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Faculty of Mathematics and Computer Science

Line Follower - example simulation of performance
based on mathematical model of robot

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1 Introduction - problem description

Linefollowing robot mission is to pass the race as fast as possible. The trajectory of movement is in general closed curve on plane parallel to earth surface. Mathematical model and simulation of an object response under influence of physical phenomena should help us understand the meaning of given parameters in reference to performance of robot. In few examples this kind of modelling will let us to do inverted process i.e. calculating the values of parameters for given project specification. Model of a system will be simplified with omitting some phenomena which do not have big influence on passing race. The most important processes in physical world in reference to movement of a robot will be linked with mathematical statements resulting from laws and theorems of Physics.

Modelling and simulation should let us find answers for questions like:

- How to choose motors for robot in reference to radius and friction of wheel?
- What are the natural frequencies of mechanical model?
- What is the influence of track defects on response of model?
- Which information could be possibly lost in ADC and DAC conversion based on light sensor model?
- How to protect the information in signal from aliasing effect?

2 Solution of a problem

Model of static friction between wheel and route surface has been build by applying laws and theorems of Physics, especially Mechanics:

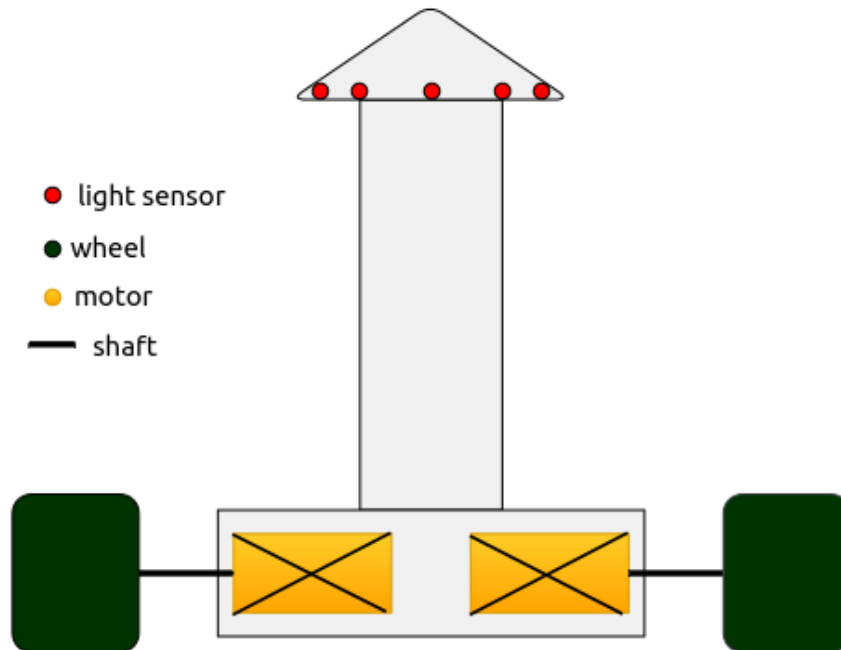


Figure 1: Simplified scheme of a robot

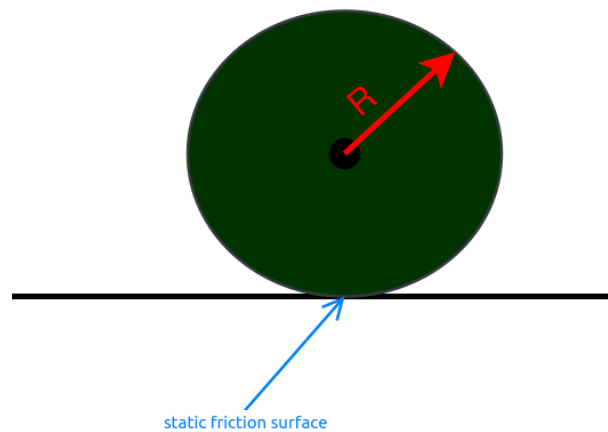


Figure 2: Friction surface

While passing the boundary value of static friction force the wheels will start to slide. The designer of robot objective due to achieving good control of movement is to find relationship between geometrical and material parameters of wheels and friction process.

