2024 Summer Seminar

A simulation model for calculating solvency capital requirements for non-life insurance risk

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Information

- **Title:** A simulation model for calculating solvency capital requirements for non-life insurance risk
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- **Journal:** Scandinavian Actuarial Journal
- **Year:** 2014

목차

Framework

Framework of this study

- Distribution of $p_{ij}^{(n)}$
- Distribution of $Y_i^{(n)}$
- Trend
- Correlations

$\rightarrow$ **Simulation** $\xrightarrow{\text{Calculate}}$

- BE (Best estimate)
- Technical provisions
- Risk margin
- SCR

Real data
- V (Volume measure)
- ρ (correlations)

$\rightarrow$ **Solvency II standard model formulas** $\xrightarrow{\text{Calculate}}$ SCR

Compare

Contribution of this study

- Compared SCR, best estimates, risk margins and technical provisions of different scenarios
- Compared the SCR calculated using the Solvency II standard model with the SCR values that simulation model generates
- Discussed pros and cons of using the simulation model for determining SCR, and what impact the results may have on future insurance-risk modelling

2.1 Notation and assumptions

Notation	Explanation	Notation	Explanation
$\boldsymbol{N}$	• Types of insurance, $N \in \{1, 2, 3\}$	$\boldsymbol{p}_{\mathbf{i}}^{(\boldsymbol{n})}$	• payment pattern for accident period i and insurance type n • $p_{ij}^{(n)} = \frac{X_{ij}^{(n)}}{C_{iJ}^{(n)}}$
$\boldsymbol{K}$	• accident periods of equal length	$\boldsymbol{Y_i}$	• the normalized ultimate claim amounts of accident period i • $Y_i^{(n)} = \frac{C_{iJ}^{(n)}}{E_i^{(n)}}$
$\boldsymbol{I}$	• the most recent accident period	Q_{kl}	• the interest rate at time k of a non-defaultable zero-coupon bond maturing at time l
$\boldsymbol{J}$	• ultimate development period • represents how the amount of damages or claims changes over time.	B_{kl}	• **discount factor**, from time k to time l
$\boldsymbol{X_{ij}^{(n)}}, \boldsymbol{C_{ij}^{(n)}}$	• incremental and cumulative claim amounts for accident period i, development period j and insurance type n • *($X_{ij}^{(n)}$ can be interpreted as a liabilities in this paper)*	F_t	• the information available at time t
M	• the number of policies of type n at time i	X_t	• the amount to be paid to policyholders at time t
$E_i^{(n)}$	• the insurer's exposure in terms of one-year policy equivalents • $E_i^{(n)} = \frac{M_{i-1}^{(n)} + M_i^{(n)}}{2K}$	L_I, A_I	• the value today of the insurer's liability and assets, respectively.

2.2 Solvency risk

Solvency Risk?

- It is the risk that it, given some valuation methods, will **not have enough assets to cover its liabilities** at some future point in time (based on 1 year)
  - They only considered what can happen to the values of assets and liabilities **on year into the future**, so they focused their attention on **what we know today (at time I)** and **what we will know in one year (at time $I+K$).**

- BE, which is an unbiased best estimate of the **present value of the outstanding-loss-liability** cash flows, is given by:
  - $BE = \sum_{t=I+1}^{I+J+K} B_{It}\, \hat{E}[X_t|F_I]$
  - $\hat{E}[X_t|F_I]$ is to be interpreted as an unbiased prediction of X_t given the information F_I using some (predefined) actuarial method.

- Let L_I and L_{I+K} denote the value today and in one year, respectively, of the **insurer's liabilities**.
  - $L_u = \sum_{t=u+1}^{I+J+K} B_{ut}\ \hat{E}[X_t|F_u]$ for $u = I, I+K$
- Let A_I and A_{I+K} denote the value today and in one year, respectively, of the **insurer's assets**.
  - $A_{I+K} = \frac{A_I}{B_{I,I+K}} - \sum_{t=I+1}^{I+K} \frac{X_t}{B_{t,I+K}}$

2.2 Solvency risk

- Value-at-risk at the level 0.005 will be our risk measure, as proposed in the Solvency II framework.
- If X is the value of a stochastic portfolio at time t, and $\alpha \in (0,1)$, then value-at-risk at the level α is define by:
  - $VaR_\alpha(X) := B_{It}\, F_{-X}^{-1}(1-\alpha),\ t \geq I$
  - where F_{-X}^{-1} is the inverse of the distribution function (i.e. the quantile function) of $-X$ (note the minus sign), and B_{It} is the price today of a zero-coupon bond with principal 1 maturing at time t.

- In one year, the value of the insurer's portfolio will be $A_{I+K} - L_{I+K}$, and the regulator will conclude that the insurer has enough assets to cover its liabilities if $VaR_{0.005}(A_{I+K} - L_{I+K}) \leq 0$, which is equivalent $A_I \geq L_I + VaR_{0.005}(\Delta)$
  - $\Delta := A_{I+K} - \frac{A_I}{B_{I,I+K}} - (L_{I+K} - \frac{L_I}{B_{I,I+K}})$
  - The random variable Δ tells us how the insurer's balance (i.e. the difference between assets and liabilities) changes over the coming year.

