The Actuarial Education Company

on behalf of the Actuarial Profession

General Insurance – Pricing

Subject ST8

COMBINED MATERIALS PACK

FOR EXAMS IN 2013

2013 Study Guide

Subject ST8

Introduction

This Study Guide contains all the information that you will need before starting to study Subject ST8 for the 2013 exams. **Please read this Study Guide carefully before reading the Course Notes,** even if you have studied for some actuarial exams before.

When studying for the UK actuarial exams, you will need a copy of the **Formulae and Tables for Examinations of the Faculty of Actuaries and the Institute of Actuaries, 2nd Edition (2002)**. These are often referred to as simply the yellow **Tables** and are available separately from the Publications shop of the Actuarial Profession. You will also need a 'permitted' scientific calculator from the list published by the Profession. Please check the list carefully since it is reviewed each year. You will find the list of permitted calculators and a link to the Publications shop in the profession's website at **www.actuaries.org.uk**.

Contents:

1 The Subject ST8 course structure

There are six parts to the Subject ST8 course. This should help you plan your progress across the study session. The parts cover related topics and have broadly equal lengths. The parts are broken down into chapters. The following table shows how the parts, the chapters and the syllabus items relate to each other. The end columns show how the chapters relate to the days of the regular tutorials.

Part	Chapter	Title	No of pages	Syllabus items	3 full Days
1	1	Insurance companies	45	-	1
	2	Insurance products – background	43	(b)	
	3	Insurance products – types	72	(b)	
	4	Tackling an unusual product	11	-	
	5	Reinsurance products (1)	30	(c)	
	6	Reinsurance products (2)	63	(c)	
2	7	General insurance markets	30	(d)	
	8	External environment	53	(d)	
	9	Risk and uncertainty in pricing	42	(e)	
	10	Data	63	(f)	
3	11	Aggregate claim distribution models	44	(h)	2
	12	Rating methodologies and bases	45	(i),(j)	
	13	Further considerations when rating	25	(i),(j)	
	14	Rating using frequency-severity and burning cost approaches	41	(k),(l)	
4	15	Rating using original loss curves	51	(m)	
	16	Generalised linear modelling	75	(n)	
	17	Use of multivariate models in pricing	50	(o)	
5	18	Credibility theory	54	(p)	3
	19	Actuarial investigations	42	(g)	
6	20	Reinsurance pricing	49	(q)	
	21	Use of catastrophe models	15	(r)	
		Glossary	52	(a)	

2 ActEd study support

This section lists the study support available from ActEd for Subject ST8.

Course Notes

The Course Notes will help you develop the basic knowledge and understanding of principles needed to pass the exam. They incorporate the complete Core Reading and include full explanation of all the syllabus objectives, with worked examples and short questions to test your understanding.

Each chapter includes the relevant syllabus objectives, a chapter summary and, where appropriate, a page of important formulae or definitions.

Question and Answer Bank

The Question and Answer Bank provides a comprehensive bank of questions (including some past exam questions) with full solutions and comments.

The Question and Answer Bank is divided into seven parts. The first six parts include a range of short and long questions to test your understanding of the corresponding part of the Course Notes. Part seven consists of 100 marks of exam-style questions.

Assignments

The six Series X Assignments (X1 to X6) cover the material in Parts 1 to 6 respectively. Assignments X1, X2 and X3 are 80-mark tests and should take you two and a half hours to complete. Assignments X4, X5 and X6 are 100-mark tests and should take you three hours to complete. The actual Subject ST8 examination will have a total of 100 marks.

Combined Materials Pack (CMP)

The Combined Materials Pack (CMP) comprises the Course Notes, the Question and Answer Bank and the Series X Assignments.

The CMP is available in eBook format, enabling it to be downloaded in a rights-protected PDF format and read on your PC, laptop and ipad (but NOT currently on Kindles). CMP eBooks can be ordered separately or as an addition to a paper CMP. Visit **www.ActEd.co.uk** for more details about compatibility, printing restrictions and software requirements.

Mock Exam

A 100-mark mock exam paper (Mock Exam A) is available for students as a realistic test of their exam preparation. The mock is based on Mock Exam A from last year but it has been updated to reflect any changes to the Syllabus and Core Reading.

Additional Mock Pack (AMP)

The Additional Mock Pack (AMP) consists of two further 100-mark mock exam papers – Mock Exam B and Mock Exam C. This is ideal for students who are retaking and have already sat Mock Exam A, or for those who just want some extra question practice. If you are retaking this subject you should note that the mock exams in the AMP use many questions from Mock Exam B for the 2011 exams. Therefore, if you purchased Mock Exam B that year you may not wish to purchase the AMP as many of the questions will be duplicated.

ActEd Solutions with Exam Technique (ASET)

The ActEd Solutions with Exam Technique (ASET) contains ActEd's solutions to the previous four years' exam papers, *ie* eight papers, plus comment and explanation. In particular it will highlight how questions might have been analysed and interpreted so as to produce a good solution with a wide range of relevant points. This will be valuable in approaching questions in subsequent examinations.

A "Mini-ASET" will also be available in the summer session covering the April Exam only.

CMP Upgrade

The CMP Upgrade lists all significant changes to the Core Reading and ActEd material so that you can manually amend your 2011 study material to make it suitable for study for the 2012 exams. The Upgrade includes replacement pages and additional pages where appropriate. If a large proportion of the material has changed significantly, making it inappropriate to include all changes, the upgrade will still explain what has changed and if necessary recommend that students purchase a replacement CMP or Course Notes at a significantly reduced price. The CMP Upgrade can be downloaded free of charge from our website at **www.ActEd.co.uk**.

Flashcards

Flashcards are a set of A6-sized cards that cover the key points of the subject that most students want to commit to memory. Each flashcard has questions on one side and the answers on the reverse. We recommend that you use the cards actively and test yourself as you go.

Flashcards may be used to complement your other study and revision materials. They are not a substitute for question practice but they should help you learn the essential material required.

Flashcards are available in eBook format, enabling them to be downloaded in a rights-protected PDF format and read on your PC, laptop, ipad and many smart phones (but NOT currently on Kindles). Flashcard eBooks can be ordered separately or as an addition to a paper copy. Visit **www.ActEd.co.uk** for more details about compatibility, printing restrictions and software requirements.

Marking

We are happy to mark your attempts at any of the currently available assignments, Mock Exam A or the mock exams included within AMP. Marking is not included with the products themselves and you need to order it separately. You can submit your scripts by email, fax or post.

Series Marking and Mock Exam marking

Series Marking (for the Series X Assignments) and Mock Exam Marking (for Mock Exam A) apply to a specified subject, session and student. If you purchase Series Marking or Mock Exam Marking, you will **not** be able to defer the marking to a future exam sitting or transfer it to a different subject or student.

If you order marking at the same time as you order the assignments or mock exam, you can choose whether or not to receive a copy of the solutions in advance. If you choose not to receive the solutions in advance, we will send the solutions to you when we return your marked script (or following the deadline date if you don't submit).

If you are having your attempts at the assignments marked by ActEd, you should submit your scripts regularly throughout the session, in accordance with the schedule of recommended dates set out in the summary at the end of this document. This will help you to pace your study throughout the session and leave an adequate amount of time for revision and question practice.

Any script submitted after the relevant final deadline date will not be marked. It is your responsibility to ensure that scripts are posted in good time.

Important information

The recommended submission dates are realistic targets for the majority of students. Your scripts will be returned more quickly if you submit them well before the final deadline dates.

Marking Vouchers

Marking Vouchers give the holder the right to submit a script for marking at any time, irrespective of the individual assignment deadlines, study session, subject or person.

Marking Vouchers can be used for any assignment, Mock Exam A, or the mock exams contained within the AMP. Please note that attempts at the AMP can **only** be marked using Marking Vouchers.

Marking Vouchers are valid for four years from the date of purchase and can be refunded at any time up to the expiry date.

Important information

Although you may submit your script with a Marking Voucher at any time, you will need to adhere to the explicit Marking Voucher deadline dates to ensure that your script is returned before the date of the exam. The deadline dates are given at the end of this study guide.

If you live outside the UK you must ensure that your last script reaches the ActEd office earlier than this to allow the extra time needed to return your marked script.

Tutorials

ActEd tutorials are specifically designed to develop the knowledge that you will acquire from the course material into the higher level understanding that is needed to pass the exam. We expect you to have read the relevant part of the Course Notes before attending the tutorial so that the group can spend time on exam questions and discussion to develop understanding rather than basic bookwork.

ActEd run a range of different tutorials at various locations. Full details are set out in ActEd's *Tuition Bulletin*, which is sent regularly to all students based in the UK, Eire and South Africa and is also available from the ActEd website at **www.ActEd.co.uk**.

Regular and Block Tutorials

You can choose ***one*** of the following types of tutorial:

- **Regular Tutorials** (three full days) spread over the session.
- A **Block Tutorial** (three full days) held 2 to 8 weeks before the exam.

The Regular Tutorials provide an even progression through the course. Block Tutorials cover the whole course.

Queries and feedback

From time to time you may come across something in the study material that is unclear to you. The easiest way to solve such problems is often through discussion with friends, colleagues and peers – they will probably have had similar experiences whilst studying. If there's no-one at work to talk to then use ActEd's discussion forum at **www.ActEd.co.uk/forums** (or use the link from our home page at **www.ActEd.co.uk**).

Our online forum is dedicated to actuarial students so that you can get help from fellow students on any aspect of your studies from technical issues to study advice. You could also use it to get ideas for revision or for further reading around the subject that you are studying. ActEd Tutors will visit the site from time to time to ensure that you are not being led astray and we also post other frequently asked questions from students on the forum as they arise.

If you are still stuck, then you can send queries by email to **ST8@bpp.com** or by fax to 01235 550085 (but we recommend that you try the forum first). We will endeavour to contact you as soon as possible after receiving your query but you should be aware that it may take some time to reply to queries, particularly when tutors are away from the office running tutorials. At the busiest teaching times of year, it may take us more than a week to get back to you.

If you have many queries on the course material, you should raise them at a tutorial or book a personal tuition session with an ActEd Tutor. Information about personal tuition is set out in our current brochure. Please email **ActEd@bpp.com** for more details.

If you find an error in the course, please check the corrections page of our website (**www.ActEd.co.uk/Html/paper_corrections.htm**) to see if the correction has already been dealt with. Otherwise please send details via email to **ST8@bpp.com** or send a fax to **01235 550085**.

Each year ActEd Tutors work hard to improve the quality of the study material and to ensure that the courses are as clear as possible and free from errors. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses. If you have any comments on this course please email them to **ST8@bpp.com** or fax them to 01235 550085.

The ActEd Tutors also work with the profession to suggest developments and improvements to the Syllabus and Core Reading. If you have any comments or concerns about the Syllabus or Core Reading, these can be passed on via ActEd. Alternatively, you can address them directly to the Profession's Examination Team at Napier House, 4 Worcester Street, Oxford, OX1 2AW or by email to **examinations@actuaries.org.uk**.

3 Core Reading, the Syllabus and the Profession

Core Reading

The Syllabus for Subject ST8 has been written by the profession to state the requirements of the examiners. The relevant individual Syllabus Objectives are included at the start of each course chapter and a complete copy of the Syllabus is included in Section 6 of this Study Guide. We recommend that you use the Syllabus as an important part of your study. The Syllabus is supplemented by Core Reading, which has also been written by the profession. The purpose of Core Reading is to give the examiners, tutors and students a clear, shared understanding of the depth and breadth of treatment required by the Syllabus. In examinations students are expected to demonstrate their understanding of the concepts in Core Reading. Examiners have the Core Reading available when setting papers.

Core Reading deals with each Syllabus objective. Core reading covers what is needed to pass the exam but the tuition material that has been written by ActEd enhances it by giving examples and further explanation of key points. The Subject ST8 Course Notes include the Core Reading in full, integrated throughout the course. Here is an excerpt from some ActEd Course Notes to show you how to identify Core Reading and the ActEd material. **Core Reading is shown in this bold font.**

Note that in the example given above, the index *will* fall if the actual share price goes below the theoretical ex-rights share price. Again, this is consistent with what would happen to an underlying portfolio.

This is ActEd text

After allowing for chain-linking, **the formula for the investment index becomes:**

This is Core Reading

$$\boldsymbol{I(t) = \frac{\sum_i N_{i,t} P_{i,t}}{B(t)}}$$

where $\boldsymbol{N_{i,t}}$ is the number of shares issued for the *i*th constituent at time *t*;

$\boldsymbol{B(t)}$ is the base value, or divisor, at time *t*.

Core Reading accreditation

The Institute and Faculty of Actuaries would like to thank the numerous people who have helped in the development of the material contained in this Core Reading, and are grateful to the Casualty Actuarial Society for permission to use some of its educational material for sections within the Units.
In particular various sections of unit 14 have been used by permission of the Casualty Actuarial Society. Republished with minor modifications from *Foundations of Casualty Actuarial Science*, Fourth Edition, (Arlington, Virginia, USA: CAS, 2001).

Changes to the Syllabus and Core Reading

The Syllabus and Core Reading are updated as at 31 May each year. The exams in April and September 2013 will be based on the Syllabus and Core Reading as at 31 May 2012.

We recommend that you always use the up-to-date Core Reading to prepare for the exams.

The Profession's Copyright

Past exam papers

You can download some past papers and reports from the profession's website at **www.actuaries.org.uk**.

Calculators

Please refer to the profession's website for the latest advice on which calculators are permitted in the exams.

4 Study Skills

The ST Subject exams

It is important to recognise that the ST subject exams are very different from the CT Subject exams in both the nature of the material covered and the skills being examined.

Both the Core Reading and the exam papers themselves are generally much less numerical and more "wordy" than the typical CT subject. The exam will primarily require you to explain a particular point in words and sentences, rather than to manipulate formulae. Numerical questions typically account for only a small part of each exam paper. If you haven't sat this type of exam for some time, you need to start practising again now. Many students find that it takes time to adjust to the different style of the ST subject exam questions. As ever, practice is the key to success.

The aim of the exams is to test your ability to apply your knowledge and understanding of the key principles described in the Core Reading to specific situations presented to you in the form of exam questions. Therefore your aim should be to identify and understand the key principles, and then to practise applying them. You will also need a good knowledge of the Core Reading to score well and quickly on any bookwork questions.

We recommend that you prepare for the exam by practising a large number of exam-style questions under exam conditions. This will:

- help you to develop the necessary knowledge and understanding of the key principles described in the Core Reading
- highlight exactly which are the key principles that crop up time and time again in many different contexts and questions
- help you to practise the specific skills that you will need to pass the exam.

There are many sources of exam-style questions. You can use past exam papers, the Question and Answer Bank (which includes many past exam questions), Assignments, the Mock Exam and ASET.

Overall study plan

We suggest that you develop a realistic study plan, building in time for relaxation and allowing some time for contingencies. Be aware of busy times at work, when you may not be able to take as much study leave as you would like. Once you have set your plan, be determined to stick to it. You don't have to be too prescriptive at this stage about what precisely you do on each study day. The main thing is to be clear that you will cover all the important activities in an appropriate manner and leave plenty of time for revision and question practice.

Aim to manage your study so as to allow plenty of time for the concepts you meet in this course to "bed down" in your mind. Most successful students will probably aim to complete the course at least a month before the exam, thereby leaving a sufficient amount of time for revision. By finishing the course as quickly as possible, you will have a much clearer view of the big picture. It will also allow you to structure your revision so that you can concentrate on the important and difficult areas of the course.

A sample ST subject study plan is available on our website at:

www.ActEd.co.uk/Html/help_and_advice_study_plans.htm

It includes details of useful dates, including assignment deadlines and tutorial finalisation dates.

Study sessions

Only do activities that will increase your chance of passing. Try to avoid including activities for the sake of it and don't spend time reviewing material that you already understand. You will only improve your chances of passing the exam by getting on top of the material that you currently find difficult.

Ideally, each study session should have a specific purpose and be based on a specific task, *eg* "*Finish reading Chapter 3 and attempt Questions 1.4, 1.7 and 1.12 from the Question and Answer Bank*", as opposed to a specific amount of time, *eg "Three hours studying the material in Chapter 3*".

Try to study somewhere quiet and free from distractions (*eg* a library or a desk at home dedicated to study). Find out when you operate at your peak, and endeavour to study at those times of the day. This might be between 8*am* and 10*am* or could be in the evening. Take short breaks during your study to remain focused – it's definitely time for a short break if you find that your brain is tired and that your concentration has started to drift from the information in front of you.

Order of study

We suggest that you work through each of the chapters in turn. To get the maximum benefit from each chapter you should proceed in the following order:

1. Read the Syllabus Objectives. These are set out in the box on Page 1 of each chapter.

2. Read the Chapter Summary at the end of each chapter. This will give you a useful overview of the material that you are about to study and help you to appreciate the context of the ideas that you meet.

3. Study the Course Notes in detail, annotating them and possibly making your own notes. Try the self-assessment questions as you come to them. Our suggested solutions are at the end of each chapter. As you study, pay particular attention to the listing of the Syllabus Objectives and to the Core Reading.

4. Read the Chapter Summary again carefully. If there are any ideas that you can't remember covering in the Course Notes, read the relevant section of the notes again to refresh your memory.

You may like to attempt some questions from the Question and Answer Bank when you have completed a part of the course. It's a good idea to annotate the questions with details of when you attempted each one. This makes it easier to ensure that you try all of the questions as part of your revision without repeating any that you got right first time.

Once you've read the relevant part of the notes and tried a selection of questions from the Question and Answer Bank (and attended a tutorial, if appropriate), you should attempt the corresponding assignment. If you submit your assignment for marking, spend some time looking through it carefully when it is returned. It can seem a bit depressing to analyse the errors you made, but you will increase your chances of passing the exam by learning from your mistakes. The markers will try their best to provide practical comments to help you to improve.

It's a fact that people are more likely to remember something if they review it from time to time. So, do look over the chapters you have studied so far from time to time. It is useful to re-read the Chapter Summaries or to try the self-assessment questions again a few days after reading the chapter itself.

To be really prepared for the exam, you should not only know and understand the Core Reading but also be aware of what the examiners will expect. Your revision programme should include plenty of question practice so that you are aware of the typical style, content and marking structure of exam questions. You should attempt as many questions as you can from the Question and Answer Bank and past exam papers.

Active study

Here are some techniques that may help you to study actively.

1. Don't believe everything you read! Good students tend to question everything that they read. They will ask "why, how, what for, when?" when confronted with a new concept, and they will apply their own judgement. This contrasts with those who unquestioningly believe what they are told, learn it thoroughly, and reproduce it (unquestioningly?) in response to exam questions.

2. Another useful technique as you read the Course Notes is to think of possible questions that the examiners could ask. This will help you to understand the examiners' point of view and should mean that there are fewer nasty surprises in the exam room! Use the Syllabus to help you make up questions.

3. Annotate your notes with your own ideas and questions. This will make you study more actively and will help when you come to review and revise the material. Do not simply copy out the notes without thinking about the issues.

4. Attempt the questions in the notes as you work through the course. Write down your answer before you check against the solution.

5. Attempt other questions and assignments on a similar basis, *ie* write down your answer before looking at the solution provided. Attempting the assignments under exam conditions has some particular benefits:

   - It forces you to think and act in a way that is similar to how you will behave in the exam.
   - When you have your assignments marked it is *much* more useful if the marker's comments can show you how to improve your performance under exam conditions than your performance when you have access to the notes and are under no time pressure.
   - The knowledge that you are going to do an assignment under exam conditions and then submit it (however good or bad) for marking can act as a powerful incentive to make you study each part as well as possible.
   - It is also quicker than trying to write perfect answers.

6. Sit a mock exam four to six weeks before the real exam to identify your weaknesses and work to improve them. You could use a mock exam written by ActEd or a past exam paper.

5 Frequently asked questions

Q: What knowledge of earlier subjects should I have?

A: The Course Notes have been written assuming that you have already studied, or been exempted from, Subjects CT1–CT8. The key area that you will need in studying Subject ST8 is the material in Subject CT6.

Q: What is your advice if I am simultaneously studying Subject ST7?

A: There is a fair amount of duplication between Subjects ST7 and ST8, particularly in the early part of the course. We suggest you review the material in both subjects simultaneously.

Q: What is your advice if I am simultaneously studying Subject SA3?

A: Subject SA3 builds on the common principles developed in Subjects ST7 and ST8, but requires a much greater depth of knowledge and understanding. Consequently, there is some overlap between the subjects, particularly in the types of questions that are likely to appear on the exam papers. It is therefore important to assimilate the key ideas presented in Subjects ST7 and ST8 before tackling the same ground in Subject SA3.

We suggest that you aim to cover the Subject ST7 and ST8 courses as quickly as possible, so as to get a general feel for the principles underlying general insurance together with an overview of the course content. From time to time over the study session, and particularly at the revision stage, it might also be a good idea to review the Subjects ST7, ST8 and SA3 Course Notes at the same time, along with the Question and Answer Banks. In particular, it is always worth thinking about how each idea or principle is presented in each of ST7, ST8 and SA3 and hence how it might consequently be examined in any exam.

Q: What calculators am I allowed to use in the exam?

A: Please refer to the Profession's website for the latest advice.

Q: ***If I have passed ST3 (but not SA3) before 31 December 2009, do I need to also sit and pass both ST7 and ST8 to be able to go on to do SA3?***

A: There is an article on page 15 of the April 2009 edition of the Actuary magazine which summarises the situation.

Briefly:

If you have achieved ST3 then neither ST7 nor ST8 will be able to count as your second ST subject due to the amount of overlap in the subject matter.

You can, however, choose to discount your ST3 pass and take both ST7 and ST8 to count as your two ST subjects.

For the new SA3 exam, you will be expected to know the material from ST7 and ST8 and so, even if you don't need to sit the ST7 and ST8 exams, you will still need to be familiar with (*ie* study) these subjects.

Q: ***Which past Subject ST3 exam questions are relevant to Subject ST8?***

A: A full cross-reference grid can be found in the Subject ST8 ASET.

As the material has changed significantly, there is not always a direct link between the Subject ST3 exam questions and the Subject ST8 material.

However, students studying Subject ST8 might like to attempt:

- April 2006 Questions 1, 2, 3, 4, 6
- September 2006 Questions 2, 3, 4, 5, 6, 8
- April 2007 Questions 1, 3, 4, 5
- September 2007 Questions 2, 3, 4, 5
- April 2008 Questions 1, 3, 5
- September 2008 Questions 2, 3, 4, 6, 7
- April 2009 Questions 1, 2, 4, 5, 6
- September 2009 Questions 1, 3, 6.

Q: What should I do if I discover an error in the course?

A: If you find an error in the course, please check our website at:

www.ActEd.co.uk/html/paper_corrections.htm

to see if the correction has already been dealt with. Otherwise please send details via email to **ST8@bpp.com** or send a fax to 01235 550085.

6 Syllabus

The full Syllabus for Subject ST8 is given here. The numbers to the right of each objective are the chapter numbers in which the objective is covered in the ActEd course.

Aim

The aim of this General Insurance Specialist Technical subject is to instil in successful candidates the ability to apply, in simple pricing analysis situations, the mathematical and economic techniques and the principles of actuarial planning and control needed for the operation on sound financial lines of general insurers.

Links to other subjects

Subject CT3 – Probability and Mathematical Statistics: provides a basic grounding in statistics.

Subject CT4 – Models: covers some stochastic models used in general insurance.

Subject CT6 – Statistical Methods: covers some of the mathematical methods relevant for general insurance.

Subject CA1 – Actuarial Risk Management: covers the general underlying principles affecting all specialisms.

Subject ST7 – General Insurance – Reserving and Capital Modelling Specialist Technical.

Subject SA3 – General Insurance Specialist Applications: will use the principles of general insurance developed in this subject to develop a deeper understanding of general insurance business and United Kingdom practice.

Objectives

On completion of this subject the candidate will be able to:

(a) Define the principal terms in use in general insurance *(Glossary)*

(b) Describe the main types of general insurance product in terms of:

- the needs of customers
- the financial and other risks they pose for the general insurer including their capital requirements and possible effect on solvency *(Chapters 2 and 3)*

(c) Describe the main types of general reinsurance products and the purposes for which they may be used *(Chapters 5 and 6)*

(d) Describe the implications of the general business environment in terms of:

- the main features of the general insurance market
- the effect of different marketing strategies
- the effect of the regulatory and fiscal regimes
- the effect of inflation and economic factors
- the effect of legal, political and social factors
- the effect of the climate and environmental factors
- the general effect of professional guidance
- the impact of technological change *(Chapters 7 and 8)*

(e) Describe the major areas of risk and uncertainty in general insurance business with respect to pricing, in particular those that might threaten profitability or solvency *(Chapter 9)*

(f) With regard to the use of data in pricing:

(i) describe the types of data that are used

(ii) describe the main uses of data

(iii) describe the requirements for a good information system

(iv) outline the possible causes of data errors

(v) understand the effects of inadequate data *(Chapter 10)*

(g) Outline the major actuarial investigations and analyses of experience undertaken with regard to pricing for general insurers, including the monitoring of business being written *(Chapter 19)*

(h) (i) Describe the Individual Risk Model and its applications in a general insurance environment

(ii) Describe the Collective Risk Model and its applications in a general insurance environment

(iii) Understand the derivation of the Aggregate Claim Distribution for the Collective Risk Model, and its approximations using stochastic simulation *(Chapter 11)*

(i) (i) Understand the various components of a general insurance premium

(ii) Describe the basic methodology used in rating general insurance business

(iii) Appreciate the various factors to consider when setting rates *(Chapters 12 and 13)*

(j) Develop appropriate rating bases for general insurance contracts, having regard to:

- return on capital
- underwriting considerations
- reinsurance considerations
- investment
- policy conditions such as self retention limits
- the renewal process
- expenses *(Chapters 12 and 13)*

(k) (i) Describe the burning cost approach to rating

(ii) Understand the assumptions required when using this approach

(iii) Outline the practical considerations when using this approach *(Chapter 14)*

(l) (i) Describe the frequency / severity approach to rating

(ii) Understand the assumptions required when using this approach

(iii) Outline the practical considerations when using this approach *(Chapter 14)*

(m) (i) Describe how Original Loss Curves can be used in rating

(ii) Understand the assumptions required when using this approach

(iii) Outline the practical considerations when using this approach *(Chapter 15)*

(n) Understand the applications of Generalised Linear Models to the rating of personal lines business and small commercial risks *(Chapter 16)*

(o) (i) Understand the uses of multivariate models in pricing

(ii) Outline the different types of multivariate models *(Chapter 17)*

(p) (i) Outline the fundamental concepts of credibility theory

(ii) Describe and compare the Classical and Bayes credibility models

(iii) Describe the practical uses of credibility models in a general insurance environment *(Chapter 18)*

(q) (i) Outline the similarities and differences between pricing direct and reinsurance business.

(ii) Describe how to determine appropriate premiums for each of the following types of reinsurance:

- proportional reinsurance
- non-proportional reinsurance
- property catastrophe reinsurance
- stop losses

(iii) Describe the data required to determine appropriate premiums for each of the above types of reinsurance *(Chapter 20)*

(r) (i) Outline the basic structure of a catastrophe model

(ii) Describe the key perils that can be modelled *(Chapter 21)*

7 Assignment deadlines – Subject ST8

For the session leading to the April 2013 exams – ST Subjects

Marking vouchers

Subjects	*Assignments*	*Mocks*
ST4, ST5, ST7, ST8	20 March 2013	26 March 2013
ST1, ST2, ST6, ST9	26 March 2013	3 April 2013

Series X Assignments

Subjects	*Assignment*	*Recommended submission date*	*Final deadline date*
ST1, ST2, ST4 – ST9	**X1**	**21 November 2012**	16 January 2013
ST1, ST2, ST4 – ST9	**X2**	**5 December 2012**	30 January 2013
ST1, ST2, ST4 – ST9	**X3**	**16 January 2013**	13 February 2013
ST1, ST2, ST4 – ST9	**X4**	**30 January 2013**	27 February 2013
ST1, ST2, ST4 – ST9	**X5**	**13 February 2013**	13 March 2013
ST4, ST5, ST7, ST8	**X6**	**20 February 2013**	20 March 2013
ST1, ST2, ST6, ST9		**27 February 2013**	26 March 2013

Mock Exams

Subjects	*Recommended submission date*	*Final deadline date*
ST4, ST5, ST7. ST8	**13 March 2013**	26 March 2013
ST1, ST2, ST6, ST9	**20 March 2013**	3 April 2013

We encourage you to work to the recommended submission dates where possible. Please remember that the turnaround of your script is likely to be quicker if you submit it well before the final deadline date.

For the session leading to the September/October 2013 exams – ST Subjects

Marking vouchers

Subjects	***Assignments***	***Mocks***
ST4, ST5, ST7, ST8	28 August 2013	4 September 2013
ST1, ST2, ST6, ST9	4 September 2013	11 September 2013

Series X Assignments

Subjects	***Assignment***	***Recommended submission date***	***Final deadline date***
ST1, ST2, ST4 – ST9	**X1**	**5 June 2013**	24 July 2013
ST1, ST2, ST4 – ST9	**X2**	**19 June 2013**	31 July 2013
ST1, ST2, ST4 – ST9	**X3**	**3 July 2013**	7 August 2013
ST1, ST2, ST4 – ST9	**X4**	**17 July 2013**	14 August 2013
ST1, ST2, ST4 – ST9	**X5**	**31 July 2013**	21 August 2013
ST4, ST5, ST7, ST8	**X6**	**14 August 2013**	28 August 2013
ST1, ST2, ST6, ST9		**21 August 2013**	4 September 2013

Mock Exams

Subjects	***Recommended submission date***	***Final deadline date***
ST4, ST5, ST7, ST8	**21 August 2013**	4 September 2013
ST1, ST2, ST6, ST9	**28 August 2013**	11 September 2013

We encourage you to work to the recommended submission dates where possible. Please remember that the turnaround of your script is likely to be quicker if you submit it well before the final deadline date.

Chapter 1

Insurance companies

This chapter touches on many syllabus objectives. There is no core reading in this chapter, however it provides an important introduction to topics that are addressed in more detail later in the course.

0 Introduction to Subject ST8

This chapter aims to give you a broad understanding of many of the key topics in general insurance. Subsequent study of individual topics in the later chapters should then be clearer because you will understand the importance of the topic in the context of the whole subject. Because we cover so many fundamental topics in this chapter, we recommend that you work slowly and carefully. If you are simultaneously studying Subject ST7, most of the material in this chapter is covered there too.

The sections in this chapter are as follows:

Section 1: Introduction to general insurance

Section 2: A general insurer's balance sheet

Section 3: Technical reserves

Section 4: Free reserves

Section 5: Premium rating

Section 6: Capital modelling

Section 7: Investments

Section 8: Profitability and cashflow

Section 9: Reinsurance

Section 10: Glossary items.

1 Introduction to general insurance

1.1 Why do general insurers exist?

To meet a need

Centuries ago merchants were encouraged to hazard great journeys by the existence of insurance: if they took the risk and disaster struck, then they would not be ruined if they were insured. The same social advantage is still there today. The exciting ventures have changed somewhat, but the ability to insure against various perils still enables individuals and companies to take on risks that they would not otherwise undertake.

The ability to make a known, small outlay to insure against the risk of a potentially large loss is justified by economists' utility theory, empirical evidence and common sense. People will pay more for insurance than the expected recovery from insurance (*ie* "the risk premium"), because they are risk-averse and prefer the more certain outcome.

To make money

Most general insurance companies exist primarily to make money. Get this firmly in your mind at this early stage. A moneymaking opportunity exists for Insurer *A,* because of the last sentence of the previous section. The constraints on *A*'s profitability are how much the customer is prepared to pay, any statutory controls on insurers and the competition from other insurance companies.

These concepts apply to most industries, not just general insurance. All companies weigh up the potential risks of their business and invest their capital so as to maximise return to shareholders. For most of Subject ST8 (and ST7 and SA3) you need very commercial, profit-driven thinking.

Question 1.1

Why might some insurance companies quote much higher premiums than other insurance companies for car insurance for the same individual and period of cover? (Try to give several different ideas.)

1.2 What is general insurance?

In simple terms, general insurance is any type of insurance that is not life insurance.

General insurance therefore encompasses a wide range of types of insurance. In most cases a general insurance policy is a contract of indemnity, *eg* if a loss occurs of an insured article, the insurer will reimburse the value of the insured article. Other policies might pay specified amounts on specified contingencies only (*eg* £10,000 if you lose the use of an eye) or if the loss is unclear, the amount might be determined by a court of law.

1.3 Risks, uncertainty and general insurance

Policyholders reduce their uncertainty by passing risks to insurance companies. It is not surprising, therefore, that insurance companies themselves are subject to risk and uncertainty. How to quantify and control this uncertainty is the underlying theme throughout this course. As you work through the different areas of the course think about how product design, capital requirements, pricing, reserving, reinsurance and accounts are influenced by the different risk and uncertainties involved in writing insurance.

Most of the major uncertainties centre around *how many* claims will occur and *how much* the insurer will have to pay to settle them. These uncertainties have a big influence on how much the insurer will charge for the protection provided (covered in Subject ST8) and how much the insurer needs to reserve for future claims payments (covered in Subject ST7). Other risks to the insurer include:

- failure to recover fixed expenses
- failure of other parties (*eg* brokers or reinsurers)
- falls in asset values
- the insurance cycle (also known as the underwriting cycle).

The size of the free reserves will influence the ability of the insurer to cope with these risks, as will the insurer's reinsurance cover and investment policy.

1.4 Actuaries and general insurance

That's all very well, but what about actuaries?

Rather surprisingly, actuaries are relatively recent arrivals in the general insurance world. It's clear to us as actuaries that we should have lots to offer this industry, but the industry has, until recently, been a little slow to recognise that.

The main actuarial roles have traditionally been in reserving and setting premiums. More recently, some actuaries have moved into much wider areas within general insurance. For example:

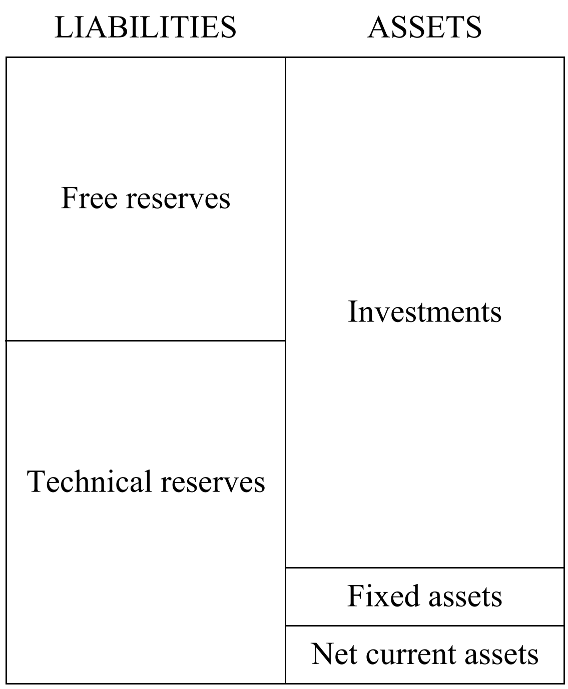
- capital allocation
- risk assessment, *eg* modelling catastrophic events
- strategic management of the business
- determining a suitable investment strategy
- assessing reinsurance requirements
- expense allocation
- assessing the effectiveness of marketing campaigns
- assisting with the early settlement of liabilities in the event of a wind-up.

Many of the topics that we study in general insurance are, in fact, areas where actuarial involvement is increasing.

2 *A general insurer's balance sheet*

The balance sheet of any organisation gives a snapshot of the finances of that organisation as at the date of the balance sheet. The balance sheet of a general insurer is therefore a summary (strictly an estimate) of the financial status of the company at a particular moment. The balance sheet is summarised in the diagram shown below.

The right-hand side is a statement of everything the insurer owns, and the left-hand side is a summary of what it owes. (Free reserves are "owed" by the insurer to the owner(s).)



Free reserves:	The balancing item equal to the excess of assets over insurance liabilities. Discussed in Section 4.
Technical reserves:	These are the amounts set aside in respect of expected claim payments to or on behalf of policyholders, plus related expenses. Discussed in Section 3.
Investments:	These might be bonds, equities, property *etc*. Discussed in Section 7.
Fixed assets:	For example, the office building and equipment. Not discussed much in this course, but you should be aware that they exist.
Net current assets:	Excess of current assets over current liabilities, *eg* money due from brokers.

Different types of balance sheet

The values placed on each of the items in the balance sheet are not always uniquely defined. Firstly, there may be an element of judgement used in deciding the values, so that a balance sheet based on a very prudent approach could be very different from a balance sheet produced with less conservative assumptions. Secondly, balance sheets produced for different purposes will be assessed using different approaches. For example:

- Statutory accounts, if required, may be in a prescribed format or there may be certain principles that must be applied.
- Internal management accounts may be produced to assist internal decision-making. These are likely to be on an on-going, realistic basis, although a variety of "what-if" scenarios are also likely to be produced.

Question 1.2

Why might the latest balance sheet of an insurance company (as given in the company accounts) not give a true indication of the financial strength of the company?

3 Technical reserves

The technical reserves are held to cover the liabilities relating to policies already written. The technical reserves might also be called *insurance reserves* or *insurance provisions* because they relate to the liabilities arising from writing insurance business.

These liabilities can be split into two main categories:

(a) Past: liabilities (claims plus expenses) in respect of accidents or losses from events that have occurred prior to the accounting date. Liabilities for *outstanding claims* are discussed in Section 3.2.

(b) Future: liabilities (claims plus expenses) in respect of future insurance cover from policies for which premiums have already been received. Unexpired risks are discussed in Section 3.3.

First, in Section 3.1, we explain how the claim characteristics of the major insurance classes influence the degree of uncertainty in the reserving process.

3.1 Major claim characteristics

Claim characteristics refer to the ways in which and speed with which claims originate, are reported, are settled and are on occasion reopened.

Imagine the rather extreme scenario in which insurance companies make claim payments as soon as a claim event occurs. For example, the instant that a policyholder has a car accident, there is an immediate transfer to the accounts of the affected parties. In this scenario, what is the insurance company's liability at any one time for outstanding claims?

The answer is zero. If all claims are settled the instant the event occurs, then the company never has any outstanding claims liability. In reality, insurance companies *do* have outstanding claims liabilities because there *are* delays between claims occurring and being settled. Hence insurance companies will hold reserves for outstanding claims. Before looking at reserves, let's consider the delays more fully.

There are two main types of delays: reporting delays and settlement delays.

Reporting delays

The reporting delay is the time from when the event occurs through to the time that the insurance company is notified of the event. Sometimes policyholders may be slow in getting round to advising the insurer – possibly because the amount involved is quite small. Other times the policyholders do not submit claims because they do not realise there is cause for claiming. For example, in the case of a number of industrial diseases (*eg* asbestosis, industrial deafness) it may be many years before the condition emerges. In these cases, reporting delays are often considerable.

Settlement delays

The settlement delay is the period between notification to the company and the payment of the claim. These delays are due to:

(a) initial administrative processing

(b) establishing whether the insurer is liable

(c) waiting for a condition to stabilise (*eg* will the injured party recover, or is the disability permanent?)

(d) establishing how much should be paid.

In a few cases where the insurer and the claimant cannot agree, the case may go to court.

In general (but certainly not always):

- property damage claims are settled more quickly than claims in respect of bodily injury
- large claims take longer to settle than small claims.

Short tail and long tail

Different classes of business are referred to as “short tail” or “long tail”, where:

- short tail means that claims are generally reported quickly and settled quickly by the insurer, and
- long tail means that there is a sizeable proportion of total claim payments that take a long time for the insurer to settle.

When looking at an individual class of business, or type of claim, you should make a point of noting whether it is short tail or long tail (or in-between!). This is important in developing your understanding of an insurance class, and may influence your answer to a question in the exam. The claim characteristics of the main insurance classes are detailed later in the course.

3.2 Reserves for outstanding claims

The outstanding claims reserve is the first of the two main components of the technical reserves. It may be given as a total figure or, alternatively, it may be split into anything up to five separate components:

(a) Reserve for outstanding reported claims: this is the estimated reserve needed to settle the claims that the company knows about at the accounting date.

(b) Reserve for *incurred but not reported* (IBNR) claims: the IBNR reserve is needed to cover the claim payments for incidents which have happened, but have not been reported to the insurance company.

(c) Reserve for *incurred but not enough reported* (IBNER): the IBNER reserve is needed to cover expected increases (or decreases) in estimates for reported claims.

(d) Reserve for re-opened claims: this is an additional reserve which may be explicitly shown to allow for claims that the insurance company treats as being fully settled, but which might one day require further payments. In practice, insurers differ significantly over when they "close" a claim.

(e) Reserve for claims handling expenses: in settling claims in each of the above classes, the company will incur some additional expenses (*eg* legal fees). The reserve for these expenses may be held separately.

Even if the reserves are not shown split into these categories, an insurance company should still hold reserves to cover all of these items. For example, if the reserves are shown split into the first two components only, then the reserve for re-opened claims might be within the outstanding reported claims reserve, the reserve for IBNER might be within the reserve for IBNR and the reserve for claims handling expenses would be split between the two.

Question 1.3

There are two main reasons for needing outstanding claims reserves: reporting delays and settlement delays. Which of the components of the outstanding claims reserve is linked to each type of delay?

Estimating outstanding claims reserves

There is great uncertainty about the payments an insurer will need to make in respect of outstanding claims. The amounts which will be paid are not known, so the insurer must use estimates when deciding how much to set aside in respect of these liabilities. Two distinct approaches are:

(a) making estimates of the liability for each individual outstanding claim ("case estimates", or "case by case estimation")

(b) using statistical techniques to estimate the total outstanding payments for the portfolio.

In practice, insurers use a combination of the two. We study this topic in detail in Subject ST7. In the meantime, the following observations should be apparent:

- individual estimates cannot be used for IBNR because the insurer does not yet know about the claim
- statistical techniques are more useful for classes of insurance where there are lots of claims (*eg* private motor), and where there is stability in the numbers and amounts of claims.

One important consequence of the uncertainty about the liability for outstanding claims is that any aspect of the insurer which relies upon estimates for outstanding claims (*eg* profitability) is, as a result, also subject to uncertainty. This is an important point that you should be aware of at all times when working with figures that rely upon technical reserves.

The degree of this uncertainty will vary from class to class. Generally, there is much more uncertainty with long-tail classes, where the reserves for outstanding claims are larger in relation to premium income.

Question 1.4

(a) The technical reserves can be split into two main components. What are they?

(b) The outstanding claims reserves might be split into two, or perhaps even five components. What are they?

Question 1.5

An insurance company splits its outstanding claims reserves into two components: reported claims and IBNR.

The company writes two classes of business. For each class, the outstanding claims reserve is split as follows:

	Class 1	Class 2
Reserve for reported claims	90%	50%
Reserve for IBNR	10%	50%

The reserves for Class 2 are a much bigger proportion of premium income than are the reserves for Class 1.

Suggest two common classes of insurance this company might write.

3.3 Reserves for unexpired policies

We have discussed reserves in respect of claim events that have already happened. We now turn to the other main component of the technical reserves, the liabilities in respect of existing policies with some unexpired exposure, *ie* future claim events from policies with future periods of cover remaining at the accounting date.

Unearned premium reserves

The usual basis for determining the reserves in respect of the unexpired exposure is to hold a portion of premiums in respect of that exposure. For example, on a straight averaging basis, for a policy with half of its term still to run, it might be reasonable to hold a reserve of half of the premium that was charged. For an annual policy with one month to run, a reserve of one twelfth of the premium might be held.

The name given to the portion of premiums held in respect of unexpired exposure is the *unearned premium reserve* (UPR). The title is quite logical: the UPR is simply the premiums that have been received which have not yet been earned. This is a retrospective assessment of the reserve.

The straight averaging approach used above has a number of fundamental weaknesses in practice:

- it ignores the fact that the risk from the policy may not be spread evenly over the period of cover, *eg* seasonal effects
- it ignores the fact that the expenses of setting up and servicing the policy may not be incurred evenly over the period of cover.

For the purpose of this discussion, we will assume that risk is even over the period of cover. However, a similar assumption about expenses would not be appropriate because there is clearly a large element of expense that is generally incurred at the commencement of a policy, *eg* commission paid to the sales outlet.

The expenses that are incurred by the insurer at the start of a policy are called *acquisition costs*. Commission is generally the major component of acquisition costs. But how should these acquisition costs be allowed for when setting the reserves in respect of unexpired exposure? The best way to see this is through a simple example.

Example

Suppose a policy has acquisition costs of 20% of the premium. Then 80% of the premium is available to meet claims, on-going expenses and profit. If we assume that the risk and on-going expenses are spread evenly over the period of cover, then the 80% of premium (*ie* after deduction of acquisition costs) could also be spread evenly. If the policy were half-way through its life at the accounting date, the UPR would be 40% of the premium (*ie* half of the 80% which is to be spread over the life of the policy).

This approach to establishing the UPR can be extended to a more general formula:

$$\text{UPR} = \text{proportion of risk unexpired} \times (\text{premium} - \text{acquisition costs})$$

The UPR calculated in this way is the net UPR. The gross UPR doesn't allow for the acquisition expenses.

Question 1.6

Explain in layman's terms what is meant by UPR.

Question 1.7

An insurance company calculates its UPR using an individual, policy by policy approach. Calculate the UPR as at 31 December 2011 for the following annual policies:

(a) Premium £1,000. Commission £120. Started 1 October 2011.
Risk assumed to be uniform over the year.

(b) Premium £3,500. Commission 10%. Started 1 July 2011.
Risk starts at zero, and increases daily by a constant linear amount over the policy year.

(c) Premium £2,200. Commission £200. Started at 11.59pm on 31 December 2011.
Risk fluctuates throughout the policy year.

(d) Premium £10,000. No acquisition costs. Started 1 November 2011.
All risk assumed to be in April.

Unexpired risk reserve

So far, our assessment of the reserve to be held in respect of unexpired exposure has been retrospective by looking at the premiums we have received in the past. We have set up a reserve equal to the unearned premium net of acquisition costs, *ie* the net UPR. A prospective approach should also be considered by thinking about possible future claim events. What reserve do we think we need to hold to cover the unexpired risks?

The *unexpired risk reserve* (URR) is the name for this prospective assessment. Again, the title is quite logical. Remember that such a reserve would need to cover all the claims and all the expenses that are expected to be incurred in the future by the unexpired portion of existing policies.

We would normally expect the unearned premium reserve to be bigger than the unexpired risk reserve. This is effectively the same as saying we expect the premiums to be big enough to cover the claims and expenses – which is what a profit-seeking insurer would generally want. In the cases where the UPR is greater than the URR, there is no need for the insurer to keep reserves greater than the UPR for unexpired policies. Because of the accounting accruals principle we would generally hold at least the full UPR. Here, holding a reserve equal to the UPR, we would expect some profit to emerge over the coming months from these policies.

However, for cases where URR is greater than UPR, the calculations become more complex. These cases imply that the company expects to make a loss on the unexpired policies because it expects to pay out more in claims and expenses than the amount of premium held back for the unexpired period. Therefore, the UPR will be insufficient to meet the expected payments, and the insurer should set up additional reserves to meet this strain.

These additional reserves are known (again, quite logically) as the *additional unexpired risk reserves* (AURR), or the *additional reserves (or provision) for unexpired risks*.

It should not surprise you that:

$$\text{AURR} = \text{URR} - \text{UPR} \quad \text{(minimum of zero)}$$

You will need to be particularly careful with the expression *unexpired risk reserve*. In some contexts practitioners use it to mean the total URR (as defined above), and others may use it to mean AURR. So:

- stick to the definitions given in this chapter
- be careful when reading *unexpired risk reserve*, and check which is meant in the context
- be clear in your own work as to whether you mean total unexpired risk or additional unexpired risk.

Question 1.8

State in layman's terms the key difference between the UPR and the URR. Which of the two calculations, UPR or URR, is open to most uncertainty?

3.4 *Other types of technical reserve*

So far in this section we have looked at reserves in respect of outstanding claims and reserves in respect of unexpired policies. We now look briefly at two further types of technical reserve which do not fall easily into either category: claims equalisation reserves and catastrophe reserves.

Claims equalisation reserve

Because insurance business is volatile, the year-to-year profits of an insurance company can be very volatile.

One way to reduce the volatility of profits over time is to hold a *claims equalisation reserve* (CER). This is a reserve that is used to smooth the profits from one year to another. In a good year when profits are large, money is transferred to the claims equalisation reserve, thereby reducing the initial assessment of profit. In a bad year, money is transferred from the equalisation reserve, thereby increasing the initial assessment of profit.

Catastrophe reserve

In the context of general insurance, a catastrophe is a single event that gives rise to an exceptionally large aggregation of losses. Examples of catastrophic events range from natural catastrophes such as floods, windstorms earthquakes, to man-made catastrophes such as aircraft crashes, explosions or oil spillages.

An insurance company may choose to set aside an additional reserve to cover the losses that might arise from a catastrophe. Whereas an insurer would expect to use the reserve for, say, outstanding claims, an insurer would not expect to have to pay out from the catastrophe reserve. It is genuinely a contingency reserve that would be held just in case something awful were to happen.

If an insurer did hold a large explicit catastrophe reserve, there would be less need for the insurer to hold extensive free reserves (*ie* the excess of assets over liabilities). Conversely, the free reserves for a company that does not hold a catastrophe reserve need to be sufficiently big to cover the possibility of a catastrophe (or two!).

Question 1.9

What are the two main approaches to estimating outstanding claims? Which of the two should be used for estimating IBNR? Which of the two is more likely to benefit from actuarial input?

Question 1.10

An insurance company that recognises that it has been writing business unprofitably for the last six months shows just three different types of technical reserve in its accounts. What do you think they might be? How would your answer change if the question had said *very profitably* instead?

4 Free reserves

The free reserves are the excess of the assets over the technical reserves (as shown in the diagram of the balance sheet on page 5).

You need to be particularly careful here because lots of different expressions are used by different practitioners in different circumstances to refer to the excess of assets over liabilities. The following expressions are all commonly used as alternatives to free reserves:

- free assets
- solvency margin
- shareholders' funds
- capital employed.

The word "solvency" has several different possible interpretations for a general insurance company. The most common is the concept that the assets exceed the liabilities. Hence the excess of assets over liabilities may be called the solvency margin. Where a solvency margin *ratio* is discussed in general insurance, it is the solvency margin divided by the written premiums, not the solvency margin divided by total assets. This is another area where terminology sometimes differs. Some practitioners use the term solvency margin when referring to the ratio defined above.

4.1 Significance of free reserves

Without free reserves, an insurance company would not be able to operate.

Firstly, you may regard the free reserves as the pool of funds being used to provide the backing for insurance risks. If the insurer did not have an adequate level of free reserves, policyholders would have no reason to believe that the insurer would be able to meet claims in the event of disaster. Meeting claims after adverse events is the whole purpose of insurance.

Secondly, there may be a legal requirement for an insurance company's free reserves to exceed a statutory minimum level. In the UK, this minimum capital amount is often called the *Minimum Capital Requirement (MCR),* or *Required Minimum Margin (RMM)*.

The extent of the free reserves is very important for the management of the insurance company. For example, it is closely linked to the following:

- The maximum amount of business the company should write: free reserves are required to provide a cushion against unexpected adverse results. The more business that is written, the bigger the required cushion. Conversely, there is a maximum amount of business that a given level of free reserves should support.
- The classes of business written: some classes of business have more variable claims experience than others and some classes involve bigger risks. A higher level of free reserves can support more variable and larger risks.

Question 1.11

Other aspects of the management of a general insurer are also influenced by the size of the free reserves. What do you think the effect of higher free reserves will be on an insurer's:

- reinsurance programme?
- investment strategy?
- pricing policy?

5 Premium rating

5.1 Introduction

The process of setting appropriate premium rates is key to the operation of a successful insurance company, and is a major area of actuarial involvement.

Premium rating forms a core part of the syllabus for Subject ST8. Some of the topics covered will be familiar to you from your study of previous subjects, *eg* Subject CT6. However, Subject ST8 will require you to apply your commercial awareness to specific scenarios, as well as to understand the more technical aspects of the course.

5.2 Technical methods revisited

Throughout all areas of general insurance work, be it reserving, pricing or capital modelling, actuaries will be interested in the frequency and severity distributions of claims. In particular, the estimation of claim distributions will allow a pricing actuary to calculate the pure "risk premium", *ie* the amount of premium that is required to exactly cover the expected cost of claims alone.

Subject ST8 revisits some of the methods used to model aggregate claim distributions, and some of the approximations that can be used where analytical methods are impractical. You should recognise this from your study of Subject CT6.

The pricing actuary will also wish to ensure that different types of policyholders are charged the most appropriate premium given their own risk characteristics. For example, female drivers are considered on average to be lower risk and therefore can expect to be charged a lower motor insurance premium than male drivers. The actuary will therefore want to identify those characteristics of a policyholder that have the greatest impact on the amount of risk taken on. Generalised Linear Models (GLMs) and multivariate analyses are key tools in the identification of these risk factors. Again, you should be familiar with much of this material from your study of previous subjects.

5.3 Other pricing techniques

A "burning cost" approach is a simple method often used as a starting point to price certain types of insurance and reinsurance. It calculates the risk premium as actual cost of claims during a past time period, expressed as an annual rate, per unit of exposure. You might therefore like to think of it as just an average claim cost per year, per unit of exposure.

The "frequency-severity" approach to pricing calculates the risk premium as the expected average claim cost multiplied by the expected average number of claims in the period.

Original loss curves are often used in general insurance to derive premium rates where past claims data is too sparse to derive a credible price using more traditional techniques. They are often used to price certain types of reinsurance arrangements. However, a key problem of original loss curves is the difficulty of estimating and/or selecting appropriate curves, since the resulting expected claim cost is often very sensitive to specific model selected. This topic will be covered in detail in Chapter 15

5.4 Pricing methodology

A pricing actuary is likely to estimate the expected cost of claims based on past data. However, it would be naïve to assume that past claims experience will always be an accurate predictor of claims experience on a future policy. Adjustment may have to be made to allow for:

- unusually heavy / light experience in the past data
- large or exceptional claims – the actuary may exclude these from the past data and then make an explicit loading in the risk premium later for catastrophes and large claims
- trends in claim experience which mean the past data will be out of date by the time the new rates are in service
- inflation between the date of the data and the date the policy will be in force
- changes in the risk and/or cover provided
- changes in environmental factors, *eg* legislative / technological changes
- incomplete past data arising from the fact that these claims might not yet be fully developed.

A corresponding analysis and projection of exposure data will also need to be made.

The premium actually charged to a policyholder will not only be based on the expected cost of claims on that policy, however. Further adjustments are made, to arrive at the "office premium" *ie* the actual market price for a policy. These adjustments include:

- expected reinsurance costs and recoveries
- expense loadings such as commission, policy administration costs, claim handling costs, overheads, levies *etc*
- loadings for profit
- allowance for investment income.

Further adjustments will also be made for practical consideration such as:

- the insurer's business objectives *eg* desire to build up market share
- competitive pressures
- the level of brand or customer loyalty amongst the insurer's target market
- the state of the insurance cycle
- market acceptability, *ie* the level of cross-subsidy between policies, for example between new business and renewals business, and the likely reaction of policyholders to these differences
- the use of no-claims discounts.

6 Capital modelling

6.1 Introduction

Capital modelling is a relatively new field in general insurance. Recent changes in regulation, and the increasing focus on risk modelling in all sectors of the financial services industry has meant that this is a growing area of actuarial work.

Any resources held by the company in excess of the value of its liabilities are called its capital, and we saw in Section 4 that a general insurer will hold capital well in excess of these liabilities. In fact, it will hold enough capital to be sure of meeting all its obligations to policyholders with a certain degree of confidence. The process by which this confidence level is obtained is called "capital modelling". A capital model will also be used to help allocate capital between classes, products and individual policies.

6.2 Structure of a capital model

A capital model will most often take the form of a cashflow model, and will project all aspects of a general insurer's business into the future, to assess the company's ability to meet its financial obligations. A range of assumptions will therefore be required, including:

- future written premium income
- future claims, on current and future business, and the timing of these claims
- future reserving basis
- future expenses
- investment return
- reinsurance arrangements, costs and recoveries, potential reinsurance exhaustion
- economic variables such as future inflation and interest rates
- likelihood / cost of catastrophes
- the insurance cycle
- operational losses
- tax
- dividends.

It may help when developing ideas on this in the exam, to think about the components of a general insurer's income statement.

A capital model will usually analyse the general insurer's business in a considerable level of detail. We would expect at least that each class of business will usually be projected separately, but projections may also include separate analyses for different sources of business, locations, claim type, asset types, asset categories *etc*. The level of detail, or granularity, built in to the capital model will depend on the uses to which the model is put.

Given the degree of volatility in general insurance business, a range of methods will be used, including stress testing and scenario testing. It is also likely that stochastic models will be used since the tail end of an insurer's claims distributions will be of particular interest in determining confidence levels.

6.3 Allowance for risk in a capital model

The modern approach to assessing the level of capital that general insurers are required to hold is to allow for the inherent riskiness of the insurer's activities. The greater the risk taken on by the insurer, the greater the level of capital that it is required to hold. Conversely, if an insurer is considered to be less risky, this can be reflected in the capital model, and a lower capital level will be required. Thus, companies can be rewarded for maintaining a sound risk management policy.

The types of risk that may be taken into account in the capital modelling process include:

- insurance risks – the uncertainty arising from the amount and timing of claims, expenses and premiums
- reserving risk – the risk that the level of technical reserves held is inappropriate to the level and type of business written
- market risk – risks relating to changes in investment market values
- credit risk – the risk of failure of third parties to repay debts, including the failure of reinsurers
- operational risk – the risk of loss due to failures of people, processes and systems, *eg* fraud or mismanagement within the general insurer itself
- liquidity risk – the risk that the company is unable to meet its obligations as they fall due as a consequence of a timing mismatch between its assets and liabilities
- group risk – the risk that the company experiences as a result of being part of a group *eg* a subsidiary of a parent company, as opposed to a stand-alone entity

- other risks such as strategic risks, political risks and enterprise risk management risks (ERM). You may have come across ERM if you have studied Subject CA1, where it was also called integrated risk management. ERM is discussed further in Subject ST7.

An insurer's required level of capital can be adjusted for the fact that some of the risks outlined above will be negatively correlated. A good capital model will make allowance for this diversification effect. Conversely however, should the insurer be exposed to accumulations of risk, the level of capital will need to be increased.

7 Investments

As with any investing institution, a general insurance company will want to achieve the maximum possible return from its investments, without exposing itself to an undesirable level of risk.

Here's a quick summary of the major factors influencing the company's investment strategy.

Nature of liabilities

Some of the liabilities will be in fixed monetary amounts. Investments producing fixed monetary returns are appropriate for these liabilities.

However, many of the liabilities will need to be settled in prices applicable at the time of settlement. This means that there is an element of inflation underlying most of the liabilities. The type of inflation the liabilities are exposed to varies by class and peril. This becomes important for those classes of insurance where there are considerable reporting and settlement delays. Investments that tend to maintain their real value are desirable for such liabilities.

Term of liabilities

Investments should have similar terms to those of the liabilities. The appropriate term of investments is very dependent upon the classes of business written. Compared with life assurance and pension funds, many liabilities tend to be short term, and the investments tend to reflect this. For long-tail classes of business (*eg* employers' liability), some medium-term and even some long-term investments are appropriate.

Currency of liabilities

In some classes of insurance, for example, marine, travel, product liability, insurers may have liabilities in several currencies. Assets should be held to match.

Uncertainty of liabilities

In general insurance there is often a great deal of uncertainty about the amounts that will be necessary to settle claims and the timing of the claim payments. The proportion of the assets held in a form that is liquid must be sufficient to cover this uncertainty.

Size of free reserves

The bigger the free reserves (relative to the size of the company), the greater the extent to which the investment strategy can be aimed at maximising returns.

Legislative influences

The need to maintain the free reserves above a particular minimum solvency margin means that insurers may be very wary of holding too many investments that have volatile market values. The extent to which this is a concern depends on the size of the free reserves relative to any statutory minimum solvency margin that may exist.

Taxation

Insurers will want to maximise their post-tax investment returns, therefore the taxation basis for insurers is relevant to the choice of investment strategy.

Question 1.12

An insurance company produces provisional accounts, as at 31 December four days later on 4 January. These accounts do not include an AURR for its domestic household account.

When the draft accounts are drawn up two months later, the balance sheet *does* include an AURR for household business. Assuming that there was no change in the reserving or accounting basis, why might the accounts have been modified?

Question 1.13

Insurance companies are described as "capital intensive". Explain why they need so much capital in relation to payroll, size of premises *etc*.

Question 1.14

Which two aspects of an insurance company's operations will have the most impact on the company's investment strategy?

8 Profitability and cashflow

8.1 Basic measure of profits

In this section we will look at the items that make up profits.

Intuitively, it should be clear that:

$$\text{post-tax profits} = \text{premiums} - \text{claims} - \text{expenses} + \text{investment return} - \text{tax}$$

(In case you are wondering why reserves are not explicitly shown in this equation, all is explained below.)

With the premiums, claims and expense items, we need to be careful that the figures we use are sensible and consistent. Suppose we are trying to calculate the profit earned in 2012. Then, if we write £10,000 of new business on 30 December 2012, does this mean that the 2012 profits will be bigger by £10,000? If we follow usual accounting principles, the answer is NO!

The accounting principle that matters here is *accruals*. Income and expenditure should accrue over the period to which they relate.

Let's consider various measures of premiums and claims and then select the measures that are consistent with the accruals principle.

Written premiums

This is the total amount of premium income written in the year. So for policies starting in the year, the whole premium will be included within written premiums (also expressed as premiums written).

Earned premiums

This is the amount of premium income relating to insurance cover provided during the year (also expressed as premiums earned). For example, if a new annual policy is started on 1 December 2012 for a premium of £1,200, the earned premium in 2012 from this policy would be £100 (assuming that the risk and expenses are even over the policy year). This policy would then contribute £1,100 to earned premiums in 2013.

So, which of these two measures of premiums is consistent with the accruals concept?

Hopefully, you will agree that it is earned premiums because this tells us how much premium has accrued during the year.

Paid claims

This is the total amount of claim payments made by the insurer during the year (also expressed as claims paid).

Incurred claims

This is the amount of claims paid (as above) *plus* the increase in the total reserve for outstanding claims (also expressed as claims incurred). For example, suppose that a claim for £1,000 is reported on 20 December 2012, but payment is delayed until 2 January 2013. The effect of this delay is to decrease the 2012 claims paid by £1,000 but the 2012 *claims incurred* is unchanged (assuming that the reserve for outstanding claims as at 31 December 2012 is increased by £1,000).

It's claims incurred that is consistent with the accruals principle.

Question 1.15

An insurance company (which writes remarkably little business!) writes six-monthly policies, each for a fixed premium of £1,200. Policies commencing in 2011 and 2012 were written on the following dates:

1/3/11, 1/4/11, 1/8/11, 1/10/11, 1/12/11, 1/1/12, 1/3/12, 1/5/12, 1/6/12, 1/8/12, 1/11/12.

At the end of 2011, the reserve for outstanding claims was £6,000. By the end of 2012, the reserve for outstanding claims was £7,300. A total of £5,500 was paid in claim settlements in 2012.

Calculate (for the 2012 accounting year):

- premiums written
- premiums earned
- claims paid
- claims incurred.

Expenses incurred

When policies are written the insurer pays commission and other initial expenses. The acquisition costs already paid for unexpired policies at the accounting date are known as *Deferred Acquisition Costs* (DAC).

To be consistent with the treatment of premiums and claims above, the expenses item will also need to be based on an incurred basis rather than just showing expenses paid.

Underwriting result

The underwriting result (or underwriting profit) is the term given to the excess of premiums over claims and expenses:

	Earned premiums
–	Claims incurred
–	Expenses incurred
=	Underwriting result

Note that the whole of this account is based on the accruals concept. The underwriting result shown in the revenue account of a general insurance company is analogous to the operating profit of non-insurance companies. The issues covered in this section are discussed in more detail in Subject ST7.

Question 1.16

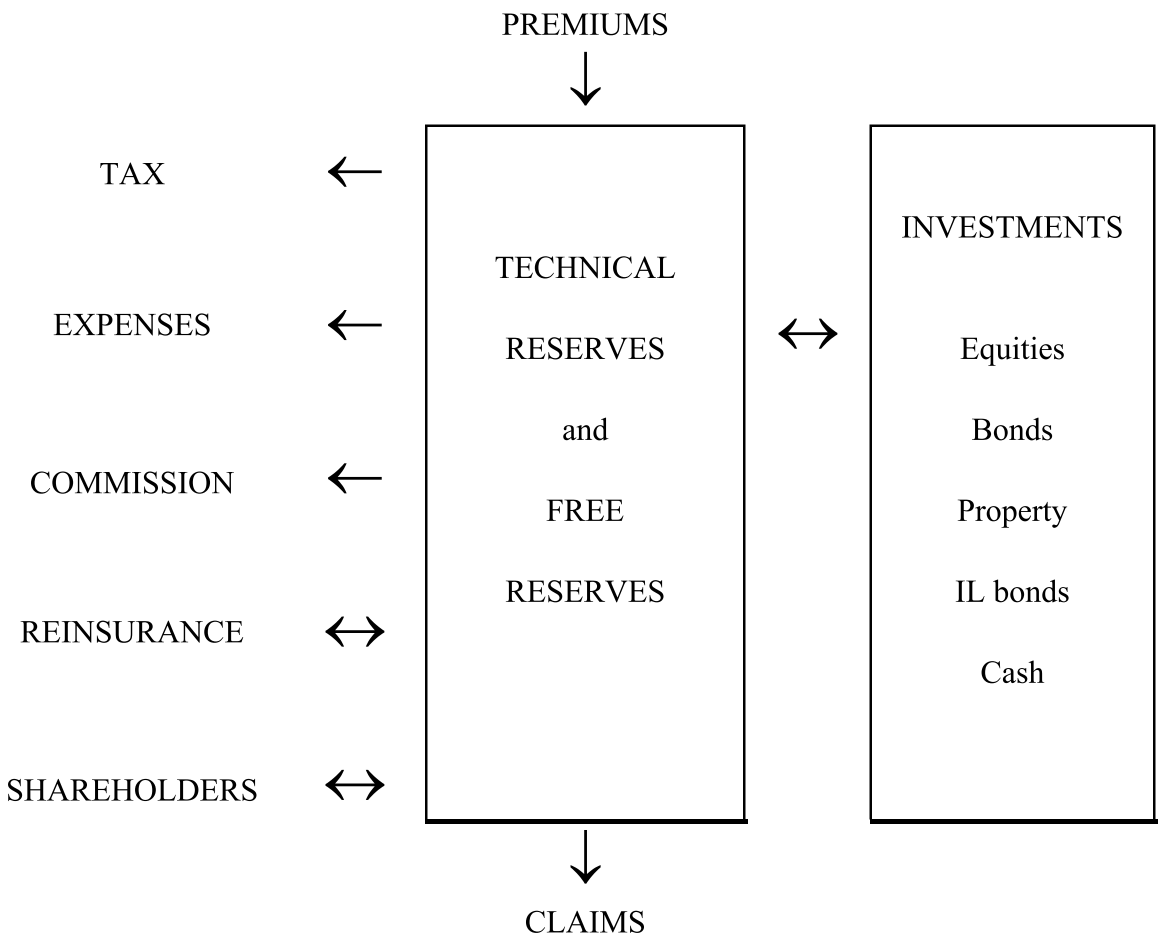
The following data is available for a general insurance company for an accounting year (in £ million):

premiums receivable:	165
unearned premium brought forward at 1 January:	75
unearned premium carried forward at 31 December:	83
claims paid:	103
outstanding claims reserve at 1 January:	124
outstanding claims reserve at 31 December:	133
expenses incurred:	31

Produce the revenue account showing the underwriting profit (or loss) for the year.

8.2 Cashflow diagram

When considering the mechanics of an insurance operation, it is often useful to consider all the various cashflows that may take place. A simple diagram summarising the cashflows is:



Most of the expressions in this diagram should now be familiar to you, and the mechanics should be self-evident. However, the following points are worth bringing out:

Reinsurance

This is the insurance company's own insurance. That is, the insurance company pays out reinsurance premiums to a reinsurance company and the reinsurer covers part of the risk the insurer has taken on. The insurance company will, as a result of this insurance, sometimes make reinsurance recoveries from the reinsurer.

Investments

The reason for the double arrow is to highlight that money may be lost on investments as well as being gained. Hopefully the investment income and investment gains will far outweigh the losses!

Shareholders

Shareholders will hope to receive dividends (hence the "out" arrow). From time to time shareholders may be asked to put more money into the company through rights issues.

Finally, thinking of the cashflow diagram and the parties involved in each cashflow is often useful in helping to generate ideas in the exam.

Question 1.17

List seven different types of reserve which, when added together, make up the total technical reserves. Which of these is most likely to be zero, and why?

Question 1.18

What would happen to the post-tax profits of a general insurer if the company decided to reduce the reserve it had been holding for IBNR by £20 million? Assume tax is charged at a rate of 30%. Suggest (*ie* use some imagination and make something up) a single event which might cause this to happen.

Question 1.19

Complete the following statements by filling in the blanks.

AURR is the _______ of _______ over _____, subject to a _______ of ________.

UPR is the portion of __________ that relates to __________ cover.

Claims incurred is defined as the __________ __________ plus the __________ in outstanding claims reserves.

The main determinants of the investment strategy will be:

- the __________, __________, __________ and __________ of the __________
- the relative size of the __________ __________.

A reserve used to smooth profits from year to year is called a claims _______ _______.

The excess of __________ premiums over __________ claims and __________ is called the ________ __________.

9 Reinsurance

The final topic that we wish to introduce here is reinsurance. Individuals and companies take out insurance when they perceive a need for it, so as to reduce risk to themselves. Insurance companies do precisely the same.

Reinsurance is the name for the insurance of insurance companies.

We study reinsurance in Chapters 5 and 6, but it is helpful early on to look at a couple of situations in which it may be useful.

Enormous risks

Insurers like to do business, but there are some risks, *eg* large industrial or commercial properties, that are so large that no one insurer could possibly take them on, without endangering their own solvency position. A way for them to do some business without taking on undue risk is to reinsure the portion of the risk that they cannot cope with.

Proportional reinsurance or coinsurance can help with this situation.

Accumulations of risk

Accumulations of risk occur when the insurer has an unbalanced portfolio of risks. This imbalance may be due to the nature of the classes of business written (*eg* too much exposure to liability claims), the geographical areas covered or the types of policyholder attracted. The unbalanced portfolio exacerbates the problems of non-independent risks. An example would be the severe flooding in Yorkshire in 2007 leading to many domestic property insurance claims.

The risk of accumulations can be mitigated, in certain circumstances, by the insurer arranging some reinsurance to enable them to cope with the possibility of a catastrophic event occurring.

Question 1.20

Imagine you work for an insurance company specialising in selling travel insurance, through travel agents, to parties of over 50's going on skiing holidays in the USA.

Give some examples to explain why your company is likely to want to take out some reinsurance.

10 Glossary items

At the end of each chapter we will include a section like this one which lists the Glossary items that have either been introduced in the chapter or are related to the material in the chapter. You will not have met all of the terms given here in the chapter, but it is a good idea to become familiar with these terms at this early stage. To study actively you should attempt to explain each of the terms that you have come across and then check your definition against the Glossary. You should now read the definitions of the following Glossary items:

- 24ths method
- 365ths method
- Accumulation of risk
- Acquisition costs
- Additional reserve (provision) for unexpired risk
- Annual basis of accounting
- Asset liability model
- Atafs – Age to age factors
- Atufs –Age to ultimate factors
- Average cost per claim method
- Balance of a reinsurance treaty
- Bornhuetter-Ferguson method
- Break-up basis
- Burning cost
- Case by case estimation
- Casualty insurance
- Catastrophe
- Catastrophe reserve
- Chain ladder method
- Claim
- Claim ratio
- Claim size distribution
- Claims equalisation reserve
- Claims incurred
- Clash cover
- Coinsurance
- Combined ratio
- Cover note
- Deep pocket syndrome
- Deferred acquisition costs (DAC)
- Dynamic financial analysis
- Earned premiums
- Eighths method
- Equalisation reserve and equalisation provision
- Event
- Expense ratio
- Experience account
- Franchise
- Free reserves
- Incurred but not reported (IBNR) reserve
- Incurred claims (or claims incurred)
- Indemnity, principle of
- Insurance certificate
- Insured
- Integrated Lloyd's Vehicle
- Knock-for-knock agreement
- Long-tailed business
- Loss
- Loss expense reserve
- Loss ratio

- Loss reserve
- Model uncertainty
- Non-technical account
- No-claim discount (NCD)
- One-year accounting
- Office premium
- Outstanding claims reserve (OCR) or outstanding claims provision
- Operating ratio
- Partial payment
- Period of unexpired risk
- Points rating system
- Rating
- Rating factor
- Recoveries
- Reinsurance
- Reinsurance to close (RITC)
- Reinsurer
- Reporting year
- Reopened claim
- Retroactive date
- Risk based capital (RBC)
- Risk factor
- Risk premium
- Run-off triangle
- Short-tailed business
- Solvency margin
- Solvency ratio
- Technical reserves (provisions)
- Unearned premiums
- Unearned premiums reserve (UPR) or provision for unearned premiums
- Unexpired risks reserve (URR) or provision for unexpired risks
- Written premiums.

Chapter 1 Summary

The existence of general insurance is good for society as a whole and for individuals.

On one side of the balance sheet are free reserves and technical reserves. On the other we have all the assets.

