시계열분석 과제 1

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1. Suppose that a time series follows a following horizontal process:

$X_t = c + a_t$

where a_t is a white noise with mean 0 and variance σ_a^2.

Show for a simple moving average with span N that

a) $Cov[M_t, M_{t-k}] = \sigma_a^2 (N-k)/N^2$, $k < N$

b) $Corr[M_t, M_{t-k}] = 1 - \frac{k}{N}$, $k = 1, \ldots, N-1$

1-a)

$$Cov[M_t, M_{t-k}] = \frac{1}{N^2} Cov[(X_t + X_{t-1} + \cdots + X_{t-N+1})(X_{t-k} + X_{t-k+1} + \cdots + X_{t-k-N+1})]$$

$Cov[X_i, X_j]$ 에서 $i=j$ 일 때 σ_a^2, $i \neq j$ 일 때 0.

$k<N$ 일 때 같은 x항이 $(n-k)$개 있으므로

$$Cov[M_t, M_{t-k}] = \frac{N-k}{N^2} \cdot \sigma_a^2$$

1-b)

$$\rho[M_t, M_{t-k}] = \frac{Cov[M_t, M_{t-k}]}{Var[M_t]\ Var[M_{t-k}]} \quad \left(Var[M_t] = Var[M_{t-k}] = \frac{\sigma_a}{\sqrt{N}} \right)$$

$$= \frac{\cancel{\sigma_a^2}(N-k)/N^{\cancel{2}}}{\sqrt{\frac{\cancel{\sigma_a^2}}{\cancel{N}}} \cdot \sqrt{\frac{\cancel{\sigma_a^2}}{\cancel{N}}}} = \frac{N-k}{N} = 1 - \frac{k}{N}$$

2. Suppose that a time series follows a following linear trend process:

$$X_t = c + bt + a_t$$

where b and c are constants. The term a_t is a white noise with mean 0 and variance σ_a^2.

a) Prove for a simple exponential smoothing

$$E[S_T] = c + bT - \frac{1-\alpha}{\alpha} b = E[X_T] - \frac{1-\alpha}{\alpha} b$$

b) Prove for a double exponential smoothing

$$E[S_T^{(2)}] = E[S_T] - \frac{1-\alpha}{\alpha} b$$

2-(a)

$$S_T = \alpha \cdot \sum_{i=0}^{\infty} (1-\alpha)^i X_{T-i}$$

$$E[S_T] = \alpha \cdot \sum_{i=0}^{\infty} (1-\alpha)^i E[X_{T-i}] = \alpha \sum_{i=0}^{\infty} (1-\alpha)^i [c + b(T-i)]$$

$$= \alpha(c+bT) \sum_{i=0}^{\infty} (1-\alpha)^i - \alpha \cdot b \sum_{i=1}^{\infty} i(1-\alpha)^i$$

$$= \alpha(c+bT)(1/\alpha) + b\alpha \cdot (1-\alpha) \frac{d}{d\alpha} \sum_{i=1}^{\infty} i(1-\alpha)^{i-1}$$

$$= \alpha(c+bT)(1/\alpha) + b\alpha(1-\alpha)(-1/\alpha^2)$$

$$= c + bT - b(1-\alpha)/\alpha = E[X_T] - \frac{1-\alpha}{\alpha} b$$

2-(b)

$$E[S_T^{(2)}] = E[S_T] - \frac{1-\alpha}{\alpha} b$$

$$\Rightarrow S_T^{(2)} = \alpha \cdot S_T^{(1)} + (1-\alpha) S_{T-1}^{(2)} = \alpha \cdot \sum_{j=0}^{n-1} (1-\alpha)^j S_{T-j}^{(1)} + (1-\alpha)^T S_0^{(2)}$$

• 이때 $T \to \infty$에 따라

(i) $\lim_{T\to\infty} \alpha \cdot \sum_{j=0}^{n-1} (1-\alpha)^j = \frac{\alpha}{1-(1-\alpha)} = 1$