- To set the change in balance (i.e. the profit or loss) in relation to the size of the insurer's liability portfolio, we construct the normalized loss statistic U.
  - $U := -B_{I,I+K} \frac{\Delta}{BE}$

2.2 Solvency risk

- The SCR are the minimum amount by which the present asset value must exceed the present liability value.
  - $SCR := VaR_{0.005}(\Delta) = BE\ F_U^{-1}(0.995)$
- An external investor willing to take over the insurer's outstanding loss liabilities would demand an amount of assets to balance these liabilities.
- Assuming that the investor must hold capital equal to the calculated SCR for the duration of the liabilities, one could argue that they would demand an amount of assets that is $C * T * SCR$, which is *risk margin* (RM).
  - C is the investor's cost-of-capital rate (e.g. 6%)
  - T is the estimated duration of the liabilities, and $T = \sum_{t=I+1}^{I+J+K} \frac{t-1}{K} \hat{E}[X_t|F_I] / \sum_{u=I+1}^{I+J+K} \hat{E}[X_u|F_I]$.

- The technical provisions(TP), i.e. the estimated 'market value' of the liabilities, are the sum of the best estimate and the risk margin, $TP \coloneqq BE + RM$.

2.3 Actuarial prediction methods

Individual-line method

- We consider a specific insurance type (or an individual line of business), say n.
- We denote the chain-ladder predictions of C_{ij} and X_{ij} at time I by $\hat{E}\left[C_{ij}\middle|F_I\right]$ and $\hat{E}\left[X_{ij}\middle|F_I\right]$, respectively.
- To use the chain-ladder method, we need at least one payment, so we cannot use it directly for future accident periods.
    - Instead, we look at the amounts $Y_1, \dots, Y_I$.
- The incremental claim amounts for $i > I$ are now predicted by $\hat{E}\left[X_{ij}\middle|F_I\right] = \widehat{p_j^I}\ E_i\ \hat{E}[Y_i|F_I]$
    - $\widehat{p_j^I}$ is the chain-ladder estimate of the proportion paid in development period j given the information F_I.

Aggregate method

- We treat all insurance policies as if they were of the same type, i.e. we let $X_{ij} = \Sigma_n X_{ij}^{(n)},\ C_{ij} = \Sigma_n C_{ij}^{(n)}$, and $E_i = \Sigma_n E_i^{(n)}$.

2.3 Actuarial prediction methods

Chain-ladders method

- It is a prominent actuarial loss reserving technique.
- Its intent is to estimate incurred but not reported claims and project ultimate loss amounts.
- Its assumption is that historical loss development patterns are indicative of future loss development patterns.

2.3 Actuarial prediction methods

Chain-ladders method (example)

Example 5.6

Consider the run-off triangle from Example 5.5:

Cumulative claim payments		Development Year			
		0	1	2	3
Accident Year	2011	600	680	720	740
	2012	620	695	730	
	2013	680	760		
	2014	720			

development triangle

Use the chain ladder method to estimate the cumulative claim payment in Development Year 3 for Accident Years 2012 to 2014.

1. Calculate the **developments factors**

Development Year 2 to Development Year 3	$\frac{740}{720} = 1.027778$
Development Year 1 to Development Year 2	$\frac{720 + 730}{680 + 695} = 1.054545$
Development Year 0 to Development Year 1	$\frac{680 + 695 + 760}{600 + 620 + 680} = 1.123684$

2. Estimate the **cumulative claim payment** in Accident Year 2012, 2013, 2014

Cumulative claim payment on 2012	$730 * 1.027778 = 750.2778$
Cumulative claim payment on 2013	$760 * 1.054545 * 1.027778 = 823.7172$
Cumulative claim payment on 2014	$720 * 1.123684 * 1.054545 * 1.027778 = 876.8823$

3. Data analysis

Purpose

- Analysis of historical payment patterns($p_{ij}^{(n)}$) and normalized ultimate claim amounts($Y_i^{(n)}$) → **Suggest Distribution.**

