Chapter 4.5 - Multilevel Categorical Variables

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Review

Let y_i and $x_{1,i}$ be quantitative variables and $x_{2,i}$ be a categorical variable with two levels.

Model 1: $y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \epsilon_i$

Model 2: $y_i = \alpha_0 + \alpha_1 x_{1,i} + \alpha_2 x_{2,i} + \alpha_3 x_{1,i} x_{2,i} + \epsilon_i$

For each of the following tests, which model and parameter would you use?

1. There is a linear association between x_2 and y after adjusting for x_1.
2. There is a linear association between x_1 and y for subjects in the reference group of x_2.
3. There is a linear association between x_1 and y for subjects not in the reference group of x_2.
4. The effect of x_1 on y differs by level of x_2.

Let's say you fit Model 2 and you find α_2 is not sign-ficant. Should you conclude there is no association between x_2 and y? Explain.

Diamonds

When buying a diamond, you focus on the 4 C's: cut, clarity, color, and carat (weight). In this example, we will investigate the association between price, weight, and clarity. There are five clarity categories (IV, VVS1, VVS2, VS1, VS2).

```
diamonds = read.table(file = "http://www.isi-stats.com/isi2/data/diamonds.txt",
                      header = T)
```

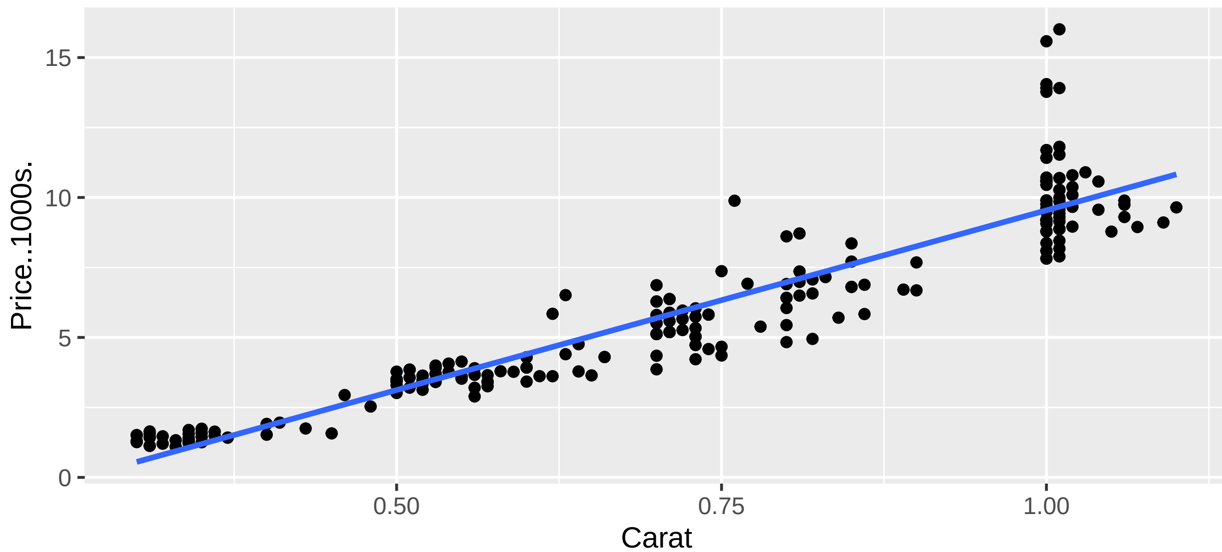
One variable analyses

Price vs Weight

Let y_i be the price and x_i be the weight (carats) of diamond i. Write a linear model for price as a function of weight.

```r
diamonds %>% ggplot(aes(x = Carat, y = Price..1000s.)) +
  geom_point() + geom_smooth(method = "lm", se = F)
```

```
## `geom_smooth()` using formula 'y ~ x'
```




```r
model_weight = lm(Price..1000s. ~ Carat, data = diamonds)
```