Technical reserves (or insurance reserves) might be split into:

Past events:	–	o/s reported claims
	–	IBNR and IBNER
	–	re-opened claims
	–	claims' handling expenses
Future events:	–	UPR
	–	AURR

Insurers may also hold catastrophe reserves or other claims equalisation reserves.

The claims reserves might be shown as one amount for outstanding claims and one for unexpired risks.

Claims reserves occur because there are reporting delays, settlement delays and premature closure of claims files. Claims reserves are generally larger for long-tail classes of business.

Estimates for outstanding claims reserves are carried out by estimates of individual outstanding claims or by using statistical methods for the totals. Individual estimates can't be used for IBNR!

Claims reserves are estimates. Therefore any work which is based on claims reserves should recognise the uncertainty underlying the estimates. This uncertainty is generally greater for long-tail classes.

UPR is the portion of premiums set aside to cover the claims and expenses for future accounting periods for which premiums have already been received.

URR is a prospective assessment of the amount required as at the accounting date to cover the claims and expenses from the unexpired risks.

AURR is the excess of URR over UPR, subject to a minimum of zero.

Free reserves are the excess of assets over technical reserves. They may also be referred to as free assets, the solvency margin, shareholders' funds or capital employed.

The size of free reserves is an important determinant of:

- the amount of business the company can reasonably write
- the size of risks written
- the amount of risk within the investment strategy
- the need for reinsurance.

Premium rating can involve techniques such as:

- a burning cost approach
- a frequency severity approach
- modelling aggregate claims distributions
- identifying appropriate rating factors *eg* using GLMs and other multivariate analyses
- original loss curves, often used to price reinsurance contracts where past data is too sparse to be credible.

Adjustments will be made to past data to ensure it remains a good predictor of future experience. Further adjustments will be made to arrive at an "office premium". For example:

- allowance for reinsurance and investment income
- loadings for profit and expenses
- commercially-driven adjustments *eg* no-claims discounts, allowance for the insurance cycle, policyholders' reactions, the company's strategic objectives and the state of the insurance cycle.

Capital modelling is the process by which general insurers ensure they hold enough capital to meet all their obligations subject to a given degree of confidence, and allocate this capital to different areas of the business.

A good capital model will include assumptions on all aspects of the business which will affect its future financial strength, and can be a considerably detailed exercise. It may include stress testing, scenario testing and stochastic modelling.

The level of capital held by a general insurer will depend on the riskiness of the insurer's activities. Allowance should be made for the level of correlation between different risks.

The main influences on the investment strategy will be the term, nature and level of uncertainty of the liabilities (these factors are determined mainly by the classes of business written), the size of the free reserves, and legislative factors.

The profit of an insurer is the excess of premiums and investment returns over claims and expenses.

Earned premiums, rather than written premiums, should be used to determine underwriting profit. Similarly, we should use claims incurred rather than claims paid.

Claims incurred is claims paid plus the increase in outstanding claims reserves.

Underwriting profit equals earned premiums less claims incurred less expenses.

The cashflow diagram is a useful way to study the mechanics of a general insurer. The diagram includes all the main monetary flows:

- premiums
- claims
- expenses (including commission)
- investment (in & out)
- reinsurance (in & out)
- dividends (*ie* out to shareholders)
- rights issues (*ie* in from shareholders)
- tax.

Reinsurance can protect insurance companies from various risks that may otherwise be too large for them to bear.

This page has been left blank so that you can keep the chapter summaries together to use for revision purposes.

Chapter 1 Solutions

Solution 1.1

There are lots of possible ideas, including:

- different estimates of the expected claim amount
- different levels of expenses, commission, reinsurance costs, investment returns and profit
- some insurers don't want the business, so quote too high
- others may undercharge, hoping the policyholder will stay for years and that they will make money later
- different forms of cover may have been requested, *eg*:
  - comprehensive or third party fire and theft only
  - as the only named driver, or possibly with family / friends as drivers too
  - different excess levels
- quotations could have related to driving in different countries *etc, etc, etc.*

Welcome to Subject ST8. This sort of careful question reading plus common sense / lateral thinking and the generation of a lot of relevant ideas is often just what is needed.

Solution 1.2

Some judgement is required in setting values for assets and liabilities. So, for example, when assessing the financial strength of the company a prudent (not realistic) basis may be used.

Also, the balance sheet is a snapshot at a given moment. Circumstances may have changed since the date of the balance sheet.

Solution 1.3

Outstanding reported claims reserve is in respect of settlement delays.

IBNR reserve is in respect of reporting delays.

The IBNER reserve exists to cover expected increases (or decreases) in outstanding reported amounts. Such changes could result from either reporting or settlement delays.

Re-opened claims are caused by premature closure of a claims file. The cause of the closure is needed to determine whether a reporting or settlement delay is the reason.

Reserves for claims' handling expenses can be in respect of both types of delay.

Solution 1.4

(a) Outstanding claims and unexpired risks.

(b) Outstanding reported and IBNR

or

outstanding reported, IBNR, IBNER, re-opened claims and claims handling expenses.

Solution 1.5

Class 1 must be a short-tail class of business (because total claims reserves are relatively low), with little in the way of reporting delays, *eg* household contents.

Class 2 is a long-tail class (because the total claims reserves are a much bigger proportion of premiums), with extensive reporting delays. A class such as employers' liability is possible, where some illnesses may not emerge for many years which will make IBNR significant.

Solution 1.6

UPR is the portion of premiums due or received in respect of policies already taken out, that is, set aside for future accounting periods.

Solution 1.7

(a) £660, *ie* 75% of (1,000 – 120)

(b) £2,362.50 (75% of risk is outstanding at year-end, so the UPR is 75% of 90% of £3,500)

(c) £2,000, *ie* 100% of (2,200 – 200)

(d) £10,000, *ie* 100% of 10,000

Solution 1.8

Whereas UPR is the portion of premium set aside for unexpired risks, the URR is our estimate of how much we need to cover the claims and expenses from unexpired risks.

The URR is probably open to more uncertainty. (We know what premiums we charged, but we don't know what the claims experience will be next year.)

Solution 1.9

(a) Estimates of individual outstanding reported claims.

(b) Statistical estimation of totals.

Use statistical methods for IBNR.

Actuarial input is more likely to be used in statistical estimation of totals.

Solution 1.10

- Outstanding claims reserves (includes IBNR *etc*)
- UPR
- AURR (needed because URR expected to exceed UPR as unprofitable)

If the question had said that the company was writing business very profitably, then we would not have needed an AURR. So the answer would have been:

- Outstanding reported claims reserves
- IBNR
- UPR.

Solution 1.11

Higher free reserves result in:

- less need for reinsurance
- greater investment freedom, *ie* scope to mismatch
- greater scope for competitive pricing.

Solution 1.12

Perhaps the accounts were modified because the company revised its view on the unexpired risks. After two months of the new year, the company would know more. Perhaps there was a run of large claims in January or February.

Solution 1.13

This is the same as asking why they need large free reserves:

- it may be a legal requirement
- to meet normal fluctuations in claims experience
- to protect against unexpected adverse experience (catastrophes, large claims *etc*)
- to support and attract new business
- to protect against failure of a third party, *eg* a reinsurer.

We discuss these ideas in more detail later in the course.

Solution 1.14

- classes of business written, hence nature / term / currency and uncertainty of liabilities
- size of free reserves

Solution 1.15

- premiums written $= 6 \times 1{,}200 = £7{,}200$
- premiums earned $= 1{,}200/6 \times (0+0+1+3+5+6+6+6+6+5+2) = £8{,}000$
- claims paid = £5,500 (given)
- claims incurred $= 7{,}300 - 6{,}000 + 5{,}500 = £6{,}800$

Solution 1.16

Earned premiums	157	$(= 165 + 75 - 83$ or $165 - (83 - 75))$
Claims incurred	112	$(= 103 + 133 - 124$ or $103 + (133 - 124))$
Expenses incurred	$\underline{31}$	
Underwriting profit	14	

Solution 1.17

- Outstanding reported claims
- IBNR
- IBNER
- Re-opened claims
- Claim handling expenses
- Unearned premium reserve
- Additional reserve for unexpired risks

The AURR is most likely to be zero. If it is not zero, it indicates that we think that we have recently been writing business on unprofitable terms.

Solution 1.18

This year's pre-tax profits would be increased by £20 million, so assuming that this goes straight into the assessment of tax, we would pay an extra £6 million tax. Post-tax profits would be increased by £14 million.

IBNR might be revised downwards if it included a big allowance for claims that might come through from a particular cause, *eg* employees claiming against their employers for back-ache from sitting at desks. If a court case has just gone through in favour of the employer, then the insurer will give a big sigh of relief and reduce its IBNR.

Solution 1.19

AURR is the excess of URR over UPR, subject to a minimum of zero.

UPR is the portion of premiums which relates to unexpired cover.

Claims incurred is defined as the claims paid plus the increase in outstanding claims reserves.

The main determinants of the investment strategy will be:

- the nature, term, currency and uncertainty of the liabilities
- the relative size of the free reserves.

A reserve used to smooth profits from year to year is called a claims equalisation reserve.

The excess of earned premiums over incurred claims and expenses is called the underwriting result (or underwriting profit).

Solution 1.20

It depends critically on how wide the cover provided is, but examples of some potentially scary things, for which reinsurance may help the insurer sleep at night, are:

- One or more whole parties could be affected by the same claim event, *eg*:
  - delayed departure from airport
  - no snow on arrival, or other sub-standard holiday features
  - plane / coach crash
  - avalanche.
- Similarly, if one or more travel agents became bankrupt the insurer might, conceivably, suffer a large loss from many claims.
- One or more of the skiers may suffer injury, with potentially enormous medical expense claims, especially with older skiers (complications?) and especially in the USA.

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Chapter 2

Insurance products – background

Syllabus objective

(b)(i) Describe the main types of general insurance product in terms of:

- *the needs of customers*
- *the financial and other risks they pose for the general insurer including their capital requirements and possible effect on solvency.*

Covered in part in this chapter.

0 Introduction

This chapter and the following chapter give an overview of the essential features that distinguish the main types of general insurance products and so help to identify the crucial aspects that influence the nature and extent of the risk to be covered by the insurance. Reinsurance products are described in Chapters 5 and 6 and health products are covered elsewhere in the Syllabus (*ie* in other subjects)**.**

In this chapter we look at general insurance principles and introduce the broad categories of insurance products. In the next chapter we look at the different general insurance products in detail.

1 General insurance principles

Before considering the main types of insurance and reinsurance, it is worth noting some overriding features of all types of insurance.

You might recognise some of the following material from Subject CA1, however, the material here is *slightly* different.

1.1 Insurable interest

For a risk to be insurable:

- **the policyholder must have an interest in the risk being insured, to distinguish between insurance and gambling**
- **a risk must be of a financial and reasonably quantifiable nature**
- **the amount payable by the insurance policy in the event of a claim must bear some relationship to the financial loss incurred.**

1.2 Insurable risk

Ideally risk events also need to meet the following criteria if they are to be insurable:

- **Individual risk events should be independent of each other.**

  However, as we have seen in Chapter 1, reinsurance is available to help cope with situations where risks are not independent. In practice we won't often get strict independence but a low correlation is desirable.

- **The probability of the event should be relatively small. In other words, an event that is nearly certain to occur is not conducive to insurance.**

  For example, a house would not be insured if it stood on the edge of a crumbling cliff. The premium to cover such a risk would be exorbitantly high.

- **Large numbers of similar risks should be pooled to reduce the variance** (of the average claim size) **and hence achieve more certainty.**

  With a large enough number of similar risks, we can appeal to the law of large numbers, and see that the insurer will benefit from more predictable claims experience than the policyholders. The similar risks should still be independent.

- **There should be an overall limit on the liability undertaken by the insurer.**

  This would help the risk event meet the above criteria that it must be of a reasonably quantifiable nature.

- **Moral hazards should be eliminated as far as possible because these are difficult to quantify, result in selection against the insurer and lead to unfairness in treatment between one policyholder and another.**

> ***Question 2.1***
>
> What is *moral hazard*?

See also "uberrima fides" below.

- **There should be sufficient existing statistical data / information to enable the insurer to estimate the extent of the risk and its likelihood of occurrence.**

  We will see later in the course that there are a number of sources of data that might be used if the insurer doesn't "own" any relevant data itself.

However, the fact that these ideal criteria are not always met in practice does not necessarily mean that insurance cannot be found. Insurers may be prepared to underwrite simply to generate income, to build a relationship or develop a new market entrepreneurially. All risks were once insured for the first time.

1.3 Other terms

Other important general insurance terms you will come across are given in this section. Many others are given in the Glossary – remember you will need to know the Glossary definitions for the Subject ST8 exam.

Uberrima fides

Latin for "utmost good faith". This honesty principle is assumed to be observed by the parties to an insurance, or reinsurance, contract. An alternative form is uberrimae fidei: "of the utmost good faith".

The principle of honesty underlies all insurance business. For example, misrepresentation or non-disclosure of any material fact in the proposal can make the policy void. Each renewal of a general insurance policy actually constitutes a new proposal, and the insured should disclose to the insurer any material changes during the period covered by an insurance policy.

The insurer faces a variety of moral hazards because insureds may not abide by this legal principle and, if they do not, it may be hard for the insurer to detect.

Multiple claims

Unlike life insurance policies, many general insurance policies allow the insured to claim as many times as necessary during the period of cover, usually a year, provided by the policy.

Nil claim (or zero claim)

A claim that results in no payment by the insurer, because, for example:

- **the claim is found not to be valid**
- **the amount of the loss turns out to be no greater than the excess**
- **the policyholder has reported a claim in order to comply with the conditions of the policy, but has elected to meet the cost in order to preserve any entitlement to no-claim discount.**

Again, unlike simple life insurance policies, for which the claim event, *eg* death, is fairly clear cut, a significant number of general policies generate claims which are subsequently settled by the insurer with no payment to the insured. This may be, for example, because the insurer is not liable (*eg* the policy wording excludes that type of claim). However, nil claims still invariably result in administrative expenses for the insurer.

Underinsurance

If the contents of your home are worth £30,000 then you should take out a home contents policy with a sum insured of £30,000. However, you may choose a policy with a sum insured of only £20,000 because you have under-estimated the value of your belongings, or because you think the chance of losing more than £20,000 is very small. This is known as underinsurance.

Question 2.2

Why is underinsurance a *hazard* to the insurer?

Average

In order to prevent underinsurance, some property insurance policies where premium rates are based on sum insured contain an average clause. This provides that, if the sum insured is less than the full value of the property at the time of a loss, the insurance payment will only be a proportion of the value of the loss – the same proportion as the sum insured bears to the full value. This is known as the *principle of average*.

Example

If you insured the contents of your house for £20,000 but the actual value of the contents was £30,000 then the principle of average may apply if you make a claim. A claim for £600 may result in a payment of only £400 because the claim is scaled down by 20,000/30,000.

Warning! The term "average" has different meanings in different contexts within the general insurance world.

In non-marine insurance, the term relates to the practice of reducing the amount of a claim in proportion to the extent of underinsurance.

In marine insurance, the term is generally used to describe damage or loss.

The word *average* derives from a Latin word *havaria* which broadly means "loss". There are many uses of this word within marine insurance. For example, a *general average loss* is a loss resulting from a sacrifice or expenditure made by an individual for the benefit of others at a time of peril, *eg* throwing cargo overboard from a boat to stop it sinking, thereby saving the remaining cargo and the vessel.

Subrogation

The substitution of one party for another as creditor, with a transfer of rights and responsibilities. It applies within insurance when an insurer accepts a claim by an insured, thus assuming the responsibility for any liabilities or recoveries relating to the claim. For example, the insurer will be responsible for defending legal disputes and will be entitled to the proceeds from the sale of damaged or recovered property.

Subrogation means that the insurer replaces the policyholder in law and acquires all rights and responsibilities in legal matters regarding the loss suffered, be it before or after the claim has been settled.

> ### *Example*
>
> If you receive a payment from an insurer for replacement of your boat, following serious damage or loss, then the original boat becomes the insurer's property. The insurer may then be able to recover a salvage value, for its own benefit.

Discovery period

A time limit, usually defined in the policy wording or through legislative precedent, placed on the period within which claims must be reported. It generally applies to classes of business where several years may elapse between the occurrence of the event or the awareness of the condition that may give rise to a claim and the reporting of the claim to the insurer: for example employers' liability or professional indemnity.

Employers' liability and professional indemnity insurance will be described in the next chapter.

The discovery period prevents claims being made to insurance companies many years after the event that caused the claim. In principle it allows insurance companies to write off IBNR liabilities from a contract once the discovery period has elapsed, although this has sometimes been overridden by the courts.

The discovery period is often defined in the sunset clause. Both terms relate to the same topic. The discovery period is the actual time limit; the sunset clause is a clause that defines the time limit.

Underwriting

The process of consideration of an insurance risk. This includes assessing whether the risk is acceptable and, if so, the appropriate premium, together with terms and conditions of the cover. It may also include assessing the risk in the context of the other risks in the portfolio. The more individual the risk (for example most commercial lines), the more detailed the consideration. The term is also used to denote the acceptance of reinsurance and, by extension, the transacting of insurance business.

Underwriting is the process of assessing the risk for individual policies. As well as setting premium rates, underwriters will specify excesses or exclusions to cover or possibly required improvements to the risk before cover is provided. For small and standard homogeneous risks, insurers will often provide insurance automatically, without referring the individual risks to the underwriters.

1.4 The policy document

The policy document is important. It sets out the terms and conditions under which an insurer is liable to pay insurance claims in specific circumstances and must, therefore, be carefully worded to cover all possible circumstances under which payment will and will not be made.

The relationship between the insured and the insurer is governed by the policy document. It is a legal and binding contract and hence subject to contract law.

Policy forms are normally standard for all personal lines business and small commercial policies, in the sense that an insurer will issue the same wording to all policyholders. Items that vary between policyholders will be included in a schedule.

Examples of common items in a schedule are:

- **details of vehicle / property / people covered**
- **excess applied**
- **any limits to the cover**
- **exclusions**
- **time limits (such as hours clause)**
- **whether or not any optional covers have been taken**
- **details of insurance premium paid.**

An hours clause would be found in some types of reinsurance treaty. This will be discussed in Chapter 6.

Larger commercial risks tend to have policies that are individually made for the particular policyholder, possibly assembled from a library of standard clauses.

1.5 Exclusions

Exclusions are clauses in a policy that limit the circumstances in which a claim may be made.

Some examples of common exclusions are:

- suicide or self-inflicted injury
- dangerous pastimes
- loss resulting from illegal activity by the policyholder
- war, terrorism, civil riot.

Question 2.3

Can you think of some more examples of common exclusions?

Exclusions can apply to certain perils (*eg* terrorism) or particular types of loss (*eg* cash, whatever the peril).

Question 2.4

List the exclusions that are likely to be applied on a private motor policy.

Exclusions are used to avoid payment by the insurer in situations where:

- **the policyholder is at an advantage through possessing greater personal information about the likelihood of a claim**
- **the claim event is largely under the control of the policyholder**
- **the claim event would be very difficult to verify**
- **the loss occurs as part of the normal course of events and could be considered to be depreciation.**

Without an exclusion there would be a very high probability of a claim or that the risk could not be reasonably estimated.

Exclusions might also be used in other situations where the risk cannot be reliably estimated by the insurer, regardless of whether or not the policyholder has better information, or when the probability of loss is very high. Remember from Section 1.2 that one of the criteria for an event to be insurable is for the probability of the event to be relatively small.

Exclusions are also used where the risk is covered by a third party such as the government.

Example

Terrorism is an example of such a risk in the UK.

Exclusions are also used to limit the scope of the policy to make it more appropriate for a particular target market or to reduce the premium for competitive reasons.

Exclusions are sometimes used to reduce the risk of moral hazard and fraud. For example, theft of a vehicle when the keys have been left in the ignition might be excluded from a motor policy. This reduces the moral hazard of policyholders' carelessness, which might result from the existence of insurance cover.

Indemnity for loss of cash might be excluded from a household contents policy. This reduces the risk of fraudulent exaggeration of loss amounts, as it would be very easy for a policyholder to claim, but difficult to verify, that large sums of cash had been stolen when a burglary has taken place.

Question 2.5

List the reasons for applying exclusions to an insurance policy.

2 Types of product

The types of insurance cover provided by general insurance products can be classified under four main headings:

- **liability**
- **property damage**
- **financial loss**
- **fixed benefits.**

This section gives a brief overview of these categories of insurance product. They are described in detail in the next chapter.

2.1 Liability

The essential characteristic of liability insurance is providing indemnity where the insured is legally liable to pay compensation to a *third party*.

Examples of liability insurance include:

- employers' liability – where the insurance indemnifies the employer against compensation payable to employees for losses that they suffer as a result of negligence of the employer
- motor third party liability – where the insurance indemnifies the owner of a motor vehicle against compensation payable to third parties for personal injury or damage to their property (*eg* their vehicle)
- product liability – where the insurance indemnifies a manufacturer against compensation to a third party for losses that they suffer as a result of a product fault.

2.2 Property damage

The main characteristic of property damage insurance is providing indemnity to the *insured* for loss of, or damage to, the policyholder's *own* property.

Examples of property damage insurance include:

- motor insurance
- buildings insurance (which includes both residential buildings (houses) and commercial buildings, such as offices, shops and industrial buildings)
- contents insurance.

2.3 Financial loss

Financial loss indemnifies the insured against financial losses arising from certain causes.

Examples of financial loss insurance include:

- creditor insurance – where the policy will make regular loan repayments if the policyholder becomes disabled (so that they cannot work) or otherwise unemployed
- business interruption cover – where the policy will pay out to compensate the policyholder for not being able to conduct their business, *eg* as a result of a fire in the building.

2.4 Fixed benefit

The defining characteristic of fixed benefit insurance is that the benefits are usually specified, fixed amounts, payable on certain losses occurring.

An example is personal accident insurance, where the insured receives a fixed payment on suffering a specified injury, *eg* the loss of a limb.

2.5 Combining categories of insurance

Insurance policies may comprise elements of one or more of these types of cover.

Example – comprehensive motor insurance

A typical comprehensive motor insurance policy will provide cover for:

- **compensation for personal injury to third parties and damage to their property**
- **compensation for loss of or damage to the insured's vehicle**
- **fixed benefits in the event of defined categories of personal accident to the insured.**

Therefore, a typical motor policy may comprise elements of liability, property and fixed benefit cover.

Example – household contents insurance

A typical policy covering household contents will provide cover for the financial loss, property damage and liability of the insured (and any member of the insured's family living at the same address) to third parties.

Therefore a household contents policy may comprise elements of property and liability cover.

Question 2.6

Give an example of how, under a household contents policy, the insured may be liable to a third party.

Policies that are likely to be combined as a single product are described further in Chapter 3.

2.6 The customer

Personal lines and commercial lines business

General insurance may be sold to both individuals and businesses.

Insurance products sold to individuals are known as *personal lines* business. They include private motor, domestic household, personal accident and travel insurance.

Insurance products sold to businesses are known as *commercial lines* or *group business*.

Package policies

Policies for small businesses often include all types of cover the business needs apart from motor; appropriately, they are called "package policies".

Question 2.7

List the types of cover a small retailer might require as part of a "package policy".

3 Cover provided

For each of the four main types of cover, the features will be discussed in the following sections under six headings:

- **benefits**
- **insured perils**
- basis for cover
- **measures of exposure to which premiums are related**
- **claim characteristics**
- **risk factors and rating factors.**

In the next chapter, the products are discussed under the headings of the main types of cover that they provide.

Any unusual words or phrases, for example *risk factors* used in the above paragraphs will be explained fully later in this chapter.

The following *general* comments relate to these six headings.

3.1 Benefits

The benefits provided by an insurance policy will vary between types of insurance and between insurers. Typically, the intention is to provide the insured with money to cover his or her financial loss as a result of an insured event, although policies that provide benefits in kind are also possible.

The provision of a courtesy car under a motor policy (while your car is being repaired) is an example of a benefit in kind provided by an insurance policy.

Question 2.8

Describe the benefits you would expect to receive for each of the following claims:

- loss of luggage whilst on holiday
- loss of a finger
- loss (to your vehicle and injury to yourself) as a result of a car accident.

In some territories, certain insurance cover is compulsory.

3.2 Insured perils

A *peril* is a type of event that may cause losses, *eg* theft or flood.

In virtually all types of insurance, it would be impossible to list all possible perils against which a policyholder might wish to be protected. Claims can result from a very large number of perils; not all of them will be currently apparent and not all of them will be standard to every insurance product on the market.

The perils are also likely to vary by country. For example, in some countries it is more likely (or necessary) for perils such as volcanic activity or stampeding animals to be included in the cover.

The precise form of the cover provided in respect of any insured peril may vary between insurers, and the benefits provided may be defined in any one of a number of ways.

> ***Exam Tip***
>
> **The examiners will expect candidates to apply some common sense to extend the examples or groupings given for the products mentioned in this chapter and the following chapter.**

3.3 Basis for cover

This section describes the basis on which different types of cover may be written. Insurance policies may be written as:

- losses-occurring policies
- claims-made policies.

Losses-occurring policy

A losses-occurring policy is a policy providing cover for losses occurring in the defined period no matter when they are reported. Note that a losses-occurring policy may also be referred to as a "claims-occurring policy".

Claims-made policy

A claims-made policy covers all claims reported to an insurer within the policy period irrespective of when they occurred. Note that a claims-made policy may also be referred to as a "claims-reported policy".

Question 2.9

An individual purchases a one-year motor insurance policy from Cars 'R' Us on 1 January 2012. The following year, he buys another one-year policy from Cars 4 You.

On 27 December 2012, the individual has an accident, which causes significant damage to his car. He decides to wait until the New Year to report the claim.

Which insurer should the individual contact if the policies are written on a:

(a) losses-occurring basis

(b) claims-made basis?

Which basis do you think is more common for this type of insurance?

Question 2.10

What characteristics of claims might make a claims-made basis appropriate?

Question 2.11

What is the key problem for an employer in switching insurance from an insurer giving cover on a claims-made basis to an insurer giving cover on a losses-occurring basis?

3.4 Measures of exposure to which premiums are related

This section discusses the underlying principles involved in choosing exposure measures. More specific comments on the various measures used for different insurance classes are given later in the chapter. As you read the Core Reading think actively about how the practical choices made have been influenced by the principles.

One of the objectives of the insurer when setting premium rates is to charge a premium which accurately reflects the amount of risk. The *pure risk premium* is the premium required to cover the expected claim amount only. No allowance is made for expenses or profit. For example, if a particular policy gives rise to a claim of £1,000 with a probability of 10% and no claim otherwise, the pure risk premium would be £100. The *actual premium* charged would then need to cover this level of risk and also include an allowance for expenses, profit and investment income and any other loadings.

Selection, anti-selection

If an insurer does not charge premiums that accurately reflect the amount of risk, the insurer may suffer from selection (strictly called anti-selection in this context). If in the example above the insurer charged a premium based on a risk premium of £120, they may not get the business if other more accurate insurers used a risk premium of £100. On the other hand, if they allowed for only £80 they might get the business but make a loss.

In general, insurers who fail to charge premiums that reflect the amount of risk run the danger of getting little business (premiums too high) or lots of loss-making business (premiums too low). Assessing the amount of risk involved with a particular policy is a critical aspect of an insurer's work.

Question 2.12

Do the dangers of selection still exist if all insurers charge the same premiums?

Exposure

In practice, insurance proposals do not come with neat little labels stating the probability of claim and the expected cost per claim. The amount of risk underlying an individual policy is often largely unknown. In fact, the amount of risk is *never* known exactly.

For the purpose of setting premiums, insurance companies try to determine measures which give an indication of how much risk there is within each policy. These measures are called *measures of exposure*.

In practice, the chosen measure of exposure should meet two key criteria:

(a) It should be a good measure of the amount of risk, allowing for both the expected frequency of claim and the expected severity of claim (*ie* the average claim amount). In other words, the total expected claim amount should be proportional to the exposure.

(b) It should be practical. This criterion embraces several aspects. The measure should be objectively measurable and should be easily obtainable, verifiable and not open to manipulation.

There are rarely perfect measures of exposure which completely define the amount of risk underlying each policy. For most classes in fact, the exposure measure used is a basic principal indicator of risk. Given a measure of exposure, risk can be further classified by rating factors. These are discussed further in Section 3.6.

In some types of insurance there is a degree of choice as to the measure of exposure, as it will not be immediately obvious which reliable and measurable factor bears the closest relationship to the expected claim amounts. Examples of measures used in some classes are given in the next chapter.

An exposure measure isn't just a factor that is used to calculate a premium. It is a measure, which when added over all policies, gives an indicator of the "volume of business" or the total amount of risk. As such, the expected total claim amount should be proportional to the exposure. Clearly the best measure of exposure is then the expected claim amount on each policy but exposure should also be a simple measure that is verifiable *etc*. The number of policies always has this property but there may be another quantifiable factor on each policy that works better.

Question 2.13

Give another possible exposure measure, other than the number of policies, that would give a reasonable indicator of the total level of risk on any portfolio of business.

3.5 Claim characteristics

This topic was introduced in Chapter 1.

As well as the amount that becomes payable, the claim characteristics refer to the ways in which and speed with which the claims:

- **originate**
- **are notified**
- **are settled and paid**
- **are, on occasion, reopened.**

Claim characteristics also refer to the frequency with which claims are made. For some lines of business, such as private motor insurance, claims occur with a far higher frequency than other lines, such as various liability classes.

When assessing claim characteristics, it is therefore necessary to consider:

- delays (reporting, settlement *etc*) and hence whether claims are short-tail or long-tail claims
- claim frequency
- claim severity / amount (allowing for accumulations, catastrophes *etc*).

Claim frequencies and claim costs are often discussed in terms of a claim frequency / claim cost distribution. This is just a statistical distribution for claim frequency / individual claim cost.

All these features have implications for the assessment of risk borne by the insurer.

3.6 *Risk factors and rating factors*

The fact that exposure measures are never perfect measures of the amount of risk means that there is scope for refinement of the premium even after applying the exposure measure. An example will help to illustrate the point.

Example

Consider a special class of insurance whereby policyholders who bang their heads on doorframes or low ceilings of homes are entitled to an insurance payment. The worse the injury, the more the payment.

What exposure measure should we use to determine premiums? There are several candidates, but let's suppose that we decide to use the policyholder's height as the basis for setting premiums. Our measure of exposure would then be centimetres. Exposure measures often incorporate time units to reflect the fact that policies for two years should be charged twice the premium of one-year policies. Hence our exposure measure is *centimetre-years*. If the premium is £0.10 per centimetre-year, a policyholder of height 175*cm* would be charged an annual premium of £17.50.

Question 2.14

Do you think that centimetre-year is the *best* exposure measure to use? If not, explain why and suggest an alternative.

In this example, our insurance company may still be exposed to the possibility of selection. The premium rating structure has done nothing to incorporate allowances for the following factors:

- the non-linearity of the relationship between height and risk
- short-sighted people may hit their heads more often
- clumsy people may hit their heads more often
- people who move around quickly will hit their heads harder
- people living in homes with low doorframes and ceilings will hurt themselves more often.

These further considerations of risk are called *risk factors*. Note that the exposure measure itself is also a risk factor. Risk factors are any factors that have a bearing on the amount of risk.

To prevent selection, the insurer must try to incorporate these factors into the premium rating process. If our 175*cm* policyholder turned out to be short-sighted, clumsy, fast and the inhabitant of a 16th Century cottage, the premium might be increased from £17.50 to, say, £40 each year.

Risk factors will depend on precisely the cover provided, but the factors applicable in most cases are given in the next chapter.

Sometimes direct use of the risk factors in the rating process is not practical (*eg* the risk factors may not be easily measured). In these cases, other factors that are more easily identified may be used as proxies for the underlying risk factors. The expression for the factors actually used in the premium rating process is *rating factors*.

Rating factors will be either objectively measurable risk factors or other factors that can be used as reliable proxies for the risk factors. Where credible exposure and claims data exist, experience rating can be used to take account of residual risk factors.

This is saying that we can use the actual claims experience of the insured in the past to help set an appropriate premium for the future. For example, the number of claims you have made on your motor policy is likely to affect your insurance premium. Experience rating is discussed in more detail later in the course.

Note that in some countries, certain rating factors are not allowed to be used. For example, the European Court of Justice recently ruled that a person's sex can soon no longer be used to calculate insurance premiums.

You may hear the term *underwriting factors* used by some practitioners. There are some differences in how this term is used with some taking it as synonymous with rating factors. However most, (and importantly for Subject ST8, the Core Reading), take it to mean rating factors plus subjective factors that, although they cannot be measured, the underwriter takes into account in setting premiums or policy conditions. A subjective factor in our head-banging example might be how accident-prone the person appeared to be.

3.7 Combining exposure measures, risk factors and rating factors

The following statements follow on from the discussion above:

(a) The more heterogeneous the class of insurance and the types of risk covered, the greater the number of risk factors needed to identify or define the amount of risk.

If, for example, we were dealing with a class of business where all the risks were identical, there would be a single premium rate that applied to all policyholders. In this case there would be no additional rating factors.

At the other extreme is a class of insurance like private motor where the risks are very heterogeneous. The amount of risk will vary from one policyholder to another according to a great number of risk factors. We will see later that for motor insurance, many rating factors are used to calculate the premium.

(b) The better the measure of exposure in identifying the amount of risk, the lesser the importance of other rating factors.

(c) The choice of rating factors will depend on the choice of exposure measure.

Sometimes an exposure measure used in practice does not satisfy the ideal property of proportionality exactly. The exposure measure can be used as a rating factor as well. A real-life example that you will see is domestic property insurance in which the size of the property (*ie* the sum insured) is taken as the exposure measure.

For contents insurance the main peril is theft but the sum insured includes the value of items that are rarely stolen, such as carpets, curtains, the toilet brush *etc*. If the value of the goods likely to be stolen, as a proportion of the total sum insured, tends to decrease as the total sum insured increases then the expected loss increases less than proportionately with the sum insured. Therefore an insurer might use sum insured as a rating factor as well as an exposure measure.

It may do this by charging different rates for different bands of sum insured. An example is shown in the following table.

Sum insured band	*Premium rate per 1,000 sum insured*
0 – 10,000	18.6
10,001 – 20,000	17.5
20,001 – 30,000	16.9
> 30,000	16.7

In one sense, the exposure measure is always a "rating factor" as it always affects the premium. However, "rating factor" is usually used to mean a factor that affects the premium rate, hence these comments.

The rating factors used in practice will vary between different insurers as they attempt to find a competitive edge. The more common rating factors, some of which have been justified by statistical analysis, are given in the next chapter.

Question 2.15

(a) What are the main criteria in the choice of an exposure measure?

(b) What is the difference between a risk factor and a rating factor?

(c) Why are rating factors needed?

(d) What is the difference between a rating factor and an underwriting factor?

Question 2.16

Under what circumstances would an insurer's underwriters not look at the risk and rating factor details for new policy proposals?

4 How does the insurer's risk vary by class?

Some classes of business cause insurers more risk and uncertainty than others because of the nature of the risks involved and the claims that can arise from those risks.

4.1 Homogeneity of risks

Where there is a lack of homogeneity of risks within a class of business, there is greater risk to the insurer.

Even within a given rating category, exposures can be very variable and dissimilar.

In commercial buildings insurance, for instance, properties can vary considerably by size, construction and value. Moreover, you may have a mixture of properties, ranging from small shops to large chemical factories.

You can therefore have policyholders with very different risk potential within the same rating category. This will be reflected in the subsequent claims experience, and its inherent variability. This will be particularly true for some of the liability classes.

For certain other classes, the risks within each rating category tend to be more homogeneous, and the experience will therefore tend to be more predictable from year to year. Private motor is an example of this, due to the large number of rating factors used to categorise risks. Some insurers also insure vast numbers of (independent) cars, which reduces the relative variability of the experience, and so also increases the predictability.

4.2 Non-independence of exposures

The variability is increased where exposures are not independent, as this can lead to an accumulation of risk. For example, if the majority of policyholders in a household insurance portfolio live in a certain area of the country, there will be a disproportionate claim cost if there is a local catastrophe. If the book has exposures spread across the country (and so more independent geographically), the claim cost will not be affected so greatly by a local catastrophe.

Some classes will lend themselves more to independence than others. For example, a personal motor insurance portfolio should have a reasonable spread of exposures, whereas creditor insurance will be heavily linked to the state of the economy and unemployment.

The policyholders of a motor insurer are not generally concentrated in one geographical region because business tends to be sold nationally.

Creditor insurance provides cover to insureds who are subject to obligations to repay credit advances or debt. It is described in detail in the next chapter.

4.3 Changing risk

Another feature leading to risk uncertainty is that the nature of the risk may change, both during a particular policy year and in succeeding years.

Examples include:

- **a change of fire precautions within a building**
- **a change of drivers or location under a motor policy**
- **a change in economic conditions under a mortgage indemnity policy.**

In each example above, the insurer's risk portfolio will change over time from that originally written, leading to difficulties in pricing and managing that portfolio.

In some cases a change in the underlying risk should be notified to the insurer by the policyholder. For example, a motor policyholder should inform the insurer if he or she moves house; this is a rating factor.

In extreme cases, failure to notify the insurer could make the cover void.

Changes in background conditions, such as economic conditions, would not normally need to be notified.

Some classes of risk vary more than others in this respect. Employers' liability can also fluctuate markedly in certain years, due to the turnover of employees, or through business acquisitions. Other classes of risk can be fairly stable in the short term.

Question 2.17

How stable would the risk be for a typical simple industrial buildings and contents policy (with no business interruption cover) for a manufacturing company?

4.4 How do claim characteristics affect risk?

Numbers of claims

In general insurance, as distinct from life assurance, there is often no limit to the number of claims that can arise from a policy while it is in force.

Some classes, such as motor and household contents, have a relatively high claim frequency, with sometimes 15% or more of policies having a claim each year. Other types of cover can have much lower claims frequency, particularly commercial classes with high excesses.

Examples of such commercial classes include public and product liability and commercial buildings.

Claim cost

The cost of a claim from any given policy cannot be predetermined except in a very small number of classes – such as personal accident – and is often very variable.

This applies when the basis of cover is indemnity rather than fixed benefit. Personal accident is a case where cover is generally on a fixed benefit basis.

While there will often be a maximum sum insured stated (or implied) in the policy, relatively few claims will be settled for that maximum sum. The usual principle of general insurance is to indemnify the insured for any losses or claims made upon him or her. Most claims will therefore be for only a portion of the maximum cover, depending on the circumstances of the incident.

For most classes, a large proportion of claims will be for small amounts and there will be only a small number of large claims. The precise distribution of claim amounts will, however, vary greatly by class, in particular between property and bodily injury claims, and also year by year.

When a probability distribution is used to represent the distribution of the sizes of claims in a class of business it is conventional to use highly skewed distributions with no theoretical upper limit, such as lognormal or Pareto.

Exposures are not all identical, even within the same class of business, and each exposure has its own claim cost distribution. Exposures are often too small to yield useful information individually so their experience is aggregated to derive parameters of the claim cost distributions when the risks are homogenous enough and attritional. The shape of these claim cost distributions depends on risk characteristics demonstrated by different classes of business and the insured's risk profile, among other factors.

Claim inflation

Where a class is exposed significantly to the risk of inflation, any unexpected change in inflation, for whatever reason, will affect the risk profile of that business.

Different classes of business are affected by inflation in different ways:

- **Property insurance responds mostly to the cost of property, and claims will tend to increase in line with general inflation, although repair costs can be linked to earnings.**
- **A large proportion of motor claims cost is for the repair of vehicles, and will be affected by the level of earnings since these determine labour costs.**
- **Liability classes are often subject to higher levels of inflation, especially on personal injury claims, as there is a trend to more generous compensation in many markets. However, this may arise in steps rather than as a continuous process of inflation, as landmark legal judgements are handed down or legal reform comes into effect.**

Delay patterns

An insurer's ability to manage a general insurance account is further complicated, and hence the degree of risk increased, by the length of time that it takes for claims to emerge, to be reported and to be settled.

Claim delays can arise for various reasons, for example:

- **a delay between the incident occurring and the policyholder becoming aware of it**, *eg* the time between a burglary occurring in a property and the policyholder returning home to discover it
- **a delay between the insured becoming aware of the loss and reporting it**, *eg* the policyholder may be slow to report a claim if it is quite small
- **a delay before sufficient details of the incident can be gathered to assess the value of the claim**
- **a delay until an injured party's condition stabilises to the extent that assessment of damages is appropriate**, *eg* to assess whether the injured party will recover or is now permanently disabled
- **delay in agreeing the actual value at which the claim is to be settled, and the payment of this amount to the insured.**

The typical extent of such delays will differ according to the class of business.

Bodily injury cases tend to have the longest delay tails, owing to the contentious issues of many of the claims involved, often with the need for legal proceedings. This may be worsened by the greater likelihood of latent claims or claims for industrial disease where the delay from event to reporting can be considerable.

For example, some liability claims relating to disease and pollution have taken decades to emerge. In many cases, the insurers were largely ignorant of the potential risks at the time that cover was given. Such claims are known as *latent claims*. The possibility of latent claims increases the level of risk for insurers.

By contrast, property damage classes have a much shorter delay tail, and hence in this respect a lower degree of risk, since the losses are more immediately apparent and can usually be valued reasonably accurately by a competent assessor.

Until all claims have been settled from a given exposure period, you are uncertain both as to the number of such claims and, more importantly, their cost. This will have consequences for setting reserves when drawing up your accounts, and for evaluating future rating needs.

Also, while claims are outstanding they will be subject to increases in cost due to inflation, sudden jumps in court awards, changing legislation and indirect taxes. This further complicates your ability to estimate future claim settlement costs.

Question 2.18

One of your student colleagues says that for motor third party cover, liability claims are long tailed and property claims are short tailed. Do you agree?

Variability of experience

Depending on the class involved, the numbers of claims can vary according to such features as unusually bad weather, the economic situation and catastrophes.

Some classes are more susceptible than others to individual claims that are large enough to affect the results of the whole class.

The pattern of claims arising within a year for any given class will rarely, if ever, equate with the theoretical claims cost distribution for that class. One or more very large claims can easily swamp the normal cost.

Accumulations

The insurer can be exposed to accumulations of risk if the portfolio is unbalanced. There is a possibility of many claims arising from a single event or a single cause.

Some classes of business are (particularly) **subject to accumulations of risk.**

Property classes are prone to catastrophes: external events that affect a large number of policies at the same time, possibly causing mainly small claims, but in aggregate giving rise to a very large total loss. The most important examples are extreme weather, earthquakes and civil disturbance.

Such accumulations tend to be based on single events affecting a number of risks in the same geographic region.

However, such accumulations need not be single incidents; for example, a very dry summer may cause a large number of subsidence claims, although there is no single event to link them.

Other forms of accumulation are not necessarily based on geographical concentration, *eg* for creditor insurance, there may be many claims triggered by high unemployment.

Liability insurance is less susceptible to large single incident accumulation losses, but a single cause may give rise to a large number of claims. The most obvious example is exposure to asbestos, which has given rise to claims under liability policies that are expected to exceed any single event catastrophe.

Fraudulent claims

Certain classes are more exposed than others to the risk that the insured will make false or invalid claims, or exaggerate the amount claimed following a loss. Often, it will be difficult or uneconomic for the insurer to check whether the claims are genuine or not. At the extreme such false claims could include arson and embezzlement. The rate of fraudulent claims has been observed to increase in times of economic stringency.

Examples under personal lines cover can include:

- Motor: a policyholder, with non-comprehensive cover, suffers non-claimable damage to the insured vehicle. The policyholder then drives the vehicle to a remote spot, destroys it by fire and claims under the fire and theft sections of the policy.
- Household contents: claims are made for loss of fictitious articles, or the values are exaggerated.
- Buildings: claims are made for defective roofs, which are really due to natural wear and tear.
- Buildings: after a genuine insured loss, a builder may offer to add the amount of the insurance excess to his invoice. The insurer then unwittingly pays the full amount of the actual repair cost.

Under commercial lines of business, examples can include:

- Buildings: the insured deliberately sets fire to the insured premises because the business is losing money, and the insured stock is actually of little value.
- Liability: malicious or extravagant claims are made by third parties because they have seen people with valid claims receive substantial awards.
- Financial loss: embezzlement, fraud or falsification of accounts is carried out by the insured, in order to exaggerate a loss.

In all cases, these risks tend to be greatest during periods of economic downturn and depression. The insured sees the insurer as an illegal means of recovering losses that may threaten the insured's own financial position.

Question 2.19

If you were the owner and manager of a general insurance company, what would you do to try to reduce the fraudulent claims against you?

5 Capital requirements and impact on solvency

General insurers require resources beyond those needed to cover their technical liabilities in respect of the business they have written. The excess of the value of the assets over the value of the liabilities is known as the capital or solvency margin of the company.

You may hear it referred to as the *free reserves* of the company.

The amount of capital held is subject to minimum requirements if a company is to be allowed to continue to trade. These requirements are laid down in legislation that varies from one country to another, although all countries in the EU have a common minimum solvency regime. This solvency margin will act as a safety margin against the uncertainty surrounding the ultimate future cost of liabilities and the value of the assets supporting them.

If the number or cost of claims is greater than the reserves held to meet the claims, then the solvency margin (free reserves) will be needed to pay the claims. Unexpected claims may result in the need to realise assets to pay those claims. Free reserves will guard against the danger of volatile asset values.

Question 2.20

What are the other reasons for a general insurer holding free reserves?

Determining the capital that a general insurer ought to hold to cater adequately for the risks associated with the business it transacts is a complex issue. The longer the tail of the business written the greater the uncertainty and hence, other things being equal, the more capital will be required. In setting its capital requirements, beyond those specified by law, the general insurer will need to take into account the uncertainty and variability of the business it writes.

Generally, for two classes where the same amount of business – measured by premium income – has been written, the capital requirements should be larger for the class with the greater uncertainty and variability in its future claims experience and in the run-off of reserves.

> ***Question 2.21***
>
> Rank the following classes in decreasing order of uncertainty and variability of future claims experience:
>
> - employers' liability
> - household contents
> - motor liability.

When considering its capital requirements a general insurer will need to consider each class of business individually. However, its overall capital requirement will be more important. An insurer that writes a variety of classes of business with a good spread of risks is likely to be exposed to less overall uncertainty than one that writes limited classes of business in a limited market.

You will probably already be familiar with the principle of diversification of investments. Holding equities, fixed interest bonds and property is usually considered less risky than just holding property. This principle can also be applied to the liabilities too. An insurer that writes lots of different classes of business is exposed to less risk compared with an insurer that concentrates on one class.

6 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Anti-selection
- Average
- Claim amount distribution
- Claim cost inflation
- Claim frequency
- Claim frequency distribution
- Claims made policy
- Claims reported
- Commercial lines
- Discovery period
- Excess
- Exclusion
- Expiry date
- Exposure
- Exposure unit / measure
- Inception date
- Liability insurance
- Moral hazard
- Nil claim
- Peril
- Personal lines
- Salvage
- Subrogation
- Sunset clause
- Uberrima fides
- Underinsurance
- Underwriter
- Underwriting
- Underwriting factor
- Zero claim.

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Chapter 2 Summary

General insurance principles

For a risk to be insurable the policyholder should have an interest in the risk, it should be quantifiable and the claim amount payable should be commensurate with the size of the financial loss.

Ideally, risk events should:

- be independent
- have low probability of occurring
- be pooled with similar risks
- have an ultimate liability
- avoid moral hazards.

In addition, there should be sufficient data to enable the insurer to estimate the size of the risk and likelihood of occurrence.

Principles underpinning an insurance contract include assessing the insurable risk, the details of the proposal form and contract itself, identifying an insurable interest and *uberrima fides*.

Exclusions are used to avoid payments by the insurer for a variety of reasons.

Types of product

The generic types of general insurance cover:

- liability to third parties
- property damage
- financial loss
- fixed benefits.

Cover provided

The benefits usually aim to indemnify the insured for any financial losses suffered as a result of an insured event, although fixed benefits may sometimes be provided. A peril is a type of event that may cause losses, *eg* theft, flood. Policies may be written on a losses-occurring or a claims-made basis.

The exposure measure is the principal measure of risk for an individual policy. Premiums will be set according to the measure of exposure.

The claims characteristics refer to the ways in which and speed with which they originate, are notified, are settled and paid and are, on occasion, reopened. Claim frequency and amount are also relevant. Claims characteristics vary by class.

Risk factors are any factors that have a bearing on the amount of risk. *Rating factors* are factors that are actually used in the premium rating process. Rating factors are either measurable risk factors or proxies for the underlying risk factors (*ie* where it is not practical to use the true risk factor).

Underwriting factors are rating factors, together with subjective factors that cannot be measured, but will still be taken into account when setting premiums / policy conditions.

Selection against the insurer may occur where an insurer's premium rating structure does not reflect the underlying risks, especially if premiums differ from those offered by the rest of the market.

How risk varies by class

Some classes of business cause insurers more risk and uncertainty than others because of the nature of the risks involved and the claims that can arise from those risks. Factors that affect the level of risk and uncertainty include:

- homogeneity of risks
- non-independence of risks
- changing risks
- numbers of claims
- claim cost
- claim inflation
- delay patterns
- variability of experience
- accumulations
- fraudulent claims.

Capital requirements and solvency

General insurers will be required to hold capital over and above their technical reserves. This capital is known as solvency capital.

Generally, the capital requirements will be larger for a class with a greater uncertainty and variability in its future claims experience.

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Chapter 2 Solutions

Solution 2.1

Moral hazard is defined in the Glossary as the risk that an insured may act differently because of being insured, *ie* the policyholder may become less risk averse. For example, a policyholder may start to leave spare keys under the doormat after taking out household contents insurance because they feel less concerned about possible adverse consequences.

Solution 2.2

An insurer will base its premium rates on the expected claim amounts. This will take into account the expected frequency of claims and the expected size of the claims.

The higher the sum insured the higher the expected size of claims. Therefore if a policyholder quotes a sum insured lower than the actual value of the contents, there is a risk that the premiums will be inadequate.

Solution 2.3

- nuclear or radio-active risks
- earthquakes
- unseaworthiness of vessels
- loss of money and documents

Solution 2.4

- certain specified uses, *eg* for business (unless explicitly allowed), racing
- depreciation, wear and tear, or damage to car tyres
- where there was an element of illegality, *eg* the driver did not hold a driving licence or was under the influence of alcohol or drugs
- insured's personal accident benefits where the insured is very old
- losses arising in consequence of earthquakes, war, riot or civil commotion

Solution 2.5

Exclusions might be used:

- to avoid payment by the insurer in situations where:
  - the policyholder is at an advantage through possessing greater personal information about the likelihood of a claim
  - the claim event is largely under the control of the policyholder
  - the claim event would be very difficult to verify
  - loss occurs as part of the normal course of events, and could be considered to be depreciation
- where the risk cannot be reliably estimated by the insurer, regardless of whether or not the policyholder has better information
- when the probability of loss is very high
- the risk is covered by a third party such as the government
- to limit the scope of the policy to make it more appropriate for a particular target market
- to reduce the premium for competitive reasons
- to reduce the risk of moral hazard and fraud.

Solution 2.6

Cover may be provided in respect of visitors' belongings that may for example be damaged in a fire. Also the insured may be indemnified for personal liability arising out of accidents to members of the public somehow caused by the property of the insured, *eg* a garden fence falling on a pedestrian.

Solution 2.7

Other than motor (which is not usually included in "package policies", a small retailer might require cover to:

- compensate employees for accidents occurring due to negligence of the employer (employers' liability and personal accident)
- compensate third parties, *eg* customers, for accidents occurring in the shop / on the shop premises (public liability)
- protect against damage to the shop (commercial buildings cover)
- compensate for lost revenues should the shop be unable to trade (business interruption cover)

- protect against loss of or damage to stocks while in the shop (moveable property (contents) cover)
- protect against loss of or damage to stocks while in transit (goods in transit cover)
- protect against bad debts by suppliers of goods / materials (credit insurance)
- protect against bad debt by customers buying goods on trade credit (credit insurance)
- protect against financial losses due to dishonest actions of employees (fidelity guarantee insurance)
- pay any legal expenses as a result of legal proceeding being initiated against the shop or the shop needing to initiate legal expenses against another party (legal expenses cover).

Don't worry if you didn't get all (or even nearly all!) of these points. However, hopefully by the time you've finished reading the next chapter, it won't seem like such a tall order!

Solution 2.8

Loss of luggage whilst on holiday

Your insurance is likely to cover you for the cost of the luggage, subject to any limits that may apply. This is an example of the benefit being money to cover financial loss as a result of an insured event.

Loss of a finger

Your insurance is likely to provide a fixed benefit that is intended to go (at least) some way towards compensating you for the loss. It is usually impossible to quantify exactly how much the payment should be.
Loss (to vehicle and self) as a result of a car accident

Your insurance is likely to cover you for the cost of damage to your vehicle and will also compensate you for your discomfort or inconvenience. The former should be fairly straightforward to determine (as it is should cover the financial loss incurred). The latter may be more complicated: the insurance policy may specify the amount of the payment, which would depend upon the type of injury, or alternatively the payment may have to be determined by the courts.

Solution 2.9

(a) The accident occurred when the individual was insured by Cars 'R' Us. Therefore for a losses-occurring basis, the individual should contact Cars 'R' Us to report the claim.

(b) The individual is reporting the claim while the insurance cover is being provided by Cars 4 You. Therefore for a claims-made basis, the individual should contact Cars 4 You to report the claim.

A losses-occurring basis seems more sensible, as this is not affected by when the individual chooses to report the claim. In other words, under this basis, the individual cannot *choose* which insurer should pay the claim.

Solution 2.10

A claims-made basis may be appropriate when it is not clear when the loss actually occurred. This might be true for certain types of liability classes, where the loss emerges gradually over time, *eg* deafness caused by continual exposure to loud noises at work under an employers' liability product.

Solution 2.11

There is the potential for a gap in the cover. For example, suppose, as an employer, we switch on 1/1/09 from a policy on a "claims-made" basis to a policy on a "losses-occurring" basis. Then on 2/1/09 we discover that our foreman, Bert, who suffered an injury at work last year which appeared to have completely healed, has had a major relapse and is instituting proceedings for compensation.

We cannot claim for this under our current policy, since the event occurred last year. We cannot claim for it under our previous policy either, because the claim was not reported last year.

Solution 2.12

Yes. If insurance is optional, policyholders may select against the insurance market. Those least likely to claim may choose not to purchase insurance and vice versa.

Solution 2.13

Premiums. The total premium should be a good indicator of the total risk on a portfolio of business.

Solution 2.14

Probably not. The reasons why include:

- The risk does not increase linearly with the exposure measure. For example, a child whose height is $110cm$ is no more likely to hit their head than one whose height is $90cm$ (and possibly less likely, if they are a couple of years older).
- It is not very practical. It would not be easy to verify (cheaply) and will be continuously increasing for the young.
- A better measure of exposure is probably person-year. It is much more practical and when all other factors are constant, it will be proportional to the expected claim amount.

Solution 2.15

(a) An exposure measure should be:

- a good measure of the amount of risk (*eg* doubling the exposure measure should double the amount of risk)
- measurable
- easily obtainable and objectively verifiable
- not capable of manipulation.

(b) A *risk* factor is a factor that affects the level of risk for a particular policy.

A *rating* factor is a factor used in the rating process, either because it is a measurable risk factor or because it is a proxy for a risk factor.

(c) Rating factors are needed because different policies have different levels of risk and because the exposure measure is rarely good enough by itself to gauge the level of risk.

(d) An *underwriting factor* is one that is used to determine the premium, terms and conditions for a policy. It may be a rating factor or some other risk factor that is accounted for in a subjective manner by the underwriter. Remember that rating factors must be measurable, verifiable *etc*.

Solution 2.16

- the potential claim amount is very small
- risks are very homogeneous, so standard book rates apply to all risks

Solution 2.17

Manufacturing companies are unlikely to change the nature of their business markedly during any given year, although in the longer term they may do so. The value of the stocks and output may vary, however, depending on the economic climate and the time of year.

Overall, the risk will be fairly stable unless stock levels are significant and variable.

Solution 2.18

Not quite. Many liability claims involve property damage only (for example, if you accidentally scrape against somebody else's car in a car park). These claims will also be relatively short tailed (although there may be some delay whilst you argue whose fault it was). The student has therefore confused the term *liability* with *bodily injury*, a common mistake. This will be discussed in more detail in Chapter 3.

Solution 2.19

Who knows what you may have dreamt up! Doing unspeakable things to the body, or family, of anyone you catch probably wouldn't go down too well. Things that insurers have tried include:

- making the policy wording as tight as possible, and reviewing it regularly in the light of market and judicial changes
- devising contracts which minimise the risks, *eg* minimum and indexed sums insured for household contents policies
- working together with others to try to identify and punish persistent offenders
- random spot checks on claims, even smaller ones
- having repairs done by a small number of approved firms (rather than at the choice of the claimant)
- insisting on the police being involved before paying out on a theft claim
- publicity to advise against it, *eg* "It's a crime to ..." and "Look what happened to this fraudster ...".

Solution 2.20

- it may have to by law
- to guard against catastrophes / accumulations / latent claims
- to guard against lower than expected investment return
- to give more investment freedom
- to guard against higher than expected expense or expense inflation
- to guard against failure of a third party, for example a reinsurer or broker
- to reduce the need for reinsurance
- to attract new business
- to demonstrate financial strength, *eg* to regulators, shareholders, analysts

Solution 2.21

employers' liability, motor liability, household contents

Motor liability has a longer tail than household contents because of the occasional bodily injury claim.

Chapter 3

Insurance products – types

Syllabus objective

(b)(i) Describe the main types of general insurance product in terms of:

- *the needs of customers*
- *the financial and other risks they pose for the general insurer including their capital requirements and possible effect on solvency.*

Covered in part in this chapter.

0 Introduction

The range of general insurance products is very wide and continually changing and therefore it is difficult to set out the features of all types of product.

The sections below provide a general indication and examples of the knowledge that examiners would expect a candidate to have in relation to the features of the major types of general insurance product. However, the examiners will also expect candidates to apply this knowledge to any other products that may exist.

This chapter is not intended to be an exhaustive description of all the main products available. However, it should give you:

- a sound introduction to the main product types
- a good understanding of the framework within which any other product could be analysed.

In this chapter, the products are discussed under the headings of the main types of cover that they provide. The candidate will need to bear in mind the possible effects of combining different types of cover within a single policy. For example, the risk factors relevant to the liability cover will not necessarily be the same as those that are relevant to the cover for damage to property.

This chapter concludes with a discussion of how products may be combined or "packaged" up.

As well as being a sound introduction, this chapter will be very useful to you in your revision.

This chapter is very long. You may therefore wish to use two or three study sessions to work through it.

The examiners do not only examine the "basic" products such as motor and household. Sometimes, more unusual products are examined. If you would like to know more about some of the more unusual products, you may want to have a look at the following book:

Insurance Law Handbook (4th edition)
ISBN: 9781847660930
By Barlow Lyde and Gilbert LLP
To order, call Tottel Publishing on +44 (0)1444 416119

This textbook is part of the recommended reading for the Chartered Insurance Institute. It gives a comprehensive factual introduction to the legal and coverage sides of many classes of insurance, and so would be useful for students who wish to find out more about the more "unusual" classes of business. It also covers other issues such as insurance markets (*eg* Lloyd's) and regulation. However, the book is quite UK-specific and is not part of the Core Reading for this subject.

1 Liability

The essential characteristic of liability insurance is providing indemnity where the insured, owing to some form of tort (private or civil wrong, such as negligence), is legally liable to pay compensation to a third party.

The main types of liability insurance are:

- **employers' liability / workers' compensation**
- **motor third party liability**
- **marine and aviation liability**
- **public liability – often linked to other types of insurance such as property and marine**
- **product liability**
- **professional indemnity**
- **Directors' and Officers' (D&O) liability**
- **environmental liability.**

(Motor third party liability is a particular type of public liability insurance, but is treated separately because of its importance in personal lines business.)

Other forms of liability cover are variations of one or more of these types.

> ***Question 3.1***
>
> Give an example of the sort of negligence that you think may result in a claim under a professional indemnity contract.

> ***Question 3.2***
>
> Suggest what Directors' and Officers' insurance might cover.

The extent of any legal liability may depend on the prevailing legislation. For marine and aviation liability, international law is likely to prevail. For classes such as motor and employers' liability, national laws are likely to apply.

1.1 Benefits

The basic benefit provided by any liability insurance is an amount to indemnify the policyholder fully against a financial loss. However, subject to any statutory requirements, this benefit may be restricted by:

- **a maximum indemnity per claim or per event (this may involve more than one claim), or an aggregate maximum per year**
- **an excess.**

Subject to the details of any reinstatement clause, payment of any benefits may result in a cancellation of cover or the need for a further premium.

Question 3.3

What are the aims of having an excess?

Any legal expenses relating to such liability are usually also covered. (Note that an illegal act of negligence will often invalidate the cover.)

Example

An insurer is unlikely to pay out compensation if the insured caused a crash whilst drink driving or whilst driving a stolen car. However, the insurer would pay out if the crash was due to a punctured tyre or just a complete accident.

Employers' liability

This insurance indemnifies the insured against legal liability to compensate an employee or his or her estate for bodily injury, disease or death suffered, owing to negligence of the employer, in the course of employment. Loss of or damage to employees' property is usually also covered.

Employers are liable if they are negligent in providing their employees with safe working conditions. Generally speaking the employer would be liable if failing to:

- provide a proper working place with proper equipment in which the employee can work,
- properly maintain the working place as well as tools and equipment, and
- create and enforce proper working procedures and methods.

The benefit can be in the form of regular payments to compensate for disabilities that reduce the employee's ability to work, lump sum payments to compensate for permanent injuries to the employee, and benefits under the legal framework.

Legal costs will also be covered. Other costs such as care costs can also be included.

Compulsory cover

In many countries cover is compulsory.

> ***Example***
>
> **The Employers' Liability (Compulsory Insurance) Act 1969 requires that every employer in the UK (except some bodies such as local authorities) must maintain, with an authorised insurer, an employers' liability insurance policy covering all its employees. The policy must have a minimum level of indemnity of £5 million per event, but will usually provide a higher level of cover. Most authorised insurers will provide cover of at least £10 million.**

Most countries have similar requirements to the UK, although the details vary from country to country.

The most important distinction is between countries that have a system of employers' liability (in which losses must arise from the employer's negligence if they are to form the basis of compensation), and workers' compensation (in which losses merely have to be suffered in the course of employment).

In some countries, for example Australia, employers' liability insurance is more frequently referred to as workers' compensation insurance.

Motor third party liability

This insurance indemnifies the owner of a motor vehicle against compensation payable to third parties for personal injury or damage to their property.

The benefits will include compensation for loss of earnings, hospital costs and damage to property costs, and can be paid in a lump sum or periodically to the injured party.

Compulsory cover

In most countries such cover is compulsory, although precise rules vary, for example in the amount of cover required. The cover provided may or may not be limited to that required by legislation.

Example

Third-party liability cover is required for all motor vehicles in the UK, under the Road Traffic Act. The cover must be unlimited in relation to personal injury claims, but is required only up to £250,000 for damage to a third party's property.

Since motor liability insurance is a legal requirement for vehicle owners in most countries, policies that provide liability cover only are widely available.

In South Africa, motor liability insurance is not compulsory unless the car has been bought on credit, in which case comprehensive insurance is compulsory.

Marine and aviation liability

The insured is indemnified against the legal liability to compensate a third party for bodily injury, death or damage to property arising out of operation of the vessel or aircraft. The third parties include, but are not limited to, passengers.

Various additional covers have often been included in aviation insurance, including:

- **manufacturers' hull and liability (covers accidents involving aircraft in the possession of the manufacturer, for example test flights)**
- **grounding risks (business interruption due to the grounding of a fleet for safety reasons)**
- **hangarkeepers (liability arising from damage to aircraft / property in the insured's care, custody or control).**

A large proportion of marine liability insurance is provided not by commercial insurance companies but through Protection and Indemnity Clubs. These are mutual associations of ship owners that provide cover for their members on a pooled basis.

Protection and Indemnity Clubs are described in Chapter 7

Public liability

The insured is indemnified against legal liability for the death of or bodily injury to a third party or for damage to property belonging to a third party, other than those liabilities covered by other liability insurance.

Example

Suppose you are a recently qualified actuary, married with young children, walking past a construction site underneath the covered pedestrian pavement. Despite the overhead protection, a brick falls through and knocks you on the head, causing instant death. Your dependants could sue the building contractor for their loss of future support because of your death as both spouse and parent. The contractor would then rely on public liability insurance to meet the claim.

Compulsory cover

In some countries – and for certain individuals or institutions – public liability is compulsory.

Example

Statute requires public liability policies to be held by horse riding establishments and nuclear installations in the UK. However, there is no general requirement in the UK for individuals or businesses to hold public liability insurance.

There are two main types of cover, namely:

- **the risk at insured's own premises (*eg* a commercial premises such as a warehouse)**
- **the risk when work is carried out by the insured away from their own premises (*eg* on the site where builders are working, away from their official place of work).**

Product liability

This insurance indemnifies the insured against legal liability for the death of or bodily injury to a third party, or for damage to property belonging to a third party, that results from a product fault.

The policy will usually also cover legal costs. Some policies will include the costs of recalling faulty products that have not actually caused damage.

Professional indemnity

This insurance indemnifies the insured against legal liability for losses resulting from negligence in the provision of a service, for example unsatisfactory medical treatment or incorrect advice from an actuary or solicitor. The insured will be a professional person or a professional firm.

Like companies selling products, enterprises offering a service may also be concerned to protect themselves against possible liability claims.

In a company take-over, for example, considerable weight may be given to professional reports from accountants, merchant bankers and even, where relevant, actuaries. Negligence by one of the advisors could cause their client to suffer large losses.

The USA is notorious for the high cost of professional indemnity cover for medical doctors, because they can be sued for millions of dollars by patients who are not satisfied with treatment.

There are several types of professional indemnity insurance sold, including Directors' and Officers' liability (D&O) and Errors and Omissions cover (E&O). The fundamental cover is the same – the difference is due to the parties that buy the cover. E&O is typically sold to professional services firms, whereas professional indemnity cover is wider-reaching.

Furthermore, E&O is primarily concerned with performance failures and negligence with respect to products and services, whereas D&O is concerned with the performance and duties of management.

Compulsory cover

Holding professional indemnity insurance is often a legal or regulatory condition of being allowed to practise a profession or may be imposed as a condition by a professional body.

Directors' and Officers' liability

D&O insurance indemnifies the insured against the legal liability to compensate third parties owing to any wrongful act of the insured in his or her capacity as a director or officer of a company. The insurance is personal to the director or officer, but is usually bought for him or her by the company.

Deliberate fraud by directors and officers will typically be excluded.

Environmental liability

The insured is indemnified against the legal liability to compensate third parties as a result of bodily injury, death and damage to property as a result of unintentional pollution for which the insured is deemed responsible.

The costs of cleaning up the pollution and regulatory fines may also be covered. Gradual and sudden environmental pollution will generally both be covered.

1.2 Insured perils

Employers' liability

The perils can be largely grouped into the following:

- **accidents caused by the negligence of the employer or by other employees**
- **exposure to harmful substances (*eg* chemicals, coal dust, asbestos)**
- **exposure to harmful working conditions (*eg* loud noises or repetitive strain).**

At any given time there is often a particular cause which gives rise to a large number of claims, *eg* asbestosis, industrial deafness, repetitive strain illness (RSI) and stress have each been common.

Motor third party liability

The perils for motor third party can be grouped into the following categories:

- loss or damage to the property of third parties caused by the insured vehicle
- bodily injury and death of third parties caused by the insured vehicle.

Marine and aviation liability

The perils can be grouped into the following:

- **loss of or damage to passengers' property, including luggage**
- **bodily injury and death of passengers either while on board the vessel or aircraft or when boarding or leaving the aircraft**
- **bodily injury caused by the vessel or aircraft**
- **damage to property caused by the vessel or aircraft.**

There are likely to be a number of exclusions on aircraft liability insurance, including perils such as terrorism, war and illegal activities, although some of these may be covered under extensions to cover and may be a legal requirement in some countries.

In December 2007, US law was changed to provide a transparent system of compensation for insured losses resulting from acts of terrorism. Under the law, compensation would be shared between the public and private sector.

The insurance market in the US offers terrorism insurance coverage to the majority of large companies that request it. Depending on the required level of coverage and the location of the company, the cover may be expensive.

In France, it is mandatory for insurers to include terrorism coverage when insuring commercial risks.

Public liability

As this type of insurance forms part of many types of insurance policy, the insured perils will relate to the type of policy. For example, compensation for a dog bite may be covered by a household policy, while compensation for injury from a falling object may be covered by a commercial policy held by a builder.

In general the policy will not be restricted to named perils, although some perils may be excluded.

Product liability

Here the perils depend greatly on the nature of the product being produced, but include:

- **faulty design**
- **faulty manufacture**
- **faulty packaging**
- **incorrect or misleading instructions.**

Examples could include:

- An electrical appliance with faulty wiring could injure people using it.
- A pharmaceutical company concerned about future unwanted side effects emerging in those taking a new drug. Two real life examples are:
  - Thalidomide – in the early 1960s this sedative, known for its apparent lack of toxicity was found to relieve the symptoms of morning sickness, and so was given to pregnant women. The drug was later found to have caused the arrested development of arms and legs in thousands of babies.
  - Opren (also known as Benoxaprophen) was prescribed in the early 1980s to relieve the symptoms of rheumatoid arthritis. Dozens of elderly patients died as a result of its unpleasant side effects.

Professional indemnity

The perils here depend on the profession of the insured. Examples include:

- **wrong medical diagnosis**
- **error in medical operation**
- **error in an actuarial report.**

Directors' and Officers' liability

The perils include the following:

- **allowing a company to continue operating in circumstances when it should have been declared insolvent**
- **any act resulting in the insured being declared unfit for his or her role**
- **allowing false financial statements to be published.**

Environmental liability

The insured perils can be regarded as any incident causing gradual or sudden environmental pollution.

1.3 Basis for cover

Employers' liability

Most classes of business are written on the basis of when the loss was incurred, *ie* cover relates to the date of the accident rather than the date of reporting the accident. So if a policy provided cover for 2012, say, the insurer would be liable for claims resulting from accidents in 2012.

With employers' liability, this gives rise to a few problems:

- Where an industrial disease results from prolonged exposure, it is hard to define the "accident date". In practice, where the company has changed insurer over the period, the claim is split between the insurers in proportion to the time span each provided cover over the relevant period of exposure.

- Because many diseases take a very long time to develop fully, there have been extensive reporting delays with employers' liability. This makes it more difficult for the insurer to manage the business.

To overcome these difficulties, several insurers devised policies on a claims-made basis whereby the insurer was liable for claims reported in the period of insurance cover. This approach to insurance has led to a number of difficulties (*eg* overlaps in insurance cover), so many policies are still (currently) operating on the original losses-occurring basis.

Note that, in the UK, employers' liability insurance cannot currently be written on a claims-made basis.

Motor third party liability

Cover is usually provided on a losses-occurring basis.

Marine and aviation liability

Policies are usually written on a losses-occurring basis.

Public liability

This is likely to depend on the exact cover being provided, although is likely to be on a losses-occurring basis.

Public liability insurance may be combined with other insurances, for example residential buildings and contents products may include public liability cover. In such cases, the basis for cover would be as for the other insurance cover provided.

Product liability

Policies are likely to be written on a claims-made basis.

Question 3.4

Why might a claims-made basis be an appropriate way of writing product liability cover?

Alternatively, an insurer could give cover relating to sales in a given year.

Professional indemnity

Professional indemnity is usually written on a claims-made basis.

Directors' and Officers' liability

D&O liability is usually written on a claims-made basis.

Environmental liability

Policies are likely to be written on a claims-made basis.

1.4 Measures of exposure to which premiums are related

Most policies that cover the various types of commercial liability will be underwritten on the basis of the perceived risk with the premium adjusted in the light of the insured's turnover.

Question 3.5

(i) List the different types of *commercial liability* insurance.

(ii) Which types of liability cover may be written as *personal lines* business?

Employers' liability

Person-hours worked could be the best exposure measure when assessing claim frequency. However, claim settlements are often related to loss of earnings so, in practice:

The main measure of exposure for employers' liability is payroll, or total wage and salary costs.

Question 3.6

Why?

The premium rate for an employer will be agreed when a policy is taken out and a premium will be paid on the basis of the estimated payroll for the year. At the end of the year an adjustment premium will be paid or refunded on the basis of the actual total payroll for the year.

This is because the annual payroll cannot usually be determined accurately in advance.

Motor third party liability

For motor third party liability the measure of exposure is the vehicle-year. In other words an agreed monetary premium is charged for the insurance of a single car for a year.

Marine and aviation liability

For marine and aviation liability insurance alternative measure of exposure can include:

- **passenger kilometres**
- **passenger voyages**
- **in-service seats**
- **in-service vessels / aircraft.**

However, turnover is still the most commonly used measure, although it is not an accurate measure of the risk.

> ***Question 3.7***
>
> Why not?
>
> *Hint: Recall that marine and aviation liability is usually written on a losses-occurring basis.*

For example, a manufacturer could have zero turnover in a policy year and yet still have significant exposure as coverage is based on accident year (*ie* on a losses-occurring basis), **irrespective of when the craft was made or sold. Therefore previous years' turnover can give rise to claims on the current year's policy.**

Some underwriters see past production as a key exposure measure. A better measure might be rolling average turnover. For example, if a craft or vessel spends an average of 20 years in service, the average turnover in the last 20 years might be used as an exposure measure.

Or the historical annual turnover could be weighted by a risk profile. This could be derived from the age of craft or vessels in accidents.