(ii) $\lim_{T\to\infty} (1-\alpha)^T S_0^{(2)} = 0$

$\therefore$ By (i), (ii), $S_T^{(2)} = \alpha \sum_{j=0}^{T-1} (1-\alpha)^j S_{n-T}^{(1)}$

$$E[S_T^{(2)}] = E[\alpha \sum_{j=0}^{T-1} (1-\alpha)^j S_{T-j}^{(1)}] = \alpha \sum_{j=0}^{T-1} (1-\alpha)^j E[S_{T-j}^{(1)}]$$

$$= \alpha \sum_{j=0}^{T-1} (1-\alpha)^j E[c + b(n-j+1) - \frac{1}{\alpha} b]$$

$$= \underbrace{\alpha \sum_{j=0}^{T-1} (1-\alpha)^j E[c + b(n+1) - \frac{1}{\alpha} b]}_{①} - \underbrace{\alpha b \cdot \sum_{j=0}^{T-1} (1-\alpha)^j j}_{②}$$

①: By (i), ① $= E[c + b(n+1) - \frac{1}{\alpha} b] = E[S_n^{(1)}]$

②: $A \overset{\text{let}}{=} \sum_{j=0}^{\infty} (1-\alpha)^j j = 0 + (1-\alpha) + 2 \cdot (1-\alpha)^2 + \cdots$

$$\begin{array}{r} \quad\ \alpha \cdot A = \quad\quad\quad\quad\quad\quad\quad\quad\quad (1-\alpha)^2 + \cdots \\ \hline (1-\alpha) A = (1-\alpha) + (1-\alpha)^2 + \cdots \end{array}$$

$\therefore A = \frac{1-\alpha}{\alpha^2}$.

$\therefore$ ② $= \alpha \cdot b = \frac{1-\alpha}{\alpha^2} = b \cdot \frac{1-\alpha}{\alpha}$

따라서 $E[S_T^{(2)}] = E[S_T^{(1)}] - \frac{1-\alpha}{\alpha} b$.

3. Consider series J05 (gold prices in US dollars).

a) Obtain double moving averages with N=3 and N=6, respectively and make graphs comparing with the original series.

b) Obtain double exponential smoothing with $\alpha = 0.1$ and $\alpha = 0.3$, respectively and make graphs comparing with the original series.

c) Apply Holt's model with $\alpha = 0.1$ to the data until 1999 and make one-step-ahead forecasts from 2000 to 2017. Calculate forecasting performance measures based on the forecast errors.

3-a) See the R code (HW1-3.r)

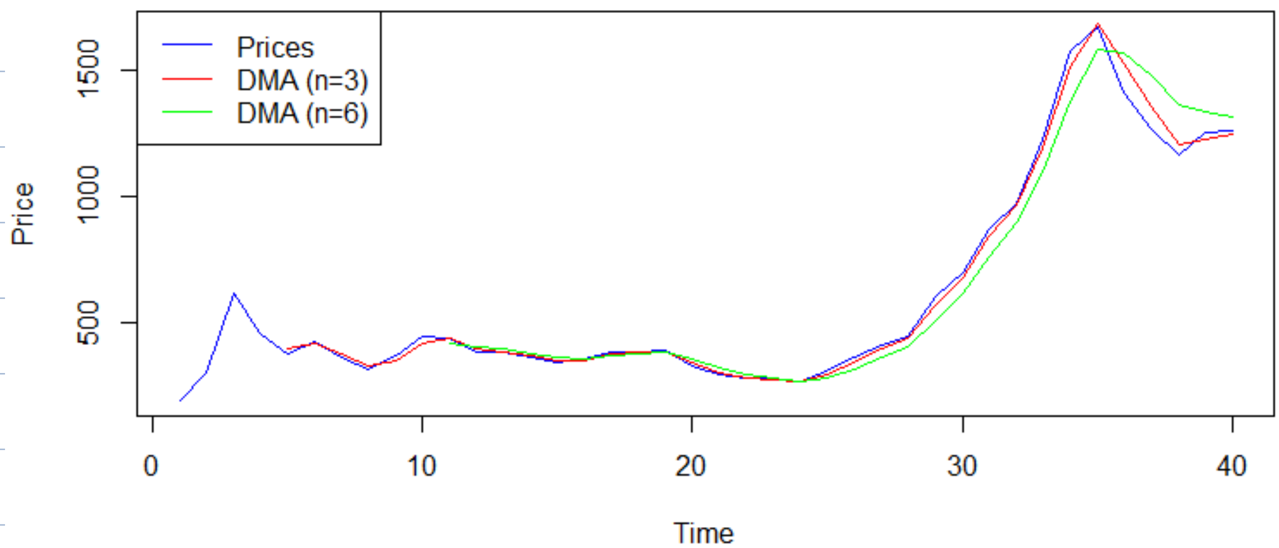
DMA (n=3)

```
> df_ts_dma.3
 [1]        NA        NA        NA        NA  437.7911  453.9689  429.6689  391.0778
 [9]  367.4900  364.3378  380.9144  405.1333  412.8967  399.0867  379.7389  364.7156
[17]  360.4133  364.6633  374.6678  376.3667  363.6144  335.6011  307.6622  287.1578
[25]  282.2544  292.5078  320.6700  360.3500  417.3889  490.8822  596.9022  717.1589
[33]  864.4056 1041.8800 1255.8044 1431.6822 1495.9289 1426.2256 1317.9511 1242.5511
```