Describe two interesting features of the plot above.

```r
summary(model_weight)
```

```
##
## Call:
## lm(formula = Price..1000s. ~ Carat, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.2819 -0.6242 -0.0978  0.3977  6.3380
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -3.3010     0.2543  -12.98   <2e-16 ***
## Carat        12.8426     0.3355   38.28   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.195 on 228 degrees of freedom
## Multiple R-squared:  0.8653, Adjusted R-squared:  0.8647
## F-statistic:  1465 on 1 and 228 DF,  p-value: < 2.2e-16
```

```
anova(model_weight)
```

```
## Analysis of Variance Table
##
## Response: Price..1000s.
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## Carat       1 2092.26 2092.26  1465.1 < 2.2e-16 ***
## Residuals 228  325.59    1.43
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

How accurately does this model typically predict diamond price?

Based on this model, what's a reasonable price per carat?

Explain how confounding by clarity could occur in this scenario.

Price vs clarity

We can express models for clarity and price in different ways. The models below are equivalent.

Write a separate means model for price and clarity.

Write a model for price and clarity using effect encoding.

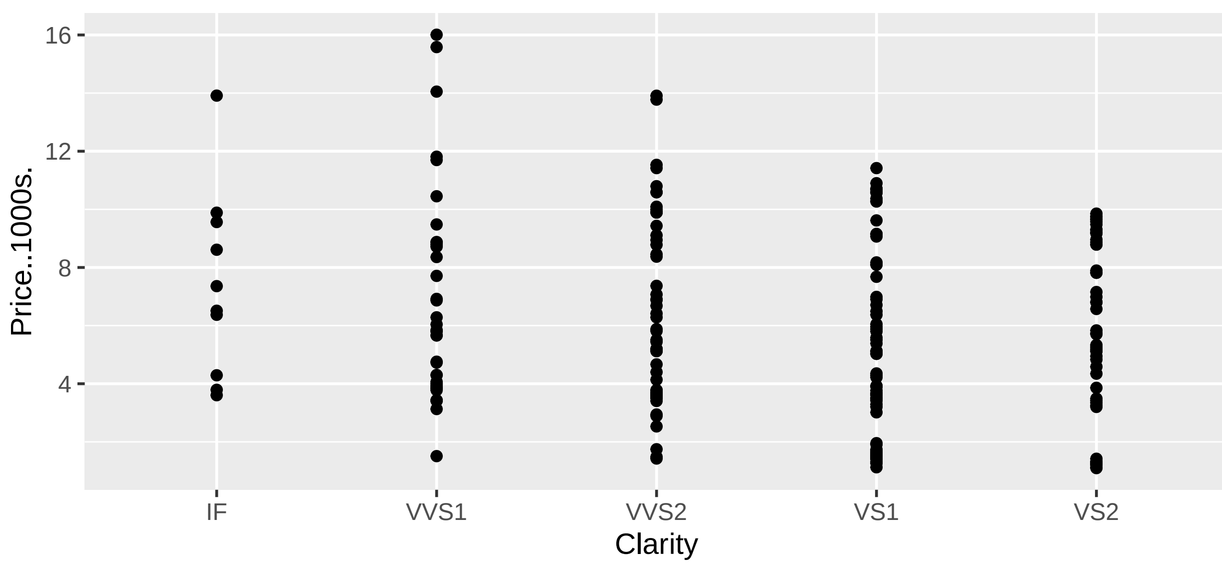
Write a model for price and clarity using indicator encoding.

```
#reorder the levels from highest to lowest clarity
diamonds <- diamonds %>%
  mutate(Clarity = factor(Clarity,
                          levels = c("IF", "VVS1", "VVS2", "VS1", "VS2")))
#summary statistics
diamonds %>%
  group_by(Clarity) %>%
  summarize(mean = mean(Price..1000s.),
            sd = sd(Price..1000s.),
            n = n())
```

```
## # A tibble: 5 x 4
##   Clarity  mean    sd     n
##   <fct>   <dbl> <dbl> <int>
## 1 IF       7.39  3.23    10
```

```
## 2 VVS1      6.72  3.56    38
## 3 VVS2      6.29  3.16    57
## 4 VS1       5.14  3.21    74
## 5 VS2       5.90  2.99    51
```

```
#plot
diamonds %>% ggplot(aes(x = Clarity, y = Price..1000s.)) +
  geom_point()
```



Let's move forward using indicator encoding. Let's fit the model.