The main problem with these exposure measures is their lack of uniformity. For example, a jet engine as one of four on a wide-bodied airliner represents a very different risk to a turboprop engine on a cargo aircraft. All of the exposure measures used need large adjustments derived from rating factors before they can be of any use for rating.

Public liability

The most commonly used measure of exposure is turnover.

An alternative is payroll. The argument for this is that if you are a furniture maker paying £200,000 *pa* in wages you would present a far greater public liability risk than if you paid wages of only £50,000 *pa* because you will probably have a larger workforce.

Turnover is sometimes considered a better measure since the relative cost of materials and equipment used is also indicative of the public liability risk.

For specific types of policies, such as for hospitals' public liability, the number of beds may be a more appropriate measure.

Product liability

The most commonly used measure of exposure is turnover.

Despite the problems with heterogeneity, annual turnover of the company is normally used because no better alternative exists.

Professional indemnity

The most commonly used measure of exposure is turnover. However, alternatives can be used in some situations such as amount of funds under management.

This would be appropriate if the insured is a fund manager.

Directors' and Officers' liability

As a type of professional indemnity insurance, the exposure methods will be similar, *eg* turnover. Another measure that may be used is net assets and liabilities.

Environmental liability

The measure of exposure will depend heavily on the nature of the industry carried out by the insured. For example, the number of power stations or the amount of energy generated could be used for a power company.

Alternatively a measure related to the value of the land may be used.

1.5 *Claim characteristics*

Several issues need to be settled before a liability can be settled:

- **whether or not there has been a loss**
- **whether or not the insured is liable**
- **the quantum (amount) of the loss.**

The loss may also be reduced if there is contributory negligence on the part of the victim.

Reporting delays

Reporting delays might arise because, say, an injury gets worse over time whereas the sufferer had expected the problem to be only temporary, or because the seriousness of the condition only comes to light in a later routine check-up.

Some liability claims remain undetected for some years. This means that loss may not become evident until many years after it was caused. A classic example of this is mesothelioma – a disease that arises from exposure to asbestos – where the first symptoms are usually not evident for at least twenty years after exposure and sometimes not for fifty years or more. This exacerbates the long-tailed nature of the associated liability insurance if it was written on a losses-occurring basis.

Question 3.8

Would this exacerbate the long-tailed nature of the associated liability insurance if it was written on a claims-made basis?

A further complication is that claims may arise over a long period; for example a storage tank may leak contents into the ground, causing pollution, for some years before the damage is discovered, and typically those with asbestos-related diseases were exposed to asbestos for many years before their condition became manifest. This means that it may not be clear which insurance should respond to the loss, and often some form of allocation is necessary.

Settlement delays

Settlement delays may be due to the delays caused by establishing liability, or the time to establish the extent of injuries and to assess the speed of recovery.

If the loss is personal injury, it may take some years before the victim's condition has stabilised and can be assessed for damages. Claim settlement will involve litigation for any claim of significant size, even though most cases are resolved out of court before they go to trial.

Settlement for motor liability **is often faster than for other types of liability claims, although claims arising from serious bodily injury do typically take several years to settle. Even so, motor liability claims, and particularly bodily injury claims, are significantly longer tailed than other motor claims.**

The legal environment has a significant effect on the settlement of claims, and this varies from country to country. Therefore claims development patterns for similar classes of business may vary significantly between countries.

These characteristics (both reporting and settlement delays) **mean that most liability classes of business are long-tailed, whether the cover is written on a claims-made or a claims-occurring basis.**

Claim frequency

Claim frequencies on liability classes of business tend to be low.

Motor liability claims tend to be more frequent than claims on other liability policies, both for property damage and for bodily injury to third parties.

Amount of the claim

Claim cost distributions tend to be more widely spread than for property classes, and there can be some extremely large individual claims that take many years to settle.

Accumulations of risk arise in many liability classes as a result of an unbalanced portfolio. For example, almost by definition, employers' liability insurance produces a concentration of risk. Because one policy will provide cover for an employer, there will be groups of employees together in the same office or factory so a single event (such as the Piper Alpha oil rig explosion) can lead to many claims.

Accumulations can arise in other ways too:

- If there is a court award in favour of a claimant, it may trigger a large number of similar claims from individuals with the same complaint against their employer.
- An insurer who provides cover for several employers in the same industry is exposed to the possibility that a large number of claims will emerge from a common cause. Previous examples of this are asbestosis, industrial deafness and repetitive strain injury.

1.6 Risk factors and rating factors

Employers' liability and public liability

For employers' liability and public liability, the main factor influencing the risk is the type of industry or the occupation(s) of the insured.

Another important factor is the extent to which the employer implements safety measures.

The underwriting factors are as follows:

- **type of industry or occupation**
- **exposure and claims experience**
- **location of the workforce**
- **frequency of visitors to the site**
- **the materials handled**
- **the processes involved**
- **safety precautions in place, for example sprinklers in a factory or office**
- **turnover**
- **size of deductible***
- payroll
- level of staff training and safety standards (linked to quality of management)
- provision of first aid facilities.

These are factors that underwriters will take into account, some quantitatively, some qualitatively in assessing an appropriate premium. In addition, underwriters should vary the premium according to a general assessment of the risk management standards of the insured.

For very large companies, the previous claims experience will also be relevant.

* A deductible is a portion of a loss that is paid by the policyholder. It may be an amount or a percentage. For example, a 5% deductible will restrict the size of any claim payment to 95% of the loss.

Question 3.9

Give a simple example to illustrate the difference between the operation of an excess and a deductible on an insurance policy.

Motor third party liability

Since motor third party cover is usually provided in the same policy as cover for damage to the insured vehicle, the risk factors and rating factors are discussed in the section below on property damage.

In some European countries motor liability insurance and motor own-damage insurance are sold as separate policies, but the rating factors are likely to be similar.

This is the case in Hungary for example.

Marine and aviation liability

A wide range of different rating factors are used by insurers. Some form the basis of actuarial rating models whereas other, more subjective, rating factors are used by underwriters as part of the rating process.

Some of these are:

- **loss history**
- **type of craft or vessel (as accident rates and types / sizes of claims will vary), for example, jets, turboprop and rotor wing in the case of aircraft**
- **commercial category (commercial, private, military)**
- **satellites and missiles also attract separate rating structures**
- **use of craft or vessel (passenger / cargo, leisure / business)**
- **geographic region (jurisdiction of litigation).**

Product liability

The nature of the products produced by the insured will be an important factor in assessing the level of premium. Other risk factors include:

- **the distribution channel of the product**
- **how much US-exposure the product has – in general, products sold in the US show a different claims distribution** (since there tends to be a different attitude to claiming), **and more claims are likely to be made**
- **its usage**
- the general trade of policyholder
- any potentially dangerous components within the product, and how quickly they deteriorate.

However, each policy will be underwritten individually, and the underwriter will need to assess the risks arising from a policy subjectively.

Professional indemnity

The nature of the profession will be an important factor in assessing the level of premium. However, each policy will be underwritten individually, and the underwriter will need to assess the risks arising from a policy subjectively.

The risks may also vary between size of professional firm, with sole practitioners presenting a very different risk profile from large professional-services firms.

Directors' and Officers' liability

The nature of the company will be an important factor in assessing the level of premium. However, to some extent each policy will be underwritten individually, and the underwriter will need to assess the risks arising from a policy subjectively.

Other than the type of business, typical risk / rating factors are past experience of the company and the state of the economy.

Environmental liability

Each policy will be underwritten individually, and the underwriter will need to assess the risks arising from a policy subjectively.

This will take into account:

- **the processes carried out by the insured**
- **the likely effects of any accident**
- **the likely cost of clean-up**
- **a general assessment of the risk management practices of the insured.**

> ***Question 3.10***
>
> You propose to market a new type of policy to small shopkeepers selling general groceries and other goods in suburban neighbourhoods. You want to develop a package that will cover all of their insurance needs. What types of peril would your package cover?

2 Property damage

The main characteristic of property damage insurance is indemnifying the policyholder. However, here the indemnity is against loss of or damage to policyholders' *own* material property.

The main types of property that are subject to such damage are:

- **residential buildings** (for example, house)
- **commercial and industrial buildings (for example, offices, shops and factories)**
- **moveable property (for example, the contents of a home or of commercial premises)**
- **land vehicles (for example, cars, buses, taxis)**
- **marine craft**
- **aircraft**
- **goods in transit**
- **construction**
- **engineering plant and machinery**
- **extended warranty**
- crops.

Land vehicles can be further divided into:

- private motor
- commercial vehicle
- motorcycle
- motor fleet.

Policies are commonly available under these sub-headings.

A motor fleet policy provides cover for a number of different vehicles belonging to an individual owner, *eg* a company. A small fleet might have five vehicles in it, a large fleet could have 500 or more.

A motor fleet may extend beyond cars to include other types of commercial vehicle (*eg* lorries and vans). In these cases, the fleets may be very heterogeneous.

Marine, aviation and goods in transit are often labelled as "MAT": Marine, Aviation and Transport.

2.1 Benefits

The benefit is usually the amount needed to indemnify the insured against the value of the loss or damage, subject to any limits or excesses.

Household buildings property

For buildings insurance, the benefit, by the principle of indemnity, will be the amount required to fully reinstate the property, rather than the market value of the property.

The amount paid under the policy is usually the amount needed to restore it to its previous condition, subject to any excess or deductible. Ancillary costs, such as the provision of alternative accommodation while the insured property is uninhabitable, may also be covered, but are not standard.

The policy will define precisely what is included within the definition of buildings. For example, the fixtures and fittings, patio, drive, swimming pool, walls, fences and gates may all be included with the building covered by a household policy.

Household policies normally include insurance for liabilities arising from the insured property.

For example, household insurance may also contain public liability cover in respect of the house.

Commercial buildings property

The cover is likely to be for the full value of the property, however, alternatively, it maybe on a *first loss* basis.

First loss is a form of insurance cover for which the chosen sum insured is restricted, with the insurer's agreement, to a figure less than the full reinstatement-as-new value of the property. The insured therefore has to bear any loss in excess of the sum insured.

This may be appropriate in circumstances where:

- the insured considers that a loss in excess of the sum insured is extremely unlikely
- the item is effectively priceless, *eg* a stately home, for which there may be no possibility of the building being reinstated
- the insurance is against water damage (where only the ground floor areas may be affected).

Moveable property

For buildings, the sum insured is based on the total cost of rebuilding the property. For contents, it is based on the value of the contents.

The policy will define precisely what moveable property is covered by the insurance. For example, under a household contents policy, the insured's household goods and personal possessions plus visitors' personal effects.

Household policies may have a number of extensions of cover, which may be optional or may be used by companies to distinguish their own products; examples of these would include food in a freezer spoiled during a power cut, pet insurance (which is also widely available as a separate product), bicycles and the contents of sheds.

The amount paid on a claim can be:

- **the replacement value, which is the cost of a new item reduced to allow for the depreciation on the lost item, or**
- **on a "new for old" basis, under which the cost of an equivalent brand new item is provided.**

The insurer may retain the right to provide a replacement item rather than provide monetary compensation.

New for old cover would not be normal in commercial policies but is common in household policies.

For commercial contents the cover is usually for pure indemnity, *ie* the full depreciated value of the contents is covered. As with household insurance, the insurer may apply the principle of average to scale down claim amounts in cases of underinsurance.

The effect of inflation

Insurers believe that, during times of high inflation, many policyholders become underinsured because they fail to increase the total sum insured sufficiently from year to year. To help prevent this, many insurers have introduced policies with automatic indexation of the sum insured. Each year the level of cover is automatically increased in line with an appropriate inflation index. The inflation index should reflect the increased cost in claims (*eg* an index of goods for contents and an index of building cost inflation for buildings insurance). The indexation facility is detailed in the *escalation clause*.

Motor property

The three main levels, or types, of cover usually available are:

(a) third party

(b) third party, fire and theft

(c) comprehensive.

However, in some countries there may be slightly differing levels of cover to this.

The most basic level is *third party* insurance described in Section 1. This basic level of cover is not commonly used.

The next level of cover is *third party, fire and theft* (TPF&T). This provides the same cover as third party insurance plus:

- losses or damage caused to the insured's vehicle from fire and/or theft.

The highest level of cover is *comprehensive*. This gives the cover provided by third party, fire and theft, and additionally:

- accidental or malicious damage to the insured's own car.

The maximum benefit is the depreciated value of the vehicle. In most cases of damage the insurer will pay to have the vehicle repaired.

The vehicle is insured for its replacement value. However, if repair costs exceed the market value of the vehicle at the time of an incident, it will be declared a complete "write-off" and only the second-hand market value at the time of the accident will be paid to the insured.

Comprehensive policies will also usually include cover for:

- specific injuries to the insured and/or spouse usually a defined payment (*eg* £2,500)
- personal medical expenses (up to a defined amount, *eg* £150 per person)
- the insured's legal expenses in relation to any claim
- loss or damage to personal effects that were in the car (up to a defined amount, *eg* £100)
- replacement of locks after theft of the car
- replacement / repair of broken windscreens.

Marine and aviation property

There is a wide variety of types of cover available for marine insurance risks. However, the principal forms of insurance likely to be needed by a ship-owner are as follows:

- loss of or damage to the craft (often called "hull insurance")
- loss of or damage to cargo ("cargo insurance" is for the actual contents, "freight" covers the money payable for shipment of the cargo).

In addition, other types of insurance cover could be included, such as:

- compensation for loss of use of the craft
- insurance against damage to a vessel still under construction.

In marine property insurance, definitions are used to define the extent of the loss. In particular, the Marine Insurance Act defines *actual total loss* and *constructive total loss*:

Actual total loss is deemed to occur in one of three ways:

(1) where the insured item is totally destroyed

(2) where it is so damaged that it can no longer be classed as the type of object originally insured

(3) where the insured is irretrievably deprived of the insured item.

Constructive total loss is where the insured abandons the insured item because an "actual total loss" is unavoidable or because the costs of preventing a total loss exceed the value saved.

Goods in transit

This is a commercial insurance cover against loss of or damage to goods whilst being transported in vehicles specified in the policy (*eg* the company's vehicles or perhaps also by a carrier). The periods of loading and unloading are usually covered in addition to the journey. Losses due to business interruption are not usually included and a special extension is usually required if livestock are to be covered. The sum insured is likely to be the value of the goods.

Construction and engineering

Large construction and engineering **projects can take several years, and the associated policies will last until the end of the project. Since projects often overrun their schedules the policies are usually extendible. A policy for a large project may continue for some years after its completion to cover the late discovery of construction faults.**

Extended warranty

Extended warranty insurance covers losses arising from the need to replace or repair faulty parts in a product (usually electrical goods, furniture or motor vehicles) beyond the manufacturer's normal warranty period. Policies may have a term of several years.

Crop insurance

This will indemnify the insured (*eg* a farmer or orchard owner) against losses to the crop due to specified perils.

2.2 Insured perils

Household buildings property

In respect of buildings, fire is the principal peril insured against but policies can cover many other perils such as explosion, lightning, theft, storm and flood.

Question 3.11

Explain why theft is listed as an insured peril on a buildings policy despite it being very unlikely that someone will steal your house.

Subsidence is a peril that is commonly covered under household insurance.

Subsidence has become a fairly big problem in the UK.

Damage to the insured property caused by measures taken to put out a fire is also covered.

More examples are:

- earthquake (not so important in the UK, but vital in many countries)
- burst pipes (*eg* damage caused by water or oil leaking from pipes)
- vandalism
- impact (*eg* from falling branches, aircraft, cars, stampeding animals).

Commercial buildings property

Commercial buildings will be subject to the same sorts of perils as household buildings, *eg* fire, explosion, lightning, theft, storm and flood.

Commercial property insurance is often referred to as "fire insurance", but conventionally covers other perils as well, such as weather-related losses and malicious damage. A reference to "fire" as an insurance class should not be taken to mean that the policy is limited to fire risks unless the context clearly indicates that it is, but rather it should be assumed to include other property damage perils.

"All-risks" policies are also common, especially for household policies; these cover all causes of damage that are not explicitly excluded.

Moveable property

As with buildings insurance, the policy will set out the perils covered.

The most important perils covered under property insurance are fire and theft. Malicious damage and damage arising from weather events are usually covered. Accidental damage is often provided under household policies, but often as an optional extra.

Motor property

The perils include accidental or malicious damage to the insured vehicle, and fire or theft of that vehicle.

Marine and aviation property

The following perils relate specifically to marine hull cover, but similar perils are covered for marine cargo and aviation insurance:

- **perils of the seas (or other navigable waters)**
- **fire**
- **explosion**
- **jettison**
- **piracy *etc*.**

Goods in transit

Damage, loss and theft are the main perils for goods in transit.

Construction

The main insured perils are damage, destruction and failure to finish the construction project.

Engineering

Machinery breakdown, explosion and electronic failure are the main insured perils.

Extended warranty

The main peril is faulty manufacture.

Crop insurance

The main perils include disease, fire and weather-related perils (such as storms or drought).

2.3 Basis for cover

Property damage insurance is likely to be written on a losses-occurring basis.

2.4 Measure of exposure to which premiums are related

Household property (buildings and contents)

For household property the exposure is frequently measured in sum insured years: the sum for which a property is insured multiplied by the period at risk.

The sum insured should be the cost of rebuilding the property should it be destroyed, including the clearing away of debris. This is usually less than the market value of the property, since the value of the land itself is not normally impaired by the destruction of the building and the loss of the land, for example through coastal erosion, is not normally an insured risk.

It is generally accepted that the bigger the property, the greater the total amount of risk. Exposure measures are therefore linked to the size of the property. The most commonly used exposure measure, sum insured per year, can be used for both buildings and contents.

Other measures of the size of the risk have been used. For example, some insurers have used number of bedrooms for both buildings and contents. The logic behind this is that both the rebuilding costs and the value of contents are a function of the number of bedrooms.

> ***Question 3.12***
>
> List the perils that might be covered by a home contents insurance policy.

Commercial property (buildings and contents)

For commercial property as with household property, the measure of exposure is usually the sum insured year.

The sum insured should cover the full value of the property, unless first loss cover is intended.

The sum insured for buildings should reflect the cost of rebuilding, not the market value.

However, the sum insured year is complicated for two reasons:

- **The amounts of stocks held may vary considerably over the period of the insurance. Hence the stock may be covered on a declaration basis, determined retrospectively with an adjustment premium.**

- **There is no standard way of allowing for inflation in the policy. Hence policies with different types of inflation treatment need to be considered separately to determine the exposure.**

However, (if first loss cover is used), **the premium for large commercial property policies will often be determined by reference to the maximum amount that the underwriter believes could feasibly be lost in a single incident, either because complete destruction is an event so remote that it need not be provided against or else because the policy covers a number of properties, sufficiently far apart that the possibility of damage to more than one in a single incident is remote. This amount is known variously as the estimated maximum loss (EML) or probable maximum loss (PML).**

Virtually every commercial property is unique, at least in respect of location, use, size and construction. It is therefore almost impossible to group them in a homogeneous manner.

Motor property

For motor insurance, vehicle-miles would probably be the best measure of exposure for damage claims, as the chance of an accident depends on the extent to which the vehicle will be used. However it will be difficult to verify miles travelled as stated by the proposer. Therefore the measure usually used is the vehicle-year, which has the advantage of ease of calculation.

A motor policy that insures one vehicle for one year is exposed for one vehicle-year.
For a motor fleet, the amount of exposure will probably not be available in advance, since the number of vehicles in the fleet will change during the year. At the end of the year an adjustment will be made to the premium for the year just gone to reflect the number of vehicles actually insured.

Example

In 2006, Norwich Union (now Aviva) launched a "Pay as you drive" motor insurance product under which policyholders had a "black box" installed in their vehicle, which logged the details of their journeys – in particular, the distance driven and the time of day.

Policyholders were charged a fixed basic monthly premium, based on the standard motor insurance rating factors (such as age and sex, and vehicle make and model) plus a variable monthly premium based on information provided by the black box. Lower premiums would be charged to those with low mileage clocked up during daylight hours.

Using actual mileage as a measure of exposure (for the variable part of the premium) led to significant premium reductions for many customers.

In 2008, Norwich Union withdrew this product to its private customers, however it is still offered to its fleet customers. More recently, it has decided to consider re-entry into the private market.

Marine and aviation property

For marine property, the insured value of the hull or the value of the cargo might be used as exposure measures.

Similarly, for aviation, the insured value of the aircraft or the value of the goods carried might be used.

Goods in transit

The consignment value is most commonly used. Specific types of cover can exist, for example, relating to specie (high-value items), fine art, jewellery and so on.

Construction

The value of the contract may be used as an exposure measure.

Engineering

The sum insured value or value of the contract may be used as an exposure measure.

For both engineering and construction, the risk is not uniform along the timeline of the project. For example, if the site is destroyed by a storm at the start of the project, little may be lost. Whereas, just before completion, the value of the loss will be much higher (tending towards the rebuild value). During the rating process, the risk profile will be taken into account and a percentage rate will be loaded for this, before it is applied to the exposure measure.

Extended warranty

The number of appliances, or appliance-years (number of appliances times number of years), could be used as an exposure measure.

2.5 Claim characteristics

Property claims are generally reported and paid quickly compared to liability claims. Losses arise from incidents that are observed at the time and the value of the loss is usually straightforward to establish. There are exceptions, and the reinstatement of a major industrial plant after a major incident can be a long project, but generally the property damage classes are among the shorter-tailed classes of insurance. Comments are included below where there are specific features to note.

Household and commercial property

The event giving rise to a claim for damage to buildings or contents usually occurs suddenly (as for fire or burglary) and the cause is easily determinable.

Notification is then also made promptly and a reasonably good estimate of the claim amount can be made. (Subsidence claims are an exception to this.)

Settlement is in many cases by a single payment but larger claims can take longer and may be settled with intermediate payments as the building project to repair or replace the building proceeds. Delays may be greater, however, where it is necessary to verify the value of stock held in a commercial property.

Domestic property claims will tend to be fairly consistent in size and distribution, with a small number of larger, total loss claims. Commercial property claims have similar features, although they tend to have a more scattered cost distribution, owing to the singular nature of the properties insured.

These classes are the ones that are most exposed to moral hazard.

Question 3.13

What is the difference between "indemnity" and "replacement" cover? Which would expose the insurer to the greater risk of fraudulent claims?

There is also the risk of exposure to catastrophe, for example after severe storms or floods.

Question 3.14

Give three other examples of how a household insurance portfolio could be exposed to accumulations.

Motor property

Claims for damage to the insured's vehicle are usually reported and settled quickly.

Claims for damage to the property of third parties may take slightly longer to settle than claims for damage to the property of the insured.

There may be some delay in settling property claims while the liability for settlement is established. Generally, delays for settlement of property claims should be in weeks or months rather than years.

As you might know from bitter personal experience, motor claims have a relatively high frequency. The average claim frequency for different insurers will depend very much on the type of business they write. As a rough indicator, many insurers will have claim frequencies of about 25% on a comprehensive portfolio, *ie* 1 claim for each 4 vehicles each year. Claim frequencies for a non-comprehensive portfolio may typically be about 15%.

In general, private motor insurance does not give rise to extreme accumulations. Because business is sold nationally, an insurer is unlikely to have a concentration of risks in one geographical area.

> ***Question 3.15***
>
> How do you think the claims characteristics of a motor fleet portfolio will differ from the characteristics of a private motor portfolio?

Marine and aviation property

Among the claim characteristics for this business, the following are important:

- **claims are usually reported as soon as the vessel reaches a major port (or indeed the event might take place in port)**
- **settlement delays might be long if there is a dispute over legal liability or the amount that should be paid**
- **minor damage is often repaired when the vessel goes into dock for routine maintenance**
- **claim amounts can vary from relatively small amounts for minor hull damage to small vessels to very large amounts in respect of the complete loss of a large vessel and its cargo.**

Accumulations of risk are quite possible with marine insurance:

- Geographical concentration would make an insurer exposed to the possibility of lots of claims arising from one incident. Storms or tidal waves could affect many ships in one harbour. A good example of this would be where an insurer sells lots of policies to a particular yacht club and a severe storm then damages all the yachts.
- If a ship spills hazardous material in a populated coastal area, there may be many liability claims.

Goods in transit

There may be reporting delays if claims are not reported until vehicles or vessels reach their destination. However, these generally amount to months rather than years.

Construction and engineering

Property claims can generally be reported and settled quickly, although as with other classes serious damage may take a relatively long time to repair. Liability claims will take longer to settle as liability needs to be established before amounts of loss can be determined. However, the claims from a single underwriting year can take many years to emerge because of the long-term nature of the policies and the fact that policies often cover a multi-year contract period.

Extended warranty

Claim costs are fairly uniform by product, as they are related to the cost of the original product. Where repairs are covered, there is a risk of multiple repairs being needed, which may increase the claim cost above the original price of the goods.

Claims can arise because a proportion of goods sold turns out to have been faulty, which may be stable, or there could be a large number of claims caused by a defective product design. This would add some volatility to the reporting patterns.

2.6 Risk factors and rating factors

Household property (buildings and contents)

A measure of the scale of the risk (for example the amount of sum insured or number of rooms) is the key risk factor. The remaining principal risk factor is location because such risks as theft, subsidence and flooding vary by location.

In this class many risk factors have been used as rating factors. Regression analysis has shown their suitability to differentiate between levels of risk.

Common rating factors include:

- **sum insured**
- **number of rooms**
- **location**
- **the voluntary or compulsory use of excesses**
- **whether there is any business use of the property**
- **whether the policyholder owns or rents the property**
- **if the property is normally unoccupied during the day**
- **whether it is a house or flat or some other construction**

- **type and standard of construction**
- **age of the building**
- **type of locks and/or burglar alarms fitted.**

This might seem like enough but there are many more that are frequently used, for example:

- whether smoke alarms have been fitted
- high risk contents
- type of cover (“new for old” or indemnity)
- family composition
- smoker / non-smoker
- type of heating
- age of policyholder.

By using a no-claim discount or other experience-rating system the insurer may be able to make an adjustment for any other influences on the level of risk. This is, however, comparatively rare in household insurance.

It is important to realise that the rating factors used may vary by country. For example, in countries where the risk of burglary is relatively high, the insurer may ask prospective policyholders if the windows of the property are barred, or if the homeowner is a member of an armed response unit. (This is an armed unit that would come to the property in an emergency to try to prevent the burglary and ensure that the residents were safe.)

> ***Question 3.16***
>
> Consider the rating factor “whether the property is normally unoccupied during the day”. Does it affect the claim frequency or the claim severity and how?

Commercial property

Apart from the monetary value of the property and the surveyor’s report of the property, the trade or business is the key risk factor.

Regression analysis has shown that the following rating factors might also be taken into account:

- **estimated maximum loss**
- **age of building**

- **fire protection equipment**
- **construction type**
- **excesses**
- **location of building.**

The estimated maximum loss is the largest loss that is expected to arise from a single event. This may well be less than either the market value or the replacement value of the insured property and is used as an exposure measure in rating certain classes of business.

For larger risks, the underwriters will take into account further factors:

- qualitative impressions: the nature of adjacent buildings, how well the company is run (there will be less risk in a tightly-managed factory with disciplined procedures)
- previous claims experience, especially with smaller claims (large claims are so rare that past experience of them is unlikely to be credible).

Motor property

Motor insurance is a good example of a class of insurance where possible risk factors can be identified, but it is difficult to regard them as meeting the criteria needed to use them as rating factors.

For example the following are all risk factors:

- **the number of miles driven**
- **the density of the traffic where the car is driven**
- **the ability of the driver**
- **the speed at which the vehicle is usually driven and its general level of performance**
- **the ease with which the vehicle can be damaged and the cost of repairing it**
- **the theft risk**
- weight of the vehicle
- fire risk.

However, these are not all measurable or quantifiable statistics.

Therefore, the insurer cannot depend on information on these risks received from the policyholder as there is considerable scope for the policyholder to stretch the truth in his or her favour or be too subjective about his or her own skills.

Question 3.17

Name some other risk factors for motor insurance that can also be used as rating factors.

Question 3.18

State whether the following risk factors are likely to affect the frequency of claims, the size of claims, or both:

(i) where the car is driven

(ii) how expensive the car is to replace / repair

(iii) how fast the car is driven.

Type of cover is the most important rating factor, as varying the type of cover can exclude an entire class of claims.

For example, third party cover will not include any claims for accidental damage to the insured's own vehicle.

Policy excess is also an important rating factor as it too will affect claim sizes.

Many small claims may be eliminated altogether leading to a reduction in claim frequency and expense savings.

An excess may be compulsory (for example for a young driver) or optional to secure a reduction in the premium. Typically insurers will offer proposers a choice of excess levels.

Other rating factors are proxies for those risk factors for which direct information is unreliable. These include:

- **the use to which the vehicle is put** (*eg* for business use)
- **the age of the vehicle**
- **the occupation of the policyholder and other drivers**
- **whether there are additional drivers of the vehicle as well as the policyholder**
- **sex of main driver**
- **age of policyholder and other drivers**

- **whether or not driving is restricted to certain named drivers**
- **make and model of vehicle**
- **the extent of any modification to the engine or body**
- **location of policyholder** (*eg* postal code)
- **where the vehicle is kept overnight: on the road / on a driveway / in a garage *etc***
- whether or not the driver has any driving convictions
- past experience.

> ***Question 3.19***
>
> The age of the policyholder and the address of the policyholder are proxies for which risk factors?

> ***Question 3.20***
>
> (i) Why is mileage not used as the exposure measure in private motor insurance?
>
> (ii) Which of the primary rating factors may potentially act as proxies for mileage?

For fleet motor, the rating factors detailed above, other than those that apply to individual drivers, are relevant. The important factors will be:

- types of vehicles
- type of cover
- level of excess
- types of use and goods carried.

Age of driver, used for private motor, would not be practical for anything other than the smallest fleets. Insurers make use of the fleet's own claims experience. In simple terms, the larger the fleet, the more weight the insurer will put on the fleet's own claims experience when setting premiums.

For many larger fleets, experience rating will take the form of a profit-sharing arrangement, whereby the insured receives a refund, if claims experience is good, or pays a further premium, if claims experience is poor, at the year end.

Marine and aviation property

The main risk factors for marine and aviation property are the size, type and condition of the craft or vessel and the nature of the cargo.

There are no definitive rating factors. Factors used might well depend on the rating factors deemed appropriate for any associated marine liability cover.

Important factors will usually include:

- the type and value of the craft
- the scope of the voyages
- the areas covered or the destination
- the number and experience of the crew
- previous claims experience.

Goods in transit

The main rating factors are:

- **mode of transport**
- **nature of goods**
- **type of storage used (for example refrigerated)**
- **time period of transit and number of stages**
- **length of time spent at warehouse.**

Construction and engineering

The main rating factors are:

- **type of project (for example, block of flats, airport, tunnel)**
- **term of project**
- **contracting firm**
- **materials and technologies used**
- **location of project.**

Extended warranty

The premium will depend on the:

- **make and model of the item being covered**
- **the length of the manufacturer's guarantee**
- **the term of the warranty.**

3 Financial loss

Financial loss insurance can be categorised as follows:

- **fidelity guarantee**
- **credit insurance**
- **creditor insurance**
- **business interruption cover, also known as consequential loss**
- **legal expenses cover.**

3.1 Benefits

The benefit provided is indemnity against financial losses arising from a peril covered by the policy.

Fidelity guarantee insurance

Fidelity guarantee covers the insured against financial losses caused by dishonest actions of its employees (fraud or embezzlement). These will include loss of money or goods owned by the insured or for which the insured is responsible and reasonable fees incurred in establishing the size of the loss (paid to auditors or accountants, for example).

Fraud is wrongful or criminal deception intended to result in financial or personal gain.

To embezzle is to steal or misappropriate money or other assets placed in one's trust or under one's control.

Credit insurance

Credit insurance covers a creditor against the risk that debtors will not pay their obligations. The principal types are:

- **trade credit**
- **mortgage indemnity.**

Trade credit may cover uncollectible debts and be sold on an annual basis; cover may also be for the length of a project, for example a ship built for a customer who does not pay for it at the end of construction.

Mortgage indemnity covers the lender (mortgagee) in a mortgage loan against the risk of the borrower (mortgagor) defaulting and the value of the property on which the loan is secured not being sufficient to repay the loan. These policies may last for the duration of the mortgage and have terms of many years.

The perils leading to default are not specified under credit insurance, so non-payment for *any reason* is covered. Claim payments are usually single lump sums.

When a lender (*eg* a building society) provides a mortgage to an individual for house purchase, the lender will be worried that:

- the borrower may default on the interest payments
- in taking possession of the property, the proceeds from the sale of the property may be insufficient to cover the amount of the mortgage and outstanding interest.

Mortgage indemnity insurance protects the mortgage lender against the risk that the borrower (who sometimes pays the insurance premium) defaults on the loan and a loss is made by the lender.

Where the amount of mortgage is low in relation to the value of the property, the lender should be quite relaxed because the sale proceeds will almost certainly cover the mortgage and interest. However, where the mortgage is a large percentage of the property value, then the lender will want a mortgage guarantee insurance policy to protect against the risk described above.

Typically, lenders require this insurance when the mortgage exceeds 75% of the valuation of the property at purchase. The lender takes out the policy and then sometimes makes the borrower pay the premiums. With most insurance policies, the beneficiary of the policy cover pays the premiums. Mortgage guarantee is unusual in this respect because sometimes the borrower pays the premiums but the lender is the beneficiary.

Question 3.21

What are the unusual features of mortgage indemnity insurance compared to other insurance classes?

Credit insurance is also known as *pecuniary loss* insurance.

Creditor insurance

Creditor insurance provides cover to insureds who are subject to obligations to repay credit advances or debt. Most policies are made to individuals to cover personal loans, mortgage loans or credit card debts.

The cover is usually against disability and unemployment, on the basis that these perils may prevent the insured getting an income. The policy will pay the regular loan payments until the borrower is recovered or obtains new work or until the loan is fully repaid or a maximum number of payments made.

Creditor insurance for personal loans and credit cards is normally sold alongside an associated life assurance policy that will repay the whole outstanding balance of the loan if the insured borrower dies.

Creditor insurance is also known as payment protection insurance (PPI).

Business interruption cover

The financial consequences of a fire to a company can be much more significant than the cost of repairs to premises. If the company's production lines are hit, income from customers will be much reduced until alternative production arrangements can be made. If this income stream was being used to pay off loans from a bank, the accumulating interest charges can put further financial strain on the company.

Business interruption cover indemnifies the insured against losses made as a result of not being able to conduct business.

Methods used to achieve this need to take account of the following factors:

- Turnover will drop off dramatically after a major fire, but will (hopefully) start to build up again as soon as production facilities can be restarted.
- When the policy is taken out, the proposer will need to assess how long it would take to rebuild the business in the event of a major disaster. This period will then be specified in the policy, and is known as the indemnity period. The insured will then be indemnified for loss of profit over this period.
- Profit will depend on the proportions of fixed and variable costs incurred by the business.
- Additional temporary costs may also be incurred as a result of the fire.
- Assessing what the turnover figure would have been without the fire will also have to take account of:
  - any inherent seasonal variation in the business
  - any organic growth (or shrinkage) in the business itself.

A fixed sum insured per day will normally be specified in the policy and the insured will receive this until the property can be occupied again or until the term specified in the policy has expired. This cover is also known as *consequential loss* and *loss of profits*.

> ***Question 3.22***
>
> What is *suretyship*?

Legal expenses cover

Legal expenses cover indemnifies the insured against legal expenses incurred as a result of:

- **legal proceedings being initiated against the insured**
- **the need for the insured to initiate proceedings.**

Legal expenses policies will normally cover the payments to legal representatives.

Personal legal expenses insurance is often sold as an optional add-on to a household policy. Commercial legal expenses insurance can be sold as a stand-alone policy or as an add-on to a policy such as employers' liability or commercial property.

Such policies might cover legal expenses associated with a neighbourly dispute (personal) or defending against a consumer complaint in respect of goods and services provided (commercial).

3.2 Insured perils

For financial risks the insured perils will depend on the precise cover. The perils might include:

- **dishonest actions by employees (fidelity guarantee)**
- **failure of third parties specified in the policy (credit)**
- **accident or sickness resulting in loss of income with which to repay debt (creditor)**
- **fire at the insured's own property (business interruption cover)**
- **fire at a neighbouring premises causing loss of access to own property (business interruption cover)**
- **legal proceedings being brought against the insured (legal expenses cover).**

Business interruption cover is normally sold in conjunction with property insurance and will cover the same perils: fire, weather losses and so on, and the business interruption claim will follow a property damage claim.

3.3 Basis for cover

Financial loss insurance is likely to be written on a losses-occurring basis.

Legal expenses insurance is sometimes written on a basis combining losses-occurring and claims-made. Claims must both occur and be reported within the policy period.

3.4 Measure of exposure to which premiums are related

Fidelity guarantee and credit insurance

For fidelity guarantee and credit insurance, the measure of exposure will depend on the precise cover provided.

A possible, though imperfect, measure of exposure for mortgage indemnity guarantee insurance is the excess of the amount of the loan over a certain percentage of the value of the property.

This defined percentage is often referred to as the "normal advance".

For example, if premium rates were £5 per £100 above the normal advance, a borrower had a mortgage for £132,000 and the normal advance was £114,000 (75% of the property valuation, £152,000), then the premium would be £900.

Creditor insurance

The exposure measure for creditor insurance on personal loans is normally the amount of the loan or the total amount repayable. On a mortgage it will be the insured monthly benefit. On a credit card it will normally be the outstanding balance at the latest monthly statement date.

Business interruption cover

The exposure measure for consequential loss is the annual profit or turnover.

Obviously this cannot be known in advance so adjustment premiums are common.

The amount payable in the event of complete cessation of the business will normally be set out in the policy. This should be set so that the insured has an incentive to get the business running again as soon as possible.

Legal expenses cover

The number of policyholders or policyholder-years are possible exposure measures for legal expenses insurance.

3.5 Claim characteristics

Financial loss insurance covers a wide variety of risks that can give rise to a financial loss. Frequency and claim cost distributions tend to vary by class, but in general these risks tend to be short tailed, even if the policies concerned have relatively long terms for general insurance.

Fidelity guarantee insurance

It may take time to discover that there has been fraudulent behaviour by employees, so there may be reporting delays for fidelity guarantee insurance. Settlement delays could be significant, as it may take time to establish the size of the loss (including the resulting fees, *eg* to auditors).

The potential size of fidelity guarantee insurance claims could be large (especially for large companies), however claim sizes may also depend on the level of regulation in the territory.

Credit insurance

The claims experience on mortgage indemnity guarantee insurance depends heavily on economic factors, as was illustrated in the UK in the early 1990s when some insurers incurred severe losses on this type of business.

Question 3.23

Which economic factors is claims experience likely to depend on and how will it vary with them?

Creditor insurance

Creditor insurance claims will normally be a series of payments made until the insured recovers or the limit on payments is reached:

- **The payments on personal loan policies will be the monthly repayment specified in the loan agreement; these loans are normally made on a rate of interest that does not vary after the loan is taken out.**
- **The monthly benefit on mortgage policies is normally a set amount selected by the insured when he or she takes the policy out and linked to the monthly repayment, although it may sometimes be varied if interest rates change.**
- **The monthly benefit on a credit card policy is usually the minimum monthly payment on the balance as at the monthly statement date preceding the claim.**

The frequency of claims depends on factors such as the rate of unemployment (higher if there is a recession) and the likelihood of policyholders becoming sick or having accidents.

Business interruption cover

For business interruption cover claims, the reporting delays are directly linked to any associated property claim. Settlement, however, is likely to be slower than for property claims, owing to the greater need for verification, although once a loss is established payments can be made regularly.

3.6 Risk factors and rating factors

The risk factors and rating factors for financial loss depend on the cover provided and the nature of the policy.

Fidelity guarantee and credit insurance

Fidelity guarantee and trade credit are commercial policies that will be individually underwritten, and the nature of the business and the size of the sums at risk will be taken into account.

Mortgage indemnity insurance premium is usually based on the amount of loan in excess of a certain proportion of the value of the property and will take into account the quality of loan underwriting by the lender.

With mortgage indemnity insurance, in practice insurers do not incorporate the circumstances of the individual borrower into the rating structure. They rely upon the lender to be prudent in granting mortgages.

Also, the effect of house price falls will be factored into the premium rates on a global basis rather than different rates for different houses.

The term of the mortgage might be used for rating. Otherwise, standard rates will apply for all policies issued through particular mortgage lenders.

Question 3.24

What are the main measures of exposure used in practice for the following types of insurance:

(i) Employers' liability?

(ii) Household contents?

(iii) Motor liability?

Creditor insurance

Creditor insurance cover for personal loans is usually provided at a fixed price percentage of the amount lent or the total amount repayable. Premium rates will vary with the term of the personal loan. The premium will either be paid as a single amount at the start of the loan (in which case it is usually financed as an addition to the loan) or as a monthly addition to the repayment, in which case it will also be covered by the insurance.

The premium for mortgage cover is a proportion of the insured monthly benefit, payable each month.

The monthly premium for credit card cover will be a proportion of the outstanding balance.

No allowance is made for the profile of the individual being covered.

In reality, the risk factors include:

- **age**
- **sex**
- **employment status**
- **occupation**
- **state of health**
- **amount of monthly benefit.**

When setting the premium rates for a lender's business the underwriter will consider the composition of the lender's book of business in relation to these variables.

Business interruption cover

The risk and rating factors are likely to be similar to those for commercial property damage insurance. Depending on the exposure measure, annual profits and/or turnover may also be used.

The insurer may also want to incorporate the insured company's dependence on the economic cycle into the rating process, since companies that are dependent in the economy will face significantly larger (smaller) claim sizes during a boom (recession).

Legal expenses cover

For legal expenses, the rating is normally based on the sum insured.

4 Fixed benefits

Fixed benefit claims arise under personal accident insurance.

A personal accident contract may specify an amount to pay in the event of the insured suffering the loss of a limb, £50,000 for losing an arm, for instance. It is very hard to quantify the value of the loss of a limb and so a fixed benefit is provided.

There are also other fixed benefits insurance products, examples of which are described below.

4.1 Benefits

Personal accident cover

Benefits are usually specified fixed amounts in the event that an insured party (this may include the policyholder's family as well as the policyholder) suffers the loss of one or more limbs or other specified injury, or accidental death. This is not indemnity insurance because it is not possible to quantify the value of the loss, for instance, of an arm.

Cover may be offered on a group basis by an employer to all employees. Alternatively / additionally, it might be combined with a sickness policy.

An individual is usually assumed to have an unlimited financial interest in his or her own life, and this may be extended to personal injury, although an insurer should take care to avoid moral hazard by granting large sums insured for relatively minor injuries.

Example

An individual is fairly unlikely to risk his life just because he is safe in the knowledge that death will lead to a life insurance payout. Similarly, an individual is unlikely to take less care in a particular activity and risk losing a limb. However, loss of a finger or toe might be considered to be minor and so an individual may not take as much care not to lose one of these, especially given that it might lead to an insurance payout of several thousand pounds.

Often a policyholder will be able to select his or her level of cover, which will be paid in full if he or she suffers the complete loss of or loss of use of a major limb or permanent total disablement. Often there is a reducing scale of benefits, known as a continental scale, for lesser injuries, in which each specified injury is worth a set proportion of the full sum insured.

Policies such as this are often sold as units of sum insured; for example a unit might be £25,000 and the policyholder may select a number of units of cover, up to a maximum.

Other fixed benefits cover

Fixed benefits may also be granted for periods of disability through accident or injury or for periods of hospitalisation. In this case the benefit is usually a fixed amount per day, week or month until recovery, possibly after a waiting period for benefit.

4.2 Insured perils

Personal accident cover

Here the perils are any form of accident that results in the loss of limbs or other specified injury.

For example, some motor policies cover fixed personal accident benefits to the policyholder in respect of injuries arising from the peril of motor accidents caused by the policyholder.

4.3 Basis for cover

Personal accident insurance will typically be written on a losses-occurring basis.

4.4 Measure of exposure to which premiums are related

For personal accident insurance the true measure of exposure is the person-year multiplied by the sum insured.

This is because the cover is defined in relation to the level of cover normally required by one person. However, in many cases, the member- or employee-year may be all that is available.

In other words, if family members are covered under the group or individual plan then ideally we would need more details of these members to calculate the exposure measure. However, in practice this is often not available and so member-year is used instead.

The insurance may cover the whole family at a standard rate but the insurer may or may not be made aware of number of family members. The insurer charges a rate assuming an average family unit size and, hopefully, gets it right on average.

Question 3.25

Describe the risk that the insurer will be exposed to as a result of using a standard rate to cover the whole family.

For group policies the number of people covered by a policy may need to be adjusted at the year-end and the premium adjusted in line with their risk characteristics.

Under personal accident insurance if policyholders can select more than one unit of sum insured, this too will need to be reflected in the exposure.

For group schemes, where the scale of benefits is linked to salaries, the exposure measure is often the sum insured or total salaries.

4.5 Claim characteristics

Claims are usually reported quickly. The incidence of an event is usually very clear (so reducing reporting delays), although with accidental death claims the insured's dependants may not always know the policy exists and may discover their entitlement after an extended period.

The claims may be settled quickly, although if a claim is for permanent total disability it may be necessary to wait some years for a claimant's condition to stabilise. As the claim cost is known for these claims the settlement delays are reduced.

As well as paying out on specified injuries, many personal accident policies will make a payment if the insured is "permanently and totally disabled". It may, however, take several months or years before establishing that this is the case for these claims.

The claim frequency tends to be reasonably stable.

Claims can be large: cover of several hundred thousand pounds per person is not uncommon.

4.6 *Risk factors and rating factors*

Personal accident cover

Apart from sum insured, the prime factor affecting the risk is usually occupation. Those employed in more dangerous occupations such as demolition work will certainly need to be distinguished from those in safer occupations such as clerical work.

Other factors may be:

- age – **age is not usually a major determinant of risk until old age is reached, and this cover is not normally sold to the elderly**
- sex – **sex may be a factor: women tend to be less prone to accidents than men**
- health condition may also be a determinant of risk – those in worse health may be higher risk than average
- dangerous pastimes – **those following hazardous hobbies may also be higher risk than average.**

The rating factors are the same as the risk factors, because they can all be measured.

5 How policies may be combined

5.1 Motor insurance policies

Recall that motor insurance cover may be for:

- third party liability, or
- property damage.

However, it is common for motor liability cover and cover for damage to the vehicle to be provided in a single policy.

Example

In the UK, a motor policy that provides cover for liability and damage to the vehicle from all perils including accident is known as "comprehensive". In some other European countries it is common for motor liability and damage policies to be provided separately.

Recall that the main perils for motor insurance are:

- accidental or malicious damage to the insured vehicle
- fire or theft to the insured vehicle.

In many countries, including the UK, this cover is typically provided together with motor third party cover within a single policy, whilst in other countries it may be provided in a separate policy.

For example, in some states in Australia, the three main types of policy are broadly similar to the UK:

- third party property damage
- third party fire and theft
- comprehensive.

However, none of the options include third party bodily injury claims. This is split into a separate category and is sometimes insured by the state with the premiums forming part of the annual registration fee.

Motor policies will usually also include public liability cover. A motor policy may also include personal accident cover.

5.2 Marine and aviation policies

Aviation policies will usually also include public liability cover.

Commercial marine policies, however, usually exclude public liability cover, which is provided by Protection and Indemnity Clubs.

Marine and aviation are often considered separately from other forms of insurance because of their wide range of cover, the other peculiarities they demonstrate and because historically, market practice has been to deal with them separately. However, the types of cover are essentially the same as for other forms of transport such as land vehicles.

5.3 Buildings and contents insurance policies

Household property insurance

Household policies are often sold as a package, covering both buildings and contents.

> ***Question 3.26***
>
> Before reading on, suggest:
>
> (i) what the main advantage of this is, and
>
> (ii) when such a package might not be appropriate.

One advantage of this is that the separation between the two is sometimes not entirely clear: which fixed items are part of the fabric of the property and which are contents? However, tenants are not normally responsible for arranging buildings insurance and will need only contents insurance.

Public liability cover may also be included in such policies. Buildings and contents insurance are also provided as separate contracts.

Commercial property insurance

Small businesses are conventionally covered by "package" policies, which include buildings, contents and liability insurance. They may also include business interruption.

5.4 Contractors' All Risks (CAR) policies

Construction risks are typically insured under a contractors' all risks (CAR) policy. This will include the contractor's liability for losses caused to third parties.

The principle of "all risks" policies is to cover all perils not specifically excluded. Examples of possible exclusions for a CAR policy include:

- liability to employees working on the project (as this would often be covered by a separate employers' liability policy)
- damage to the work-in-progress resulting from faulty workmanship
- damage to any property which existed on the contract site prior to the work commencing (*eg* if an extension to an existing building was being constructed)
- damage due to wear and tear.

6 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Accident year
- Adjustment premium
- All risks
- Business interruption insurance
- Cancellation
- Consequential loss insurance
- Deductible
- Deposit premium
- Escalation clause
- Estimated maximum loss (EML)
- Fidelity guarantee insurance
- First loss
- Fleet
- Latent claims
- Loss of profits
- Minimum and deposit premiums
- New for old
- Probable maximum loss (PML)
- Replacement
- Suretyship.

This page has been left blank so that you can keep the chapter summaries together for revision purposes.

Chapter 3 Summary

Liability

The essential characteristic of liability insurance is to provide indemnity – subject to maximum levels of benefit and/or excesses – where the insured, owing to some form of negligence, is legally liable to pay compensation to a third party.

The extent of any legal liability may depend on the prevailing legislation. Legal expenses relating to the liability are usually also covered.

Significant reporting and settlement delays mean that most liability classes are long tailed. Claims tend to be relatively infrequent and claim cost distributions tend to be widely spread.

The main types of liability insurance are:

- employers' liability / workers' compensation
- motor third party liability
- marine and aviation liability
- public liability
- product liability
- professional indemnity
- Directors' and Officers' (D&O) liability
- environmental liability.

Employers' liability indemnifies the insured (the employer) against legal liability to compensate an employee / their estate for bodily injury, disease or death suffered, owing to negligence of the employer. The benefit may be regular payments or a lump sum. The perils include accidents and exposure to harmful substances or working conditions. The main measure of exposure is payroll. The main risk factor is the type of industry.

Workers' compensation is another name for employers' liability. In some countries, however, workers' compensation may refer to losses suffered by employees during the course of employment regardless of whether the employer has been negligent.

Motor third party liability indemnifies the owner of a motor vehicle against compensation to third parties for personal injury or damage to their property. Cover is usually on a losses-occurring basis. The measure of exposure is the vehicle-year. The risk and rating factors are as for motor property damage.

Marine and aviation liability indemnifies against the legal liability to compensate a third party for bodily injury, death or damage to property arising out of operation of the vessel or aircraft. Exclusions are likely to include terrorism, war and illegal activities. Cover is usually on a losses-occurring basis. Possible measures of exposure include passenger kilometres / voyages and in-service seats. A wide range of rating factors is used in practice.

Public liability indemnifies the insured against legal liability for the death of or bodily injury to a third party or for damage to their property from any other cause. The perils will depend on the type of policy. The most common measure of exposure is turnover.

Product liability indemnifies the insured against legal liability for the death of or bodily injury to a third party or for damage to their property that results from a product fault. Cover is likely to be on a claims-made basis. Perils include faulty design, faulty manufacture, faulty packaging and incorrect or misleading instructions. The most common measure of exposure is turnover. The nature of the product is an important risk factor, however each policy will be individually underwritten.

Professional indemnity indemnifies the insured against legal liability for losses resulting from negligence in the provision of a service. Cover is usually on a claims-made basis. Perils may include wrong medical diagnosis, and errors in medical operation or actuarial report. The most common measure of exposure is turnover. The nature of the profession is an important risk factor, however each policy will be individually underwritten.

D&O liability is a type of professional indemnity that indemnifies the insured against the legal liability to compensate third parties owing to any wrongful act of the insured director. Cover is usually on a claims-made basis. Perils include wrongfully allowing a company to continue operating, acts resulting in the insured being declared unfit for the role and allowing the publication of false financial statements. The nature of the profession is an important risk factor, however each policy will be individually underwritten.

Environmental liability indemnifies against the legal liability to compensate third parties for the death of or bodily injury to a third party or for damage to their property as a result of unintentional pollution for which the insured is deemed responsible. Cover is usually on a claims-made basis. Perils are any incident causing environmental pollution. Each policy will be individually underwritten.

Property damage

The main characteristic of property insurance is to indemnify policyholders against loss of or damage to their own material property.

The main types of property that are subject to such damage are:

- residential, commercial and industrial buildings
- moveable property (contents)
- land vehicles
- marine and aircraft
- goods in transit
- construction and engineering
- extended warranty.

Property claims are usually reported and paid quickly. They are often more predictable in frequency and size than liability claims.

Residential, commercial and industrial buildings insurance usually pays the amount needed to restore the property to its previous condition, subject to any excess / deductible. The main perils are fire, explosion, lightning, theft storm, flood, subsidence and damage caused in putting out fires. The exposure measure is usually the sum insured year. The sum insured should be the cost of rebuilding / restoring the property. Subsidence claims may suffer reporting delays. Many rating factors are likely to be used.

Moveable property can pay an amount equal to the replacement value of the item, or may be on a "new for old" basis. The main perils are fire and theft. Malicious and accidental damage may also be covered. The exposure measure is usually the sum insured year, although this may be less appropriate for commercial covers, where the level of stocks is very variable. Determining levels of stocks may lead to settlement delays. Many rating factors are likely to be used.

Land vehicle insurance provides a benefit to repair the vehicle, subject to a maximum of the replacement value of the vehicle. The exposure measure is the vehicle-year. Many rating factors are likely to be used, including the level of excess.

Marine and *aircraft* may cover loss of or damage to the craft and the cargo. Perils include fire and explosion. The exposure measure may be the insured value of the hull or aircraft and may allow for cargo. Claims may not be reported until the vessel reaches a port. There are no definitive rating factors.

Goods in transit is sometimes sold alongside marine and aviation insurance. The main perils are damage, loss and theft. The exposure measure is typically the consignment value. Claims may not be reported until the end of the journey. The main rating factors are mode of transport, nature of goods, type of storage, time period / number of stages of journey ad length of time spent at warehouse.

Construction insurance may be provided for projects over the course of their lifetime (and sometimes beyond), and so may last for longer than a year. The main perils are damage, destruction and failure to finish the construction project. The exposure measure may be the value of the contract, which is unlikely to be uniform over the lifespan of the project, so allowance must be made for this. The main rating factors are type and term of project, contracting firm, materials and technology used and location of project.

Engineering is similar in nature to construction. The main perils are machinery breakdown, explosion and electronic failure. The exposure measure may be the sum insured value of the contract, which is unlikely to be uniform over the lifespan of the project, so allowance must be made for this. Claim costs are likely to be fairly uniform, although there may be accumulations. Rating factors are similar to those for construction.

Extended warranty covers losses arising from the need to replace / repair faulty parts in a product beyond the manufacturer's normal warranty period. Such policies may have a term of several years. The exposure measure is usually the number of appliances or appliance-years. The main rating factors include make / model of item, length of manufacturer's guarantee and term of warranty.

Financial loss

Financial loss insurance can be categorised as: pecuniary loss, fidelity guarantee and business interruption cover also known as consequential loss.

- fidelity guarantee
- credit insurance (trade credit and mortgage indemnity guarantee)
- creditor insurance
- business interruption cover (also known as consequential loss)
- legal expenses cover.

The benefit provided is indemnity against financial losses arising from a peril covered by the policy. Claims tend to be relatively short tailed.

Fidelity guarantee covers the insured against financial losses caused by dishonest actions of employees. Contracts will be individually underwritten – the nature of the business and the size of the sums at risk will be taken into account.

Trade credit covers uncollectible debts. The term of cover may be annual or may depend on the length of a project. As above, contracts will be individually underwritten.

Mortgage indemnity guarantee covers the lender against the borrower defaulting on the loan. The exposure measure may be the excess of the amount of the loan over a certain percentage of the value of the property. Claims experience depends heavily on economic factors. The premium will be affected by the quality of the lender's loan underwriting.

Creditor insurance generally covers individuals who cannot make loan repayments due to disability or unemployment. The loans may be personal loans, mortgages or credit card debts. Benefits are usually limited to a maximum number of payments. The exposure measure is usually the amount of the loan / the total amount repayable. Claim frequency depends on the rate of unemployment and the likelihood of policyholders becoming sick / having accidents. The premium will vary with the term of the loan. No allowance is made for the profile of the individual being covered.

Business interruption cover indemnifies against losses made as a result of not being able to conduct business. Perils are similar to those for property damage. The exposure measure is usually the annual profit or turnover. Settlement may be slower than for the associated property damage claim.

Legal expenses cover indemnifies against legal expenses from court proceedings being brought against / by the insured. The exposure measure may be the number of policyholders. Premiums will depend on the sum insured.

Fixed benefits

Fixed benefit claims arise under personal accident insurance.

Personal accident cover usually provides fixed benefits in the event that the insured party (or parties) suffers one of a specified set of injuries / losses or death. The individual may be able to select the level of cover. The exposure measure is the person-year multiplied by the sum insured. Claims tend to be reported and settled quickly. The main rating factor tends to be occupation, although age, sex and hobbies may also be used.

Packaged products

Different types of general insurance products may be combined under a single policy. For example:

- motor third party liability and motor property
- residential buildings and contents (with some public liability cover)
- commercial buildings, contents and business interruption (with some public liability cover).

You will need to be able to use a variety of different approaches to produce good solutions for Subject ST8 exam questions.

Chapter 3 Solutions

Solution 3.1

Incorrect actuarial advice.

Solution 3.2

Directors and Officers cover is a form of liability insurance which will indemnify directors or senior managers of a company against them, or their company, being sued for acts that they have committed. For example, they may have "unfairly" dismissed an employee.

Solution 3.3

An excess:

- reduces the amount of each claim (by the excess)
- reduces the number of claims (all claims less than the excess are eliminated)
- in particular eliminates the small claims just above the excess, where the policyholder may feel it's not worth claiming and results in expense savings
- arguably encourages policyholders to be more careful and so helps prevent claims
- may allow the company to reduce premiums and so make them appear more competitive.

Solution 3.4

Customers may not always remember exactly when they noticed a product fault or when they bought the product, and health problems caused by product faults (*eg* pharmaceuticals) may take a long time to arise. In these cases, it is likely to be easier for this business to be written on a claims-made basis.

If a claims-made basis is used for certain products, such as motor insurance, the insured might strategically choose when to report a claim, for example to retain the next year's no-claims discount. However, for product liability, it is the customer who will initiate the claim, but it is actually the company that is the insured. The customer therefore has no personal interest in the exact time at which the claim is reported.

Solution 3.5

(i) ***Commercial liability***

- employers' liability
- motor third party liability (individual vehicles or motor fleets)
- marine and aviation liability
- public liability
- product liability
- professional indemnity
- D&O liability
- environmental liability

(ii) ***Personal lines liability***

- motor third party liability
- public liability (this is likely to be part of another type of insurance)
- environmental liability (this may be part of another type of insurance, *eg* household buildings)

Solution 3.6

Payroll is used as the exposure measure for employers' liability because the compensation payable to employees who claim is often related to salary. For example, if an employee is unable to work again due to injury, the main component of compensation would be loss of earnings.

Solution 3.7

Consider a manufacturer that had very low turnover in a particular policy year, *eg* as a result of a downsizing exercise.

Marine and aviation liability is usually written on a losses-occurring basis, therefore there may be claims made this year in respect of goods produced (and hence turnover) last year.

The manufacturer may therefore have significant exposure to risk this year, even though turnover is very low. In other words, previous years' turnover can give rise to claims on the current year's policy.

Solution 3.8

No – if the business was written on a claims-made basis, then the reporting delay would be irrelevant (as the claim is effectively deemed to occur when the claim is made).

Solution 3.9

Suppose we have two policies. Policy A has an excess of \$50 and policy B has a deductible of \$50. The policies are identical in all other ways. The policies have a sum insured of \$1,000.

For any claim less than \$1,000 then both policies pay the same amount to the insured, *ie* claim – \$50.

The amount paid by the insurer on the two policies is different if the claim exceeds the sum insured. Suppose the claim is \$1,500.

Policy A pays \$1,000. Policy B pays \$950 (*ie* the amount is reduced by the deductible).

Solution 3.10

Assuming that the shopkeeper does not own the shop itself, the shopkeeper is likely to require some or all of the following:

- protection for the stock and fixtures and fittings against theft, damage from fire, flood, lightning, hold up or malicious persons.
- consequential loss of profits insurance in case the shop suffers a major fire
- plate glass insurance for his shop window
- employers' liability insurance if anyone else is employed in the business
- protection for frozen food in freezers against spoilage as a result of freezer breakdown
- product and/or public liability insurance to protect against claims from the public in respect of faulty goods sold
- insurance against theft of cash
- protection for goods in transit.

In practice, most insurance companies would be able to offer the small trader a "package policy" incorporating all of these benefits, together with a number of other "optional extras".

Solution 3.11

It refers to the damage done to buildings in the course of forced entry by thieves, *eg* windows broken.

Solution 3.12

- fire, lightning, explosion, earthquake
- storm, flood
- burst pipes
- theft, vandalism, civil commotion
- subsidence or land heave
- impact (*eg* from falling trees, aircraft)
- damage caused by measures to put out a fire
- accidental damage

Solution 3.13

- Indemnity cover — the insurer compensates the insured for the value of items as at the date of loss.
- Replacement cover — the insurer pays for new replacement items.

The risk of fraudulent claims will be greater with replacement cover. Having shelled out their (admittedly higher) premium, policyholders with replacement cover have more to gain from such claims, *eg* it may save them going shopping to buy a new TV to replace their 20 year old set.

Solution 3.14

Subsidence, fire, explosions.

Solution 3.15

The motor fleet claim frequency will be higher. Claim frequencies of 40% or more (per vehicle-year) are not unusual on fleet policies. Fleet cars tend to be driven long distances by a variety of drivers, some of whom may not be totally familiar with the vehicles they are driving. Some people with company cars might also think "it's only a company car so ..."!

Many motor fleets will be subject to greater accumulation of risk than exists for private motor insurance, *eg* vehicles may regularly be parked in the same place.

Solution 3.16

It affects the claim frequency because a house that is unoccupied during the day is more likely to be burgled than an occupied house. Also, a fire is less likely to be noticed at an early stage and be put out to prevent a claim being made.

It also affects the claim severity. Burglars will have more time to steal more items, increasing the claim size. A fire may have more time to spread and cause more damage. Also a burst water pipe will cause more damage if not noticed and stopped immediately.

There may also be an impact on perils other than theft and fire.

Solution 3.17

- type of cover
- excess
- value of the car and its contents (*eg* car radio)

Solution 3.18

(i) Where the car is driven will have a big impact on the claim frequency.

(ii) How expensive the car is to replace / repair will have most effect on the claim size.

(iii) How fast the car is driven will probably influence both the frequency and the size of the claims.

Solution 3.19

Age of policyholder

- the ability of the driver (very young drivers tend to be more accident prone)
- the number of miles driven (some age groups tend to drive more than others)

The address of the policyholder

- the density of the traffic where the car is driven
- the number of miles driven
- the theft risk

Solution 3.20

(i) It is administratively less convenient (*eg* there would be year-end adjustments). Verifying mileage is deemed to be impractical.

(ii) Most of these factors might be acting as a proxy for mileage in one way or another:

- drivers with "business use" will probably do more miles than drivers with "social and domestic use"
- cars with very low rating groups tend to do fewer miles per year
- new cars are usually driven further than older cars
- young drivers may drive further than older drivers
- drivers in rural locations may drive further
- drivers with high mileage may have more claims and have worse NCD ratings.

Solution 3.21

The policy is usually single premium but the cover may last for say 25 years. The policy is of no direct benefit to the person often paying the premium. In other words the beneficiary of any claim is the lender.

The risk of a claim is closely linked to the economic cycle, with its impact on interest rates and employment levels.

The claim amount is the difference between the resale value of the property and the amount of the loan outstanding, including any interest due. This should reduce over time, although this also depends heavily on economic conditions (*eg* house prices).

Solution 3.22

The Glossary defines suretyship to be a product "that provides a guarantee of performance or for the financial commitments of the insured".

It is a specialised class of insurance where one party (known as the surety) guarantees the performance of an obligation by another party (the insured). There are three parties to the agreement:

1) the party that undertakes the obligation

2) the surety that guarantees that the obligation will be fulfilled

3) the obligee, who receives the benefit of the surety bond.

It is therefore a form of financial guarantee, although, unlike "normal" insurance the party that undertakes the obligation still retains the risk, because it's the obligee that gets the benefit.

Although detailed knowledge of this product is perhaps beyond the Subject ST8 syllabus, you can find out more at www.rkabonds.com/basics.asp.

Solution 3.23

Claims experience is likely to depend on:

- interest rates – higher interest rates are more likely to lead to default on mortgage payments (for variable rate mortgages)
- house prices – falling house prices might lead to negative equity; if the insurer is forced to take possession of the property, then the property value may be insufficient to cover the amount of the mortgage and any outstanding interest
- rising unemployment – individuals will be unable to afford mortgage payments.

Solution 3.24

(i) annual payroll

(ii) sum insured years

(iii) vehicle-years

Solution 3.25

The insurer will be exposed to the risk of selection. The policy might only be attractive to large families. This would particularly be the case if other insurers *do* use the actual family details in the rating process. The insurer would usually market the policy very carefully to minimise the selection risk.

Solution 3.26

See the Core Reading that follows this question.

Chapter 4

Tackling an unusual product

0 Introduction

The examiners may require students to apply the concepts covered in Chapters 2 and 3 to other, more unusual, products. In many ways this is a truer test of understanding than asking questions on a familiar product.

The purpose of this exercise is to enable you to:

- apply the principles covered in Chapters 2 and 3 to think of perils and risk factors for an unusual product
- recognise when these risk factors are usable for rating, underwriting or exposure measurement.

This chapter covers a (very) unusual product by way of a question. To get the most from this chapter you should attempt the question yourself before looking at our ideas.

1 Question

The National Football League is about to be rocked by a breakaway of the biggest 22 clubs. They are going to form their own Ultimate League, with substantial extra finance promised to them by a consortium of cable and satellite TV companies. At the end of each season the bottom three clubs in the Ultimate League will be demoted to the top division of the remaining Football League and replaced by the three teams who have finished at the top of that division.

The chief executives of the 22 prospective breakaway clubs have approached the large general insurance company that you work for. They are seeking to negotiate a new form of insurance for the clubs against the risk of demotion from the Ultimate League to the (impoverished) Football League, in which their share of the money from television fees is likely to be minimal.

A student actuary who is very keen on football has provided you with the following background information:

"There is a reasonably clear hierarchy of football clubs. The big ones (like Bigclub United, AC Winsagain and Yeovil Town) have been winning trophies for years, while others of the breakaway clubs (like Notsogood United, Real Trouble and Manchester City) are expected to struggle in the new regime. Football is a funny old game though, and these expectations may not be fulfilled. At the end of the day it's just eleven players against eleven players, after all."

You have been asked to prepare a report for the directors of your company on this proposed new form of insurance.

Read the question

Reading the question carefully is an important skill that you must develop. As you read, you may find it helpful to underline key words and annotate the question paper, or rough paper, with your ideas[1]. Having read it quickly a few moments ago, go back to the question again now before reading our thoughts on the question, which are given below.

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[1] If this seems a touch patronising we apologise, but a slightly different style is required for the later exams and for some students this could be their first attempt at one of the later subjects.

Thoughts on the information given

"The National Football League is about to be rocked by a breakaway of the biggest 22 clubs. They are going to form their own Ultimate League, with substantial extra finance promised to them by a consortium of cable and satellite TV companies. At the end of each season the bottom three clubs in the Ultimate League will be demoted to the top division of the remaining Football League and replaced by the three teams who have finished at the top of that division."

- Note that 3 teams from 22 are demoted each season. All other things being equal this would imply a claim probability of 3/22.
- *"Substantial extra finance"* means that any club demoted will lose significant revenue for the following year.
- Can these TV fees be predicted at the start of the season?
- Is there a schedule of who will be on TV and when, or would we need an end of year adjustment to reflect actual exposure?
- Do all clubs get an equal share of the fees or do some get more than others?

"The chief executives of the 22 prospective breakaway clubs have approached the large general insurance company that you work for. They are seeking to negotiate a new form of insurance for the clubs against the risk of demotion from the Ultimate League to the (impoverished) Football League, in which their share of the money from television fees is likely to be minimal."

- Note that your company is *large*. This may mean that you have the resources available to consider such a one-off arrangement. On the other hand, it may mean that you are not interested in odd one-off arrangements that will presumably imply considerable per policy expenses. Conversely, there may be prestige and free publicity in being involved with the project.
- Note this is a *new* form of insurance. This means there is no direct historical data to use in rating. However we could consider league positions in the previous season as some indication of the likelihood of demotion this season.
- We need to decide what form the new insurance will take.
- This is a niche market with a maximum of 22 policies per annum. We would want to know how many clubs will take out the cover.

"There is a reasonably clear hierarchy of football clubs. The big ones (like Bigclub United, AC Winsagain and Yeovil Town) have been winning trophies for years, while others of the breakaway clubs (like Notsogood City, Real Trouble and Manchester City) are expected to struggle in the new regime. Football is a funny old game though, and these expectations may not be fulfilled. At the end of the day it's just eleven players against eleven players, after all."

- There is clearly a difference in the likelihood of demotion between the big clubs and some of the others likely to struggle. This would imply that we cannot just treat all clubs as equals.

- A probable assumption, in the absence of better information, is that the big clubs appear on TV more often and get a higher share of the money than the smaller clubs.

Now try some structured thinking

So far, all you have done is read the question, think about it and annotate the question paper. But note how many initial points have been raised. The next stage is to do some more structured thinking about the question. There are a number of valid approaches. The order in which you apply them is not important.

Ask yourself as many basic generic questions as you can sensibly devise about the product described. Then try to answer them.

(a) *Questions about the cover to be provided*

- Who is covered?
- What perils are to be covered?
- When does the cover apply, and when does it cease?
- Why should the club purchase cover?
- Are there any exclusions that should be applied?
- What is the sum insured?
- Are we providing indemnity cover?
- Do we want to impose a maximum amount of cover?

Comments

Football clubs are to be indemnified against potential loss of TV fees following demotion.

The cover should be purchased before the start of the season, so a cut-off date may be appropriate. The sum insured should be limited to a percentage, perhaps 100%, of the current year's fees.

The insurer needs to decide whether the claim period should be for one year only or if claims payments should be made until the club regains promotion. The former is likely to be the only option acceptable to the insurer. The latter option is too uncertain and open-ended. The risk to the insurer is likely to be perceived as too large. There would also be the concern that the club would have no incentive to regain promotion to the Ultimate League. This is an example of potential moral hazard.

It may well be desirable to place a monetary maximum on the amount of cover provided. This could provide protection against one of the big clubs, with a large share of the fees, being demoted. This is an example of how the insurer could limit its exposure to large claims and control the risks it writes.

(b) *Questions about the method of sale*

- What is our distribution method and target market?
- What commission is payable?
- What likely business volumes can we expect?
- Can we provide a bundled package of cover for other insurance too?
- What implications are there for administration and computer systems?

Comments

The target market is clearly defined here as the 22 clubs of the Ultimate League. There is no guarantee that all clubs will take out the cover, or that we are the only insurer that has been approached to provide cover. Hence, our potential sales volume ranges from 0 to 22 policies.

(c) *Questions on the method of rating*

- What exposure measure should we use?
- What are the risk factors?
- Can these be used for rating or do we need to use proxies?
- Will we need an end of year adjustment premium?
- What allowance should be made for expenses, commission, investment income, profit and contingencies?
- Is experience rating (*ie* the system by which the premium of each individual risk depends, at least in part, on the actual claims experience of that risk) applicable?

Comments

Possible exposure measures are:

- TV fees
- club season.

TV fees may not be known until the end of the season. Hence, if this is used, an end of year adjustment premium will be required. The initial premium paid could be based on an estimate, perhaps based on last season's share of the TV money.

Club season is a simple measure, but would leave a lot of residual heterogeneity requiring the use of further rating factors. Note that this is similar to the situation with private motor insurance.

Both these measures meet the criteria for exposure measures to be objective, measurable, simple and practical. They are also related to the underlying risk, *ie* two club-seasons would be double the risk of a one club-season.

Possible risk factors leading to demotion are:

- poor performance
- injuries
- lack of finance
- poor management
- the sale of good players.

Note that the last item is an example of possible moral hazard against the insurer. We will need to consider this when forming the policy conditions.

None of these factors are easily measurable or verifiable, so we will need to consider possible rating factors. These might include:

- past experience, *eg* position last year, as affecting likelihood of a claim
- recent share of TV fees, or number of TV appearances last season, as affecting the severity of any possible claim.

We should expect per policy expenses to be high, and can load for any commission we expect to pay.

Assuming premiums are received at the start of the season, and claims are paid after the end of the season, we should allow for the investment income that we can earn on the premium net of expenses, for that period.

Experience rating, in terms of position in previous years, is likely to be useful in predicting the probability of a claim.

(d) Questions about the possible claim profile

- What is the likely claim frequency and average amount?
- What sort of reporting and settlement delays do we expect?
- Is this a long- or short-tailed class of business?
- Are claims fixed or unlimited?
- What potential is there for large claims or accumulations?
- Do we need to purchase any reinsurance protection?
- What investigations are we able to carry out?
- What potential is there for selection against the insurer?
- Is there any potential for moral hazard?

Comments

Depending on number of policies sold, the claim frequency suffered by the insurer could be anything between 0% and 100%.

