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Question 1.

1a. sample space = {BF, LF, BW, LW}

1b. $P[W] = 1 - P[L \cap F] - P[B \cap F] = 1 - 0.5 - 0.2 = 0.3$

$P[B] = P[B \cap F] + P[B \cap W] = 0.2 + 0.2 = 0.4$

$P[W \cup B] = P[W] + P[B] - P[B \cap W] = 0.3 + 0.4 - 0.2 = 0.5$

Question 2

2a. sample space = {HF, MF, HW, MW}

2b. $P[W] = 1 - P[F] = 1 - 0.5 = 0.5$

$P[M \cap F] = P[F] - P[H \cap F] = 0.5 - 0.2 = 0.3$

$P[H] = 1 - P[M \cap F] - P[M \cap W] = 1 - 0.3 - 0.1 = 0.6$

Question 3

3a. $P[A] = P[9] + P[10] = \frac{1}{11} + \frac{1}{11} = \frac{2}{11}$

3b. $P[F] = P[0] + P[1] + P[2] + P[3] = \frac{1}{11} + \frac{1}{11} + \frac{1}{11} + \frac{1}{11} = \frac{4}{11}$

Question 4

4a. $P[H0] = P[LH0] + P[BH0] = 0.1 + 0.4 = 0.5$

4b. $P[B] = P[BH0] + P[BH1] + P[BH2] = 0.4 + 0.1 + 0.1 = 0.6$

4c. $P[L \cup H2] = P[LH0] + P[LH1] + P[LH2] + P[BH2] = 0.1 + 0.1 + 0.2 + 0.1 = 0.5$

Question 5

5a. $P[R3 \mid G1] = P[R3G1] / P[G1] = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$

5b. $P[R6 \mid G3] = P[R6G3] / P[G3] = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$

5c. $P[G3 \mid E] = P[G3E] / P[E] = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

5d. $P[E \mid G3] = P[EG3] / P[G3] = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

Question 6

6a P[E2 | E1] = P[E2E1] / P[E1] = $\frac{\frac{2}{3}*\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2}$

6b P[E1E2E3] = 0

6c P[E2 | O1] = P[E2O1] / P[O1] = $\frac{\frac{1}{3}}{\frac{1}{3}} = 1$

6d P[O1O2] = 0

Question 7

7a P[LH] = P[L] + P[H] - P[L U H] =0.16 + 0.1 – 0.1 = 0.16

7b. P[LH] / P[L] = (0.16)/(0.16) = 1

Question 8

8a. 1 = $\frac{c}{2} + \frac{c}{4} + \frac{c}{8}$

c = $\frac{8}{7}$

8b. P[X = 4] = $\frac{\left(\frac{8}{7}\right)}{4} = \frac{2}{7}$

8c. P[X < 4] = P[X = 2] + 0 = $\frac{\frac{8}{7}}{2} = \frac{4}{7}$

8d. P[3 ≤ X ≤ 9] = 1- P[X = 2] = $1 - \frac{4}{7} = \frac{3}{7}$

Question 9

let a be the probability of single

P[B = 1] + P[B = 2] + P[B = 3] + P[B = 4] = P[B > 0]

a + 0.5a + 0.25a + 0.125a = 0.3

1.875a = 0.3

1875a = 300

a = 4/25

$$PB(b) = \begin{cases} \frac{4b}{25} & , \ if\ b = 1,2,3,4 \\ 0, & otherwise \end{cases}$$

Question 10

10a. $PB(b) = \begin{cases} \frac{\left(\frac{T}{5}\right)^b e^{-\left(\frac{T}{5}\right)}}{b!}, \; if\ b = 0,1,2,\ldots \\ 0 \quad , \; otherwise \end{cases}$

10b. T = 2

PB(3) = $\frac{\frac{2}{5}^3 e^{-\frac{2}{5}}}{3!}$ = 0.0072

10c. T = 10

PB(0) = $\frac{\frac{10}{5}^0 e^{-\frac{10}{5}}}{0!}$ = 0.135

10d. P[B > 0] = 1- P[B = 0]

0.99 = 1 – $e^{-T/5}$

T = 23minutes

Question 11.

11a. P[T > 32] = 1 – P[T ≤ 32] = 1 – f$\left(\frac{(32-10)}{15}\right) = 0.071$

11b. P[T < 0] =$f\left(\frac{0-10}{15}\right) = 1 - f\left(\frac{2}{3}\right) = 0.251$

Question 12.

