JAGIELLONIAN UNIVERSITY

Faculty of Mathematics and Computer Science

# Line Follower - example simulation of performance based on mathematical model of robot

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# Contents

# 1 Introduction - problem description

Linefollowing robot mision is to pass the race as fast as possible. The trajectory of movement is in general closed curve on plane parallel to earth surface. Mathematical model and simulation of an object response under influence of physical phenomenas should help us understand the meaning of given parameters in reference to performance of robot. In few examples this kind of modelling will let us to do inverted process i.e. calculating the values of parameters for given project specification. Model of a system will be simplified with omitting some phenomenas which do not have big influence on passing race. The most improratant processes in physical world in reference to movement of a robot will be linked with mathematical statements resulting from laws and theorems of Physics.

Modelling and simulation should let us find answers for questions like:

- How to choose motors for robot in reference to radius and friction of wheel?
- What are the natural frequencies of mechanical model?
- What is the influence of track defects on response of model?
- Which information could by possibly lost in ADC and DAC conversion based on light sensor model?
- How to protect the information in signal from aliasing effect?

## 2 Solution of a problem

Model of static friction between wheel and route surface has been build by applying laws and theorems of Physics, especially Mechanics:

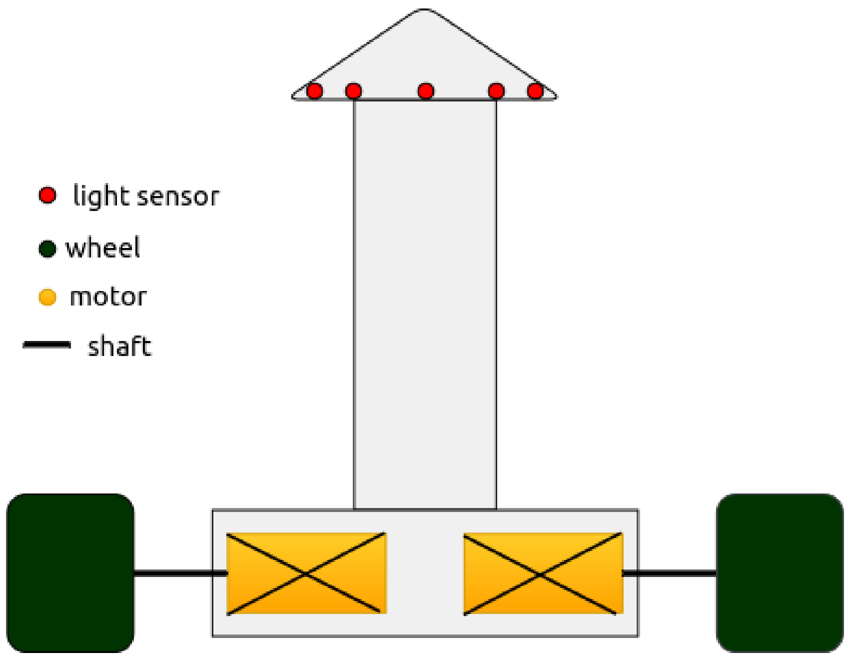


Figure 1: Simplified scheme of a robot

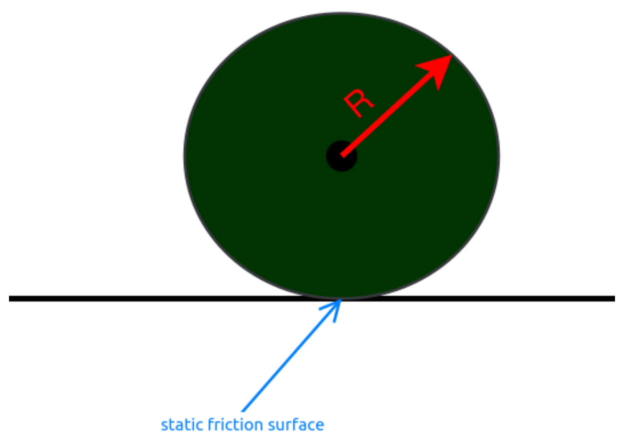


Figure 2: Friction surface

While passing the boundary value of static friction force the wheels will start to slide. The desinger of robot obejctive due to achieving good control of movement is to find relationship between geometrical and material parameters of wheels and friction process.