Data

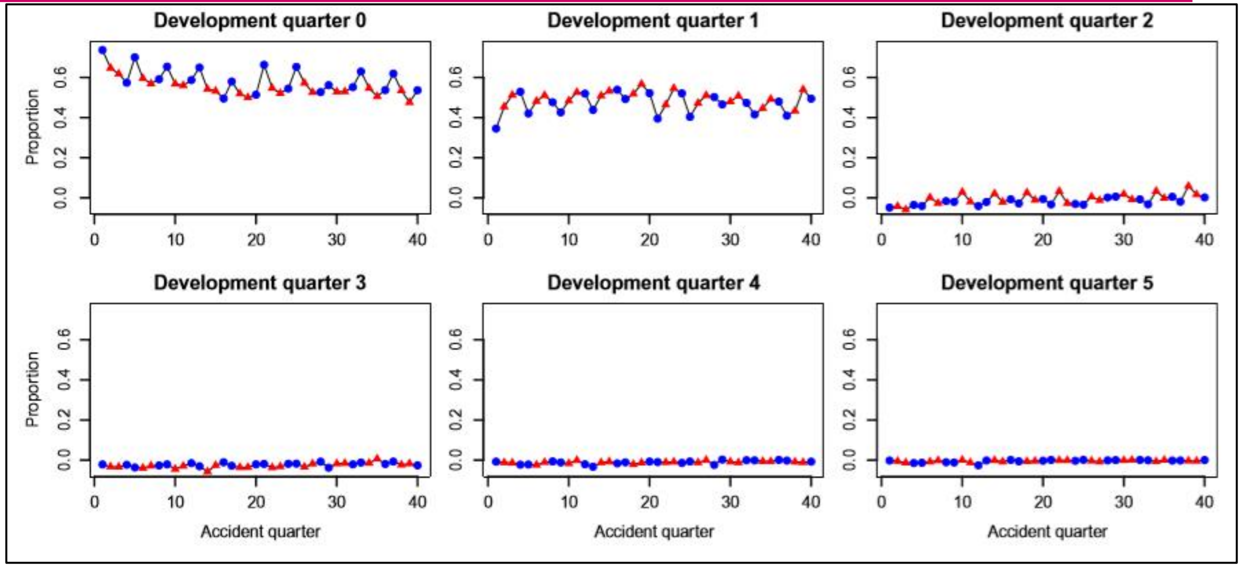
- Folksam's claim payments and exposures for three motor insurance types
  - collision insurance ($n = 1$), major first-party insurance ($n = 2$), and third-party property insurance ($n = 3$).
  - all accident quarters from 1998 to 2007 (40 accident periods, 9 development periods)

- Car Insurance types
  - Collision insurance: it is a coverage that helps pay to repair or replace your car if it's damaged in an accident with another vehicle or object. After paying a deductible, received the rest of expenses.
  - First party: it covers damages sustained by the owner or driver of a vehicle in an accident. This coverage is more comprehensive than collision insurance (n=1).
  - Third party: it covers damage an insured causes to another's property in an accident (i.e. repair amount on other's car)

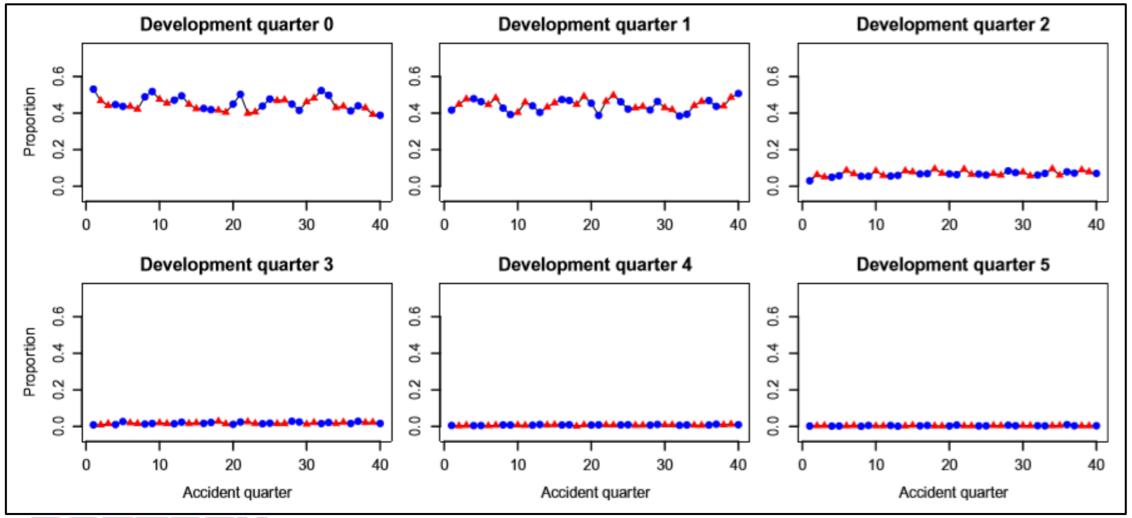
3.1 Payment patterns

Analysis of historical payment patterns($p_{ij}^{(n)}$)