Claim amounts, assuming we limit indemnity to only one season's loss of fees, will be no higher than that of the largest club. We could reduce this amount further by imposing a monetary maximum.

Delays should be short. Teams will soon know when they have been demoted. Indeed, we may well be aware that this is likely several matches before the end of the season. Settlement delays should also be short. Hence this will be a short-tailed insurance product.

There is potential for very poor experience. We may only sell three policies, and all three clubs may be demoted. The insurer could use some form of non-proportional reinsurance to protect itself from this risk. How can it prevent itself from being selected against? What if only the clubs likely to be demoted take out cover?

Question 4.1

The chief executive designate of the Ultimate League Executives is also, coincidentally, a director of your company. He has unofficially proffered the following additional information:

(a) Taking out this insurance might be made compulsory for all clubs as a condition of joining the Ultimate League.

(b) Given the poor international record of the National Football team, there is a growing opinion amongst some of the players, managers and tabloid press in favour of a smaller league. There may, as a result, be a gradual reduction in the size of the Ultimate league from 22 to, say, 18 clubs over four seasons.

Comment on the implications for the insurance company, of these options in turn, assuming that all other details of the new form of insurance are agreed between the company and the chief executives.

Other valid approaches you could use to generate ideas here are:

- Working from a checklist of ideas, to check that you have not missed a possible different angle.

- Role playing. Imagine you are trying to calculate a premium for this policy. Think about the questions you would want answers to, and ways in which the insurer could be selected against. Alternatively, imagine you are the football club and think about your insurance needs or things you might do for your own benefit.

What we have set out here is far more than you might expect to generate on the basis of what you have seen so far. However, it does highlight the sort of thought processes required for Subject ST8.

The task, then, is to present the ideas generated in the time allowed, and in a suitable form for the examiner. This is a skill that you need to practise on Assignments, the Question and Answer Banks and past paper questions. Try to devise new forms of insurance, then think about how you would make them work, what the perils are, how you would rate them, *etc* ...

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Chapter 4 Solutions

Solution 4.1

(a) The big concern about selection is much reduced if all clubs have to take out this insurance. However, we do not know if the cover is compulsory with this insurer, or whether it has competition. If we have competition we may still be faced with the situation of insuring 3 clubs and seeing them all relegated. Alternatively, if we are insuring all 22 clubs we know that the claim frequency will be 3/22. The only uncertainty for the insurer will be the claim amounts.

(b) We will need to know how this is to be accomplished. Is it to be by demoting 3 and promoting 2 clubs, or demoting 4 clubs and promoting 3 for a period of years? Ultimately, would the claim frequency expected be 3/18 if we insured all clubs? At a first glance it might be assumed that there is no effect on potential claim amounts. However, if the same pot of money is to be shared amongst 18 rather than 22 clubs then average claim amounts could increase.

Chapter 5

Reinsurance products (1)

Syllabus objective

(c) Describe the main types of general reinsurance products and the purposes for which they may be used.

Covered in part in this chapter.

0 Introduction

0.1 What is reinsurance?

Insurance companies provide insurance to individuals and corporations against many risks (hurricanes, earthquakes, lawsuits, collisions, sickness and death, and so on).

Reinsurers also provide reinsurance to reinsurance companies (retrocession).

Chapter structure

In this chapter, we discuss reinsurance in general terms. In particular:

Section 1: Insurance for insurers and reinsurers

Section 2: Other participants in the reinsurance market

Section 3: Reasons for purchasing reinsurance

Section 4: Ways of writing reinsurance

Section 5: Traditional reinsurance – an overview

Section 6: Other types of reinsurance and alternatives to reinsurance – an overview

In the next chapter, we will consider the different reinsurance products in detail, as well as some alternatives to traditional reinsurance.

0.2 Reinsurance language

There is quite a lot of specialist language in reinsurance. Although you will soon get used to it, you will find it useful to be familiar with the following terms immediately:

cede	=	"pass on" or "give away", as in "cede some risk to a reinsurer"
facultative	=	"individual", as in an individually negotiated arrangement
treaty	=	covers a group of policies – reinsurance that the reinsurer is obliged to accept, subject to conditions set out in a treaty
direct writer	=	the insurer with a direct contract with the insured (as opposed to a reinsurer, who has a contract with the direct writer), also called the primary insurer or cedant.

1 Insurance for insurers and reinsurers

1.1 Reinsurance

Reinsurance is a form of insurance. It is a means by which an insurance company obtains from other insurance companies (reinsurers) protection against the risk of losses.

Reinsurance is very important, both in practice and as part of Subject ST8! It is important that you take your time over this chapter and the next chapter to make sure you are comfortable with what reinsurance is and why an insurer may purchase reinsurance, as well as the different types of reinsurance product.

1.2 Retrocession

Retrocession is also a form of insurance. However, rather than being bought by individuals (insurance) or insurers (reinsurance), it is bought by reinsurers.

Retrocession is reinsurance of reinsurance and is a very specialist area of the reinsurance market. A retrocession protection is necessary where a reinsurer that accepts substantial amounts of reinsurance business requires reinsurance protection itself.

The ceding reinsurer in a retrocession is called the *retrocedant* and the assuming reinsurer is called the *retrocessionnaire*.

2 Other participants in the reinsurance market

2.1 Brokers

Brokers primarily fulfil a sales role. Their purpose is to act for their client – the insurer – to obtain reinsurance on its behalf.

Reinsurance brokers may specialise in the reinsurance markets only, or else deal with insurance, reinsurance, retrocession and other financial products.

2.2 Direct reinsurance placements

Reinsurance need not only be handled by reinsurance brokers. In some international markets, professional reinsurers often deal directly with insurers in placing their reinsurance. These relationships could have come about historically based on business culture, customs, practices or language.

Therefore, it is not uncommon for reinsurance intermediaries (*ie* brokers) **to compete with professional reinsurers who offer the service of a broker directly to the client.**

2.3 Fronting

Fronting occurs when an insurer, acting as a mere conduit, underwrites a risk and cedes all (or nearly all) of the risk to another insurer which is technically acting as a reinsurer. The ceding or "fronting" insurer will typically receive a fee for its involvement to cover its expenses and profit.

Despite the additional fee, there are a number of reasons why the assuming insurer may use a fronting arrangement rather than underwrite the risk directly. Here are some examples:

- **It may not be licensed to write a specific line of business in a particular country or state of the USA.**

  In some territories, it is a legal requirement for either all or certain classes of insurance to be written by a local insurer. A fronting arrangement allows insurers outside the territory to effectively cover risks in such a territory.

  Sometimes, a single insured may have insurance coverage needs in many different states and the insurer may not be licensed in all of them. Fronting arrangements could then be used to "fill in the gaps" in the states where the insurer was not licensed.

- **Its credit rating may be inadequate to satisfy the insured's minimum requirements; for example because the insurer suffered a downgrade just prior to renewal.**

  If the fronting insurer has a higher credit rating than the assuming insurer, then the assuming insurer will be able to attract and retain more business.

- **There may be tax advantages in issuing the policy via the fronting insurer.**

  This could be the case if the fronting insurer operates in a different tax jurisdiction, *eg* Bermuda (which has a favourable tax environment).

Although the fronting insurer is not generally concerned with the profitability of the underlying business it does remain legally liable to the original insured. If the assuming insurer fails to meet its obligations as part of the reinsurance agreement, then the fronting insurer would still be liable for any claims incurring to the policy.

The fronting insurer should assess the credit risk of the assuming insurer before entering into the fronting agreement. The fronting company may require the risk-bearing party to provide some form of collateral to help mitigate the credit risk, for example a letter of credit.

Question 5.1

What other factors might the fronting insurer consider when assessing credit risk?

The Glossary defines a letter of credit as:

A financial guarantee issued by a bank that permits the party to which it is issued to draw funds from the bank in the event of a valid unpaid claim against another party.

In some jurisdictions, regulators do not look favourably on fronting agreements. Prevailing regulation should be considered before entering into a fronting agreement.

Question 5.2

How is fronting different from selling insurance business through an intermediary?

2.4 Captives

A captive insurance company may be established to self-insure insurance risk. Captive insurance companies are generally allowed more flexibility than traditional insurers in the creation of their reinsurance arrangements, taking advantage of local regulatory or tax regimes.

These will be described in detail in Chapter 7, General insurance markets.

3 Reasons for purchasing reinsurance

We introduced reinsurance in Chapter 1 and mentioned a couple of reasons why insurers might use reinsurance: to enable them to write large risks and to protect against accumulations. In this section we will revisit these reasons and consider some additional reasons for using reinsurance.

This section mentions types of reinsurance, including facultative, proportional, excess of loss and stop loss. You may have met some types of reinsurance (*eg* excess of loss (XL) reinsurance) in your earlier studies (*eg* Subject CT6), but if not, all will be revealed later in this chapter and in the next chapter.

3.1 Why purchase reinsurance?

This section considers the following reasons for using reinsurance:

- limitation of exposure to risk or spreading of risk
- avoidance of large single losses
- smoothing of results
- increasing profitability
- improving solvency margin
- increasing capacity to accept risk
- financial assistance
- availability of expertise.

You may find it useful to come back and re-read this section after you have completed this chapter and the next.

3.2 Limitation of exposure to risk or spreading of risk (single risks, aggregations of single risks, accumulations, multi-class losses)

An insurer with a portfolio of business faces an uncertain future. Even with the best advanced statistical analysis and experienced underwriters, the claims outgo from any period of risk cannot be predicted to any degree of accuracy. This is the reason for insurance: to cover the unknown financial consequences of possible future perils.

In general, where an insurer fears a loss or combination of losses that would materially (adversely) affect the results within its portfolio, it will consider reinsurance.

This adverse effect may be caused by a single event or from related events across portfolios. It may arise because an insurer is particularly exposed to a particular peril or has a concentration of risk in a particular territory.

These are the accumulations of risk described in Chapter 1.

Accumulations may be:

- geographic, *eg* as a result of floods, hurricanes, earthquakes, volcanic activity
- by peril, *eg* asbestosis claims from employers' liability business.

Thus the geographic location of a risk or group of risks is an important consideration in reinsurance purchases. An insurer must control its aggregations of many individual risks in one specific area or zone, especially if those risks are subject to natural peril events.

An insurer might also accept business from different sources.

Different sources might include fronting and reciprocal arrangements, which are discussed in the next chapter.

Therefore an insurer could have an accumulation of different interests on the same risk and reinsurance offers protection against these eventualities.

The above considerations could apply to any line of business.

Factors affecting an insurer's appetite to limit risk

An insurer's appetite for offsetting its risks through reinsurance is influenced by:

- **the size of the insurer**
- **the insurer's experience in the marketplace**

> ***Question 5.3***
>
> Describe briefly how the size of the insurer and the insurer's experience in the marketplace are likely to affect the amount of reinsurance a general insurer uses.

- **the insurer's available free assets**

  The larger a company's available free assets, the bigger cushion it has to absorb claims costs that are higher than expected. Hence the higher the free reserves, the less the need for reinsurance.

- **the size of the insurer's portfolio**

  Generally, everything else being equal, the larger the portfolio of business, the more credible the past claims experience will be and the more predictable the business outcome will be. Hence, the less the need for reinsurance.

- **the range within which the business outcome (or profit) can be forecast with confidence.**

Be aware that we have made some very sweeping statements here. There are many factors affecting the amount and type of reinsurance used and they should all be considered together. An overriding factor in one case may be a minor factor in a different case. For example, a company with relatively large free reserves may use more reinsurance than a company with relatively low free reserves because it writes business with very volatile claims experience.

Cost and availability of reinsurance

Against the above factors, the cost and availability of reinsurance has to be considered. For some risks at certain phases of the insurance cycle or after particularly large losses (for example, the terrorist attacks on New York and Washington on 11 September 2001), reinsurance cover may be restricted or only available at unattractive terms. Reinsurers may be reluctant to offer cover to an insurer because of its particular approach to underwriting and claims containment.

Just like insurance companies, reinsurers are in the business to make money, usually for their shareholders. They generally do not wish to write loss-making business. If there is great uncertainty about the future claims cost then reinsurers may be inclined not to offer reinsurance cover or at least charge much higher premiums than had previously been the case.

Question 5.4

When might reinsurers knowingly write loss-making business?

3.3 Avoidance of large single losses (for example, liability claim)

The size of a particular risk is very important. Reinsurance can help an insurer to reduce its involvement in very large risks to containable levels.

What is large to an insurer will depend on the size of the free assets available. Many risks in insurance have very high payout limits; some may even offer unlimited cover. Many small- to medium-sized insurance companies will cede a top layer of potentially large payouts to reinsurers as cover against this eventuality. This is especially true in liability lines of business where excess of loss reinsurance is commonplace.

We will introduce excess of loss reinsurance in Section 5.2. It is described in more detail in the next chapter.

A simple example of such use of reinsurance is for bodily injury claims on motor insurance. Liability claims can be very large, running to millions of pounds, euros, dollars *etc*. Insurance companies may want to limit or cap their exposure to such claims to help control the impact on the free reserves or profits.

3.4 Smoothing of results

By reinsuring the larger risks or accumulation of smaller risks above certain limits, the development of financial results (or profit) can be smoother year-on-year, especially when the portfolio is relatively immature.

For a portfolio that is relatively immature, experience will be uncertain and potentially volatile. Even for classes where claims are fairly stable and predictable, an immature portfolio may be relatively small and so may lack diversification.

Reinsurance provides stability by reducing the potential for fluctuations or variations from the planned result when losses are subject to variations in number and/or size. Purchasing reinsurance will usually reduce profits, but it might also protect an insurer against severe losses.

The insurer pays a premium to mitigate these fluctuations, and its net result is more predictable. This predictability may be more acceptable to shareholders and regulators. Stop loss is a form of reinsurance that could be used for these purposes.

Stop loss will be described in more detail in the next chapter, but as its name suggests it reduces extreme losses directly and hence helps to smooth profits. However, stop loss isn't the only type of reinsurance that will smooth results. Nearly all types of reinsurance will smooth profits to some degree. This might be achieved by simply limiting claim amounts (as mentioned above) using excess of loss, or by enabling the insurer to write a larger number of smaller risks and hence achieve a greater diversification of risk.

3.5 Increasing profitability

Reinsurance can increase the opportunity for an insurer to make a profit and plan its business more accurately.

In the long term, reinsurance premiums are likely to be higher than the reinsurance recoveries (payments from the reinsurer to the insurer in respect of claims) – the difference covering the reinsurer's expenses, profit margin *etc*. However, in any one year, the insurer's profits might be higher as a result of having reinsurance, *eg* in years where claims recoveries are high.

As discussed in the previous section, reinsurance should smooth results year-on-year by reducing claim fluctuations. Increased certainty should enable the insurer to plan more effectively, which may increase profits in the future.

Example

Increased certainty may reduce the amount of capital the insurer needs to hold, and so enable it to write a greater volume of profitable business. Similarly, increased certainty might enable the insurer to adopt a riskier investment strategy, and increase the expected return on investments.

3.6 *Improving solvency margins*

Solvency depends on the relationship between an insurer's solvency margin and the total premium income that insurers generate from all the lines of insurance business written. Governments everywhere use financial regulation to protect consumers. Calculating an insurer's solvency margin is an important part of financial regulation and allowance is often made for reinsurance in this calculation.

Reinsurance is an accepted management tool to improve an insurer's solvency margin. It helps an insurer to plan its operations more effectively and strengthen its balance sheet.

If we interpret "solvency margin" as the excess of the value of the assets over the value of the liabilities, then we can see that in order to improve the solvency position of the insurer, it is necessary to either:

- increase the value of the assets
- decrease the value of the liabilities
- decrease the regulatory minimum difference between the assets and liabilities
- some (or all!) of the above.

Reinsurance can help to achieve this.

Increasing the value of the assets

We shall see later that by obtaining certain types of reinsurance, the insurer may be able to increase the value of the assets. This is described in Section 3.8.

Decreasing the value of the liabilities

By reinsuring the business, the insurer is reducing the value of its liabilities (as some if its liabilities are ceded to the reinsurer). Therefore, as a result of the reinsurance, it will hold smaller reserves, and so the solvency position of the insurer will improve, although of course assets will be reduced by the size of the reinsurance premiums paid.

Decreasing the regulatory minimum difference

The required solvency level is often calculated with reference to the proportion of business reinsured. In other words, more reinsurance means a lower solvency requirement, and therefore a stronger solvency position. However, this reduction may be subject to a limit, since there may be a legal requirement for an insurance company's free reserves to exceed a *Required Minimum Margin (*RMM). (We mentioned this in Chapter 1.) In other words, you can only reduce the RMM so far and there will be a minimum level that you still need to hold.

Question 5.5
Why might the allowance for reinsurance be limited?

3.7 *Increasing capacity to accept risk (that is, underwriting capacity: singly or cumulatively)*

Owing to insufficient capital backing, an insurer may be reluctant to accept, or be incapable of accepting, particular risks by sector or by volume.

For example, an insurer may be unable to accept a risk that is larger than its financial strength allows.

An insurer may also be reluctant to accept a particular risk if it would be exposed to an accumulation of risk as a result.

In order to accept new business, the insurer must have adequate capital to set aside to cover the risk. Larger risks will require more capital to be set aside. Therefore very large risks may require more capital than an insurer has available, in which case the insurer would be unable to accept the risk.

Reinsurance cover can prevent this situation from occurring. The solvency requirements for a particular line of business are normally reduced in line with the proportion ceded, although this may be subject to a limit.

Reinsurance can increase the size of an insurer's underwriting capacity, allowing it to compete more effectively in the market.

3.8 Financial assistance (new business strain, bolstering free assets, merger / acquisition)

Reinsurance funds can assist financially with particular business propositions. Where a particular distribution strategy would involve substantially more cash outflow in the initial stages than premium income, reinsurance commission may be available in return for future surplus streams. In effect, the reinsurer lends now (by paying commission) against the predicted future flows of premiums less expenses and claims.

Proportional reinsurance allows this reimbursement. As companies grow and become stronger, they tend to use more excess of loss and less proportional reinsurance.

Proportional reinsurance and excess of loss are described later in this chapter. We will also see that there are also other types of reinsurance that can be used to improve the free asset position that involve little or no risk transfer from the insurer to the reinsurer. Such contracts are referred to as financial reinsurance or finite risk reinsurance and are described in the next chapter.

An insurer could use a reinsurer as a partner to support its new operations. It can share developmental and operational costs with the other party.

New business strain / financing projects

Here, the reinsurer is paying some money to the insurance company to improve its cash balance, for example, to:

- enable it to write more business (*ie* to help meet new business strain)
- set up a new distribution channel or computer system (or both if using the internet!).

This payment is effectively a loan but is "disguised" as a commission payment for the reinsurance. In return, the insurer will repay the loan out of the profits it makes on the underlying business.

This type of arrangement may be more attractive to the insurer than, say, a simple bank loan, because the repayments will only be made if the company does well and profits are made. There is also an accounting advantage, since the repayments are contingent on profits arising so that such reinsurance arrangements do not appear as debt on the balance sheet, whereas a bank loan does.

Bolstering free assets

Where a block of renewing business which is producing regular profits is identified, capital can be found for the insurer to bolster the free asset position by reinsuring this portfolio of profitable policies. The reinsurer would pay an initial commission in return for which the reinsurer would be entitled to the future surplus of premiums over claims for as long as the arrangement remained in place.

The arrangement mentioned here is identical to that described above except that it's for a different reason. The reinsurer still loans some money to the insurer and the loan is repaid out of future profits. Because the insurer has no liability to repay the loan unless a surplus has been made, it does not have to reserve for the future payments. So it has increased its assets by the amount of the loan but not increased its liabilities and hence has improved its free asset position.

However, the extent to which this is possible will depend on the precise requirements of the supervisory regime concerned.

Merger / acquisition

In a similar way, reinsurance can be used to facilitate the acquisition of an insurance company. In this case, a profitable subset of business (or all policies) is identified within the company being acquired so that the reinsurer is prepared to advance funds in anticipation of future surplus.

In these last two cases, quota share reinsurance is the norm, though any form of proportional reinsurance might be used.

3.9 Availability of expertise to develop new markets and products

International reinsurers often have considerable knowledge and can provide technical services and expertise in many insurance products in different countries around the world.

Sometimes an insurer wishes to enter a new insurance market at home or overseas. It could use a reinsurer's knowledge in the development and operation of any new insurance products.

In addition, if an insurer is considering underwriting new risks, unusual risks, or risks in new territories, the reinsurance broker can provide assistance.

Often it is the broker who is the first port of call for reinsurance expertise.

For example, when an insurer adopts a strategy that will take it into new risk areas where it has little previous experience, the reinsurance broker can sometimes help with rating, underwriting and claims management.

When monitoring the policy content, pricing, marketing, sales, sources of acquisition and results of any new insurance product, it can use valuable knowledge to help future development of its business.

In each of these cases, quota share reinsurance is usual, though any form of proportional reinsurance might be used.

3.10 Reasons for retrocession

Question 5.6

Without reading on, which two of the reasons for purchasing reinsurance do you think are most relevant in the context of retrocession?

The retrocedant purchases the retrocession to gain additional capacity or to contain or reduce its risk of loss on a specific or aggregate basis. In particular, a reinsurer would require a retrocession protection if it has limited capital and is heavily exposed to specific geographical regions and/or classes of business.

Typically, retrocession protections are written on an excess of loss basis. Retrocession protections can protect the retrocedant's facultative, proportional or excess of loss business, or a combination of the three, which may lead to spiral of cover.

A spiral of cover can occur when reinsurers write retrocession business on risks that they already have exposure to through other reinsurance arrangements. They can therefore (sometimes unknowingly) end up reinsuring themselves.

4 Ways of writing reinsurance business

How are insured risks reinsured?

- **Individually, (that is, facultative reinsurance).**
- **Groups of similar risks, (that is, treaty reinsurance).**

Insurers cede risks either facultatively or by treaty according to how they weigh up the advantages and disadvantages of each arrangement.

4.1 Facultative reinsurance

Facultative reinsurance is the reinsurance of a single risk.

When each individual risk on which reinsurance is required is offered separately to a reinsurer, the risk is said to be offered facultatively. There is no obligation for the ceding company to offer the business, nor is the reinsurer obliged to accept it. Each case is considered on its own merits and the reinsurer is free to quote whatever terms and conditions it sees fit to impose for that risk.

For example, the direct writer may be asked to provide cover for a large risk. While negotiating terms with the potential policyholder, the direct writer will seek out a willing reinsurer that is prepared to accept a portion of the risk on the best terms.

Advantages and disadvantages

The main advantage of facultative reinsurance is the flexibility that both parties have within the process. For example, the direct writer is under no obligation to use a particular reinsurer. The direct writer can approach several reinsurers in search of the best terms for each risk individually. Similarly, the reinsurer is under no obligation to accept risks.

The main disadvantages to the insurer of facultative reinsurance include:

- **it is a time-consuming and costly exercise to place such risks**
- **there is no certainty that the required cover will be available when needed**
- **even if cover is available, the price and terms may be unacceptable**
- **the primary insurer may be unable to accept a large risk until it has been able to find the required reinsurance cover. This means the insurer cannot accept business automatically when it is offered, and consequently its standing in the market may be reduced.**

This form of reinsurance placing is therefore usually limited to large one-off risks, or to risks that fall outside the scope of any treaty reinsurance arrangements available to the primary insurer.

4.2 Treaty reinsurance

Arranging reinsurance for each individual risk is administratively messy. Therefore direct writers may instead set up *treaties* with reinsurers. This allows them to place business with the reinsurer automatically. The terms and conditions of the treaty are carefully laid down so that both parties know exactly where they stand. An insurer will also wish to use reinsurance to control its solvency and growth requirements. This can be done more readily by treaty than by facultative reinsurance.

Treaty reinsurance is the reinsurance of a group of similar risks under one reinsurance agreement.

Treaties are usually arranged so that the ceding insurer is obliged under the terms of the treaty to pass on some of the risk in a defined manner and the reinsurer is obliged to accept it.

This is known as an obligatory / obligatory (oblig / oblig) basis: in other words, the insurer is obliged to pass the risk on and the reinsurer is obliged to accept it. Such an arrangement is common in quota share treaties, which are covered in the next chapter.

Occasionally, treaties may be written on a facultative / obligatory basis. Under such an arrangement, for each risk, the insurer has the choice of whether to include it in the treaty, but the reinsurer is obliged to accept all the requested risks. Such treaties are normally associated with reciprocal arrangements, whereby each insurer reinsures a block of business with the other.

Here, the direct writer is under no obligation to use a particular reinsurer to cover the risk, but the reinsurer has to accept the risk if offered it, subject to the terms of the treaty. Potentially, with fac / oblig treaties there is scope for a direct writer to select against a reinsurer because the direct writer can pick and choose which risks to keep and which to reinsure. Such an arrangement is common in surplus treaties, which are covered in the next chapter.

Question 5.7

What is likely to happen if the direct writer manages to select against the reinsurer so that the reinsurer's experience is much worse than the direct writer's experience?

Advantages and disadvantages

A treaty does not have the disadvantages mentioned above for facultative reinsurance and insurers therefore tend to place the bulk of reinsurance cover on a treaty basis.

The features of treaties are very much the reverse of the features of facultative reinsurance:

Efficient: Risks are reinsured automatically. This is administratively quicker and cheaper.

Certain: With a treaty, the direct writer knows that reinsurance is available (if the risk falls within the limits of the treaty) and on what terms.

Inflexible: Once the treaty is set up, then both parties must operate within the terms of the treaty (so each should ensure it is happy with the terms before it signs the treaty).

Terms within the treaty

It is essential for both the direct writer and the reinsurer that the treaty is absolutely precise in its definition of:

- what is and what is not covered
- the financial arrangements (*ie* premiums, commissions, timing of payments)
- the obligations of both parties.

The treaty document sets out all the relevant details and obligations under the arrangement, though the wording differs for different types of reinsurance. There will also be wide differences in wording between treaties in any one category.

Question 5.8

Try to list twenty items that might be included in a reinsurance treaty.

5 Traditional reinsurance – an overview

So far we have discussed facultative and treaty methods of reinsurance, *ie* the methods by which reinsurers agree to accept risks. We have not discussed the mechanisms by which the premium and claims payments under a reinsurance contract are passed to the reinsurer. These are now discussed below.

There are two main methods of reinsurance:

- **proportional** – whereby the direct writer cedes a proportion of the risk, passing on a proportion of the premium, and the reinsurer pays that proportion of the claims
- **non-proportional** – whereby the direct writer pays the reinsurer an agreed premium, and in return the reinsurer pays out for claims between specified limits.

In each case, the reinsurer will usually obtain business through specialist brokers who will receive commission for placing the business.

This commission is paid by the reinsurer and is usually called brokerage.

5.1 Proportional reinsurance

Under proportional reinsurance, the reinsurer covers an agreed proportion of each risk and the reinsurance premium is proportional to this risk ceded.

The proportion may be constant for all risks covered (that is, quota share) or may, to some extent, be at the discretion of the ceding insurer (that is, surplus). Both forms have to be administered automatically, and therefore require a treaty.

Quota share and surplus are discussed in detail in Chapter 6.

Proportional reinsurance reduces the size of the ceding insurer's net account, and so it can be used as a means of accepting a larger size of risk than would otherwise be possible.

By purchasing proportional reinsurance, the insurer can retain a smaller share of a larger number of risks and hence increase the diversification within its portfolio, reducing the volatility of the overall result.

5.2 Non-proportional reinsurance

Although proportional reinsurance can be used to spread risk and to reduce pro rata the size of risk retained, it does not cap the cost of very large claims that occur.

Under non-proportional reinsurance, the reinsurer covers the loss suffered by the insurer that exceeds a certain amount, called the *retention* or *priority* up to the policy limit.

Under excess of loss (XL) reinsurance, the cost to an insurer of a large claim is capped with the liability above a certain level being passed to a reinsurer. However, if the claim amount exceeds the upper limit of the reinsurance, the excess will revert back to the insurer. Variations of this form of reinsurance cover exist to limit an insurer's loss from a single event or over a given period.

Non-proportional types of reinsurance include risk excess of loss, stop loss or aggregate excess of loss reinsurance.

Within non-proportional:

- The limit might operate on individual claims (*eg* to cover against large individual risks), or on aggregations of claims (*eg* to cover against widespread damage from one event).
- There might also be an upper limit, above which the reinsurer's liability ends.
- The reinsurer might pay for all claims within the limits or perhaps a proportion of the claims within the limits (*eg* 90%). This could also be expressed as having a deductible of 10%.
- The limits might be linked to inflation.

Question 5.9

For what types of individual risk is non-proportional reinsurance likely to be particularly useful?

5.3 Different reinsurance bases

The natural basis for writing proportional reinsurance policies is to provide cover for those policies written by the primary insurer during the term of the reinsurance contract (that is, a policies-incepting basis). Claims-made policies and losses-occurring policies may also be written, although the latter in particular are more commonly used for non-proportional reinsurance.

Claims-made and losses-occurring policies were discussed in terms of insurance bases in Chapter 2.

Policies-incepting basis

With this type of arrangement the reinsurer provides cover to the direct writer for the claims arising from all policies written under the treaty over a period, *ie* corresponding to an "underwriting-period" cohort. This is the natural arrangement with all proportional types of reinsurance: all policies written during the same period will be subject to the same proportion ceded.

Question 5.10

Why is proportional reinsurance likely to be arranged on a policies-incepting basis?

Losses-occurring basis

This basis provides the direct writer with cover for any claim incident(s) under the treaty occurring within a defined period, *ie* corresponding to an "accident-period" cohort. This is commonly used with non-proportional types of reinsurance: any claims occurring within a certain period will be subject to the same retention level and upper limit.

Claims-made basis

This basis would provide the direct writer with cover for any claims reported under the treaty reported to them within a defined period, *ie* corresponding to a reporting-period cohort.

Question 5.11

What is the main issue that the reinsurer should consider before providing cover on a claims-made basis?

6 Other types of reinsurance and alternatives to reinsurance – an overview

Other reinsurance products include:

- finite risk reinsurance
- run-off reinsurance.

Capital market products may also be available as an alternative to reinsurance.

These are discussed in detail in the next chapter.

7 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Broker
- Captive
- Ceding company (cedant)
- Co-reinsurance
- Direct business
- Excess of loss (XL or XOL) reinsurance
- Facultative-obligatory reinsurance
- Facultative reinsurance
- Inwards reinsurance
- Letter of credit
- Net premium
- Non-proportional reinsurance
- Outwards reinsurance
- Primary insurer
- Proportional reinsurance
- Reciprocity
- Retrocession
- Retrocessionaire
- Treaty reinsurance.

Chapter 5 Summary

Reinsurance

Reinsurance is a means by which an insurance company can protect itself against the risk of losses by ceding the risk to other companies (reinsurers).

Retrocession is a means by which a reinsurance company can protect itself against the risk of losses with other reinsurance companies (retrocessionaires).

Participants in the reinsurance market

Reinsurance business is often sold by reinsurance intermediaries, *ie* brokers. Their role is primarily a sales role, using their specialist knowledge and their contacts in order to get the best reinsurance price for their clients. They may also provide expertise.

An alternative to reinsurance intermediaries is professional reinsurers dealing directly with buyers.

Fronting occurs when an insurer underwrites a risk and cedes it all (or nearly all) to another insurer. The fronting insurer will receive a fee.

A captive insurance company may be established to self-insure risk. This often has regulatory and tax advantages.

Reasons for reinsurance

The main reasons for reinsurance are:

- *limitation of exposure to risk or spreading of risk* (single risks, aggregations of single risks, accumulations, multi-class losses) – the need for reinsurance will depend on the size of the insurer, its portfolio, its level of free assets, its experience in the marketplace and the type(s) of business written
- *avoidance of large single losses* (*eg* liability claim) – what is "large" will depend on the size of the insurer and the free assets available
- *smoothing of results* – reinsurance reduces the potential for fluctuations if losses are subject to increases in number and/or size
- *increasing profitability* – reinsurance can increase the opportunity for an insurer to make a profit and plan its business more accurately
- *improving solvency margin* – reinsurance helps an insurer to plan its operations more effectively and strengthen its balance sheet

- *increasing capacity to accept risk* (singly or cumulatively) – this can enable the insurer to compete more effectively in the market
- *financial assistance* (new business strain, bolstering free assets, merger / acquisition) – the reinsurer can lend the insurer cash now, which will be repaid out of the insurer's future profits
- *availability of expertise to develop new markets and products* – expertise may be needed if the insurer is entering a new market or writing new risks, unusual risks, or in new territories.

The benefits provided by reinsurance must be weighed up against the cost and availability of the reinsurance.

The reasons for retrocession are to gain additional capacity or to contain or reduce risk of loss (singly or cumulatively).

Ways of writing reinsurance business

Facultative reinsurance is the reinsurance of a single risk. There is no obligation for the ceding company to offer the business and no obligation for the reinsurer to accept it. It is a flexible way of writing reinsurance business, but is time consuming and uncertain in terms of availability of reinsurance and price.

Treaty reinsurance is the reinsurance of a group of similar risks under one reinsurance arrangement. The reinsurer is obliged to accept risks that fall under the terms of the treaty. The treaty document sets out all relevant details and obligations.

Traditional reinsurance

Reinsurance may be:

- *Proportional* – the reinsurer covers an agreed proportion if each risk and the reinsurance premium is proportional to the risk ceded
- *Non-proportional* – the reinsurer covers the loss suffered above the insurer's retention, possibly subject to an upper limit.

Proportional reinsurance is usually written on a policies-incepting basis. Non-proportional reinsurance is often written on a losses-occurring basis. A third option is a claims-made basis.

Chapter 5 Solutions

Solution 5.1

A fronting insurer might also consider the following factors relating to the assuming insurer:

- size
- solvency level
- attitude / strength of management
- existence of a parent company
- types of business written
- level of expertise / experience with different classes of business.

Solution 5.2

A fronting insurer remains legally liable to the original insured, however an intermediary does not. An intermediary may also provide expertise to the seller of reinsurance.

Solution 5.3

Size of insurer

Larger insurance companies will generally have larger free reserves and a relatively more diversified portfolio of business and hence will be less exposed to random fluctuation of claims experience. As a result they may require less reinsurance.

However, it could be argued that larger companies are more likely to write types of business that "need" reinsurance – *ie* those with large and/or unusual risks. Also, large companies may be more prone to accumulations.

Experience in the marketplace

If a company has a lot of experience in the marketplace then it will hopefully have sufficient credible data to be able to make an objective estimate of the expected claims outgo when setting premium rates and reserves. It should also be able to set sensible policy conditions to help control the risk. As a result the company will be less likely to use reinsurance to help with these areas as well as generally limit the claims exposure.

Solution 5.4

A reinsurance company may write loss-making business if it expects either to obtain compensating higher future profits or profits from other connected business (from the same cedant, or perhaps the same reinsurance broker).

At the bottom of the (re)insurance cycle, premiums across the whole market will be low and so in order to retain market share reinsurance companies will be forced into accepting loss-making business. Reinsurers will hope that premiums will soon start to increase and the business becomes profitable once more.

Also, it may write some products as a loss leader knowing that it will also be able to sell other more profitable business on the back of the initial sales.

Solution 5.5

The limit might exist to protect against the risk of reinsurer default. If the reinsurer was unable to pay its claims then the insurance company would still be liable to pay the original claims to its policyholders. Therefore, even if an insurance company reinsured all of its business, it would still need to hold some free reserves in case things went wrong.

Solution 5.6

The most relevant reasons are:

- limitation of exposure to risk (single risks, aggregations of single risks, accumulations, multi-class losses)
- increasing capacity to accept risk (singly or cumulatively).

Since reinsurers tend to be larger than insurers:

- they have less need to avoid large single losses completely
- their business is likely to be more diversified, leading to smoother result
- their solvency margin may already be adequate – in fact, it may be subject to different regulations, so that solvency requirements are not as onerous
- they are less likely to need financial assistance.

They are also likely to have a fair amount of their own expertise.

Solution 5.7

The reinsurer is likely to increase premiums or change the terms of the treaty in favour of the reinsurer at the next renewal date of the treaty.

The reinsurer may even decide not to reinsure the direct writer again.

Solution 5.8

Depending on the type of reinsurance contract, the details set out in the treaty will include some of the following:

- names of the parties to the treaty
- period of cover
- territorial limits
- class(es) of business covered
- exclusions to the cover
- definition of loss occurrence
- retention of the ceding company
- cover granted automatically by the reinsurer
- reinstatement provisions
- a stability clause
- premium rate
- premium payment arrangements
- ceding commissions payable
- profit commission payable to the direct writer and the method of calculation
- commissions payable to reinsurance brokers
- claim notification arrangements
- claim payment arrangements, including special arrangements for large claims
- rendering and settlement of accounts
- a currency clause (if more than one currency is involved)
- access by the reinsurer to risk details
- terms for termination of the treaty (period of notice, *etc*)
- sunset clause (specifies the discovery period)
- an arbitration clause, in case of disagreements arising.

Check the Glossary if you are not familiar with any of the terms above.

Solution 5.9

For classes where claims are potentially very large or even unlimited. Claims from liability classes of business fall into this category.

Solution 5.10

With proportional reinsurance the direct writer cedes a specified portion of a risk to the reinsurer. It is natural that they provide cover in the defined proportions throughout the duration of the risk. That way both the division of cover into different accounting periods and the dates when any claims are reported become irrelevant.

Solution 5.11

Unexpectedly large numbers or amounts of claims may be reported in a given period. This could be as a result of latent types of claim, *eg* asbestosis and industrial deafness, that have come to light some time after the claim incidents occurred.

Chapter 6

Reinsurance products (2)

Syllabus objective

(c) Describe the main types of general reinsurance products and the purposes for which they may be used.

Covered in part in this chapter.

0 Introduction

In this chapter, we look in detail at the different types of reinsurance product.

We start by looking at traditional reinsurance products. Proportional and non-proportional reinsurance were introduced in the last chapter.

Section 1: Quota share reinsurance (proportional)
Section 2: Surplus reinsurance (proportional)
Section 3: Excess of loss reinsurance (non-proportional)
Section 4: Stop loss (non-proportional)

We then go on to discuss other types of reinsurance products, which have been developed more recently to meet specific needs of insurers.

Section 5: Finite risk (or financial) reinsurance
Section 6: Run-off reinsurance

Reinsurers may not have the capacity to meet all an insurer's needs in terms of limiting exposure to risk and providing financial assistance, therefore the insurer may have to seek assistance elsewhere. We conclude the chapter by looking at some alternatives to reinsurance, available from the capital markets.

Section 7: Capital market products

1 Quota share reinsurance

1.1 How quota share operates

Quota share reinsurance is a form of proportional reinsurance where the apportionment of claims and premiums between the insurer and reinsurer is constant for all risks covered by the treaty.

Quota share is always by treaty. It is a simple mechanism for ceding a portion of the business.

Suppose the direct writer decides that for a particular class it does not want to retain all the business it writes. So the direct writer enters into a quota share treaty with a reinsurer. Then a constant proportion of each and every risk within the scope of the treaty is automatically passed to the reinsurer. The treaty will specify the proportion to be ceded to the reinsurer, R% say. This is often referred to as an R% quota share treaty.

Premiums

The direct writer passes to the reinsurer R% of the policyholders' premiums for risks covered within the treaty. (In general insurance, under proportional reinsurance everything is shared with the reinsurer in the specified proportion. Note that this is not always the case for life and health products, where the premium may not be split in the same proportion to the claims.)

Claims

The direct writer recovers from the reinsurer R% of the claims from risks covered by the treaty.

Commission

The direct writer (the cedant in the reinsurance arrangement) will have suffered expenses in order to sell the original policies (for example, commission to brokers). The reinsurer will reimburse the direct writer with some percentage of the premium to help cover the acquisition expenses. This payment is often called *return commission*.

For example, consider a policy with an original insurance premium of £100. If a 40% quota share treaty is in place with a return commission of 4%, then the return commission (paid from the reinsurer to the insurer) is £1.60 (4% of 40% of £100).

To compensate the direct writer for the extra work that the direct writer will have carried out in attracting and administering the business, the reinsurer may pay a further commission payment to the direct writer. The commission over and above the return commission is sometimes called *override commission*. The sum of the return and override commission is often called the *ceding commission* (although sometimes the return commission alone is called the ceding commission).

To add to the confusion, the Glossary states that sometimes the return commission is called the override commission. Whichever terminology you use in the exam, be sure to define it!

In practice, the ceding commission is rarely split up between return and override commission but hopefully the total will be sufficient recompense for the expenses incurred by the insurer. However, market forces (*eg* competition from other reinsurers) and the profitability of the reinsurance contract will be the most important factors that influence the total amount of commission that is paid.

There might also be a form of *profit commission*, which the reinsurer pays the direct writer as a reward for passing on good business. This would be calculated (on a pre-agreed basis) at the end of the year or end of the treaty when the claims experience is known.

In addition, brokerage would usually be paid out by the reinsurer if the reinsurance has been arranged through a reinsurance broker.

Question 6.1

A direct writer has a 50% quota share treaty with a reinsurer.

For a risk with a gross premium of $10,000, the direct writer pays commission of 15% and receives 15% return commission from the reinsurer. The reinsurer also pays 5% override commission.

Calculate how much premium is received by the direct writer and the reinsurer *after* allowing for commissions. Do these relative levels of premium look fair?

If there is subsequently a claim for $50,000, how much will the direct writer be able to reclaim from the reinsurer?

Practical considerations

The administration of quota share is very straightforward.

Because each risk has the same proportion reinsured, there is no need for anything more than a list of all the risks with total premiums and total claims. The premiums and claims payable between the direct writer and the reinsurer are a straight proportion of the totals.

1.2 Cedant's claims experience

The cedant's loss ratio for the account will be approximately the same before and after reinsurance, since the premiums and claims are ceded in the same proportion, depending on the treatment of commissions.

Note that the commissions paid by the reinsurer to the insurer can vary depending on, for example, market practice and the relationship between the reinsurer and insurer. Therefore, the reinsurer's loss ratio can be different from the insurer's loss ratio. However, it is common practice to ignore commission (reinsurance or otherwise) when calculating loss ratios, in which case the loss ratios would be identical.

1.3 Reinsurer's claims experience

The cedant and reinsurer will have proportionately the same overall underwriting experience on the business included in the treaty, apart from differences in expenses and commissions. The reinsurer will therefore be concerned at the outset to establish:

- **the nature of the business being offered**
- **the cedant's attitude to underwriting and claims settlement**
- **any previous experience of this business.**

If the reinsurer is unhappy with any of these aspects, it can refuse to participate in the treaty or ensure that the level of commission written into the treaty is low in expectation of poor experience. On occasions, however the reinsurer may reserve the right in the treaty to be involved in the approval and settlement process for claims above a certain size. The reinsurer might also negotiate for part of the commission payment to the cedant to be a profit commission that is payable only if the business ceded meets specified profitability criteria.

Since the capacity of the reinsurer to write such business may be limited, the treaty may specify a limit on the amount of business that may be ceded to the treaty. This limit will normally be expressed in terms of the original gross premium income of the cedant for that business.

1.4 Advantages of quota share

The main advantages of quota share to the direct writer are that:

- **It spreads risk, enabling insurers to write larger portfolios of risk and encourage reciprocal business.**

  At one level, the principle is very simple. An insurer's risks are better spread by retaining 50% of 1,000 risks than 100% of 500 risks.

  Quota share can also help spread risks through *reciprocity*. This is where two insurers agree to swap some business on a reciprocating basis.

Example

Company *A* may have a 40% quota share treaty with Company *B* for business it writes directly, while Company *B* has a similar quota share treaty with Company *A*. Provided the two portfolios are not perfectly correlated, both companies will have achieved a better spread of risks. An example of this would be two household insurers, covering different geographic areas, spreading risk to limit the impact of storms.

  A large, established insurer will be far less dependent (if at all) on quota share as a means of spreading risk than a small, new insurer. However, large insurers often employ quota share extensively for marine and some aviation business to spread the risk, or reciprocate business in smaller classes.

- **It directly improves the solvency ratio and helps the insurer to satisfy the statutory solvency requirement.**

  The solvency ratio is normally defined as free assets divided by net written premiums. This will be looked at in more detail later in the course. Using quota share will reduce net written premiums and hence increase the solvency ratio in the short term.

Many statutory solvency requirements are based on net written premiums. The legislation may require that the company has a solvency margin at least as great as a defined percentage of net written premium. For example, in the UK, the solvency margin requirement is close to 16%. Quota share can be used to directly reduce this type of solvency margin requirement.

In addition:

- **it is administratively simple**
- **the commission may help with cashflow.**

1.5 *Disadvantages of quota share*

The main disadvantages to an insurer of ceding business by quota share are that:

- **It cedes the same proportion of low-variance and high-variance risks.**

  It is often totally inflexible, since a set percentage of all risks must be passed to the reinsurer, whether or not the direct writer wants to cede the particular risk.

- **It cedes the same proportion of each risk, irrespective of size. The insurer may, however, wish to cede a greater proportion of the larger risks than the smaller ones, owing to their greater loss potential.**

  It is not a good way of protecting against catastrophes or very large claims. Note that it does not cap very large claims, since insurers are still liable for a percentage of that very large claim.

- **It passes a share of any profit to the reinsurer.**

Question 6.2

Define, briefly, quota share reinsurance.

2 Surplus reinsurance

2.1 How surplus operates

Surplus reinsurance is a form of proportional reinsurance where the proportions are determined by the cedant for each individual risk covered by the treaty, subject to restrictions defined in the treaty.

The mechanics for operating surplus can at first be a bit confusing. However, if you take a common sense approach, then it becomes much clearer. In the following explanation, the principles are described without using the usual terminology for surplus reinsurance. We will introduce these afterwards.

"Common sense" approach

Let's start by imagining that there is a direct writer who specialises in industrial fire risks. The company's size and free reserves are such that risks greater than £200,000 are too big. A problem faced by the company is that the number of buildings the company can insure is severely limited. Without any form of reinsurance, the company would have to decline all risks over £200,000.

So it sets up a surplus reinsurance treaty which allows it to cede to a reinsurer the share of the risk that it doesn't want to keep.

Suppose that it insures a property with a maximum loss of £400,000. The direct writer might choose to keep only 50% (so that the retained risk is limited to £200,000) and 50% of the risk is ceded to the reinsurer. For this particular risk, the direct writer passes on 50% of the premium. The reinsurer is then obliged to cover 50% of all claims from this risk.

The next property has a maximum loss of £1,000,000. The direct writer passes on 80% of the risk to the surplus reinsurer, retaining 20%. In this way, the direct writer has effectively engineered another property risk of size £200,000.

Note that the insurer is not paying all claims up to £200,000 and passing the excess to the reinsurer – since that would be excess of loss reinsurance. It is instead using the £200,000 to determine the proportion of risk it wishes to keep. So in the second example, the reinsurer would pay 80% of all claims, whatever their size.

For some risks, the insurer may choose to keep *less than the maximum* retention. With the maximum loss of £1,000,000, it might choose to retain only 10% of claims and engineer the risk down to £100,000. The retention does not have to be the same for all risks – which differs from the surplus reinsurance that you may have met in *life* reinsurance.

Question 6.3

A direct writer has a limit of £50,000 exposure to any one risk. Assuming there is surplus reinsurance that will absorb all the risks not wanted by the direct writer, and that the direct writer retains the maximum for each risk, calculate the premiums due to and the claims due from the reinsurer for the following risks:

Risk	Size	Gross premium	Claim information
1	£300,000	£4,500	no claims
2	£450,000	£6,000	one claim for £80,000
3	£60,000	£1,500	no claims
4	£40,000	£700	no claims
5	£1,000,000	£12,000	no claims
6	£500,000	£7,200	one claim for £50,000

If you got the right answer to the last question, you probably understand the principles of surplus reinsurance. Let's now define surplus reinsurance using the correct terminology.

Measuring the risk: estimated / expected maximum loss (EML)

We have used the expression "size" in the previous examples when describing different risks. But how would size be measured?

It *could* be sum insured. However, this is not necessarily the basis a company would use when deciding on whether a risk is too large. Consider two different risks:

Risk 1: 30 individual buildings spread around a site. Total sum insured of £15 million.

Risk 2: One large office block. Total sum insured of £10 million.

A single insurer could conceivably insure all 30 buildings, but decline the office block on the grounds that it was too big. So how can £10 million be "bigger" than £15 million? The office block is bigger in that there is a much more concentrated risk. The possible loss from one incident with *Risk* 2 may be much greater than the possible loss from one incident involving *Risk 1,* which might typically involve a loss of about £½ million.

For the purpose of determining the proportion of each risk to be retained or ceded, the ceding insurer needs to have a satisfactory measure of risk. For most classes this can simply be the sum insured specified in the original policy. However, for certain classes (and in particular for commercial fire), the full sum insured is very unlikely to be paid out in the event of a claim under a large risk, especially if the sum insured is spread over several sites. For such classes the EML is used as the measure of the risk.

Definition – Estimated maximum loss (EML)

The "estimated maximum loss" is the largest loss that is reasonably expected to arise from a single risk.

The EML will be based on the cedant's view as to the maximum loss that could arise from a single event (*eg* a fire or explosion). The EML could be considerably less than the sum insured for policies that cover several buildings. Even for a single tower block, the EML might be based on the value of several floors rather than the cost of rebuilding the whole block. This will depend on the capability of the fire precautions, such as sprinkler systems.

Defining the limits

The treaty will specify the limits for any risk to be included in the treaty. In particular, it might specify the:

- insurer's minimum retention
- insurer's maximum retention
- reinsurer's maximum level of cover.

Question 6.4

Why might a reinsurer require an insurer to have a:

- minimum retention level
- maximum retention level?

Definition – Minimum retention

This is the minimum level of retention the reinsurer requires to prevent the insurer from having too little interest in the risk. This requires the insurer to retain all risks that fall below the minimum retention.

This is the minimum level of retention for the *insurer*.

Definition – Maximum retention (R)

This is the maximum level of retention for any risk to be included in the treaty and will be specified in the treaty.

This is the maximum level of retention for the *insurer*.

Definition – Number of lines of cover (L)

This is specified in the contract and is used to calculate the maximum cover available from the reinsurer. The maximum cover available from the reinsurer is calculated as *L* multiplied by *R*.

This defines the maximum level of cover provided by the *reinsurer*. Reinsurers are typically much larger than insurers and can usually withstand higher levels of risk. Therefore the reinsurer is likely to be able to offer cover at a higher level than the insurer. This is why the reinsurer's maximum cover may be expressed as a multiple of the insurer's maximum cover.

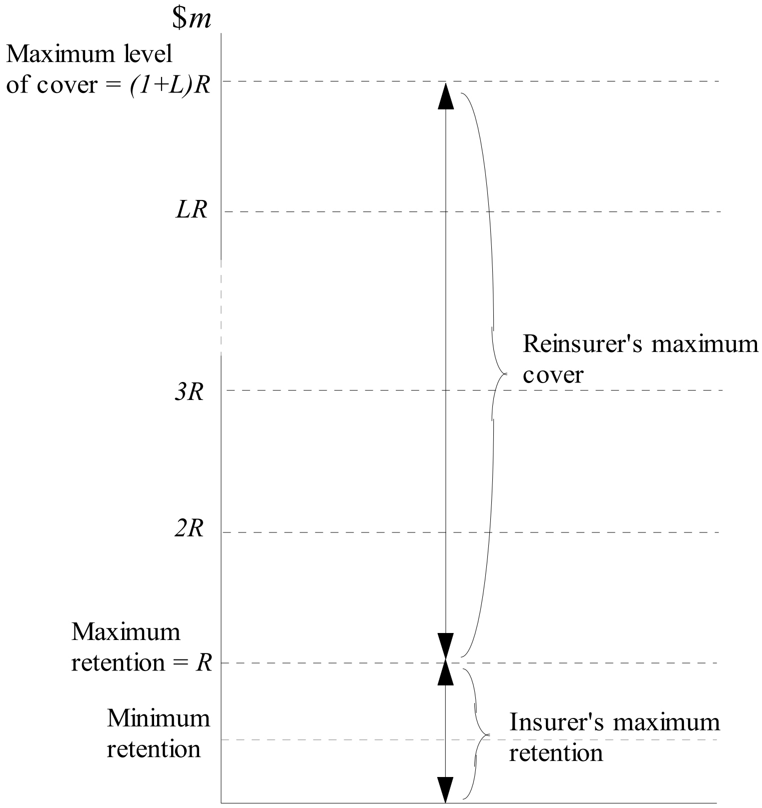
The maximum number of lines is the biggest multiple of the direct writer's chosen retention that the surplus reinsurer would accept, *ie* effectively this defines the cedant's minimum proportion as $1/(1+L)$ and the reinsurer's maximum proportion as $L/(1+L)$.

The insurer's maximum retention (R) plus the reinsurer's maximum level of cover ($L \times R$) will determine the maximum size of risk that can be placed on the treaty.

The maximum size of risk that can be placed on the treaty (including the cedant's retention) will be:

$$(1+L)\times R$$

It may help to visualise what's going on here. The diagram on the following page shows the limits that the treaty will specify.



Determination of proportion ceded

The treaty lays down a framework within which the insurer can select, for each individual risk, the amount that will be ceded to the reinsurer and the amount that it wishes to retain.

So far, we have just looked at what the treaty will specify – we have not considered how much of each risk the insurer will actually *choose* to retain, and how this will be specified in terms of lines. Let's look at this now.

Given any risk, E, normally measured by the EML, up to a maximum size of $(1+L)\times R$, **the insurer will analyse the nature of the risk and select a retention to keep (call this r). Note that $r \le R$ and r is not less than any treaty-stipulated minimum.**

Question 6.5

Mark on the diagram above:

- an EML (E) which satisfies the condition in the previous paragraph
- the region in which r may be chosen.

Note also that retentions will vary from case to case and indeed a number of risks, though covered by the scope of the treaty, may not be ceded under the arrangement.

> ***Question 6.6***
>
> Give two examples of risks that may not be ceded to the reinsurer.

Where the risk does not exceed the capacity of the treaty, the balance of cover $(E-r)$ will be ceded to the treaty (*ie* the reinsurer). **This will require $(E-r)/r$ lines of cover, with each whole line being of the same value (r) as that retained by the insurer. Call this number of lines k (where $k \le L$). The original premium and all claims for that risk will then be shared between the insurer and the reinsurer in the proportion 1:*k*.**

It's often useful to remember that for a specific risk:

$$E = r \times (k+1)$$

and also note that k does not have to be an integer.

Where, for example a treaty specifies: $R = \$2m$, $L = 5$ lines, the maximum that could be written (retention plus full coverage provided by the treaty) would be $(1+5) \times \$2m = \$12m$. If a particular risk has an EML of only \$6*m*, the insurer has a number of options available:

(a) retain the maximum of \$2*m* and place the remainder by two lines of cover (each \$2*m*)

(b) use the maximum lines of cover, five, each \$1*m*, and retain only \$1*m*, provided that this is not less than the minimum retention

(c) adopt any intermediate approach between the two extremes.

> ***Question 6.7***
>
> With reference to self-assessment Question 6.3:
>
> (a) What is the direct writer's retention?
>
> (b) How many lines of surplus reinsurance were used for each risk?
>
> (c) What is the "size" of the largest risk that is covered by this surplus reinsurance treaty if the maximum number of lines the reinsurer will accept is 12?

Premiums

Surplus is proportional. (You must hang on to that fact, even if "EMLs" and "lines" worry you.) Therefore, once a risk has been accepted under the treaty, the premiums for that individual risk are shared in the specified proportion, *ie* $1{:}k$, where k is the number of lines used for the risk in question.

Unlike quota share, you will not be sure at the time the treaty is set up how the total premiums will be split between the two parties. It will depend on what risks are actually written and what proportion of each risk is ceded.

Claims

For the individual risk, the claims are split in the same proportion as the premiums, irrespective of the size of claim. Even if there was a disaster and the total loss was greater than the EML, the claim would be split in the specified proportions.

Because the proportions vary from risk to risk, there is no way of telling what the split of total claims will be between the two companies. It is possible for the overall experience of the direct writer and the reinsurer to be quite different.

Question 6.8

What is the main difference between quota share and surplus reinsurance?

Question 6.9

A direct writer has a surplus reinsurance treaty with a maximum retention of £80,000 (there is no minimum retention level) and a maximum of 8 lines to be passed to the reinsurer. The following risks all gave rise to claims. In each case, calculate the range of possible amounts paid by the reinsurer to the direct writer:

Risk	EML	Claim
1	£240,000	£12,000
2	£480,000	£12,000
3	£100,000	£80,000
4	£720,000	£600,000
5	£80,000	£100,000
6	£960,000	£200,000
7	£160,000	£200,000

Commission

There may be return commission, override commission and profit commission in the same way as with quota share reinsurance (and brokerage if the reinsurance was arranged by a broker).

> ***Question 6.10***
>
> What should the insurer do if the risk exceeds the capacity of the treaty?

Second surplus treaty

Some of the larger risks may exceed the capacity of the first surplus treaty, and therefore need another reinsurance facility to protect them. It is common, therefore, to find a second surplus treaty in place with another reinsurer, in order to accommodate automatically that part of any risk exceeding the limit of the first surplus. There may also be a third, or even fourth, surplus treaty for some of the larger accounts.

The second surplus acts concurrently with the first surplus for those risks that involve both treaties. The size of line used under each treaty will be the same, both being based on the cedant's retention.

Therefore even if a claim does not exceed the capacity of the first surplus treaty, the risks will still be shared in the pre-defined proportions between the cedant and the two reinsurers.

The usual practice would be to use the whole of the first surplus capacity for a risk, before placing the balance on the second surplus treaty. Some treaties may require this, but in other cases the treaty may stipulate that at least as much risk would have to be placed to the first surplus treaty as to the second, to avoid selection against the second surplus reinsurer.

> ***Question 6.11***
>
> An insurance company has a 10-line surplus reinsurance treaty.
>
> Describe how the overall nature of the portfolio of retained risks will differ from the portfolio of ceded risks.

2.2 *The cedant's claims experience*

With surplus reinsurance the cedant can choose which individual risks it wishes to cede and in what proportion. The cedant may retain varying proportions of each risk.

Therefore, depending on the claims outcome the cedant and reinsurer will have different outcomes.

2.3 *The reinsurer's claims experience*

The smaller risks may not reach the reinsurer because they are within the retention. The cedant and reinsurer will not therefore share the same portfolio of risks. The risks that are ceded to the reinsurer will vary with the proportion of risk reinsured. Hence the underwriting experience of the risks to which the treaty applies will vary between the two parties. As with quota share, there will also be differences in expenses and in interest earnings between the two on their respective portions of the total portfolio.

2.4 *Advantages of surplus*

The main advantages to the direct writer are that:

- **It enables an insurer to write larger risks, which might otherwise be beyond its writing capacity.**

  Surplus reinsurance is particularly suitable for large property risks, *ie* where the direct writer feels that some risks are too "big" to be retained fully, and that it is better to share the risk with a reinsurer. The risks will be big enough to merit the additional individual attention given to each risk.

- **It enables the insurer to choose, within limits, the size of risks that it will retain.**

  It allows the direct writer to increase its exposure by a chosen amount (within limits), helping control overall business volumes, fine-tune its experience and control potential accumulations of risk.

- **It is useful for those classes where a wide variation can occur in the size of risks.**

  Insurers use surplus reinsurance, rather than quota share, extensively for those classes where a wide variation can occur in the size of risks. For example, it is used with commercial fire risks. It is also suitable for some marine and aviation insurance (*ie* property cover, like hull and cargo losses, rather than liability cover).

- **It helps to spread the risks.**

  It helps the direct writer spread its exposure (*ie* diversify). Retaining a portion of lots of risks is better than retaining all of a few risks.

- **The commission may help with cashflow.**

2.5 Disadvantages of surplus

The main disadvantages to the direct writer are that:

- **The administration is more complicated than for quota share, owing to the need to assess and record separately for each risk the amount to be ceded.**

- **It is unsuitable for:**
  - **unlimited covers, *eg* motor liability**
  - **personal lines cover where potential losses are small compared to the insurer's resources.**

In addition, the treaty terms may not be flexible enough, so that it may not cover the largest risks without the need for extra negotiation

3 Excess of loss reinsurance

3.1 How excess of loss (XL) operates

With XL reinsurance, the reinsurer agrees to indemnify the cedant for the amount of any loss above a stated excess point. More usually, the reinsurer will give cover up to a stated upper limit, with the insurer purchasing further layers of XL cover – which stack on top of the primary layer – from different reinsurers. The higher layer cover(s) come into operation on any particular loss only when the lower layer cover has been fully used (or "burnt through").

The top layer of excess of loss reinsurance might be unlimited (*ie* have no upper limit).

The layers of reinsurance should be arranged so that there are no gaps, *ie* the lower limit of the second layer of reinsurance starts at the upper limit of the first excess of loss reinsurance.

The expression generally used to describe the cover provided under an excess of loss reinsurance treaty is:

Amount of layer in excess of *lower limit*

So a treaty that provides cover for claim amounts between an excess point of £50,000 and an upper limit of £200,000 would be described as:

£150,000 in excess of £50,000, or simply as £150,000 *xs* £50,000.

Question 6.12

A direct writer has three excess of loss treaties covering its employers' liability portfolio:

£140,000 in excess of £60,000
£300,000 in excess of £200,000
£2 million in excess of £700,000

What is silly about this reinsurance programme?

How much will the direct writer be able to recover in respect each of the following claims?

(a) £80,000 (b) £280,000 (c) £2,400,000 (d) £4,000,000 (e) £680,000
(f) £50,000 (g) £500,000

Retention / deductible

This is the loss amount that is retained and paid by the cedant.

Working layer

A working layer is the first layer above the cedant's retention where moderate to heavy loss activity is expected by the cedant and reinsurer. Working layer reinsurance agreements often include adjustable features (for example, premiums that increase to some extent with adverse claims experience) to reflect actual underwriting results.

An adjustable feature is one that changes with the experience of the contract. Profit commission is an example.

Indexed limits

Where inflation has a significant effect on the cost of claims, a stability clause may be applied to the excess point. This is so that the reinsurer does not receive a higher proportion of the risks purely because of inflation. The cedant will normally be required to pay an extra premium to compensate the reinsurer for the added risk if the excess point is not indexed.

The upper limit (where one exists) may similarly be indexed to preserve the original real value of the cover.

The basis for indexation should be a reliable inflation index that bears some relation to the inflationary effects on the claim sizes.

Question 6.13

A direct writer has two layers of risk XL cover. The first is 100,000 in excess of 100,000, the second is 300,000 in excess of 200,000. All limits are indexed, and the chosen index starts the treaty at a value of 100.

Two separate claims are made: one for 350,000 when the index was 105 and the other for 600,000. The index on the dates the claim amounts were agreed was 105 and 110 respectively.

Assuming that the treaties cover these claims, calculate how much each insurer pays for each.

Note that in practice, the actual operation of indexation clauses might vary, depending on the exact policy wording, so any reasonable approach should earn you marks.

Commission

Return commission and override commission are not normally relevant to excess of loss reinsurance. This is because the reinsurer charges the insurer a premium to cover the risk and so a commission payment back to the insurer would be equivalent to simply charging a lower premium.

Profit commission is possible, for example, for working layer reinsurance. However, profit commission would be totally inappropriate for the very high layer excess of loss treaties, where the reinsurer expects to pay out only in exceptional circumstances.

Brokerage is very likely to be paid, since this type of reinsurance is usually arranged through brokers.

Reinstatements

Under excess of loss reinsurance arrangements, the treaty will often include reinstatements; that is, after a number of separate events have collectively exhausted the XL treaty limit, the reinsurer will allow one or more reinstatements of the coverage. Often, further premiums (reinstatement premiums), which may be more or less than the original premium and may be scaled down for the unexpired risk term, are payable.

A reinstatement is the restoration of full cover following a claim. After a reinsurance claim is made, the cedant may be required to pay an additional premium for the insurance cover to continue. The reinstatement premium and the number of reinstatements allowed will be set out in the treaty document – once agreed, both parties are bound by the arrangement and there are no options.

Limited reinstatements usually apply to higher layers of reinsurance cover. For lower layers of cover, there may be a limited number of free reinstatements before an additional premium is required. For very low working layers, there may be unlimited free reinstatements.

In practice, reinstatement premiums are often deducted from the claim recoveries, as this simplifies the administration.

The precise way that reinstatements work in practice depends on the wording in the treaty. A common approach would be to charge a proportionate premium according to the proportion of the layer that has been burnt through (often known as *pro-rata as to amount*). The period of cover remaining may also affect the premium (often known as *pro-rata as to time*).

Example

If a claim is made which uses half of the original layer and half the original period of cover is remaining, a premium of a quarter of the full reinstatement premium may be deducted from the claim recovery in order that cover can be restored.

Question 6.14

A \$1.5*m* *xs* \$1*m* excess of loss reinsurance treaty has the following terms and loss history in the year it was written:

- up front premium of \$300,000
- 1 reinstatement at 140% additional premium.

In chronological order the only losses to potentially impact this treaty are:

1. \$2.0*m*
2. \$5*m*
3. \$1.8*m*
4. \$10.2*m*

Ignoring the issue of the period of cover remaining, calculate how much of each loss is recoverable and the reinstatement premiums generated by each loss.

Also, calculate the "rate on line" of this contract. (Hint: You need to know your Glossary definitions to pass this subject!)

Increased limit factors

These are factors that define the expected loss cost for a limit *L* as a multiple of the loss cost for a loss with the basic limit *B*. For example, for a basic limit of *B* the factor is 1.0, for a limit of 2*B* the factor may be 1.6 (must be ≤ 2) and for a 3*B* limit the factor may be 2.0 (must be ≤ 3). Thus the expected loss costs are 60% more and 100% more than the cost of a basic limit respectively.

The theory of increased limit factors (ILFs) is covered in Chapter 15.

Proportional within non-proportional

For all forms of excess of loss, it is possible that the reinsurer will cover only a proportion of the claims within the layer, by applying a deductible. For example, the cover might be set as 90% of £150,000 in excess of £50,000. In this case, which has a 10% deductible, a claim of £120,000 would generate a recovery of £63,000 (*ie* 90% of £70,000).

The reason for this type of arrangement is to give the direct writer more incentive to keep the claim settlements low. Otherwise the reinsurer may feel exposed to the "moral hazard" of the direct writer having sloppy settlement procedures for large claims.

> ***Question 6.15***
>
> Why in practice would the direct writer be keen to handle claims above the excess point prudently, even if 100% of the claim above the excess was being paid by the reinsurer?

3.2 Cedant's claims experience

The cedant's experience will differ from the reinsurer's experience and will be better or worse depending on the distribution of large losses.

This is only to be expected, since (unlike proportional reinsurance) it is only once claims reach a certain size that the reinsurer has to pay anything.

For an unlimited XL cover (although this is rarely available), the reinsurer is likely to suffer a higher claims ratio than the cedant when more large losses occur than expected, and a lower claims ratio when the account experiences a high frequency of low severity losses that are less than the excess point. Note that for a limited XL cover (the more usual scenario), the claim amount exceeding the upper limit of the reinsurance will revert back to the cedant.

3.3 Reinsurer's claims experience

Reinsurance claims can differ markedly in shape and size from those of the original writing office. Typically on smaller claims, below the treaty limit, the reinsurer will not be liable at all. On larger claims, the reinsurer may have to provide the majority of the loss, depending on the limits in the arrangement's wording.

The degree to which reinsurers can influence the underwriting and claims management will depend on the wording of the treaty and the nature of the relationship between the parties.

3.4 Types of excess of loss (XL) reinsurance

There are three main types of excess of loss reinsurance:

- **risk XL**
- **aggregate XL**
- **catastrophe XL.**

In practice the boundaries between different forms of XL reinsurance are not distinct. In any given case the treaty will spell out exactly what is covered.

3.5 Risk (or individual) XL

This is a type of XL reinsurance that protects against large individual losses.

The reinsurer indemnifies an insurance company for the amount of an individual loss in excess of the excess point.

Risk XL should protect an insurer adequately against losses that affect only one insured risk, provided that the upper limit is sufficiently high.

The direct writer pays a premium to the reinsurer in return for protection against large individual claims (*eg* a large liability claim on an individual motor policy).

Risk XL can be written facultatively or by treaty.

3.6 Aggregate XL

Aggregate XL is a very simple extension of risk XL. However, rather than operating on large individual claims, the excess point and the upper limit apply to the aggregation of multiple claims. As you will see, there are several different ways that the claims might be aggregated.

The reinsurer indemnifies an insurance company for a cumulative, that is, aggregate amount of losses in excess of a specified aggregate amount.

As with risk XL, there will normally be an upper limit to this cover. This and the excess point will usually be at a higher level than for the working layer. There will also be a limit on the aggregate amount payable in the year from all events, usually by offering a strictly limited number of reinstatements. The reinsurer will usually also require payment of a reinstatement premium each time.

There are several ways it can be written:

- **excess of a monetary amount (for example, $500,000 in the aggregate excess of $500,000 in the aggregate)**
- **excess of a percentage (loss ratio) amount (for example, 50 loss ratio points excess of 75 loss ratio points)**
- **with an interior deductible; that is, to apply only to losses excess of a stated dollar amount (for example, $500,000 in the aggregate excess of $500,000 in the aggregate applying only to losses greater than $50,000 per loss).**

This form of contract would usually be used to protect an account that would not normally be exposed to major "event" losses but could be subject to major attritional losses (that is, a large number of small losses), for example, a medical expenses account.

Aggregation by event

Events can occur that cause losses to several insured risks at the same time. Depending on the insurer's risk portfolio, that event could lead to an aggregation of claims. Individually, each claim might not be of an exceptional size, but collectively the aggregate cost might be damaging to the insurer's gross account.

In this situation, the conventional risk XL treaty, by treating each claim as a separate loss, will fail to protect the cedant adequately against the aggregate cost of such losses, thus the need for aggregate XL reinsurance.

In this case, all the claims arising from that event might be added together. If the total amount exceeds the lower limit, then the direct writer can make a recovery from the reinsurer.

Usually an event covered by aggregate XL will be sudden and obviously identifiable.

Example

The event might be a motorway pile-up caused by a single driver. The insurer of this driver might then be faced with dozens of individual injury claims, none of which in isolation is too worrying, but which in total could damage the company.

A common form of reinsurance for motor insurers is "claim and event XL", under which cover is provided for a single large claim or all claims from a single event.

However, in some cases, the time span of the event may be less easily identifiable and therefore require careful wording in the treaty definition.

For example, the event could be a major storm. In this instance, the event would need to be defined very carefully so that there is no doubt as to whether particular claims are to be included. This form of aggregation by event is generally known as *catastrophe excess of loss*. Catastrophe excess of loss is described in the next section.

The boundary between aggregate and catastrophe excess of loss is not distinct.

Aggregation by peril or cause

Alternatively, cover may be defined in terms of a common cause or peril, rather than a single event, over a particular period of time.

For example, a direct writer may seek aggregate excess of loss reinsurance for all claims from asbestosis in its employers' liability account.

Aggregation by class

The direct writer may get aggregate excess of loss reinsurance to cover all claims from a particular class. This form of aggregate excess of loss is called *stop loss*. It is described in Section 4.

3.7 Catastrophe XL

In its most extreme form, an event may be of catastrophic proportions, involving losses to many hundreds, or even thousands, of different insured risks with a potential cost falling beyond the normal capacity and intention of aggregate treaties and possibly even beyond the finances of a large insurer.

The scope of aggregate XL therefore needs to be extended, to catastrophe XL, to cope with such disasters.

Catastrophe XL is a form of excess of loss reinsurance which, subject to a specified limit, indemnifies the ceding company against the accumulated loss. This is in excess of a specified retention, resulting from a catastrophic event or series of catastrophic events.

Definition – Hours clause

A clause within a catastrophe reinsurance treaty that specifies the limited period during which claims can be aggregated for the purpose of one claim on the reinsurance contract. Common examples are 24 or 72 hours.

How catastrophe excess of loss operates

The main difference between catastrophe XL and aggregate XL is that catastrophe XL operates at a much higher level of aggregate cover. A catastrophic event also needs to be defined carefully because the cause may not always be instantaneous and it may be spread over a wide geographical area. A severe windstorm, for instance, may last for several days; it may even abate only to return to cause further waves of damage.

For this reason, the hours clause of a treaty usually limits windstorm claims to all losses occurring within a consecutive period of 72 hours, whilst freeze claims are usually limited to any period of 96 hours. The cedant is allowed to choose the starting point of the period to which the reinsurance claim will apply.

The purpose of the hours clause is to prevent any disputes occurring in cases where, for example, two different storms occur four days apart (in which case the two storms are treated as separate events). The starting point will be chosen by the insurer in order to maximise the reinsurance recoveries.

Question 6.16

Apart from brokerage, what types of reinsurance commission might sensibly apply for catastrophe excess of loss reinsurance?

3.8 *Advantages of excess of loss*

The main advantages of excess of loss to the direct writer are that:

- **It allows an insurer to accept risks that could lead to large claims. "Large" here means large relative to the insurer's solvency margin, annual premiums, expected profits, or the results of the impacted classes.**

- **It reduces the risk of insolvency from a catastrophe, a large claim or an aggregation of claims.**

- **It stabilises the technical results of the insurer by reducing claim fluctuations.**

  By reducing claim fluctuations (and hence the variance of the claims outgo), the annual profitability of the direct writer will be less volatile. (Shareholders are not known for their love of profit volatility!)

- **It helps make more efficient use of the capital by reducing the variance of the claim payments.**

  High volatility of outgo means that the company must hold large free reserves. If the volatility is reduced, lower free reserves are required. This means that the company can write the same amount of business with less capital by using excess of loss reinsurance.

3.9 *Disadvantages of excess of loss*

The insurer pays a premium to the reinsurer that in the long run, if priced accurately, will be greater than the expected recoveries under the treaty.

