A Fuzzy/Probabilistic Approach to Uncertain Interval Algebra

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***Abstract*— A novel approach to uncertain temporal inference is presented. Allen's Interval Algebra is extended to fuzzy time-intervals via representing the latter as trapeziums with distinct beginning, middle and end. An uncertain version of the Interval Algebra composition table is developed via running a computer simulation in which a large number of fuzzy time-intervals are drawn from an assumed probability distribution, and using machine learning to induce uncertain compositional rules that hold approximately across the corpus of simulated intervals.**

I. Introduction

BEN can write a couple intro paragraphs [2]

A. Allen Interval Algebra

One way to approach temporal reasoning is to represent the events as *intervals*. An interval captures some portion of time within which some features hold, then a temporal event could be represented as **[a, b]** where a is the beginning time-step of the event and b is the ending. There has been many works focused on addressing interval handling for temporal inference, most notable of these are Allen's Interval Algebra (IA) [1] and Interval Temporal Logic (ITL) [4], [3].

The foundation that IA is built on in treating intervals is to set aside the actual time frames in which events occur, and instead, work with the *relation* between two intervals. To this end, IA defines thirteen basic relations that address three desired qualities:

- Distinct: the set of 13 relations constitute a partitioning over the relation space
- Exhaustive: any pair of intervals can only be described by only one of the relations
- Qualitative: the ground for comparing two intervals is how they relate, as opposed to the time frames they lay in (quantitative)

Each Allen relation poses four constraints, for two intervals to be qualified as holding the relation, they should satisfy all four. For instance, assuming two intervals a and b are given, the relation *{s}* (starts) is defined as 1) $beginning_a = beginning_b$ 2) $beginning_a < ending_b$ 3) $ending_a > beginning_b$ 4) $ending_a < ending_b$. The set of thirteen Allen relations are shown in Table I.

Having the relations in place, IA continues with defining operations on relations:

- Complement: denoted as ~*r*, complement of set *r* is the set of all relations not present in *r*
- Converse: or !*r*, is the set of relations obtained from substituting intervals *a* and *b* in the original relation *a{r}b*

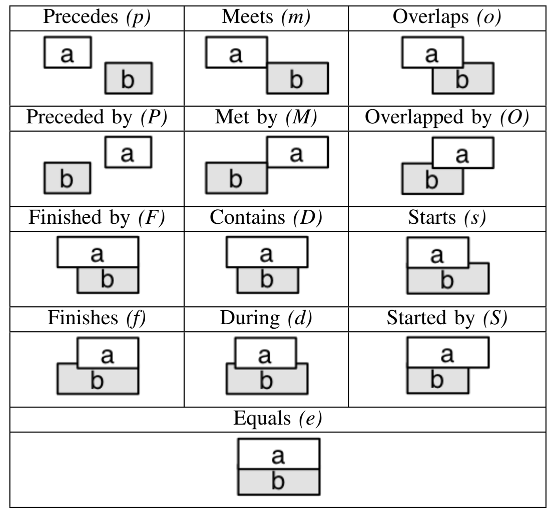

TABLE I

Temporal relations in Interval Algebra

Complement	Converse
~(p) = (moFDseSdfOMP)	!(p) = (P)
~(pmoFD) = (seSdfOMP)	!(pmoFD) = (dfOMP)
~() = (pmoFDseSdfOMP)	!(mM) = (mM)
	!() = ()
Intersection	**Union**
(pmo)^(FDseS) = ()	(pmo)+(FDseS) = (pmoFDseS)
(pFsSf)^(pmoFD) = (pF)	(pFsSf)+(pmoFD) = (pmoFDsSf)
(pmo)^(pmo) = (pmo)	(pmo)+(pmo) = (pmo)
Composition	
(m).(m) = (p)	
(pm).(pm) = (p)	
(oFD).(oFDseS) = (pmoFD)	

TABLE II

Examples of operations on intervals

- Intersection: shown by *r^s* is the set of relations present in both *r* and *s*
- Union: *r+s*, set of relations present in either *r* or *s*
- Composition: *r.s*, the set of all possible relations for intervals *a* and *c* having *a{r}b* and *b{s}c*

Examples for these operations are presented in Table II and Table III shows composition of *a{r}b* and *b{r}c* for singular *r* and *s*.