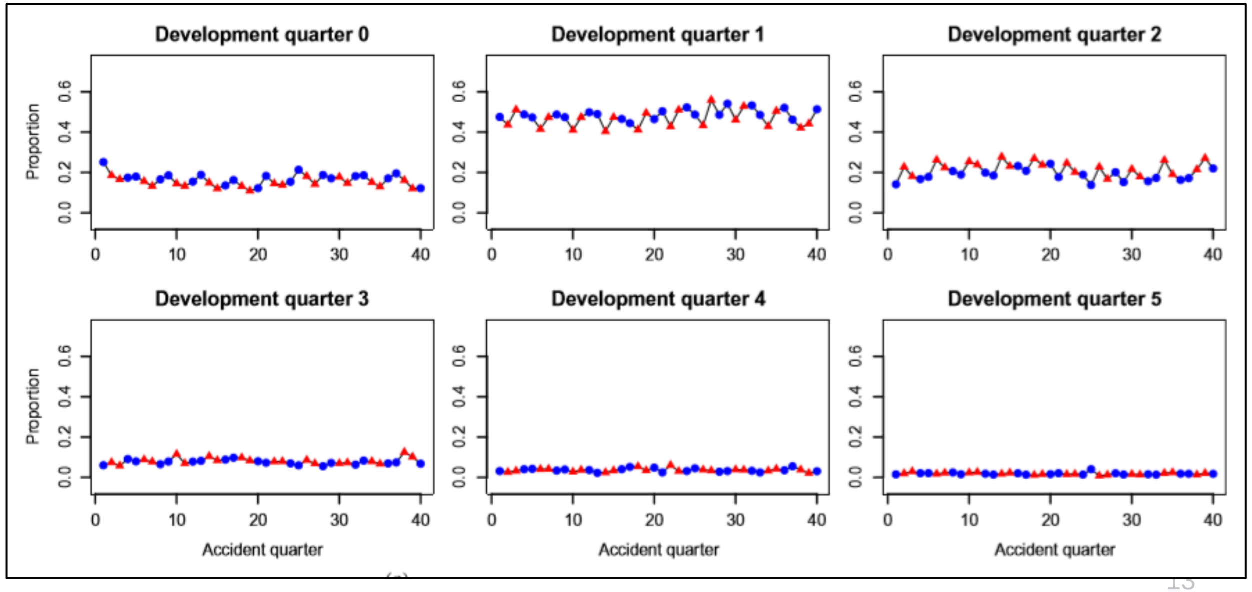
- Empirical payment proportions ($p_{ij}^{(n)}$) by considering the data plots.
  - **Distinct seasonal pattern**, especially for **collision insurance**: where accidents in the first quarter of each year are handled more quickly than later in the year. (quarter 0 > quarter 5)
  - **Negative proportions** in **development quarter 2,** especially **n=1**: Because Folksam pays their policyholders before it is clear who caused the accident.
  - Variation between different accident years, but in general there are no clear trends over time.



Collision insurance ($n = 1$)



Major first-party insurance ($n = 2$)



Third-party property insurance ($n = 3$)

3.1 Payment patterns

Distribution of historical payment patterns$(\boldsymbol{p_{ij}^{(n)}})$

- We will use the **Dirichlet distribution** to simulate payment patterns.
  - This distribution only handles non-negative payment proportions, we must modify the sample means in order to be able to use them as distribution parameters
  - The (J+1)-dimensional Dirichlet distribution has a parameter vector $\lambda\pi$, where $\lambda > 0$ is an inverse variability parameter and $\pi = (\pi_0, \dots, \pi_J)^T$ is a mean vector with constraints $\pi_j \geq 0$ and $\Sigma_{j=0}^{J}\pi_j = 1$.

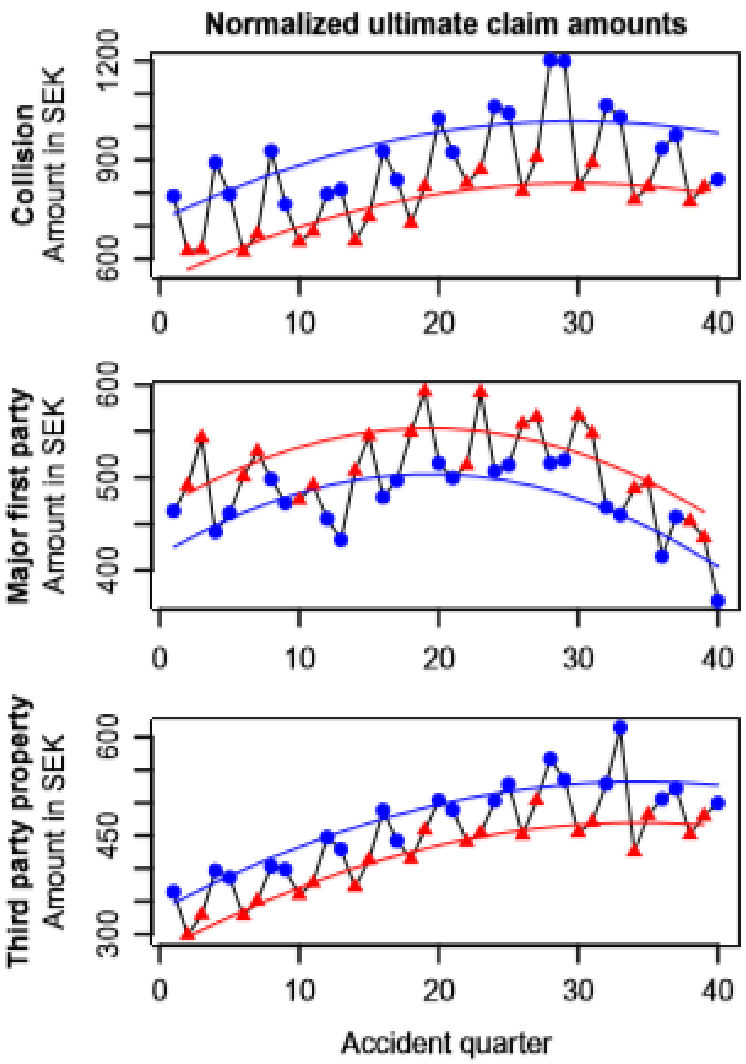
- Dirichlet distribution
  - A random vector $X = (X_1, \dots, X_K)^T$ is distributed according as the Dirichlet distribution with parameters $\alpha_1, \dots, \alpha_K > 0$, denoted $Dir(\alpha_1, \dots, \alpha_K)$, if its pdf is $p(x) = \frac{\Gamma(\alpha_1 + \dots + \alpha_K)}{\Gamma(\alpha_1) * \dots * \Gamma(\alpha_K)} x_1^{\alpha_1 - 1} \dots x_K^{\alpha_K - 1}$, for $x_k > 0$ and $x_1 + \dots + x_K = 1$.