This is because the excess of loss reinsurer's premium will load the expected claims for expenses, profit and contingency margins.

From time to time, excess of loss premiums may be considerably greater than the pure risk premium for the cover. For example, after reinsurers have had a few years of poor results, the supply of reinsurance falls and premiums rise, as reinsurers attempt to restore their solvency positions.

Excess of loss cover may be hard for the insurer and reinsurer to price.

4 Stop loss

As the name suggests, stop loss reinsurance is designed to stop losses. In fact, it doesn't actually stop losses altogether, but it can help make bad losses a bit less dire. With stop loss reinsurance, the aggregation applies to …

… all claims arising …

… in a defined account(s) (*eg* a specified class) …

… during a defined period (*eg* a year).

Stop loss is a form of XL reinsurance that indemnifies the ceding company against the amount by which its losses incurred during a specific period (usually 12 months) exceed either:

- **a predetermined monetary amount or**
- **a percentage of the company's subject premiums (loss ratio) for the specific period. Subject premiums are the premiums for the account that is being reinsured.**

Sometimes a particular class of business gives rise to large variations in the levels of total claims payable in any one year. Aggregate XL only provides cover for either one cause over the year or one event. An insurer may therefore wish to protect the class of business by a form of reinsurance that extends cover to all causes or all events during the year. Stop loss does this by covering the total losses for the whole account, above an agreed limit, for a 12-month period. The whole account can be one or several classes of insurance.

The excess point and upper limit for stop loss are often expressed as a percentage of the cedant's premium income for that account. Cover might typically be given from an excess point of 110% claims ratio up to an upper limit of 130% or 140%.

Question 6.17

Why is stop loss cover often quoted in terms of claim ratios rather than monetary amounts?

Example

A company that writes just one class of business has a stop loss reinsurance treaty for business in 2012:

- lower limit of 105% of earned premium
- upper limit of 125% of earned premium
- reinsurer covers 80% of the claims in the layer.

Suppose that the earned premiums for 2012 were £292 million and the total incurred claims were £333 million (*ie* a loss ratio of 114%).

The reinsurance recovery would be $0.8 \times (333 - 1.05 \times 292) = £21.12$ million.

Question 6.18

(a) List several risks relating to the direct writer's operations that a reinsurer would face if they made stop loss cover available.

(b) Given these, why would the direct writer probably not want stop loss in practice?

(c) Suggest two broad responses to the risks in (a) that the direct writer may well find preferable in practice.

Question 6.19

(a) Explain why reinsurers are often not prepared to provide stop loss cover.

(b) If the reinsurer does provide stop loss cover, what conditions are they likely to impose on the business covered?

Question 6.20

If available, why is stop loss reinsurance normally arranged on a losses-occurring basis?

5 Finite risk (or financial) reinsurance

5.1 Purpose of finite risk (or financial) reinsurance

Traditional risk transfer involves standard reinsurance contracts (proportional and non-proportional). Risk financing moves away from this, however, the risk may still be transferred to a reinsurer. Alternatively it may be transferred to the capital markets. This section and the next describe products that involve a reinsurer. Section 7 describes products that involve the capital markets.

The aim of finite risk reinsurance tends to be risk financing rather than traditional risk transfer. This is typically a multi-year contract aimed at reducing the cedant's cost of capital by means of earnings smoothing. The contract reduces year-to-year earnings volatility but it provides limited risk transfer over the whole contract period.

A wide variety of financial reinsurance contracts exist, although all have been devised primarily as a means of improving the apparent accounting position of the cedant.

5.2 Features of finite risk (or financial) reinsurance

The main feature of a financial reinsurance contract is that it involves only a small element, if any, of transfer of insurance risk from the cedant to the reinsurer. The contract, however, does involve investment type risks. Many forms of financial reinsurance are, in fact, often viewed as being more similar to investment than to reinsurance. Usually the effective "return" that the contracts provide is low in comparison to conventional investments.

Typical features might be:

- **limited assumption of risk by the reinsurer**
- **multi-year contract term**
- **explicit inclusion of investment income in the contract**
- **sharing of the results with the cedant**
- **risk transfer and risk financing are combined; for example, time and distance deals.**

Time and distance deals are described in Section 5.4.

5.3 *Types of finite risk (or financial) reinsurance*

There are two types of finite risk reinsurance:

- *Pre-funded* arrangements, whereby the insurer pays premiums into a fund held by the reinsurer (which earns interest), and claims are paid from the fund.
- *Post-funded* arrangements, whereby the reinsurer pays the losses and the insurer pays back the losses over time.

> ***Question 6.21***
>
> What is the main difference between these two arrangements as far as *managing risk* is concerned?

Note that while the two of these may involve reinsurers, they may also be classed as capital market products.

Specific types of finite risk (or financial reinsurance) include:

- time and distance deals
- spread loss covers
- financial quota share
- structured finance
- industry loss warranties.

5.4 *Time and distance deals*

Time and distance deals are designed to discount technical reserves for the time value of money.

An insurer pays a single premium in return for a fixed schedule of future payments matched to the estimated dates and amounts of the insurer's claim outgo. The purpose of such contracts was to achieve the effect of discounting in arriving at the reserves for outstanding claims.

> ***Example***
>
> For a payment of £10 million on 1 January 20013 the financial reinsurer may agree to pay to the direct writer £1.2 million on 1 January from 2013 – 2022 inclusive. The reinsurance premium is £10 million and the reinsurance recoveries form an annuity of £1.2 million *pa*, payable annually in arrears for ten years.

This does not at all look like a reinsurance policy. The insurer is effectively purchasing an annuity from the reinsurer. The policy is more similar to an investment than a reinsurance policy. However, this type of policy has been used in various parts of the world in order to improve the apparent solvency position of the insurance company. You may be asking how.

If the relevant authorities can be convinced that the arrangement constitutes reinsurance then the annuity payments may be treated as reinsurance recoveries (*ie* negative claim payments) rather than investment proceeds. If future claim payments and hence reinsurance recoveries can be shown at face value in the balance sheet, *ie* the payments don't have to be discounted (and the present value shown), then in this example the payments would be taken into account at face value, *ie* £12 million. Hence, the company has swapped a cash asset of £10 million (the reinsurance premium) for an asset of £12 million (the reinsurance recoveries) and its disclosed solvency position appears to increase by £2 million.

Please note that we have deliberately used the words "if" and "may" in the above paragraph. There is no guarantee that the local supervisory authority will allow such contracts to be valued at face value. If the insurer is obliged to calculate the present value of the future annuity payments then there might not be any benefit to the insurer at all, depending on the discount rate used.

They were useful in the past to insurers who were not permitted to discount their reserves (*eg* Lloyd's syndicates).

Similarly, these contracts do not count as insurance contracts for the purpose of accounting in the US, so they no longer achieve the desired effect on balance sheets.

Since Lloyd's changed its rules so that the credit allowed for time and distance policies in a syndicate's accounts was limited to the present value, such policies have become less popular.

However, some companies in certain countries still have these policies on their books.

A finite risk reinsurance contract has to have a reasonable level of risk transfer if it is to be treated as reinsurance under most systems of accounting principles; otherwise it is treated as an investment and may thus lose its appeal.

Few financial reinsurance policies are as simple as the one described above. Often they are "disguised" to look like normal reinsurance contracts. If you come across a reinsurance policy that seems to be transferring very little claims risk from the insurer to the reinsurer then it is probably a financial reinsurance policy.

Example

Insurer A has the following aggregate excess of loss arrangement with reinsurer B.

This covers all claims due to storm and flood damage on any household property policies written by A during the year 2010 in excess of £0.8 million, with an upper limit of £2 million (*ie* "£1.2*m* *xs* £0.8*m*").

At the start of 2010, a reinsurance premium of £1.2 million is paid by A. Return commission of £200,000 is paid by the reinsurer – giving a net reinsurance premium of £1 million.

A 98% profit share is payable to A at the end of 2013. This is calculated as 98% of profit, where profit is calculated as the original premium of £1.2 million less any recoveries made under the reinsurance treaty.

At the end of 2013, the arrangement is then terminated. At this time, all reinsurance recoveries and profit share must be paid to insurer A.

What type of reinsurance arrangement is this?

In order to answer this question, we need to consider the cashflows that insurer A will experience.

In return for a net premium of £1 million paid on 1/1/10, insurer A receives (on 31/12/13):

- £1.176*m* (*ie* 98% of 1.2*m*), if no recoveries are made
- £1.2*m* as recoveries, if recoveries exceed £1.2*m* (hence the profit is zero), or
- between £1.176*m* and £1.2*m*, if the layer is partially burnt through.

You can see that claims experience has very little effect on insurer A's finances, so there is very little transfer of claims risk between the two parties. It is therefore actually a financial reinsurance contract.

Question 6.22

Discuss the reasons why insurer A may wish to take out this particular arrangement.

Note that here, the initial payment is from the insurer to the reinsurer. In the majority of other financial reinsurance arrangements, the initial reinsurance commission is from the reinsurer to the insurer.

5.5 Spread loss covers

Spread loss covers involve the insurer paying annual or single premiums to the reinsurer for coverage of specified claims. These accumulate with interest (contractually agreed) in an experience account, the balance of which is settled at the end of the multi-year period.

These might be useful where the insurer is exposed to a potentially large risk that may occur from time to time, for example an earthquake. A spread loss cover would help to spread the effect of a possible big loss over several years.

Example

An insurer covering an earthquake risk may expect that in nine years out of ten there will be no claims, but in the tenth year, there will be a very large claim. In that year (whenever it turns out to be), there may be a severe adverse effect on the profits of the company. To deal with this, the insurer may pay a series of premiums to a reinsurer, and in return the reinsurer will pay a claim when it is needed.

The definition of the claim will be such that it cannot be much greater than the sum of the premiums, while if low or no claims occur, the premium can be refunded. In this way the reinsurer is only taking limited timing risk, and so can keep his charges down.

However, the reinsurer holds the credit risk of the insurer, if the balance on the experience account turns negative.

These types of contracts involve very limited underwriting risk (limited practical risk transfer), but provide the insurer with the liquidity and security of the reinsurer.

Again, there often has to be sufficient (but minimal) risk transfer in order for this contract to count as "reinsurance".

As well as liquidity and security, this arrangement helps reduce the volatility of the insurer's reported results, which may be important at least for presentation purposes.

> ***Example***
>
> Suppose that when a claim in respect of this earthquake risk occurs (in one year in ten) it is for \$1*m*. Instead of reporting results that reflect claims outgo of \$0*m* in nine years and \$1*m* in one year, an insurer may prefer its reported results to reflect claims outgo of \$100,000 every year.

At a more strategic level, reduction in the volatility of reported results should reduce the requirement for capital allocation and therefore improve return on the capital employed.

Less volatile results will reduce the *buffer* capital that needs to be held, thereby freeing up this capital for other uses, *eg* in development of the business, that will earn better returns than would be achieved by having to keep it invested as solvency capital.

5.6 Financial quota share

Financial quota share is an arrangement where the purpose lies in the commissions paid for financing purposes, rather than the transfer of risk.

This is a traditional quota share arrangement, but written for the primary purpose of a financial arrangement involving the commission payment. Financing is achieved by overcompensating, (*ie* paying more than a normal reinsurance commission), in the initial period and undercompensating, (*ie* paying less than a normal reinsurance commission), over a period thereafter.

It is therefore a type of financial reinsurance contract.

5.7 Structured finance

Structured finance is a broad term used to describe a sector of finance that was created to help transfer risk using complex legal and corporate entities.

Reinsurers became involved in structured finance through their finite reinsurance business and the increasing need of financial guarantee insurers and investment banks for additional capacity.

The typical financing solution provided by the reinsurer is a credit enhancement in which the reinsurer provides a financial guarantee or credit insurance wrap to the institution borrowing from the capital market.

Credit enhancements involve insurance companies insuring loan portfolios or providing credit protection to companies to improve the creditworthiness of debt instruments. These solutions use derivative products available in the capital markets, in addition to variations on traditional credit insurance.

Credit insurance wraps are insured or guaranteed by a third party. The third party may provide a promise to reimburse losses up to a specified amount. Deals can also include agreements to advance principal and interest or to buy back any defaulted loans. The third-party guarantees are typically provided by AAA-rated financial guarantors.

Credit enhancement or financial guarantees lower the cost of borrowing.

5.8 Industry loss warranties (ILWs)

Industry loss warranties (ILWs) are a type of reinsurance contract where the basis of cover is not indemnity, *ie* repayment of actual losses suffered.

Here one party will purchase protection based on the total loss arising from an event to the entire insurance industry rather than their own losses.

The original size of the industry loss is used as a trigger for a recovery.

The contract pays a specified fixed amount to the insured if:

- there has been an insured loss of a particular type, *eg* a hurricane, to the insurance industry of a particular size, and
- a second indemnity-based trigger is breached, based on the value of the losses incurred by the insured.

The "industry loss" size referred to in the first bullet point is usually determined using a recognised standard of some sort. For example, for US property events the common source is PCS (Property Claim Services), which is an organisation in the US that collates and publishes industry catastrophe loss information for US catastrophe events, such as tornadoes, hailstorms and floods as well as hurricanes and earthquakes. The loss amount published by PCS is often used to determine whether an event has breached the industry loss trigger for an ILW. In other regions a cat model might be used, which calculates a model industry loss size based on wind speed, earthquake magnitude *etc*.

The second condition is necessary to ensure that the insured has an insurable interest in the cover.

The payout to the insured may be fixed, so there is a potential mismatch that works in favour of – or against – the insured.

Typically, reinsurer payment should be quite quick once the insurer makes a claim.

Note that the name Original Loss Warranty (OLW) is sometimes used for the same concept.

6 Run-off reinsurance

6.1 Run-off solutions

Run-off reinsurance is not a term generally used in the market place and so the meaning would be described in any question. It could in practice involve any treatment or processing of any closed book of business. The reinsurance of a closed tranche is just one possible run-off solution.

The aim of run-off reinsurance is the transfer of reserve development risks. It provides cover against the insurer's earnings volatility arising from past activities. It may be sought in circumstances such as:

- **corporate restructuring**
- **mergers and acquisitions**
- **closing lines of business**
- **economic changes in the value of the liability**
- **regulatory, accounting or tax changes**
- **legal developments, for example court decisions.**

The "book" is sold to the reinsurer who assumes all remaining premiums and all of the risk. The claims reserves are also transferred from the insurer to the reinsurer.

In the US, for example, there have been many run-off solutions applied to accounts with exposure to absestos-related claims, in view of the uncertainty (and deterioration) of that claims experience.

There are two main types of run-off reinsurance:

- adverse development cover
- loss portfolio transfers.

6.2 Adverse development cover

Adverse development cover is a reinsurance arrangement whereby a reinsurer agrees, in return for a premium, to cover the ultimate settled amount of a specified block of business above a certain pre-agreed amount.

It protects the cedant from significant reserve deterioration on run-off business. This caps the liability and protects the balance sheet from any further development on existing losses, and from future losses in respect of old business.

The premium that is payable for the cover will depend on the risk appetite of the market.

Usually it is only possible to reinsure a layer above a specified amount. This specified amount may be in excess of the current level of reserves. There could be an upper limit. If the ultimate cost of losses is in excess of this, the insurer is liable for the excess. The reinsurer may also insist that the insurer has a small participation in the layer.

Claims are usually still handled by the insurer and hence there are the associated expenses. Reserves are maintained by the insurer and it receives all investment income generated from the investments backing these reserves.

There is no transfer of reserves from the insurer to the reinsurer. The insurer simply pays a premium for the reinsurer to take on responsibility for the development of reserves beyond a specified position.

Question 6.23

What sort of traditional reinsurance arrangement would achieve this?

The insurer is exposed to the credit risk of the reinsurer. Legally, the insurer remains liable to the insured parties for all claims within the block reinsured. Hence, some but not all of the risk from adverse run-off of the reserves is removed.

6.3 Loss portfolio transfers (LPTs)

LPTs are an arrangement whereby the liability for a specified book of business is passed in its entirety from one insurer to another. Policyholders will be informed of this "novation" and the deal may need to be approved by a court. This enables the original insurer to concentrate on any remaining book of business.

The Glossary defines a "novation" as "The transfer of the rights and obligations under a contract from one party to another." Note that this definition is strictly only part of the Subject SA3 Core Reading. Strictly speaking, since the responsibility for the book of business passes entirely to the new insurer, this is not reinsurance.

Novation is not strictly reinsurance since the new insurer is responsible for the liabilities in total from the date of the transfer.

The original insurer will transfer the reserves and the remaining exposure to the new insurer. It is likely that there will be a premium in addition to the existing reserves. This would normally include a claims handling service.

All adverse claims risks and the investment income will be passed to the new insurer.

Advantages of LPTs

- **They can improve the credit rating of the original insurer.**
- **The new insurer will gain diversification if not already in this area and achieve a larger client database. There are specialist players in the market that can possibly run-off such portfolios more profitably than the original insurer.**

Disadvantages of LPTs

- **Assets may need to be realised to pass across the value of the reserves to the accepting insurer which is particularly important if there is mismatching or if tax gains / losses would be crystallised.**
- **If the new insurer defaults, this could damage the reputation of the original insurer.**
- **The transfer may require the buy-in of reinsurers where there are existing reinsurance arrangements covering the portfolio.**
- **There will be an associated cost to the original insurer of the risk transfer, which will depend on the current risk appetite of the market. This cost would be any premium payable plus the "lost" investment income.**

The "premium payable" referred to above is an amount to compensate the new insurer both for taking on the risk and for expenses associated with the transfer. This would be paid on top of the value of the reserves. The "associated cost" referred to is therefore this premium plus the value of any investment income effectively lost if the transferred value of the reserves uses a discount rate which turns out to be too low.

Example

Suppose a general insurer has a block of business and the discounted value of expected future claims in respect of this business is \$100*m*. If the regulations prohibited the discounting of future claim reserves, the regulatory provisions would be somewhat higher, say \$150*m*.

The general insurer could seek to reinsure this block of business. It would pay the reinsurer a premium. This should be sufficient to meet expected claims (*ie* \$100*m* in this case) plus the reinsurer's fee (say \$10*m*). So, the insurer's assets would decrease by \$110*m*. However, its liabilities would decrease by the amount of the provisions now passed to the reinsurer, *ie* \$150*m*. Therefore, the general insurer's reported financial position has been improved.

If the actual claims experience is such that the \$100*m* is more than is required to meet the actual claims, the excess could be returned as a profit commission. If the \$100*m* is less than the actual claims, then (depending on the terms of the arrangement) the insurer might be required to pay an additional amount to the reinsurer.

7 Capital market products

7.1 Types of capital market products

In this section we discuss products where risk is transferred to the capital markets rather than insurance markets. We will discuss the following products:

- committed (or contingent) capital
- securitisation
  - insurance-linked securities
  - credit securitisation
  - motor securitisation
- weather derivatives.

7.2 Committed (or contingent) capital

This was introduced in Subject CA1.

Committed capital or contingent capital is based on a contractual commitment to provide capital to an insurer after a specific adverse event occurs that causes financial distress. The insurer purchases an option to issue its securities at a predetermined price in the case that the defined situation occurs, on the understanding that the price would be much higher after such an event.

If the defined event occurs, leading to financial distress of the insurer, then the price of the insurer's securities will fall (*ie* it will be more expensive to raise capital by issuing securities). The option will allow the insurer to sell its securities after the adverse event at a higher price than their market price.

Example

If the securities might have a current market value of \$100, then the insurer might fix the predetermined price (*ie* the strike price of the option) at \$100. Following the adverse event, the market value of the securities might fall to \$80, however, the insurer will still be able to issue such securities at the higher price of \$100.

There may be one or more triggers that have to occur before the option can be exercised, in order to avoid moral hazard.

Contingent capital provides a mechanism of ensuring that, should a particular risk event happen, capital will be provided. As such, it is a cost-effective method of protecting the capital base of an insurance company. Under such an arrangement, capital would be provided as it was required following a deterioration of experience (*ie* it is provided when it is needed).

7.3 Securitisation

You may have met securitisation in Subject CA1.

Purpose of securitisation

Securitisation has two main purposes:

- Risk management – to transfer insurance risk to the banking and capital markets.

  It is often used for managing risks associated with catastrophes, as the financial markets are large and capable of absorbing catastrophe risk.

  It involves turning a risk into a financial security, *eg* as in a catastrophe bond.

- Capital management – to convert illiquid, inadmissible assets into liquid admissible assets, hence improving the balance sheet.

  Almost any assets that generate a reasonably predictable income stream can in theory be used as the basis of a securitisation. Examples of illiquid assets that could be securitised are:
  - future profits, *eg* on a block of in-force insurance policies
  - mortgages (and other loans).

  Each of these could be securitised into tradeable instruments (*eg* bonds), in order to raise capital. The owner of the assets issues bonds to investors (*eg* pension funds, insurance companies and banks) and the future cashflow stream generated by the secured assets is then used to meet the interest and capital payments on the bonds.

  There is typically risk transfer as the repayments on the bonds are made only if, for example, the future profits emerge or mortgage repayments are made.

Example

A portfolio of mortgage loans owned by a bank could be pooled together and the cashflows from these mortgages used to service the interest and capital payments on a bond. Securitisation of this type, that had been backed by sub-prime mortgages in the US, was the focus of much attention during the recent sub-prime crisis and credit crunch.

Operation of securitisation

In simple terms, a securitisation works as follows:

1. An investor purchases a bond from the insurance company and therefore provides a sum of money to the insurer.

2. The repayment of capital (and possibly of interest) is contingent on:
   - a specified event *not* happening, *eg* an earthquake measuring 6.5 on the Richter scale *not* happening, or
   - the portfolio of insurance business (upon which the bond is securitised) producing adequate profits.

3. If the event does happen (*eg* the aforementioned earthquake occurs), or inadequate profits are made from the securitised business, the insurer may default on the interest and capital payments due under the bond:
   - in the case of securitising a particular risk, the insurer can use the sum of money provided from the investor (in purchasing the bond) to cover the cost of claims arising from the earthquake
   - in the case of securitising a block of business, the poor experience of the business has been passed directly to the investor.

4. If the event does not occur or the business makes adequate profits, the investor gets his interest and capital back in the normal way.

In practice, the direct link between the investor and the issuer is broken by a special purpose vehicle (SPV), which is a separate legal entity that sits between the parties. Where it is a portfolio of business that is being securitised, the securitised assets are transferred into this vehicle.

Question 6.24

Why?

This may seem like a particularly high-risk investment. It is! However, as long as the expected return on the investment is commensurate with the investor's required (risk-adjusted) rate of return, then a market for such an investment will exist.

The rationale is that insurance catastrophe risk or the risk of underperformance of the securitised business, is not correlated with investment market risks and so there is a benefit to the capital market in the diversification of risk achieved in purchasing such investments.

The banking and capital markets are used because of capacity issues and because the risks involved are ones with which the banking and capital markets are comfortable.

A key point to note about securitisation is that it is making insurance products look much more like banking products. The reverse, often called *insuritisation*, is making banking products look more like traditional insurance.

We now look at three specific types of securitisation:

- insurance-linked securities
- credit securitisation
- motor securitisation.

Insurance-linked securities (ILS)

Insurance-linked securities (ILS) are an innovative way of increasing insurance capacity. Many ILS issues have been catastrophe (cat) bonds, a mechanism for insurers to transfer catastrophic risks to the capital markets via a bond issue. If the specified catastrophic risk is triggered, the bondholders typically forfeit the interest and principal on the bond to the insurer.

CAT bonds developed primarily in response to the hard market (*ie* high premiums) of traditional catastrophe reinsurance in the 1990s.

This is an example of the first type of securitisation, *ie* as a risk management tool.

There can be many variations, and many types of trigger event.

The basic advantages of ILS are that they:

- increase insurance capacity by transferring risk to the capital markets
- mitigate counterparty risk as funds are held in a secure independent vehicle.

Credit securitisation

Though not usually involving reinsurance, insurance companies have been active in the credit securitisation markets.

Their main roles have been:

- **enhancing the creditworthiness of debt instruments**
- **providing capital relief to banks by insuring loan portfolios**
- **providing credit protection to companies.**

Enhancing the creditworthiness of debt instruments / providing capital relief to banks

Consider a bank securitising some of its loan portfolios. The interest and capital repayments under the loans will be securitised and used to pay the interest and capital repayments under the debt instruments (*ie* bonds).

Investors will require a return on the bonds that is adequate to compensate them for the risk of default. The bank may want to keep the return on the bond as low as possible, therefore it must try to ensure that the bond is relatively secure. In order to do this, it must either securitise its best quality loans (*ie* the loans with the lowest risk of default), or it must securitise a large number of loans relative to the number of bonds issued (in which case, even if the loans default, there will still be an adequate number of bonds left with which to make payments on the bonds).

The first of these options may not be available if the bank does not have (or has already securitised) a portfolio of "safe" loans. The second option may be undesirable, because effectively, the bank is using up a lot of its business in the securitisation, which will reduce the profits it receives from the loans that it does not securitise.

A third option is to use insurance to reduce the credit risk of the bonds. The bank insures the bonds so that their return is guaranteed (as long as the insurer does not default!). If the payments under the loan portfolio are not sufficient to meet the interest and capital payments under the bond, then the insurance will kick in and make the payments to the investors.

Having insurance as an underlying guarantee will enhance the creditworthiness of the debt. This will help to ensure that the bank does not need to pay a very high rate of return on the bonds, or, equivalently, does not need to sell them cheap. It should therefore be able to sell the bonds at a relatively high price, thus maximising the capital relief provided by the securitisation. This needs to be weighed up against the cost of insuring the bad debt.

This type of arrangement falls into the category of capital management, as described above.

Providing credit protection to companies

There are numerous types of credit securitisation arrangement, although the basic contract is a *credit default swap*, which is essentially an agreement to compensate the "insured" (*ie* the buyer of the swap) if a specified credit event occurs (*eg* bankruptcy or loan default of another company).

Note that for each of these arrangements, the insurer is not usually one of the two parties involved in the securitisation itself. Instead, it is a third party providing insurance against the risk of default by another party.

These alternative risk transfer (ART) solutions use derivative products available in the capital markets, in addition to variations on traditional trade credit insurance.

Motor securitisation

Another capital market product is motor securitisation (where certain aspects of a motor insurer's portfolio risks are passed to the investment market).

The insurer issues a bond where the coupon payments depend on the claims experience of the insurer's motor portfolio. If the insurer experiences poor claims experience, it may forego some or all of its repayments. Thus, the insurance risk is transferred to the capital markets instead of to the reinsurance market.

As with other debt issues, these bonds are tradable financial instruments.

7.4 *Weather derivatives*

Weather derivatives are another example of the insurance industry's response to a hard and uncertain insurance market. Strictly speaking, weather derivatives are *insurance* rather than *reinsurance*.

This is where standard derivatives techniques, such as put and call options and swaps, are then used to make a derivative contract based on the weather.

Example

Energy companies' earnings are very dependent on the weather, and the companies are likely to want to reduce the resulting volatility of their profits.

This can be achieved by using weather derivatives. The solution can be based on any weather-related peril, but perhaps temperature is the most common.

Here, the payment is based on *heating degree days*, or *cooling degree days.* A heating degree day, for example, is the number of degrees by which the day's average temperature falls below some reference temperature. Payments from weather derivatives are based on the accumulated value of degree days over a period of time.

The advantages of using a derivative approach as opposed to traditional insurance are:

- there is no need for an insurable interest
- there is no need to understand the underlying business for which cover is being purchased
- there is no need to prove the extent of the loss to a claims handler.

8 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Adverse development cover
- Aggregate excess of loss reinsurance
- Balance of a reinsurance treaty
- Catastrophe reinsurance
- Excess and surplus lines insurance
- Expected maximum loss
- Financial engineering
- Financial risk reinsurance, finite risk insurance or reinsurance
- Hours clause
- Increased limit factors
- Line
- Loss portfolio transfer
- Original gross premium income (OGPI)
- Over-riding commission
- Possible maximum loss (PML)
- Profit commission
- Quota share reinsurance
- Rate on line
- Reinstatement
- Retention
- Return commission
- Risk excess of loss reinsurance
- Stability clause
- Stop loss reinsurance
- Surplus lines insurance
- Surplus reinsurance
- Time and distance reinsurance
- Working layer.

8.1 End of Part 1

You have now completed Part 1 of the Subject ST8 Notes.

Review

Before looking at the Question and Answer Bank we recommend that you briefly review the key areas of Part 1, or maybe re-read the summaries at the end of Chapters 1 to 6.

Question and Answer Bank

You should now be able to answer the questions in Part 1 of the Question and Answer Bank. We recommend that you work through several of these questions now and save the remainder for use as part of your revision.

Assignments

On completing this part, you should be able to attempt the questions in Assignment X1.

Reminder

If you have not yet booked a tutorial, then maybe now is the time to do so.

This page has been left blank so that you can keep the chapter summaries together for revision purposes.

Chapter 6 Summary

Quota share

Quota share is proportional treaty reinsurance whereby the premiums and claims for all risks covered by the treaty are split in a fixed proportion. The reinsurer pays return and override commission to the insurer. Profit commission may also be payable.

The cedant's experience (in terms of loss ratios) will be the same before and after reinsurance. The reinsurer will have proportionately the same underwriting experience as the cedant.

Quota share:

+ spreads risk, increasing capacity and encouraging reciprocal business
+ directly improves the solvency ratio (without losing market share)
+ is administratively simple
+ may provide commission that helps with cashflow
– cedes the same proportion of low and high variance risks
– cedes the same proportion of risks, irrespective of size
– passes a share of any profit to the reinsurer
– is unsuitable for unlimited covers.

Surplus

Surplus is proportional treaty reinsurance whereby the proportion of risk covered varies from risk to risk depending on the size and type of risk.

The EML for a risk is used in assessing the proportion of the risk to reinsure, defined in terms of "lines". If k lines are used for a risk then premiums and claims are split in the proportion $1:k$.

The *width* of one line represents the amount the insurer would pay if a claim equal to the EML occurred. This amount is called the retention (r). Therefore: $EML = (1+k)\times r$.

A surplus treaty will usually specify a maximum number of lines and a minimum and maximum retention. Higher levels of cover can be obtained by purchasing a second (and third, and fourth) surplus treaty.

The cedant and reinsurer will have different experience: smaller risks may be retained in full by the cedant, whereas larger risks may be covered primarily by the reinsurer.

Surplus:

+ enables the insurer to fine-tune its experience
+ enables the insurer to write larger risks
+ is useful for classes where wide variation can occur in the size of risks
+ helps to spread risks
+ may provide commission that helps with cashflow
– requires more complex administration
– is unsuitable for unlimited covers and personal lines cover.

Excess of loss

The reinsurer covers the risk (or a proportion of it) between defined layers, the limits of which are often indexed for inflation (using a *stability clause*). The insurer may choose to have a number of layers of cover with different reinsurers.

Once the layer of cover has been "burnt through", it will need to be reinstated, which might require a further reinsurance premium to be paid.

The cedant's and reinsurer's experience will be different and will depend on the distribution of large losses.

There are three main types of excess of loss reinsurance:

- *Risk XL* – this relates to individual losses and is usually written by treaty
- *Aggregate XL* – this relates to cumulative losses, where the aggregation may be by event, by peril or by class
- *Catastrophe XL* – this is a form of aggregate XL covering severe losses within the *hours clause* that result from a specified event.

Excess of loss:

+ allows the insurer to accept risks that could lead to large claims
+ reduces the risk of insolvency from a large claim, an aggregation of claims or a catastrophe
+ reduces claim fluctuations (and so smooths results)
+ helps to make more efficient use of capital.

Stop loss

Stop loss is a specific type of aggregate XL, which covers against very bad experience across a whole account over a defined time period. The limits are usually defined as loss ratios (*ie* as percentages of premiums).

Finite risk (or financial) reinsurance

The main feature of a financial reinsurance contract is that it involves only a small element, if any, of transfer of insurance risk from the cedant to the reinsurer. Financial reinsurance was devised primarily as a means of improving the apparent accounting position of the cedant.

The following are examples of finite risk reinsurance products:

- *time and distance policies* – the insurer pays the reinsurer a premium and in return, the reinsurer pays an agreed schedule of claim payments; this has the effect of discounting the reserves of the insurer for the time value of money
- *spread loss covers* – the insurer pays an annual or single premium to the reinsurer for the coverage of specified claims; these may be used to provide liquidity and security to the insurer and may be used for catastrophes
- *financial quota share* – this is quota share (as described above) purchased in order to obtain reinsurance commissions for financing assistance
- *structured finance* – these often provide the insurer with a credit enhancement, which lowers the cost of borrowing
- *industry loss warranties* – these are a type of reinsurance that pay out based on industry losses rather than losses to individual insurers.

Run-off reinsurance

Run-off reinsurance solutions focus on the full-scale risk transfer of reserve development risks.

Adverse development covers involve the purchase of reinsurance cover for the ultimate settled amount of a block of business above a certain pre-agreed amount. Reserves are maintained by the insurer.

Loss portfolio transfers involve the purchase of reinsurance cover for the ultimate settled amount of a block of business in its entirety. Reserves are transferred to the reinsurer along with all remaining exposure to the business.

Capital market products

Committed (or contingent) capital is a contractual commitment to provide capital to an insurer should a specific adverse event occur that causes the insurer financial distress.

Securitisation may be used to manage risk or capital. Examples of securitisation include:

- *insurance-linked securities (ILSs)* – often in the form of catastrophe bonds, where the issuer can default on the interest and capital payments on the bond if the catastrophe occurs
- *credit securitisation* – these may enhance the creditworthiness of debt instruments and provide capital relief to banks / credit protection to companies
- *motor securitisation* – certain aspects of a motor insurer's portfolio risks are passed to the investment market.

Chapter 6 Solutions

Solution 6.1

The first split of the premium is $5,000 to each of the direct writer and the quota share reinsurer. But the direct writer pays commission of $1,500 (*ie* 15% of $10,000) and receives $1,000 (*ie* 20% of $5,000) from the reinsurer.

This leaves the net of commission premiums as $4,500 for the direct writer and $4,000 for the quota share reinsurer. This split doesn't look unfair. (You would expect the direct writer to have a bigger share to compensate for the initial work.)

The direct writer would be able to reclaim $25,000 from the reinsurer on a claim of $50,000.

Solution 6.2

Quota share is a form of treaty reinsurance whereby all the premiums and claims (for risks within the terms of the treaty) are split in a fixed proportion.

Solution 6.3

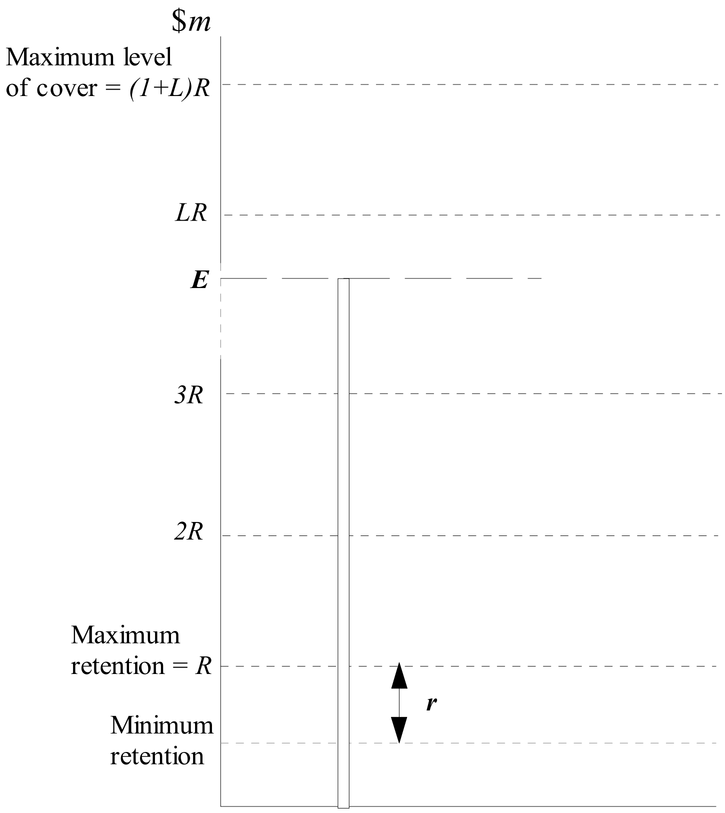
	Direct Writer		Reinsurer	
Size	Premium	Claim	Premium	Claim
£300,000	£750	–	£3,750	–
£450,000	£667	£8,889	£5,333	£71,111
£60,000	£1,250	–	£250	–
£40,000	£700	–	–	–
£1,000,000	£600	–	£11,400	–
£500,000	£720	£5,000	£6,480	£45,000

Solution 6.4

The reinsurer might require a minimum retention level in order to prevent the insurer from having too little interest in the risk. Having too little interest in the risk could lead to poor underwriting or claims management with respect to that risk.

The reinsurer might require a maximum retention level in order to prevent very small risks being passed on, as these are likely to be administratively expensive (compared to the size of the risk).

Solution 6.5



Note that if E exceeds (1+L)R then the insurer will have to arrange further reinsurance, eg a *second surplus treaty.*

Solution 6.6

Risks may not be ceded to the reinsurer if:

- the EML (*E*) falls below the insurer's retention (*R*)
- the EML (*E*) is above the maximum size of a risk $(1+L)\times R$ (unless a second surplus treaty can be arranged).

Solution 6.7

(a)	The direct writer's retention is:	£50,000
(b)	The number of lines used was (in the order given):	5, 8, 1/5, 0, 19, 9
(c)	The largest allowable risk would be:	£650,000 if 12 lines were available

Solution 6.8

Whereas quota share has the same proportion of every risk ceded to the reinsurer, the proportion ceded will vary from risk to risk with surplus reinsurance.

Solution 6.9

Assume (a) that the direct writer kept £80,000 of risk in each case, and then (b) that they ceded the largest possible proportion. The amount recovered could then be anywhere between these limits:

1. (a) £8,000 (b) £10,667
2. (a) £10,000 (b) £10,667
3. (a) £16,000 (b) £71,111
4. (a) £533,333 (b) £533,333
5. (a) zero (b) £88,889
6. This could not be placed on the treaty unless other cover (*eg* prior XL) was in place.
7. (a) £100,000 (b) £177,778

Solution 6.10

If the risk exceeds the capacity of the treaty, then the insurer may be able to find a "second surplus" treaty to cover risks over and above the maximum covered under the first surplus treaty.

Other options are not to cede the risk at all, or to find non-proportional reinsurance on a facultative basis for amounts above the EML.

Solution 6.11

The smallest risks may be retained in full. A large proportion of each of the risks in the next size up will be retained. Only a small portion of the largest risks will be retained.

In contrast, the portfolio of ceded risks will have no exposure to the smallest risks and a much higher proportionate exposure to the larger risks (*eg* it could have 10 times the direct writer's exposure for the biggest risks).

Solution 6.12

There is a "hole" in the cover between £500,000 and £700,000. The layers of reinsurance ought generally to be arranged so that there are no gaps.

Recoveries from the treaties are:

Claim	1st layer	2nd layer	3rd layer	Total
(a)	£20,000	–	–	£20,000
(b)	£140,000	£80,000	–	£220,000
(c)	£140,000	£300,000	£1,700,000	£2,140,000
(d)	£140,000	£300,000	£2,000,000	£2,440,000
(e)	£140,000	£300,000	–	£440,000
(f)	–	–	–	–
(g)	£140,000	£300,000	–	£440,000

Solution 6.13

Claim of 350,000 (when index was 105)

When the index is 105, the layers of reinsurance cover become 105,000 in excess of 105,000 and 315,000 in excess of 210,000. Therefore:

Direct writer pays the first 105,000

First XL reinsurer pays the next 105,000

Second XL reinsurer pays the remaining 140,000.

Claim of 600,000 (when index was 110)

When the index is 110, the layers of reinsurance cover become 110,000 in excess of 110,000 and 330,000 in excess of 220,000. Therefore:

Direct writer pays the first 110,000

First XL reinsurer pays the next 110,000

Second XL reinsurer pays the next 330,000.

Direct writer is left with the remaining 50,000, *ie* paying 160,000 in total.

Solution 6.14

Assume that the reinstatement premium will be calculated according to the proportion of cover burnt through. We are told to ignore the issue of the period of cover remaining, although in practice it could be a significant issue.

First claim: recover \$1*m* (*ie* \$2.0*m* less \$1*m* excess) from the reinsurer. Note that \$0.5*m* of the reinsurance cover, *ie* a third, is yet to be used. Pay a reinstatement premium of \$0.28*m* (*ie* two-thirds of 140% $\times$300,000). In practice this is likely to be offset from the reinsurance recovery.

Second claim: recover \$1.5*m* from the reinsurer, and pay a further reinstatement premium of \$0.14*m* (the remainder of the reinstatement premium). Again this is likely to be offset from the reinsurance recovery. This gives cover remaining of one-third of the layer, *ie* \$0.5*m* in excess of \$1*m*.

Third claim: recover \$0.5*m* (the remaining cover). No further reinstatement premium can be paid.

Fourth claim: No reinsurance recoveries and no further reinstatement premium.

The rate on line is defined, for non-proportional reinsurance, as the total premium charged for the reinsurance divided by the width of the layer covered. Note by total premium we are only referring to the initial premium, the reinstatement premium should be excluded.

Hence the rate on line is $\frac{0.3}{1.5}$, *ie* 20%.

Solution 6.15

- The direct writer will want to maintain a good long term relationship with the reinsurer.
- There might be some profit commission.
- Future premiums will be related to past claims experience.

Solution 6.16

None! There's no need for return or override commission because the reinsurer charges a premium, so the commission would simply act to reduce this premium. Profit commission would be silly because the reinsurer would expect claims rarely.

Solution 6.17

If it was not, then the direct writer could, after taking out the cover, write loads more business, by cutting premiums, and trigger the stop loss limits that way.

By using claim ratios, the limits (and the premium charged for the cover) rise in proportion to the amount of business written by the direct writer.

Solution 6.18

(a) ***Risks to a stop loss reinsurer***

- the direct writer's premiums may be under-priced (*eg* a competitive market)
- poor underwriting by the direct writer
- poor premium rating structure leading to adverse selection
- poor claims experience (*eg* catastrophes and/or large claims)
- generally adverse claims experience (*ie* random event)
- poor claims control

(b) ***So why not take stop loss in practice?***

The reinsurer would need to charge the insurer a very high premium to cover themselves against the risks.

(c) ***Better moves for the direct writer***

- Set up tight internal controls to ensure good premium rating, underwriting and claims control.
- Buy particular types of reinsurance to guard against specific events (*ie* buy individual excess of loss and catastrophe excess of loss as needed).

Solution 6.19

(a) Reinsurers are often not prepared to provide stop loss cover because:

- the reinsurer has only limited control over initial underwriting and claim payments made
- historically some stop losses have been loss making.

(b) Conditions the reinsurer may impose before providing stop loss cover are:

- impose a deductible so that the insurer retains a proportion of the risk
- maintain some control over underwriting, premium rates and claims.

Solution 6.20

Stop loss cover protects the direct writer against many claims arising *over an accounting period*. For a company using normal accident-year accounting the corresponding exposure period is the accident year. Hence a losses-occurring basis would be used (except when using a funded accounting basis, *eg* at Lloyd's, when a policies-incepting basis would be used).

The concepts behind accident-year and funded accounting will be described in detail later in the course.

Solution 6.21

With pre-funded arrangements, the insured bears the risk of the reinsurer's default, and vice versa for the post-funded arrangement.

Solution 6.22

The insurer may wish to take out this arrangement:

- to improve its published solvency margin. Depending on local accounting regulations, it may be able to show the expected return of (at least) £1.176*m* as an asset and this could improve its published solvency margin by £0.176*m*.

- as an investment. It will make between 17.6% and 20% return on the reinsurance premium over 4 years, and this may be considered to be a good return.

The insurer will also need to consider whether or not four years is a good match to the term of the liabilities covered by the arrangement.

Note that the reinsurer will also want to benefit from the deal. Benefits for the reinsurer might include:

- *an improvement in the published solvency position*
- *a good investment (if it can make a better return than it is giving the insurer)*
- *tax efficiency*
- *a good relationship with the cedant, giving the opportunity for cross-selling.*

Solution 6.23

A non-proportional arrangement, *eg* a stop loss or excess of loss contract.

Solution 6.24

The existence of a separate vehicle with separate ownership of the securitised assets provides better security and greater transparency for investors in the securitisation.

Chapter 7

General insurance markets

Syllabus objective
(d) Describe the implications of the general business environment in terms of the:
– main features of the general insurance market
– effect of different marketing strategies
– effect of the regulatory and fiscal regimes
– general effect of professional guidance.
Covered in part in this chapter.

0 The general insurance market

So far in this course we have looked at the different types of general insurance products. In this chapter and the following chapter we consider some important features of the general business environment and the implications that they have for general insurance business.

This chapter begins by discussing the market for general insurance, *ie* who are the providers of general insurance?

Section 1 describes different types of insurance companies, and the role of the London Market, which includes Lloyd's syndicates. It also covers various forms of self-insuring groups.

The general insurance market is very diverse. World-wide, general insurance is provided by insurance companies and Lloyd's syndicates. A corporate entity may also manage its self-insurance through a captive insurance company.

Cover is sometimes provided also by governments as insurers of last resort – though rarely in these situations will there be prices, reserves and capital established as in the traditional format.

In turn, insurance providers will wish to obtain reinsurance for some of their risks. This can be obtained:

- **from the London Market**
- **from Lloyd's**
- **from specialist reinsurance companies**
- **from direct insurers who also write reinsurance**
- **in some cases, from the capital markets.**

Depending on the size of a risk, a combination of these options may be used.

Section 2, on marketing strategies, describes in more detail the methods used to obtain insurance business. This section focuses on methods used by insurance companies and in Lloyd's.

Sections 3 and 4 discuss the effect of regulatory and fiscal regimes, and of professional guidance on general insurance business.

The next chapter describes further external influences on general insurance business, such as economic, legal, political and environmental factors.

1 The major providers

1.1 The companies

Direct insurance companies

Direct insurance companies provide insurance for individuals and companies.

In this context "direct" means a company that writes insurance directly for an insured person or company as opposed to one that writes reinsurance.

Direct market companies are very active in "personal lines" insurance, which is business sold to individuals. For example, you might be a user of private motor, household, medical insurance or travel insurance.

In the context above "direct" means "not reinsurance". However, the term *direct* can be open to some misinterpretation. It has historically been taken to mean that there is a direct contract between the insurer and the policyholders, as distinct from reinsurance where the policyholder may not even have heard of the "final insurer". The term *direct* has also recently come to describe insurance sold direct to the public, often by phone, without going through a broker.

The alternative meaning of a "direct" insurance company – one that deals directly with its customers rather than through an intermediary such as a broker – is not meant in this context**.**

Direct insurers can be divided into the following three groups:

- **Composite insurance companies — insurance companies that write both general insurance and life insurance.**
- **(Pure) general insurance companies that write many classes of general insurance.**
- **Insurance companies that specialise in writing business in a selection of classes of general insurance (or even just a single class).**

Writing both life and general insurance business

Many groups contain both life insurance companies and general insurance companies. While these groups are conveniently referred to as "composite insurers", the risk-taking companies themselves will usually be pure life assurance companies or pure general insurance companies.

Suppose Company X wishes to provide its customers with both life insurance and general insurance. It may, for example, establish itself as a life insurance company and write life insurance business, and set up an arrangement with a general insurer (or insurers) that will "underwrite" any general insurance business that Company X sells.

Question 7.1

Discuss why Company X might want to do this, rather than write general insurance business directly.

Alternatively, Company X may:

- write general insurance but have its life insurance business underwritten by a life insurer
- set itself up as a composite insurer so it can write both types of business (although new composites haven't been allowed in the UK for many years now).

Corporate structure of general insurers

General insurance companies may be:

- proprietary (*ie* owned by shareholders in order to make profit)
- mutual (*ie* owned by policyholders).

Most insurance companies are proprietary companies limited by shares. However, some mutual insurance companies do exist; they are more common in some markets than in others.

Reinsurance companies

Reinsurance companies provide cover for insurance providers. Again, some reinsurance companies will specialise in only writing some types of reinsurance, while others write many different classes of reinsurance. Some insurance groups write both direct insurance and reinsurance.

1.2 The London Market

When UK general insurance is discussed you will often hear of "the London Market". This does not include all general insurance companies with London phone numbers! It does mean a specialised group of insurers, writing business along broadly similar lines, which is based in an area within the City of London.

You can simplistically think of it as "Lloyd's plus company equivalents". That is, Lloyd's is an important part of the London Market, and the rest of the market is made up of companies with buildings near Lloyd's offering insurance similar to Lloyd's.

The London Market is that part of the insurance market in which insurance and reinsurance business is carried out on a face-to-face basis in the City of London. These companies tend to be physically located close to each other.

The London Market concentrates mainly on providing insurance and reinsurance cover to companies. It specialises in:

- **the larger direct insurance risks – both property and liability – that are beyond the capability of other direct insurance companies (for example, energy and aerospace risks)**
- **international risks**
- **reinsurance.**

Most UK insurers who transact this type of business, or foreign insurers who transact this business in the UK, therefore operate within the London Market. Consequently, the participants in the London Market would include:

- **Lloyd's syndicates** (see Section 1.3)
- **UK subsidiaries or branches of overseas insurance or reinsurance companies**
- **the reinsurance departments of UK composite companies, or reinsurance subsidiaries of such** companies
- **small professional reinsurance companies set up by (or acquired by) large broking firms for the specific purpose of transacting London Market business**
- **captives** (see Section 1.4)
- **P&I Clubs** (see Section 1.4)
- **companies owned by a group of insurance or reinsurance companies**
- pools (see Section 1.4).

Over 100 companies operate within the London Market, as well as many syndicates in Lloyd's.

Although the London Market concentrates mainly on providing insurance and reinsurance cover to companies. Lloyd's does also provide private motor insurance and some other personal lines cover.

1.3 *Lloyd's syndicates*

What is Lloyd's?

Lloyd's origins lie in a coffee shop run by Edward Lloyd in the late 1680s. Lloyd's coffee shop became an informal meeting place for merchants, ship owners and ships' captains. The shop soon became known as a source of reliable shipping news, coffee and as a place for merchants to find private individuals who would share in the risks of proposed ventures.

Edward Lloyd did not carry out any insurance himself. His role was to provide the premises (and the coffee!) for his clients. Similarly today, Lloyd's itself does not carry out insurance business. Instead it provides the facilities for its members to carry out insurance.

Lloyd's of London is a unique insurance institution. It began in Edward Lloyd's coffee shop in the late 1680's before being incorporated by the Lloyd's Act of 1871. It is not an insurance company – it is a marketplace made up of members who provide capital and accept liability for risks that are underwritten in return for their share of any profits that are earned on those risks.

Names

Historically, all the members of Lloyd's were individuals, known as "Names". However, in 1994, companies were allowed to become Names for the first time. This is called "corporate capital" and the companies themselves are known as "corporate Names". A corporate Name is a limited-liability company whose only business is to provide capital to Lloyd's.

"Limited liability" means that the corporate member cannot lose any more than the capital it has provided. This is discussed in more detail below.

Nowadays, corporate capital accounts for more than 90% of the capital in the Lloyd's market.

Some insurance companies and reinsurance companies are active in Lloyd's through corporate Name subsidiaries.

Syndicates

Lloyd's members conduct their insurance business in "syndicates", which are groups of members who collectively co-insure risks. At the end of 2011 there were 88 syndicates at Lloyd's. The syndicates employ underwriters to write insurance and reinsurance business on behalf of the members. Individual syndicates often specialise in particular types of insurance. Each member who belongs to a particular syndicate will contribute capital to that syndicate and will accept a portion of the insurance risks written by the syndicate; the share of each member being predetermined according to the amount of capital they have contributed. The profit or loss made by the syndicate is then shared among its members in these proportions. The member's share of a syndicate is fixed during an underwriting year but may change from year to year.

Limited liability

Traditionally, individual Names could not limit their liability in respect of their exposures at Lloyd's: they were potentially liable up to the full extent of their personal wealth. However, when corporate capital was admitted, this was on a limited-liability basis so that a corporate member's maximum loss was limited to the amount of capital it provided to Lloyd's. It subsequently became possible for individual Names also to limit their liabilities and many have opted for this approach.

Size of syndicate

Lloyd's members often spread their exposure by belonging to a number of different syndicates. However, some corporate members only underwrite through a single syndicate and some syndicates have only one Name; a corporate member that provides all the syndicate's capital.

Syndicates vary in size from just one Name to over a thousand. A syndicate is analogous to an insurance company and the Names belonging to the syndicate are broadly analogous to shareholders in the insurance company.

Question 7.2

From what we have covered on Lloyd's so far, what do you think is the key difference between an individual Name's involvement in a syndicate and a shareholder's involvement in an insurance company?

> ***Question 7.3***
>
> State the types of insurer that provide insurance in the direct insurance market and the ways they may be owned.

Access to global licences

One key advantage that operating a Lloyd's syndicate has is access to Lloyd's global licences that enable Lloyd's syndicates to write business almost anywhere in the world.

Lloyd's is licensed or eligible to undertake insurance business in around 80 countries. This, together with the strong international brand that Lloyd's has, enables syndicates to start writing business in new territories more easily than most insurance companies.

Capital efficiency

Another key advantage arises from the capital efficiency of writing business through Lloyd's (capital structure, FAL, Central Fund, *etc*). This is considered further in Subject SA3.

Funds at Lloyd's (FAL) is the capital fund of a member, *ie* the amount of capital, specified by Lloyd's, that each member must provide. This may be lodged either through physical assets or via a letter of credit.

The Central Fund is a further layer of protection for policyholders. It is built up from contributions by members and held centrally by Lloyd's.

1.4 Self-insuring groups

> ***Question 7.4***
>
> Define self-insurance.

Some insurers are set up with the primary purpose of providing self insurance for their owners' or members' insurance risks.

Captives

A captive insurance company is an insurer that is wholly owned by an industrial or commercial enterprise and set up with the primary purpose of insuring the parent or associated group companies and retaining premiums and risk within the enterprise.

There is another use of the term "captive" in the insurance market, but this is not what is meant here:

The term "captive insurance company" is often applied to an insurance company set up to sell insurance to the parent's customers. This usage is sometimes made, but cannot be considered to be technically correct.

If you come across captives in the exam, you should assume the first interpretation, *ie* an insurer set up for risk retention purposes, *unless* an alternative meaning is clear from the question.

Question 7.5

Without reading ahead, suggest conditions under which a company may consider setting up a (correctly defined) captive.

The usual reasons for setting up captives include the following:

- **To fill gaps in insurance cover that may not be available from the traditional insurance market.**
- **To manage the total insurance spend of large companies or groups of companies.**
- **To enable the enterprise to buy cover directly from the reinsurance market rather than direct insurers.**
- **To focus effort on risk management.**
- **To gain tax and other legislative or regulatory advantages.**

To facilitate the last aim, captives are usually set up in a location where it is possible to gain such advantages. There are a number of locations that host a significant captive-insurance industry. However, this has now perhaps become a secondary consideration, with the primary purposes being the other ones mentioned above.

In addition to accepting the risks of their parent companies, captives may also accept external risks on a commercial basis. In some territories this is necessary in order for insurance premiums paid by the parent to be tax-deductible.

An authorised captive is free to provide insurance to risks other than those of its parent, providing this does not change its main purpose. Those who do, called "open market captives", often provide insurance to the parent company's customers.

Reinsurance may be used by the captive to limit the extent of self-insurance. For example, the captive may obtain reinsurance to cover very large losses whilst retaining most of the risk itself. Alternatively, the captive may be set up in order to gain direct access to the reinsurance market with all (or most) of the risks being passed to the reinsurer.

Growth areas, in volumes of business being put through captives, include professional liability, extended warranty, mortgage indemnity, employers' liability and product liability. Growth is strongest in areas where the traditional market is:

- expensive
- volatile (the premiums are very variable)
- simply not offering the cover, as with pollution, for example.

Protection and Indemnity (P&I) Clubs

P&I Clubs are mutual associations of ship owners that were originally formed to cover certain types of marine risks (mainly liability), that could not be covered (at an acceptable price) under a commercial marine policy.

Today, insurance for many of these risks may be found in the commercial market. However, owing to their mutual nature and technical expertise, P&I Clubs still currently provide around 90% of the world's coverage against marine liability claims.

One of the main points here is that if a mutual can do the job just as well as an external proprietary insurer, the mutual would usually be preferred since, other things being equal, there's no "insurer's profit margin" to pay for.

In addition to providing insurance, P&I Clubs also provide ship owners with technical assistance in the marine market and advice on issues relating to the shipping industry.

Some of the largest P&I Clubs themselves mutualise in respect of very large claims.

The mutual organisation being referred to is the International Group of P&I Clubs which consists of 13 P&I Clubs, representing approximately 90% of the world's ocean-going tonnage. The member clubs use the International Group as a mechanism to pool their larger risks and to arrange reinsurance to help deal with very large claims. Not all P&I Clubs belong to the International Group.

Pools

A pool is an arrangement under which the parties agree to share premiums and losses for specific insurance classes or types of cover in agreed proportions. To some extent, all insurance is pooling but specific pooling arrangements are sometimes used, particularly where the risks are very large (*eg* atomic energy risks) or through mutual associations that cater for an industry.

P&I Clubs, mentioned above, are a type of pool.

The critical difference between insuring with a conventional insurer and insuring with a pool is that the insured's liability to an insurer is limited to the premium charged, whereas the liability to a pool will be related to the insured's share of the total claims and other costs that arise.

1.5 Non-UK Markets

All insurance markets tend to comprise the same participants: mutual and joint-stock (*ie* proprietary) **insurance companies. Pools are also found in a number of markets, although not all.**

The most developed insurance markets tend to be in large developed economies, notably the USA, Japan, Canada, France, Germany, Italy and Spain.

Many markets have reinsurance companies, but reinsurance is more likely to be placed internationally than direct business; notable reinsurance markets are the USA, Switzerland and Germany.

Bermuda

One market that should be mentioned separately is Bermuda, which has become a major international centre for insurance and reinsurance, despite not being a large economy in its own right.

Question 7.6

Why do you think this is?

A number of large insurance companies and groups are domiciled there, and some UK insurance groups have moved their principal domicile there in recent years.

Bermuda had been the world's most important domicile for captive insurance companies for some decades; other important captive centres include the Cayman Islands, Vermont, the British Virgin Islands, Guernsey, Barbados, Luxembourg and Dublin.

Insurance and reinsurance groups are not limited to a single market and many operate out of multiple markets.

1.6 Non-traditional markets

The transfer of insurance risk to the banking and capital markets is known as securitisation. Some examples of this were covered in Chapter 6 and in Subject CA1.

The capital markets are increasingly involved in taking insurance risk through Industry Loss Warranties (ILWs), catastrophe (cat) bonds, "sidecars" and even traditional reinsurance contracts. These often need more complex legal arrangements involving transformer vehicles and interest rate swap arrangements.

Industry Loss Warranties, which were mentioned in Chapter 6, are types of reinsurance or derivative contracts through which one party will purchase protection based on the total loss arising from an event to the entire insurance industry, rather than their own losses.

Question 7.7

Explain how a catastrophe bond differs from a traditional bond, give examples of relevant trigger events and explain how catastrophe bonds transfer risk from the insurer to the purchasers of the bond.

A "sidecar" is a financial structure that is created to allow investors to take on the risk of a group of insurance policies. It is a means of allowing investors exposure to the reinsurance market without having to invest in existing reinsurance companies, which may have losses from previous years. A sidecar acts like a reinsurance company but it reinsures only one cedant and the investors need to place sufficient funds in the entity to ensure that it can meet any claims that arise.

The above structures are commonly purchased by hedge funds and other investors.

Question 7.8

Outline possible advantages of using securitisation products, such as those mentioned in this section, to cede insurance risk.

They tend to have a very high reliance on third-party models and as a result are predominantly (though not exclusively) involved with catastrophe risk.

For example, the trigger event for a catastrophe bond may be linked to the extent to which the modelled loss exceeds a specific threshold once the model's parameters have been updated to reflect the event (*eg* an earthquake), rather than being based on the insurer's actual losses.

2 Marketing strategies

We now consider methods of acquiring insurance business.

2.1 Non-London Market business

Insurance business is obtained:

- **through intermediaries such as brokers, banks and so on**
- **through staff directly employed by the insurance provider**
- **through internet, telesales, post and off-the-page advertising.**

If any of these terms are unfamiliar to you then read on – they will be explained below.

Intermediaries

Brokers

Brokers act as intermediaries between the seller and buyer of a particular insurance or reinsurance contract without being tied to either party. They are likely to be paid by commission (*brokerage*) from the insurer, but when placing business legally (under the "law of agency") they are the agent of the insured.

Brokers may also carry out some functions on behalf of insurers (for example, operating binding authorities / line slips). When carrying out these functions they are legally agents of the insurer.

Note the above distinction between when the broker is an agent of the policyholder and when it is an agent of the insurance company.

Binding authorities (also called "binders") **are contractual agreements setting out the scope of delegated authority, allowing cover holders to enter into contracts of insurance and to issue insurance documents on behalf of Lloyd's managing agents.**

The contract will specify the period for which insurance can be placed, the classes of business covered and the policy wordings that are to be used.

Many agencies are paid a percentage of premiums as commission. This causes a potential conflict of interest for the agency because it has an incentive to increase premiums without sufficient regard for the profitability of the business.

An agency can increase its commission income by writing a very high volume of business. This can happen, especially for price-sensitive classes, when premium rates are too low and the business is loss-making.

Underwriting agencies represent a very large source of London Market business. Many of these agencies were formed by brokers. In some cases, a company may establish a specialist agency to underwrite risks on behalf of an insurer. Some agencies have been formed with specialist risk management functions in-house to write specialist business on behalf of an insurer.

Outside of Lloyd's, binding authorities can be used to allow a broker to enter into contracts of insurance and issue insurance documents on behalf of an insurance company.

Similarly, a line slip is a facility under which underwriters delegate authority to accept a predetermined share of certain coinsured risks on their behalf. The authority may be exercised by the leading underwriter on behalf of the following underwriters; or it may extend to the broker or some other agent authorised to act for all the underwriters.

Tied agents

Organisations such as banks and building societies are sometimes tied to a particular insurer (perhaps part of the same group) and sell that insurer's products alongside their own. They are usually paid by commission for this service. They may also act as brokers or have an insurance-broking subsidiary.

These options are not exclusive – some banks own insurance companies that write some lines (for example, property and creditor), but act as broker or tied agent for other lines (for example, motor).

Sale of a particular line of business through a tied agent is exclusive to a particular insurer. If Insurer A sells motor policies through a tied agent, the tied agent will not also sell motor policies from any other insurer. The tied agent may, however, sell property policies from Insurer B.

Staff directly employed by the provider

Staff employed by the provider may be paid a fixed salary, or entirely by commission, or, more usually, by a mixture of these two methods.

Direct marketing

Some insurers employ staff in direct sales, where potential policyholders are invited, through advertising, to make proposals by telephone or the internet, or are attracted through cold-call selling by post or telephone.

Methods used

All of the above methods of acquisition are used to some extent across most lines of business, although the main method varies by country and by type of insurance. Mass advertising in the media tends to be used for personal lines and small commercial lines of insurance. For larger commercial risks, personal contact through the insurer's sales force or specialist insurance and reinsurance brokers are the more usual methods of acquisition.

The type of intermediary will often depend on the class of business. With classes such as domestic buildings insurance, mortgage guarantee insurance and travel insurance, insurance brokers may not be the natural intermediaries. With buildings insurance and mortgage guarantee insurance, much of the business is sold through the building society or bank that supplied the mortgage for the house purchase. In the case of travel insurance, the travel agent who arranges the travel will often sell the insurance. Some insurance companies have tried to develop this principle for the motor market by arranging for car insurance to be provided on new car purchases.

It is unlikely that the methods of sale of insurance will stay static. In the quest for improved profitability, insurers constantly review their methods of selling business. In the UK within the last decade there have been increased attempts to sell insurance directly to the public, avoiding the intermediaries altogether. Direct sales to the public can be made through advertising in the media *(eg* TV, newspapers), by direct mail, by phone or using the internet. Making direct sales using the phone is often called telesales. Placing advertisements in newspapers, magazines or phone books *etc* is called off-the-page. The main classes affected by the growth of direct sales in recent years have been private motor and domestic property.

In some respects, the sale of insurance is quite different from the sale of, say, toothpaste. For insurance companies, reviewing the efficiency of intermediaries extends beyond an analysis of the number of sales made and the expenses incurred (*ie* commission). The insurer will also need to keep a close eye on the quality of the business sold, *eg* what was the claim experience for business sold through each of the sales outlets?

2.2 London Market business

The London Market insurance providers, including Lloyd's, have traditionally acquired business through specialist brokers and, in particular, international brokers using the slip system (also called the *subscription* system)**.**

Historically, Lloyd's syndicates could only write risks that they received through Lloyd's brokers. However, for certain standard proposals in personal lines a syndicate may now deal directly with the insured or a non-Lloyd's broker.

Under the slip system in the subscription market:

1. The insured approaches a London Market broker.

2. **The broker prepares a slip that shows, in a standard format, the main features of the risk to be insured.**

3. **The broker shows the slip to one or more quoting underwriters, who, on the basis of the slip and further information as appropriate, quote a premium.** The broker will know which of the many companies and Lloyd's syndicates are the experts in this particular type of risk. Note the high level of trust that the underwriter places in the broker by relying upon the details given in the slip, although they could sue if they have been misled. The underwriters trust the brokers to have disclosed all the facts that will be relevant in assessing the risk.

4. **The cedant (with the broker's advice) will then select a lead underwriter and a "firm order" price for the broker with which to approach the market. This firm order price may be below any of the quoted prices.**

5. **The lead underwriter accepts a share of the risk by stamping and signing the slip.** On accepting a portion of the risk, the underwriters write their names (and the proportion they are willing to accept) on the slip under the description of the risk, hence the term *underwriter*.

6. **The broker then approaches other underwriters (the following market) to accept the risk on the same terms. The follow underwriters indicate the share that they are willing to take by stamping and signing the slip under the lead underwriter line. (Hence the etymology of the term *underwriting*.) All the underwriters act as coinsurers with several liability.**

   Even though the underwriters share the risk, "several liability" means each underwriter is separately liable for its obligations, and so could be sued separately for any loss that it is due. If one underwriter went bankrupt, the other underwriters would not be liable for that underwriter's share of the losses.

7. **The broker continues until he or she has finished placing the risk (that is, received offers for 100% or more of the risk).**

   Note the importance of the rate quoted by the lead underwriter. As all the underwriters follow the premium rate set by the lead underwriter (except in exceptional circumstances), it is important that the rest of the market respects the lead underwriter.

Also, it is important that the premium rate set by the lead underwriter is not so low that no other underwriter is prepared to follow.

8. **If the written lines exceed 100% then, in agreement with the insured, they are reduced (or "signed down") so that the signed lines total 100%.**

   The shares of the underwriters may total something in excess of 100%. (This assists in the continuity of placing the risk in future years.) The total is then adjusted by reducing all the percentage shares (often proportionately, but not necessarily so). The underwriter's acceptance of an initial percentage of the risk means that this is the maximum they are prepared to accept on those terms.

9. **If it is not possible to find capacity to place 100% of the risk, an additional shortfall cover may need to be placed at different terms.** So the premium rate is increased or the cover or terms renegotiated.

If the risk is over-placed, this indicates that the firm order price was probably too high. Conversely, if the risk is not fully placed the firm order price was probably too low. Not placing the full risk, unless intentional, is usually a bigger issue for the insured and the broker.

In general, all (re)insurers on the slip receive the same terms. However, there are some markets where the lead underwriter may receive a higher rate to reflect the additional work that they carry out on behalf of the following market.

3 Regulatory and fiscal regimes

3.1 The need for supervision

Why should insurance business suffer more legislation than, say, umbrella manufacturers? One of the reasons is that there is more scope for the purchaser to lose out financially. When you buy an umbrella, you have a look at it, and if you like it, you pay the price. However, with insurance, you pay the price at the start of the contract and you have to trust the insurer to pay valid claims as and when they arise in the future.

The uncertainty underlying insurance business means that it is not just a question of trusting the honesty of the insurer. The insurer may be very well meaning, but if the insurer's business is not soundly managed, you may find that the insurer has collapsed by the time you need to make a claim.

In many countries, therefore, there are specific rules and regulations that apply to general insurers. Different countries adopt different approaches to the regulation of insurers' operations.

3.2 Effect of the regulatory regime

The following regulatory restrictions on the actions of a general insurer may be encountered in one or more countries of the world:

- **Restrictions on the type of business that a general insurer can write or classes for which the insurer is authorised.** An authority could prevent an insurer from writing volatile classes of business or classes where it had little expertise.

- **Limits or controls on the premium rates that can be charged.**

  For example, the authorities in some US states, *eg* Massachusetts, set the personal motor premium rates that must be charged. An authority could also set a maximum or minimum premium or restrict the way in which the premiums are calculated. For example an authority could set a maximum allowance for expenses defined as a percentage of the gross premium.

- **Restrictions on the information that may be used in underwriting and premium rating** (perhaps to avoid unfair discrimination)**.**

- **A requirement to deposit assets to back claims reserves.**

- **A requirement that the general insurer maintains a minimum level of solvency, measured in some prescribed manner,** *ie* a minimum level of free assets. This might, for example, be calculated as a proportion of premiums written.

- **Restrictions on the types of assets or the amount of a particular asset that a general insurer can take into account for the purposes of demonstrating solvency.** This might be with the possible aim of avoiding risky investments or increasing diversification.

- **A requirement to use prescribed bases for calculating premiums or for valuing the general insurer's assets and /or liabilities when demonstrating solvency.**

- **Restrictions on individuals holding key roles in companies.**

- **Licensing of agents to sell insurance and requirements on the methods of sale and disclosure of commission / broking terms.**

- **A requirement to pay levies to consumer protection bodies.**

- **Legislation to protect policyholders if a general insurer fails.**

Question 7.9

Suggest possible legislation that could be used to protect policyholders if a general insurer fails.

Other regulations that could be imposed on general insurers include:

- requirement to provide detailed reports and accounts at prescribed intervals
- requirement to purchase reinsurance
- requirement to hold a claims equalisation reserve
- limits on contract terms
- restrictions on the rating factors used or on the ability to decline cover (to avoid discrimination)
- advertising restrictions
- prescription to hold certain assets.

Regulation is considered further in Subject ST7.

3.3 *Effect of the fiscal regime*

In most countries the taxation of general insurers broadly follows that for other businesses although there may be special features, such as allowing equalisation reserves to be held to allow for the uncertain nature of general insurance business.

For example, transfers to equalisation reserves or catastrophe reserves may be allowable against taxable profit.

Some countries impose a tax on general insurance premiums for some or all classes of business.

4 Professional guidance

When carrying out work for a general insurer or reinsurer an actuary should always bear in mind any professional guidance relating to the work being carried out and the professional body to which he or she belongs.

For members of the Institute and Faculty of Actuaries practising in the UK, there are four Technical Actuarial Standards (TASs) relating to general insurance (as at April 2012):

- Technical Actuarial Standard M: Modelling
- Technical Actuarial Standard D: Data
- Technical Actuarial Standard R: Reporting Actuarial Information
- Insurance Technical Actuarial Standard

Although the content of the TASs is not officially part of the Core Reading, a familiarity with them may help you pass the exam.

Question 7.10

Name the six sections of the Insurance TAS.

An actuary should also bear in mind guidance on professional standards in addition to guidance on technical issues.

All members of the Institute and Faculty who are regulated by the UK Actuarial Profession are subject to The Actuaries' Code. This gives general guidance on professional conduct to which all actuarial members must conform in both the spirit and the letter.

In addition to formal guidance, the professional body may issue advice from time to time on specific issues.

This may be done by issuing an Advisory Note. A recent Advisory Note provided advice on the wording that should be used to convey uncertainty in relation to specific large losses in Lloyd's opinions.

The details of all the latest professional guidance issued by the Actuarial Profession can be obtained via its website: **www.actuaries.org.uk**.

Professional guidance is covered in more detail in Subject SA3.

5 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Benchmark
- Binding authority
- Capacity
- Central Fund (Lloyd's)
- Closed year
- Committee of Lloyd's
- Council of Lloyd's
- Composite insurer
- Funds at Lloyd's
- Going-concern basis
- Lead underwriter
- Line slip
- Lloyd's broker
- Lloyd's deposit
- Lloyd's managing agent
- Lloyd's members' agent
- Lloyd's special reserve fund
- LMX on LMX
- LMX spiral
- London Market
- London market excess of loss
- Managing agent
- Members' agent
- Mutual insurer
- Names (Lloyd's)
- Open year
- Pooling
- Premium income limit
- Premiums trust fund (PTF)
- Proprietary insurer
- Protection and Indemnity (P&I) Cubs
- Self-insurance
- Slip system
- Syndicate (Lloyd's)
- UK Guarantee Fund
- Underwriting agent
- Underwriting ratio
- Underwriting year
- Value at Risk
- Wind-up basis.

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Chapter 7 Summary

General insurance is provided by insurance companies and Lloyd's syndicates. A corporate entity may also manage its self-insurance through a captive insurance company. Reinsurance can be obtained from the London Market, Lloyd's, specialist reinsurance companies or direct insurers who also write reinsurance.

Direct insurers (as opposed to reinsurers) write insurance business for individuals and companies. They may be composites or they may write general insurance only (either specialising in certain classes or writing all classes of GI business). Most insurers are proprietary, but there are some mutual insurers.

The London Market is that part of the insurance market in which insurance and reinsurance business is carried out on a face-to-face basis in the City of London. It includes Lloyd's and the specialist companies who write business in the City. Much of this insurance is for large and/or international risks and reinsurance, although Lloyd's does provide certain personal lines cover.

