CompSci 161
Spring 2021 Review :
Hierarchy of Running Times
Lower Bounds on Sorting

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Hierarchy

1. Constant Time push to a stack : $O(1)$
2. Poly-logarithmic $\log n$ $\log^3 n \leftarrow (\log n)^3$
3. Polynomial n n^2 n^3 . . . $\hat{}$ $n \log n$ $\log n \approx n^{\varepsilon}$ $\varepsilon > 0$ but small
4. Exponential 2^n 1.01^n $n!$

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Sort the running times

- $f_A = 2^{100n}$
- $f_B = 2^{n^2}$
- $f_C = 2^{n!}$
- $f_D = 2^{2^n}$
- $f_E = n^{\log n}$
- $f_F = n \log n \log \log n$
- $f_G = n^{3/2}$
- $f_H = n \log^{3/2} n$
- $f_I = n^{4/3} \log^2 n$

exponentials

? where within

$2^{\log n} = n$ base 2

$4^{\log n} = n^2$

$\log n \approx n^{\varepsilon}$

$100n,\ n^2,\ n!,\ 2^n$ order

in

$100n, n^2, 2^n, n!$

ABCD is $\boxed{EABDC}$

$n^{\log n} \ ?\ 2^n$

$\log(n^{\log n}) \ ?\ \log(2^n)$

$\log^2 n \ ?\ n$

FHIGEABDC

$f_F : n^{1+\varepsilon+\varepsilon^2}$

$G : n^{3/2}$

$f_H : n^{1+1.5\varepsilon}$

$f_I : n^{\frac{4}{3}+2\varepsilon}$

$n\ \cancel{\log n}\ \log\log n$

$\cancel{n}\ \cancel{\log^{3/2}} n\ \sqrt{\log n}$

$n^{4/3}\ \log^2 n$

$\cancel{n}\ \log n\ \sqrt{\log n}$

$\cancel{n} \cdot n^{1/3} \cdot \log^2 n$

$n^{1/3} = \sqrt[3]{n}$

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Lower Bound on Sorting

$\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

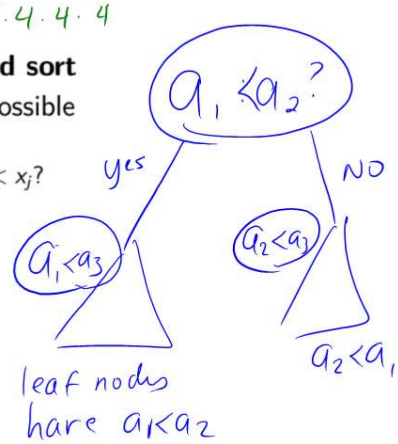
$4 \cdot 4 \cdot 4 \cdot 4$

- This applies to any **comparison-based sort**
- Every permutation of input must be possible
- Let's build a **decision tree**
  - Questions asked are comparisons: is $x_i < x_j$?
- What are leaf nodes of decision tree?

  permutations
- What are internal nodes?

  questions
- What is height of the tree?

  worst case running time



$$\log\left(\frac{n}{2}\right)^{n/2} \le \log(n!) \le \log(n^n) = n \log n$$

"nlogn shaped"