```
model_Clarity = lm(Price..1000s. ~ Clarity, data = diamonds)
summary(model_Clarity)
```

```
##
## Call:
## lm(formula = Price..1000s. ~ Clarity, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.2052 -2.6522 -0.7788  2.5254  9.2928
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    7.390      1.016   7.278 5.62e-12 ***
## ClarityVVS1   -0.675      1.141  -0.591   0.5548
## ClarityVVS2   -1.102      1.101  -1.001   0.3179
## ClarityVS1    -2.246      1.082  -2.076   0.0391 *
## ClarityVS2    -1.489      1.111  -1.341   0.1814
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.211 on 225 degrees of freedom
## Multiple R-squared:  0.04039,    Adjusted R-squared:  0.02333
## F-statistic: 2.368 on 4 and 225 DF,  p-value: 0.05363
```

```r
anova(model_Clarity)
```

```
## Analysis of Variance Table
##
## Response: Price..1000s.
##            Df  Sum Sq Mean Sq F value  Pr(>F)
## Clarity     4   97.66  24.415  2.3676 0.05363 .
## Residuals 225 2320.19  10.312
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

What is the predicted price for an IF diamond?

How accurately does this model typically predict diamond price?

Using the results above, find estimates of the parameters in the effect encoding model.

Weight vs. Carat

The last thing we might want to look at before proceeding to the two-variable analysis is the relationship between clarity and weight. For time sakes, we'll just look at the means.

```r
diamonds %>%
  group_by(Clarity) %>%
  summarize(mean = mean(Carat),
            sd = sd(Carat),
            n = n())
```

```
## # A tibble: 5 x 4
##   Clarity  mean    sd     n
##   <fct>   <dbl> <dbl> <int>
## 1 IF      0.748 0.170    10
## 2 VVS1    0.731 0.198    38
## 3 VVS2    0.753 0.215    57
## 4 VS1     0.656 0.254    74
## 5 VS2     0.764 0.253    51
```

Two variable analysis

In this section, we want to:

- Estimate the price per carat after adjusting for clarity.

- Determine whether the clarity is associated with price after adjusting for diamond weight.

Using indicator encoding, write a model for price as a function of weight and clarity.

```
model_ClarityWeight = lm(Price..1000s. ~ Carat + Clarity, data = diamonds)
summary(model_ClarityWeight)
```

```
##
## Call:
## lm(formula = Price..1000s. ~ Carat + Clarity, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9982 -0.6078 -0.0376  0.4914  5.6904
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.2787     0.4265  -5.343 2.24e-07 ***
## Carat        12.9264     0.3196  40.450  < 2e-16 ***
## ClarityVVS1  -0.4593     0.3970  -1.157  0.24850
## ClarityVVS2  -1.1619     0.3829  -3.034  0.00270 **
## ClarityVS1   -1.0629     0.3774  -2.816  0.00529 **
## ClarityVS2   -1.6997     0.3863  -4.400 1.67e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.117 on 224 degrees of freedom
## Multiple R-squared:  0.8844, Adjusted R-squared:  0.8819
## F-statistic: 342.9 on 5 and 224 DF,  p-value: < 2.2e-16
```

```
anova(model_ClarityWeight)
```