Starting torque of motor should be enough high to start the slide of wheel, but later controlled in a proper manner.
Let's denote $\mu$ as a static friction coefficient. newline Static friction force is proportional to load generated by robot. $T = \mu N$, where $N$ - load of robot.

Force on tread wheel is given by equation:

$$F = \frac{M_n}{R} \tag{1}$$

Assuming that the force generated by motors should be higher than static friction resistance final torque statement is:

$$M_n > R\mu N \tag{2}$$

Mass-spring-damper system model. Let's assume viscous damping between mass elements and apply Newtons laws of movement for given system.

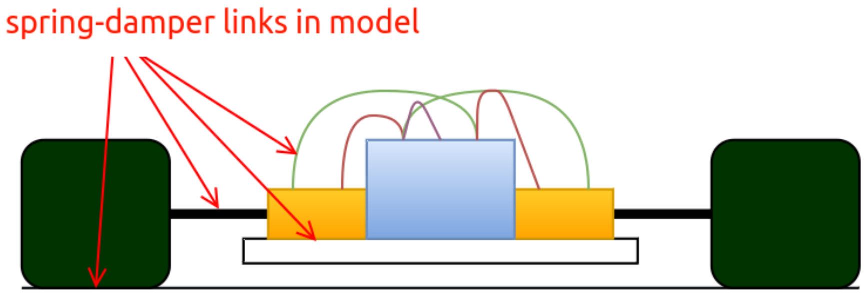


Figure 3: mass-spring-damper in reference to physical system

Model of robot is symetrical, so we will calculate only one half of it. Discretize mass elements and model the connections between masses:

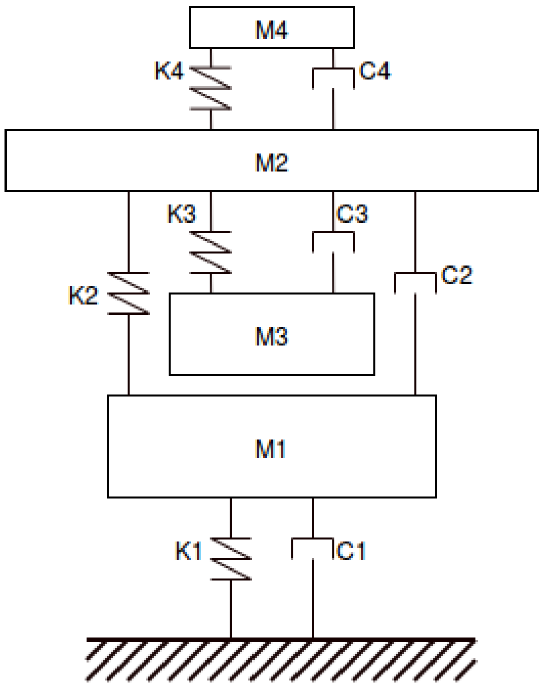


Figure 4: mass-spring-damper model

Element 1 - wheel of robot

Element 2 - drive and controler

Element 3 - the support element with light sensors

Element 4 - cables and trunk lines

System of dynamic equations:

$$F_1 - m_1\ddot{x_1} - c_1\dot{x_1} - k_1x_1 - (x_1 - x_2)k_2 - (\dot{x_1} - \dot{x_2})c_2 = 0$$

$$F_2 - m_2\ddot{x_2} - c_2(\dot{x_2} - \dot{x_1}) - k_2(x_2 - x_1) - k_3(x_2 - x3) - c_3(\dot{x_2} - \dot{x_3}) - k_4(x_2 - x_4) - c_4(\dot{x_2} - \dot{x_4}) = 0$$

$$F_3 - m_3\ddot{x_3} - k_3(x_3 - x_2) - c_3(\dot{x_3} - \dot{x_2}) = 0$$

$$F_4 - m_4\ddot{x_4} - k_4(x_4 - x_2) - c_4(\dot{x_4} - \dot{x_2}) = 0$$

State variables:

$h_1 = x_1$, $h_2 = \dot{x_1}$, $h_3 = x_2$, $h_4 = \dot{x_2}$, $h_5 = x_3$, $h_6 = \dot{x_3}$, $h_7 = x_4$, $h_8 = \dot{x_4}$