3.2 Normalized ultimate claim amounts

Analysis of historical normalized ultimate claim amounts$(Y_i^{(n)})$

- $Y_i^{(n)} = C_{ij}^{(n)} / E_i^{(n)}$
  - Total amount paid (for claims in a fixed accident period) by the insurer to its policyholders divided by the exposure in terms of number of ‘one-year policy equivalents’.

- **(n=1,3)** the amounts are consistently **higher in the winter quarters** than in the summer quarters of the same accident years: More slippery roads in Sweden during the winter
- **(n=1,3)** **upward trend over time**: due to claims inflation (i.e. price increases for repair work and spare parts) + changes in deductibles
- **(n=2)** the amounts are **higher in the summer quarters** than in the winter quarters: The number of car thefts and fires are more common during the summer.
- **(n=2)** **clear downward trend** in the amounts over the last few years: New cars are much more difficult to steal than older ones + number of car thefts decreases as people buy new cars.



3.2 Normalized ultimate claim amounts

Analysis of historical <u>seasonally adjusted</u> normalized ultimate claim amounts$(Y_i^{(n)})$

- The amounts are clearly non-stationary, so to be able to compare different accident quarters, we must adjust for both seasonal variation and trends over longer time frames.
- The trend and seasonally adjusted amounts seem rather stationary over time.
- The normal QQ plots indicate that the data are close to being normally distributed with the means and standard deviations.

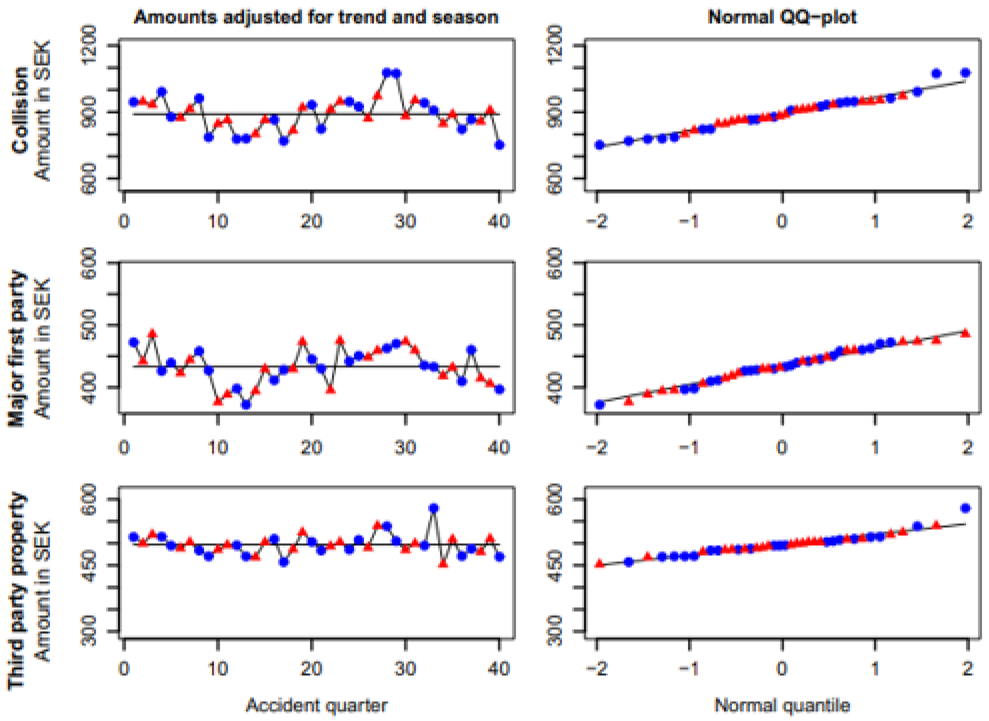


Table 2. Sample means and sample standard deviations of the trend and seasonally adjusted (normalized ultimate claim) amounts ($Y_i^{(n)}$'s). Values in SEK.

	Collision	Major first-party	Third-party property
Mean	887.3	432.7	492.8
SD	71.6	25.1	23.6

3.2 Normalized ultimate claim amounts

Linear dependence between each pair of insurance types

- The sample correlation coefficients are:
  - **0.67** between collision and major first-party
  - **0.61** between collision and third-party property
  - **0.41** between major first-party property

