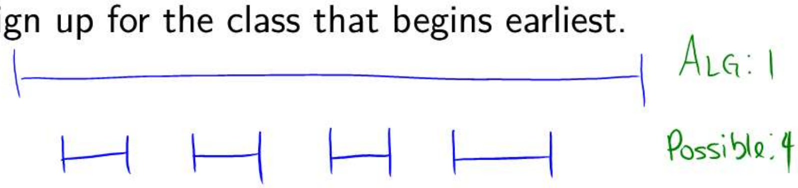
CompSci 161
Spring 2021 Lecture 20:
Greedy Algorithms:
Interval Scheduling

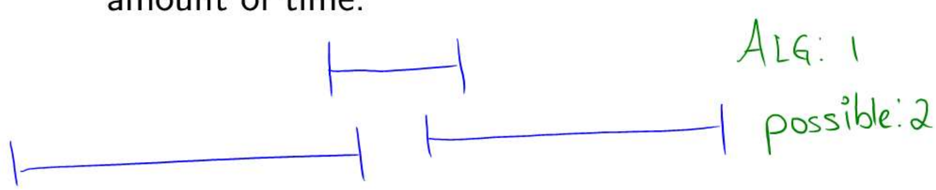
Unweighted Interval Scheduling Problem

Two possible algorithms (four on handout):

- Sign up for the class that begins earliest.



- Sign up for the class that meets for the least amount of time.

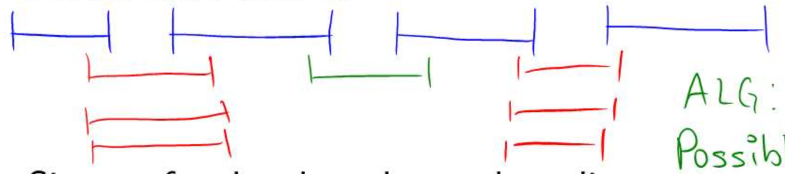


3

Unweighted Interval Scheduling Problem

Two more algorithms (four on handout):

- Sign up for the class that conflicts with the fewest other classes.



ALG: 3
Possible 4

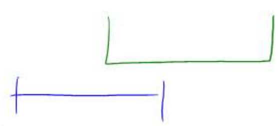
- Sign up for the class that ends earliest.

4

Interval Scheduling Problem (proof)

Correct Algorithm:

- Sign up for the class that ends earliest.
- Remove it and all overlapping classes from the set of available classes.
- Repeat this process until no classes remain.



Claim: There is an optimal solution that includes the first-ending class.

Proof of Claim: Suppose all optimal solutions do not. Select an arbitrary optimal solution OPT.

$x \leftarrow$ OPT's first ending interval

$c \leftarrow 1^{st}$ ending interval in input

if $x \neq c$

we know $s_x < f_c$

$OPT' = \underbrace{OPT - \{x\} + \{c\}}$

5 ## Proof of Correctness

- We began with an arbitrary optimal set OPT
  - Its first element was not first-ending.
  - We removed that one
  - We added our first one: the first-ending.
  - This forms a set we'll call OPT$'$
- **Claim:** OPT$'$ is an optimal solution.
  - Is it the same size as every optimal solution?

    $|OPT'| = |OPT| + 1 - 1 = |OPT|$
  - Is it a valid solution?

    $f_c \leq f_x$ und $\forall y \in OPT - \{x\}$

    $f_x < s_y$

    So c overlaps none in OPT$'$ others

6 ## Proof of Correctness

- We proved that an optimal solution exists that includes the first-ending class.
- What does the full proof look like?