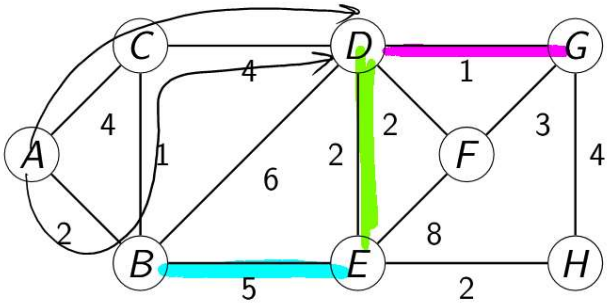
CompSci 161
Spring 2021 Lecture 19: Greedy Algorithms: Introduction/Review Dijkstra SSSP, Prim MST

Paths in Weighted Graphs



- What is the cost of the path $B - E - D - G$?
  $5 + 2 + 1 = 8$
- Shortest (lowest-cost) path from A to D?

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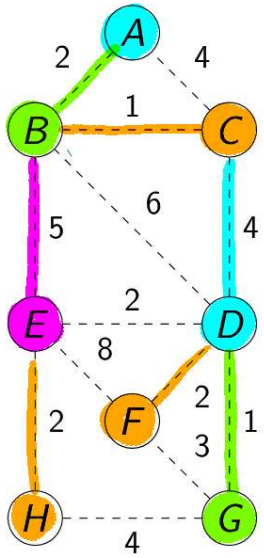
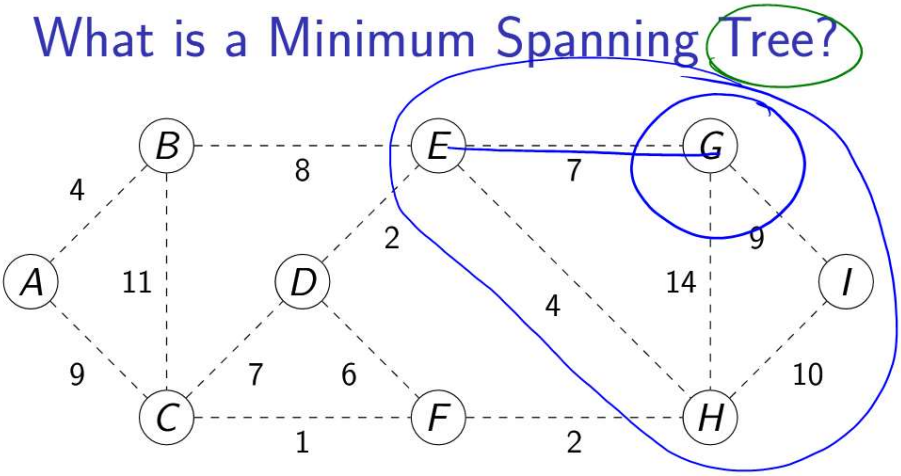
Single Source Shortest Path

v	intree(v)	parent(v)	dist(v)
A	✓	N/A	0
B	✓	A	2 ~~∞~~
C	✓	~~A~~ B	~~4~~ 3 ~~∞~~
D	✓	~~B~~ C	~~8~~ 7 ~~∞~~
E	✓	B	7 ~~∞~~
F	✓	E D	~~15~~ 9 ~~∞~~
G	✓	D	8 ~~∞~~
H	✓	E	9 ~~∞~~



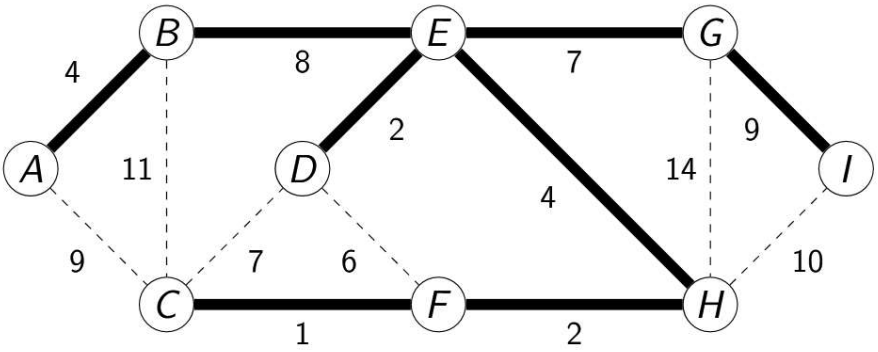
4

What is a Minimum Spanning Tree?



- Given a weighted connected graph G
- All weights are positive
- Keep smallest *sum* of edges such that the graph is still connected.

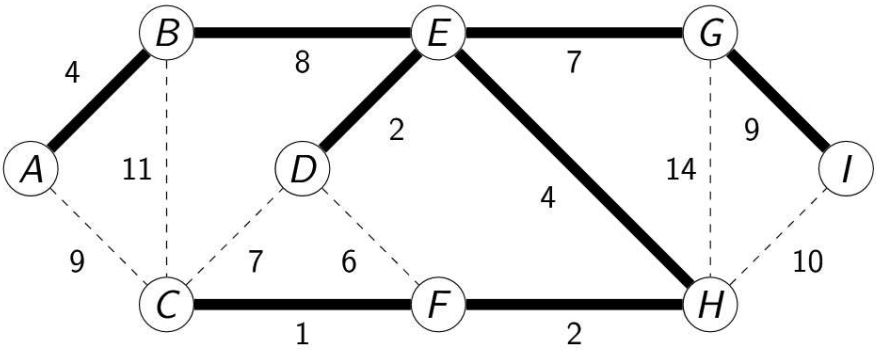
5 What is a Minimum Spanning Tree?



- Could any correct solution have edge (B, C)?

Cycle property

5 What is a Minimum Spanning Tree?



- Could any correct solution omit edge (E, G)?

Cut property

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Finding a MST (Prim/Jarnik/Dijkstra)

v	intree(v)	parent(v)	dist(v)
A	☑	N/A	0
B	☑	A	4 ~~∞~~
C	☑	~~A~~ D F	~~9~~ 7 ~~∞~~
D	☑	F	2 ~~∞~~
E	☑	B	8 ~~∞~~
F	☑	~~D~~ H	~~2~~ ∞
G	☑	E	7 ~~∞~~
H	☑	E	4 ~~∞~~
I	☑	H G	~~10~~ ~~∞~~