1-a) 20232863 배기웅

$X_t = c + a_t$

$E[M_T] = E[X_T] = c,\ Var[M_T] = \frac{\sigma_a^2}{N}$

$Cov[M_t, M_{t-k}] = \frac{\sigma_a^2}{N} - \frac{\sigma_a^2}{N} \times \frac{k}{N} = \sigma_a^2 (N-k)/N^2$

1-b)

$\rho[M_t, M_{t-k}] = \frac{\sigma_a^2 (N-k)/N^2}{\sqrt{\frac{\sigma_a^2}{N} N} \cdot \sqrt{\frac{N\sigma_a^2}{N^2}}} = \frac{N-k}{N} = 1 - \frac{k}{N}$

2-(a)

$E[S_T] = c + bT - \frac{1-\alpha}{\alpha} b = E[X_T] - \frac{1-\alpha}{\alpha} b$ (simple ES)

proof) $E[S_T^{(1)}] = \alpha \sum_j (1-\alpha)^j E[X_T]$

$= \alpha \sum_j (1-\alpha)^j E[c + bt + a_t]$

$= \alpha \cdot \frac{1}{1-(1-\alpha)} E[c + bt + a_t]$

$= E[c + bt + a_t]$

$= E[c] + b \cdot E[t] + 0 = c + bT$

2-(b)

$E[S_T^{(2)}] = E[S_T] - \frac{1-\alpha}{\alpha} b$

$\Rightarrow S_T^{(2)} = \alpha \cdot S_T^{(1)} + (1-\alpha) S_{T-1}^{(2)} = \alpha \cdot \sum_{j=0}^{n-1} (1-\alpha)^j S_{T-j}^{(1)} + (1-\alpha)^T S_0^{(2)}$

• 이제 $T \to \infty$ 일 때

(i) $\lim_{T\to\infty} \alpha \cdot \sum_{j=0}^{n-1} (1-\alpha)^j = \frac{\alpha}{1-(1-\alpha)} = 1$

(ii) $\lim_{T\to\infty} (1-\alpha)^T S_0^{(2)} = 0$

$\therefore$ By (i), (ii), $S_T^{(2)} = \alpha \sum_{j=0}^{n-T} (1-\alpha)^j S_{n-T}^{(1)}$

$$E[S_T^{(2)}] = E[\alpha \sum_{j=0}^{T-1} (1-\alpha)^j S_{T-j}^{(1)}] = \alpha \sum_{j=0}^{T-1} (1-\alpha)^j E[S_{T-j}^{(1)}]$$

$$= \alpha \sum_{j=0}^{T-1} (1-\alpha)^j E[c + b(n-j+1) - \frac{1}{\alpha} b]$$

$$= \underbrace{\alpha \sum_{j=0}^{T-1} (1-\alpha)^j E[c + b(n+1) - \frac{1}{\alpha} b]}_{①} - \underbrace{\alpha b \cdot \sum_{j=0}^{T-1} (1-\alpha)^j j}_{②}$$

① : By (i), ① $= E[c + b(n+1) - \frac{1}{\alpha} b] = E[S_n^{(1)}]$

② : $A \overset{let}{=} \sum_{j=0}^{\infty} (1-\alpha)^j j = 0 + (1-\alpha) + 2 \cdot (1-\alpha)^2 + \cdots$

$\alpha \cdot A = \quad (1-\alpha)^2 + \cdots$

$(1-\alpha) A = (1-\alpha) + (1-\alpha)^2 + \cdots$

$\therefore A = \frac{1-\alpha}{\alpha^2}$.

$\therefore$ ② $= \alpha \cdot b = \frac{1-\alpha}{\alpha^2} = b \cdot \frac{1-\alpha}{\alpha}$

따라서 $E[S_T^{(2)}] = E[S_T^{(1)}] - \frac{1-\alpha}{\alpha} b$.

3-a) See the R code (HW1-3.r)

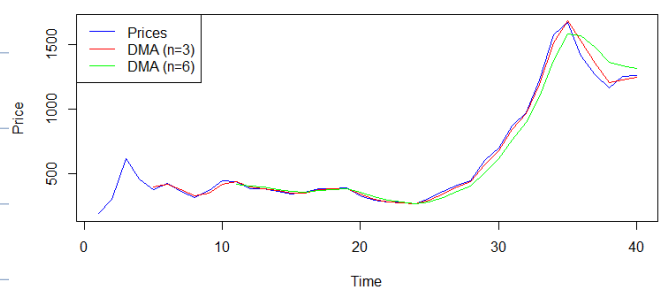
DMA (n=3)

```
> df_ts_dma.3
 [1]        NA        NA        NA        NA  437.7911  453.9689  429.6689  391.0778
 [9]  367.4900  364.3378  380.9144  405.1333  412.8967  399.0867  379.7389  364.7156
[17]  360.4133  364.6633  374.6678  376.3667  363.6144  335.6011  307.6622  287.1578
[25]  282.2544  292.5078  320.6700  360.3500  417.3889  490.8822  596.9022  717.1589
[33]  864.4056 1041.8800 1255.8044 1431.6822 1495.9289 1426.2256 1317.9511 1242.5511
```