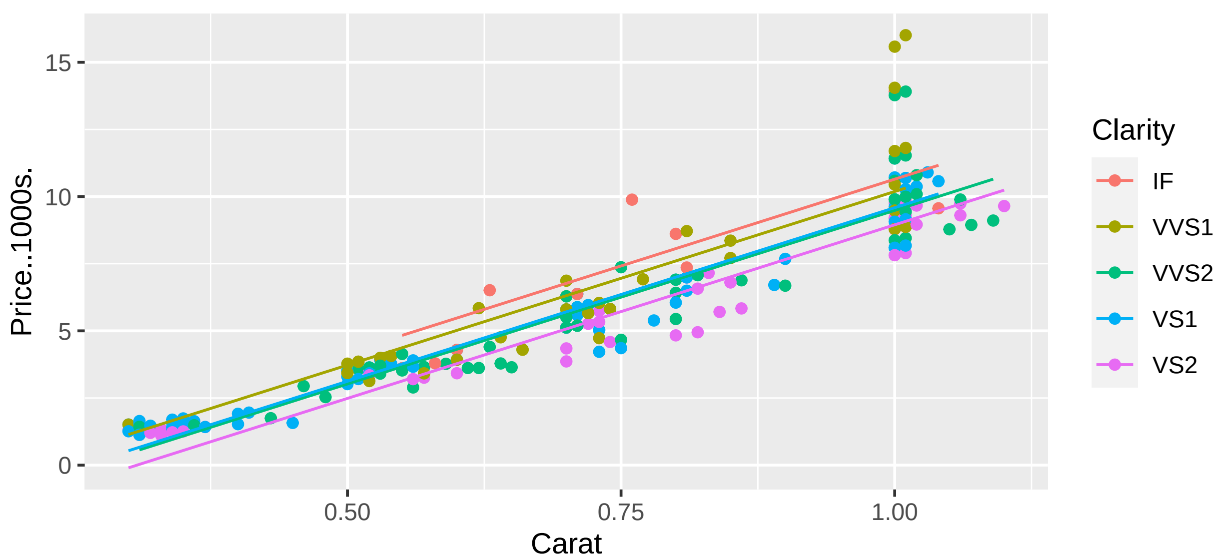
```
## Analysis of Variance Table
##
## Response: Price..1000s.
##            Df  Sum Sq Mean Sq   F value    Pr(>F)
## Carat       1 2092.26 2092.26 1677.4324 < 2.2e-16 ***
## Clarity     4   46.20   11.55    9.2601 6.084e-07 ***
## Residuals 224  279.39    1.25
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on this model, what's a reasonable price per carat? Has it changed much after adjusting for clarity?

How much does clarity improve your predictions of price?

Write out the regression equations for the five categories of clarity. What is the relationship among these regression lines?

```
diamonds = diamonds %>%
  mutate(predicted2 = predict(model_ClarityWeight, diamonds))
diamonds %>% ggplot(aes(x = Carat, y = Price..1000s., color = Clarity)) +
  geom_point() + geom_line(aes(y = predicted2))
```



Draw inference.

H_0 : There is no linear association between clarity and price, after adjusting for diamond weight.

H_a : There is a linear association between clarity and price, after adjusting for diamond weight.

Why not use the p-values for each indicator variable?

Perform the partial F-test (pg 344).

What do the p-values of the indicator variable coefficients tell us?

Interaction

Write a research question we could answer by including the interaction between clarity and weight.

H_0 : There is no interaction between clarity and weight.

H_a : There is an interaction between clarity and weight.

```r
model_interaction = lm(Price..1000s. ~ Carat*Clarity, data = diamonds)
summary(model_interaction)
```

