Astronomy Exercise 4

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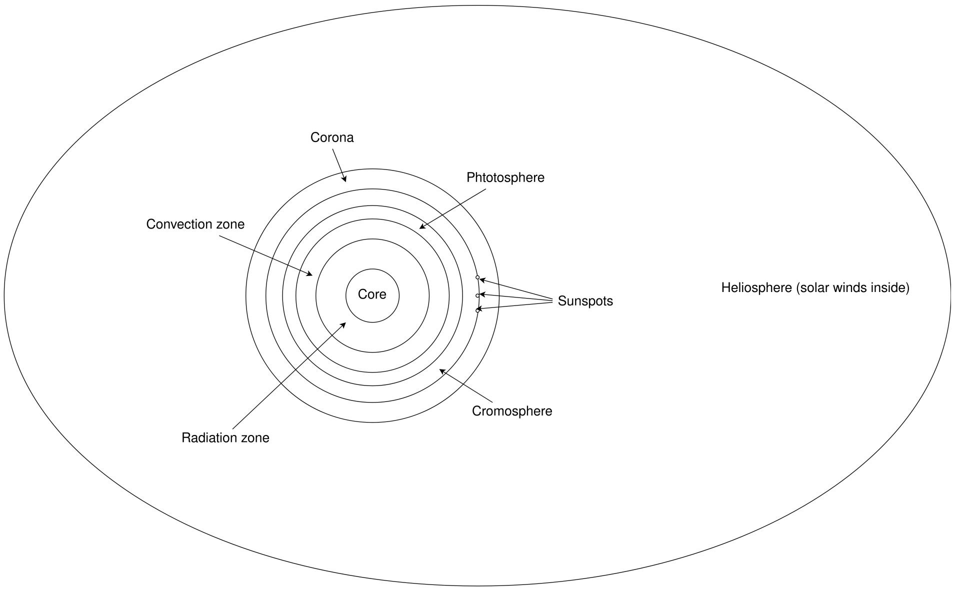
1. Basic solar structure

a) Draw an illustration diagram of the Sun with the following components and write a short description of each component: solar wind, corona, chromosphere, photosphere, convection zone, radiation zone, core and sunspots.



Solar wind is a stream of charged particles, predominantly electrons and protons, ejected from the Sun's outer layers into space.

Corona is the outermost layer of the Sun's atmosphere, characterized by extremely high temperatures. It can be seen as a halo during a solar eclipse.

Convection zone is a layer of the Sun where heat is transported outward by the rising of hot plasma and sinking of cooler plasma. Energy transfer occurs through convection currents.

Radiation zone is where energy is transported by electromagnetic radiation (photons) rather than the movement of material. Photons can take thousands to millions of years to travel from the core to the outer layers in this zone.

Core is the central region of the Sun where nuclear fusion reactions occur. High temperatures and pressures enable hydrogen atoms to fuse into helium, releasing huge amounts of energy.

Chromosphere is a layer of the solar atmosphere located just above the photosphere. It appears as a red glow and is visible during a solar eclipse.

Photosphere is the visible surface of the Sun that emits light and heat. It is the layer where most of the Sun's energy is radiated into space as sunlight.

Sunspots are temporary phenomena on the Sun's photosphere that appear as dark spots. They are caused by magnetic activity and are cooler than their surroundings.

b) Use Wien's Law to calculate the wavelength of peak thermal emission from the average temperature of the surface of the Sun. What color does this correspond to? How does that compare to the color of the Sun that the human eye perceives?

Wien's displacement law gives us the following formula to calculate the wavelength of th peak thermal emission:

$$\lambda_{\text{peak}} = \frac{b}{T}$$

The sun surface average temperature is around 5772 K. Inserting that and the constant of proportionality we receive:

$$\begin{aligned}\lambda_{\text{peak}} &= \frac{2898\mu\text{m} \cdot \text{K}}{5772\text{K}} \\ &= \frac{2898\mu\text{m}}{5772} \\ &\approx 0.5021\mu\text{m} \\ &= 502.1\text{nm}\end{aligned}$$

Usually the Sun looks yellow - reddish. But light from radiation with a wavelength of 502.1 nm would be perceived as a bright blue by the human eye.

c) The corona has a temperature of around 1 million K. How does this compare to the temperature of the surface of the Sun?

If we insert the temperature of the corone into the previous equation from b). We can see that the wavelength of it's radiation is way below 380 nm, which is why the light is not visible to the human eye and we don't see it's plasma usually.