Lloyd's, a major component of the London Market, is a special insurance market where wealthy individuals and corporate bodies, known as *Names*, group together in syndicates to collectively coinsure risks. Lloyd's itself does not provide the insurance.

Most members are companies (corporate names) and have limited liability. Private members may have unlimited or limited liability. Lloyd's syndicates can write business almost anywhere in the world.

A number of large enterprises have set up a captive insurance company to provide for their own insurance needs, retaining premiums and risk within the enterprise. The main reasons for captives include:

- focusing effort on risk management
- managing the overall insurance spend
- access to the reinsurance market
- providing insurance cover not available elsewhere.

Also, there may be tax or regulatory advantages. In addition to accepting the risks of their parent companies, captives may also accept external risks on a commercial basis.

Protection and Indemnity (P&I) Clubs provide insurance to ship owners. They are a popular way to cover marine liability claims due to their mutual nature and the provision of technical assistance. The International Group of P&I Clubs can be used to pool very large risks and to obtain reinsurance.

Some of the largest P&I Clubs themselves mutualise in respect of very large claims and the purchase of some reinsurance via the International Group.

Specific pooling arrangements, where the parties agree to share premiums and losses for specific insurance classes or types of cover in agreed proportions, are sometimes used particularly where the risks are very large (*eg* atomic energy risks) or via mutual associations that cater for an industry.

Markets in different territories tend to differ in the concentration of insurers (by market share), the sales methods used, the importance of mutuals and whether composites are permitted.

Reinsurance tends to be placed internationally. Many reinsurers are based in the USA, Switzerland and Germany. Bermuda has become a major international centre for insurance and reinsurance, with a number of captive insurers being based there.

Non-London Market business can be sold:

- through intermediaries, such as brokers or tied agents (*eg* building societies or travel agents)
- by staff employed by the provider
- via direct marketing methods (*eg* internet or telesales).

The main sales method varies by class of insurance.

Traditionally, business is placed in Lloyd's and the rest of the London Market by specially accredited brokers, using the slip system.

A regulator could restrict the actions of a general insurer in many ways including limiting premium rates, restricting investments and requiring a minimum level of solvency on a prescribed basis.

When carrying out work for a general insurer or reinsurer an actuary should always bear in mind any professional guidance or advice relative to the work being done and the professional body to which he or she belongs.

Chapter 7 Solutions

Solution 7.1

The company may prefer another company to underwrite its general insurance business because:

- it does not need to establish itself as a general insurer, and thus avoids the administrative expense of going through the authorisation process
- there is less need for specific general insurance expertise, *eg* in underwriting, claims control or pricing, if the general insurance risks are borne elsewhere
- management can focus solely on life insurance risks, rather than spreading its expertise too thinly
- it may not be cost-effective to set up a general insurance operation if sales volumes are uncertain or expected to be low
- there may be tax or regulatory capital requirement advantages in this arrangement, *eg* there is less need to hold capital if risk is transferred.

Solution 7.2

Many Names are liable for the whole of their personal wealth. If an insurance company has problems, the shareholders may lose their original investment, but they will have no further liability. However, Names would have to use their entire personal wealth to bale out their share of the losses of a syndicate in trouble. Now that most individual Names have opted for limited liability, there are fewer individuals in this position.

Solution 7.3

The insurers within the direct market can be subdivided into:

- mutuals *and* proprietaries

  or into

- composites, specialist insurers or multi-class insurers.

Mutuals are owned by the policyholders.

The proprietaries may be:

- publicly owned (*ie* a listed company with many shareholders)
- owned by other companies *(eg* a bank)
- owned by a mutual life office
- privately owned.

Solution 7.4

Self-insurance is the retention of risk by an individual or organisation, as distinct from obtaining insurance cover.

A more detailed definition is given in the Glossary.

Solution 7.5

- cover required is not available in the traditional insurance market
- large enough (by amount of premiums) to go to the effort of setting up a captive
- large enough (by amount of capital) to bear some risk
- insurance market isn't satisfactory, *eg* too expensive, too restricted or tax-inefficient
- able to obtain captive management and reinsurance, as required, at reasonable price

Solution 7.6

Reasons why Bermuda has become a major international centre for insurance and reinsurance, despite not being a large economy in its own right, include:

- a favourable tax environment
- regulatory advantages – the aim is to have a regulatory environment that meets international standards (*eg* on solvency standards) yet is attractive for (re)insurers based in Bermuda, and so has relatively few restrictions (*eg* rules regarding authorisation, investment and disclosure are relatively relaxed)
- a government that actively encourages and promotes growth of the country's financial industry
- as a major tourist location with (usually) good weather, it is an attractive place to live and work!
- strong historical and cultural links with the UK (which can help attract business from UK-based companies including Lloyd's business)
- relatively close to the USA, with which it has strong business, cultural and tourist links (and so it can gain access to this major insurance market).

Solution 7.7

A catastrophe bond is like a traditional bond issued by an insurance or reinsurance company (usually via a separate legal entity known as a Special Purpose Vehicle), but where the repayment of capital (and possibly of interest) is contingent on a specified event *not* happening.

The trigger event may be related to the insurer's losses (*eg* flooding resulting in claims to the insurer of over £100*m*), industry-wide losses (*eg* storm damage causing the insurance industry losses in excess of $200*m*) or objective measures of the peril's severity (*eg* an earthquake measuring six on the Richter scale or a windstorm reaching a speed of 50 metres per second).

The company has effectively passed on the insurance risk to the purchasers of the bond because:

- if the trigger event does happen, the investor does not receive the remaining coupon payments, and so the insurer can use the sum of money provided from the investor (in purchasing the bond) to cover the cost of claims arising from the earthquake
- if the trigger event does not occur, the investor gets his interest and capital back in the normal way.

Solution 7.8

Possible advantages of using securitisation:

- insurance risk may be uncorrelated with other risks that capital markets are exposed to, and so capital markets are prepared to take on this risk
- no reinsurance default risk
- may be cheaper than conventional reinsurance
- provide cover that reinsurers may not be willing to, or have the capacity to, accept
- more effective or tailored provision of risk management
- source of capital
- tax advantages.

Solution 7.9

A fund could be set up to pay claims from failed general insurers. It could be funded by the government or by charging a levy on other insurers.

Companies could be required to deposit a large initial sum to a central governing body. This could be used to pay claims on default.

Solution 7.10

A Purpose of the insurance TAS.

B Interpretation.

C Scope.

D General principles.

E The exercise of discretion in long-term insurance business.

F General insurance business written by Lloyd's syndicates.

Chapter 8

External environment

Syllabus objective

(d) Describe the implications of the general business environment in terms of the effect of:

- *inflation and economic factors*
- *legal, political and social factors*
- *the climate and environmental factors*
- *the impact of technological change.*

Covered in part in this chapter.

0 Introduction

In this chapter, we continue our discussion of various features of the general business environment and their implications for general insurance business.

This chapter discusses the impact of:

- claims inflation and other economic factors, such as the underwriting cycle, investment conditions and currency movements
- legal, political and social factors – in particular, court awards, legislative changes, and trends in society's behaviour and attitudes
- the climate and environmental factors, such as the weather, other catastrophic events (both natural and man-made) and various classes of latent claims
- changes in technology.

Question 8.1

What is a latent claim?

1 Inflation and economic factors

1.1 Introduction

General economic conditions can affect insurance business in many ways. For example, in times of low economic growth and high unemployment, the following effects can be observed:

- a greater number of claims on mortgage indemnity guarantee insurance, as more homes become repossessed
- more claims for theft and arson, as crime rates increase
- more attempts to make fraudulent or exaggerated claims
- increased demand for pecuniary loss cover, as businesses become more concerned about the risk of suppliers, debtors, *etc* becoming bankrupt.

This section discusses the following economic influences:

- different types of claims inflation and how inflation (including expense inflation) affects insurance business
- the underwriting cycle – how it arises, influences on it and ways in which insurers deal with it
- investment conditions – in particular, the importance of investment income to insurers and allowance for investment income in underwriting / pricing
- currency movements – situations when this is and isn't important and dealing with exchange rate movements (*eg* by analysing business by currency and having an appropriate investment strategy).

1.2 Claims inflation

The general levels of inflation in the economy affect the cost of providing insurance and are therefore likely to be reflected in premiums.

Except for fixed benefit claims, inflation will affect the amount of each claim (severity). It will also affect the level of expenses incurred by the insurer. This section focuses mainly on inflation of claim amounts.

Different types of claims inflation

However, different types of insurance are affected by different types of inflation, and so the overall cost may not be affected in the same way.

Question 8.2

Without looking ahead, try to list four different types of inflation and indicate how they might affect insurance claims.

Failure to anticipate appropriate levels of inflation can lead to insurers charging premiums that are likely to be less adequate than they believe them to be. They can also lead to under-reserving if reserving is done using methods that require an explicit assumption about inflation.

The following discussion assumes that inflation is positive, *ie* increases claims costs, which is usually the case. However, it should be borne in mind that there have been periods in some territories where some types of inflation have been negative.

Household contents insurance

The most commonly quoted inflation indices are those for general consumer goods; these are commonly known as retail-price inflation or consumer-price inflation. These may reasonably be expected to affect the cost of claims of household contents policies, although they may not be an entirely accurate predictor of claims costs since consumer goods are likely to be the subject of claims whereas the inflation indices may include items such as food and housing costs.

Buildings insurance

Private, commercial and industrial property claims will generally be for the repair and rebuilding of property, particularly buildings. These costs will be linked to the cost of building materials, but labour will be a major part of the claims costs. This cost is more likely to be linked to wages than prices; wages tend to increase more than prices so the cost of this insurance is likely to increase more than general prices. If wages change differently in different industries, the link to an index may be less direct.

Another aspect of inflation is "loss amplification" or "demand surge". This arises where there is a temporary increase in costs of labour or raw materials due to a large number of claims being made at the same time, for example, as a result of a single large catastrophic event.

Medical expenses

One area, important for general insurance, in which inflation has been persistently higher than the most quoted indices, is medical expenses. A specialist inflation index is almost certainly necessary for projecting this type of business.

This will apply to claims on medical expense insurance (*ie* private medical insurance) and bodily injury claims (*eg* on motor insurance).

In many territories medical inflation can be significantly higher than price or wage inflation because it is a combination of several factors:

- more advanced medical treatments are used, which are more expensive
- better treatment means patients survive for longer, but still require medical care
- in some cases, doctors and consultants salaries can rise in excess of average wage inflation.

Motor property damage

Motor property damage claims are generally for repair. This will include the supply of parts, but motor repair is labour-intensive and will be related to wage indices. Large claims will generally be for the replacement of vehicles.

The cost of providing a replacement vehicle may vary quite differently from a general price inflation index. It will be influenced by factors such as the competitiveness of the motor insurance market, availability of cars from overseas and efficiencies in the manufacture of vehicles. Specialist indices, or adjustments to a prices index, may be required. This may vary according to the type of vehicle (*eg* car, van, bus).

Fixed benefits

A few types of insurance are immune from inflation because they provide fixed benefits. These include some personal accident policies. We should expect sums insured to increase in line with inflation, (either because policyholders will regularly reassess their needs, thus **without a link to a particular index,** or because some insurers increase sums insured on some policy types in line with an index, as a default); **but as premiums will be proportional to sums insured, premium rating would not be affected by inflation.**

Under fixed benefit contracts, *eg* a personal accident policy providing a specified payout on loss of a limb, the benefit amount is known throughout the period of cover. There is therefore no risk to the insurer that the claim amount will be greater than expected.

Of course the number of claims might be greater than expected, but remember that this section is just discussing *inflation* of claim *amounts*, rather than trends in claim frequency.

The sum insured may increase from one year to the next due to:

- policyholders choosing a higher level of cover, or
- the insurer increasing the benefit amounts.

In either case, the increase may be in response to the increasing cost of living, for example. However, there could be other factors involved. These increases will also tend to be *ad hoc*. Hence, the benefit level may not be increased for several years and might be expressed in round figures.

For example, the standard level of benefit for loss of limb on a personal accident policy might progress as follows:

Year	2006	2007	2008	2009	2010	2011
Sum insured ($)	25,000	25,000	30,000	30,000	30,000	40,000

Question 8.3

Outline other possible factors that could prompt a policyholder to increase the level of cover on a personal accident policy.

If the benefit amounts are increased on renewal, there will be a proportionate increase in the premium (unless the pricing basis is also changed). There is a slight risk to the insurer that if it increases the benefit amounts, and so demands an increased premium, this might discourage policyholders from renewing. However, as the Core Reading points out, premium rates are often expressed as a premium proportional to the sum insured. The premium *rate* would therefore be unchanged, which may be more acceptable to policyholders.

However, if sums insured, and therefore premiums, did not increase over time then the cost component of premiums would need to be increased from time to time.

This is referring to the allowance for expenses in the premium. As discussed below, expenses will be subject to inflation, which is most likely to be salary-related inflation.

Liability insurance

Liability insurance presents a generally different relationship with inflation. Liability for property damage may develop in line with the same indices as property damage claims. However, claims for personal injury are more complex. They generally have several components.

The components of personal injury claims are sometimes called "heads of damage". The three components that will be discussed here are:

- compensation for loss of income
- cost of medical and nursing care
- awards for pain and suffering.

There may be compensation for loss of income, which will be subject to the same inflation indices as other income-related costs. However, in the UK several recent reforms have increased the cost of this component by more than general wage costs, by allowing for discounting through the "Ogden tables", discounted at low rates of interest and the possibility of structured settlements being imposed.

Claim payments that are intended to represent the future lost earning of an individual following an accident are likely to be based upon the present value of that future income. The courts may from time to time change the rate of interest at which insurers are allowed to discount future earnings.

The *Ogden tables* are actuarial tables that provide multipliers that are widely used by UK courts in assessing the present value of future losses in personal injury or accident death cases. They are based on assumptions for life expectancies, employment risk and a set of discount rates. The discount rate actually used is set by the Government. A low discount rate will result in a high lump sum award.

The discount rates to be used (and indeed other assumptions in the tables) will be changed from time to time. Some changes could have a dramatic effect upon the overall claim payments. The unpredictability of these court decisions makes it difficult to estimate this type of inflation.

Question 8.4

What do you think the employment risk assumption is?

A *structured settlement* is also known as a "periodical payment". It is where compensation is paid in the form of an annuity rather than a lump sum. In the UK, since 2005, courts have had the power to insist that claims for compensation of future earnings are given in this form.

Paying claims as a structured settlement is expected to increase the cost of claims to insurers, because:

- annuities can be expensive to purchase, especially if the market for impaired life annuities is limited
- the courts might decide to increase the regular payment amount at a later stage
- it will be more expensive to administer the regular payments compare to a lump sum.

The reluctance of insurers to have a structured settlement may encourage them to accept a higher lump sum settlement than they would otherwise, and so increase their costs.

A second component of personal injury claims **is the cost of medical and nursing care, which is more likely to develop according to medical expenses inflation, and may also be affected by changing approaches to discounting.**

Finally, awards for pain and suffering have tended to become more generous in many countries and have grown at a faster rate than general price or wage inflation. These awards tend to be set judgementally rather than according to fixed criteria. These and other factors often called "social inflation" or "superimposed inflation" are further discussed below.

These components of personal injury claims are discussed later in this chapter, in Section 2.1, where we cover court awards.

Economic influences

The frequency of losses can also depend on economic factors.

Example

In times of recession, there might be increases in the number of claims on creditor business, in respect of arson on property business and in theft generally.

Expense inflation

The cost loading for an insurance policy is also subject to inflation. Insurance is a relatively labour-intensive industry, although technology may improve productivity and reduce unit costs. This suggests that this component of policy costs may be likely to increase in line with wage inflation.

1.3 *The underwriting cycle*

The underwriting cycle (also known as the "insurance cycle") is a cycle of high then low premiums. When insurance premiums are high and profitable, competition tends to increase as more general insurance companies want a share of the profits. The competition then drives down premiums. Low, unprofitable premiums lead to insurers struggling to stay afloat. Some insurers become insolvent and leave the market. The reduced competitive pressures then lead to an increase in premiums, and so the cycle continues.

Although it might seem difficult to believe, there has been a surprisingly clear cyclical pattern of insurance profits in the past. Despite this clear pattern the cycle is quite difficult to predict going forwards.

In the past it has been observed that insurance premium rates have varied in ways that do not reflect the underlying cost of providing the insurance. This is most common in large commercial and industrial insurance; for example that placed in the London Market, but it affects all classes of insurance.

On many general insurance products there will be times when insurers can make a large allowance for profit in their premiums and other times where they need to make a much smaller allowance (if any!). This is due to the effect of the underwriting cycle, which can often be observed, not only on large commercial and industrial insurance, but also on many personal lines products such as household and private motor insurance, particularly in competitive markets.

Stages of the underwriting cycle

In general, the cycle can be described in the following terms, although describing it as starting from a position of general profitability is purely arbitrary: the sequence could be entered at any point.

1. **Insurance is generally highly profitable. This position is commonly known as a hard market.**

   Insurance premiums are generally high (or "hard") at this point. You can think of it as being a time when it is ***hard*** for policyholders to buy insurance cheaply.

2. **The level of profits attracts new entrants to the market and encourages existing insurers to write more business.**

   These insurers are attracted by the prospect of good returns.

3. **To fill the extra capacity, premium rates are reduced to attract business.**

   Premium rates start to "soften" during this phase as insurers compete for market share.

4. **Eventually premium rates fall to the extent that insurance is generally loss-making. This position is commonly known as a soft market.**

   Insurance will often merely be less profitable rather than actually loss-making. This is the bottom of the cycle.

5. **Insurers leave the market in response to the level of losses, or reduce the amount of business they write.**

6. **With restricted availability of insurance, premium rates increase.**

7. **Eventually premium rates rise to the extent that insurance is generally highly profitable.**

   We're back at stage 1 again, and so the cycle continues…

Question 8.5

Why don't companies enter at the bottom of the cycle and leave at the top?

Note that different classes of insurance business will tend to be at different points of the cycle at different times. At any point in time, profits from one class of business will subsidise another, less profitable, class.

It is important that an insurer is aware of the position in the underwriting cycle of each of its classes of business when making strategic decisions.

Influences on the underwriting cycle

It should be noted that an insurer's ability to write insurance is limited by the amount of capital that it holds. While the prospect of an extremely profitable market will attract new capital that may be subscribed to existing companies and new companies, a profitable market in itself increases insurers' capital bases as retained profits increase capital holdings. Since the same effects apply to reinsurers, reinsurance is also likely to become available on easier terms, which increases insurers' ability to write business.

So stage 2 above (an expanding market) is a natural consequence of stage 1 (a profitable market).

The actual mechanisms that reduce the size of the market when it is unprofitable will be:

- **companies becoming insolvent**
- **companies withdrawing as a reaction to unprofitability because of unwillingness to accept continuing loss**
- **reinsurance being less readily available.**

In the past, soft markets have often ended when a major disaster triggered severe losses at a time when premium levels would not support the normal level of claims. Examples of this are Hurricane Andrew in 1992 and the terrorist attacks of September 2001 (as well as some other substantial losses earlier in that year).

Possible reasons why the underwriting cycle exists

The reasons for the existence of the cycle are much debated.

Four factors that encourage the cycle's progress are now discussed:

- the ease with which new entrants can join insurance markets
- the delay between writing business and knowing how profitable it is
- simplistic regulatory capital requirements that encourage insurers to write more business when premium rates are falling and less business when they are rising
- economies of scale, which encourage marginal costing.

Low barriers to entry

Insurance is an industry in which barriers to entry are generally low. Authorisation is a significant process in most countries, but if a new company can demonstrate capital sufficiency and technical competence, it will usually be authorised fairly quickly. Setting up in business does not require the establishment of specialised plant and equipment or much development of resources. This leads to a situation in which capital providers can quickly move into the sector.

Delay until profitability of business written is known

Another key factor contributing to the existence of the cycle is the delay between writing business and knowing how profitable it is.

Simplistic capital regimes

Simplistic capital regimes may exacerbate the cycle. In many jurisdictions, at least until recently, the capital required to write an insurance policy depended on the premium.

For many years, the EU regulatory minimum capital requirement for general insurers was approximately 16%–18% of annual premiums (but modified if claims were very high).

Question 8.6

What do you think was the rationale behind this approach?

A similar measure to the above is still used in EU countries as a *minimum* regulatory capital requirement. However, additional capital requirements are being (or have been) introduced using risk-based measures – for example, Solvency II developments, which are covered in Subjects CA1 and SA3.

Using only simple measures, such as the EU minimum, does mean that if an insurer cuts its premium rates, it does not need to hold so much capital. This does seem counter-intuitive: if it cuts its rates, it is more likely to become insolvent (*eg* due to inadequate premiums) and it would need to hold a higher level of capital to cover this risk!

This meant that it required less commitment of capital to write a policy if it were underpriced than it would have done if it had been overpriced, the exact opposite of what risk-based considerations would merit. This means that companies can write more business – in terms of the amount of risk taken on rather than the amount of premium written – as premium rates fall. Conversely, as premium rates rise they must restrict the amount of risk taken on unless they can raise more capital. This exacerbates the difficulty of finding cover and will tend to drive premium rates even higher.

Economies of scale

The economics of insurance business may also help to enforce the cycle. Insurers' overheads tend to be, if not fixed, then less variable than premium rates. There may be little or no cost saving (apart from commission) from an insurer not writing a policy. Therefore if business at least covers its claims cost it may be marginally profitable for an insurer to write it, even if business overall makes losses. However, in the depths of soft markets, it is common for business to fail to do even this. Insurers sometimes do not want to lose market share because of the cost of acquiring the business again in the future, loss of reputation and other reasons.

In a competitive market, an insurer may set premiums at a level that makes an insufficient contribution to its fixed expenses. This may be justified on the grounds that there is still *some* contribution to fixed expenses, as opposed to the zero contribution which would result from trying (and failing) to sell an uncompetitive product with a "correct" contribution built in.

So, it is important that an insurer is aware of the true underlying profit or loss of its business. Otherwise it is all too easy for it to get carried away with making premium cuts in line with the competition and end up in a position where premiums do not even cover claim costs, and this can result in unprofitable business.

1.4 Investment conditions

Insurance companies take premiums from customers and hold them until they have to be paid out in claims; these monies will be placed in financial investments during this period. Most of their capital is also available for investment.

An insurance company needs to decide how to invest the money that it has at any one point in time. It will be exposed to the investment conditions of the assets it invests in. For example equity investments will be exposed to changes in the underlying market values (of the individual shares or of the market overall).

The capital that the insurance company holds can be split into two broad categories:

- that required to meet the liabilities, *eg* the statutory reserves, plus
- the free assets, which is the excess of the company's assets over its liabilities.

Generally, an insurer will try to hold assets to match its liabilities (by term, nature, certainty and currency), but will have greater freedom over how it invests its free assets.

In this, insurers are unlike non-financial companies, whose capital is usually tied up in capital goods or stock. This means that income from invested securities is an integral part of insurance business.

Only a small proportion of an insurance company's assets is likely to be tied up in fixed assets, such as the office buildings or machinery. Therefore, where the assets are invested, and the investment return those assets make, are more important decisions for an insurance company to make than they are for other companies, such as manufacturing firms.

Significance of investment conditions

Question 8.7

What factors will influence the significance of investment conditions for a general insurance company?

The amount of investment income that is generated by insurance business depends on the characteristics of that business: the longer-tailed the business, the greater the amount of investment income likely to be generated.

Some personal lines business, such as home contents, is very short-tailed, and premiums may be paid monthly; very little investment income is generated in these circumstances. Liability insurance will not pay claims, on average, until several years after premiums are received; in this case the investment income will be a significant proportion of the premiums.

Question 8.8

Explain why claim delays on liability business can be for "several years".

Allowing for investment return in pricing

Traditionally, underwriters of all lines of business aimed to achieve an underwriting profit without taking the benefit of investment income, although in the 1990s an appreciation of the high levels of investment income obtainable led to so-called "cashflow" underwriting, which was ultimately associated with high losses.

In simple terms, an insurer makes an *underwriting profit* if its premiums are sufficient to cover its claims and expenses. In addition, the insurer will make investment returns on the assets that it holds, which will contribute to its *overall* profit.

The rationale behind the "cashflow" underwriting approach was that, although premiums were set at a level below that required to cover claims and expenses, the underwriting loss would be made up by the investment return earned on the premiums (before claims were settled). The insurer should still make an overall profit. It was argued that, unless premiums were set at a competitive level, no premiums would be received to make any investment return. As competitive pressures increased, the allowance some insurers made for investment income became increasingly significant.

In some cases, the investment return actually received ended up being much less than that anticipated, and so overall losses were made.

These approaches reflect a lack of sophistication in the pricing and underwriting process. A more sophisticated approach allows for the expected level of investment income under current investment conditions.

Investment income can be allowed for in the pricing calculation by discounting expected future claims and expenses to the date at which the premium is received. Under the approach described here, the discount rate would reflect current investment conditions on the assets backing the policy – for example, the current gross redemption yield for fixed-interest bonds might be used.

However, a number of points should be made here:

- **A high rate of interest may indicate that expected inflation rates are high. If these interest rates are allowed for in pricing then it is important that the projected claims reflect a consistent level of inflation.**

- **Insurance companies typically place funds at shorter durations than the term of their longer-tailed liabilities. This is partly because they need to ensure liquidity and partly because any reductions in the market value of assets, even if they do not have to be realised, may be reflected in solvency margins. Therefore current rates, if high, may not continue to be available as funds are rolled over.**

An alternative (more common) approach is to discount at a risk-free rate of return. This rate can be defined as the rate at which money is borrowed or lent when there is no credit risk, so that the money is certain to be repaid. In practice, the return on bonds issued by a (secure) government is often used as the risk-free rate.

If investment income is taken into account in pricing it would be normal to use a risk-free rate. Insurers may invest in more risky assets than risk-free, but since the insurers themselves assume the investment risk it is appropriate that they receive the extra income associated with it.

Question 8.9

An insurer is earning 6% *pa* return on its assets, and the risk-free rate of return is 4% *pa*. Explain why it should use a discount rate of 4% rather than 6% in the premium calculation.

Pricing should also take account of the required profit loading. If calibrated to a return on capital it should take account that the invested capital will already earn the risk-free rate. This may be done either by:

- **taking as a target return on capital the target in excess of the risk-free rate**

- **taking account of investment income on the capital in the calculation of projected profitability.**

Also, target return on capital will generally be higher on long-tail lines of business because they are generally more risky than short-tail lines. This is likely to reduce the effect of longer-tailed lines' greater investment income.

The mechanics of pricing, including allowance for investment return, is covered in more detail later in the course, from Chapter 12 onwards.

1.5 *Currency movements*

The impact of movements in currency exchange rates on an insurer partly depends on where in the world its business is written and where claims are made.

Country where business is written and claims are made

Most small and many medium-sized insurers write business in only one country. All their premiums are likely to be in that country's currency and claims in other currencies are likely to be few. The effect on these companies of movements in exchange rates is very limited.

As discussed below, the exposure to currency movements for these insurers will be limited to a few overseas claims on certain policies, such as travel insurance.

The insurers that sell business in more than one territory tend to be the larger companies.

On the other hand, most large companies write business in a number of territories as do many smaller companies, notably London Market and reinsurance companies. In these companies, business may be written in a number of currencies, and the currency of a policy may not be the currency in which all that policy's claims are incurred.

Examples of where claims may be incurred in a country that is different from that in which the policy was written are commercial property, marine insurance and some personal lines policies (such as private motor and travel insurance).

A single policy may cover a company's property in several countries, for example, or a ship may incur claims in several currencies at it sails from country to country. This is not a problem that is limited to major industrial and commercial policies; for example, motor policies may lead to foreign claims and travel insurance will do so in obvious ways.

The exposure to currency movements will also depend on any territorial limits applying to policies. For example, travel insurance that is restricted to journeys within Europe (where the Euro currency dominates) will have less exposure to currency risk than worldwide travel insurance.

Business written through Lloyd's

Traditionally, Lloyd's has accounted for its business in three currencies: sterling, US dollars and Canadian dollars. The euro was added when it was launched. Most Lloyd's syndicates will analyse their business in these currencies, although there is no reason why other currencies should not be treated separately if they are important to the syndicate concerned and added to the appropriate reporting currency when drawing up official returns.

Analysing business by currency

Analysing business by currency avoids distortions that can arise if there are changes in exchange rates. For example, if a company writes a policy in a foreign currency that appreciates against its home currency then claims will cost more when measured in the home currency and a profitable policy may appear to have been sold at a loss.

Example

Suppose a UK insurer writes a policy in the USA for a premium of $1,000, assuming that claims will be 80% of premiums. If claims on the policy turn out to be $800, in US dollar terms, this would leave the insurer with $200 towards expenses, profit, *etc*.

Suppose the value of the dollar increased from £0.50 when the policy was written, to £0.70 when the claim was made.

If the values were converted into the insurer's domestic currency, then:

- the premium would be valued at $1{,}000 \times 0.50 = £500$
- the claims would be valued at $800 \times 0.70 = £560$
- and so the policy appears to have made a loss.

This policy would make a loss as shown in the insurer's published (Sterling) accounts. However, this loss is due to exchange rate movements, rather than poor pricing or adverse claims, and so this would not be a good reason to increase premiums. A better approach would be to have used some form of currency hedging (we'll discuss this later).

Currency movements can also cause problems when reserving.

Development factors will be distorted if the underlying claims are converted to home currency at different rates of exchange within a single dataset such as a triangle.

You may recall from Subject CT6 (or Subject ST7) how claims can be grouped according to the year of origin (*eg* accident year) and year of development (*eg* claim payment) in a claim triangle. Statistical methods, such as the basic chain ladder, can be applied to the triangle to project future claims.

Development factors are the ratios of claims in successive development periods from chain ladder calculations. If different exchange rates apply at different times, this will distort the development factors for each year of origin. This distortion can be removed if only one exchange rate is applied to all the data.

For this reason the actuarial analysis of historical loss data is usually carried out with all data converted at current exchange rates.

However, there are exceptions. In some international industries a single currency may dominate worldwide.

Example

Major claims in the oil and aerospace industries may be determined in dollars because new equipment and expertise in control and repair is bought in that currency, even though claims may be paid in the currency of a local company.

Judgement needs to be used in deciding how exchange rates should be applied in any analysis.

Currency hedging

Insurers can make real profits and losses through currency movements. A basic assumption is that an international insurer should hold assets in currencies that can match its liabilities; in this way the value of both assets and liabilities move together and exchange gains on one offset losses on the other, or a similar effect might be achieved through a currency-hedging strategy.

For example, forward currency contracts could be used.

It is not possible to match currency precisely, since, as mentioned above, it is not always possible to know in advance in which currencies claims will arise, and reserves may run-off favourably in one currency and unfavourably in another. Insurers may also depart from a matched position for strategic reasons.

Even a well-matched portfolio that performs as expected can give rise to profits or losses: if the home currency strengthens, the profits and capital made and held in foreign currencies will simply be worth less after the change in exchange rates.

2 Legal, political and social factors

Insurance does not stand by itself in a country's economy. It is affected by many factors in the way society is ordered.

In this section we discuss:

- court awards for compensation claims in liability insurance – how the legal basis of "negligence" has arisen, how the size of a compensation award is decided, recent trends in such awards, and the impact on liability insurance
- the impact of changes in legislation on insurers – this could impact insurers directly (such as making insurance compulsory) or indirectly, where legislation affects the cost or frequency of claims
- trends in society's behaviour and attitudes, such as the propensity to claim, driving whilst drunk, crime rates, fraudulent or exaggerated claiming, and organisations (*eg* claim management companies) encouraging people to claim.

2.1 Court awards

Judicial decisions, or court awards, have most impact on claims under liability insurance business. Consider an employer who is being sued for compensation by one of its employees claiming to be ill due to working in hazardous conditions. If the claim goes to the courts, the courts can decide:

- whether the employee is entitled to compensation, *ie* whether the employer is liable or not
- the amount of (any) compensation the employee is entitled to.

Courts determine liability and award compensation for wrongs suffered by organisations or individuals.

Hence they can directly affect both the frequency and severity of liability insurance claims.

In many territories, a decision made by a court can set a precedent for future court awards, *ie* in similar cases, and for cases where the parties involved agree to settle out of court. Most liability claims are settled out of court.

> ***Question 8.10***
>
> Why would the parties involved decide to agree a compensation case out of court?

Claims are usually made:

- **on the basis of some breach of contract between the injured party and the allegedly responsible party, or**
- **(more usually) on the basis of negligence.**

Negligence

A *tort* is a legal term to mean a civil wrong or injury, not arising out of any contract, for which action for damages may be sought. Note that this is a civil wrong or injury, as opposed to a criminal offence. Those that conduct such wrongs are known as *tortfeasors*.

The tort of negligence was developed in the 1930s in response to a case in which a customer alleged that she had been made ill by drinking ginger beer from a bottle sold to her that contained a decomposing snail. She could not sue the café owner under breach of contract as she had no contract with him: the drink had been bought for her by a friend (the drink was sold as he had received it and was in an opaque bottle, so he had no opportunity to inspect it and the case might have failed on this also), and the case against the manufacturer failed because she had no contract with him. The judge concerned defined the new tort of negligence as a legal requirement not to harm a neighbour, defined as "persons who are so closely and directly affected by my act that I ought reasonably to have them in contemplation as being so affected when I am directing my mind to the acts or omissions that are called in question".

This case (*Donoghue vs Stephenson*) took place in Scotland and it took almost four years from the initial visit to the café before it was settled, in 1932.

It is now generally accepted that those who cause harm to others or to their property (known legally as tortfeasors) are liable to compensate the victim. Negligence may arise even where there is a contract; for example, an actuary who is negligent could be sued for professional negligence even if there is a contract in place between the actuary and the client.

This has a direct effect on all types of liability insurance. Although most liability claims are decided by negotiation between the insurer and the representatives of the victim and few are decided in court, the decisions of courts set benchmarks for negotiators.

Jurisdiction shopping

One result of different legal systems and awards is "jurisdiction shopping". This is where the claimant will try to launch proceedings in the most claimant-friendly jurisdiction in order to maximize any potential award. It is a particular issue in aviation insurance with changes to the Montreal Convention.

The Montreal Convention is an international treaty which sets out airlines' liabilities for passengers (*eg* for death or injury) and their baggage. Changes to this treaty are thought to have raised awareness of the fact that countries that have not signed up to the agreement may have arrangements that are more favourable to claimants, and these are the jurisdictions that people will tend to try and claim under.

Compensation

We now discuss factors that are considered when determining the size of a compensation award.

Compensation is supposed to put the victim in the position he or she would have been had there been no incident of negligence.

This is known as the principle of indemnity.

Property damage or loss

This may be straightforward in the case of the loss of some property: it is easy to replace many things and their cost is easily found. However, even here complications arise:

- **how should depreciation be calculated?**
- **to what extent should the inconvenience that results from the loss of the item be compensated?**

An example here would be compensation for the loss of luggage, which a passenger may claim from an airline.

Bodily injury – "heads of damage"

Compensation for bodily injury is much more complicated: the tortfeasor cannot put the victim right again, and a monetary award must suffice.

Compensation for major injuries is divided into several "heads of damages": different amounts are provided for different aspects of the victim's loss. For example:

- **loss of income**
- **medical and nursing costs and**
- **compensation for pain and suffering**

are likely to be recognised separately in an award.

Loss of income is often split between loss of past income and loss of future income, with each considered separately. Special head of damages will apply in particular cases.

The courts will consider each head of damage separately and decide on the amount of compensation for each. The total award is the sum of all the components. In some cases, new heads of damage have emerged, such as "bullying" and "post traumatic stress disorder", each of which contribute to the total award amount. This has contributed towards a general increase in overall claim costs (see below).

Punitive or exemplary damages are intended to punish the tortfeasor over and above the cost of compensating the victim; these are very rare in English or Scottish law, but quite common in the USA.

The increasing cost of compensation

In general it has been noted in many markets that compensation for negligence has become more generous. There are two ways in which this is observed:

1. **courts may be more willing to accept that there is liability for a victim's suffering (and even that suffering exists)**
2. **given that they decide there is liability, the courts may award larger amounts for similar losses.**

So, from a liability insurer's point of view, this acts to increase both the frequency and the severity of claims.

Whether or not this is a good thing is a matter of controversy that will not be discussed here, but there seems to be general agreement that the trends are real. They may arise through judicial decision or through legislation. In the USA tort trials are decided by juries who are also responsible for setting awards; they are another source of this trend.

In other territories, judges may decide the amount of the award. One could argue that the people on juries are more likely to be emotionally affected by the victim's case and to decide in favour of them. This is in comparison to judges, who might be more objective.

In some states of the USA, there are "tort reform" laws that constrain the amount of freedom that juries can exercise. For example, the state may place a monetary limit on the amount of compensation that may be payable (*eg* per head of damage or in total). However, some US tort reforms have increased claims costs, *eg* in the 1980s when it became easier to claim compensation for injury due to childhood vaccination.

Examples

An example of an increase in awards that came about by judicial decision in the UK is the use of the "Ogden tables" for compensation for serious personal injury.

An example of legislation is the Courts Act (in the UK)**, which empowered courts to impose structured settlements (regular payments rather than lump sums) as compensation even when both parties preferred lump sums.**

The Ogden tables and the impact of structured settlements were discussed in Section 1.2.

The effects on insurance are to some extent obvious: if the cost of awards increases, compensation will increase and liability claims will be more expensive. This will feed through into increased premiums.

However, it is important to note that awards are made on claims that are covered by insurance whose risk period has expired. An unexpected increase in compensation will affect insurance for which no more premiums can be taken and so will, other things being equal, cause loss.

While a general trend towards higher compensation can be included in premium rates, the rate of change is hard to predict and unexpected changes may cause claims reserves to be inadequate.

2.2 Legislation

Impact of legislative changes on claims experience

Changes in legislation may affect insurers in the same way as court awards.

In other words, the changes can directly affect the frequency or severity of claims on policies that were written some time ago, and so there is no scope for the insurer to increase the premium of these policies.

It is possible that legislation can be made to apply to events that occur only after the date of the legislation, but if it concerns the way that amounts of compensation should be determined in court this is rare.

A change in legislation that affects all claims that have not finally been settled at the date the legislation comes into force will affect the adequacy of claims reserves in the same way as an unexpected judicial decision, although as legislation takes some time to prepare and enact, it may be less unexpected.

It is therefore important for insurers to keep abreast of relevant legislative proposals and developments so that they can assess their expected impact, and allow for this in their pricing, reserving, financial planning, *etc*. Insurers may also be able to influence governments in setting legislation – for example, through industry bodies, such as the Association of British Insurers (ABI) in the UK.

Legislation may affect the expected losses suffered by insurers.

> ***Example***
>
> **Health and safety legislation might be expected to reduce the frequency and severity of accidents, which should reduce claims costs, although if it reflects a higher expected level of safety, courts may be more inclined to grant large awards to the victims of tortfeasors who infringe the standards.**

Insurers might expect to see fewer employers' liability claims overall, but some third party liability claims may be for higher amounts. In many cases the impact of new legislation is difficult to predict.

> ***Example***
>
> New noise regulations were introduced in the UK in 2006, which reduced the number of decibels at which employers must provide hearing protection for their workers. Although this measure has increased health and safety levels, it may have actually increased the number of employers' liability claims. This is because some employees who are being exposed to a level of noise that is below the old threshold may now be able to claim against their employer for negligence.

Other changes can affect the cost of insurance. For example the strictness of seatbelt rules, drink-driving rules and motorspeed limits – and the strictness with which they are enforced – should affect the number and severity of road accidents. It is usually impossible to estimate accurately the effect of any such changes before they come into effect.

The insurer could make use of some information, such as the experience of overseas countries that have already introduced similar legislation, to estimate the impact on its business. However, as the Core Reading says, any estimate is likely to be fairly crude. In particular, it will be difficult to predict:

- the degree to which legislation will be enforced, *eg* what proportion of cars are the police going to check that people are wearing seatbelts and what penalties (if any) will they impose?
- the extent to which people will adhere to the new laws, which will depend on factors such as social attitudes of people towards law enforcement or safety measures and the extent to which the new laws are publicised.

Therefore, it will be important that insurers regularly monitor the impact of new legislation as soon as it is introduced, so that they can reflect this in their business processes, *eg* premium rates.

Legislation that affects insurance directly

Legislation may be aimed explicitly at affecting insurance.

Example

One example is when insurance is made compulsory for people or organisations doing particular things. Examples of this in most countries include:

- **motor third-party liability insurance for car owners**
- **employers' liability or workers' compensation insurance.**

Question 8.11

Describe possible impacts on general insurers of making certain lines of business compulsory.

Example

A different example is the introduction of the Personal Injuries Assessment Board in the Republic of Ireland. This was designed to make compensation amounts payable for personal injury more predictable. It was intended that this would make legal action over such injuries unlikely, thereby reducing the legal costs associated with them and ultimately reducing the cost of insurance without reducing the compensation paid to injured people.

The impact on the Irish insurance market since the PIAB was introduced in 2005 has generally been positive. Claims have been settled more quickly and at lower cost. Insurers have reduced premiums as a result.

The previous chapter describes examples of the effect of the regulatory regime on the actions of a general insurer.

Impact of legislative changes on the litigation process

Legislation also affects the process of litigation. Reforms that make it easier or cheaper to bring or defend litigation or that affect the payment of legal costs may affect people's willingness to bring cases for compensation, impacting the costs of liability insurance.

No-win-no-fee arrangements

An example is when lawyers in the UK were allowed to conduct cases on the basis of no-win-no-fee, with an enhanced fee for success. This led, as might have been expected, to a large number of extra legal cases seeking compensation for injuries. It also led to the emergence of a new class of insurance – after-the-event legal cover – in which the legal costs that are foregone when a case is lost are recovered through the policy.

Under a "no-win-no-fee" arrangement, the lawyer will only charge the claimant (*eg* an injured employee) a fee if the claim is successful. The fee charged is more than it would be otherwise to compensate for the fact that the lawyer does not get a fee if the case in not successful. Lawyers can take out after-the-event insurance to cover the defendant's legal fees where the case is unsuccessful. (Often the courts will make the losing party pay the other party's legal costs.)

The claimant is therefore not subject to any financial risk:

- if he loses the case, no fees are paid
- if he wins the case, he will receive the court award – the legal fees may be deducted from the court award or may be recoverable from the defendant.

The introduction of no-win-no-fee arrangements has therefore increased the popularity of claiming for compensation. This was particularly marked in the UK when changes in the law made it possible to recover legal fees from the losing party so that the claimant is entitled to the full court award, with no deduction.

2.3 Trends in behaviour and awareness

The behaviour of people and organisations directly affects the cost of insurance, and trends in claims costs can often be traced to various trends in society. These may be in the things people do or in the way they view their place in society, their rights and their obligations.

Increasingly litigious society

It has been observed that a large proportion of cases that might successfully be litigated are never brought; possibly because victims:

- **are unaware of their rights**
- **do not know how to go about the process**
- **do not want to dwell on past miseries, or**
- **are not sufficiently motivated to go through the process.**

The risk of having to pay legal costs, which was the case in the UK before no-win-no-fee arrangements were introduced, would also deter would be claimants from attempting to obtain compensation.

If the proportion of those eligible to seek compensation who do so is small, a change in the proportion can have a significant effect on the amounts of claims paid. The general tendency to seek compensation when it is likely to be available is known as propensity to claim.

You may have heard this tendency referred to as an increasing "compensation culture". It is believed that it is becoming more acceptable for people to look for someone to blame for their suffering and seek compensation from them. This is being seen more in some countries (*eg* USA) than in others.

Societal attitudes

Other societal attitudes are also important.

A societal attitude is the way in which we think of, and behave towards, others in our community. Here we discuss changes in attitudes towards drink driving, crime and insurance.

Drink driving

For example, driving while drunk has long been illegal in most countries but in others it was long considered to be socially acceptable; those who were prosecuted being regarded as unlucky rather than ill-behaved. In many countries attitudes have changed: this is generally regarded in many countries as something that responsible people do not do; consequently it may now be done less than before, with corresponding effects on insurance claims.

There should be fewer, and less serious, accidents as a result of this change in attitude, reducing the cost of claims on motor insurance policies.

This trend was reinforced in many places by legislative changes; for example

- **a reduction in the level of alcohol in the blood that was permitted while driving**

  In some territories it is illegal to drive with *any* alcohol in your blood stream.
- **the introduction of random breath tests**
- **the penalties for offending**, which are generally being increased
- **the resources devoted to enforcement**, *eg* more police checks are being carried out.

Insurers themselves may have helped in this trend; for example by introducing or enforcing strictly policy clauses that removed the insurance cover (except what was required by law) when a driver was drunk. This would have had a direct effect on claims. It would also have reinforced the idea that driving while drunk was socially unacceptable and would have raised the cost of so doing, further deterring people.

Crime rates

Crime rates are another obvious area in which societal trends have a direct effect on insurance costs. Crime often leads to insurance claims since theft, burglary and malicious damage are often insured perils.

Crime rates tend to vary greatly from year to year, and trends in crime rates will also vary considerably between countries. General insurance companies might actively engage in trying to encourage policyholders to take steps to reduce crime, in the hope of reducing claim costs.

Attitude of people towards insurance

The attitude of people towards insurance will always affect the cost of insurance. Surveys have often suggested that people view fraudulent claiming of insurance as a minor offence, if an offence at all, especially when it takes the form of exaggerating a genuine claim rather than presenting a claim that is wholly fraudulent. (This is particularly relevant with certain products like single trip travel insurance.)

Question 8.12

Exaggerating a genuine claim is an example of what sort of risk to the insurer?

This gives rise to claims cost, as it is often impossible to detect these claims and it increases the cost of claims handling. While this is a normal cost of insurance any change in public attitudes will change the general level of claims cost.

Suppose you are using claims data for pricing (*eg* own insurer data) that already contains some level of fraudulent or exaggerated claims. If there is no reason to expect this to change over time, then this data could be used for projecting future claims. If, however, you expect a *change* in the extent of fraudulent or exaggerated claims (*eg* due to changing attitudes), then you should make adjustments to allow for this.

Organisations encouraging the placing of claims

Some organisations encourage the placing (making) **of claims, and increases or reductions in their activity will affect the cost of claims.**

These organisations are called claim management, or accident management, companies (or sometimes, "ambulance chasers"). Their aim is to help individuals make compensation claims. For example, they will arrange for lawyers, medical experts and expert witnesses on the claimants' behalf. They will often actively encourage individuals to pursue compensation claims, although the extent of this encouragement will depend on any marketing or advertising restrictions in place.

In the USA some firms of lawyers specialise in personal injury claims, advertising widely for people who may be able to make claims. A number specialise in particular diseases or allegedly harmful substances, such as asbestos or silicone implants.

In other countries similar roles may be taken by lawyers, trade unions or specialist firms set up for the purpose, as happened in the UK after it became legal for lawyers to work on a no-win-no-fee basis, as mentioned earlier. These organisations tend to concentrate on liability for causing latent claims (ones that manifest themselves some years after the cause), which means that the effect on insurance is on reserves, sometimes reserves for years of account that were thought to be settled in full, rather than on current business.

We discuss latent claims later in this chapter, in Section 3.3. Reserving is covered in detail in Subject ST7.

Staged accidents

There are also types of fraud that result in claims to insurance policies of innocent third parties, for example, staged motor vehicle accidents.

An example of this would be where the car in front deliberately and suddenly brakes in order to cause an accident. The owner of the car then pretends to have sustained serious, but difficult to prove, injuries (such as neck whiplash or stress) and also fraudulently claims for damage to the vehicle, which existed before the accident. The people committing staged accidents are often part of organised crime gangs. Staged accidents are a big issue for motor insurers in many territories, but this practice is gaining awareness among the general public.

3 Climate and environmental factors

The natural environment leads to many insurance claims, including some of the most spectacular ones. Recent concerns about global warming have raised the prominence of this issue, but normal trends in weather and the incidence of spectacular events, means that weather-related losses are inherently unpredictable from year to year. The man-made environment can also be a cause of claims in ways that are not always obvious in advance.

This section covers:

- the impact of the weather, including seasonal effects (which will depend on characteristics of the property being insured), and the possible implications of global warming
- catastrophes, including weather-related events (particularly hurricanes), and factors affecting the financial impact of these; earthquakes and examples of man-made catastrophes are also discussed
- latent claims, giving lots of examples, and how insurers have tried to deal with latent claims.

3.1 Weather

Seasonality

The most obvious way in which weather varies, in most countries, is seasonality. In temperate climes there is the spring / summer / autumn / winter pattern; in tropical climes there may be a dry season / wet season pattern or a monsoon season. The precise pattern and the dangers associated with each phase will vary from country to country even within geographic zones, with differences in weather patterns and building codes, among other things.

Building codes are the standards to which houses, offices, bridges, *etc* must be constructed. For example, in territories prone to earthquakes, such California and Japan, there are regulations to ensure that all new buildings are built to standards to withstand earthquakes of a specified intensity.

In areas where the standard of building construction is high, insurers should bear lower losses.

In general, winter weather is harsher and for some classes is more likely to give rise to claims: storm damage is more likely and driving conditions are likely to be more treacherous, including the fact that there are fewer hours of daylight. This is rarely of concern to insurers, since most policies are issued for a year and will be in force through all four seasons. However, in extreme cases it may influence patterns used to earn premiums.

The unearned premium reserve (UPR) is usually calculated by taking a portion of premiums in respect of the unexpired exposure period. Often, this is done on a straight averaging basis, *eg* for an annual policy with six months to go it might be reasonable to take half of the premium. However, if the risk is not uniformly spread over the year of cover, *eg* where the claim costs vary according to the time of the year, the proportion of premium taken should reflect the expected risk in the unexpired period.

In calculating UPR, an allowance might be made for initial expenses. The calculation of the UPR is discussed in detail in Subject ST7.

Subsidence and land heave

Although the weather in summer is generally more benign than in the winter the problems of subsidence and heave are generally more likely to arise in the summer. This first became a problem in the UK in 1976 in a very dry hot summer. The shrinkage of land on which houses were built as the ground became desiccated (dried up) **led to damage to the houses, which was exacerbated when the drought broke and the ground expanded again. This led to a large number of claims for structural damage to property, especially domestic property, many of which were for large amounts. Also, because the damage was not caused over a short period of time, as it might have been with a storm, catastrophe XL reinsurance may not have responded to these losses.**

Heave is essentially the opposite of subsidence. Whereas subsidence involves a downward movement of land due to a reduction in hydration, heave involves an upward movement due to increased soil hydration. Heave could occur after trees are felled in an area because they will no longer absorb moisture from the soil.

Question 8.13

Why do you think catastrophe XL (excess of loss) reinsurance may not have covered many of the subsidence claims?

Location of property

Different areas are obviously subject to different climates, but the vulnerability of particular properties to weather events will vary in ways that are not always obvious, and make underwriting difficult for a mass product. Places close to each other will suffer almost the same weather, but some locations are more sheltered than others, some will be more prone to being flooded. Obviously, properties built on flood plains are prone to flooding, as are those on low-lying lands near the coast, but other vulnerable places may not be so obvious: where water is channelled as it runs downhill it may make some hillside properties vulnerable to flooding.

So two properties might be in the same small town but have a very different weather-related risk because:

- one is in a sheltered, dry spot under a hill
- the other is in an exposed area, next to a river.

Some insurers are dealing with this issue by, for example:

- requesting more precise details about the location of the property and previous claim history
- rating according to a more precise measure of location – *eg* using full postal code, rather than just the first few characters.

Global warming

A number of serious weather losses have been linked to global warming.

The theory behind "global warming" is that increased carbon emissions is leading to increased temperatures. This, in turn, is leading to climate change. Melting of the polar ice caps may also lead to increased sea levels. Many experts now expect an increase in claims due to severe weather-related events, such as floods, hurricane, storms and droughts, as a result. However, the links between carbon emissions, global warming and the climate changes have not been proven with certainty.

Whether or not this is the true explanation is debatable, and there is some evidence that major storms in the Caribbean and the Gulf of Mexico have fluctuated in number and intensity as long as records exist.

Having said that, in a survey conducted by UK Actuarial Profession in 2007, 83% of the actuaries that responded felt that climate change is real. However, even if you assume that it will happen, quantifying the *rate* of climate change is a difficult challenge and open to debate.

The problem for insurers is two-fold:

- **Firstly, it is not possible to be sure that long-term trends exist or, if they do, where they will lead.**
- **Secondly, the effect on insurance is unclear.**

It has been suggested that even small changes in climate can have a more than proportional effect on insurance losses, but this is impossible to prove.

Some people consider that global warming will increase the volatility / unpredictability of weather-related claims, which will impact insurers (even without a change in the mean).

Again, the nature of general insurance as an annual contract means that companies will be able to adapt gradually to changes in the claims environment.

Question 8.14

Climate change is seen as a long-term effect. Suggest some short-term measures that general insurers could take each year in response to the resulting adverse claims experience.

However, if changes mean that some properties become substantially more vulnerable to loss, it may affect their insurability and this can lead to political differences.

Many people feel that governments have a moral obligation to protect householders from flooding. This is demonstrated through state-funded construction of flood defences, such as the Thames barrier in London. In some territories, such as France and the United States, flood protection is provided through government insurance or pooling arrangements. In others, such as the UK, if flood defences fail or are inadequate, the householder may be left paying, and so will need to buy private insurance.

The concern in the UK is that some people, *eg* those living in low lying properties or coastal areas, will not be able to afford to insure their houses because the insurance premiums will be too expensive. The ABI, representing the insurance industry, has therefore been in discussions with the UK governments (*eg* English and Scottish) so that affordable insurance will be available to those on low incomes, and the governments will progress steps to improve flood risk mitigation measures.

This has resulted, for example in the UK, in a statement of principles from the ABI committing member insurers to continue to provide flood insurance coverage in defined circumstances until 2013.

3.2 Catastrophes

Catastrophic losses can take the form of one immense loss, such as an oil-rig explosion. Alternatively, there may be many smaller insured losses, all stemming from a common, identifiable event such as a hurricane.

One way to reduce the impact of catastrophic losses is to write business in a wide range of geographical locations and across many classes. Catastrophe reinsurance will also help.

Catastrophes may be either natural or man-made in origin.

Examples of natural catastrophic losses include:

Ice, snow, frost: Widespread property damage may arise from water damage caused by burst pipes. There will also be many more claims for accidents from the motor account.

Storms: Severe storms (*eg* wind, hail or rain) can cause extensive damage to property. There may be a large number of claims from agricultural or motor policies in a region hit by a hailstorm or household property damage from wind storms or flooding (*eg* parts of the UK in the summer of 2007).

Earthquake: Potentially massive damage to property classes (*eg* Los Angeles earthquake in 1994).

Examples of man-made catastrophic losses include:

Fire: A large fire especially in hot, dry territories (*eg* forest fires on California in October 2007).

Air crash: This could affect the aviation or public liability classes. If the problem is a design fault, claims could fall on the manufacturer's product or public liability cover.

Explosion: *Eg* oil depot at Buncefield (UK) in 2005, the Piper Alpha oil rig in 1988. Losses could hit property, employers' liability, public liability and/or consequential loss policies.

Terrorism: *Eg* terrorist attacks on the World Trade centre in New York on 11 September 2001.

Hurricanes, storms

Although the USA is cited for having the most expensive weather incidents, more recently, events in New Zealand, Australia and SE Asia have reminded us that "weather" is a worldwide phenomenon.

The pre-eminence of the USA in this regard is partly because of the concentration of high insured risks and partly because of the vulnerability of the coast of the Gulf of Mexico and the Atlantic states' littoral area to hurricanes.

Littoral means pertaining to the coast.

As mentioned before, the high total cost of insurance claims from weather-related events in the USA compared to other countries can be partly attributed to:

- the large number of properties in certain areas, particularly in cities along the eastern coast
- the high proportion of properties that are insured
- the high average value of these properties.

In addition, the US states of Alabama, Florida, Georgia, Louisiana, Mississippi, North Carolina, South Carolina and Texas are particularly prone to hurricanes.

It is normal seasonal behaviour for tropical storms to form over the Atlantic ocean and track in an easterly direction; some forming hurricanes in the Caribbean, the Gulf of Mexico and on the USA's south-eastern coast. They tend to cause damage – sometimes very serious damage – in Caribbean states, but the concentration of insured values there is low; whether or not they are serious events in global insurance terms depends partly on their strength, but more on whether they affect the USA and where in the USA they land.

The most expensive weather incident ever recorded (as the time of writing) **is Hurricane Katrina, which hit Louisiana in the autumn of 2005. Although it was a very strong storm it was not uniquely strong, and not in fact the strongest storm to hit the USA in 2005 (that was Hurricane Wilma slightly later that year). However, Hurricane Katrina passed almost precisely over the city of New Orleans, which proved to be particularly vulnerable. This type of effect adds to the uncertainty of underwriting property insurance in loss-prone areas.**

Part of the reason for there such being extensive damage was that Hurricane Katrina weakened a main levee (breakwater) protecting New Orleans. Six days later the levee broke, resulting in flooding to approximately 80% of New Orleans and subsequent loss of lives, damage to property and much looting in and around the city.

Rising cost of major losses

It has been observed that the cost of major losses has risen substantially. This is largely due to economic development: some of the areas in the USA most vulnerable to storm losses have been at the forefront of development. In particular, Florida is a major holiday destination, which has led to a great deal of development near the coast; it is also a low-lying state which makes it vulnerable to hurricanes.

Another cause of increases in the cost of disasters is a general trend towards insurance. Insurance cover is not universal even in developed economies; in less-developed economies it can be the exception rather than the rule. However, in almost all countries property is more likely to be covered by insurance than it was some years ago. This means that the proportion of economic loss covered by insurance in any catastrophe is higher than it used to be. An obvious consequence of the difference in the proportions of properties insured in various places is that a catastrophic event in a less-developed country can cause only modest insurance losses whereas a similar event in a developed country can give rise to very heavy losses.

Example

The 1994 Los Angeles earthquake resulted in fewer than 70 deaths but cost almost \$2 billion in insured losses. The 2008 earthquake in the Sichuan province of south-western China killed around 70,000 people but insured losses are only expected to be around \$1 billion because only around 5% of the losses were insured.

Earthquakes

Earthquakes are occasional events that may lead to heavy insured losses. Geological structures determine an area's vulnerability to earthquakes in general, and the most vulnerable areas are well known, although small events are not unknown elsewhere. Most areas of greatest vulnerability are areas of low insurance intensity, but there are important exceptions; notably Japan, including Tokyo, and the San Francisco and New Madrid areas of the USA.

On the other hand, the Asian tsunami of December 2004 – caused by an offshore earthquake that was one of the largest ever recorded – caused about 230,000 deaths and widespread devastation, but relatively little insured loss. In March 2011, an earthquake hit the northeast coast of Japan, causing a 10-metre tsunami; over 20,000 lives were deemed dead or missing and over 100,000 buildings were totally destroyed.

Other natural perils

Other natural catastrophe perils include flood, typhoon, hail and volcanic eruption.

In the UK the most important catastrophe perils are extra-tropical cyclones (windstorms) and flooding (including coastal surge, riverine and drainage related).

In some countries there are nationally-administered insurance schemes that may effectively provide some or all of the cover for certain catastrophe perils.

Man-made catastrophes

Man-made catastrophes consist mainly of terrorist incidents, industrial accidents and conflagrations.

A conflagration is a large destructive fire, which can be difficult to control.

Terrorism may or may not be covered by insurance, depending on local practice and law. Terrorist incidents give rise mainly to property damage claims, but may give rise to liability claims if security measures are found to have been inadequate.

In some territories, such as in the UK, claims arising from terrorist attacks are covered by the government-backed arrangements. In these circumstances, insurance contracts tend to specifically exclude claims resulting from terrorism. (In some cases, such as in the UK prior to the 2001 WTC attacks, insurers would cover claims up to a specified monetary limit, and the government-backed arrangement would pay claims in excess of this.)

3.3 Latent claims

Latent claims derive from perils that were unforeseen when the policy concerned was signed. Students should note however that the term is also applied to any insurance claims that become known about some years after the cause of loss. Most of these arise from diseases caused by products or industrial processes, but faulty construction of buildings is another possibility. Claims arising from the sexual molestation of people, particularly children, is another example.

Most latent claims will therefore arise under product liability and employers' liability insurance. The latter two examples in the above paragraph will usually be covered under public liability insurance.

Examples of types of latent claim

The most notorious classes of claims of this nature are those arising from asbestos. These have been projected to cost insurers $200 billion in claims.

Numerous employers' liability (and some product liability) claims are arising in respect of workers that handled asbestos materials and products. Although some of those affected were exposed to asbestos from as early as the 1940s, the resultant lung conditions (*eg* asbestosis and mesothelioma) did not begin to materialise until about 1980.

Some other latent claim classes are listed below. These include claim causes that appear to have run their course, others that are still in the process of manifesting themselves and others that have caused concern, but may or may not develop into significant sources of loss:

- **Agent orange.**

  This was a chemical defoliant that was sprayed over Vietnam by the US army in the 1960s. Many of the soldiers based there have sued against the manufacturers of the chemical for consequent health problems to themselves and their families (*ie* for birth defects). Most of these claims were made in the 1980s. More recently (2006), Vietnamese victims have tried unsuccessfully to make claims.

- **Radiation from mobile phones.**

  It has been suggested that this might be linked to an increased risk of some cancers, particularly in children, headaches or sleeping problems. To date, no definite link has been proven, but research continues.

- **Benzene.**

  Exposure to benzene can cause serious health problems, including some cancers. The chemical has been found in some carbonated soft drinks, which were, of course, immediately withdrawn from sale.

- **Diethylstilbestrol** (DES).

  This is a drug, which was given to millions of women in the US in the middle of the 20th century to reduce the likelihood of premature births. However, it has been linked to genital abnormalities in daughters, and even potentially in granddaughters.

- **Electromagnetic fields.**

  These are linked to the increased risk of leukaemia and other cancers.

- **Pollution.**

  Exposure to pollution may last several months or years. The impact on health of being exposed to polluted conditions may not be apparent for many years.

- **Guns.**

  In the US, there have been attempted claims for compensation against gun manufacturers:
  - by victims of accidental shootings
  - by city councils, for the increase in gun crime
  - by gun users, where accidental injury has been causes.

  To date, most of these claims have been unsuccessful.

- **Noise-induced deafness.**

  Most commonly, this is due to working with, or being exposed to, noisy machinery (*eg* with pneumatic drills or beside aircraft).

- **Blood products infected with HIV or hepatitis.**

  During the late 1970s and early 1980s, large numbers of haemophiliacs became infected with HIV or hepatitis after receiving tainted blood-clotting substances.

- **Sick-building syndrome.**

  The building people work in can be blamed for a range of illnesses, such as irritation to the nose, throat and eyes, fatigue or headaches. This could be attributed to micro-organisms within the air conditioning or the humidity of the building, but the specific causes of these conditions are generally difficult to prove.

- **Latex gloves.**

  Some people are severely allergic to latex rubber. Examples of compensation claims are:
  - under employers' liability, from hospitals sued by workers that have been made to wear latex gloves
  - under product liability, from glove manufactures sued by customers wearing, or patients treated by medical staff wearing, latex gloves.

- **Lead paint.**

  Lead is added to some paint to improve its performance, *eg* in drying quickly. It can be damaging to health, in particularly hindering the development of young children. Now that the dangers are known, it should only be used in certain circumstances, *eg* for painting road surfaces. There are potential product liability claims against paint manufacturers, and some public liability claims, *eg* against landlords.

- **Bovine spongiform encephalopathy** (BSE).

  BSE is commonly known as "mad cow disease" as it affects the brains of cattle. It was first found in the mid-1980s, mainly in parts of Europe. However, 10 years later a brain disease (known as vCJD) was found in humans, causing several deaths, and there is evidence to suggest that some victims may have caught the disease by eating meat from BSE-infected cattle.

- **Toxic mould.**

  There have been houses and other buildings in the US, particularly in Texas, where types of mould that emit toxins have been claimed to cause health problems and damage to property. Among the parties being litigated against are builders, architects and owners of buildings (such as schools).

- **Dalkon shield.**

  This is a contraceptive intrauterine device that was found to cause severe injury to a disproportionately large number of its users.

- **Repetitive strain injury** (RSI).

  RSI is a generic term used to describe a range of painful conditions of the muscles, tendons and other soft tissues. It can affect the upper limbs, neck, spine, or other parts of the musculoskeletal system. They are generally caused by performing work-related, usually repetitive, tasks, and so they can lead to employers' liability claims. Vibration white finger (from using vibrating machinery, such as pneumatic drills) is a traditional example, although conditions related to computer use (*eg* poor posture) are more prevalent nowadays.

- **Silica dust.**

  A fine silica dust can be produced when certain types of rock are cut, drilled, *etc*, which can cause lung diseases if inhaled. Foundry workers and people working with the products produced (*eg* potters and sandblasters) are most at risk unless proper precautions are taken. It can take, say, 10 to 15 years following exposure before symptoms develop.

- **Tobacco.**

  Smokers and their families have taken tobacco companies to court for illnesses, injury or death caused by long-term smoking. Most cases have been in the US, where some medical insurance providers have also claimed compensation from the companies.

- **Year 2000 computer systems.**

  Towards the end of the 1990s, there was a huge fear that many computer systems and products that relied on microprocessors would fail in the year 2000. This concern arose because early computer programs often use a two digit code for the year component of dates and the ambiguity of the date "00" may lead to incorrect calculations. Products that may have failed include computers, machinery, lifts and safety equipment. Failure on safety equipment may also have led to employers' liability claims. Companies and organisations all over the world checked and upgraded their computer systems in preparation for the "millennium bug", and no significant computer failures occurred when the time came.

- **Nanotechnology.**

  Nanotechnology is the ability to work with materials on an extremely small scale, *eg* 100 billionths of a metre or less. This is still a developing field, but nanomaterials are already being incorporated into many products worldwide, including cosmetics, paints, medicines and food products. However, there is very little knowledge about how nanomaterials may affect the long-term health of workers and consumers.

  Any toxic tort litigation arising from nanomaterials could impact manufacturers, distributors, secondary users (*ie* producers who incorporated nanomaterials into other products) and retailers. Insurers writing employers' liability, general liability and product liability could therefore be affected.

Problems with latent claims

One problem with latent claims is that it is impossible to know where the potential claim is lurking. Also, if the claim does materialise, the future claim cost is completely unknown.

For example:

- Will there be future employers' liability claims for damage to people's eyes from using computers too much?
- If so, how much will the claim amounts be and how many people will be able to claim?

There is also the problem of identifying when exactly the claim event occurred, especially if exposure (*eg* to the harmful substance or working conditions) was over many years.

Most latent claims arise in liability insurance. The normal form of these policies was the occurrence basis in which a claim would always be paid from the year of account in which the damage was caused. This leads to problems of definition: if a person who worked with asbestos for a number of years, possibly with several different employers, contracts mesothelioma some decades later, how can it be traced to a particular year of insurance? The answer is that a legally-imposed or industry-agreed method of allocation must be found.

It may be difficult to identify the claim event date. However, the claim notification date should be readily identifiable and objective.

Partly as a response to this the claims-made policy was developed in the 1980s. This is now the standard form of policy for professional indemnity insurance and some other liability classes. It is intended to cover all claims that were first notified in the year of insurance. However, the cover granted may be unsatisfactory from the claimant's point of view, and even more so from the point of view of a claimant who depends on the tortfeasor's insurance to obtain redress. When a claim arises the tortfeasor may no longer exist, and if latent claims are emerging he or she may have trouble obtaining continued cover.

So, for example, say an employer took out liability insurance on a claims-made basis. If it is known that the employer exposed its workers to hazardous conditions, it will be difficult for it to get cover that is either affordable or comprehensive enough to cover future claims. This is because insurers will fear a large number of claims being notified in the coming year.

For reasons such as this occurrence cover may be required in areas where insurance is compulsory, such as UK employers' liability.

Liability insurance is intended to protect the insured against the cost of having to pay compensation, rather than to protect the third-party victim. However, such insurance is compulsory to ensure that victims can be compensated. It is therefore common for victims to be able to claim directly from insurers where the tortfeasor no longer exists, having ceased to trade for example.

Latent claims are covered in much more detail in Subject SA3.

4 Technological change

The business world changes at an accelerating pace. For insurers and their customers, it has brought about increasing efficiencies but, as yet, it does not seem to have reduced the need for human input, nor indeed hastened the arrival of the mythical "paperless office".

It is now possible to do more complicated analysis than ever before, using vast quantities of data. But however powerful the computers have become, they can't (yet) do the thinking, and still need to be operated by people who have an understanding of the problems to be solved.

Among the evolutions that are currently taking place, one might cite:

- **increased computer power enabling much more complex models to be processed**
- **reduced cost of data storage, allowing more and more detailed information to be stored and hence improving the accuracy of predictive models**
- **increasing availability of external data**
- **better strategy, planning and capital forecasting**
- **growing market in modelling software solutions, including the increasing sophistication of natural catastrophe models**

  Catastrophe models are covered in Chapter 21.

- **increasing technological awareness of customers, enabling different approaches to sales platforms, including web-based data transfer**
- **the growth in, and acceptance of, the electronic broker, the comparison websites**

  These comparison websites are commonly known as *aggregators*.

- **the growing ability to handle claims online**
- **the potential in telematics, the insurer's eye in the insured vehicle, which can transfer personal underwriting information on driving skill and usage**

  This is where an electronic device is installed in an insured vehicle in order to monitor the location, movements and behaviour of a vehicle and its driver. The data received by the device can be used in pricing the insurance risk more accurately.

- **the growing ability to invest and switch, where not only are the products more complex but even the decisions to buy and sell are generated electronically.**

An insurer will need to embrace these new technologies and their associated implications because their competitors will be doing the same. Any insurer that fails to do so is likely to be at a competitive disadvantage very quickly.

Technological change does not always mean increasing costs. The "information super highway" puts detail, data and comparative analysis at the fingertips of supplier and end user, often at zero-cost, including some of the software solutions mentioned above (downloadable free).

In the exam, you will be expected to be able to give examples of technological advancements and also describe how these might impact the operations and experience of an insurer.

For example, price comparison websites have increased the transparency and competitiveness of the market and the sensitivity of consumers to changes in the rating structure.

Question 8.15

Give three more examples of how the technological advances listed above might affect an insurer.

5 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Hard premium rates
- Insurance cycle
- Losses-occurring (losses occurring during [LOD]) policy
- Ogden tables
- Risk attaching basis
- Soft premium rates.

Chapter 8 Summary

Among the ***economic factors*** affecting insurance business are claims inflation, the underwriting cycle, investment conditions and currency movements.

Different insurance claims types are affected by different types of inflation, *eg*:

- household contents – mainly price inflation
- buildings and motor damage repair costs – mainly wage inflation
- medical expenses – medical cost inflation
- liability for personal injury – court award inflation (split into components).

Expense loadings are also subject to inflation – mainly salary-related. Failure to anticipate appropriate levels of inflation can lead to inadequate premiums or reserves.

The underwriting cycle is a continuous cycle of hard (profitable) premiums, then increased competition, followed by soft (less profitable) premiums, and then insurers leave the market. Soft markets have often ended when a major disaster has triggered severe losses. Low barriers to entry, the delay until profitability of business written is known, simplistic capital regimes and marginal costing have facilitated the cycle.

The significance of investment conditions depends on the characteristics of the business sold, *eg* greater for long-tailed business. If significant, investment return should be allowed for in pricing, often using a risk-free rate.

The impact of currency movements depends partly on where business is sold and where claims are made. Analysing business by currency can avoid distortions where there are changes in exchange rates, although judgement needs to be applied in deciding how to apply exchange rates in any analysis. Currency hedging can be used.

Among the ***legal, political and social factors*** affecting insurance business are court awards, legislative changes and trends in behaviour and awareness.

Court awards have most impact on liability insurance claims. Claims may be on the basis of breech of contract but are more usually on the basis of negligence. Although most cases are settled out of court, court decisions set benchmarks for the negotiations.

Compensation awards for major bodily injuries can be complicated, and are split into several "heads of damage". Punitive or exemplary damages are quite common in the USA. In many markets, compensation awards are becoming more generous, leading to increases in liability insurance premiums. Unexpected increases in claims may lead to inadequate reserving.

Legislative changes can directly impact insurers, *eg* making certain types of insurance compulsory. Changes in regulations to improve health and safety (*eg* at work or for motorists) or the litigation process (*eg* introducing no-win-no-fee arrangements) will also affect insurance claims.

Changes in the attitudes and behaviour of people and organisations directly affect the cost of insurance. Examples of this are a greater propensity of people to seek compensation (helped by the encouragement of the placing of claims for compensation with claim management companies, for example), lower social acceptance of drink driving and a greater tendency towards fraudulent or exaggerated claiming.

Among the ***climate and environmental factors*** affecting insurance business are the weather, catastrophes and latent claims.

The seasonality of weather-related claims, if significant, may need to be allowed for in earned premium payment patterns. Subsidence can be a problem in very dry, hot summers, and can lead to large property damage claims, arising over a long period of time. A number of serious weather losses have been linked to global warming.

An individual property's location may need to be considered in some detail to be able to assess its exposure to weather conditions accurately. However, if insurers fully reflect the risk in their premiums, this could cause political difficulties, *eg* leading to some properties being insurable against flood damage.

Major natural catastrophes include hurricanes and earthquakes. The largest insurance losses tend to be in the USA. This is due to the concentration of high insured risks and the vulnerability of some areas to hurricanes. The cost of these claims is rising due to economic development and greater use of insurance.

Human-made catastrophes consist mainly of terrorist incidents, industrial accidents and conflagrations.

