Astronomy Exercise 2

Kilian Calefice (796461)

November 2023

1. Kepler's laws

a) State the three laws of Kepler

1. Law of ellipses: Bodies move around a body with way greater mass in an ellipitcal path and the greater body being at one of the focal points of the ellipses.

2. Law of equal areas: The body moving around the other doesn't move with a constant velocity around in the ellipitcal path as it covers always the same area that is drawn on the ellipses in the same time, meaning if the body is further away from the focal point it moves slower as it would when it is close to it.

3. Law of harmonic time: The planet's orbital period is proportional to the cube of the length of the semi-major axis of the ellipses.

b) The comet Tschurjumow-Gerassimenko has a perihelion of 1.21 AU and an orbital period of 6.43 yr. What is the aphelion of the comet's orbit? Give the answer in both AU and in km.

With Kepler's third law we can determine the lenght of the semi-major axis since the orbital period is given:

$$\begin{aligned}
\frac{a^3}{T^2} &= \frac{G(M+m)}{4\pi^2} \approx \frac{GM}{4\pi^2} \\
a^3 &= \frac{GMT^2}{4\pi^2} \\
a &= \sqrt[3]{\frac{GM}{4\pi^2} \cdot T^2} \\
&= \sqrt[3]{\frac{T^2}{yr^2}} \cdot \sqrt[3]{\frac{GM}{4\pi^2} \cdot yr^2}
\end{aligned}$$

The term $\sqrt[3]{\frac{GM}{4\pi^2} \cdot yr^2}$ is the definition of the astronomical unit AU, which is why we receive:

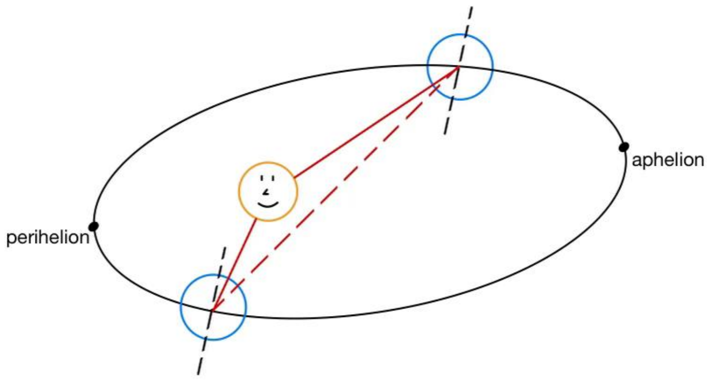
$$\begin{aligned}
a &= \sqrt[3]{\frac{T^2}{yr^2}} AU \\
&= \sqrt[3]{\frac{6.43^2 \cdot yr^2}{yr^2}} AU \\
&= \sqrt[3]{6.43^2} AU \\
&\approx 3.4579 AU
\end{aligned}$$

The aphelion we receive by subtracting the perihelion from the full major axis:

$$\begin{aligned}
d_{aphelion} &= 2a - d_{perihelion} \\
&= 2a - d_{perihelion} \\
&= 2 \cdot 3.4579 AU - 1.21 AU \\
&= 5.7058 AU \\
&= 5.7058 \cdot 1,496e+8 km \\
&\approx 8,535755e+12 km
\end{aligned}$$

c) This and next year's spring equinox happend on March 20, while this year's fall equinox occured on September 22. If you count the days, there are roughly 186 days between the spring and fall equinox, but only 179 days between fall and the next spring equinox. Using Kepler's laws, explain why the nothern winter seems to take less time than the northern summer. During which season is the Earth closer to the Sun?

An equinox occurs when the Sun's declanation is $0°$. The Earth's axis is tilted in perspective to the Sun. Also the orbital eccentricity of the Earth is about 0.017, which means that the Sun is at one of the foci and there is aphelion and perihelion which the Earth reaches. The two equinox points draw a line throught the ellipses of Earth's orbit deviding the ellipses in a not mirror symmetrical but symmetrical manner. That means that the two lengths which we receive from the circumference are equal. The perihelion and aphelion are each on one of these lengths. Due to Kepler's second law these to lengths will be run with two different velocities, one being faster and one slower. Due to the slight ellipitcal shape of the orbit this difference will be noticed in only a few days as stated in



the problem setting.
During the northern winter the Earth is closer to the Sun, which means that the Earth has a higher velocity on it's circular path and which is why this season takes less time.

An equinox occurs when the Sun's declanation is $0°$, so that the Sun rises exactly at east and sets exactly west. Since the Earth's orbital eccentricity is about 0.017 it's orbital shape is not a circle but a slight ellipses. Also that means that the Sun is at one of the foci. Also the Earth is titled. Due to that and the elliptical shape of the Earth's orbit these points are diagonal to each other in perspective to the ellipses, meaning that one half of the ellipses is longer close to the perihelion and the other longer closer to the aphelion. Because of Kepler's second law

2. Time systems

a) Define sideral time, true solar time and mean solar time.

b) Calculate the difference between the civil time and mean solar time for the following cities: Berlin $\lambda = 13.40°$ E; Barcelona $\lambda = 2.16°$ E; Warsaw $\lambda = 21.012°$ E. Note all these cities follow the same time zone (CET) defined by meridian with longitude $\lambda = 15°$ E.

3. The Virial Theorem

a) In space there are many gravitationally bound systems. If a system is roughly in equilibrium, the Virial Theorem states that the kinetic energy is equal to minus one half the potential energy ($\langle T \rangle = -\frac{1}{2}\langle V \rangle$). Consider a light particle in a circular orbit around a heavier one. Prove the Virial Theorem from this system's equation of motion.

4. Luminosity

a) What do you understand by irradiance? Calculate the irradiance received from the Sun above the absorbing atmospheres of planet Mercury, Earth, and Uranus. Given, luminosity of the Sun $L_\odot = 3.839 \times 10^{26} W$.

b) The irradiance received by the Sun above the Earth's atmosphere per unit area, is also known as the "solar irradiance". Find the distance from where a 60-Watt lightbulb has its irradiance equal to the solar irradiance.

c) The energy emitted per second by a star is $L = 4\pi R^2 \sigma T_{eff}^4 = S\sigma T_{eff}^4$, where S is the surface area of the star. A person also emits radiation. Under normal conditions, the temperature of the human body is around $37°C$ or $T \approx 310K$. An area of a person's body is on the order of $S \approx 1.7m^2$. What is the energy emitted per second by a person and what is the characteristic wavelength of this emission?