```
##
## Call:
## lm(formula = Price..1000s. ~ Carat * Clarity, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.6811 -0.5578 -0.0072  0.5191  4.8095
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       -5.043673   1.605603  -3.141  0.00191 **
## Carat             16.622824   2.098298   7.922 1.15e-13 ***
## ClarityVVS1       -0.005889   1.740908  -0.003  0.99730
## ClarityVVS2        1.067706   1.687518   0.633  0.52758
## ClarityVS1         2.110958   1.642652   1.285  0.20011
## ClarityVS2         2.131208   1.676210   1.271  0.20491
## Carat:ClarityVVS1 -0.535659   2.278844  -0.235  0.81438
## Carat:ClarityVVS2 -2.985106   2.200818  -1.356  0.17637
## Carat:ClarityVS1  -4.319286   2.155500  -2.004  0.04631 *
## Carat:ClarityVS2  -5.091072   2.181988  -2.333  0.02054 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.07 on 220 degrees of freedom
## Multiple R-squared:  0.8958, Adjusted R-squared:  0.8915
```

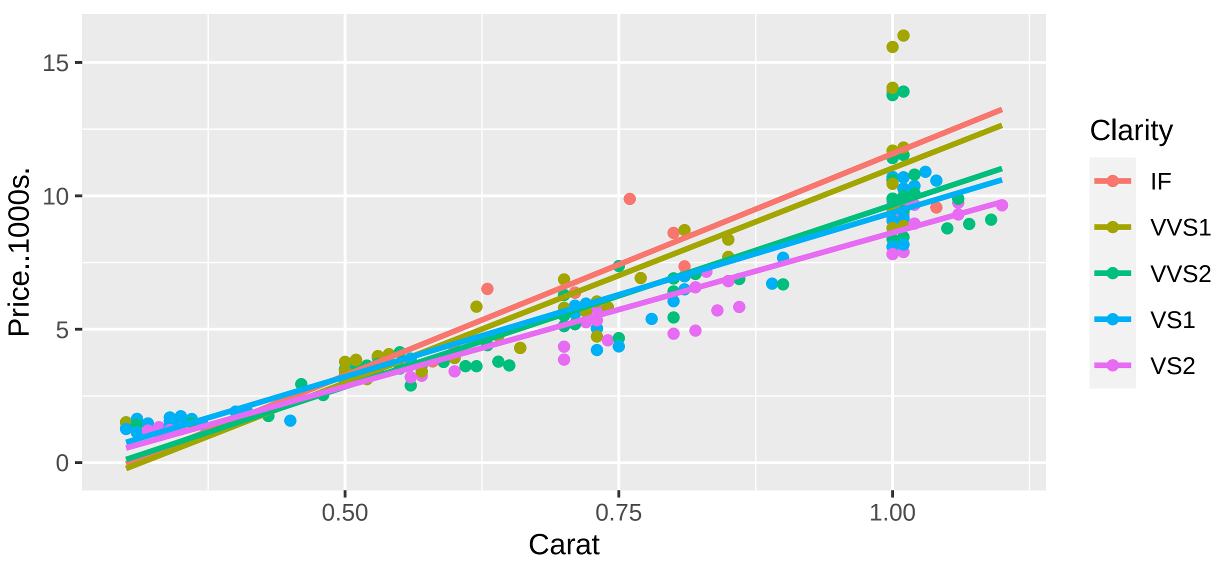
```
## F-statistic: 210.1 on 9 and 220 DF,  p-value: < 2.2e-16
```

```r
anova(model_interaction)
```

```
## Analysis of Variance Table
##
## Response: Price..1000s.
##                Df Sum Sq Mean Sq   F value    Pr(>F)
## Carat           1 2092.3 2092.26 1826.5840 < 2.2e-16 ***
## Clarity         4   46.2   11.55   10.0835 1.638e-07 ***
## Carat:Clarity   4   27.4    6.85    5.9793 0.0001384 ***
## Residuals     220  252.0    1.15
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
diamonds %>% ggplot(aes(x = Carat,y = Price..1000s., color=Clarity)) +
  geom_point() + geom_smooth(method = "lm", se = F, fullrange = T)
```

```
## `geom_smooth()` using formula 'y ~ x'
```



Perform the partial F-test.

Confidence Intervals

```r
confint(model_interaction)
```

```
##                        2.5 %      97.5 %
## (Intercept)        -8.208004 -1.87934173
## Carat              12.487486 20.75816258
## ClarityVVS1        -3.436879  3.42510158
## ClarityVVS2        -2.258064  4.39347497
## ClarityVS1         -1.126390  5.34830568
## ClarityVS2         -1.172277  5.43469216
## Carat:ClarityVVS1 -5.026816  3.95549878
## Carat:ClarityVVS2 -7.322490  1.35227777
## Carat:ClarityVS1  -8.567357 -0.07121415
## Carat:ClarityVS2  -9.391346 -0.79079804
```