Latent claims are insurance claims that become known about some years after the cause of loss. Most of these arise from diseases caused by products or industrial processes, and so arise in liability insurance. The most notorious classes of claims of this nature are those arising from exposure to asbestos, but there are many other examples.

It can be difficult to identify the occurrence date of a latent claim. Claims-made policies can make it difficult to obtain continued cover, and so the claim-occurring basis is usually required for compulsory type of liability business.

There are also ***technological factors*** affecting insurance, largely arising due to the increasing capacity of computers and widespread use of the internet.

Chapter 8 Solutions

Solution 8.1

A latent claims is a claim resulting from a cause that the insurer was unaware of when it wrote the policy, and for which the potential for claim to be made many years later had not been appreciated.

In common parlance, latent claims are also those that generally take many years to be reported.

Solution 8.2

Four different types of inflation that might affect insurance claims are:

- price inflation, which will affect the replacement cost of goods
- earnings inflation, which will affect repair costs and loss of earnings claims
- medical inflation, which will affect medical expense claims
- court inflation, which will affect claims where the claim amount is decided by a court, *eg* on some liability claims.

Solution 8.3

The following factors could prompt a policyholder to increase the level of cover on a personal accident policy:

- a change in family circumstances, *eg* marriage or birth of a child
- increasing salary, in particular due to a change in job or promotion
- a response to marketing material from the insurer, *eg* promoting the benefits of, or offering discounts on, an increased level of cover.

Solution 8.4

The employment risk assumption in the Ogden tables allows for the risk that the individual may have been unable to work (and thus earn an income) in some future periods, assuming the accident did not happen. For example, this may be due to sickness, unemployment or retirement.

Solution 8.5

It may be difficult for an insurer to act against the cycle because:

- It is far from obvious when the peak and trough have been reached.
- Even if it were known, such a policy is not practical for several reasons, *eg*:
  - the insurer would need to get a large market share from a standing start, this may involve offering very low premiums or spending lots on advertising
  - the insurer would incur high expenses on closing down and starting up again (acquiring new business incurs higher expenses than processing renewals).

Solution 8.6

The rationale behind the EU minimum margin was that the amount of capital an insurer holds relates to the amount of business it writes. It is a simple measure and it easy to calculate. It is easy for a regulator to enforce and verify, and for third parties to make comparisons between insurers.

Solution 8.7

The significance of investment conditions is influenced by:

- the size of the company's free assets
- the amount of business the company writes (*eg* annual premium income)
- the size of its in-force business (*eg* size of the reserves)
- the length of claim delays (both reporting and settlement delays)
- the term of the contracts (*eg* investment is less important for weekly travel insurance contracts than for five-year extended warranty policies)
- the premium payment method and distribution channels (premiums might be more subject to delay under some methods / channels).

Solution 8.8

The claim may not be notified for some years after the claim event, *eg* the policyholder may not be aware that he or she has been exposed to a harmful substance until symptoms of a disease emerge many years later. Also, the exposure may be over a long time period.

There may be a significant delay before the claim is fully settled. It may take time to agree whether the policyholder is liable or not, and to decide on the amount of compensation, which may involve court proceedings and may be disputed. Some claims may be settled using partial payments.

Solution 8.9

The insurer is earning higher than the risk-free rate because it is taking on some of the risks associated with the assets it is invested in, such as the risk that the issuer of the asset defaults. If it discounts at a *higher* rate (*ie* 6%), it will calculate a *lower* premium, which may be insufficient if the risks do materialise. Using the lower discount rate of 4% allows an adequate premium to be charged for the risks that the insurer is exposed to.

Solution 8.10

Reasons why the parties involved may decide to settle out of court include:

- to avoid (or at least significantly reduce) legal and court fees
- to obtain an earlier settlement
- to avoid spending the time and incurring the emotional upset of a (potentially lengthy) court case
- to avoid possible adverse publicity (*eg* a high-profile product manufacturer)
- to have more control over the final outcome, *ie* they can negotiate rather than leaving the decision to the courts
- it may be less risky – if the case goes to court, one party may end up with nothing!
- to avoid a structured settlement (as discussed in Section 1.2)

Solution 8.11

Possible impacts of making certain lines of insurance business compulsory are:

- increased volume of business overall, which should lead to increased overall profits and lower per-policy expenses
- may need to reduce premiums to maintain market share if market for product becomes more competitive as more insurers join the market
- less opportunity for anti-selection, if people are forced to buy a policy
- possibly greater risk of moral hazard – *eg* if people are forced to buy a policy, they may be tempted to claim fraudulently in order to get something back for their premiums.

Solution 8.12

This is an example of fraud.

Solution 8.13

Catastrophe excess of loss reinsurance covers catastrophic events. An event is defined as a number of claims occurring within a short period of time (defined in the hours clause in the reinsurance treaty). Many subsidence claims will have been deemed to occur outside of this period, and so would not be counted in the total claim amount used to determine the reinsurance recovery.

Solution 8.14

A general insurer could respond to adverse claims experience by:

- increasing premiums, either overall or for certain risks
- changing its benefits, *eg* exclude specific perils
- strengthening underwriting or claims control measures
- no longer selling certain products
- restricting or attracting a different target market, *eg* stopping marketing activities in certain locations
- strengthening its reinsurance programme.

Solution 8.15

Increased computer power and more complex models could lead to:

- more accurate pricing
- higher costs, *eg* relating to IT costs and the hiring of expertise
- spurious accuracy and an over-reliance on the model's output.

Better strategy, planning and capital forecasting could lead to more accurate reserves being held, thereby increasing consumer protection and/or reducing the cost of capital.

The ability to handle claims online might tend to:

- accelerate claim settlement patterns
- reduce claim handling costs, *eg* stationary and postage costs
- improve customer satisfaction.

Chapter 9

Risk and uncertainty in pricing

Syllabus objective

(e) Describe the major areas of risk and uncertainty in general insurance business with respect to pricing, in particular those that might threaten profitability or solvency.

0 Introduction

Insurance contracts transfer elements of risk and uncertainty from customers to insurers. The insurers accept these risks and uncertainties and therefore it is important that they are able to control these if they are to survive and be profitable.

There are a number of risks surrounding the profitability of the insurance business written, which are discussed in this chapter. As well as profitability, volumes of business and capital must also be carefully managed such that the insurance company remains able to support the business it has written, as well as that which it is writing – that is, staying solvent.

There is a risk that volumes of business may be lower than expected which could lead to lower profits than expected and a failure to cover fixed expenses. There is also a risk that volumes may be higher than expected. This would lead to strain on available resources (*eg* staff to handle policy applications and claims) and higher capital requirements to back the expanding business volumes. If capital held is insufficient, there is a risk of intervention by regulators or, in extreme cases, insolvency.

0.1 Uncertainty

Uncertainty is the inability to predict the future with confidence. Because of the presence of uncertainty, we need to consider the effects of possible deviations from the projected figures.

0.2 Risk

Risk is the possibility of variations in financial results, positive or negative.

It is important that a general insurer is able to identify the risks that it faces and assess the suitability of methods available for managing those risks.

The greater the uncertainty, the greater the risk.

0.3 Premium rating

An insurer (or reinsurer) intends the premium to cover all cash outgoes (including claims, expenses, reinsurance premiums, commissions and so on), net of all inflows (including reinsurance recoveries, subrogation and third party recoveries, profit commissions, investment income and so on), and produce an eventual surplus that meets the profit targets.

There is substantial uncertainty about the quantum and timing of the income and the outgo.

> ***Question 9.1***
>
> Suggest ways in which each type of cashflow mentioned above may be uncertain regarding amount and timing.

To decide on an appropriate premium to charge, insurers tend to analyse past experience of similar relevant business, while adjusting the resulting figures to reflect the commercial realities of the prevailing market conditions.

0.4 Allowing for risk in premium rates

When it sets a premium, the general insurer should assess the level of risk being accepted and decide a suitable premium to cover this risk.

Therefore the insurer should assess the expected cashflows and ensure that the premium covers the expected net outgo. It is very unlikely that the insurer will get these estimates exactly right and so the premium should also allow for any uncertainty surrounding the estimate of the net outgo.

We can summarise the risk in this context as the risk that the cashflows into and out of the company do not match those expected when it determined the premium.

In some theoretical approaches to pricing, we include a risk loading in the premium, so that two insured risks with the same expected outcome may be charged different premiums, with a higher premium for the risk with the greater variability of outcome.

We can express this mathematically in terms of the variance or skewness, for example. We can achieve this in other ways, such as by setting a return on capital employed target, which will also result in a higher premium for more risky policies.

There are many possible ways to subdivide the risks and uncertainties facing a company at this point, for example:

- ***Data errors from systems and procedures*: This covers the uncertainty arising from the way past claims are reported and stored.**
- ***Model error*: This covers uncertainty in the selection of the model and how it is used.**
- ***Random error*: This covers the fundamental uncertainty (the insurable risk) that is insured by the policyholder.**
- ***Adjustment factors*: It is necessary to make a number of assumptions about how to adjust observed data to reflect the future experience, each of which is uncertain.**
- ***Market conditions*: Insurance is not written in isolation. Legal, regulatory and competitive effects can all influence the risk and uncertainty.**
- *Portfolio movements*: Unexpected changes in the volume and mix of business can cause an insurer uncertainty.

Risks and uncertainties are discussed further in the rest of this chapter. However, the Core Reading does not use the subdivisions above.

If you are studying Subject ST7, you may notice a significant overlap with the material in the equivalent chapter in that subject. In fact, the Core Reading in this chapter usually mentions the effects on reserving and capital modelling rather than the effects on pricing. Remember, however, that reserving (*ie* estimating ultimate claims) is part of the pricing process.

Another way to subdivide the risks facing a company is to split them into different items of experience, for example:

- claim frequencies
- claim amounts
- expenses and commission
- investment income
- new business volumes
- lapses.

Question 9.2

How do these categories fit into the categories listed in the Core Reading above?

However, there are many possible areas of overlap between different subdivisions, and no one correct way of organising your thoughts.

The examiners would expect students to be able to describe all significant risks and uncertainties relevant to an exam question, no matter which subdivisions or structure you choose.

1 Process error – specific risk

Process error refers to the inherent randomness underlying a book of business.

In Sections 1 and 2, we discuss a number of sources of process error. Please note that this is not intended to be an exhaustive list and that other sources of process error exist.

An insurer's specific process uncertainties stem from:

- general claims uncertainty relating to the specific business written
- internal influences, *eg* changes in reserving philosophy or mix of business.

1.1 Inherent uncertainty in individual claims

The amount and timing of an individual claim is always going to be uncertain. Similar historic claims may give some idea as to how an individual claim may develop, but there is no guarantee that the development will be similar.

General insurance claims are extremely unpredictable. Even with a reasonable quantity of reliable, relevant past data, it is impossible to predict claims accurately. Uncertainty exists over:

- claim frequency
- claim amounts
- claim payment patterns.

Claim frequency

Claim frequencies may be subject to random fluctuations, and may also change over time.

Changes over time may be due to a changing attitude of policyholders to claiming.

> ***Example***
>
> In the United States, policyholders have shown an increase in the propensity to submit claims.

This change in attitude could lead to a spiral effect. If an individual manages to make a successful, but unusual, claim then due to the publicity that this is likely to attract, other policyholders in the same position are also likely to claim.

It is often not clear if it is the attitude of policyholders that is fundamentally changing, or whether the actions of lawyers, in encouraging people to make such claims, is the underlying cause of the trend.

The interpretation of policy wordings can also affect claim frequencies.

Question 9.3

How?

The policy document is a legal agreement between the insurer and the insured. Amongst other things, it describes the events and circumstances under which losses are or are not covered. It is, therefore, important to word the policy as precisely as possible, if it is to give the cover intended and to prevent claims from being paid that were never intended to be included in the cover given. There may nevertheless be some residual uncertainty as to whether certain types of event could give rise to a legitimate claim.

Claim amounts

Claim amounts may also be subject to considerable variability.

For some types of business the size of possible claims covers a very wide range, and there is consequent uncertainty as to whether the claims that have actually occurred can properly be regarded as typical of what might be expected to occur.

Example

In private motor business, the variability of property damage claims is relatively small, even allowing for the possibility of a number of cars being damaged in, say, a motorway pileup. However, the size of injury claims could range (in the UK) from a few hundred pounds to a few million pounds.

The variance of aggregate claim amounts will also increase if there is non-independence of risks. Therefore accumulations of risk will increase the uncertainty relating to the variability in claim size.

Accumulations of risk may occur due to the insurer's business acquisition strategy (*eg* it might target policyholders of a particular type) or they may arise inadvertently (*eg* there may be a large concentration of policies taken out by individuals living close to the insurer's head office). Accumulations may also arise as a result of a catastrophe event.

Due to the variability in the size of claims, there may be uncertainty as to whether changes in claim costs from year to year are due to changes in underlying risks or are simply the result of random variation.

The level of random variation will be higher, the smaller the portfolio of business. This problem is therefore greater for small companies (or small classes of business) where we would expect a larger variation from year to year.

Claim payment patterns

There are typically delays between:

- the date of the claim event
- the date of reporting the claim
- the final settlement of the claim.

In addition, the date of the claim event itself may be uncertain.

Question 9.4

What sort of claim events may be uncertain in their timing?

The delay between the claim event and the reporting of the claim will depend on the type of claim and the speed at which policyholders report potential claims.

The delay between the claim being reported and settled will also vary by type of claim. Large claims are likely to suffer the longest delays, especially in liability classes where the claims may need to be settled by the courts. Large claims are discussed later.

Changing development patterns

Claims development patterns can be expected to change over time. This may be due to a number of factors.

Examples

There may be political pressure on insurers to speed up the payment of claims following a natural disaster, or payment / recording of claims may slow down due to staff shortages.

Where development patterns change due to a known cause, such as those mentioned above, then allowances can be made in the reserving (and pricing) **process. However, there will be occasions where the development pattern changes without explanation, and the impact of this may not have been included within the claims reserving** (or pricing) **process.**

> ***Question 9.5***
>
> Suggest two examples of factors that could change the development pattern that the insurance company would be unaware of.

Demand surge

Following a major catastrophe, there will be increased demand for goods and services in the affected areas.

> ***Example***
>
> **The demand for builders may increase following a flood. This increase in demand could force up the price for such goods and services in an unpredictable way.**

Higher prices could mean higher claim amounts.

1.2 Internal sources of process uncertainty

Planned or unplanned changes in mix

Different risks will exhibit different claims characteristics, *eg* claim frequency, severity, volatility, timing of payments *etc*. The degree of uncertainty inherent in the business will therefore depend on the mix of risks that have been written.

In addition, any changes in the mix of business will increase this uncertainty.

> ***Question 9.6***
>
> (i) Suggest possible strategies that might lead to a change in business mix.
>
> (ii) Suggest other reasons why the business mix might change.

Booked reserves different to best estimate

Although the reserving actuary will calculate the best estimate reserves, it is often the case that the final decision will rest with the company directors. If the directors are under pressure (for example, from shareholders) to deliver a particular insurance result, they may take a more optimistic view than the best estimate. Conversely, the directors may decide to book a result that is more pessimistic than the best estimate. Such management decisions can have an unpredictable effect on published reserves.

The pricing actuary would normally use best estimate reserves to calculate premium rates. Any prudence to allow for profit, contingencies, *etc* should be allowed for explicitly as a loading, so that the effects can be quantified and monitored more accurately.

Expenses uncertainty

Specific influences on expense uncertainty stem from:

- uncertainty over commission and other sales-related expenses, *eg* for existing and new distribution channels
- uncertainty arising from changes in operations, *eg* new markets and off-shoring.

New markets

Entering a new market or territory will incur expenses for the insurer, including set-up fees, accommodation costs, fees to the regulator and legal costs. Some of these will be known in advance with relative certainty. Others will be unpredictable.

New types of investment

With capital markets becoming increasingly complex, there is now a much wider range of investments in which the insurer may choose to invest. Some of these investments may be less certain than more traditional asset classes.

Example

An example of this is securitised bonds, where the interest and capital repayments are contingent on some event *not* happening, or on the business on which the bond is securitised making adequate profits. The return on such investments is dependent on what it has been securitised on and so could be very uncertain.

Some of these new asset classes may be restricted or prohibited by local regulation.

2 Process error – systematic risk

In this section we discuss a number of sources of process error relating to systematic risk. Please note that this is not intended to be an exhaustive list and that other sources of process error in this context exist.

Systematic (or systemic) risk is undiversifiable (or market) risk. It is covered further in Subject CA1.

2.1 Uncertainty of the economic environment

The investment markets are affected heavily by the economic environment, both because there is a natural link between the two, and because the government or central bank may use various policies to control economic variables, such as inflation and economic growth.

> ***Example***
>
> During recessions, the value of many asset classes falls. This is due to a fall in demand for the assets, and an increase in the uncertainty of risky assets.
>
> In order to stimulate economic activity, the government might reduce interest rates in order to increase spending (by reducing saving and increasing borrowing). Lower interest rates will affect the value of many different asset classes.
>
> Lower interest rates might increase inflation, which would then impact returns from various asset classes.

The effect of economic conditions on claims

Many types of incident giving rise to claims are influenced by economic conditions, whose changes are difficult to predict as regards both timing and extent. There is therefore a continuing uncertainty as to the number and cost of the claims that will occur when conditions change.

Examples

Mortgage indemnity is one class that is heavily affected by economic conditions.

The claims experience of other insurance classes such as motor, household and commercial fire is also correlated to the economic conditions in some countries.

A number of economic variables could have a direct impact on claims. For example:

- inflation – this will directly affect claim amounts

Question 9.7

Outline three types of inflation that can affect claim amounts.

- unemployment – this could lead to certain sections of society being unable to afford insurance, and so produce a different mix of business
- economic growth – this could lead to more sections of society being able to afford insurance (and higher levels of cover in some cases), and so produce a different mix of business
- change in the value of the exchange rate – for business transacted in a currency other than that of the country in which the insurer is based, there is a risk that the insurer's results will be adversely affected by changes in the exchange rate between the two currencies; there will also be uncertainties stemming from currency mismatching between assets and liabilities, and because it may be impossible to predict the currency in which a claim will have to be settled.

Question 9.8

Give three examples of general insurance classes in which there is likely to be a high level of uncertainty relating to the currency of the claim payments.

In addition, the economic conditions can have a wider impact on the environment; for example, crime rates may increase during recessions.

The rates of crimes such as theft and arson have shown considerable variation from year to year and from country to country. General insurance companies might actively engage in trying to encourage policyholders to take steps to reduce crime, in a hope to reduce claim costs.

The effect of economic conditions on investment return

The general economic cycle is difficult to predict and has a significant effect on investment markets, most notably on investment returns.

Poor investment returns include:

- lower than expected investment income, which can lead to the reserves that are held being inadequate to meet the liabilities
- movements in asset values, which can jeopardise an insurer's solvency, (since solvency is measured by comparing the value of assets and liabilities).

Movements in asset values can impair solvency if there is:

- a fall in certain sectors of the market in which investments are held, such as Japanese equities
- a failure or adverse performance of individual assets or investments, such as Poorco PLC
- an adverse movement in currency rates, such as the Rand versus the Euro.

Investment return can sometimes be an important source of income for an insurer, especially if it writes long-tailed classes of business.

Question 9.9

Why?

The best way to minimise asset risks is to ensure that the assets match as closely as is practical to the corresponding liabilities, *eg* by nature, term and currency. Unfortunately, with general insurance business, this is usually impossible due to the uncertainty surrounding claim timings and amounts.

Exchange rates

If the insurer holds investments in foreign currencies then future exchange rates will be a further source of uncertainty.

Influence of other investment markets

With increased globalisation comes increased interdependence within investment markets. Uncertainty in a foreign investment market can easily spread to the domestic investment market.

Example

A major stock market crash in the USA will almost certainly affect European investment markets.

2.2 The insurance cycle

The insurance cycle is the observed tendency of insurance prices and hence profitability to vary in a cyclical fashion over a period of several years.

The insurance cycle can be very difficult to predict, as the frequency and amplitude of the cycle can change every time the market turns.

How the insurer reacts to the cycle can also be an important factor.

Example

An insurer may decide not to follow the market down during a softening period of the cycle, instead opting to lose some business and hopefully retain profitable business.

If this is the case, the insurer's historic experience may not be a good indicator of future outturn.

3 Specific risk – parameter error

Parameter error refers to the uncertainty arising from where the parameters of the model are set incorrectly. Given that a capital model is an artificial representation of a real life situation, there will always be a certain degree of parameter error.

One of the requirements of a good model is that the parameter values used should be accurate for the classes of business being modelled. However this is easier said than done. Indeed, there might be several possible selections for a parameter, each appearing to be equally reasonable, yet a judgement has to be made as to which to select.

Other requirements of a good model are discussed in Subject ST7.

In this section, we discuss a number of sources of reserving uncertainty which come under a broad heading of parameter error. Please note that this is not intended to be an exhaustive list and other sources of parameter error exist.

3.1 Uncertainty arising from the data used

Data may be:

- of a poor quality
- internally inconsistent
- incomplete
- nonexistent.

Poor quality data

There will be occasions where the raw data is poor; for example, claim/policy details may be inaccurate, with perhaps claim dates recorded as being prior to policy inception.

Inconsistent data

Data may also have inconsistencies, for example, changes in claim recording procedures.

Incomplete and nonexistent data

In some instances, data may be incomplete or even nonexistent, for example, when a new class of business is being written.

Insurers may also write business in new territories where they have relatively little experience. This will lead to uncertainty in setting reserves, and hence uncertainty in setting premium rates**.**

Question 9.10

Other than for new classes of business, when might data be nonexistent?

Uncertainty at extreme values

Data may not be nonexistent for a whole class of business – an insurer may have a significant body of good quality data for the majority of risk cells, however it may lack adequate data in the tails of any distribution.

When fitting a particular distribution to a set of data, it is usually very difficult to fit the tails of the distribution. This may be because there is no data at these extreme values, or the data that exists is too volatile to be usable. Assumptions will therefore have to be made from what is available. This will give rise to uncertainty in the model output. Care should be taken when interpreting the model's output.

Certain events have little / no data

In addition to data being scarce for part or all of a business class, adequate data may not exist for certain events.

Certain claim events have insufficient data to build a reliable capital model, or historic data may be deemed to be inappropriate.

Capital modelling is discussed in Subject ST7.

Example

Global weather patterns may be changing at such a pace that renders historic weather data obsolete.

On all of these occasions, assumptions will have to be made. These may be based on similar classes of business, benchmark statistics or the modeller's subjective judgement. In any case, inadequate data will lead to uncertainty within the model.

3.2 Incorrect dependencies

A number of the variables in the model will be correlated with one another; for example, interest rates and claims inflation.

Question 9.11

How might interest rates and claims inflation be correlated?

It is important that the dependencies are programmed correctly. The correlations can be regarded as additional parameters, and it is essential that they are not overlooked.

3.3 Change in case estimate reserving philosophy

Reserving philosophy within a company will change from time to time.

Example

If claims handlers have under-reserved a case in the recent past, they may be inclined to overestimate future claims to compensate.

There may also be changes in reserving philosophy following a change in senior personnel.

This could involve a change in reserving methods, or a change in the basis used for the reserve estimates (within an acceptable range).

If changes in reserving philosophy are known, it may be possible to make adjustments.

3.4 Large and exceptional claims

Large claims

Large claims can be expected to have different frequency and severity distributions to non-large claims. They are also likely to have different development patterns. There may also be differences in development pattern based upon the *type* of large claim.

Example

A large windstorm claim may develop at a different rate to a large flood claim, although both types of claim may be expected in a property book.

Uncertainty may also arise in how a large claim is defined. They could be defined as claims over a particular threshold limit (possibly with a different threshold for different perils), or large claims may be a subjective management decision. If the threshold limit method is chosen, there is the additional uncertainty as to whether this increases over time, and at what rate.

It is normal practice to remove large claims from the development and project these separately.

On some occasions, there may be an absence of large reported claims, and the reserving actuary may wish to add a loading to reflect this fact. This will give rise to additional uncertainty.

Catastrophes

Catastrophic losses can take the form of one immense loss, such as an oil-rig explosion. Alternatively, there may be many smaller insured losses, all stemming from a common, identifiable event such as a hurricane.

Catastrophes are typically hard to predict, so are hard to allow for when pricing. Catastrophe modelling is discussed later in this subject.

One way to reduce the impact of catastrophic losses is to write business in a wide range of geographical locations and across many classes. Catastrophe reinsurance will also help (more of this later in the course).

Latent claims

Catastrophic claims can also result from sources that were unknown, or for which a legal liability was not expected, at the time of writing the business. The cost of such claims cannot be calculated with any accuracy for the purpose of setting premiums.

The first problem with latent claims is that it is impossible to know where the potential claim is lurking. Secondly, if the claim does materialise, the future claim cost is completely unknown.

> ***Example***
>
> Will there be future employers' liability claims for damage to people's eyes from using computers too much? If so, how much will the claim amounts be, and how many people will be able to claim?

The development of these claims is often uncertain: one court judgement can act retrospectively over many policies, which can result in large losses for the insurer.

The effect of judicial decisions is very similar to that of inflation. In fact, the effect of judicial decisions is often simply referred to as "court inflation".

Court inflation results from court awards. The differences between court inflation and price inflation are as follows:

- court inflation, historically, has been higher than price inflation
- court inflation tends to remain level for a period, then increase in sharp jumps when new precedents are created
- court inflation is less predictable than price inflation.

From time to time, judicial decisions will set new precedents for the admission of certain claims, and the amounts at which they will be settled.

Decisions relating to imprecise policy wordings can lead to the admission of new types of claim that had not been allowed for in the original costings. Liability claims are particularly exposed to this type of risk.

Courts also periodically set new levels of award or compensation for existing categories of claim. The effect of such awards will be to increase immediately the average amount at which all future claims of a similar nature are likely to be settled, including those that have already been reported. Such awards are very hard to predict, so it is even harder to allow for this form of inflation than normal claim inflation.

> ***Example***
>
> Claim payments that are intended to represent the future lost earnings of an individual, following an accident, are likely to be based upon the present value of that future income. The courts may from time to time change the rate of interest at which insurers are allowed to discount future earnings. This change could have a dramatic effect upon the overall claim payments. Court awards can be impacted by decisions made in other countries too.

3.5 *Inadequate data supplied by third party claims handlers*

There may be occasions when an insurer outsources its claims handling function, either to a broker or a specialist claims handling firm. On these occasions, data recording and manipulation will be out of the hands of the insurer, who may not be able to check data validity.

> ***Question 9.12***
>
> Suggest how third party claims handlers may distort an insurer's pricing data in terms of:
>
> - claim frequency
> - claim severity
> - claim delays.

If different claims handlers are employed for different classes / sources of business, the way in which data is recorded may be inconsistent.

There may also be delays in passing the data to the insurer, and these delays may also differ between claims handlers.

This will make it difficult to establish claims development patterns.

3.6 *Format of data*

Claims data can be stored in a number of different ways; for example, gross or net of reinsurance, or inclusive or exclusive of claims handling costs. It is important to have an understanding of exactly what is and what is not included in the data.

If there is any change in data storage protocols in the historic data, it should be considered whether adjustments will have to be made, since this may have an effect on the claims development pattern.

3.7 *Claims inflation not as expected*

Inflation assumptions will often be required; for example, for calculating an initial loss ratio for the Bornhuetter-Ferguson method.

The Bornhuetter-Ferguson method is discussed further in Subject ST7, although you will have met it in Subject CT6.

Uncertainty regarding future rates of inflation, and in particular the rates of escalation of claims, will affect the assessment of the financial outcome of the existing business and hence the reserves required.

If inflation turns out to be different from that expected, the resulting reserves may be too high or too low.

3.8 Incorrect distributional assumptions in modelling reserve uncertainty

It is sometimes necessary to calculate a range of possible values for a reserve, in which case distributional assumptions will be required.

Setting distributions for claim frequencies and claim amounts may be tricky (for the same reasons as discussed above), and there is scope for both the distributions and the parameters used to be wrong.

3.9 New distribution channels

Different distribution channels will have different expense profiles.

Question 9.13

Without reading on, comment on whether the main expenses are fixed or variable for:

- broker sales
- internet sales.

Example

Broker sales may have a high variable cost and a low fixed cost, whereas internet sales may have a high fixed cost and very little variable cost.

Some expenses are relatively predictable. Commissions paid to brokers and other intermediaries are almost invariably expressed as a percentage of the premiums payable. As such, they do not give rise to uncertainty in assessing the outcome of business already written. However, other expenses are less certain, *eg* underwriting costs will depend on the level of, and time spent on, underwriting.

Expense uncertainty also arises through a change in the relative proportions of business coming from existing distribution channels.

If the mix of sales differs from what was expected (either between classes or between distribution channels), so that a higher proportion of business is sold on higher commission terms, the average commission rate will increase.

Also, if different brokers are paid different levels of commission, there might be a risk that the mix of business by broker changes.

Additionally, where a differential rate of commission is paid on business acquisition to that paid on renewal, there is a persistency risk in the spreading of these different commission rates across future "level" premiums. This applies equally to any business expense which is higher at the point of policy acquisition and initial processing than it is at the renewal stage.

A further risk or uncertainty may relate to the recovery of commission on a policy proposed but subsequently not taken up, or on early lapse where the distribution agreement specifies a return of commissions paid.

While – on the whole – expenses for existing channels should be relatively certain, the expenses for new distribution channels will be far less so.

It may be difficult to predict the expense profile of a new distribution channel.

Set-up costs of a new channel must also be factored in.

3.10 Planned or unplanned changes in mix

If the mix of business changes significantly, either as a result of the company pursuing a particular strategy or through unknown causes, the development pattern is likely to change, and in an unpredictable way.

4 Systematic risk – model specification

In this section, we discuss a number of sources of systematic risk in the area of model specification. Please note that this is not intended to be an exhaustive list and that other sources of uncertainty in relation to model specification exist.

4.1 Model error

Model error arises from the choice of or specification of the model. This topic is discussed again in Subject ST7.

Model error arguably gives rise to a greater degree of uncertainty than parameter error, as it is less easy to detect.

> ***Question 9.14***
>
> Why?

4.2 Programming error

There is also the risk that errors can creep into the actual programming of the model; for example, typing errors during model construction. This risk can be greatly reduced through thorough checking and peer reviewing of the capital model.

4.3 Simulation error / too few simulations

The output of a stochastic model will be heavily influenced by the number of simulations carried out: obviously, the greater the number of simulations, the greater the accuracy of the output.

A "good" stochastic model should use at least 10,000 simulations.

However, large and complex stochastic models can take a considerable amount of time to run. Even with today's computing power, the most complex models may take 48 hours or more to run a sufficient number of simulations.

There is clearly a trade-off here. If the modeller has severe time constraints, there may have to be a sacrifice in the number of simulations or in the complexity built into the model.

5 Systematic risk – future systematic

In this section, we discuss a number of sources of systematic risk which could threaten the adequacy of premiums and profitability in the future. Please note that this is not intended to be an exhaustive list and that other sources of future systematic risk exist.

Many of the points discussed in this section could also be classed as sources of future process uncertainty.

5.1 Climate change

Over the last decade, global weather patterns have changed significantly from an insurance point of view. Various agencies have produced climate models that predict further volatility in global weather patterns.

> ***Example***
>
> **Global temperatures are slightly higher and severe weather events are becoming more frequent.**

Therefore, **historic development patterns for weather-related claims may no longer be appropriate.**

> ***Example***
>
> **Windstorm claims may now be more severe and therefore take a longer time to settle, or windstorm claims may be combined with flood claims, with unpredictable results.**

It will be important in the future to closely monitor the development of weather-related claims.

5.2 New distribution channels

Claims frequency, severity and development may be expected to vary by distribution channel.

If a business acquisition or renewal strategy tends to attract policyholders with different characteristics from those of the existing clientele, the resulting claims experience may differ from that of the past.

> ***Example***
>
> **Direct sales may be expected to develop more quickly than broker sales if claims from broker sales are reported through the broker. They may also be more frequent if the broker has a facility to filter out any fraudulent claims.**

The internet has come to light in the last few years as being a significant development channel, and this looks set to increase in the future. The lack of a face-to-face meeting or a telephone call when buying a policy certainly increases the possibility of fraud, which will affect frequency and claims development patterns.

The number of distribution channels is likely to increase in the future, as insurers pursue ever more innovative ways of attracting new business.

New channels may also create a knock-on effect on existing channels.

> ***Example***
>
> **If the internet market increases, the broker market may have to shrink to compensate, resulting in brokers offering incentives to attract business. This will have an unpredictable effect on claims patterns and stakeholders may not necessarily be aware that reserves may have been set on a less prudent basis.**
>
> In other words, it will take time for the change in mix of broker business to become apparent in the claims data. Meanwhile, the reserves may be set at an inappropriately low level, on the assumption that the mix of business is unchanged.

5.3 Increased use of profit share arrangements

Profit share arrangements may incentivise the broker to pass on better quality business to the insurer. If the use of profit share arrangements increases and no adjustments are made to reflect this, this may result in overestimated reserves.

5.4 New claims handling procedures eg online

Some insurers offer an online claims reporting service. This may increase policyholders' propensity to claim, thereby increasing claim frequency. It may also cause an increase in fraudulent behaviour; for example, policyholders reporting nonexistent claims, or exaggerating existing claims, which would increase both frequency and severity.

Both of these increases are unpredictable and introduce additional uncertainty into the reserving process.

5.5 Bodily injury claims

Some governments have introduced legislation concerning the payment of bodily injury claims. The idea is that to indemnify the policyholder, the claim payment should be in the form of income replacement, as opposed to a lump sum. This effectively places a life liability on the insurer, which introduces additional uncertainty.

Insurers could get around this problem through the purchase of an annuity. However, annuity prices can be volatile, and will also include a profit loading for the annuity provider. Therefore, the insurer may prefer not to go down this route.

This type of legislation can also act retrospectively.

This means that legislation may be applied to claims that have already been reported.

5.6 Differences in third party behaviour (eg lawyers actively seeking out asbestos claimants)

The behaviour of third parties may also impact claim characteristics for certain classes.

Example

Lawyers may actively seek out people affected by asbestosis. This would increase the claim frequency, and may also have an effect on severity.

Question 9.15

How might severity be affected by this?

5.7 Claims inflation not as expected

The uncertainty surrounding claims inflation will result in uncertainty regarding an appropriate level of reserves.

This was discussed in Section 3.7.

5.8 The effect of the external environment on claims

There may be changes in the external environment that affect either the frequency or amount (or both) of claims.

Government legislation

Legislative actions can be divided into three main types:

- fiscal changes, such as increases in tax on insured items – many claims are settled on a replacement basis (*ie* the insurer replaces the damaged item), so if the sales tax on that item increases, the cost of replacing that item will increase and the claim cost will increase

- changes in the law that increase the amount of cover being provided, such as removal of a legal limit on compensation levels

- changes in the law that restrict or forbid the use of certain factors in underwriting.

In the first two cases, an insurer is unlikely to have foreseen such changes; therefore there may be a sudden change in the reserves that need to be held. Furthermore, since premiums cannot be changed retrospectively, the changes will adversely affect profits until some time after the premiums or cover can be adjusted. The third type of change will be known about in advance, but may expose the insurer to anti-selection for which the cost cannot be accurately assessed. This may result in the need for higher reserves.

In the past, government legislation has tended to increase the burden on insurers.

Example

Legislation may change and require the insurer to pay towards the cost of an ambulance to attend a road traffic accident.

This legislation has been in force for many years in the UK, although many hospitals don't claim these costs from the insurer.

This will affect future claims and claim development patterns.

Such legislation can be retrospective, thereby including historic claims as well as incurred but not reported (IBNR) claims.

Broker mergers lead to more pricing power on brokers side

If a broker grows to a market dominant position, either through acquisition or organic growth, it may be in a position to demand increased commission payments from an insurer, in return for placing business.

This will lead to uncertainty in future commission payments.

Aggregators

During the last few years, a number of "aggregator" companies have been set up, dealing mainly with personal lines business. These companies find the best price for the customer from a pre-selected panel of insurers.

Example

Examples that operate in the UK include Compare the market, Money supermarket and Go compare.

Insurers may have to pay a fee or satisfy certain conditions to be a member of this panel, which can have an uncertain effect on expenses.

There may also be different commission arrangements to those of a traditional broker, for example, a per policy charge as opposed to a percentage of written premium.

Accounting changes

Accounting procedures may change from time to time. This may be the result of a decision by the regulator, a change in market practice or simply an internal decision. This will incur additional expenses through training staff and/or employing consultants. These costs can be unpredictable.

Off-shoring

During the last few years, there has been a trend for insurers to relocate some of their back office functions to different countries to access a cheaper labour market.

Back office functions include functions such as call centres and policy administration. In recent years, India has been a popular place to off-shore functions to.

> ***Question 9.16***
>
> Why do you think India has been popular?

Such a move is likely to have high set-up and redundancy costs.

Increased demand for labour in the target countries may lead to increased wage inflation in those countries. This begins to erode any potential benefits that the insurer was hoping to gain, and in an unpredictable way.

The insurer also faces the uncertainty of currency fluctuations, assuming the off-shoring agreements are arranged in the off-shore currency.

Changes in tax rate

From time to time, governments may vary the level of tax, which will have an effect on the insurer's post-tax profits.

Levies

Levy payments can be used by the regulator in order to compensate policyholders in the event of insurer default. If there are different levels of insurer default to those expected, there will be an uncertain effect on the insurer's levy contributions.

There may also be other (uncertain) charges, *eg* professional charges from auditors, or fines from the regulator.

5.9 Economic conditions

General economic conditions will have an effect on expenses.

> ***Example***
>
> **An increase in inflation will have an effect on the average level of expenses.**

> ***Question 9.17***
>
> Which index (or indices) are expenses likely to increase in line with?

Economic conditions are very difficult to predict in advance, and can change suddenly.

5.10 Traditional monetary policy now not as effective at controlling economy

Traditional monetary policy is becoming less effective at controlling the economy.

> ***Example***
>
> **The changing of short-term interest rates now seems to be less effective at controlling inflation. This may be due to increased use of derivative instruments by financial institutions, for example, using swaps or interest rate options to make their cost of borrowing more certain.**

Central banks may therefore attempt to control the economy in different ways, with unpredictable effects.

5.11 Globalisation of investment markets

With investment markets becoming more globalised, there is a wider range of assets in which the insurer may invest, which may not be denominated in the insurer's domestic currency. If the insurer has little experience in dealing in such assets, investment returns may be less certain.

Also, as mentioned above, if the assets and liabilities are mismatched by currency, then currency fluctuations lead to additional uncertainty.

With large volumes of foreign investment flowing across borders, the investment markets themselves may also be affected, and behave in unpredictable ways.

> ***Example***
>
> Suppose you had invested in an emerging market and were enjoying high expected returns. If there was a sudden increase in demand for investments in this emerging market, then it could affect asset prices and yields. It could also affect the exchange rate. Investor sentiment could then reduce (or further increase) demand at any time, hence reversing (increasing) this impact. Changes in investor sentiment may be hard to predict.

Globalisation of insurance markets will also increase the uncertainty for an insurer. This is discussed in the next section.

5.12 Competition

The level of competition can be affected by a number of factors. Two such factors are:

- globalisation of insurance markets
- changes / differences in regulation.

Any changes in the level of competition will have an uncertain effect on a general insurer's mix of business and claims experience.

Globalisation of insurance markets

Insurance markets are becoming far more globalised and insurers are more willing to write business originating outside their home territory. This can lead to increased competition between insurers, so similar business may not be as profitable as it has been in the past.

This can also have an uncertain effect on the insurance cycle.

Regulation arbitrage

Many financial companies will base their head office in a territory where the regulation is beneficial. This can distort competition within the insurance market (as there may be more or fewer companies competing in that market than would be the case if the regulation was the same everywhere).

6 Glossary items

Having studied this chapter, you should now ensure that you are able to explain the following Glossary items:

- Act of God
- Agents' balances
- Bonus-malus
- Model uncertainty
- Parameter uncertainty
- Process uncertainty
- Protected NCD.

Chapter 9 Summary

The premium charged by a general insurance company needs to cover the risks being accepted by the insurer.

The risks and uncertainties faced by a general insurer can be classified as:

- errors from systems and procedures
- model error
- random error
- adjustment factors
- market conditions.

Process error

Uncertainty relating to process error stems from:

- general claims uncertainty:
  - inherent uncertainty in individual claims (amount, frequency and timing)
  - demand surge
- internal sources, such as:
  - changes in business mix
  - booked reserves different to best estimate
  - uncertainty over expenses
  - new markets
  - new types of investment
- systematic sources, such as:
  - the economic environment
  - the insurance cycle.

Parameter error

Parameter error results in reserving uncertainty, and stems from:

- the data used
  - poor quality data
  - inconsistent data
  - incomplete and nonexistent data

- incorrect modelling assumptions, *eg*:
  - correlations in the model
- change in case estimate reserving philosophy
- particularly large / unusual risks:
  - large claims
  - catastrophes
  - latent claims
- inadequate data supplied by third party claims handlers
- format of data
- claims inflation not as expected
- incorrect distributional assumptions
- new distribution channels
- planned or unplanned changes in mix.

Model risk

Uncertainty in a models specification arises from:

- model error
- programming error
- simulation error / too few simulations.

Future systematic risk

The risks include:

- climate change
- new distribution channels
- increased use of profit share arrangements
- new claims handling procedures
- bodily injury claims
- differences in third party behaviour
- changes in claims inflation

- other changes in the external environment, *eg*:
  - legislation
  - broker mergers
  - aggregators
  - accounting changes
  - off-shoring
  - changes in tax rate
  - levies (and other charges)
- economic conditions
- monetary policy failing to control economic conditions
- the globalisation of investment markets
- competition factors, such as:
  - globalisation of the insurance markets
  - regulatory arbitrage.

The above risks do not act in isolation – some risks trigger others.

This page has been left blank so that you can keep the chapter summaries together for revision purposes.

Chapter 9 Solutions

Solution 9.1

Cash outgoes

Claims: Most general insurance claims are uncertain in both amount and timing, *eg* a motor insurance claim could happen at any time and the size of the claim will not be known in advance.

Expenses: Some expenses will be known contingent on another event, *eg* expenses of writing new business will be known at the point that the new business is sold; however, this will not be known in advance. Other expenses will be linked to claims, whose timing is unknown and whose amount will vary depending on the nature and size of the claim. Some other expenses may be one-offs (*eg* the replacement of a mainframe computer) and not necessarily known in advance.

Reinsurance premiums: If these are not on guaranteed terms, then the amounts of the reinsurance premiums will be uncertain. If certain risks are written on a facultative basis, then timings may also be uncertain.

Commissions: These depend on how much business is written, when it is written, and the distribution channel used, so both amounts and timings may be unknown.

Cash incomes

Reinsurance recoveries: These will depend on the amount and timing of the claims (which are not certain). They will also depend on the speed at which the reinsurer pays recoveries, and the risk of default of the reinsurer.

Other recoveries: These will also depend on the amount and timing of the claims as well as the speed at which cash is recovered from the third party.

Profit commissions: These will depend on the amount of profit made, which will be uncertain. The timing will be unknown to the extent that adequate profits to trigger the commissions may not be made in some years.

Investment income: Investment income for many assets is uncertain, *eg* dividend payments. Timings are uncertain to the extent that sometimes income may not be paid, *eg* rental defaults / voids.

Solution 9.2

Data errors from systems and procedures – This is defined as relating to claims.

Model error – Modelling could be used for any of claims, expenses (and commission), investment income, business volumes and lapses. (In fact modelling can be used for pretty much anything!)

Random error – This primarily relates to the number and amount of claims.

Adjustment factors – These will be needed whenever we are setting assumptions for future business. Assumptions may be needed for claims, expenses, investment income, business volumes and lapses.

Market conditions – Market conditions may affect:

- claim frequencies, *eg* from time to time, new claims will emerge, which will increase claim frequencies
- claim amounts, *eg* through inflation
- expenses, *eg* through inflation, regulation
- commission, *eg* through the power of brokers
- investment income, *eg* during a recession
- new business volumes and lapses, *eg* through levels of competition.

Portfolio movements – This certainly relates to new business volumes and lapses, since these will affect the volume and mix of business. However it may also be linked to:

- expenses, *eg* through per policy expense loadings
- commission, *eg* through broker deals.

Solution 9.3

Misinterpretation of policy wording leads to claims that the insurer had not intended to cover under the policy and hence claim frequencies will be higher than expected.

If the insurer is required to meet the claim payment, it may set a precedent for many other claims, further increasing claim frequency.

Even if the insurer does not pay the claim, it may be involved in lengthy and costly litigation – this will not affect claim frequencies, but will affect claim amounts and/or expenses.

Solution 9.4

For claims such as subsidence or industrial disease, it is very hard to pinpoint the exact date of the claim occurrence. In fact, it is likely that the claims arose over a period of time.

Solution 9.5

Examples include:

- a change in the propensity of individuals to report claims quickly – this may be a gradual change, *eg* due to longer working weeks (and so less free time to report claims)
- changes in processes of brokers regarding claims processing / reporting, for business sold through brokers.

Solution 9.6

(i) ***Strategies leading to a change in business mix***

- strategic change in target market
- change in distribution channels used
- change in marketing method
- change in pricing structure
- introduction of new rating structures
- change in underwriting processes
- change in claims handling procedures

(ii) ***Other reasons why the business mix might change***

- increase in anti-selection by policyholders
- change in the attitude to claiming
- change in fiscal regime, *eg* tax relief on certain groups of policyholders buying insurance
- change in regulatory regime, *eg* certain types of insurance becoming compulsory
- change in company reputation, *eg* a company becoming seen as a budget provider

Solution 9.7

- price inflation, which will affect the replacement costs of goods
- earnings inflation, which will affect repair costs and loss of earnings claims
- court inflation, which will affect claims that are settled in the courts

Solution 9.8

Marine, aviation and travel.

These are probably the most obvious but you could also mention product liability, commercial motor etc.

Solution 9.9

For long-tailed classes a considerable time will pass after the receipt of premiums before claim payments are made. Therefore allowing for investment income has a significant impact.

Solution 9.10

Data might also be nonexistent for:

- unusual risks
- new territories
- business that is not new, but has changed its terms and conditions to such an extent that past data cannot be used.

Solution 9.11

Higher interest rates tend to lead to lower levels of price inflation through:

- reduced demand-pull inflation, as borrowing becomes more expensive and saving more attractive, which is likely to reduce demand from consumers and firms
- reduced cost-push inflation, as higher interest rates are likely to increase the value of the domestic currency, making imports relatively cheap.

However, higher interest rates can also exert an upward pressure on price inflation. For example, companies will tend to pass on the higher cost of borrowing to their customers, *ie* cost-push inflation.

Other types of inflation, *eg* wage inflation and medical cost inflation, are often correlated with price inflation as they may also be affected by these factors.

Solution 9.12

Claim frequency

Third parties may not inform the insurer of claims until some time after the claim has occurred. Effectively, the insurer may hold a lower outstanding reported claims reserve and a higher IBNR reserve.

Claim amount

Third parties may not spend as long validating claims as an insurer might. For example, they may accept claim amounts submitted by the insured, without checking for reasonableness / getting several estimates of the claim size. This could lead to higher ultimate claims and thus higher premiums.

Claim delays

There are likely to be delays in passing claims information to the insurance company. These delays are likely to vary between different claims handlers.

Solution 9.13

Broker sales are primarily variable, *ie* the commission sold to the broker when the business is sold. There are unlikely to be many (fixed) setup costs for this channel.

Internet sales may have a high fixed cost, *ie* setting up and testing the website, but low variable costs.

Solution 9.14

Parameter error may be identified using sensitivity tests on individual parameters. Insight can be gained into the effect of varying different parameters, enabling the user to pinpoint the parameters that have the biggest effect. Model error may be misinterpreted as a combination of parameter errors, and hence is harder to detect.

Solution 9.15

If lawyers seek out people who have been affected by asbestosis, there may be a general increase in awareness and/or an increase in publicity over asbestosis cases. As a result, there may be an increase in awards made by courts over asbestosis claims.

Solution 9.16

India has a large pool of English-speaking people, who are technically proficient. It also met the criterion of “cheaper labour market”.

Solution 9.17

Expenses are likely to be linked primarily to wage inflation. To some extent they may also be related to price inflation.

Chapter 10

Data

Syllabus objective

(f) With regard to the use of data in pricing:

- *describe the types of data that are used*
- *describe the main uses of data*
- *describe the requirements for a good information system*
- *outline the possible causes of data errors*
- *identify checks that might be used*
- *understand the effects of inadequate data.*

0 Introduction

The availability of quality data is critical to the operation of an insurance company. Many of the functions of an insurance company have a clear and direct need for data.

***Quality* here refers to the extent of any errors or other problems arising from the data collection, grouping or final dataset used for the assessment of risks.**

Actuaries need data for two main purposes: premium rating and reserving. Premium rating is covered in this chapter. Reserving is covered in Subject ST7.

It is important to have data of sufficient quality and detail to form robust premium rates.

Reliable data is essential for the calculation of technical premium rates. However other factors, such as the stage of the insurance cycle, are also considered when determining the actual premium charged by the insurer.

In both pricing and reserving, policy and claim details covering many years will be needed. In trying to project future events, an actuary will need previous years' data in order to establish enough data for credibility and to identify trends and patterns. So the quality and quantity of reliable data is very important for the general insurance actuary. The analysis is pointless if the data is not reliable and relevant.

Whilst this is true for all actuarial investigations, the lack of adequate data seems to be more of a problem in general insurance than it is in other areas. There are probably two reasons for this. Firstly, actuaries are relative newcomers to general insurance, so there have been fewer years to establish appropriate data collection for actuarial applications. Secondly, the range and scope of the data needed is greater, in particular given the rapidly changing and competitive nature of general insurance.

Data of sufficient quality is also necessary for the uses described in Section 2.

The structure of this chapter is as follows:

- Section 1 describes the main sources of data. An insurer may have its own internal data. However it might also make use of external data, in particular data from industry-wide data collection schemes.
- Section 2 identifies the main uses and users of data. The exact use to which the data will be put is a key determinant of the level of data required. There may also be restrictions on use of data. There are a number of users of data. Our discussion focuses on the actuary.
- Section 3 considers the data available from an existing data system. Quality and quantity of data will be influenced by a number of factors, many of which may be outside the control of the insurer.
- Section 4 considers how to set up a data collection system from scratch. Since the data acquired under such a system may be used for many years to come, it is important that all relevant data is captured (*ie* relating to both premiums and claims) at an appropriate level of detail.
- Section 5 discusses errors, omissions and distortions that may arise in the claims data. Consideration is also given to ways of preventing such errors occurring.
- Section 6 identifies and discusses the specific data requirements for pricing. Note that while similar data will be needed for both reserving and pricing, each has its own specific requirements.

- Section 7 discusses the use of internal and external data for pricing. While internal data is usually preferable, there may be circumstances under which external data is the best option.

- Section 8 discusses why and how data may be grouped. Examples of the data used for some of the more common general insurance products are given.

- Section 9 discusses the situation in which data is limited.

- Section 10 concludes the chapter by describing the effect and consequences of using inadequate data for the purpose of pricing.

You should be aware that although parts of this chapter are similar to the data chapter in Subject ST7, there are significant differences. The main differences are from Section 6 onwards.

1 Sources of data

Companies may analyse either their own internal data or external data from industry sources.

Internal data is primarily the insurer's past experience of business sold.

Industry-wide (or industry-source) data is data collected and compiled by member offices of particular organisations, for example, the ABI (Association of British Insurers) **in the UK. The use of such data will increase the quantity, but not necessarily the quality, of the data available for analysis. Industry-wide data is discussed in further detail below.**

Other sources of external data exist, *eg* reinsurer's data and published accounts.

1.1 Internal data

> ***Question 10.1***
>
> Briefly, what will a company's internal data include?

The majority of this chapter focuses on the quality and quantity of internal data. Industry data is discussed below. This material may be familiar to you from other subjects, *eg* Subject CA1.

1.2 External data from industry sources

What are industry-wide data collection schemes?

In some countries, there are organisations that collect data from their member offices and then make available summaries of all the data to their members.

> ***Example***
>
> **In the UK, the ABI collects and collates a wide variety of insurance data.**

> ***Question 10.2***
>
> Why do such schemes exist?

Companies may also be required by regulators to publish data in a standard form. These can be collected to form an industry-wide database.

Examples

Examples of industry-wide data include catastrophe model datasets, UK flood maps, CRESTA zones, credit ratings and premium rates.

CRESTA stands for Catastrophe Risk Evaluating and Standardising Target Accumulations. The global CRESTA zone data is used to help assess risk relating to natural hazards, particularly earthquakes, storms and floods.

Potential benefits from using industry-wide schemes

An insurer participating in an industry-wide scheme can compare its own experience with that of the industry as a whole (or that part of it represented by the participating insurers) both at the overall level and at the level of the categories into which the data is classified. Any significant differences should be explained. Since an insurer usually seeks to expand by attracting business from its competitors, it may be important to understand the ways in which the characteristics of the business it is seeking may differ from those of the business it already has.

Industry-wide data provide a benchmark for insurers to assess their position compared to their competitors. Industry-based development factors may be valuable for reserving, especially for small insurers and insurers that have been established only a short time.

Basic problems with such schemes

There is potential for distortions within industry-wide data, particularly owing to heterogeneity.

Question 10.3

Before reading on, list as many reasons as you can why data might differ between different organisations.

Possible reasons for heterogeneity

The data supplied by different companies may not be precisely comparable because:

- **companies operate in different geographical or socio-economic sections of the market**
- **the policies sold by different companies are not identical**
- **the companies will have different practices; for example, underwriting, claim settlement and outstanding claim reserving policies**
- **the nature of the data stored by different companies will not always be the same**
- **the coding used for the risk factors may vary from company to company.**

This means that you must be very careful when interpreting industry-wide data. It may well not be relevant for your company.

Further problems

Other problems with using industry-wide data may be:

- **The data will be much less detailed and less flexible than those available internally,** hence it will be more difficult to manipulate.

- **External data is often much more out of date than internal data.** The data takes quite a while to collect, collate and then distribute to the insurers.

- **The data quality will depend on the quality of the data systems of all its contributors.** If one company makes a mistake (*eg* entering a figure in millions instead of thousands) then this invalidates the whole set of data. The more companies that contribute, the more likely that one will make a mistake!

- **Not all companies contribute.**

2 Uses and users of data

2.1 Uses of data

The main uses of policy and claims data by a general insurer are:

- **administration**
- **accounting**
- **statutory returns**
- **investment strategy and performance analysis**
- **financial control and management information**
- **risk management**
- **reserving (including unexpired risk assessment)**
- **experience statistics**
- **premium rating and product costing**
- **marketing**
- **capital modelling.**

Interactions and conflict between different uses

The interaction between these functions can be very complex and will vary from insurer to insurer. However, the underlying data requirements will normally be similar in each case. Ideally all these functions would be controlled through one single integrated data system.

This is not often achieved in practice, especially where insurers have developed through mergers of companies whose systems were not compatible with each other.

Question 10.4

Give reasons why, ideally, all functions should be controlled by "one single integrated data system".

High level data

For some purposes, data may only be required on a "big picture" basis. Here, data will be publicly available in published accounts and statutory returns. In some instances – for example, merger and acquisition work undertaken by a consulting firm – only public data may be available.

However, for the purpose of management information, the data will be needed in more detail so that strategic decisions can be made, such as deciding which classes and territories to expand.

Data on individual risks

Insurance organisations also need data relating to the individual risks for which they provide cover. The quantity and quality of these data are closely related. Without sufficient quantity, data groupings will either be non-homogeneous or lack credibility. However, even where there is plenty of data available, poor-quality data will not produce results that are reliable.

Data groupings are discussed further in Section 8.

2.2 Restrictions on the use of data

Customer information

Sufficient information will be needed to identify the policyholder of each policy and to provide basic details. However, customer information can be used for cross-selling, customer relationship management and the estimation of customer lifetime value, among other things.

To do these things it may be necessary to cross-reference different customer databases, for example, to determine which motor policyholders also have household policies. This may not always be straightforward, and if details are not recorded in exactly the same way, it may not be possible to establish these connections.

To do this, additional data may be needed that are not required for administration purposes.

Obtaining information on policyholders with different products should be made easier if all lines of business are administered on the same integrated data system.

Data should be used carefully in these ways. Data protection laws may limit the use of data. Contravening these laws can lead to criminal prosecution and extremely unfavourable publicity. Permission may need to be obtained for use of personal data when a policy is issued.

Data protection

The security of data is of paramount importance.

It may be important to both personal and commercial customers that data is protected.

It is subject to explicit law in many countries, usually to protect people's personal information. However, many commercial insureds provide data to insurance companies that is commercially sensitive, so any concern that an insurer's systems are not secure can be highly damaging to the insurer.

Data protection laws may cover what data a company may hold and for what it may be used. The laws may allow access to personal data by the subject. The subject may have a legal right to have incorrect data corrected. This will normally be to the benefit of the insurer, which will not want incorrect data on its system in any case.

Laws may also require that specified people are appointed to be responsible for certain aspects of data gathering, processing or use, or for the correctness of data held. Third parties, such as consultancies, should adhere to certain guidelines when using personal data provided by an insurer. Such guidelines include providing details of how the personal data will be used and destroying all personal data after use.

Insurers should have procedures in place to ensure that data is secure and is used only for appropriate purposes. These procedures will include limiting the use of systems to specified people, who need to identify themselves to the computer using passwords, and rules for the secure storage and transmission of data.

2.3 *Users of data*

Full development team

The full development team for a computer system should include representatives from the following departments:

- **senior management**
- **accounting**
- **underwriting**
- **claims**
- **marketing**
- **investment**
- **actuarial / statistical**
- **computing**
- **reinsurance.**

Question 10.5

What are the main uses of data for each of these potential users?

The different users of data within a general insurer will usually have different needs in terms of the quantity and quality of the data.

In developing a system, it should be noted that different departments will have different requirements.

Example

The marketing department may want data only at a very high level for their promotional material. The actuarial department, on the other hand, will want very detailed information when they undertake a rating review.

This may lead to conflict as some departments may be reluctant to bear the costs of a system that goes beyond their immediate needs and may be less attentive in supplying and supporting data which is not directly for their own purposes. Such conflicts could result in a system that does not contain data at the desired level of detail for actuarial analyses.

The role of the actuary

Actuaries are likely to be involved in the technical aspects of all the functions listed above (in Section 2.1). Ideally they should be involved in developing the information systems that support these areas.

The extent of their involvement in developing such systems will depend on the extent to which they are actively involved in each area.

Example

An actuary may be heavily involved in building and designing a database for storing a history of policy and claims information for the assessment of risks. On the other hand, actuaries may only be asked to provide verbal advice on developing a new information system for marketing purposes.

In general, the actuary must work with whatever data is available. A new insurer will not have its own historical data.

The appropriate response to data being of poor quality is discussed elsewhere in this chapter, which will depend on the nature of the task in hand.

Example

In the UK, an insurer should have kept at least enough data to be able to compile the statutory returns to the FSA (Financial Services Authority) **and its predecessors. There are similar requirements in some other countries.**

Note that the term FSA is included in the Glossary. It is the body responsible for, among other things, the supervision of insurance activity in the UK.

3 Quality and quantity of data in existing data systems

3.1 Factors that influence the quality and quantity of data

The availability of data of good quality and quantity will vary greatly:

- **between organisations**, which will depend on:
  - the size and age of the company
  - the current data system in use, including the use of legacy systems, the integrity of the data system(s)
  - the management and staff responsible for collecting and maintaining data
  - the nature of the organisation, *eg* direct insurer *vs* reinsurer
- **within organisations:**
  - depending on the distribution method of the business.
  - **between the different classes of business**.

These influences are discussed in more detail below.

3.2 Reasons for variation by organisation

Impact of size and age of company on data quantity

Large companies will have much more data available than smaller ones. They are likely to make more use of their own data, rather than rely largely on industry-wide data.

A newly established company may find that it has insufficient historical data for pricing purposes. It may need to supplement its historical data with data from industry sources.

Impact of size and age of company on data quality

The size of an organisation can also have a more direct influence on data quality, as a large company may have better data systems in place than a small one. However, a large well-established company may have an outdated computer system that is difficult to amend. A new small company may have a modern system that can be readily adapted to change.

Building a new data system is a major project. It is expensive and uses up considerable management time. The old system also has to be operated and maintained until the new one is ready and it may be difficult to transfer historical data from the old system to the new.

Section 4 below describes the information that would be kept if a new system were to be set up from scratch. In practice, the only insurers that can achieve this level of data system are new companies and those that are prepared to invest significantly to upgrade their data systems. In general, the older the insurer is, the more its data system may differ from what is ideal.

Larger companies, resulting from the merger or acquisition of other companies, can also experience legacy system difficulties. These arise from the integration of two or more data systems with different data items, and the data captured being structured in different ways.

Legacy systems

The ability to input, manage, extract and analyse data depends fundamentally on the quality of the systems. Many data analysts will be hampered in their research by legacy systems. These are systems which, for a variety of reasons, have not been updated or upgraded. Legacy systems may affect the whole company or may be limited to particular product lines (or even generations of a particular class). The problem is often at its most difficult in areas relating to portfolio transfer or company acquisition. Quite often there is an additional difficulty with systems incompatibility and communication.

As mentioned above, a common problem is that an insurer is formed from a merger of two or more insurers with their own systems. It is rare in such circumstances for it to be possible to transfer all historical data from one system onto the other. In general, we would prefer to use only the better system after the merger, but the other system is likely to lack some of the data items that the better system recorded. This is one of the things that is likely to have made it better. Usually a merged insurer will move to one of its legacy systems, but it is likely to retain the old business on the other system, and may keep renewals of the business on the other system, at least for a time.

In an insurance company, an actuary is likely to be able to influence the gathering of data and design of systems. This is especially true in a new company.

Considerations when designing systems are covered in Section 4.

However, while an actuary may be able to influence systems so they are more useful for actuarial work, it is usually not possible to enter historical data onto a new system if they were not previously collected and processed automatically. Therefore it may be some time before a new system becomes fully useful to the actuary. In the meantime, using two systems can be problematic.

We should allow for any approximations made as a result and may need to allocate more time to particular pieces of research involving data stored on legacy systems.

Integrity of systems

The quality of data depends on the integrity of the systems. In other words, checks are needed to ensure that all data is entered onto the system, entered once only, and entered correctly. Data should also be backed up regularly and securely, and have safeguards against accidental corruption. Some of these safeguards will be built into the system itself while others will be part of administrative procedures.

Checks that may be performed are discussed in Section 5.5.

Management and staff

Problems of data quality and quantity can result from poor management control of data recording or its verification processes, or from poor design of the data systems.

Management should make staff aware of the importance of maintaining good quality data records. If the administrators are not aware of this, the data input may be of a lower quality.

Management will have control over cost as well as design. Poor quality data systems may be due to cutting corners with the budget as well as bad design. The extent to which actuaries have been involved in the design of the system will also influence the quality of the data.

This may not necessarily be a reflection on the current management, as good-quality data cannot necessarily be obtained quickly. After implementing a process for maintaining extensive records, it may take many years to collect enough data for analysis purposes. This is especially true for long-tailed classes.

Reinsurer vs direct insurer

A reinsurer (especially one writing non-proportional reinsurance) will often receive aggregate (or bulk) data relating to all of its risks coming from a particular cedant. It is virtually impossible to check the accuracy to any great extent, without going through the expensive process of accessing the original records of the cedant.

For treaty reinsurance business, sometimes only grouped bordereau data may be available. Third parties, such as binding authorities or cedants may provide this information.

The Glossary defines the term *bordereau* as:

A detailed list of premiums, claims and other important statistics provided by ceding insurers to reinsurers, so that payments due under a reinsurance treaty can be calculated.

If grouped bordereau data is all that is available then the reinsurer may not be able to check and/or analyse its experience data to as high a level as it would like.

A further problem that reinsurers have regarding data is that the bulk data is usually provided by the cedant long after the claim occurred. Hence the quality of data held at any one time is poor when they try to assess future income and outgo.

There are two distinct problems for the reinsurer here. Firstly "bulk data" will not be very detailed and secondly the data may be out of date by the time the reinsurer obtains it.

In the case of excess of loss reinsurance, a cedant may fail to recognise that a claim may exceed the retention and may therefore fail to notify the reinsurer. Thus claims that have been reported to the cedant may remain IBNR for some time from the point of view of the reinsurer. In an attempt to reduce this problem, it is customary to ask cedants to notify the reinsurer of all claims that they think may exceed, say, three-quarters of the retention.

3.3 Reasons for variation by different distribution methods

Insurers distribute their business:

- **through brokers** (intermediaries)
- **through agents** (*eg* banks or building societies that sell a certain insurer's buildings and contents insurance)
- **directly to customers.**

The data needs will vary between the various distribution channels, which will not in any case be homogeneous.

Brokers and agents

Different brokers and agents will differ in:

- the role they play in the sales, administration and claims processes
- their level and form of remuneration
- the manner and speed with which they process policies / claims.

Role played in sales, administration and claims processes

Organisations that obtain large parts of their business through intermediaries, particularly where the intermediary is given responsibility for most of the administration, will have similar problems to those described in the section above on reinsurers.

> ***Example***
>
> **Some brokers will retain funds on behalf of insurers in order to pay straightforward claims while others will not be involved in claims administration. The former will provide claims details to be entered onto the insurer's systems while claims from the latter will be entered by the insurer's own staff.**

Even among brokers who have authority to administer claims, the levels of authority will vary between brokers.

> ***Question 10.6***
>
> Give an example of this sort of arrangement (with the intermediary or agent taking responsibility for most of the administration).

Remuneration

Different brokers will have different remuneration conditions. The system needs to identify them and calculate payments appropriately.

The remuneration conditions may also affect the willingness of the broker to provide good quality data to the insurer in a timely fashion. This may also depend on the state of the market.

Example

It may be more difficult to obtain data from brokers when the market is soft since brokers may be less incentivised. This is because they may receive lower commissions in comparison to when the market is hard.

The insurer may receive bordereau data in different formats and at varying levels of quality. The level of detail may also differ.

The brokers may also have requirements (*eg* on the level and form of data collected) that may impact the insurer's own data systems and processes.

Manner / speed of processing information

Brokers and agents may provide information on paper. The insurer's staff will need to enter this onto the systems or in a medium ready for computer entry or through the internet.

It may be time consuming for staff to enter such data into a computer system. It is quicker to integrate data received electronically into the system.

The system should provide appropriate management information to brokers to enable them to compile their own management information and the information they need to support their own clients.

Direct sales

For business sold direct to the policyholder, the insurer needs to enter all the data. This will be done by staff from telephone and in-branch sales, but internet sales may be processed directly into the system without human intervention.

3.4 Reasons for variation by different classes

Data can vary between classes for a number of different reasons, principally due to the different nature of risk, which leads to the following:

- big variations in claim frequency between classes
- the length of the tail of some classes means that it takes considerable time to collect the necessary claims data
- for some classes the underwriting is subjective, so it is difficult to store the details of the risk.

Claim frequencies

The more frequent the claims, the larger the available quantity of claims data.

Question 10.7

Give an example of a class of business where claim frequency tends to be very high.

Length of tail

There is a difference between the number of years of past data required for long- and short-tailed business. For longer-tailed classes of business, there may be significant delays, from the time that business is first written or the data system established, until there is sufficient data available to support an analysis of the business. This is particularly true of classes that are subject to significant delays in claim notification.

Subjectivity of underwriting

There is a difference in data quantity and quality for classes that use different forms of underwriting. Underwriting may be based on statistics or on judgement. The range and volume of data held will be greater for a class where underwriting has been largely based on statistics than for a class where the underwriting has been largely based on subjective judgement.

This does not mean that the underwriting process is necessarily haphazard. The underwriting of individual risks may be supported by excellent information, but the fact that the information will vary from risk to risk does not lend itself to systematic data capture.

Examples

Private motor business is generally rated by reference to a large number of underwriting variables, which must be held in the policy database.

Therefore, there will be more data available on private motor insurance business than for, say, marine insurance, which is more subjective. Also, for some classes (*eg* marine), the rating factors are qualitative, and therefore are more difficult to store on a computer. Industrial property is another class of business for which this may be true.

Industrial property business may be underwritten based on judgement on a policy-by-policy basis, so that large amounts of systematic underwriting information are not stored.

Problems can also arise if an insurer reorganises its class structure, and it cannot assemble historical data in the new classes. One consequence of a reorganisation of this type is that claim development triangles may comprise only the most recent diagonals and the interiors of the triangles are missing or available only in a different class structure.

4 Establishing a good information system

4.1 Introduction

The primary objective of the wider information systems is usually that the computer systems can be used effectively. For example:

- Data capture forms (*eg* proposal and claim forms) should be designed to provide precisely the information required for the computer, in the right order and with as little ambiguity or subjectivity as possible.

- Staff should be trained so that they know precisely how to use the computer and understand the importance of the data they enter onto it.

- Any new computer system should be run in parallel with existing systems for a trial period, until its reliability has been checked and optimised.

- Procedures should be set in place to monitor the performance of the systems regularly, and to improve them where necessary.

The following stages are required in the establishment of a good information system to ensure that good quality data is captured and stored:

- consideration of the users' requirements
- careful design of appropriate proposal and claim forms
- ensuring that features of premiums and claims can be recorded
- consideration of policy and claim information to be collected
- adequate training of staff.

4.2 Users' requirements

The first stage in establishing a good information system is to determine the priorities and requirements of the system for all potential users. Many of these will overlap because many of the items of data will be needed by various departments, but some may conflict, and the resultant system may have to be a compromise to some extent. The important points, however, are that the system should achieve the essential requirements, be compatible throughout the organisation and integrate the different functions within the organisation.

4.3 Policy and claim information

Information on individual risks is obtained primarily from the proposal form and the claim form.

Proposal form

The prime information source will be the details given on the proposal form. It is therefore important that it produces relevant and reliable information for the system. Questions should be well designed and unambiguous, so that the proposer will give the full correct information and the underwriting department can process the application readily, adding any coding that is necessary.

For example, storing date of birth is more useful than storing age.

However, an excessive number of questions on either the proposal form or the system may lead to poor quality data if the individuals answering the questions provide inaccurate responses in order to complete the form quickly.

This information should be held (together with any subsequent changes) so that it can be cross-checked against the claims information at the time of any claim, or for rating or accounting purposes. This should enable the automatic checking of the validity of the claim and the updating of the basic policy information (for example, the termination of cover in the event of a total loss). The cross-checking is likely to be achieved by the storage of claim reference numbers with the policy information.

When information is changed then a history of the previous information should be retained.

Question 10.8

Why?

In many policies, information is taken over the phone or through the internet. However, in these cases it is normal for the insurer to set the information out in a completed proposal form for the insured to sign as a legal contract.

Question 10.9

For a household insurance contract (covering both buildings and contents), list three pieces of information that will be required for all contracts, and six pieces of information that may only be required for some policies.

The precise information will vary from class to class and from contract to contract, since almost all policies have some optional elements. If the options are taken, details of the cover provided will be needed, such as a list of valuable items. However, all proposals will need the name and address of the proposer. Other items will vary, such as the sum insured, which may not be needed in some cases.

The sum insured may not be needed where the amount of cover that must be provided is dictated by legislation.

Example

In the UK, legislation requires unlimited cover for third party motor bodily injury cover.

Claim form

The main information source will be the details given on the claim form. Like the proposal form, it is important that this is designed with the aim of producing information that can be transferred easily to the computer system. The system should refer to the corresponding policy record and verify the existence of cover.

Again, the questions should be clear, unambiguous and objective.

4.4 Features of premium information

Premium information is necessary for reserving, pricing and monitoring. Hence it is important that the premiums received (and paid) are recorded accurately.

The features of premiums that should be recorded are:

- amounts
- timings
- adjustments to premiums, such as premium discounts and commission paid
- cross-selling information.

Written and signed amounts

Written premiums are the premiums an insurer expects to receive over the duration of the policy. The insurer should calculate the net written premiums by deducting any reinsurance premiums from the gross premiums.

In practice, the terms "written premium" and "signed premium" are used interchangeably and in fact the two figures are often the same. The difference comes when a risk is more than 100% placed. For example, let's say that a risk is 105% placed. Then the *written* premium is the total of all the insurers' combined premium, before the risk is signed down and the *signed* premium is the total after the risk is signed down. The insurer's share of the signed down amount is also sometimes called the *signed* premium.

Payment times

Premiums may be paid annually, monthly or at another frequency. The data system should record the date premium payments are due, the premium amount due and the date premiums are actually received. The system should be designed so that it is easy to check for any overdue premiums.

Premium adjustments

The data system should be flexible enough to allow for premium adjustments to be made.

Example

The system may have to record an adjustment to premiums because of endorsements, such as changes in policy limits or exclusions.

Policy limits may need to be changed on a contents insurance policy if the policyholder buys a new item of high value contents, *eg* a new television. Exclusions may need to be removed, for example if a household insurance policy previously did not cover the contents of outbuildings but the policyholder wanted to remove this exclusion.

Some endorsements may leave the premium unchanged. Others may result in an increase or decrease in the premium. The data system should be able to record the premium adjustment (and date).

If the policy is written under an NCD system, a discount may be applied to the renewal premium if the policyholder made zero claims in the previous year. The system should store both the original and discounted premiums for future reference.

The premium rating part of the system should also know (via a link) how many claims have been made in the previous year so that it can accurately calculate the renewal premium.

For treaty reinsurance business, the system should also record the dates and amounts of any reinstatement premiums (either due to be paid or already paid) for the treaty.

The system should similarly record any facultative insurance premiums with respect to individual policies.

Commissions

The system should record commissions paid to brokers and other intermediaries. The commission may be a fixed percentage of premiums. In this case, the system should record the correct percentage and update it when necessary for future reference. It should record details of the relevant broker or intermediary.

