1. Let our shortest task first solution be called *X*.

Suppose we have an alternate solution called *ALT* that does not follow the algorithm. We know that there must be at least one pair of consecutive assignment *P* and *Q* that are inverted to our shortest task first ordering. Suppose that *P* takes longer to do than some assignment *Q*, and *P* is done before *Q*. In other words, *t[P] > t[Q]*.

Then we know that in *X*, *Q* is done before *P*. We will prove that *X* is optimal by showing that *X* is no worse than *ALT*.

Let S[i] be the starting time of assignment i, and F[i] be the finish time of assignment i.

In X, let S[Q] = some arbitrary constant, we know that F[Q] = S[Q] + t[Q] = S[P]; and F[P] = S[P] + t[P] = S[Q] + t[Q] + t[P].

In ALT, we have S[P]’ = S[Q]; F[P]’ = S[P]’ + t[P] = S[Q]’; F[Q]’ = F[P].

Let the deadline be at time N. We have the following cases.

Case N >= F[Q]:

> In X, Q will not have penalty, if P is not late, then the total penalty in X and ALT is 0. If P is late, it will have a penalty equal to F[P] – N. In ALT, P may have a penalty equal to F[P]’ – N, and Q will have a penalty equal to F[Q]’ - N = F[P] – N. Therefore, the total penalty in X is less or equal to ALT. X is no worse.

Case N < F[Q]:

> In X, Q has a penalty equal to F[Q] – N, P has a penalty equal to F[P] – N. In ALT, P has a penalty equal to F[P]’ – N, Q has a penalty equal to F[Q]’ – N = F[P] – N. Therefore the total penalty in X is F[Q] – N + F[P] – N; and in ALT is F[P]’ – N + F[P] – N. Since F[P]’ = S[P]’ + t[P] = S[Q] + t[P]; F[Q] = S[Q] + t[Q]; and t[P] > t[Q], the total penalty in ALT is greater than X. X is no worse than ALT.

Here, we have shown that our algorithm produces an outcome that is no worse any other alternate ordering that does not follow our algorithm; therefore, our algorithm produces optimal outcome.

2. Suppose we have the following input:

h[1] = 6; e[1] = 5; h[2] = 8; e[2] = 2; h[3] = 100; e[3] = 5;

According to the algorithm, it will do skip day 1, do h[2], and do e[3] since h[3] is not allowed as we did homework in day 2. Producing a total point of 13.

A better solution will be do e[1], skip day 2, and do h[3]. Producing a total point of 105.

Therefore, we have shown that the algorithm does not achieve the optimal solution.