State-space model of system:

$$\begin{pmatrix} \dot{h_1} \\ \dot{h_2} \\ \dot{h_3} \\ \dot{h_4} \\ \dot{h_5} \\ \dot{h_6} \\ \dot{h_7} \\ \dot{h_8} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(k_1+k_2) & -(c_1+c_2) & k_2 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ k_2 & c_2 & -(k_2+k_3+k_4) & -(c_2+c_3+c_4) & k_3 & c_3 & k_4 & c_4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & k_3 & c_3 & -k_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & k_4 & c_4 & -k_4 & -c_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_4} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$$

$\infty$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} + \begin{pmatrix} 0 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$$

It is highly recomended to use function which imitates real processes and defects while robot attends racing as an input signal. Forcing first mass element simulate road defects while driving.
Kinematic parameters of unforced elements can be easily traced with Matlab tools from state-space model.

# 3 Description of experiments

**Title:** Calculation and analysis of relationship between required torque of motor, radius and friction coefficient of wheels.

**Description:** The experiment is about to analyze and plot relationship between parameters and their mutual influence in reference to selection of electric drive.

**Title:** Determination of hypothetical model of vibration in robot mechanical structre due to calculation of natural frequencies of system and simulating response for specified input signal.

**Description:** Bulding the mass-spring-damper model of system was a first fundamental step. After that we calculate natural frequencies and simulate kinematic parameteres of given mass elements for signal imitating defects on track.

**Title:** Analysis of sensor data in time and frequency domain. Examination of the artifacts introduced by ADC and DAC conversion errors.

**Description:** The experimental process contains generation of sample data. Generated dataset simulate sensor reading from single ride. Next step is plotting the data in time and frequency domain. Then we will show errors introduced by ADC (aliasing) and DAC(differences in spectrum) conversion.

# 4 Results

**Title:** Calculation and analysis of relationship between required torque of motor, radius and friction coefficient of wheels.

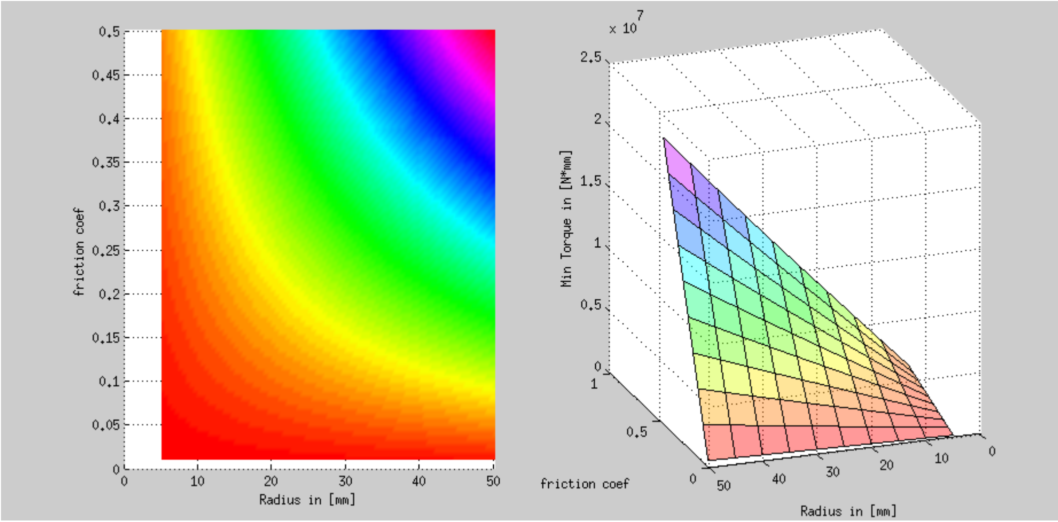


Figure 5: Radius, friction, torque dependence

**Title:** Determination of hypothetical model of vibration in robot mechanical structre due to calculation of natural frequencies of system and simulating response for specified input signal. The impulse response

