Asking the Important Questions

Since coming to UC Irvine, has anyone met a celebrity?




What is a celebrity?

- Within a group of people *G*,
  we say a person *p* is a celebrity (famous) if:
  - Everyone knows who *p* is
    (celebrities must be known by everyone)
  - Person *p* does not know who anyone else is

- Suppose I think person *p* might be a celebrity within group *G*.
  - How can you check?
  - What actions can we take?

Who is a celebrity?

- How efficiently can I check if person *p* is a celebrity?

for each $q \in G - \{p\}$
  if p knows q
    return false
  if q doesn't know p
    return false
return true

- How can I check if a group *G* has *any* celebrity?

for each $p \in G$
  ~~check~~ if (p celebrity)
    return true
return false

$O(n^2)$

Finding a Celebrity

- The previous solution was “brute force.”
- Can we do better?
- Things to look for:
  - Did we repeat work?
    (work that may have been needed, but was done twice)
  - Did we do unnecessary work?
    (work that we could have done without)

Finding a Celebrity

while $|G'| > 1$
{
pick $x, y \in G$ arbitrary and distinct
if x knows y
x not celeb (remove from G')
else
y not celeb
}
check if only $p \in G$ is celebrity // $O(n)$

n-1 iter.

$O(n)$

Finding a Celebrity

- Previous solution assumes a group has at most one celebrity.
- Could a group have zero celebrities?

Finding a Celebrity

- Previous solution assumes a group has at most one celebrity.
- Could a group have two or more celebrities?

Suppose FSOC 2^+ celebs in G
Call two celebs x, y.
Does x know y?
If so, x not celebrity →←
If not, y is not celeb →←