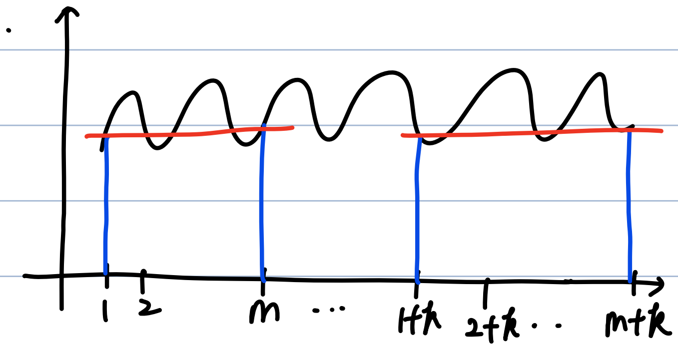
< 시계열 분석 2강 : 정상적 시계열 >　　　　2023 2863 배기웅.

## 2.1 정상성.

· horizontal pattern

*** strong stationality**




⇒ ① 시계열 기댓값 일정 $E[Z_t]=\mu$

② 〃 분산 〃 $Var[Z_t]=\sigma_Z^2=\gamma(0)$

③ time-lag 에만 의존

$Cov(Z_t, Z_{t-k}) = Cov(Z_{t+k}, Z_t)$
$= \gamma(k)$

$Corr(Z_t, Z_{t-k}) = Corr(Z_{t+k}, Z_t)$
$= \gamma(k)/\gamma(0) = \rho(k)$

*** Weak Stationary**

· time-lag 에 의존하면 Weak-Stationary

· 편의상 시계열 기댓값 $E[Z_t]=0$ 으로 가정.

*** auto covariance / auto correlation matrix**

$$\Gamma=\begin{bmatrix} Var[Z_1] & Cov[Z_1,Z_2] & \cdots & Cov[Z_1,Z_m] \\ Cov[Z_1,Z_2] & Var[Z_2] & \cdots & Cov[Z_2,Z_m] \\ \vdots & \vdots & \ddots & \vdots \\ Cov[Z_1,Z_m] & Cov[Z_2,Z_m] & \cdots & Var[Z_m] \end{bmatrix} = \begin{bmatrix} \gamma(0) & \gamma(1) & \cdots & \gamma(m-1) \\ \gamma(1) & \gamma(0) & \cdots & \gamma(m-2) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma(m-1) & \gamma(m-2) & \cdots & \gamma(0) \end{bmatrix}$$

⇒ auto covariance matrix

· $\gamma(k) = Cov(Z_t, Z_{t-k}) = Cov(Z_{t+k}, Z_t) = E[Z_t Z_{t-k}]$

↳ $Cov(Z_t, Z_{t-k}) = E[(Z_t - \cancel{E[Z_t]}^{0})(Z_{t-k} - \cancel{E[Z_{t-k}]}^{0})]$

$$P=\begin{bmatrix} 1 & \rho(1) & \cdots & \rho(m-1) \\ \rho(1) & 1 & \cdots & \rho(m-2) \\ \vdots & \vdots & \ddots & \vdots \\ \rho(m-1) & \rho(m-2) & \cdots & 1 \end{bmatrix}$$

⇒ auto correlation matrix

· $Var[Z_k] = E[(Z_k - \cancel{E[Z_k]}^{0})^2] = E[Z_k^2] = \gamma(0)$

· $\rho(k) = Corr[Z_t, Z_{t-k}] = \dfrac{Cov(Z_t, Z_{t-k})}{\sqrt{Var[Z_t]\,Var[Z_{t-k}]}} = \dfrac{\gamma(k)}{\gamma(0)}$

* 예제

- $\{Z_t, t \geq 1\}$ is a stationary time series having relation

$$Z_t = \phi Z_{t-1} + a_t$$

where $-1 < \phi < 1$ is a constant and $a_t$ is a white noise having mean 0 and variance $\sigma_a^2$.

- Find autocorrelation function.
- (Answer)
  - Autocovariance at lag 1: $E[Z_t Z_{t-1}] = \phi E[Z_{t-1}^2] + E[a_t Z_{t-1}]$

    $\gamma(1) = \phi\gamma(0)$ $\quad \rho(1) = \dfrac{\gamma(1)}{\gamma(0)} = \dfrac{\phi\gamma(0)}{\gamma(0)} = \phi$
  - Autocovariance at lag 2: $E[Z_t Z_{t-2}] = \phi E[Z_{t-1} Z_{t-2}] + E[a_t Z_{t-2}]$

    $\gamma(2) = \phi\gamma(1) = \phi^2\gamma(0)$ $\quad \rho(2) = \dfrac{\gamma(2)}{\gamma(0)} = \dfrac{\phi^2\gamma(0)}{\gamma(0)} = \phi^2$
  - Autocorrelation function

    $\rho(k) = \dfrac{\gamma(k)}{\gamma(0)} = \dfrac{\phi^k\gamma(0)}{\gamma(0)} = \phi^k \qquad k = 1, 2, \ldots$