Starting torque of motor should be enough high to start the slide of wheel, but later controlled in a proper manner.

Let's denote μ as a static friction coefficient. Static friction force is proportional to load generated by robot. $T = \mu N$, where N - load of robot.

Force on tread wheel is given by equation:

$$F = \frac{M_n}{R} \quad (1)$$

Assuming that the force generated by motors should be higher than static friction resistance final torque statement is:

$$M_n > R\mu N \quad (2)$$

Mass-spring-damper system model. Let's assume viscous damping between mass elements and apply Newton's laws of movement for given system.

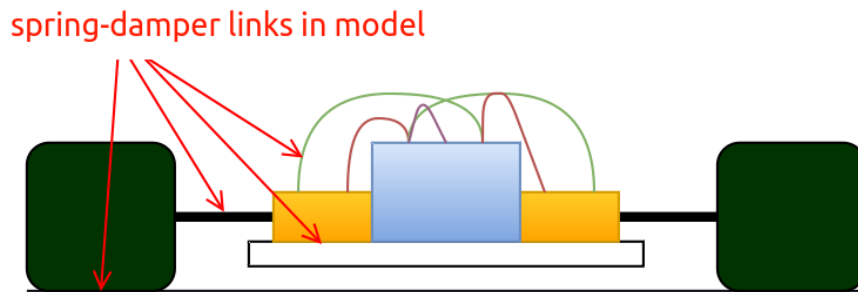


Figure 3: mass-spring-damper in reference to physical system

Model of robot is symmetrical, so we will calculate only one half of it. Discretize mass elements and model the connections between masses:

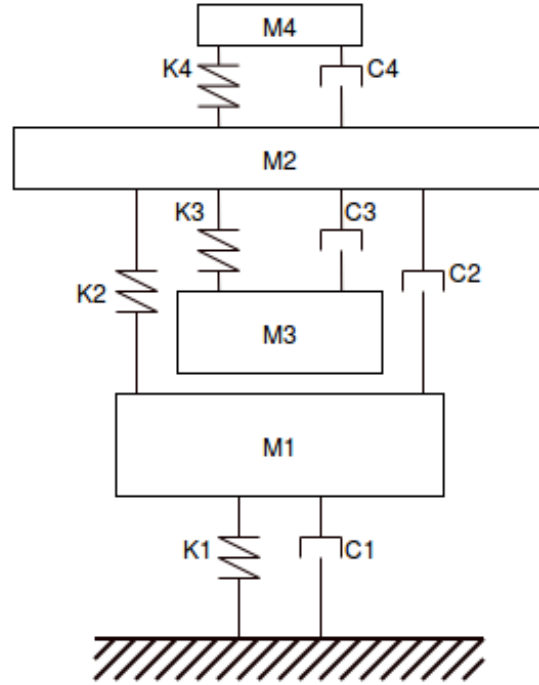


Figure 4: mass-spring-damper model

Element 1 - wheel of robot

Element 2 - drive and controler

Element 3 - the support element with light sensors

Element 4 - cables and trunk lines

System of dynamic equations:

$$F_1 - m_1 \ddot{x}_1 - c_1 \dot{x}_1 - k_1 x_1 - (x_1 - x_2)k_2 - (\dot{x}_1 - \dot{x}_2)c_2 = 0$$

$$F_2 - m_2 \ddot{x}_2 - c_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) - k_3(x_2 - x_3) - c_3(\dot{x}_2 - \dot{x}_3) - k_4(x_2 - x_4) - c_4(\dot{x}_2 - \dot{x}_4) = 0$$

$$F_3 - m_3 \ddot{x}_3 - k_3(x_3 - x_2) - c_3(\dot{x}_3 - \dot{x}_2) = 0$$

$$F_4 - m_4 \ddot{x}_4 - k_4(x_4 - x_2) - c_4(\dot{x}_4 - \dot{x}_2) = 0$$

State variables:

$$h_1 = x_1, h_2 = \dot{x}_1, h_3 = x_2, h_4 = \dot{x}_2, h_5 = x_3, h_6 = \dot{x}_3, h_7 = x_4, h_8 = \dot{x}_4$$

