Astronomy Exercise 1

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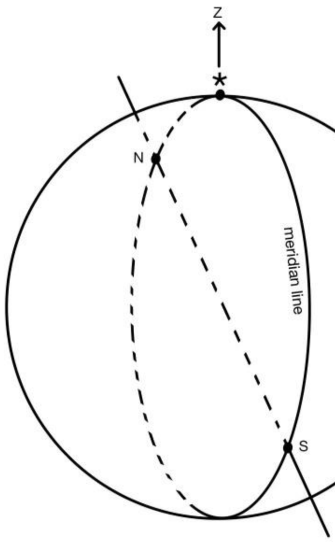
1. Coordinate system

a) Define **zenith**, **nadir**, the **celestial north** and **south poles**, and the **meridian line**. Draw the meridian plane of an observer, including the position of the observer (*), the zenith (Z), the meridian line, and the north (N) and south (S) poles.

- **zenith**: Point which is directly over the observer on a celestial sphere
- **nadir**: Point which is directly under the observer on a celestial sphere
- **celestial north pole**: Northern point of the Earth's rotation axis
- **celestial south pole**: Southern point of the Earth's rotation axis
- **meridian line**: Great circle that passes through the celestial poles



b) An observer located in the Earth's Northern Hemisphere observers the top and bottom culminations of circumpolar star. Measuring $h_i = 20^\circ\ 22'\ 32.4''$; $A_i = 180^\circ$ for the hight and azimuth of the bottom culmination and $h_s = 50^\circ\ 23'\ 08.2''$; $A_s = 180^\circ$ for the upper culmination. What is the observer's latitude ϕ?

First of all we convert the measured hights:

$$\begin{aligned}
h_i &= 20^\circ\ 22'\ 32.4'' \\
&= 20^\circ + \frac{22^\circ}{60} + \frac{32.4^\circ}{3600} \\
&\approx 20^\circ + 0.36666667^\circ + 0.009^\circ \\
&= 20.37566667^\circ \\
h_s &= 50^\circ\ 23'\ 08.2'' \\
&= 50^\circ + \frac{23^\circ}{60} + \frac{08.2^\circ}{3600} \\
&\approx 50^\circ + 0.38333333^\circ + 0.00227778^\circ \\
&= 50.3856111^\circ
\end{aligned}$$

Next I assume that the azimuth is for both measurments 0° and not 180°, because if the observer is on the Northern Hemisphere and looking south there is no way the star would be circumpolar for him at the angles 50° and 20°. With that in mind we can calculate the altitude of Polaris to determine the observer's latitude since the star circles around Polaris and its altitude will be equal to the observer's latitude. We calculate Polaris' altitude by determining the radius of the circle drawn around him on the celestial sphere and adding the lower culmination to the radius.

$$\begin{aligned}\phi &= \frac{h_s - h_i}{2} + h_i \\ &= \frac{50.3856111° - 20.37566667°}{2} + 20.37566667° \\ &= \frac{30.00994443°}{2} + 20.37566667° \\ &= 15.004972215° + 20.37566667° \\ &= 35.380638885°\end{aligned}$$

The observer's latitude is 35.380638885° north.

c) Determine the maximum height in the sky that the globular cluster ωCen (declination $\delta = -47° \, 29'$) reaches when observed from the Inter-American Observatory of Cerro Tololo, Chile (latitude $\phi = -30° \, 10' \, 20.9''$)

The Inter-American Observatory has a latitude of $-30° \, 10' \, 20.9''$, which means that the south pole of the celestial sphere is seen from that point of view at an angle of $30° \, 10' \, 20.9''$. The declanation of ωCen is $-47° \, 29'$, which means that the angle between it and the celestial equator is $47° \, 29'$ and since the celestial south pole is at $-90°$, ωCen is $90° - 47° \, 29'$ or $42° \, 31'$ away from the south pole. So at the upper culmination ωCen will be seen highest at $42° \, 31' + 30° \, 10' \, 20.9''$ or $72° \, 41' \, 20.9''$ from the Inter-American Observatory with an azimuth of 180°.

2. Greek Astronomy

a) In 265 BC, the Greek astronomer Aristarch attempted to measure the distance between the sun and the earth. To do this, he measured the angle between imaginary lines connecting the earth with the sun and the earth with the moon to be 87° at exactly half moon. Using his measurement, by how much is the distance to the sun larger than the distance to the moon? Using today's values for these distances, is his measurement accurate and in the case that it is not, what would be the angle he should have measured?

When we see a half moon from earth that means that the light of the sun is shining perpendicular on the line drawn from earth to the moon. In that case the angle between from moon to earth and moon to sun is exactly 90°. Measuring now the angle between earth to moon and earth to sun would allow to calculate how much bigger the sun is than the moon. Aristarch measured an angle of 87° which would give us $\frac{1}{cos(87)} \approx 19.1073$. Meaning the sun would be about 19.1073 times bigger than the moon. In reality the sun is about 400 times bigger than the moon.

$$\begin{aligned}400 &= \frac{1}{cos(x)} \\ 400 \cdot cos(x) &= 1 \\ cos(x) &= \frac{1}{400} \\ x &= arccos(\frac{1}{400}) \\ &\approx 89.86°\end{aligned}$$

Instead of the angle 87° he should have measured 89.86° to come up with a somewhat accurate result.

b) Around 220 BC Eratosthenes examined the fact that there is only one day each year in the city Cyrene, when the sun can reach the bottom of a deep well (meaning that the sun is located in the zenith), while this is never the case in Alexandria (which is further north and located 770 km from Cyrene). Additionally, he found that on this day, the angle between the zenith and the sun is 7.2° in Alexandria. Use this measurement to infer the radius of the earth. How well does the radius compare to today's value?