B. Fuzzy Interval Algebra

Keyvan should summarize prior papers on Allen Interval Algebra, with references

	p	**m**	**o**	**F**	**D**	**s**	**e**	**S**	**d**	**f**	**O**	**M**	**P**
p	*(p)*	*(p)*	*(p)*	*(p)*	*(p)*	*(p)*	*(p)*	*(p)*	*(pmosd)*	*(pmosd)*	*(pmosd)*	*(pmosd)*	*full*
m	*(p)*	*(p)*	*(p)*	*(p)*	*(p)*	*(m)*	*(m)*	*(m)*	*(osd)*	*(osd)*	*(osd)*	*(Fef)*	*(DSOMP)*
o	*(p)*	*(p)*	*(pmo)*	*(pmo)*	*(pmoFD)*	*(o)*	*(o)*	*(oFD)*	*(osd)*	*(osd)*	*concur*	*(DSO)*	*(DSOMP)*
F	*(p)*	*(m)*	*(o)*	*(F)*	*(D)*	*(o)*	*(F)*	*(D)*	*(osd)*	*(Fef)*	*(DSO)*	*(DSO)*	*(DSOMP)*
D	*(pmoFD)*	*(oFD)*	*(oFD)*	*(D)*	*(D)*	*(oFD)*	*(D)*	*(D)*	*concur*	*(DSO)*	*(DSO)*	*(DSO)*	*(DSOMP)*
s	*(p)*	*(p)*	*(pmo)*	*(pmo)*	*(pmoFD)*	*(s)*	*(s)*	*(seS)*	*(d)*	*(d)*	*(dfO)*	*(M)*	*(P)*
e	*(p)*	*(m)*	*(o)*	*(F)*	*(D)*	*(s)*	*(e)*	*(S)*	*(d)*	*(f)*	*(O)*	*(M)*	*(P)*
S	*(pmoFD)*	*(oFD)*	*(oFD)*	*(D)*	*(D)*	*(seS)*	*(S)*	*(S)*	*(dfO)*	*(O)*	*(O)*	*(M)*	*(P)*
d	*(p)*	*(p)*	*(pmosd)*	*(pmosd)*	*full*	*(d)*	*(d)*	*(dfOMP)*	*(d)*	*(d)*	*(dfOMP)*	*(P)*	*(P)*
f	*(p)*	*(m)*	*(osd)*	*(Fef)*	*(DSOMP)*	*(d)*	*(f)*	*(OMP)*	*(d)*	*(f)*	*(OMP)*	*(P)*	*(P)*
O	*(pmoFD)*	*(oFD)*	*concur*	*(DSO)*	*(DSOMP)*	*(dfO)*	*(O)*	*(OMP)*	*(dfO)*	*(O)*	*(OMP)*	*(P)*	*(P)*
M	*(pmoFD)*	*(seS)*	*(dfO)*	*(M)*	*(P)*	*(dfO)*	*(M)*	*(P)*	*(dfO)*	*(M)*	*(P)*	*(P)*	*(P)*
P	*full*	*(dfOMP)*	*(dfOMP)*	*(P)*	*(P)*	*(dfOMP)*	*(P)*	*(P)*	*(dfOMP)*	*(P)*	*(P)*	*(P)*	*(P)*

TABLE III

COMPOSITION IN INTERVAL ALGEBRA

Keyvan should briefly explain why we didn't just use their work

II. A TRAPEZIUM MODEL OF FUZZY INTERVALS

Keyvan will explain the modeling of events as trapeziums with beginning, middle and end, including a pretty diagram of a trapezium

III. DEFINING FUZZY INTERVAL RELATIONS VIA CONVOLUTION

Keyvan will describe the convolution approach to fuzzy interval relations, preferably using a diagram illustrating one example (could be transitivity of "precedes")

IV. PROBABILISTIC ESTIMATION OF A FUZZY INTERVAL RELATION COMPOSITION TABLE

Keyvan will explain the approach to generating a composition table probabilistically

A graph illustrating the transitivity of precedence rule should be presented, because 3D graphs are pretty and shiny

We can give example results on the precedence rule, and leave full discussion of the composition table till later..

REFERENCES

[1] James F. Allen. Maintaining knowledge about temporal intervals. *Commun. ACM*, 26(11):832–843, November 1983.

[2] B. Goertzel, M. Ikle, I. Goertzel, and A. Heljakka. *Probabilistic Logic Networks*. Springer, 2008.

[3] Joseph Halpern, Zohar Manna, and Ben Moszkowski. A hardware semantics based on temporal intervals. In Josep Diaz, editor, *Automata, Languages and Programming*, volume 154 of *Lecture Notes in Computer Science*, pages 278–291. Springer Berlin Heidelberg, 1983.

[4] Benjamin C Moszkowski. *Reasoning about digital circuits*. PhD thesis, Computer Science Department, Stanford University, 1983.