State-space model of system:

$$\begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \\ \dot{h}_4 \\ \dot{h}_5 \\ \dot{h}_6 \\ \dot{h}_7 \\ \dot{h}_8 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(k_1 + k_2) & -(c_1 + c_2) & k_2 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ k_2 & c_2 & -(k_2 + k_3 + k_4) & -(c_2 + c_3 + c_4) & k_3 & c_3 & k_4 & c_4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & k_3 & c_3 & -k_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & k_4 & c_4 & -k_4 & -c_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_4} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$$

$$\infty \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} + \begin{pmatrix} 0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$$

It is highly recommended to use function which imitates real processes and defects while robot attends racing as an input signal. Forcing first mass element simulate road defects while driving.

Kinematic parameters of unforced elements can be easily traced with Matlab tools from state-space model.

3 Description of experiments

Title: Calculation and analysis of relationship between required torque of motor, radius and friction coefficient of wheels.

Description: The experiment is about to analyze and plot relationship between parameters and their mutual influence in reference to selection of electric drive.

Title: Determination of hypothetical model of vibration in robot mechanical structure due to calculation of natural frequencies of system and simulating response for specified input signal.

Description: Building the mass-spring-damper model of system was a first fundamental step. After that we calculate natural frequencies and simulate kinematic parameters of given mass elements for signal imitating defects on track.

Title: Analysis of sensor data in time and frequency domain. Examination of the artifacts introduced by ADC and DAC conversion errors.

Description: The experimental process contains generation of sample data. Generated dataset simulate sensor reading from single ride. Next step is plotting the data in time and frequency domain. Then we will show errors introduced by ADC (aliasing) and DAC(differences in spectrum) conversion.

4 Results

Title: Calculation and analysis of relationship between required torque of motor, radius and friction coefficient of wheels.

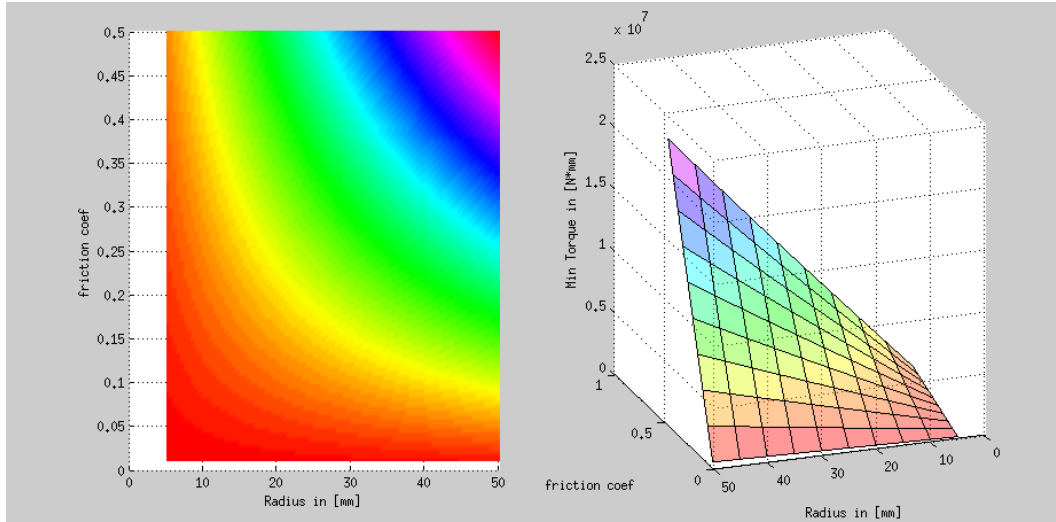


Figure 5: Radius, friction, torque dependence

Title: Determination of hypothetical model of vibration in robot mechanical structure due to calculation of natural frequencies of system and simulating response for specified input signal.