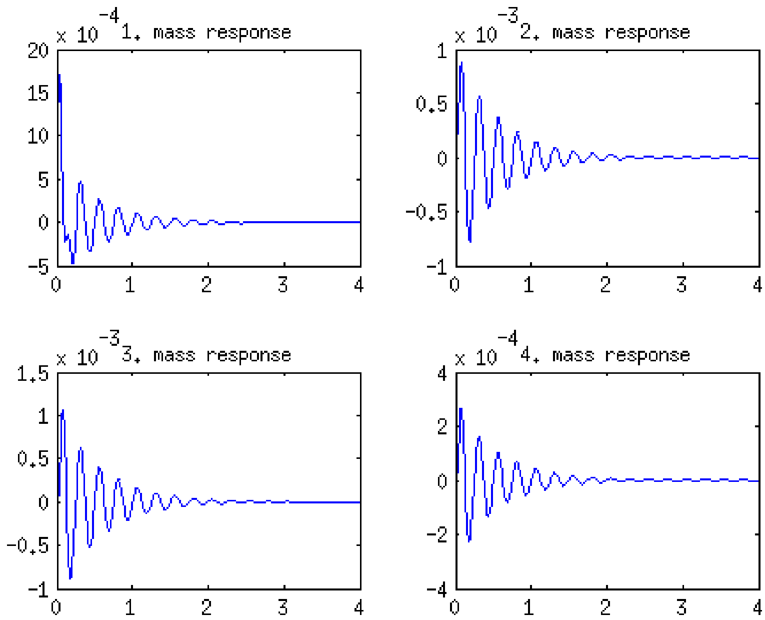


Figure 6: Impulse response of system forcing 1. element

for input on first mass element simulate the road defect. As it can be seen from plots, displacement is similar as in real-world. Units of displacement on figures are meters.

As it can be seen, system is trying to "follow" the input signal, but at the begining it hits high amplitude of displacement on every mass element.

