All quiz rules from the course syllabus are in effect for the real quiz, in addition to what follows.

If you have questions about the test, please ask *on Piazza* as a **private** post, viewable only by you and the instructors. The instructors will make an announcement if something needs to be said to the entire class.

You may use anything from lecture, discussion, or homework without proof or citation.

This quiz is to be individual effort. Students are permitted to use notes, electronics, and bring textbooks. The work you submit for each quiz is expected to be produced by *you, alone and solely for this assessment.* You may not reuse or repurpose anything you wrote at another time, nor may you ever use anything written by someone else during this quiz. However, despite being allowed notes and electronics, you may *not* seek out the answer to a question in any way, nor may you communicate with anyone during the exam, *for any reason*, with the exception of asking a question on Piazza *set as instructors-only for visibility.*

You have 40 minutes for this quiz, plus an additional ten minutes to enter your answers to the answer form and to scan and upload your free response portion. Please be careful when you enter your answers to the Google Form, as that part will very likely be graded by a computer program and not a human.

As this is the sample quiz, you may complete this any way you would like, and are not obligated to submit anything. However, I encourage you to allocate time as if this were the real quiz, as that will provide you some feedback about your preparation.

If this were a real quiz, everything except the free response question would be submitted to a Google Form, similar to what you did for the first quiz. Your response to the free response question at the end would be scanned and submitted to GradeScope.

Master Theorem Questions (3 points)

Read the entire question *carefully* before working. I will provide three recurrence relations and I will ask you to use the Master Theorem to solve them, and I will ask you about your work.

Recall that the Master Theorem is as follows; we write the recurrence in the form $T(n) = aT(n/b) + f(n)$, for some $a \geq 1$, $b > 1$, and $f(n)$ is asymptotically positive.

1. If there is a small constant $\varepsilon > 0$ such that $f(n)$ is $\mathcal{O}(n^{\log_b a - \varepsilon})$, then $T(n)$ is $\Theta(n^{\log_b a})$

2. If there is a constant $k \geq 0$, such that $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$

3. If there is a small constant $\varepsilon > 0$ such that $f(n)$ is $\Omega(n^{\log_b a + \varepsilon})$, then $T(n)$ is $\Theta(f(n))$.

Consider the following three recurrence relations:

- $a(n) = 3a(n/3) + 15$
- $b(n) = 3b(n/3) + 15n$
- $c(n) = 3c(n/3) + n^2$

1. (a) Which case do you follow for $a(n)$?
   (b) If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.

2. (a) Which case do you follow for $b(n)$?
   (b) If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.

3. (a) Which case do you follow for $c(n)$?
   (b) If your answer to the previous question was case 1 or 3, provide a value of ε that makes the conditional portion true. Write your answer using only the characters $0-9$ (inclusive) and the decimal point. Put no more than four digits after the decimal point.
   If your answer to the previous question was case 2, provide a value of k that makes the conditional true. Write your answer using only the characters $0-9$ (inclusive); your value of k will be an integer.

Additional Question(s)

This question deals with `QuickSort`, the relevant code of which is reproduced below for your convenience. Recall that the element at position q returned by the partition function is known as the *pivot*.

```
QuickSort(A, start, end)
  if start < end then
    q = partition(A, start, end)
    QuickSort(A, start, q − 1)
    QuickSort(A, q + 1, end)
```

We are sorting an array by QuickSort. A pivot was selected, the array was partitioned, and the pivot was placed in its correct position. Then, the algorithm was called recursively on the array in the range start to $q - 1$. That sub-array was partitioned, and the pivot placed in its proper position. At the moment, the array looks like this:

30	10	20	40	60	50	70	90	80

4. (2 points) What element must have been the first pivot selected? *Give us the value, not the index. For example, if you think the element currently in the first position was the first pivot, list it as '30' and not 0 or 1.*

5. (1 point) What element must have been the second pivot selected? *Give us the values, not the index.*

6. Consider the following vector:

978	167	103	386	987	335	448	298	582	215
842	640	867	943	998	784	598	739	145	595
384	594	966	724	231	948	163	578	903	748
794	928	663	702	220	556	937	569	659	520
589	502	965	457	351					

This vector is represented here with elements 1-10 in the first row, 11-20 in the second row, 21-30 in the third row, 31-40 in the fourth row, and 41-45 in the last row.

Suppose we run the deterministic selection algorithm from lecture. What is the first pivot chosen for the first partition step?

Free Response Section

This question is free response. On the real exam, you will need to write an answer on a piece of paper (typing a solution or using a tablet and software is okay too) and upload it to GradeScope, similar to how you do for homework. Please be sure that if you plan to produce your solution on paper, you can scan it and be prepared to upload it within the window allowed on an exam.

7. Suppose I have an array A of n integer elements. Given a value x, I wish to find out if there are indices i and j such that $A[i] + A[j] == x$.

   For example, my value x might be 57 and my array might be:

-5	30	2	15	96	42

   In which case, I would answer true.

   (a) (1 point) What is the running time of the following algorithm? Use Θ notation.

   ```
   for i ← 1 to n do
     for j ← i + 1 to n do
       if A[i] + A[j] == x then
         return true
   return false
   ```

   .

   (b) (3 points) Give an algorithm in pseudo-code to solve this problem in time $\mathcal{O}(n \log n)$. You may allocate $\mathcal{O}(1)$ additional memory in doing so.

   (c) (2 points) Suppose that A is already sorted. Give an algorithm in pseudo-code to solve this in time $\mathcal{O}(n)$.