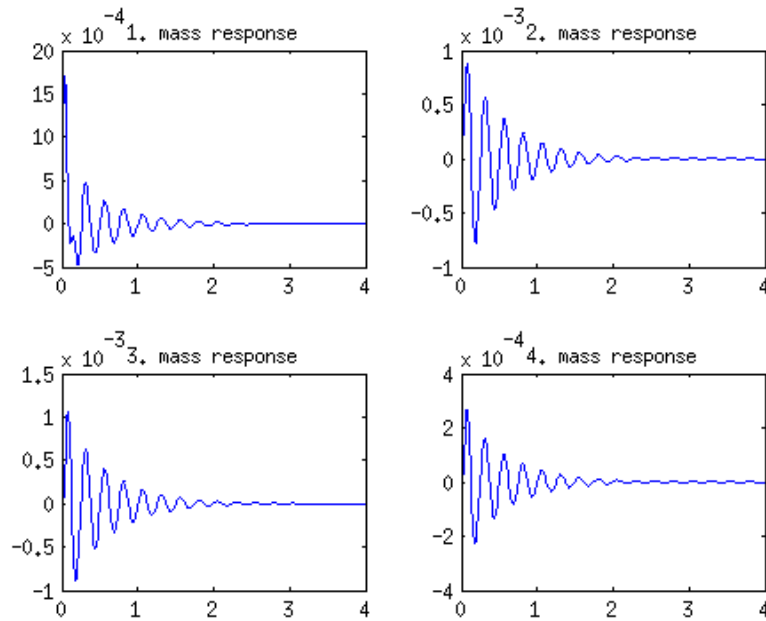


Figure 6: Impulse response of system forcing 1. element

The impulse response for input on first mass element simulate the road defect. As it can be seen from plots, displacement is similar as in real-world. Units of displacement on figures are meters.

As it can be seen, system is trying to "follow" the input signal, but at the beginning it hits high amplitude of displacement on every mass element.