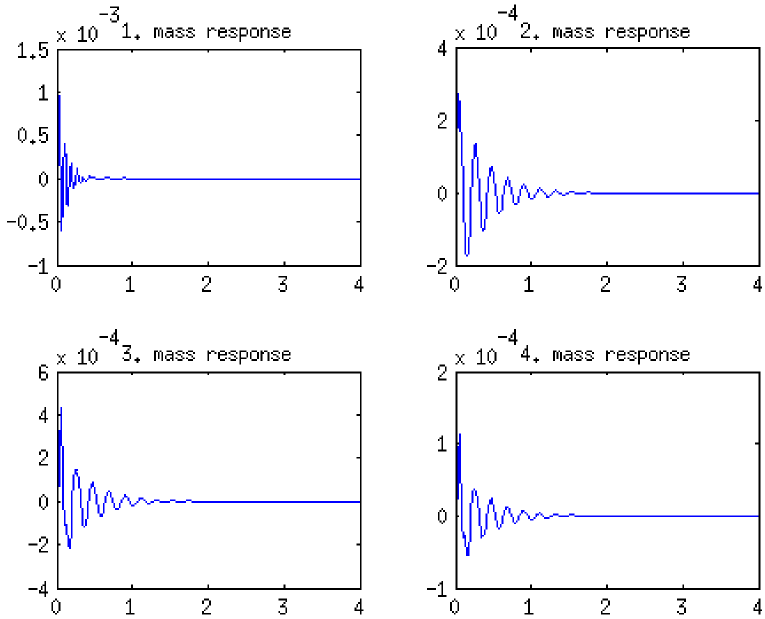


Figure 7: Impulse response - higher wheel elasticity coefficient

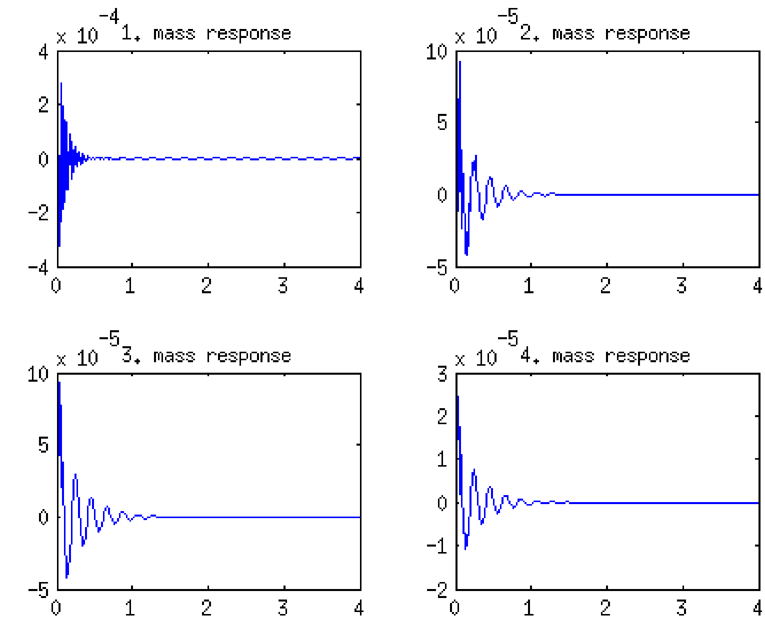


Figure 8: Impulse response - much higher wheel elasticity coefficient

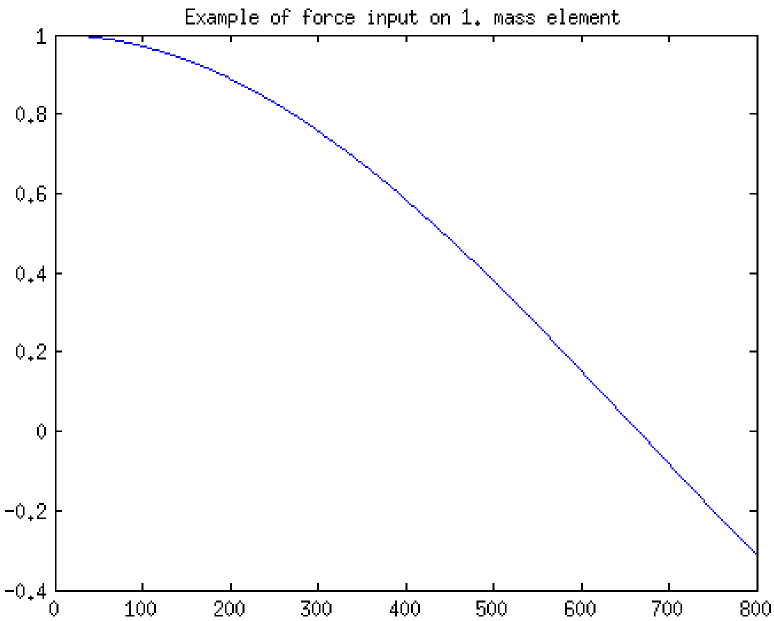


Figure 9: Other example of input signal

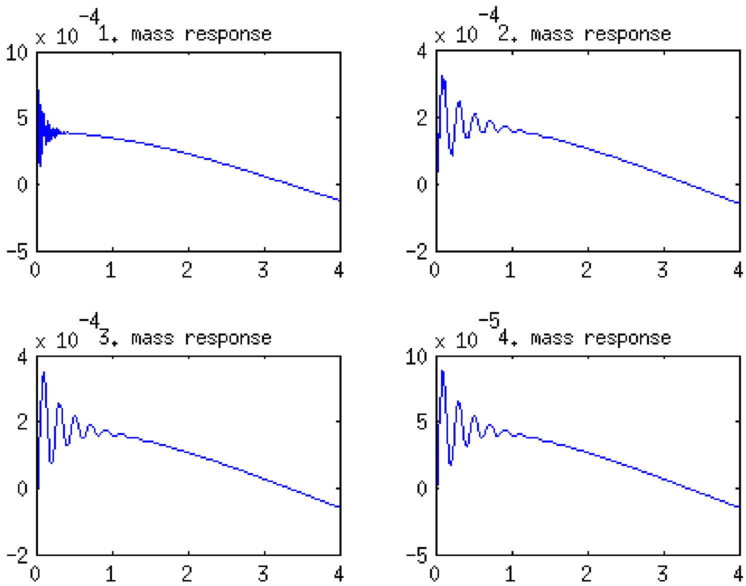


Figure 10: Response for given example input

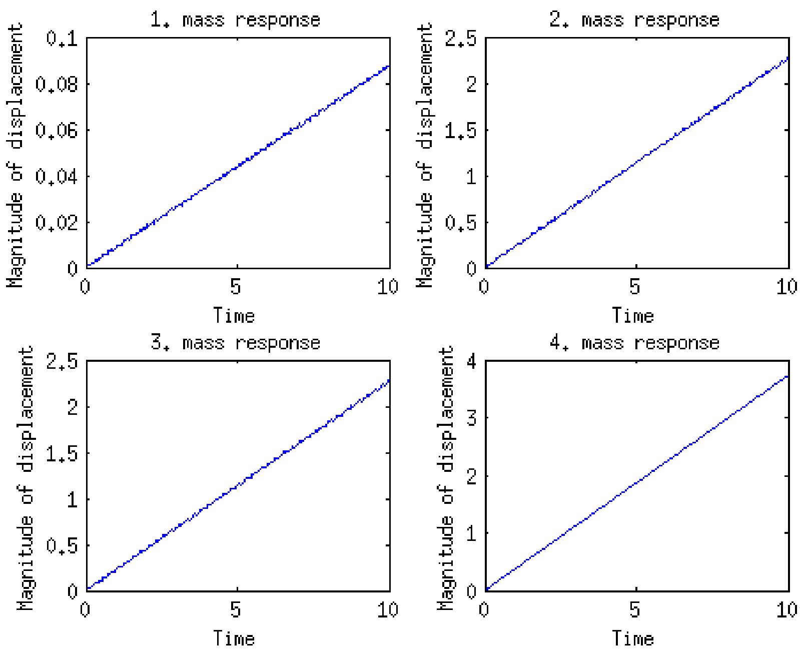


Figure 11: Response for natural frequencies of system sinusoidal signal

Magnitude of displacement is growing fast in time. To protect our robot from danger of mechanical resonance, control system and mechanical design should avoid being close to calculated frequencies on every mass element, especially this one with matched frequency.

**Title:** Analysis of sensor data in time and frequency domain. Examination of the artifacts introduced by ADC and DAC conversion errors.

