

This is due at 9:59 AM on the morning of Thursday, April 8. Scan and upload your submission to Gradescope; instructions for this are available in the course syllabus. For each question, be sure that your answer is clear to the reader, both what your answer is and which question you are answering. Any submissions that do not comply with this requirement may receive a grade penalty, up to and including a zero on the assignment.

1. For each of the following functions, determine whether it is  $\mathcal{O}(n)$ ,  $\Omega(n)$ , or both. You do not need to provide proof or justification.

Note that we are *not* asking you to provide the “best”  $\mathcal{O}$ -notation/ $\Omega$ -notation; merely to describe it with one or both of the choices provided.

$$a(n) = 1729n \quad \square \mathcal{O}(n) \quad \square \Omega(n)$$

$$b(n) = n^{\log n} \quad \square \mathcal{O}(n) \quad \square \Omega(n)$$

$$c(n) = \sqrt{n} \quad \square \mathcal{O}(n) \quad \square \Omega(n)$$

2. Rank the following functions in order from smallest asymptotic running time to largest. Additionally, identify a pair of functions  $x, y$  where  $x(n)$  is  $\Theta(y(n))$ . You do not need to show your work.

For the ordering requirement, write the function identifier **letters** in order.

The value  $e$  refers to the base of the natural logarithm;  $e \approx 2.718281828$

(a)  $a(n) = n + n^2 + \sqrt{n}$

(b)  $b(n) = \log^3 n + \log \log n$

(c)  $c(n) = 2^{n \log n}$

(d)  $d(n) = n^e$

(e)  $e(n) = (7^{e-1/e} - 9) \cdot \pi^2$

(f)  $f(n) = 2^n$

(g)  $g(n) = \sqrt{\log n}$

(h)  $h(n) = 4^{\log n}$

3. This question deals with **SelectionSort**, the relevant code of which is reproduced for your convenience:

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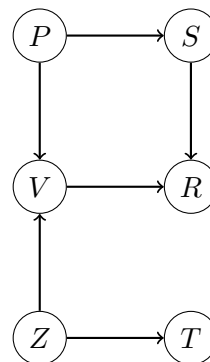
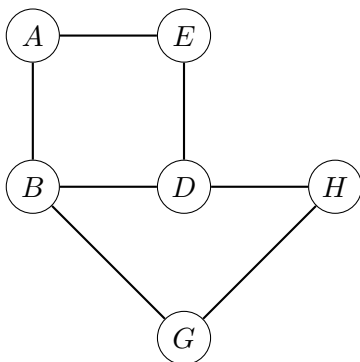
for  $i \leftarrow 1$  to  $n - 1$  do
   $\text{min} \leftarrow i$ 
  for  $j \leftarrow i + 1$  to  $n$  do
    if  $A[j] < A[\text{min}]$  then
       $\text{min} \leftarrow j$ 
  Swap  $A[i]$  and  $A[\text{min}]$ 

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The following array is being sorted by SelectionSort. Right now, the loop condition for the outer loop (for loop indexed by  $i$ ) is about to be checked.

30	39	87	96	56	84	46	58	43	55
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- (a) What is the minimum number of times the loop could have executed so far? Briefly justify (1-2 sentences) your answer.
- (b) What is the maximum number of times the loop could have executed so far? Briefly justify (1-2 sentences) your answer.
4. Give a valid breadth-first order of the following undirected graph, starting at vertex  $A$ .



5. Give a valid topological order of the above **directed** graph.

*For questions 4 and 5, be sure to use the correct graph. If you aren't sure what an undirected or directed graph is, review the video before answering the questions.*