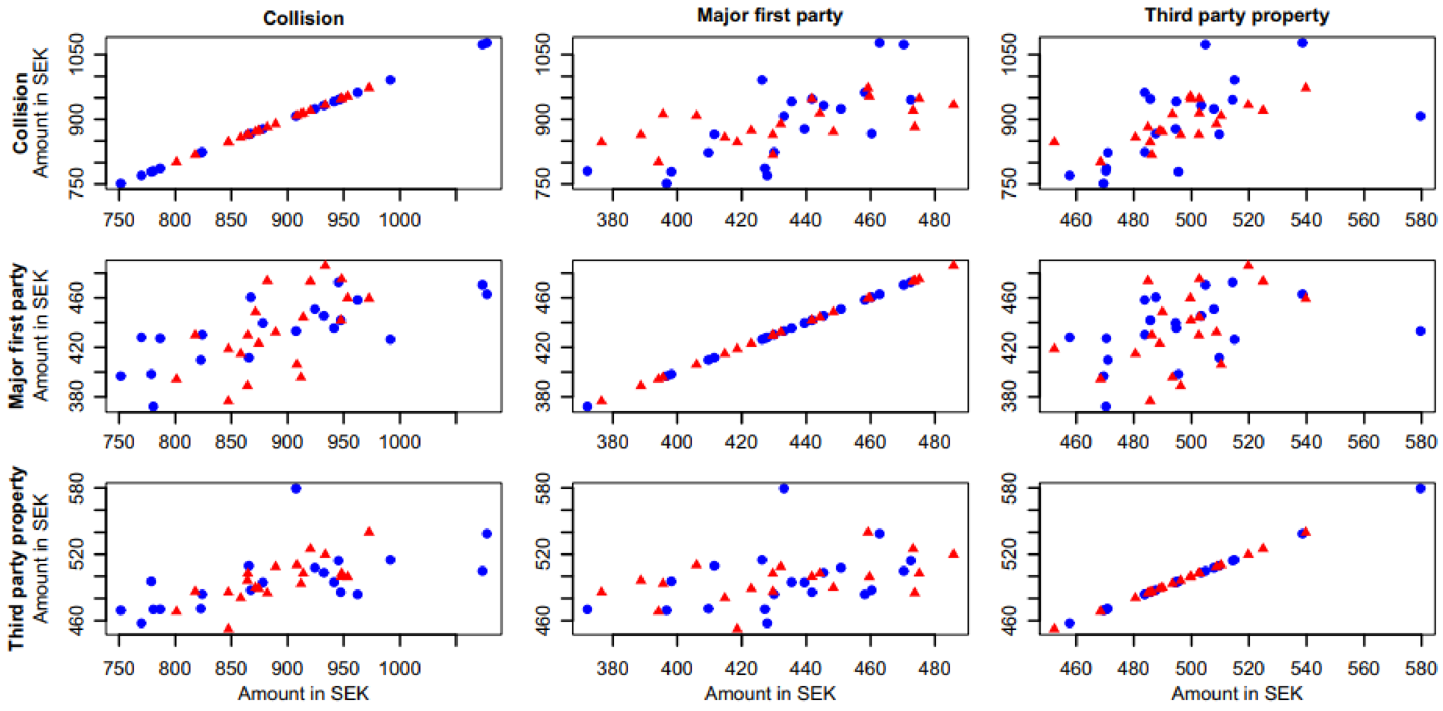


Figure 3. Scatter plots of trend and seasonally adjusted amounts.

4.1 Simplifications and general assumptions

Purpose

- We set up five different scenarios.
- For each scenarios, they specified distributions of the payment patterns$(p_{ij}^{(n)})$ and the normalized ultimate claim amounts$(Y_i^{(n)})$.

Assumptions of this simulation

1. The normalized ultimate claim amounts $(Y_i^{(n)})$ are **independent** of the payment patterns $(p_{ij}^{(n)})$.
2. Since the claims studied are handled quickly, they simplified by setting all zero-coupon rates to zero, i.e. $Q_{kl} = 0$ for all k and l, ignoring the market and interest-rate risks.
3. We set the cost-of-capital rate c to 0.06 suggested in the Solvency II framework $(C * T * SCR)$.
4. We divide each year into quarters (K=4) and simulate data for the latest 40 quarters (I=40) and the coming four quarters.
5. No payments are made later than 8 quarters after the quarter in which the accident happens (J=8).
6. The insurance types in the simulations are the same as the three.

4.2 Scenario specif

Table 2. Sample means and sample standard deviations of the trend and seasonally adjusted (normalized ultimate claim) amounts ($Y_i^{(n)}$'s). Values in SEK.

	Collision	Major first-party	Third-party property
Mean	887.3	432.7	492.8
SD	71.6	25.1	23.6

- Below follows the **five scenarios** specific distribution and parameter choices.

	Distribution of the normalized ultimate claim amounts	Correlation	Trend	In detail
1	Multivariate normal	X	X	$Y_1, \ldots, Y_{I+K}$ are i.i.d random vectors from an N-dimensional normal distribution with diagonal covariance matrix. (μ & σ on Table 2 above)
2	Multivariate normal	O	X	Covariance distribution is chosen to get the correlations between insurance types suggested by the data analysis.
3	Multivariate normal	O	O	The means are the same as the two previous scenarios up to time I, but increase thereafter by 2% per quarter.
4	Multivariate Student's t	O	X	$Y_1, \ldots, Y_{I+K}$ are i.i.d random vectors from an N-dimensional Student's t-distribution. To examine what happens to the SCR if the distribution is more heavy-tailed than real data, the degrees of freedom parameter is set to 3.
5	Multivariate log-normal	O	X	$Y_1, \ldots, Y_{I+K}$ are i.i.d random vectors from an N-dimensional log-normal distribution.