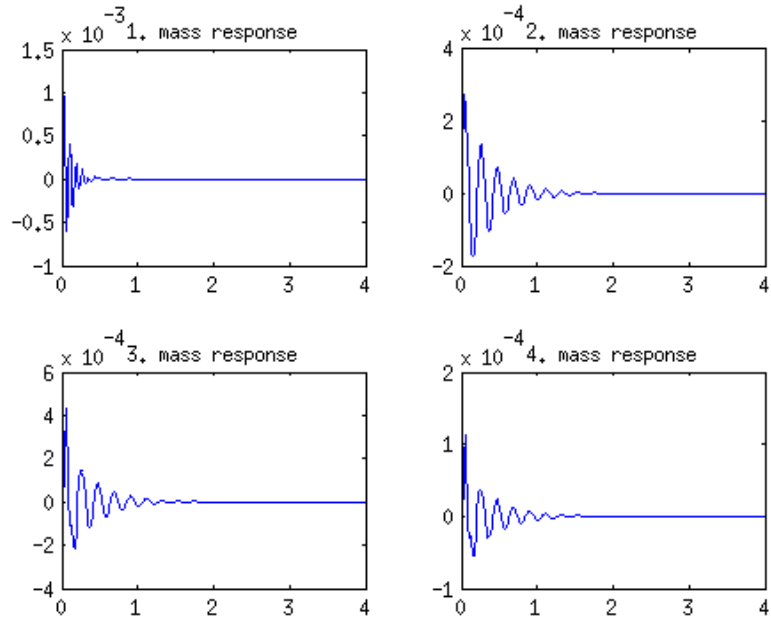


Figure 7: Impulse response - higher wheel elasticity coefficient

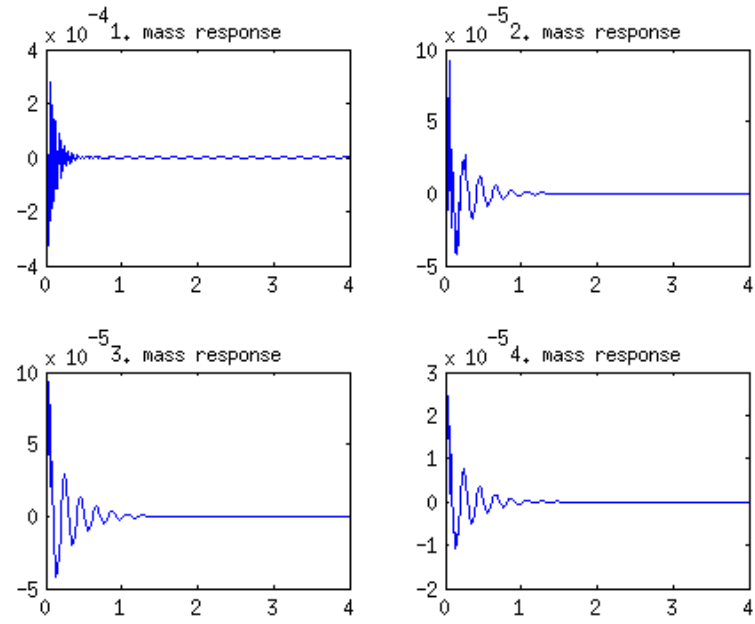


Figure 8: Impulse response - much higher wheel elasticity coefficient

Title: Analysis of sensor data in time and frequency domain. Examination of the artifacts introduced by ADC and DAC conversion errors.