1 = 2πr

r = $\frac{1}{2\pi}$

12a. $\frac{1}{2} * X * \frac{1}{2\pi}$ = Y

$\frac{X}{4\pi}$ = Y

12b. FY(y) = FX($4\pi y$)

12c. fY(y) = $4\pi\ fX(4\pi y)$

12d. E[Y] = $\int fY(y)dy$

Question 13.

13a. FX(x) = $\int_0^2 \frac{x+y}{3} dy = \frac{x}{3}[y]_0^2 + \frac{1}{3}[\frac{y^2}{2}]_0^2 = \frac{2x}{3} + \frac{2}{3} = \frac{2x+2}{3}$

E[X] = $\int_0^1 \frac{x(2x+2)}{3} dx = \frac{2}{3}\left[\frac{x^3}{3}\right]_0^1 + \frac{2}{3}[\frac{x^2}{2}]_0^1 = \frac{2}{3} * \frac{1}{3} + \frac{2}{3} * \frac{1}{2} = \frac{5}{9}$

E[X^2]= $\int_0^1 \frac{x^2(2x^2+2)}{3} dx = \frac{2}{3}\left[\frac{x^4}{4}\right]_0^1 + \frac{2}{3}[\frac{x^3}{3}]_0^1 = \frac{2}{3} * \frac{1}{4} + \frac{2}{3} * \frac{1}{3} = \frac{1}{6} + \frac{2}{9} = \frac{7}{18}$

Var[X] = E[X^2] $- E[X]^2 = \frac{7}{18} - \frac{25}{81} = 0.0802$

13b. FY(y) = $\int_0^1 \frac{x+y}{3} dx = \frac{y}{3}[x]_0^1 + \frac{1}{3}\left[\frac{x^2}{2}\right]_0^1 = \frac{y}{3} + \frac{1}{6} = \frac{2y+1}{6}$

E[Y] = $\int_0^2 \frac{y(2y+1)}{6} dy = \int_0^2 \frac{y}{6} + \frac{y^2}{3} dy = \frac{1}{6}\left[\frac{y^2}{2}\right]_0^2 + \frac{1}{3}\left[\frac{y^3}{3}\right]_0^2 = \frac{1}{3} + \frac{8}{9} = \frac{11}{9}$

E[Y^2] $= \int_0^2 \frac{y^2(2y^2+1)}{6} dy = \int_0^2 \frac{y^2}{6} + \frac{y^4}{3} dy = \frac{1}{6}\left[\frac{y^3}{3}\right]_0^2 + \frac{1}{3}\left[\frac{y^4}{4}\right]_0^2 = \frac{4}{9} + \frac{12}{9} = \frac{16}{9}$

Var[Y] = E[Y^2] $- E[Y]^2 = \frac{16}{9} - \frac{121}{81} = 0.284$

13c E[XY] = $\int_0^1 \int_0^2 xy \frac{x+y}{3} dx\, dy = \int_0^1 \frac{x}{3} [\int_0^2 xy + y^2\, dy] dx = \int_0^1 \frac{x}{3} (x * \left[\frac{y^2}{2}\right]_0^2 + \left[\frac{y^3}{3}\right]_0^2) dx$

$= \int_0^1 \frac{x}{3}\left(2x + \frac{8}{3}\right) dx = \int_0^1 \frac{2x^2}{3} + \frac{8x}{9} dx = \frac{2}{3}\left[\frac{x^3}{3}\right]_0^1 + \frac{8}{9}[x]_0^1 = \frac{2}{9} + \frac{8}{9} = \frac{10}{9}$

Cov[X,Y] = E[XY] – E[X]E[Y] = $\frac{10}{9} - \frac{5}{9} * \frac{11}{9} = \frac{10}{9} - \frac{55}{81} = 0.432$

Question 14.

14a. PX(x)= $\binom{65}{x} 0.5^x (0.5)^{65-x},\ x = 0,1,2,3,4, \ldots ,65$

14a. PY(y)= $\binom{35}{y} 0.5^y (0.5)^{35-y},\ y = 0,1,2,3,4, \ldots ,35$

14b. The event is independent. The probability of flipping a head/tail is always 0.5, event result

Does not affect the next event.

14c. PXY(x,y)= $\binom{65}{x} 0.5^x (0.5)^{65-x} * \binom{35}{y} 0.5^y (0.5)^{35-y}$