4.3 Simulation procedure and actuarial calculations

Simulation preparations

- 10,000 simulations
- Each simulation has the following steps:
  - Amounts and payments are simulated given the scenario specific distributions.
  - Development triangles are created given the information available at times I (today) and $I+K$ (in one year), respectively.
  - Calculated $\boldsymbol{\hat{E}[X_t|F_I]}$ and $\hat{E}[X_t|F_{I+K}]$ using the triangles in the previous step and the prediction methods.
  - In conclusion, calculated **best estimate (BE), balance change (Δ), loss statistic (U), and duration (T).**

- Actuarial calculation formulas
  - $BE = \sum_{t=I+1}^{I+J+K} B_{It}\, \hat{E}[X_t|F_I]$
  - $\Delta := A_{I+K} - \frac{A_I}{B_{I,I+K}} - (L_{I+K} - \frac{L_I}{B_{I,I+K}})$ $(\boldsymbol{L_u} = \sum_{t=u+1}^{I+J+K} B_{ut}\, \hat{E}[X_t|F_u],\ \boldsymbol{A_{I+K}} = \frac{A_I}{B_{I,I+K}} - \sum_{t=I+1}^{I+K} \frac{X_t}{B_{t,I+K}})$
  - $U := -B_{I,I+K} \frac{\Delta}{BE}$
  - $T = \sum_{t=I+1}^{I+J+K} \frac{t-1}{K}\, \hat{E}[X_t|F_I] / \sum_{u=I+1}^{I+J+K} \hat{E}[X_u|F_I].$

4.3 Simulation procedure and actuarial calculations

Results of Simulations

- They calculated SCR, best estimates, risk margins, and technical provisions for the different scenarios.
  - The SCR was **affected by the correlation** (it increased about 10% when correlation was added in the normally distributed scenarios) but it was even **more affected by not being able to predict a trend of 2%** per quarter (it increased about 20% when the trend was added).
  - Assuming log-normal amounts instead of normal did not change the values much. However, assuming Student's t-distributed amounts did increase the SCR, and hence increased the RM and TP.
  - The RM was small compared to the SCR, so leaving it outside the SCR calculation did not affect the results much.

Table 5. **Individual-line method:** Simulated best estimates, durations, SCR, risk margins and technical provisions (assuming the cost-of-capital rate $c = 0.06$). BE, SCR, RM and TP in million SEK, T in years.

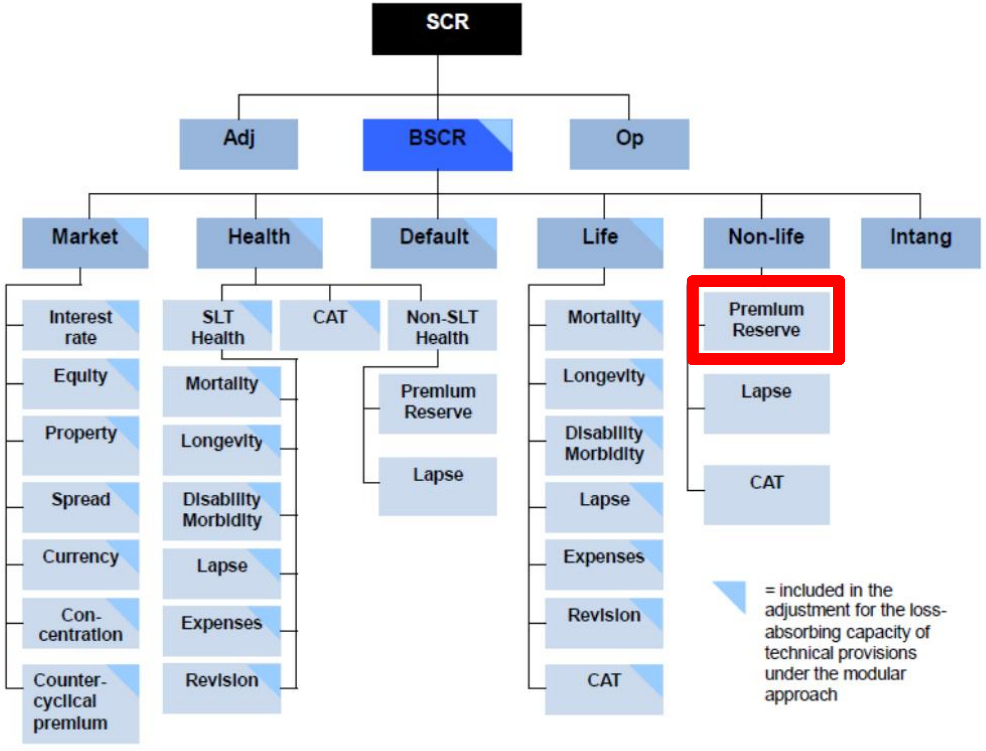
Scenario	Normal No correlation No trend	Normal **Correlation** No trend	Normal **Correlation** **Trend**	Student's t **Correlation** No trend	Log-normal **Correlation** No trend
BE	177.9	177.9	177.9	178.0	178.0
T	0.6	0.6	0.6	0.6	0.6
SCR	25.7	27.8	33.3	29.7	27.9
RM	0.9	1.0	1.2	1.1	1.0
TP	178.9	178.9	179.1	179.0	178.9

5. The Solvency II standard model

- We only consider premium and reserve risk.