Sensor output data was generated by frequency and magnitude modulation of sinewave signal plus random value set as an error and uncertainty of sensor.

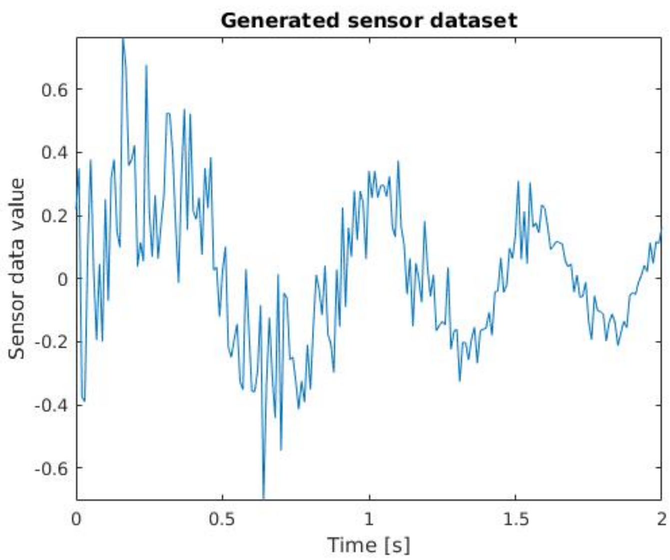


Figure 12: Generated data of light sensor model

In Analog to digital conversion there is a need to lowpass filter the input data. On the FFT we can see peaks in frequencies higher than half of sampling frequency. Two sinewaves has been added to signal, first 65 Hz, second 80 Hz. As we can see on a plot, the first artifact is at something like 40 Hz second 70 Hz, so it is not what we added. This kind of errors are called aliasing.

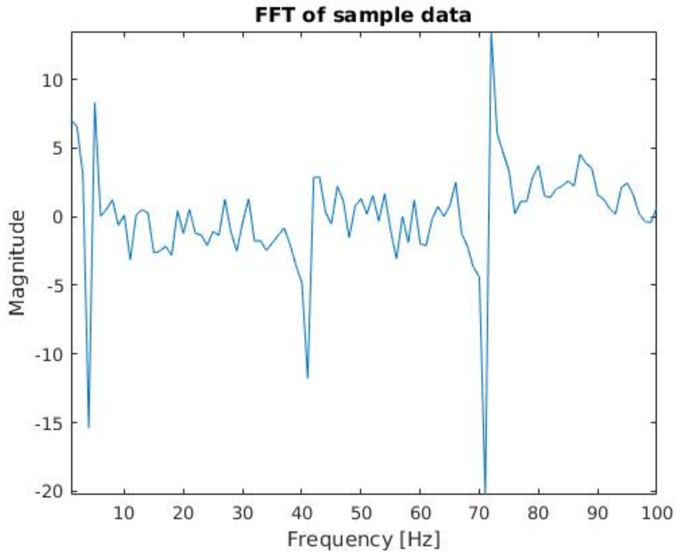


Figure 13: Higher frequencies in signal than half of sampling frequency

The easiest type of digital to analog conversion is ZOH - Zero-Order-Hold. It is about holding the previous value for all the period. This process cause errors in high frequencies.