DMA (n=6)

```
> df_ts_dma.6
 [1]        NA        NA        NA        NA        NA        NA        NA  414.4344
 [9]  410.7294  397.0033  385.9961  386.3117  388.6172  390.0006  392.4361  388.8061
[17]  379.7500  372.2011  369.6917  368.3900  364.1389  355.1344  342.0144  325.3861
[25]  308.9278  300.0850  303.9139  321.3022  354.9483  405.7761  478.6261  567.2739
[33]  677.6439  819.3911  986.4817 1148.0439 1268.9044 1341.0150 1374.8167 1369.2400
```

Graph



3-b) See the R code (HW1-3.r)

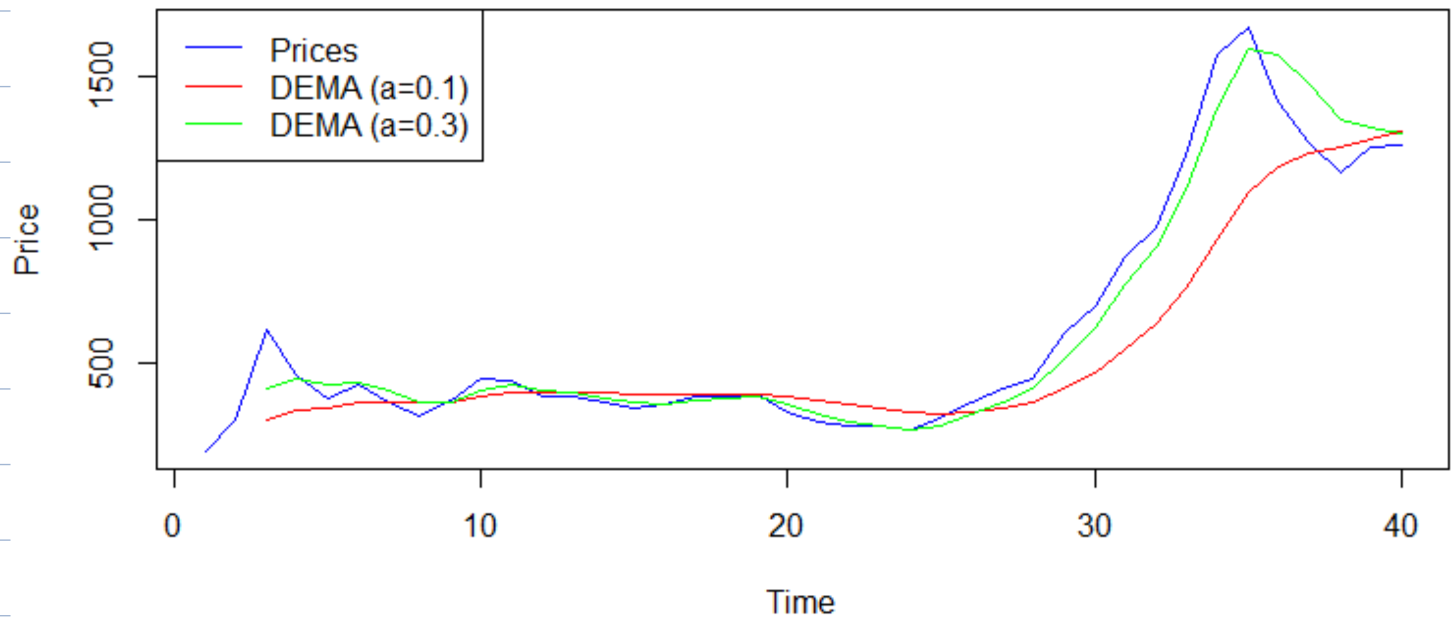
$\alpha=0.1$, $n=2$로 설정

```
> dema.0.1
 [1]        NA        NA  303.8760  335.0434  345.5788  363.5981  366.8471  361.1570
 [9]  365.7139  384.3167  398.2255  399.2304  400.3809  397.0976  390.6109  387.9053
[17]  390.0556  391.7193  393.7160  384.5247  369.4794  353.5900  340.0700  326.9778
[25]  323.1553  330.1159  344.7976  364.0367  410.6952  467.8272  549.9281  638.7242
[33]  761.8976  932.2243 1095.0992 1183.7943 1230.3939 1248.2950 1279.1532 1305.0713
```