- $SCR_{NL} = V * g(\sigma),\ \ g(\sigma) := \frac{e^{N_{0.995}\sqrt{\log(\sigma^2+1)}}}{\sqrt{\sigma^2+1}} - 1$ (European Commission (2010))
  - $N_{0.995}$ is the 0.995 quantile of the standard normal distribution ($N_{0.995} \approx 2.58$), V is a volume measure and σ is the combined standard deviation per volume unit of the non-life LoBs.

- We can rewrite $\boldsymbol{SCR_{NL} = V * F_{\tilde{U}}^{-1}(0.995)}$
  - If $\tilde{U}$ is normally distributed, then $g(\sigma) = N_{0.995}\sigma \approx 2.58\sigma$.
  - In the standard model, we have $g(\sigma)$ between 2.7 σ and 3.1σ for standard deviations in the appropriate range.
  - So, the assumption in the standard model is that insurance data have heavier tails than the normal distribution.



5. The Solvency II standard model

- The volume measure V is the sum of the volume measures of the individual LoBs.
  - $V = \sum_l V^{(l)}$
  - $\sigma = \frac{\sqrt{\sum_l \sum_m \rho_{lm} \sigma^{(l)} \sigma^{(m)} V^{(l)} V^{(m)}}}{V}$
- For an individual LoB, say l, the volume measure $V^{(l)}$ is the sum of the volume of outstanding incurred claims $V_R^{(l)}$ and the volume of claims expected to arise in the future $V_P^{(l)}$.
  - The volume of outstanding incurred claims is set to the best estimate of outstanding incurred claims, i.e. $V_R^{(l)} = BE_R^{(l)}$.
  - The best estimate of future claims multiplied by the estimated total cost to claim cost ratio $\gamma^{(l)}$ as a proxy for the volume of claims expected to arise in the future, i.e. $V_P^{(l)} = \gamma^{(l)} BE_P^{(l)}$.
  - $\sigma^{(l)} = \frac{\sqrt{\left(\sigma_R^{(l)} V_R^{(l)}\right)^2 + 2\rho \sigma_R^{(l)} \sigma_P^{(l)} V_R^{(l)} V_P^{(l)} + \left(\sigma_P^{(l)} V_p^{(l)}\right)^2}}{V^{(l)}}$

5. The Solvency II standard model

- In terms of the standard model, the insurance types collision and major first-party belong to the LoB other motor (OM), while the insurance type third-party property belongs to the LoB motor vehicle liability (MVL).
  - $\rho_{RP} = 0.5, \rho_{OM,MVL} = 0.5$

- $\gamma_{OM}^{(l)} = 1.25,\ \gamma_{MVL}^{(l)} = 1.26 \rightarrow \mathrm{V}_{\mathrm{P}}^{(OM)} = 1.25\, BE_P^{(OM)}, \mathrm{V}_{\mathrm{P}}^{(MVL)} = 1.26\, BE_P^{(MVL)}$

Table 4. Standard deviations per volume unit of the reserve risk ($\sigma_R^{(\ell)}$) and premium risk ($\sigma_P^{(\ell)}$), respectively.

	Standard deviation, reserve risk	Standard deviation, premium risk
Other motor	0.100	0.070
Motor vehicle liability	0.095	0.100

5. The Solvency II standard model

- We compare the SCR calculated using the Solvency II standard model with the SCR values our simulation model generates
  - SCR generated by the simulation model **are markedly lower than** the value calculated using the Solvency II standard model.
  - Because the LoB motor vehicle liability contains not only third-party property claims but also **third-party personal injury claims** which are often greater in size and take longer to handle.
  - The standard model assumptions about the data distribution of the insurance types studied in this paper **are different from** the distribution of real data.

Table 6. Simulated SCR for all scenarios and both methods (assuming the cost-of-capital rate $c = 0.06$), as well as the calculated SCR for the Solvency II standard model. Values in million SEK.

Scenario	Normal No correlation No trend	Normal **Correlation** No trend	Normal **Correlation** **Trend**	Student's t **Correlation** No trend	Log-normal **Correlation** No trend	Solvency II standard model
Individual line	25.7	27.8	33.3	29.7	27.9	37.8
Aggregate	25.2	28.1	33.6	30.1	28.2	–

5. The Solvency II standard model

- We discuss pros and cons of using the simulation model for determining SCR, and what impact the results may have on future insurance-risk modelling.
  - The simulation model cannot handle catastrophic risks, so these must be handled by ad hoc methods (scenario analysis), but not a disadvantage in comparison to the Solvency II standard model where ad hoc methods are also used for catastrophe risk assessment.
  - The simulation model show that **dependencies between insurance types must be taken into account** when calculating the SCR level.
  - **Trend** assumptions are also important.
  - To make the SCR value meaningful, one somehow must quantify how well an experienced actuary **can anticipate future trends** in average claim amounts.