Homework Assignment # 2

Due: Monday, February 19, 2018, 11:59 p.m.
Total Points: 40

Question 1. [10 POINTS]

Let X have an exponential density

$$f(x|\theta) = \begin{cases} \theta e^{-\theta x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Suppose that n samples $x_1, \ldots, x_n$ are drawn independently according to $f(x|\theta)$. Show that the maximum-likelihood estimate for θ is $\hat{\theta} = \frac{1}{\frac{1}{n}\sum_{k=1}^{n} x_k}$

$f(x;\theta) = \theta e^{-\theta x}$
$L(\theta) = argmax\theta e^{-\theta \sum xk}$
Taking the log:
$logL(\theta) = log\theta - \theta \sum xk$

Take derivative:
$\frac{1}{\theta} - \sum xk$
$\hat{\theta} = \frac{1}{\frac{1}{n}\sum xk}$

Question 2. [10 POINTS]

Suppose that the number of accidents occurring daily, x_i, in a certain plant has a Poisson distribution with an unknown mean λ. So $P(x_i) = \lambda^{x_i} e^{-\lambda}/(x_i!)$. Based on previous experience in similar industrial plants, suppose that our initial feelings about the possible value of λ can be expressed by an exponential distribution with parameter $\theta = \frac{1}{2}$. That is, the prior density is

$$f(\lambda) = \theta \mathrm{e}^{-\theta\lambda}$$

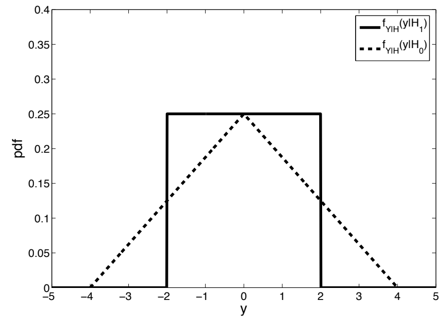
where $\lambda \in (0, \infty)$. If there are 79 accidents over the next 9 days, **determine the maximum a posteriori (MAP) estimate of** λ.

$substitute\theta = 0.5$
Taking the log:
$log0.5 - 0.5\lambda + \sum(xilog\lambda - \lambda - logxi!)$

Take derivative:
$-0.5 + \frac{1}{\lambda}\sum xi - n = 0$
$\lambda = \frac{1}{n+0.5}\sum xi$
$\lambda = \frac{1}{9+0.5} * 79$
$\lambda = 8.32$

Question 3. [10 POINTS]

Consider the binary hypothesis testing (Bayesian classification) problem and refer to the following graph (shown below) for the PDF of Y under H_0 and H_1:



(a) Using the graph below, write equations (in terms of y) for the following likelihoods $f_{Y|H}(y|H_0)$ and $f_{Y|H}(y|H_1)$

(b) For equal priors and uniform costs ($C_{00} = C_{11} = 0$ and $C_{01} = C_{10} = 1$), find the Bayesian decision rule for testing H_0 versus H_1

(c) For part (b), compute the minimum Bayesian risk

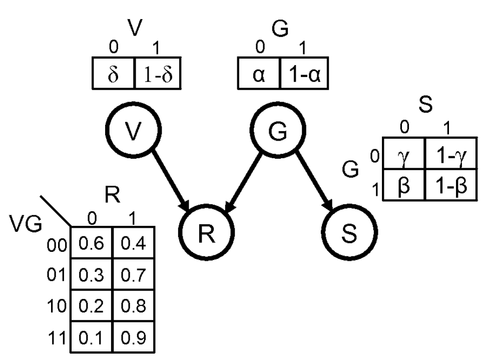
$3a) fY|H(y|H0) = -\frac{|y|}{16} + 0.25, -4 \le y \le 4$
0, otherwise
$fY|H(y|H1) = 0.25, -2 \le y \le 2$
0, otherwise

$3b fY|H(y|H0), if - 4 \le y \le -2, 2 \le y \le 4$
$fY|H(y|H1), if - 2 < y < 2$

$3c) \frac{1}{2}(\int_2^0 -\frac{1}{16}y + \frac{1}{4}dy + \int_0^{-2} \frac{1}{16}y + \frac{1}{4}dy + \frac{1}{2})$
$= \frac{1}{2}(-\frac{4}{32} + \frac{2}{4}) + \frac{1}{2}(-(\frac{4}{32} - \frac{2}{4}))$
$= 0.375 = \frac{3}{8}$

Question 4. [10 POINTS]

In this problem, there are 4 binary random variables: G = "gray", V = "Vancouver", R="rain", and S="sad". Consider the directed graphical model, shown below, which describes the relationship between these random variables:



(a) Write an expression for $P(S = 1|V = 1)$ in terms of $\alpha, \beta, \gamma, \delta$.

(b) Write an expression for $P(S = 1|V = 0)$. Is this the same or different to $P(S = 1|V = 1)$? Explain why.

4a)$P(S = 1|V = 1) = \frac{((1-\gamma)+(1-\beta))(1-\delta)}{1-\delta}$
$2 - \gamma - \beta$
4b)$P(S = 1|V = 0) = \frac{((1-\gamma)+(1-\beta))(\delta)}{\delta}$
$2 - \gamma - \beta$
They are the same, because these two are independent events, the result of V does not affect event S.