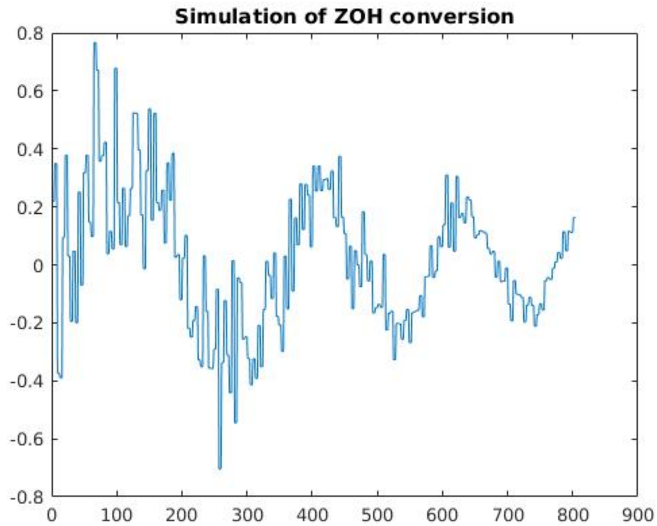


Figure 14: Simulation of ZOH on generated signal

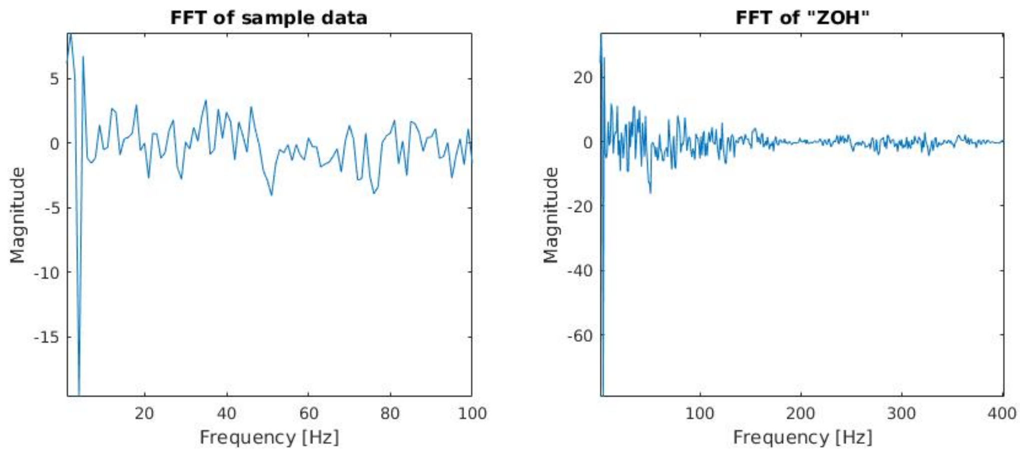


Figure 15: Non-zero magnitude above 100Hz (sampling frequency)

# 5 Conclusions

As a conclusion of first experiment. The size of wheel and friction coefficient influence minimal torque of motor in dependence seen on figure 5. Nowadays motor can easily fulfill this assumption and we should mostly choose them by the price and mass.

Damper-spring model in mechanical design is one of fundamental steps. Size of simulated mass displacement is about one milimeter. We should avoid natural frequencies in forcing system because as it is on figure 11, construction will be damaged.

Analog and digital data differs. While working with both of them we should remember about potetial errors and artifacts while processing. It is necessarily important to know fundamental theorems connected with conversion and processing.

# 6 Appendix

**Github project repo:** `https://github.com/kfarbaniec/misk.git`

To run an experiments the Matlab software is required. Simply open the .m file with experiment you are interested in with Matlab and click 'Run' icon (green triangle in Matlab editor) to run the script.

**Difficulties and design considerations**

First problem in simulating vibrations was selection of spring-damper coefs values. To choose it wisely its necessarily important to got experince and intuition. They can be measured and then set in model but to do it there must be some kind of test and measurement laboratory.

Generation of sample data is problematic because of non-deterministic real-world races. While driving, robot can possibly face many problems not included in experiment scripts.

# 7 Bibliography


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