① auto-covariance

lag 1 : $E[Z_t Z_{t-1}]$

$\gamma(1) = \phi\gamma(0)$, $\rho(1) = \phi$

lag 2 : $\rho(2) = \phi^2$

$\downarrow$

lag k : $\rho(k) = \phi^k$

## 2.3 Partial Auto-Correlation (PAFC)

- 중간 시점들 값이 미치는 영향을 뺀 순수한 $z_t$와 $z_{t-k}$의 상관계수
- $z_t = \phi_{k1} z_{t-1} + \cdots + \phi_{kk} z_{t-k} + b_t$ ( 시차 $k$의 편자기 상관계수)

$z_{t-1}, z_{t-2}, \ldots, z_{t-k-1}$ 회귀식

↓ Partial auto-correlation

- $P(k) = Corr[z_t, z_{t-k} \mid z_{t-1}, \ldots, z_{t-k+1}]$, $k = 1, 2, \ldots$

↳ 편자기상관함수, $P(1) = \rho(1)$

*** 시차 $k$의 편자기상관 계수**

→ 시차 $k$의 편자기상관계수

$$z_t = \phi_{k1} z_{t-1} + \cdots + \boxed{\phi_{kk}} z_{t-k} + b_t$$

Multiply by $z_{t-1}$ & Take expectation

$$E[z_t z_{t-1}] = \phi_{k1} E[z_{t-1} \cdot z_{t-1}] + \phi_{k2} E[z_{t-2} \cdot z_{t-1}] + \cdots + \phi_{kk} E[z_{t-k} z_{t-1}] + E[\underbrace{b_t}_{=0} \cdot z_{t-1}]$$

$$\rho(1) = \phi_{k1}\rho(0) + \phi_{k2}\rho(1) + \cdots + \phi_{kk}\rho(k-1)$$

Multiply by $z_{t-2}$ & Take expectation

$$E[z_t z_{t-2}] = \phi_{k1} E[z_{t-1} z_{t-2}] + \phi_{k2} E[z_{t-2} \cdot z_{t-2}] + \cdots + \phi_{kk} E[z_{t-k} z_{t-2}]$$

$$\rho(2) = \phi_{k1}\rho(1) + \phi_{k2} \cdot \rho(0) + \cdots + \phi_{kk}\rho(k-2)$$

↓ ....

$$\rho(k) = \phi_{k1}\rho(k-1) + \phi_{k2}\rho(k-2) + \cdots + \phi_{kk}\rho(0)$$

- $$P(2) = \frac{\begin{vmatrix} 1 & \rho(1) \\ \rho(1) & \rho(2) \end{vmatrix}}{\begin{vmatrix} 1 & \rho(1) \\ \rho(1) & 1 \end{vmatrix}} = \frac{\rho(2) - \rho(1)^2}{1 - \rho(1)^2}$$

$$\frac{\gamma(1)}{\gamma(0)} = \phi_{k1} \frac{\gamma(0)}{\gamma(0)} + \phi_{k2} \frac{\gamma(1)}{\gamma(0)} + \cdots$$

$$\Downarrow$$

$$\rho(1) = \phi_{k1} \cdot 1 + \phi_{k2}\, \rho(1) + \cdots + \phi_{kk}\, \rho(k-1)$$

*** Example**

- lag 1 : $P(1) = \rho(1)$
- lag 2 : $P(2) = \phi_{22}$ in $z_t = \phi_{21} z_{t-1} + \phi_{22} z_{t-2} + b_t$
- lag 3 :

## 2.4 시계열 표현방식

### * 자기회귀 (AR)

- $Z_t = \pi_1 Z_{t-1} + \pi_2 Z_{t-2} + \cdots + a_t$
- Backward shift operator : B를 이용.

  $Z_{t-1} = B \cdot Z_t$, $Z_{t-2} = B \cdot Z_{t-1} = B \cdot B \cdot Z_t = B^2 \cdot Z_t$

  $Z_{t-k} = B^k \cdot Z_t$
- $(1 - \pi_1 B - \pi_2 B^2 - \cdots) Z_t = a_t$ 로 변경가능.
- $Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t$ 라고 하면,

  ↳ $\pi_1 = \phi_1$, $\pi_2 = \phi_2$, $\pi_k = 0$ $(k = 3, 4, \cdots)$
- AR 형식은 stationary condition 필요 ⇒ 항상 stationary 하지 않으므로.

### * 이동평균 과정 (MA)

- $Z_t = a_t - \psi_1 a_{t-1} - \psi_2 a_{t-2} - \cdots.$

  $= (1 - \psi_1 B - \psi_2 B^2 - \cdots) a_t$
- Stationary condition : finite covariance function

  $|\gamma(k)| \le \gamma(0) < \infty$

  $\gamma(0) = Var[Z_t] = E[Z_t^2] = Var[a_t - \psi_1 a_{t-1} - \psi_2 a_{t-2} - \cdots]$

  $= Var[a_t] + \psi_1 Var[a_{t-1}] + \cdots$ $(\because a_t, a_{t-1}, \ldots,$ 독립이므로$)$

  $= \sigma_a^2 \cdot \sum_{j=0}^{\infty} \psi_j^2 < \infty$
- $Z_t = \phi Z_{t-1} + a_t$ : AR form

  $\Rightarrow Z_t = (1 - \phi B)^{-1} a_t = (1 + \phi B + \phi^2 B^2 + \cdots) a_t$ : MA form

  $\phi Z_{t-1} = -\phi B^{-1}$