$\alpha=0.3$, $n=2$로 설정

```
> dema.0.3
 [1]        NA        NA  413.5080  448.3529  423.5889  431.7722  402.9831  362.9033
 [9]  364.9405  406.1692  425.2023  406.9973  397.1183  380.1943  360.9494  358.0831
[17]  369.2433  376.0024  381.9868  356.5929  322.9006  295.8832  281.3568  269.9251
[25]  284.1304  320.7468  365.9039  409.9177  515.8455  622.3896  771.2169  904.4081
[33] 1104.3932 1390.1673 1596.2230 1572.2859 1472.1984 1350.4003 1319.8660 1301.9267
```

Graph



3-c) See the R code (HW1-3.r)

```
> forecast$accurate
          SST           SSE           MSE          RMSE          MAPE           MPE
 1.366073e+05  3.418239e+05  1.709119e+04  1.307333e+02  2.389962e+01 -3.634062e+00
          MAE            ME     R.squared R.adj.squared RW.R.squared           AIC
 9.260834e+01  1.205199e+00 -1.502238e+00 -1.627350e+00  1.597243e+19  2.163222e+02
          SBC           APC
 2.185043e+02  1.864494e+04
```

4. Consider series J04 (electricity consumption in residential & commercial use). We would like to apply Winters' model.

a) Obtain suitable initial values for this model using data in first three years.

b) Obtain smoothed values of level, trend, and seasonality using data from 2000 to 2004.

c) Make one-step-ahead forecasts from year 2004 to 2017 and calculate forecast errors.

4-a) See the R code (HW1-4.r)

$L_0 = 1583.917$

$b_0 = 21.968$

4-b) See the R code (HW1-4.r)

$L_0 = 2957.474$

$b_0 = 64.375$

$S_1 = 307.473$

$S_2 = -211.33$

$S_3 = -51.776$

$S_4 = -202.47$

4-c) See the R code.

```
> (x.2004_2017-hw.2004_2017.pred)
          Qtr1         Qtr2         Qtr3        Qtr4
6   728.677859   497.108754   456.177797  594.500000
7   774.177859   466.608754   563.677797  487.000000
8   723.677859   235.108754   233.177797  191.500000
9   490.177859     6.608754   101.677797   45.000000
10  203.677859  -211.891246   -89.822203 -262.500000
11 -228.822141  -560.391246  -505.322203 -466.000000
12 -369.322141  -735.891246  -613.822203 -756.500000
13 -521.822141  -943.391246  -536.322203 -872.000000
14 -778.322141 -1145.891246  -710.822203 -984.500000
```