If we imagine the earth in two dimensional space we would have a circle. Moving around that circle once would represent 360°. If we move 7.2° that would mean we moved $\frac{7.2°}{360°} = 0.02$ times around that circle. If the bit we moved would be 770 km that would mean that the whole circle have a circumference of $50 \cdot 770$ km $= 38500$ km with this calculation and given parameters. In reality earth's circumference is about 40030 km. The calculation is actually close to an accurate result for that time.

3. Angular size

a) The Berliner Fernsehturm is 368 m tall. If you are standing 10 km from the tower, what is it's angular size measured from your vantage point? Express the result in radians, degree, arcminutes, and in the form "x degrees + y arcminutes + z arcseconds" where x, y, z are integer numbers.

First we calculate the hypotenuse, since there is a right angle between the Fernsehturm and my distance to the Fernsehturm.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ \sqrt{a^2 + b^2} &= c \\ \sqrt{(368 \text{ m})^2 + (10000 \text{ m})^2} &= c \\ \sqrt{135424 \text{ m}^2 + 100000000 \text{ m}^2} &= c \\ \sqrt{100135424 \text{ m}^2} &= c \\ 10006.7689 \text{ m} &\approx c \end{aligned}$$

Now we can calculate the angle α between the distance from me to the Fernsehturm and the distance from me to the top of the Fernsehturm d_u with the height of the Fernsehturm h.

$$\begin{aligned} \alpha &= arcsin\left(\frac{\text{countercathete}}{\text{hypotenuse}}\right) \\ &= arcsin\left(\frac{h}{d_u}\right) \\ &= arcsin\left(\frac{368}{10006.7689}\right) \\ &\approx 0.0367834015 \\ &= 2.108° \\ &= (2.108 \cdot 60)' = 126.48' \\ &= (2.108 \cdot 3600)'' = 7588.8'' \\ &\approx 2°\ 6'\ 29'' \end{aligned}$$

b) Measure the length of your hand, the length of your arm, and calculate the angular size of your hand when you stretch out your arm. Express the result in radians and degrees. Is it larger or smaller than the angular size that you calculated in part a)?

I measured a length of 67 cm for my arm and a length of 18.5 cm for my hand. First we calculate the hypotenuse again.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ \sqrt{a^2 + b^2} &= c \\ \sqrt{(67 \text{ cm})^2 + (18.5 \text{ cm})^2} &= c \\ \sqrt{4489 \text{ cm}^2 + 342.25 \text{ cm}^2} &= c \\ \sqrt{4831.25 \text{ cm}^2} &= c \\ 69.5072 &\approx c \end{aligned}$$

Now we calculate the angle α like in the previous exercise.

$$
\begin{aligned}
\alpha &= arcsin\left(\frac{\text{countercathete}}{\text{hypotenuse}}\right) \\
&= arcsin\left(\frac{h}{d_u}\right) \\
&= arcsin\left(\frac{18.5}{69.5072}\right) \\
&\approx 0.269407 \\
&= 15.44^\circ
\end{aligned}
$$

The angular size of my hand and arm is larger then the angular size from exercise a).

4. Parallax

a) Sirius is the biggest star in the night sky. It is located at a distance of 8.61 light years from the Sun. Calculate it's trigonometric parallax in arcminutes.

The radius of the Sun is about 696340 km, which would be about 7.36031961e-8 light years. With that we can calculate the trigonometric parallax.

$$
\begin{aligned}
p &= arctan\left(\frac{\text{counter cathetus}}{\text{attatched cathetus}}\right) \\
&= arctan\left(\frac{7.36031961 \cdot 10^{-8}}{8.61}\right) \\
&\approx 4.898 \cdot 10^{-7\circ} \\
&= 0.000029388'
\end{aligned}
$$

b) Do the same for Proxima Centauri, the nearest star to the sun, which is located at a distance of 4.2 light years.

$$
\begin{aligned}
p &= arctan\left(\frac{\text{counter cathetus}}{\text{attatched cathetus}}\right) \\
&= arctan\left(\frac{7.36031961 \cdot 10^{-8}}{4.2}\right) \\
&\approx 1.004 \cdot 10^{-6\circ} \\
&= 0.00006024'
\end{aligned}
$$

c) Compare both to the angular size of the moon.

The angular size of the diameter of the moon from earth is about 31.2 arcminutes. The parallax of Sirius would be about $\frac{1}{1061642}$ of the angular size of the moon. The parallax of Proxima Centauri would be about $\frac{1}{517928}$ of the angular size of the moon.