recall that the Master Theorem is as follows; we write the recurrence in the form $T(n) = aT(n/b) + f(n)$, for some $a \geq 1$, $b > 1$, and $f(n)$ is asymptotically positive.

1. If there is a small constant $\varepsilon > 0$ such that $f(n)$ is $\mathcal{O}(n^{\log_b a - \varepsilon})$, then $T(n)$ is $\Theta(n^{\log_b a})$

2. If there is a constant $k \geq 0$, such that $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$

3. If there is a small constant $\varepsilon > 0$ such that $f(n)$ is $\Omega(n^{\log_b a + \varepsilon})$, then $T(n)$ is $\Theta(f(n))$.

Consider the following three recurrence relations:

- $a(n) = 9a(n/3) + n^2 \log n$
- $b(n) = 32b(n/4) + n^2$
- $c(n) = 8c(n/4) + n^2$

1. (a) i. Which case do you follow for $a(n)$?
   ii. If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0 - 9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0 - 9$ (inclusive); your value of k will be an integer.

   (b) i. Which case do you follow for $b(n)$?
   ii. If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0 - 9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0 - 9$ (inclusive); your value of k will be an integer.

   (c) i. Which case do you follow for $c(n)$?
   ii. If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters

$0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.

Additional Questions

Recall the partition algorithm, as described in lecture, as it was used in both QuickSort and the selection algorithms. Suppose I had an array with the following elements in it. Prior to running partition, each of the $n!$ permutations were possible. The array now is as follows:

17	18	15	25	11	14	28	35	78	86	75	80	54	58	97

2. Which value(s) could have been the pivot? List every value. *Give us the value, not the index. For example, if you think the element currently in the first position was the fist pivot, list it as 17 and not 0 or 1.*

3. Suppose I start with the array listed above. I provide it as input for the randomized selection algorithm ("quickSelect") from class. I also provide the parameter k =8. At the first stage, the pivot chosen is 25. What is the value of k for the first recursive call? Show your work.

4. Consider the following vector:

978	167	103	386	987	335	448	298	582	215	842	640	867	943	998
384	594	966	724	231	948	163	578	903	748	784	598	739	145	595
794	928	663	702	220	556	937	569	659	520	589	502	965	457	351

This vector is represented here with elements 1-15 in the first row, 16-30 in the second row, and 31-45 in the third row. For example, $A[17]$ is 594.

Suppose we run the deterministic selection algorithm from lecture. What is the first pivot chosen for the first partition step? Show your work. If you are unclear on how the algorithm works, there is a short (seven minute) video on Canvas about this that I provided.