2. Energy Production in the Sun

The sun generates its emitted energy by the fusion of hydrogen atoms to helium atoms. The dominating process to fuse helium, in sun-like stars, is generally known a the **p-p chain**. Comparing only the initial and final products, this process can be described by the equation: $4H^+ \Rightarrow He^{++} + 2e^+ + 2v + \gamma$. The liberated energy in this process is roughly equal to the mass difference between the helium and the 4 hydrogen atoms, which is also known as the mass defect.

a) Calculate the mass defect, in a p-p chain reaction, in Joule and MeV.

Mass of a proton is 1.6726×10^{-27} kg. That means that the mass of four protons is 6.6904×10^{-27} kg. The mass of a helium nucleus is 6.6447×10^{-27} kg. Subtracting the mass of the four protons from the mass of the helium nucleus we receive:

$$6.6447 \times 10^{-27} - 6.6904 \times 10^{-27} = -4.57 \times 10^{-29}$$

Inserting that in Einstin's famous equation, we have:

$$\begin{aligned}E &= mc^2 \\ &= (4.57 \times 10^{-29}\text{kg})(3 \times 10^8 \frac{m}{s})^2 \\ &= 4.11 \times 10^{-12} \frac{m\text{kg}}{s} \\ &= 4.11 \times 10^{-12}\text{J}\end{aligned}$$

We can convert the Joule to megaelectron volts like that:

$$4.11 \times 10^{-12}\text{J} \times 6.242 \times 10^{12} = 25.6546$$

b) Calculate the energy liberated, if one 1 kg of helium is produced.

In one kg of helium there are 1.5×10^{26} helium atoms. That means that the reaction has to occur 1.5×10^{26}-times to produce the needed amount of helium, which means that 1.5×10^{26}-times the energy from previous calculation will be liberated:

$$4.11 \times 10^{-12}\text{J} \times 1.5 \times 10^{26} = 6.165 \times 10^{14}\text{J}$$

c) The sun's age is roughly $4.6 \cdot 10^9$ years. Under the assumption that the solar luminosity didn't change over the sun's lifetime, how much helium was produced so far? Compare this mass with the total solar mass of $M_\circ = 2 \cdot 10^{30}$ kg.

With the assumption that the solar luminosity didn't change we can also assume that the Sun produced a steady amount of energy per year with nuclear fusion. The Sun produces 1.23×10^{35} Joules per year. With that in mind we can calculate the total amount of energy produced with nuclear fusion by the Sun:

$$1.23 \times 10^{35}\text{J} \times 4.6 \times 10^{9} = 5.658 \times 10^{44}\text{J}$$

We can devide that total amount of energy by the amount of energy, which produces one kg of helium as side product:

$$\frac{5.658 \times 10^{44}\text{J}}{6.165 \times 10^{14}\text{J}} \approx 0.9178 \times 10^{30}$$

We can estimate that the Sun produced in it's total lifespan around 0.9178×10^{30} kg of helium with nuclear fusion.

3. Resolution and the Rayleigh Criterion

The Rayleigh criterion gives the smallest possible angle between point sources or the best obtainable resolution. Therefore the Rayleigh Criterion allows to determine the best possible resolution of a telescope. This criterion for the minimum resolvable angle is $\Theta = 1.22\frac{\lambda}{D}$, with λ being the wavelength of light and D the opening diameter of an optional instrument.

a) Using the Rayleigh criterion, estimate the angular resolution limit of the human eye at the wavelength of 550 nm . Assume the diameter of the pupil is 5 mm.

b) Compare your answer in part a) to the angular diameters of the moon and Jupiter. Can we resolve features on the Moon and Jupiter with just our eyes? Use your calculations to support your argument.

c) What is the angular resolution of the radio telescope in Effenberg (D = 100 m, λ = 21 cm)? Can you use this telescope to resolve features on the moon?

4. Limiting magnitudes and light collecting power

a) While on clear nights the human eye can only observe stars down to a limiting magnitude value of 6 mag, fainter stars are observable using a telescope. This is due to the telescope's larger aperture compared to the human eye and therefore more light collection. Calculate the limiting magnitude for an observer, observing the sky through a telescope with an aperture of 50 cm. You can use that the human eye has an aperture size of 5 mm.

b) Telescopes can be compared by their power to collect light. This power directly scales with the telescope's light collecting area. How much more light will be gathered by the currently constructed Extremely Large Telescope (aperture of 39 m) in comparison to the GRANTECAN telescope (aperture of 10.4 m), which has currently the largest single telescope aperture?