DMA (n=6)

```
> df_ts_dma.6
 [1]        NA        NA        NA        NA        NA        NA        NA  414.4344
 [9]  410.7294  397.0033  385.9961  386.3117  388.6172  390.0006  392.4361  388.8061
[17]  379.7500  372.2011  369.6917  368.3900  364.1389  355.1344  342.0144  325.3861
[25]  308.9278  300.0850  303.9139  321.3022  354.9483  405.7761  478.6261  567.2739
[33]  677.6439  819.3911  986.4817 1148.0439 1268.9044 1341.0150 1374.8167 1369.2400
```

Graph



3-b) See the R code (HW1-3.r)

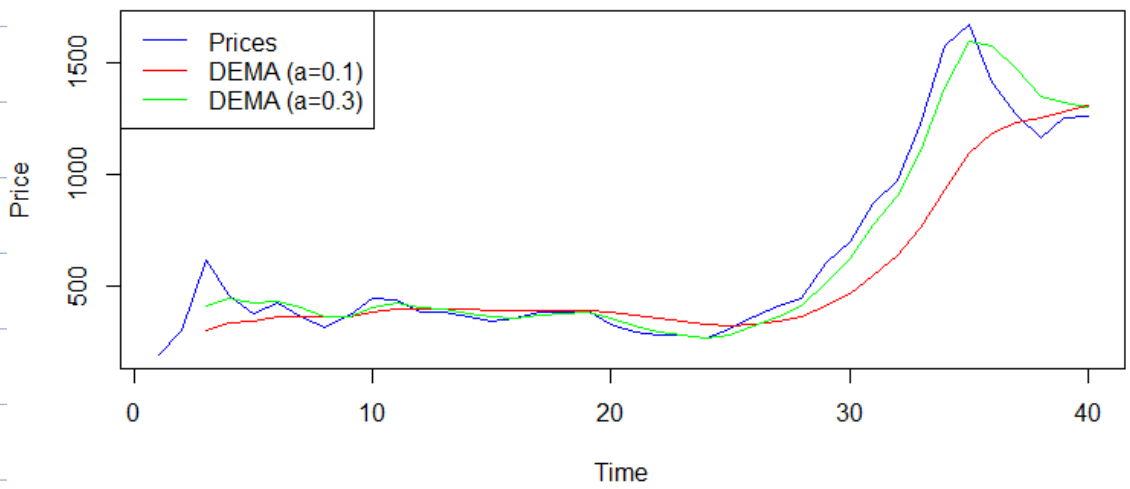
$\alpha = 0.1$

```
> dema.0.1
  [1]        NA        NA  303.8760  335.0434  345.5788  363.5981  366.8471  361.1570
 [9]  365.7139  384.3167  398.2255  399.2304  400.3809  397.0976  390.6109  387.9053
[17]  390.0556  391.7193  393.7160  384.5247  369.4794  353.5900  340.0700  326.9778
[25]  323.1553  330.1159  344.7976  364.0367  410.6952  467.8272  549.9281  638.7242
[33]  761.8976  932.2243 1095.0992 1183.7943 1230.3939 1248.2950 1279.1532 1305.0713
```

$\alpha = 0.3$

```
> dema.0.3
  [1]        NA        NA  413.5080  448.3529  423.5889  431.7722  402.9831  362.9033
 [9]  364.9405  406.1692  425.2023  406.9973  397.1183  380.1943  360.9494  358.0831
[17]  369.2433  376.0024  381.9868  356.5929  322.9006  295.8832  281.3568  269.9251
[25]  284.1304  320.7468  365.9039  409.9177  515.8455  622.3896  771.2169  904.4081
[33] 1104.3932 1390.1673 1596.2230 1572.2859 1472.1984 1350.4003 1319.8660 1301.9267
```

Graph



3-C) See the R code (HW1-3.r)

```
> forecast$accurate
           SST           SSE           MSE          RMSE          MAPE           MPE
1.366073e+05  3.418239e+05  1.709119e+04  1.307333e+02  2.389962e+01 -3.634062e+00
           MAE            ME     R.squared R.adj.squared  RW.R.squared           AIC
9.260834e+01  1.205199e+00 -1.502238e+00 -1.627350e+00  1.597243e+19  2.163222e+02
           SBC           APC
2.185043e+02  1.864494e+04
```

4-a) See the R code (HW1-4.r)

- $L_0 = 1583.917$

  $b_0 = 21.968$

4-b) See the R code (HW1-4.r)

$L_0 = 2957.414$

$b_0 = 64.375$

$S_1 = 307.413$

$S_2 = -211.33$

$S_3 = -51.776$

$S_4 = -202.47$

4-c) See the R code.