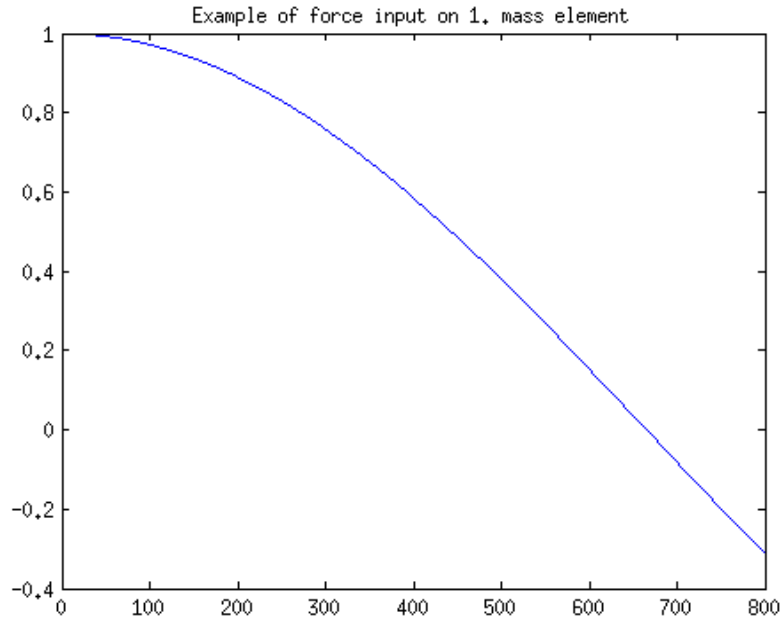


Figure 9: Other example of input signal

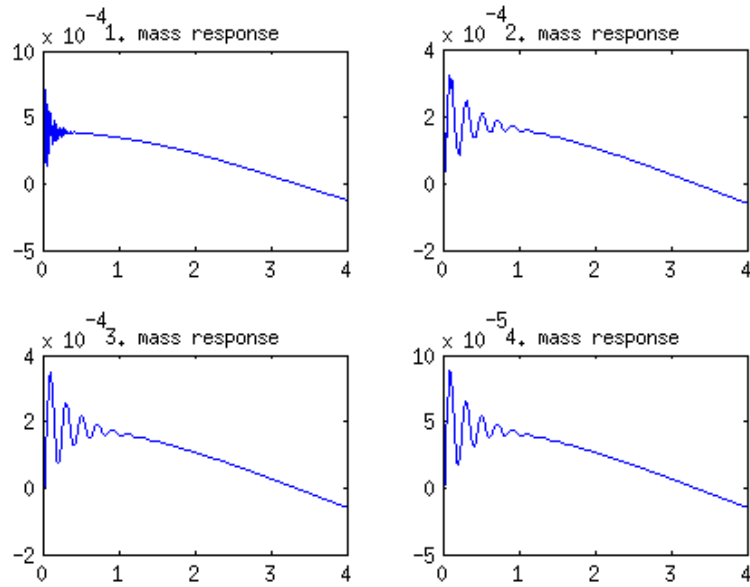


Figure 10: Response for given example input

5 Conclusions

6 Appendix

Github project repo: <https://github.com/kfarbaniec/misk.git>

To run an experiments the Matlab software is required. Simply open the .m file with experiment you

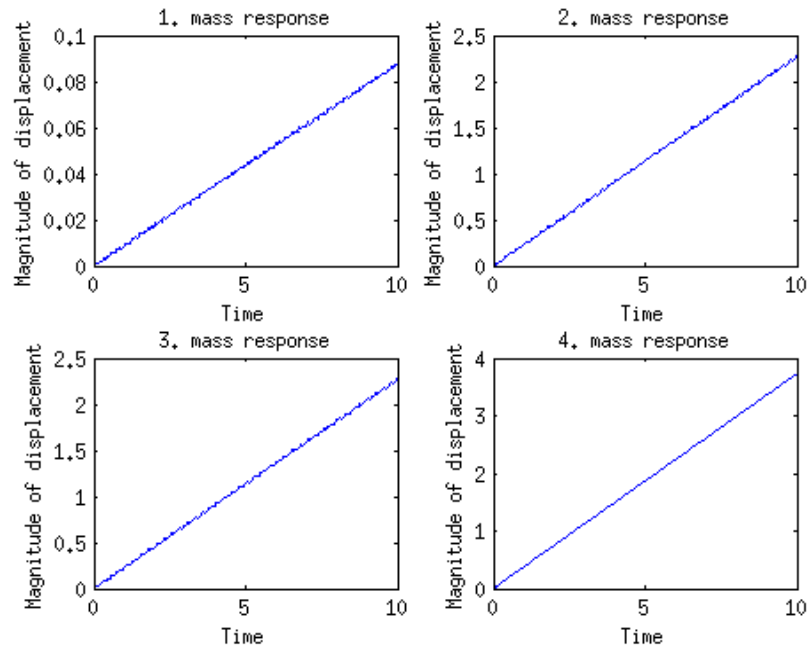


Figure 11: Response for natural frequencies of system sinusoidal signal

Magnitude of displacement is growing fast in time. To protect our robot from danger of mechanical resonance, control system and mechanical design should avoid being close to calculated frequencies on every mass element, especially this one with matched frequency.

are interested in with Matlab and click 'Run' icon (green triangle in Matlab editor) to run the script.

Difficulties and design considerations

First problem in simulating vibrations was selection of spring-damper coefs values. To choose it wisely its necessarily important to got experince and intuition. They can be measured and then set in model but to do it there must be some kind of test and measurement laboratory.

Generation of sample data is problematic because of non-deterministic real-world races. While driving, robot can possibly face many problems not included in experiment scripts.

7 Bibliography

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