

1.

(a)

Initialize 2-D array X with dimension $n \times n$

for each $i = 1 \dots n$ do

 for each $j = 1 \dots n$ do

 if $i > j$ then

$X[i, j] = 0$

 else

$p = \text{sum of } p_i \text{ from } i \text{ to } j$

$X[i, j] = p$

return X

(b)

Initialize 2-D array X with dimension $n \times n$

for each $i = 1 \dots n$ do

 for each $j = 1 \dots n$ do

 if $i > j$ then

$X[i, j] = 0$

 else if $i == j$ then

$X[i, j] = p_i$

 else

$X[i, j] = X[i, j-1] + p_j$

return X

2. Assume we have an array named **free** so that **free[n]** indicates whether we get free dinner on day n.

recursive solution:

food(n):

```
// base case
if n <= 0 then return 0
// recurrence expressions
if free[n] == true then
    cost_tonight <- food(n-1) // free food
else
    cost_tonight <- 6 + food(n-1) // buy from cafeteria
cost_grocery <- 20 + food(n-7) // alternatively, buy grocery for a week
return min(cost_tonight, cost_grocery)
```

We see that we have repeated recursive sub-problems when calling food(n-1) and food(n-7), so we can convert the algorithm to an iterative one using dynamic programming.

iterative solution:

food_iter(n):

```
initialize array cost with size n
// for simplicity, assume cost[i] return 0 if i <= 0
for i = 1 ... n do
    if free[i] == true then
        cost[i] <- min(cost[i-1], 20 + cost[i-7])
    else
        cost[i] <- min(6 + cost[i-1], 20 + cost[i-7])
return cost[n]
```

The running time of iterative solution is $\Theta(n)$, since we only have one for-loop with n iterations and each iteration takes constant time.

3. assume we are given 2 arrays **easy** and **hard** that stores the point value of each homework assignment.

recursive solution:

```
homework(n):  
    // base case  
    if n <= 0 then return 0  
    // recurrence expressions  
    do_easy = easy[n] + homework(n-1)  
    do_hard = hard[n] + homework(n-2)  
    return max(do_easy, do_hard)
```

We see that we have repeated recursive sub-problems when calling **homework(n-1)** and **homework(n-2)**, so we can convert the algorithm to an iterative one using dynamic programming.

iterative solution:

```
homework_iter(n):  
    initialize array point with size n+1, indexed from 0 to n  
    point[0] <- 0  
    point[1] <- hard[1] // hard version guarantee higher point  
    for i = 2 ... n do  
        do_easy = easy[n] + point[n-1]  
        do_hard = hard[n] + point[n-2]  
        point[i] <- max(do_easy, do_hard)  
    return point[n]
```

The running time of iterative solution is $\Theta(n)$, since the algorithm effectively filled the $(n+1)$ size array once, and filling one value in the array takes constant time.