Eigenvalues and eigenvectors are derived from the covariance matrix of the dataset.

Step 1: Covariance Matrix

The covariance matrix CC is calculated from the dataset:

$$C = \begin{bmatrix} \mathrm{Var}(x_1) & \mathrm{Cov}(x_1, x_2) \\ \mathrm{Cov}(x_2, x_1) & \mathrm{Var}(x_2) \end{bmatrix}$$

For example:

$$C = \begin{bmatrix} 1.25 & 2.5 \\ 2.5 & 5 \end{bmatrix}$$

Step 2: Eigenvalue Equation

Eigenvalues (λ) and eigenvectors (v) satisfy the equation:

$$C \cdot v = \lambda \cdot v$$

Where:

- C is the covariance matrix.
- v is the eigenvector.
- λ is the eigenvalue.

This can be rewritten as:
$(C - \lambda I) \cdot v = 0$

Where I is the identity matrix.

Step 3: Characteristic Equation

To find λ, we solve the determinant of $(C - \lambda I)$:

$$det(C - \lambda I) = 0$$

Substituting C:
$$det \begin{bmatrix} 1.25 - \lambda & 2.5 \\ 2.5 & 5 - \lambda \end{bmatrix} = 0$$

Simplify:
$$(1.25 - \lambda)(5 - \lambda) - (2.5)^2 = 0$$

Solve the quadratic equation:

$$\lambda^2 - 6.25\lambda + 6.25 = 0$$

Factoring:
$$(\lambda - 6.25)(\lambda - 0) = 0$$

Eigenvalues:
$$\lambda_1 = 6.25, \quad \lambda_2 = 0$$

Step 4: Finding Eigenvectors

For each eigenvalue λ, substitute it back into $(C - \lambda I) \cdot v = 0$ to find the eigenvector v.

For $\lambda_1 = 6.25$:

$$(C - 6.25I) \cdot v = 0$$
$$\begin{bmatrix} 1.25 - 6.25 & 2.5 \\ 2.5 & 5 - 6.25 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -5 & 2.5 \\ 2.5 & -1.25 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

From this, solve for v_1 and v_2 (any non-zero vector satisfying the equation is valid):

$$v_1 = 0.447, \quad v_2 = 0.894$$

Eigenvector for λ_1:
$$v_1 = \begin{bmatrix} 0.447 \\ 0.894 \end{bmatrix}$$

For $\lambda_2 = 0$:

Follow the same process to find the eigenvector.

Step 5: Normalize Eigenvectors

Eigenvectors are typically normalized to have a magnitude of 1:

$$\text{Normalized } v = \frac{v}{\|v\|}$$

For example:

$$\|v_1\| = \sqrt{0.447^2 + 0.894^2} = 1$$

The normalized eigenvector remains the same.