Time Series Analysis: Homework 3 (20232863 Keywoong Bae)

1. A time series $\{X_t, t=1,2,\ldots\}$ will be decomposed into three components given below:

$$X_t = T_t + S_t + R_t$$

Here, each component follows the models below:

$$(1-B)T_t = a_{1t}$$

$$(1+B+B^2)S_t = a_{2t}$$

$$(1-0.7B)R_t = a_{3t}$$

where a_{it} $(i=1,2,3)$ are white noises having mean 0 and variance σ_i^2. Which ARMA model does $\{X_t, t=1,2,\ldots\}$ follow ?

$$X_t = (1-B)^{-1} a_{1t} + (1+B+B^2)^{-1} a_{2t} + (1-0.7B)^{-1} a_{3t}.$$

$$(1-B)(1+B+B^2)(1-0.7B)X_t$$

$$= (1+B+B^2)(1-0.7B)a_{1t} + (1-B)(1-0.7B)a_{2t} + (1-B)(1+B+B^2)a_{3t}$$

$\underline{\text{ARMA}(4,3)}$.

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2. A certain time series shows that its ACF is exponentially decaying while its PACF cuts off after lag 2 with P(1)=0.5 and P(2)=0.6.
(a) What process is this time series following ?
(b) Obtain autocorrelation coefficients at lag 3 and lag 4.

(a) $\rho(1) = \dfrac{\phi_1}{1-\phi_2} = 0.5$, $\phi_2 = P(2) = 0.6.$

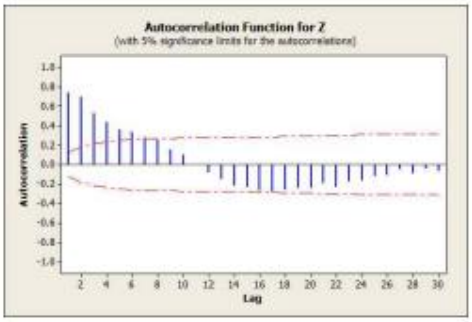
$\phi_1 = 0.2$

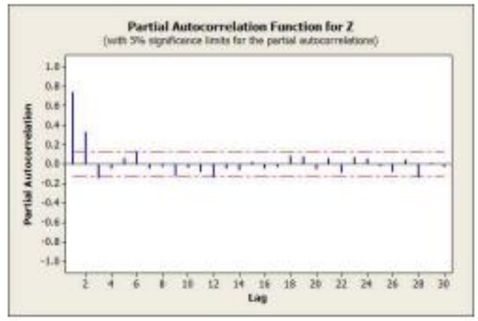
(b). $\rho(2) = \dfrac{\phi_1^2}{1-\phi_2} + \phi_2 = 0.7$

$\rho(3) = 0.2\rho(2) + 0.6\rho(1) = 0.44$

$\rho(4) = 0.2\rho(3) + 0.6\rho(2) = 0.508$

3. The sample ACF and PACF are obtained below.





(a) Describe the characteristics of ACF and PACF briefly.
(b) Identify the model.
(c) The estimates of $\rho(1), \rho(2)$ are given by $\hat{\rho}(1)=0.75$, $\hat{\rho}(2)=0.65$
Estimate the model parameters.

(a) ACF는 내리고 오르는 주기가 있어보이나 크기는 줄어드는 모습을 보임
PACF는 시차 2 이후부터 절단된 모습을 보임.

(b) AR(2) 모형.

(c) $\hat{\rho}(1) = \frac{\phi_1}{1-\phi_2} = 0.75$, $\hat{\rho}(2) = \frac{\phi_1^2}{1-\phi_2} + \phi_2 = 0.65$.

$\hat{\phi}_1 = 0.6$, $\hat{\phi}_2 = 0.2$

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4. Identify the model of the log transformed series J12 (monthly number of deaths by traffic accidents in Seoul).

AR(1) 모형.