It should record ceding commissions and profit commissions payable by reinsurers to the ceding company.

Other deductions

It should record any other increases or reductions to premiums.

Example

Insurers may award premium discounts to policyholders who pay their premiums in a timely manner.

Cross-selling

Collecting information on cross-selling may help in seeing if a particular loss-leader strategy is working.

A loss leader is an insurance product sold at a low price (usually below the technical price so that it is sold at a loss) to generate profitable sales for another insurance product.

Cross-selling is the selling of similar insurance products or the selling of products that can be incorporated with the actual insurance cover being sold.

Example

Selling home contents insurance, as a loss leader, to existing buildings insurance customers (or to new customers applying for buildings insurance) may help increase the number of home contents insurance policies sold to new policyholders.

Loss leaders may also be sold with a view to establishing the insurer in a new territory or new product class.

The system should record cross-selling information for the loss-leader product, such as type of product, actual premium charged, technical price and number of policies sold, to assess the effectiveness of the sales strategy.

4.5 Features of claim information

General insurance claims may have a number of features that must be captured by a data system and the data collection process. These include:

- the definition of a claim
- the estimated outstanding claim amount, claims paid to date and type / cause of claim
- multiple claim payments
- reopened claims
- claims handling expenses
- reinsurance recoveries
- class-level adjustments.

Definition of a claim

Setting up a claim record

Insurers will have certain rules governing when a claim record should be set up, following an alleged loss by a policyholder.

Example

A claim record could be set up on the system when:

- **a request for a claim form is received**
- **the claim form is received back from a policyholder**
- **the claim form has been assessed by a member of the claims department staff who decides that it appears likely to be valid**
- **the claims manager concludes that there is sufficient information to set a reserve.**

Changes to this rule and/or office procedures could affect:

- **the numbers of claims recorded**
- **the numbers of nil claims (claims settled at no cost to the insurer).**

Any such variations could affect subsequent analyses of claims experience for the classes concerned. They will certainly affect the development of numbers of claims reported and closed, and may affect the development of incurred claims.

Example

If the insurer changed the definition of a nil claim mid-way through the year and we were analysing average claim amount and average frequency, the analysis would be distorted by this change in definition (assuming nil claims are included in the number of claims for frequency and amount calculations).

Closing a claim

Comparable variations can exist for the definition of the settlement of a claim. Some insurers go through their claims files periodically (for example, at each year end) and only then declare claims as settled. Others close the claim record as soon as it appears that the last payment has been made. Some insurers may even close a claim file when a payment will or may be paid at some relatively distant future date (for example, where a payment is due on attainment of majority by the recipient or when a provisional damage award has been made).

An *attainment of majority* is where a payment is made once the claimant reaches a certain age that is pre-specified by the courts.

An insurer may also close claims following a one-off review with respect to a particular book of business.

Estimated outstanding amounts

Insurers' attitudes towards recording estimated amounts outstanding on each open claim vary widely, as do the frequencies with which these are updated.

So there may be differences in:

- when the estimates are first set up
- the method used to determine the amount
- how often these amounts are revised.

We should allow for these facts in designing the information system in order to advise users what information to input and what the information held means.

Setting the initial estimate

Outstanding amounts may be estimated in different ways:

- **At one extreme, an initial estimate will be put on file when the claim is first notified. This may be updated whenever a payment is made or periodically, whether or not an intervening payment has been made. The revised estimate may be the earlier estimate written down by any payments made or a completely revised amount, based on the latest known facts of the case.**

- **At the other extreme, the insurer may not attach a case estimate to any one claim individually, relying instead on a total estimated value for the risk group, based on statistical methods. However, such insurers are very rare.**

- **A more common approach is to set a standard reserve for claims that are not large until they are settled or to set a reserve based on the claim's characteristics.**

Question 10.10

Why does an insurer bother with outstanding amounts? Wouldn't the analysis be more accurate if it waits until all claims are settled?

An insurer whose only reserves are bulk reserves for the risk group does not need to record individual outstanding claim estimates. In all other cases, the reserve amount should be recorded on the system and the date when it was set. To compile loss-development statistics, they should be retained, even when they are superseded by revised estimates. It is possible to compile development statistics by retaining only the latest reserve amount and adding a further diagonal to old triangles. In the past this was sometimes done partly because of the cost of data storage. For the same reason, only an inception-to-date paid amount might be kept. However, this precludes new investigations into historical data and is much less useful to the actuary. Because of low modern data-storage costs and increased capacity, this is usually no longer a difficulty.

Triangulation methods are discussed in Subject ST7.

If the reserves are set separately by type of payment (*eg* indemnity, compensation)**, the type should be recorded. The currency also needs to be recorded.**

Determining the amount to hold

Each company will have its own case estimate philosophy and guidelines for the claims assessor to follow. For example:

- **one company may require estimates to be as realistic as possible**
- **other companies may prefer to have an element of prudence in each estimate to avoid later deterioration in the overall reserve amount.**

Another approach, although uncommon, is to set reserves deliberately low in order to provide a challenging target for claims staff to settle claims.

In the London Market, policies are often coinsured by a number of different insurers. In these cases, the lead insurer will normally be responsible for handling the claim and will advise reserves to the following insurers. It is common for following insurers to use the reserve advised by the leader, though some insurers do alter the reserve.

Large claims are often managed by loss adjusters: specialist companies that manage claims on behalf of insurers. They will usually advise the insurer on the expected ultimate amount of the claim and most insurers will adopt the adjuster's advised reserve as their reserve for the claim.

Some insurers record expected amounts to be recovered from subrogation or recovery as negative reserves attached to individual claims.

Revising amounts

Reserves should be reviewed from time to time to ensure that they remain valid. A number of claims are advised but not pursued (by the claimant)**, and the insurer may not hear further from the claimant. In such instances it may be necessary to contact the claimant to assess the current status of the claim; otherwise these reserves may remain on the books indefinitely. However, there is often a reluctance to contact the claimant, as the act of reminding him or her of the claim can be costly to the insurer.**

Multiple claim payments

Some claims will involve two or more payments, made at different times. Further, the insurer may be able to recover part of the gross cost of the claims through salvage, subrogation or reinsurance. This is likely to occur some time after the original claim payments are made and the amounts are normally recorded as negative claim payments with a code to identify the type of receipt.

Separate details should be kept for each payment made or received under a particular claim, including details of the dates, amounts, currency and type of payment involved.

Different types of payments will include:

- **indemnity payments made to policyholders**
- **compensation payments made to third parties**
- **payments to claimants' solicitors**
- **payments to loss adjusters and payments of interest.**

This is not an exhaustive list. An equivalent list of amounts recovered from reinsurers will also be needed, and it may be necessary to distinguish between different types of reinsurance recovery.

It should be noted that payments on a single claim may be made in different currencies.

Example

A UK insurer might insure a ship for a European client, with the premium being set in euros. The indemnity amount might then be payable in the currency of the country where a claim occurred and the loss-adjusting costs in sterling if a UK loss adjuster were engaged.

Even where all a company's business is done in one country, claims in a foreign currency are often possible, so it is usually desirable to be able to record currency.

This will subsequently give more scope to analyse claims experience in detail, and to establish correct run-off patterns of the claim payments. It is important to keep the approach consistent over time if data is to be useful for actuarial analysis.

This is an important general point. If you hold more detail, you will be able to perform many different, detailed analyses on the data.

The problems of inadequate and incomplete data for pricing purposes are looked at in more detail in Section 10.

Reopened claims

Some claims that the insurer regarded as settled may have to be reopened at a later date for various reasons:

- **It may be purely due to the closure definition used by the insurer.**
- **It may be because a further liability for payment, possibly of costs rather than of indemnity, has come to light.**
- **The insurer has made some recovery against a third party involved.**
- **An error was made in closing the claim originally.**

It is important that the system does not regard these as new claims, as this would cause errors in recording of claim frequency and errors in allocation of the claim by year of origin.

This is particularly a problem if there is a sudden change. If the method of recording reopened claims has suddenly changed, the distortion will be more serious than if the allowance for reopened claims has been the same for many years.

A note should also be kept of the original date of closure.

> ***Question 10.11***
>
> Why?

Claims handling expenses

Some systems will record claim payments and claims handling expenses separately, whereas others will combine the two items.

Reinsurance recoveries

We should record reinsurance recoveries that have been received from reinsurers. In many cases, reinsurance recoveries can be linked to particular claims, for example, when the recoveries arise from proportional reinsurance or single risk XL. Recoveries from catastrophe XL are not usually in respect of an individual claim. The data system should still record these recoveries in some way.

It is normal for insurers to set up a reserve for recoverable reinsurance when a recovery is expected against a gross claim. However, there is always a delay between the payment of the claim and the recovery of the reinsured amount. If this can be significant, there may be a need to distinguish between reserves for recoveries on paid claims and reserves for recovery against gross reserves. When analysing data, it may be important to understand how these have been treated. The difference may be important for accounting purposes since recoveries on payments already made will be a debtor and recoveries on reserves will be an accounting provision.

When risk XL or catastrophe XL cover has been used, it may be useful to record paid or expected amounts of reinstatement premiums against individual claims.

Class-level adjustments

It may be necessary to make adjustments at a total class level such as adjustments for IBNR claims. We should allow for reserves for pure IBNR claims at the total class level. Similarly, we should adjust for profit commission at the total class level.

We should design the system so that these class-level adjustments can be entered and updated when required.

4.6 *Information to be recorded*

The data requirements for each policy or claim record include:

- **risk definition and details of cover**

  The risk definition will include the class and subclass of business, and, together with the details of cover, will identify the cover granted. This may consist of a number of items of information.

> ***Example***
>
> **In a household policy there is usually a range of optional covers, such as garden equipment, bicycles, accidental damage and specified valuables. There may be separate sums insured under some or all of these categories and they may have different excesses. All these should be recorded.**
>
> **Commercial policies may similarly have a menu of items that may or may not have been selected.**

- **details of claim (if it is a claim record)**

  The details of a claim will normally include a claim cause code.

- **status of present record**

  The status of a record is normally in-force / expired / cancelled for a policy and open / closed / reopened for a claim.

- **control dates (start and end dates of each record, dates of claims, and so on)**

- **relevant amounts and currencies (exposure, sums insured, premiums, claims payments, and so on)**

- **administrative details.**

  Administrative details may be in narrative form for the use of the administrative departments.

The history of policy and claim records should be held, ideally, indefinitely.

This, together with the frequency of changes of information, implies holding a vast amount of data. It may, therefore, be necessary to strike a balance between:

- **the capacity of the system**
- **the cost of data storage**
- **the amount of data stored**
- **the level of detail at which it is stored.**

5 Errors, omissions or distortions in claims data

5.1 Claim estimation methods

Claims take some time to be settled from when they are incurred. Some claims may be settled very quickly, some may take longer to settle and others may take longer still.

The following terminology is commonly used:

- **We refer to the claims incurred in the same period as a *cohort*. For a particular cohort, in theory, a graph can be plotted of accumulated claims settled (or reported) against time from the date of occurrence.**
- **We sometimes refer to the total accumulated claims (when all have been settled) of a cohort as the *ultimate claims*.**
- **We sometimes refer to the rate at which accumulated claims are settled** or reported **for the cohort as the *claims development*.**
- **We sometimes refer to the graph as the *claims development pattern*. In many of our reserving methods, we rely on the claims development pattern to be stable for different cohorts.**

The errors that we discuss in Section 5.2 can distort the claims development pattern and therefore the results of the valuation. These points are discussed in more detail in Subject ST7.

Whatever method is used to estimate ultimate claims, a large degree of detail is required. The results are, therefore, highly dependent on the quality of the claims data available:

- any significant claims data errors will be projected forward, distorting estimates
- errors in claims data will also distort development patterns used for projections.

These would lead to incorrect estimates of future liabilities and to incorrect premiums being charged, the consequences of which could be serious, as explained later in this chapter.

The same is also true of wrongly grouped data or unusual data (such as a £2 million motor liability claim), which may be correct but would still distort projections, unless taken into account.

5.2 *Sources of data error*

There are many potential sources of data error.

Some examples are given below:

Wrong claim number

Details of the claim could be allocated to the wrong record, and hence to the wrong claim risk group.

Wrong policy number

If the claim record picks up its risk details from the wrong policy record, these are likely to be wrong. Again, the claim details may be allocated to the wrong claim risk group, claim year and so on.

Wrong risk details

This could happen if the current in-force details are entered, rather than those as at the date of claim. For example, a claim might be registered on the file before changes in risk factors that occurred before the date of claim.

This might happen if the policyholder has changed their car or their address in the meantime. As stated above, we want to know the details when the claim happened, not now.

An error could also occur if the policy conditions have changed, but the new policy continues to be stored in the original rating group.

Wrong claim date

This could cause the claim details to be allocated to the wrong claim year, distorting both the apparent numbers of claims for those years and their development.

A wrong claim date could also mean that the claim will relate to the wrong risk details (if these have changed).

A common cause of this is entering the date when the claim was notified, rather than the original incident date, or vice versa.

Question 10.12

Why would both of these dates be required?

For liability claims and some property claims (for example, for subsidence), where a precise date of event cannot be determined, there need to be very clear rules and procedures for the allocation of such cases to a claim year.

Wrong payment dates

Some claims are settled by several payments, made on different dates. If these are not each identified separately, then development patterns may again be distorted.

Wrong claim type

Usually an analysis of the different claim types for a particular risk group will be required, since they behave differently (that is, they may have different claims distributions or be settled over longer or shorter periods). If the claim types are not identified correctly, this distorts the separate development patterns and average values.

So in an analysis of the motor account, for example, the insurer would separate property damage and bodily injury claims.

If the different claim types are not separated, a change in the mix of types of claim also distorts the development patterns and average values.

These problems may extend to cause of claim (for example, fire, theft, explosion and so on), as claims from different causes may also behave differently.

5.3 *Sources of data distortion*

There are other factors that, whilst not exactly errors, can still distort an analysis.

Some examples are given below.

Changes in claim handling procedures

Changing practices regarding the point at which a notified loss is formally accepted as a claim on the company and is marked as such on the claim file, may distort the claim characteristics for the risk groups affected.

Similarly, the failure to mark claim records as settled on a consistent basis may affect the apparent development of claim cohorts.

Case estimates

If these are not updated correctly over time, or as payments are made, the values will be unreliable.

Similarly, the system may fail to keep a historical record of the estimates (at each calendar year-end or quarter-end, say). This will inhibit the use of this information for statistical purposes.

A change to the basis for calculating case estimates – for example, a change from prudent estimation to realistic – may distort run-off patterns (when based on reported claims).

Processing delays

If the rate at which claims are processed alters through backlogs, changes in procedures and so on, this will distort the claim development patterns and hence the analysis of them.

This leads to particular problems when setting reserves.

Large claims

The presence, or indeed the absence, of unusually large claims is likely to distort any analysis unless a suitable adjustment is made. To make adjustments in the analysis, large claims should be identified separately.

For example, an adjustment might be made which truncates claims above a certain amount and spreads them across similar policies.

Large claims are often the subject of reinsurance recoveries. If the system does not recognise these automatically, the insurer may fail to claim recoveries that are due from reinsurers.

Return premiums

Return premiums can be recorded as a claim on occasion. This may depend on the accounting principles of the insurer. However, there is a significant danger that such practices will distort all manner of analyses.

Claims inflation

Inflation of claim payments may distort the monetary amounts being used in claims data analysis unless the raw data is adjusted or the estimation method can make a suitable allowance. If the unadjusted chain ladder method is used, the claims may not have to be adjusted for inflation, since the unadjusted chain ladder method is based on the assumption that future inflation will be similar to past inflation.

The different chain ladder methods should be familiar to you from Subject CT6 (and are described in detail in Subject ST7).

Question 10.13

List the possible sources of error in a claim analysis.

5.4 The consequences of erroneous claims data

The consequences of wrong data being entered include false accounting, inappropriate reserving, pricing wrongly, failure to make recoveries and general management mistakes.

The effect of inadequate data will be discussed further in Section 10.

5.5 Prevention of errors

Integrity of systems

The key here is the avoidance of errors.

Data input should be thoroughly screened. Data management in this regard will include check digits, data field integrity checks, mandatory fields and error reports (fatal and warning). It is important to maintain consistent practices over time and over different sectors of the business.

Ways of minimising input errors should be considered. This may include the use of check digits in file numbers, numeric minimum and maximum values, and so on.

Check digits

Policy numbers are often designed so that the last digit is a check digit: it is defined by a mathematical formula based on the other digits so that the wrong entering of a policy number is likely to result in the rejection of the transaction being processed rather than it being processed to the wrong policy. The check digit might be alphabetic rather than numeric to reduce to a minimum the probability of a wrong but valid number being entered.

The check digit could work by an algorithm that generates a final letter from the policy number. If the final letter of the policy number does not tie in with the check digit then the policy number has been input incorrectly.

Similar checks can be used for other data entered, such as agent numbers and postcodes.

A check could also be made that the postcode agrees with the address.

Minimum and maximum values

The maximum and minimum values could apply to the premium size, sum insured, policyholder's date of birth, street number *etc*.

These checks improve the overall data quality.

It will also be important to ensure that, although more than one department may be responsible for use of raw data, only one department at a time is allowed access for updating the information.

Culture and training

Senior management should emphasise to employees the importance of data systems. Managers should ensure that employees receive adequate training before handling any data and that further training is undertaken if any changes are made to the systems or processes. Monitoring of staff and/or random checks may also help to ensure that high standards are maintained.

6 Data requirements for pricing

6.1 The importance of data

Setting premiums on the basis of inadequate or inaccurate data is dangerous. It is important, therefore, for the insurer to consider carefully how it can make the best use of the available data.

Possible pricing errors

If the data is inadequate or inaccurate, false conclusions would be made, and premiums could be set incorrectly. Examples of pricing errors are:

- Premium rates too high. The company will lose market share through high lapse rates and attracting too little new business. The company would make losses if it did not sell enough business to recover its fixed costs.
- Premium rates too low. Significant losses could be made since a large volume of business might be sold on unprofitable rates.
- Wrong premium structure. This could result in selection against the company, leading to losses.
- Missed opportunities. By having inadequate data, the company might miss opportunities to make profits from particular niches of the market.

So we must have good data (*ie* accurate data and plenty of it) to be confident of conducting an appropriate product pricing exercise. Using sophisticated pricing techniques on poor data is next to useless.

Using data effectively

In setting premium rates, the aim is to estimate future claim experience from past claim experience. It is therefore important that the past data to be used is reliable, credible and relevant as a basis for estimates of the future.

Reliability is established by having a sufficiently large body of data to outweigh random variation in past experience. To ensure relevance, we need to identify all possible differences between past and future experience. Sources of difference may include:

- inflation of claim amounts
- changes in claim frequency
- differences in policy conditions
- differences in mix of business
- changes in underwriting standards.

As far as possible, problems caused by differences in mix of business should be eliminated by analysing the different risk groups separately.

Dealing with differences in policy conditions needs special care, especially if we are basing our analysis on external data.

6.2 Data required

Before carrying out any form of rating exercise, we must determine the information (both quantitative and qualitative) that is relevant and establish its availability. We will need both policy data and claims data and the means of relating them to each other in a consistent manner.

Policy data

We need these to calculate the exposure within each risk group. For direct insurance and facultative reinsurance, we need the following for each policy:

- **dates on cover**
- **policy limits and excess points (current and historic)**
- **company share of total risk (for London Market business)**

  The proportion of the total risk taken by a company is referred to as that company's *signed line*.
- **all other rating factor and exposure measure details – we should be careful to identify where the rating factors for an individual policy have changed over the period of an investigation**
- **details of premiums charged, unless they can be calculated by reference to the details on rating factors and exposure**

  The premiums charged are needed if we want to assess the profitability of previous premium rates.
- **type of coverage and details of any exclusions**
- **policy number to link to claims information.**

Question 10.14

Why should we be careful to identify where rating factors have changed?

For treaty reinsurance pricing, we need information in addition to the above, such as:

- **type of reinsurance; *eg* proportional and non-proportional**
- **basis of cover; *eg* claims made and losses occurring**
- **treaty limits and excess points**
- **details of aggregate limits and excess points, including number of reinstatements and reinstatement terms**
- **other treaty terms; *eg* hours clause, sunset clause and stability clause**
- **reinsurer's share of loss or limits; *eg* with respect to quota share treaties.**

Note that for a treaty, the exposure for the reinsurer will be a specified part of the underlying exposure. The data concerning rating factors may often be of poor quality if these details are not provided to the reinsurer. The exposure may also change, as the underlying policies expire and new underlying policies are written.

Claims data

This will include:

- **date of claim event**
- **whether the claim is open, closed or reopened**
- **date closed (if applicable)**
- **date reported**
- **dates and amounts of payments**
- **payment type; *eg* indemnity cost, lawyers' fees and adjustors' fees**
- **dates and estimates, if they exist, of amounts outstanding – these may include estimates of dates of settlement**
- **currency of payments and outstanding claim amounts**
- **rating factor details**
- **type of claim** (*eg* in motor, bodily injury or property damage)
- **type of peril** (*eg* in household, storm damage or burst pipes)
- **unique claim identifier**
- **policy number / code to link to policy information.**

As with the policy data, care must be taken where the rating factors for an individual policy have changed over the period of investigation. The details should be as at the date of the accident or event.

The data on the type of claim and the type of peril are useful if the insurer wants to get a better understanding of what exactly is causing changes to claims experience. Also, the peril data are necessary for costing policy options and changes to cover.

For example, we could assess by how much premiums would be reduced if we excluded theft claims from motor insurance cover.

However, peril data is often unreliable. The reason for this is that it is very difficult to convince staff and policyholders that this data is important and necessary. If those processing the data believe that the data is of marginal use, then the data is less likely to be reliably stored.

The insurer also needs some means of relating the claims to the correct policies via policy numbers and claims numbers. The use of check digits may help to ensure accurate recording.

Question 10.15

What are check digits?

We can split the above data into:

- **required data (claim reference number, date of loss, loss description and loss amount)**
- **useful data (open claim indicator, split of loss amounts between indemnity costs and legal expenses, and date reported [useful if cover is on an occurrence basis, required if on a claims-made basis], date settled, codified cause of loss, codified type of loss, transactional development history for individual losses).**

6.3 Database principles

We discussed the requirements for a good information system in Section 4.

To price the business, we should capture as much data as possible on the system since the pricing methodology used in future may change or may become more advanced.

The data system should ensure that all entries made are of a consistent format. The system could achieve this by placing error checks to highlight entries that are in an incorrect format or by not permitting entries with invalid formats to be entered into the system.

To trace back premiums to rating factors and to see how changing the parameters impacts the premium calculation, the system should store the different rating factors, the parameter values, the calculated model price and the underwriter's price.

Only authorised individuals should have permission, after consultation with other key team members, to alter rating factors or other stored data. We should design the database so that it gives warnings and confirmations before changes can be made to the rating factors or other key data. This may help prevent accidental changes from being made.

6.4 The importance of history

It is important that the system keeps an accurate history of all policy and claims information in order that we can make appropriate analyses and adjustments to rate the insured risks appropriately. Analyses may include:

- **an analysis of claims development in order to project claims to ultimate**
- **an experience rating** exercise to help set premiums
- **an investigation into changes in the mix of business within a class.**

We may make adjustments including incurred but not enough reported (IBNR) adjustments to historical data and adjustments for trends, for example, inflation.

We discuss such analyses and adjustments in the rating methodologies chapters (see Chapter 13)**.**

7 Use of internal and external data

7.1 Internal data

An insurer that has been writing a class of business for some years should have a bank of past experience from which to derive the base values.

An appropriate base period should be selected for which relevant data is available. The factors that will be considered when selecting a base period are:

- Volume: The period should be long enough to include sufficient volume of data to be credible. Whether this means looking at one year, five years or ten years will depend on the class of business (in particular, claim frequency and tail of class).
- Detail: For a class of business where the risks are very heterogeneous, a lot of data (and hence a longer period) will be required so that we have sufficient data for each of the different risk groups.
- Trends: The period should be long enough to indicate trends in claim frequencies and claim amounts.
- Relevance: The further back we go into the past, the greater the danger that the experience is less relevant for the future. Changes in policy conditions over the period are one obvious problem.
- Unknowns: To be most relevant to future experience, we ideally want the most recent claims experience. However, this is also the experience with greatest uncertainty because we will need to rely heavily upon estimates of outstanding claims. This can be a real nuisance, especially for IBNR within long-tailed classes.

Ideally, we will use the most recent year's experience, as this is likely to represent the current situation. However, we may have to adjust this for unsettled claims and IBNR claims. It may also lack credibility if used alone. As an alternative, we can use older, more complete years of experience, although this may be inappropriate if there have been significant changes in the risk, cover, types of claims, and so on. We will need to allow for past inflation.

In any event, we will need to examine the experience of each of the most recent years' data to detect any trends that there might be in the different elements of the experience.

7.2 External data

Where an insurer has insufficient or unsuitable internal data, we will need to use external data. These may take the form of aggregate market statistics.

We could also look at competitor's rates for a similar product.

Alternatively, we may obtain data from third parties, such as reinsurers or brokers. The reinsurer, which is providing reinsurance cover for the product, may be prepared to supply data and other information about the market.

For data obtained from external sources or from third parties, we should compare with the corresponding details for the policies the insurer intends to write, as far as possible:

- **the terms of policy cover offered**
- **levels of risk underwritten**
- **the loadings included for expenses and profit in the premium.**

The insurer should also be aware of socio-economic differences.

We should establish the time period of the data so that we can make an appropriate allowance for inflation. It will be difficult to obtain much of this detail for many products.

Internal data is generally more likely to be relevant than external data. However, external data is especially useful in some circumstances:

- for a company writing a new or modified class of business
- where the company's own data is sparse (*eg* for a detailed rating group analysis)
- to provide confirmation of results derived from internal data.

Question 10.16

List the reasons why accurate past claims data might not be suitable for premium rating.

8 Data grouping and reduction of heterogeneity

8.1 What is the aim of risk classification?

The insurer will analyse the in-force data for its classes of business at regular intervals. For the purpose of such analyses the insurer will want to ensure that there is sufficient data so that the results are credible, but not so much data that heterogeneity is a significant issue.

By risk classification, we mean that we separate the insured risks into different groups suitable for our investigation, bearing in mind that we will not distinguish between risks in the same group. The main consideration in data grouping (or risk classification) is to obtain homogeneous data. By reducing heterogeneity within the data for a group of risks, we make the experience in each group more stable and ensure that the risks within each group have similar characteristics, so that we can use the data more appropriately for projection purposes.

This is important when we monitor claims experience, review rating levels, and estimate outstanding claim values. Any heterogeneity in data groups will distort the results because the average risk within the group may change in the future. This may cause us to understate or overstate the reserves and hence understate or overstate the premiums.

For example, in motor insurance, property damage and bodily injury claims should be treated separately for reserving purposes. For pricing purposes they should also be treated separately in case there is a change in the mix of business, from fully comprehensive to third party cover.

8.2 You must consider the level of detail required very carefully

We should be careful when we classify the risks to ensure that there is sufficient detail for all users. Management will perform many analyses, which may require separation of departments or intermediaries in order to assess performance.

As part of the rating process, actuaries and underwriters will need to:

- **compare actual experience, by risk factor, with that included in the premium rates**
- **calculate and apply revised premium rates to each risk**
- **introduce new rating factors, from time to time.**

Because of the last point, we should collect and record data that may become useful for rating in the future, although not used for current rating factors. Until such data are available, we cannot test statistically their appropriateness for rating. There is also the danger that administrators will take insufficient care to keep accurate data that are not currently used in determining premiums.

This process of comparing actual to expected, reviewing premium rates and refining the rating structure is an example of the use of the actuarial control cycle.

Example

Many years ago the number of vehicle rating groups used in the UK for private motor insurance was increased from 7 to 20. Some insurers were only holding the rating group (not make and model) on their system. Consequently, they were at a disadvantage to those who were storing vehicle make and model, since they could not fully analyse their data to allow for the 20 new groups.

A general point worth noting is that the higher the level of detail that you are holding, the better. For example, it is easy to add together similar makes and models to get seven rating groups. It would be impossible to disaggregate the data from rating group to give the make and model.

8.3 Examples of ways in which claim data could be grouped

Below, we give some examples of how we may group the claims data. Many of the items listed are also risk or rating factors for the particular class.

Question 10.17

Before turning the page, suggest how claims data may be grouped for:

(a) personal motor insurance

(b) household insurance.

Example – personal motor

We could group personal motor claims under the following categories:

- **claim type; *eg* property damage, bodily injury, fire and theft, and windscreen**
- **size of claim; *eg* large and non-large**
- **past claims experience; *eg* number of claims in the past year**
- **no claims discount status**
- **policyholder age**
- **policyholder gender**
- **vehicle group**
- **vehicle age.**

Example – household

We could group household claims as follows:

- **claim type; *eg* storm, subsidence, flood, theft and fire**
- **size of claim**
- **property type; *eg* detached, terraced and house / flat**
- **number of bedrooms**
- **location of property**
- **age of property**
- **past claims experience; *eg* number of claims in the past year**
- **sum insured.**

Other considerations

Personal lines classes tend to have standard policies and coverage. Hence it is usually easier to group the claims into homogeneous groups, whilst ensuring sufficient volumes of claims within each group.

Commercial lines business may include risks with tailor-made policies and coverage. In comparison to personal lines business, it may be more difficult to group claims into homogenous groups whilst ensuring sufficient claims volumes.

9 Pricing with limited data

For some classes, where there is little internal data available (for example, new lines of business or high layer excess of loss reinsurance) specific techniques have to be used.

9.1 Use of other data

If the insurer's own data contain an insufficient volume of claims, we may consider using its data from similar lines of business or data from third parties. Third-party data may include databases or publicly available data curves. We could adjust historical data – for example, for claims inflation – and use them for rating purposes in order to increase the volume of claims.

In summary, other data may include:

- own data for a similar line of business
- historical data (internal or external) – adjusted
- external data from third parties.

9.2 Margins

One way of dealing with less than ideal data is to apply loadings to outputs from the rating model. Alternatively, we could use assumptions that are more conservative than usual, or we could apply larger loadings in the calculation of technical premium rates.

However, this may not be possible in a very competitive market.

9.3 Use of ILFs

If the low claims volume is due to the insurer writing higher-layer business, we could apply increased limit factors (ILFs) or first loss curves to the premium rates calculated at a lower layer with credible experience to estimate the higher-layer premiums.

Increased limit factors and first loss curves will be explained in Chapter 15.

9.4 Qualitative methods

For some types of business, we cannot determine appropriate premiums using quantitative (numerical) data only, so we use a mixture of qualitative and quantitative information.

We may use this approach when quantitative data are:

- **incomplete (for example, because some data are held by a third party administrator)**
- **inaccurate (for example, if risk codes or classifications have not been correctly assigned)**
- **sparse (that is, only a few claims available for analysis)**
- **where risk perception plays an important element in price determination.**

9.5 Other methods

We use alternative methods of pricing for some types of business to deal with the above issues. Below are some examples of situations where we use more unusual methods of pricing:

- London Market business, which can often comprise unique risks with limited historical data.

We may need to perform pricing for London Market business quickly since underwriters may need to accept and decline risks in a short timeframe. We use the underwriter's judgement for assessing all risks, especially the larger complex risks. The analyses performed by actuaries are also considered. An actuary analyses larger risks individually using a mixture of experience and exposure rating. Actuaries are likely to assess smaller risks using an exposure rating model only.

Exposure rating is a method of calculating a premium that is based on external data or benchmarks. The premium of each individual insured does not depend on the actual claims experience of that insured, but on the amount of exposure that the insured brings to the insurer. Experience for comparable risks is then used to calculate a premium rate.

- Lines of business with more complex risk structures, such as extended warranty and creditor insurance.

We may perform pricing by pooling risks and charging similar premium rates to a wide range of policyholders.

- Reinsurance contracts

There are usually less data available for pricing reinsurance than there are for pricing direct business. For example, there could be limited information with respect to the frequency and severity of extreme events. There may also be issues regarding the credibility of models, especially since models may be recalibrated after the occurrence of losses from new events.

Methods used for pricing reinsurance are discussed in Chapter 20.

10 The effect of inadequate data on pricing

When pricing, it is important that we monitor the progress of existing experience as it develops, in order to assess the need for a review.

This again refers to the actuarial control cycle.

Thus, one effect of inadequate data is that we might make a wrong decision on the rate to be charged.

When we carry out the actual projections of the new rating requirements, inadequate data may distort the calculations. This may be due to errors in:

- **the apparent size of the business in force, and its value expressed in exposure units and premium**
- **the apparent claims experience and its trends, on which the projected future costs are being based.**

Moreover, the errors may distort the true distribution of the business between risk groups. This could have consequences if we decided to adopt a differential rating increase for each risk group. It could also affect the marketing strategy if certain risk groups appeared to be more attractive risks than they actually were.

If we adopt a deficient set of rates as a result of faulty data, the insurer might:

- **suffer underwriting losses if rates are too low**
- **suffer loss of market share if rates are too high**
- **attract undesirable risks, causing deterioration in underwriting experience if rates for such risks are too low.**

11 Finally

11.1 Glossary items

Having studied this chapter, you should now read the following Glossary items:

- Bordereau
- Claim cohort
- Credibility
- CRESTA zones
- Exposure rating
- Product costing
- Risk group.

11.2 End of Part 2

You have now completed Part 2 of the Subject ST8 Notes.

Review

Before looking at the Question and Answer Bank we recommend that you briefly review the key areas of Part 2, or maybe re-read the summaries at the end of Chapters 7 to 10.

Question and Answer Bank

You should now be able to answer the questions in Part 2 of the Question and Answer Bank. We recommend that you work through several of these questions now and save the remainder for use as part of your revision.

Assignments

On completing this part, you should be able to attempt the questions in Assignment X2.

Reminder

If you have not yet booked a tutorial, then maybe now is the time to do so.

Chapter 10 Summary

Sources of data

General insurance companies may make use of both internal and external data sources.

Industry-wide data collection schemes allow insurers to compare their own experience with industry experience. Relying on industry-wide data, when available, has several problems with its use, for example: lack of detail and flexibility, differences in policies sold, different target markets and sales methods.

Uses and users of data

Availability of a large quantity of good quality data is critical to a general insurance company. It is also critical for actuaries working in general insurance. Actuaries need data mainly to help make premium rating and reserving decisions.

Ideally, all data in a general insurance company should be controlled through one single integrated data system, although in practice this may not be possible. The actuary should be involved in technical aspects of data collection and systems design.

Customer information will be used primarily to identify policyholders. However it may also be used for cross-selling, relationship management, *etc*. Data protection is of paramount importance regarding both personal and commercial customers.

The full development team for a computer system should include senior management, accountants, underwriters, claims managers, marketing, investment, computing staff and reinsurers, as well as actuaries.

Quality and quantity of data

The quality and quantity of data varies between different insurance organisations, and, within different organisations, between different classes of business. The key factors affecting the quality and quantity of data are:

- size and age of company
- existence of legacy systems
- integrity of data systems
- management and staff
- nature of the organisation
- method of sale
- class of business.

Establishing a good information system

The system should hold a large amount of detail from the policy and claim records. Questions on the proposal and claim forms should be clear, unambiguous and objective. It is necessary to capture key features relating to both:

- premiums (including written amounts, payment times, premium adjustments, commission, other deductions and cross-selling information)
- claims (including the definition of a claim, estimated outstanding amounts, multiple claim payments, reopened claims, claims handling expenses, reinsurance recoveries and class-level adjustments).

The data to be recorded includes:

- risk definition and details of cover
- details of claim
- status of record
- control dates
- relevant amounts and currencies (sums insured, premiums, *etc*)
- administrative details.

A history of records should be held (ideally) indefinitely. A balance will need to be struck between the amount (and level of detail) of data held and the cost of the data system.

Errors, omissions and distortions in claims data

There are a large number of potential errors and distortions that must be overcome to allow accurate analysis of the data. These include: wrong policy or claim number, wrong risk details, wrong dates, wrong claim types, claims inflation, changes in procedures, case estimates, delays, large claims and return premiums.

Errors can be avoided through appropriate use of check digits, data field integrity checks (*eg* minimum and maximum values), mandatory fields, error reports and the training of staff.

Data requirements for pricing

To price policies appropriately the company needs data that is reliable and relevant. Policy and claim data is needed, along with additional data to allow adjustments and loadings to be decided. Treaty reinsurance pricing will require additional information to that needed for direct insurance and facultative reinsurance.

As much data as possible should be captured on the system since the pricing methodology used may change in the future.

Internal and external data

In order to be ideal, internal data should be credible, relevant and in appropriate detail. The latest 3-5 years worth of data may be used in order to achieve these requirements and so that trends can be ascertained.

External data (*eg* from reinsurers) should be used to supplement internal data (or may be used if the insurer has no internal data of its own). The insurer will usually need to make adjustments to such external data to make it relevant.

Where data is limited, other rating techniques must be used, *eg* using ILFs.

Data grouping

Data should be grouped into homogeneous cells, but consideration should also be given to ensuring that there is enough data in each cell for it to be credible enough for all users.

The effect of inadequate data on pricing

If the premium rates calculated from the data are incorrect, this could lead to unprofitable or uncompetitive rates, or anti-selection.

This page has been left blank so that you can keep the chapter summaries together for revision purposes.

Chapter 10 Solutions

Solution 10.1

Internal data will include:

- policy information collected from the proposal form
- premium information
- claims information.

Solution 10.2

Managers of insurance companies use the data to confirm or refute suspicions from their own data. Also, anybody managing any business should be aware of what is going on in the market place.

Solution 10.3

- different geographical or socio-economic sectors
- different target markets or sales methods
- different cover, exclusions, sizes of excess, policy conditions
- different underwriting and claim settlement practices, reserving methods
- differences in the nature and quality of data stored by different companies.

Solution 10.4

If data is controlled by one single, integrated system then:

- there is reduced chance of existing data being corrupted
- there is reduced chance of inconsistent treatment of information, between products or over time
- there is likely to be a better level of control over those who may enter information or amend information
- information will be easier to access, *eg* it will not involve collating information from several systems
- time will not need to be spent reconciling data from different systems.

Solution 10.5

Senior Management: making business decisions

Accounting: collecting premiums; paying intermediaries, claimants *etc,* preparing summaries

Underwriting: premium rating, identifying improvements, evidence of selection, portfolio monitoring

Claims: processing and settling claims

Marketing: assessing marketing performance and identifying opportunities

Investment: monitoring investment performance and opportunities

Actuarial: premium rating; reserving; assessing solvency and capital requirements and investment and reinsurance strategies; management information

Computing: writing and implementing the IT system

Reinsurance: monitoring reinsurance performance and adequacy

Solution 10.6

A large block of policies, such as mortgage indemnity business, sold through a building society, or travel policies sold through a travel agent. (In each case the insurer may only receive summarised data.)

Note that in this case the building society is only likely to do the policy administration. Since the building society is the actual recipient of claim payments, the insurer would do the claims administration.

Solution 10.7

Motor insurance claims tend to be fairly frequent. 25% on private fully comprehensive policies would not be unusual, and up to 40% for motor fleet.

Solution 10.8

It is necessary to keep a history of policy and claim records so that accurate experience analyses can be carried out. In particular, it is necessary to ensure that claims and exposure data correspond.

Solution 10.9

Information that would be required includes:

- policyholder name
- policyholder address
- some indication of the type of property, *eg* house or flat
- some indication of the size of the house, *eg* estimated rebuild cost or number of bedrooms
- the level of excess required.

Examples of information that may also be needed:

- estimated sum insured for contents
- list of valuable items to be covered
- whether the property is used for business
- whether the property is normally unoccupied during the day
- type and standard of construction
- age of the building
- types of locks / burglar alarms / smoke alarms fitted
- whether there are pets at the house
- family composition of residents
- age of residents
- whether residents are smokers / non-smokers
- type of heating
- whether claims have been made in recent years.

Solution 10.10

If it waited until all claims were settled then the data would be out of date, in particular for long-tailed classes.

Solution 10.11

To allow the insurer to analyse the percentage of claims that will be reopened and also to calculate the average time from closure to reopening.

Solution 10.12

To allow for analysis of reporting delays.

Solution 10.13

- wrong claim number
- wrong policy number
- wrong risk details
- wrong claim date
- changes in claim handling procedures
- wrong payment dates
- wrong claim type
- case estimates
- processing delays
- large claims
- claims inflation.

Solution 10.14

The aim is to calculate the exposure within each risk group. When the rating factors change, the policy exposure will be in respect of a different risk group. The correct exposure for each risk group is required to ensure correspondence between the claims and exposure.

Solution 10.15

The final digit of a policy number is often a check digit. It is defined by a mathematical formula based on the other digits so that the wrong entering of a policy number is likely to result in the rejection of the transaction being processed rather than it being processed to the wrong policy. The check digit might be alphabetic rather than numeric to reduce to a minimum the probability of a wrong but valid number being entered.

Solution 10.16

Accurate past claims data might not be appropriate for premium rating if future conditions are expected to be different from those in the past. Examples of possible differences are:

- past data is not from the class(es) we now wish to set premiums for
- we wish to change the premium structure and the past data is not credible enough to use
- the volume of past data is inadequate
- there have been sudden changes in the court treatment of certain types of claim
- policy conditions have changed.

Past data would be useful for some of these, but would need adjustment before it would be suitable.

Solution 10.17

See the Core Reading that follows the question.

Chapter 11

Aggregate claim distribution models

Syllabus objectives

(h) *(i)* *Describe the individual risk model and its applications in a general insurance environment.*

(ii) *Describe the collective risk model and its applications in a general insurance environment.*

(iii) *Understand the derivation of the aggregate claim distribution for the collective risk model, and its approximations using stochastic simulation.*

0 Introduction

This chapter extends the ideas relating to aggregate claim distributions. You may be able to remember some of these ideas from your study of compound distributions in Subject CT6. For example, you may remember questions there that involved the compound Poisson distribution.

We start this chapter by reminding you about the basic structure of the collective risk model and the individual risk model, and give you some standard results relating to these models, with which you should already be familiar.

We then look at various methods for calculating probabilities associated with the collective risk model. The first method is a recursive formula, which allows us to calculate the probabilities exactly. The other two methods are both approximations. The first of these assumes that the distribution of the total claim amount S is normal, and therefore symmetric. The second assumes that S has a translated gamma distribution, and is therefore positively skewed.

Finally, we look at methods for simulating values from compound distributions, and discuss the likely level of error involved in using a simulation approach.

1 The individual risk model

Under this model, we consider a portfolio consisting of a fixed number of risks. This model has been covered in detail in Subject CT6. Here we give a high-level overview of the assumptions underlying the individual risk model.

We will assume that:

- **risks are independent**
- **claim amounts for risks are not (necessarily) identically distributed random variables**
- **the number of risks does not change over time.**

Under the individual risk model the total claim amount S payable during a specified period in respect of a block of policies is:

$$S = X_1 + X_2 + \cdots + X_n$$

where X_i is the claim amount payable during the period in respect of risk i and n is the number of risks.

It is possible (and hopefully likely) that many of the risks will not give rise to claims at all. So many of the X_i s may be zero.

In the individual risk model, we consider separately the claims arising from each individual risk that makes up the portfolio.

For each risk the following assumptions are made:

- **the number of claims from the i th risk, N_i, is either 0 or 1**
- **the probability of a claim from the i th risk is q_i.**

It follows that each X_i has a compound binomial distribution.

If a claim occurs under the i th risk, the claim amount is denoted by the random variable X_i. For all risks, j, that do not have a claim, $X_j = 0$.

The individual risk model is very restrictive since a maximum of one claim from each risk is allowed for in the model. Most general insurance policies do not have a restriction on the number of claims that could be made in a policy year.

2 The collective risk model

This was also covered in Subject CT6.

Under the collective risk model the total claim amount S payable during a specified period in respect of a block of policies is:

$$S = X_1 + X_2 + \cdots + X_N$$

where X_i is the claim amount payable during the period in respect of the i th claim and N is the (random) number of claims during the period.

Note that the subscripts i now relate to claims, rather than risks as they did for the individual risk model.

The model assumes that:

- **the claim amounts X_i are independent and identically distributed**
- **the X_i s and N are independent of each other.**

Note three important differences between this model and the individual risk model

(1) The number of risks in the portfolio was specified in the individual risk model. In the collective risk model, there is no need to specify this number, nor to assume that it remains fixed over the period of insurance cover.

(2) The number of claims from each individual risk was restricted in the individual risk model. There is no such restriction in the collective risk model.

(3) It was assumed that individual risks were independent in the individual risk model. In the collective risk model it is individual claim amounts that are independent.

2.1 The mean and variance of S

To calculate the mean and variance of S, we use conditional expectation results, conditional on the number of claims, N.

The mean

To find $E(S)$, apply the identity:

$$E(S) = E\left[E(S \mid N)\right] = E\left[NE(X)\right] = E(N)E(X)$$

That is, the expected aggregate claim amount is the product of the expected number of claims and the expected individual claim amount.

The variance

To find $\text{Var}(S)$, apply the identity:

$$\text{Var}(S) = E\left[\text{Var}(S \mid N)\right] + Var\left[E(S \mid N)\right]$$

We can find $\text{Var}(S \mid N)$ by using the fact that individual claim amounts are independent:

$$\text{Var}(S \mid N) = N\text{Var}(X)$$

Hence:

$$\text{Var}(S) = E(N)\text{Var}(X) + \left[E(X)\right]^2 \text{Var}(N)$$

2.2 Moment generating function (MGF)

We find the MGF of S using conditional expectations. By definition, $M_S(t) = E\left[e^{tS}\right]$, so:

$$M_S(t) = E\left[E(e^{tS} \mid N)\right]$$

This equals:

$$E[\exp(t\sum_{i=1}^{N} X_i)] = E[\prod_{i=1}^{N} \exp(tX_i)]$$

which equals $\prod_{i=1}^{N} E\left[e^{tX_i}\right]$ for all i, since X_i are independent random variables.

Also, since X_i are identically distributed, they have a common MGF, $M_X(t)$, so that:

$$\prod E\left[e^{tX_i}\right] = \prod M_X(t) = [M_X(t)]^N$$

Hence:

$$E\left[e^{tS} \mid N\right] = [M_X(t)]^N$$

and

$$M_S(t) = E\left[[M_X(t)]^N\right] = E\left[e^{N\log M_X(t)}\right] = M_N[\log M_X(t)].$$

The middle equality above holds because, for any real number a, we can write:

$$a^N = \exp(N\log(a)) \qquad (2.1)$$

To see that this is true, just takes logs of both sides of equation (2.1)!

Thus, we can express the MGF of S in terms of the MGFs of N and of X_i.

3 Exact calculation of G(x) for the collective risk model

3.1 Introduction

In the remainder of this chapter, we consider calculations of and approximations to $G(x)$, the distribution function of aggregate claims for the collective risk model.

This section first considers the exact calculation. The following two sections will consider the approximations.

Recall from Subject CT6 that the aggregate claim amount random variable is:

$$S = X_1 + X_2 + \cdots + X_N$$

where N is the claim number random variable and X_i denotes the amount of the ith claim. The distribution function of S is:

$$G(x) = P(S \le x).$$

It is possible in some cases to find the distribution function $G(x)$ very simply, for example if all claims are for the same amount. However, the assumptions made in these cases will usually be too restrictive for them to be of practical interest.

Question 11.1

In a particular portfolio, all claims are for a fixed amount of £10,000. There are 100 policies, and the probability of a claim arising on any policy is 0.01, independently of the other policies. What is the value of $G(20{,}000)$?

In Section 3.2 we derive a recursion formula for the calculation of $G(x)$. In Section 4 we approximate $G(x)$ by a normal distribution fitted by moments. In Section 5 we approximate $G(x)$ by a (translated) gamma distribution fitted by moments. Finally, in Section 6, we approximate $G(x)$ using Monte Carlo simulation.

In Section 3.2 and in Section 6, we assume that the distributions of claim numbers and of claim amounts are known. In Sections 4 and 5 we assume that only the first two or three moments of these distributions are known.

3.2 A recursion formula for G(x)

Throughout this subsection we assume that the distribution of individual claim amounts, $F(x)$, is a discrete distribution on the positive integers. This means that the possible values for an individual claim are 1, 2, 3, ... , and hence that the possible values for the aggregate claims are 0, 1, 2, 3, ...

This method works best when there are only a small number of possibilities for the individual claim sizes. These claim sizes may well be measured in units of say £5,000, so that claims of 1, 2, 3,… actually correspond to £5,000, £10,000, £15,000, and so on.

On this assumption, the distribution function for individual claim amounts does not have a density function (because each X_i is a discrete random variable).

Discrete random variables have probability functions instead of density functions. An example of a discrete random variable and its associated probability function is given below.

x	100	200	500	1,000
$P(X=x)$	0.1	0.3	0.4	0.2

Recall that the probabilities must sum to 1.

We will use the following notation for the probability functions for individual claim amounts and aggregate claim amounts, respectively:

$$f_k = P(X_i = k) \qquad k = 1, 2, 3, \ldots$$

$$g_k = P(S = k) \qquad k = 0, 1, 2, \ldots$$

If, as may well be the case, the distribution of individual claim amounts is *not* discrete on the positive integers, we can always approximate it by a distribution that is.

We would do this by computer, dividing the area under the continuous distribution into strips centred on the values 1, 2, 3, and so on. The areas of each strip become the corresponding probabilities in the discrete distribution.

We say that the distribution has been *discretised*.

The problem of calculating $G(x)$ on the above assumptions is equivalent to calculating g_k for $k \le x$. We assume that:

- **the distribution of claim numbers is known**
- **the distribution of individual claim amounts (that is the f_k s) is known.**

Before proving, or even stating, this formula we need an assumption concerning the distribution of N, the number of claims. Let us denote $P(N = r)$ by p_r and assume there are numbers a and b such that:

$$p_r = \left(a + \frac{b}{r}\right) p_{r-1} \quad \text{for } r = 1, 2, 3, \ldots \tag{3.1}$$

The binomial, Poisson and negative binomial distributions all satisfy assumption (3.1).

You might think that this is a rather arbitrary requirement to impose, but in fact any discrete distribution defined on 0, 1, 2, … that satisfies assumption (3.1) must be Poisson, binomial or negative binomial.

Question 11.2

Show by considering p_r / p_{r-1} for a $Poisson(\lambda)$ distribution that the Poisson distribution satisfies equation (3.1), and determine the values of a and b.

The formulae for g_r are:

$$g_0 = p_0 \tag{3.2}$$

$$g_r = \sum_{j=1}^{r} (a + bj / r) f_j \, g_{r-j} \quad \text{for } r = 1, 2, \ldots \tag{3.3}$$

We know this result as Panjer's recursion formula.

These formulae are given on page 17 of the *Tables* and we will prove these results in this chapter. They can also be written in more familiar notation as follows:

(3.2)$\Rightarrow$ $\quad P(S = 0) = P(N = 0)$

(3.3)$\Rightarrow$ $\quad P(S = s) = \sum_{x=1}^{s} \left(a + \frac{bx}{s}\right) P(X = x) \, P(S = s - x)$ for $s = 1, 2, \ldots$

Formula (3.2) follows from the fact that the minimum claim size is one. The aggregate claims will be zero if and only if no claims occur.

To prove formula (3.3), we will use the following three formulae: for *n* = 2, 3, ...

$$E\left[X_1 \mid \sum_{i=1}^{n} X_i = r\right] = r/n \tag{3.4}$$

$$E\left[X_1 \mid \sum_{i=1}^{n} X_i = r\right] = \sum_{j=1}^{r} j\, f_j\, f_{r-j}^{(n-1)*} / f_r^{n*} \tag{3.5}$$

$$p_n\, f_r^{n*} = \sum_{j=1}^{r-1} (a + bj/r)\, f_j\, p_{n-1}\, f_{r-j}^{(n-1)*} \tag{3.6}$$

Before you worry about where these equations come from, note that f_r^{n*} is just shorthand notation for the probability:

$$f_r^{n*} = P\left(X_1 + X_2 + \cdots X_n = r\right).$$

Both (3.4) and (3.5) hold for any value of r for which f_r^{n*} is not zero; (3.6) holds for $r = 1, 2, \ldots$, whether or not f_r^{n*} is zero.

Note that $P\left(X_1 + X_2 + \cdots + X_n = r\right) = 0$ for $r < n$, since each $X_i \geq 1$.

We will now explain why equations (3.4) to (3.6) hold.

Formula (3.4) follows by symmetry: $X_1, X_2, \ldots, X_n$ are identically distributed so if their sum is r, the expected value of any one of them must be r/n.

To see why (3.5) holds, note that $f_j f_{r-j}^{(n-1)*} / f_r^{n*}$ is the (conditional) probability that X_1 equals j given that $\sum_{j=1}^{n} X_j$ equals r. (In (3.5) we assume that the probability that $\sum_{j=1}^{n} X_j$ equals r, that is f_r^{n*}, is not zero.) Given that $\sum_{j=1}^{n} X_j$ equals r, the value of X_1 cannot be greater than r. Hence, the right hand side of (3.5) is the sum, over each value X_1 can take, of the value multiplied by the probability X_1 takes this value, conditional on $\sum_{j=1}^{n} X_j$ equalling r. This then equals the left hand side of (3.5).

Using the more familiar probability notation, the argument is as follows. The conditional expectation on the LHS of (3.5) is given by:

$$E\left[X_1 \mid X_1 + X_2 + \cdots + X_n = r\right] = \sum_{j=1}^{r} j\, P\left(X_1 = j \mid X_1 + X_2 + \cdots + X_n = r\right)$$

Now using the conditional probability formula:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \qquad \text{provided that } P(B) \neq 0$$

we can write:

$$E\left[X_1 \mid X_1 + X_2 + \cdots + X_n = r\right]$$

$$= \sum_{j=1}^{r} j \frac{P\left(X_1 = j \text{ and } X_1 + X_2 + \cdots + X_n = r\right)}{P\left(X_1 + X_2 + \cdots + X_n = r\right)}$$

$$= \sum_{j=1}^{r} j \frac{P\left(X_1 = j \text{ and } X_2 + X_3 + \cdots + X_n = r - j\right)}{P\left(X_1 + X_2 + \cdots + X_n = r\right)}$$

$$= \sum_{j=1}^{r} j \frac{P\left(X_1 = j\right) P\left(X_2 + X_3 + \cdots + X_n = r - j\right)}{P\left(X_1 + X_2 + \cdots + X_n = r\right)} \qquad \text{by independence of the } X_i$$

$$= \sum_{j=1}^{r} j\, f_j\, f_{r-j}^{(n-1)*} / f_r^{n*}.$$

As the Core Reading remarked above, we are assuming that the probability term in the denominator is non-zero.

Now to derive (3.6). First of all note that (3.6) holds if f_r^{n*} is zero since, in this case, for any value of $j = 1, 2, ..., r$, either f_j or $f_{r-j}^{(n-1)*}$, or both, must be zero. Hence, if f_r^{n*} is zero, both sides of (3.6) are zero.

All the terms f (with various subscripts and superscripts) denote probabilities and so are either zero or positive. If there were a value of j for which both f_j and $f_{r-j}^{(n-1)*}$ were positive, then their product $f_j\, f_{r-j}^{(n-1)*}$ would also be positive. Since f_r^{n*} is the sum of all such products, it would follow that f_r^{n*} was positive. Therefore, the fact that f_r^{n*} is zero means that there can be no value j for which both f_j and $f_{r-j}^{(n-1)*}$ are positive.

Now suppose that f_r^{n*} is not zero. Then:

$$p_n f_r^{n*} = p_{n-1}(a + b/n) f_r^{n*} \qquad \textbf{using (3.1)}$$

$$= p_{n-1}\left[a + \frac{b}{r} E\left(X_1 \,\middle|\, \sum_{i=1}^{n} X_i = r\right)\right] f_r^{n*} \qquad \text{substituting for } n \text{ using (3.4)}$$

$$= p_{n-1} E\left[a + bX_1/r \,\middle|\, \sum_{i=1}^{n} X_i = r\right] f_r^{n*}$$

$$= p_{n-1} a f_r^{n*} + p_{n-1} \sum_{j=1}^{r} (bj/r)\, f_j\, f_{r-j}^{(n-1)*} \qquad \text{using (3.5)}$$

$$= p_{n-1} \sum_{j=1}^{r} (a + bj/r)\, f_j\, f_{r-j}^{(n-1)*} \qquad \text{since } f_r^{n*} = \sum_{j=1}^{r} f_j\, f_{r-j}^{(n-1)*}$$

$$= p_{n-1} \sum_{j=1}^{r-1} (a + bj/r)\, f_j\, f_{r-j}^{(n-1)*} \qquad \textbf{(since } f_0^{(n-1)*} = 0\textbf{).}$$

Recall that $f_0^{(n-1)*} = P\left(X_1 + X_2 + \cdots + X_{n-1} = 0\right)$, and this must be zero since each $X_j \geq 1$.

Finally, we can now derive (3.3).

That is, we can now establish the result:

$$g_r = \sum_{j=1}^{r} (a + bj/r)\, f_j\, g_{r-j} \quad \text{for } r = 1, 2, ... \tag{3.3}$$

For *r* = 1, 2, ...

$$\boldsymbol{g_r = \sum_{n=1}^{\infty} p_n\, f_r^{n*}}$$

Writing this in probability notation, we have:

$$P(S = r) = \sum_{n=1}^{\infty} P(N = n)\, P(X_1 + \cdots + X_n = r)$$

This result follows from the independence of the random variables N and X_j.

Then splitting the sum into two parts, so that the first term in the summation is separate, we have:

$$P(S = r) = P(N = 1)\, P(X_1 = r) + \sum_{n=2}^{\infty} P(N = n)\, P(X_1 + \cdots + X_n = r)$$

and setting $m = n - 1$:

$$P(S = r) = P(N = 1)\, P(X_1 = r) + \sum_{m=1}^{\infty} P(N = m+1)\, P(X_1 + \cdots + X_{m+1} = r)$$

So:

$$\boldsymbol{g_r = p_1 f_r + \sum_{n=1}^{\infty} p_{n+1}\, f_r^{(n+1)*}}$$

Now using assumption (3.1), we have:

$$p_1 = (a+b)\,p_0$$

and using result (3.6), we have:

$$p_{n+1}\ f_r^{(n+1)*} = \sum_{j=1}^{r-1}\ (a + bj/r)\ f_j\ \ p_n\ \ f_{r-j}^{n*} \quad \text{for } n = 1, 2, \ldots$$

Substituting these results into the Core Reading equation above gives:

$$\boldsymbol{g_r = (a+b)p_0\, f_r + \sum_{n=1}^{\infty}\ \sum_{j=1}^{r-1}(a+bj/r)f_j\ p_n\ f_{r-j}^{n*}}$$

Now, using the fact that:

$$p_0 = P(N=0) = P(S=0) = g_0$$

and changing the order of summation, we get:

$$\boldsymbol{g_r = (a+b)g_0\, f_r + \sum_{j=1}^{r-1}(a+bj/r)f_j\ \sum_{n=1}^{\infty} p_n\, f_{r-j}^{n*}}$$

But:

$$\sum_{n=1}^{\infty} p_n\, f_{r-j}^{n*} = \sum_{n=1}^{\infty} P(N=n)\,P(X_1+\cdots+X_n = r-j) = P(S=r-j) = g_{r-j}$$

So:

$$\boldsymbol{g_r = (a+b)g_0\, f_r + \sum_{j=1}^{r-1}(a+bj/r)f_j\ g_{r-j}}$$

$$\boldsymbol{= \sum_{j=1}^{r}(a+bj/r)f_j\ g_{r-j}}$$

and this proves formula (3.3).

In the important special case where N has a Poisson distribution, $a = 0$ and $b = \lambda$.

For a Poisson distribution:

$$\frac{p_r}{p_{r-1}} = \frac{e^{-\lambda}\lambda^r / r!}{e^{-\lambda}\lambda^{r-1}/(r-1)!} = \frac{\lambda}{r}.$$

Hence $p_r = \frac{\lambda}{r} p_{r-1}$ and so . $a = 0$ and $b = \lambda$.

The formulae therefore simplify to:

$$g_0 = e^{-\lambda}$$

$$g_r = \frac{\lambda}{r}\sum_{j=1}^{r} j f_j g_{r-j}$$

In probability notation, the last formula is:

$$P(S = r) = \frac{\lambda}{r}\sum_{j=1}^{r} j\, P(X = j) P(S = r - j)$$

Now let's look at an example to see how the formula works in practice.

Example

Under a recent issue of Premium Bonds in the UK, the allocation of small prizes can be approximated by assuming that, in each monthly draw, each bond has:

- a probability of 15/320,000 of winning £50
- a probability of 1/320,000 of winning £100.

Calculate the probability that, in any given month, a holder of 1,000 bonds will win:

(i) nothing

(ii) exactly £50

(iii) exactly £100

(iv) exactly £150

(v) at least £200.

You may assume that the Poisson distribution provides a reasonable approximation to the distribution of the number of prizes won.

Solution

The holder's total winnings during a month can be represented by:

$$S = X_1 + X_2 + \cdots + X_N$$

where X_i denotes the amount of the i th win. The probability function of X_i is given by the set of conditional probabilities (given that there is a win).

Since the probability that a given bond will win is $16/320{,}000 = 1/20{,}000$, this gives:

$$P(X = 50) = (15/320{,}000)/(1/20{,}000) = 15/16$$

$$P(X = 100) = (1/320{,}000)/(1/20{,}000) = 1/16$$

(This should be intuitive since, given that you've won a prize, you're 15 times more likely to get £50 than £100.)

The Poisson parameter is the total "rate of winning" for all the bonds in one month:

$$\lambda = 1{,}000 \times 1/20{,}000 = 1{,}000/20{,}000 = 0.05$$

Using the formula for $S = 0$:

$$P(S=0) = e^{-\lambda} = e^{-0.05} = 0.951229$$

Using the recursive formula then gives:

$$P(S=50) = \frac{50\lambda}{50} P(X=50)P(S=0) = 0.05 \times 15/16 \times 0.951229 = 0.044589$$

$$P(S=100) = \frac{50\lambda}{100} P(X=50)P(S=50) + \frac{100\lambda}{100} P(X=100)P(S=0) = 0.004018$$

$$P(S=150) = \frac{50\lambda}{150} P(X=50)P(S=100) + \frac{100\lambda}{150} P(X=100)P(S=50) + 0$$
$$= 0.000156$$

(Note that the term corresponding to $X = 150$ is zero, since there are no £150 prizes.)

So:

$$P(S \geq 200) = 1 - 0.951229 - 0.044589 - 0.004018 - 0.000156 = 0.000008$$

So the probabilities are:

(i) 95.1%

(ii) 4.5%

(iii) 0.4%

(iv) 0.02%

(v) 0.001%

Question 11.3

Write down the recursive formulae for calculating the probabilities for the aggregate claims distribution where the individual claim size distribution takes positive integer values and the number of claims has a negative binomial distribution.

Question 11.4

Individual claims from a portfolio are either for 1 unit or for 2 units with probabilities 0.4 and 0.6 respectively. The number of claims is negative binomial with parameters $k = 2$ and $p = 0.4$. Find the aggregate claim distribution up to $P(S = 2)$.

4 The normal approximation to G(x)

The recursion formula for calculating $G(x)$ discussed in Section 3.2 is a very useful tool, but it does have some drawbacks. First of all, it can still, in some applications, require a considerable amount of computer time to calculate values for $G(x)$. Secondly, we cannot use it unless we know the distributions of both N and X_i, or at least can estimate them fairly precisely.

In this section we suppose that all that is known, or can confidently be estimated, about S are its mean and variance. Since lots of different distributions have the same mean and variance, we cannot calculate $G(x)$ from just this information. One way to approximate $G(x)$ in this situation is to assume S is approximately normally distributed.

More formally, let $\Phi(z)$ be the distribution function of a normally distributed random variable with mean 0 and variance 1, so that:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left\{-\frac{x^2}{2}\right\} dx$$

Now let μ and σ^2 denote the mean and variance of S. We assume in this section that S is approximately normally distributed with mean μ and variance σ^2 so that for any x:

$$G(x) = P(S \le x) = P\left(\frac{S-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right) \approx \Phi\left(\frac{x-\mu}{\sigma}\right)$$

It is easy to obtain probability values for a normal distribution.

You'll find them in your *Tables* on pages 160 and 161.

S is the sum of a random number of independent and identically distributed (*iid*) random variables. The Central Limit Theorem suggests that a normal approximation may be reasonable. The larger the (expected) value of N (the number of random variables being summed), the better this approximation is expected to be.

"Suggests" is used here because the Central Limit Theorem really only applies to situations where the number of terms in the sum is known. However, if the random variable N does not vary too much from its expected value, we might expect the normal approximation to carry over.

Example

An insurance company has sold 10,000 personal accident insurance policies to men aged between 25 and 30. These policies provide a lump sum payment of £25,000 on death during the coming year. The probability that any individual dies during the year is 0.0015 and the mortality of different policyholders can be assumed to operate independently. Use a normal approximation to estimate the probability that the insurance company's total payout will fall in the range £300,001 to £399,999. Compare your answer with the exact answer.

Solution

Using the formulae for the compound binomial distribution where the number of claims is $Bin(10000, 0.0015)$, the mean and variance of the total claim amount S are:

$$E(S) = E(N)E(X) = 10,000 \times 0.0015 \times 25,000 = £375,000$$

$$\begin{aligned}\text{var}(S) &= E(N)\,\text{var}(X) + \left[E(X)\right]^2 \text{var}(N) \\ &= \left[E(X)\right]^2 \text{var}(N) \qquad \text{since } X \text{ is constant} \\ &= 25,000^2 \times 10,000 \times 0.0015 \times \left(1 - 0.0015\right) \\ &= (£96,752)^2\end{aligned}$$

(These formulae are given in the *Tables* on page 16.)

So we can approximate the distribution of the aggregate claim amount using a normal distribution with the same mean and variance *ie* $S \sim N(375,000, 96,752^2)$ approximately.

Since we are approximating a discrete distribution by a continuous one, we should use a continuity correction. The aggregate claim amount goes up in steps of £25,000, and we want to include all amounts between £325,000 and £375,000, inclusive. The continuity correction means that we take the values half way between £300,001 and £325,000, and half way between £375,000 and £399,999.

So:

$$P(300{,}001 < S < 399{,}999)$$

$$\approx P[312{,}500 < N(375{,}000, 96{,}752^2) < 387{,}500]$$

$$= P\left[\frac{312{,}500 - 375{,}000}{96{,}752} < N(0,1) < \frac{387{,}500 - 375{,}000}{96{,}752}\right]$$

$$= \Phi(0.1292) - \Phi(-0.6460)$$

$$= 0.551 - 0.259 = 0.292$$

The exact probability can be found by noting that the aggregate claim amount will be between £300,001 and £399,999 if the number of deaths is 13, 14, or 15. Since the number of deaths has a binomial distribution $Bin(10{,}000, 0.0015)$, the exact probability is:

$$\sum_{k=13}^{15} \binom{10{,}000}{k} \times 0.0015^k \times (1 - 0.0015)^{10{,}000-k} = 0.301$$

So, in this case, the normal distribution gives a good approximation.

Question 11.5

S has a compound Poisson distribution with Poisson parameter μ, and the claim size random variable X has a Pareto distribution with parameters $\alpha = 4$ and $\lambda = 3$. Assuming S is approximately normally distributed, calculate the values of x such that:

(a) $P(S \le x) = 0.95$

(b) $P(S \le x) = 0.99$

for the two cases:

(i) $\mu = 10$

(ii) $\mu = 50$.

5 The translated gamma approximation to G(x)

Now suppose that we know the first three moments of S, rather than just the first two, or that we can estimate them with reasonable confidence. Another way we can approximate the distribution of S is by a translated gamma distribution.

The ordinary gamma distribution takes values on the range $(0,\infty)$. A translated gamma distribution is the same except that it is shifted k units to the right, so that the range of values becomes (k,∞). The number k can be positive or negative.

Let μ, σ^2 and β denote the mean, variance and coefficient of skewness of S, respectively. For the translated gamma approximation, we assume that S has approximately the same distribution as the random variable $k+Y$, where k is a constant and Y has a gamma distribution with parameters α and δ. We choose the parameters k, α and δ so that $k+Y$ has the same first three moments as S. Note that $k+Y$ is just a gamma random variable, Y, whose values have been translated by a positive or negative amount k.

One reason why the translated gamma distribution generally gives a better fit than the normal approximation is that a gamma distribution has positive skewness as does the distribution of S in many practical situations.

Whereas the normal distribution is of course symmetrical, and so cannot reflect any asymmetry in the distribution of S.

Question 11.6

Why would you expect the distribution of S to have positive skewness?

The coefficient of skewness, β, of a gamma distribution with parameters α and δ is $2/\sqrt{\alpha}$.

Question 11.7

Show that, if $X \sim Gamma(\alpha,\delta)$, then $\text{skew}(X) = 2\alpha/\delta^3$.

From this result we can confirm that the coefficient of skewness of the gamma distribution is $2/\sqrt{\alpha}$, since by definition the coefficient of skewness is given by:

$$\frac{\text{skew}(X)}{\left[\text{var}(X)\right]^{3/2}} = \frac{2\alpha/\delta^3}{[\alpha/\delta^2]^{3/2}} = 2/\sqrt{\alpha}$$

By equating the coefficients of skewness, variances and means of S and $k+Y$, we obtain the following three formulae:

$$\beta = 2/\sqrt{\alpha}$$

$$\sigma^2 = \alpha/\delta^2$$

$$\mu = k + \alpha/\delta$$

from which we can calculate α, δ and then k from the known values of β, σ^2 and μ.

These results are given on page 12 of the *Tables*.

Alternatively, we can just use skewness, rather than coefficients of skewness for the third equation. If we equate the means, variances and skewnesses of S and $k+Y$, we will get the same answers. However in this case you will need to solve two simultaneous equations to find α and δ.

The reason for approximating the distribution of S by a translated gamma, or indeed by a normal, distribution is that it may be easier to obtain values for quantities such as $P(a < k+Y < b)$ than for $P(a < S < b)$. We can readily obtain probabilities for gamma distributions from many computer packages, including spreadsheet packages and specific statistical packages.

For example, in Excel you can use the $GAMMADIST(\ )$ function.

An alternative approach is to use the chi-square distribution. We can use the connection between a gamma distribution and a chi square distribution to calculate some translated gamma probabilities. You may recall having met this in Subject CT3.

If S has a translated gamma distribution with parameters α, δ and k, then $S-k$ has a gamma distribution with parameters α and δ, and so $2\delta(S-k)$ has a $\chi^2_{2\alpha}$ distribution.

Example

Find an approximation using a translated gamma distribution for the probability that the total claims in the example on page 19 exceed £600,000. Compare your answer with the exact answer.

You are given that the skewness of a compound binomial random variable S is:

$$skew(S) = nqm_3 - 3nq^2 m_2 m_1 + 2nq^3 m_1^3$$

where n and q are the binomial parameters and $m_k = E\left(X^k\right)$.

Solution

We have already found the mean and variance of S. The skewness is:

$$\begin{aligned}\text{skew}(S) &= nqm_3 - 3nq^2 m_2 m_1 + 2nq^3 m_1^3 \\ &= 10,000 \times 25,000^3 \times 0.0015(1 - 3 \times 0.0015 + 2 \times 0.0015^2) \\ &= (£61,563)^3\end{aligned}$$

and the coefficient of skewness is:

$$\frac{61,563^3}{96,752^3} = 0.25762$$

So, we can approximate the distribution of the aggregate claim amount using a translated gamma distribution with the same mean, variance and coefficient of skewness. This requires:

$$\frac{\alpha}{\delta} + k = 375,000, \qquad \frac{\alpha}{\delta^2} = (96,752)^2 \quad \text{and} \quad \frac{2}{\sqrt{\alpha}} = 0.25762$$

The third equation then gives:

$$\alpha = \left(\frac{2}{0.25762}\right)^2 = 60.27$$

The middle equation then gives:

$$\delta = \frac{\sqrt{\alpha}}{96752} = 0.00008024$$

The first equation then gives:

$$k = 375,000 - \frac{\alpha}{\delta} = -376,122$$

So $S \sim Gamma(60.27, 0.00008024, -376,122)$ approximately.

Using the relationship with the chi-square distribution, and using a continuity correction as before, we get:

$$\begin{aligned} P(S > 600,000) &= P[2\delta(S-k) > 2 \times 0.00008024(612,500 + 376,122)] \\ &\approx P(\chi^2_{120.54} > 158.7) \end{aligned}$$

120 degrees of freedom is beyond the end of the chi-square values given in the *Tables*. However, if we extrapolate the column for 1% (the values of which are increasing by about 12 for every 10 degrees of freedom), we can estimate that the value for 120 DF would be just under 160. So, the required probability is roughly 1%.

The exact probability can be found by noting that the aggregate claim amount will exceed £600,000 if there are more than 24 deaths. So, the exact probability is:

$$1 - \sum_{k=0}^{24} \binom{10,000}{k} \times 0.0015^k \times (1 - 0.0015)^{10,000-k} = 0.0111$$

(We worked this one out using a computer package.)

So, in this case, the translated gamma distribution gives a reasonable approximation.

Question 11.8

S has the compound Poisson distribution given in Question 11.5. For $\mu = 10$ and $\mu = 50$, calculate the parameters of the translated gamma approximation to S and use the table of χ^2 values in the *Tables* to estimate the values of x such that:

(a) $P(S \le x) = 0.95$

(b) $P(S \le x) = 0.99$

Reconcile any differences in your answers compared with the ones you found using a normal approximation.

6 Using stochastic simulation to approximate the aggregate distribution

Suppose that we know the distributions for X_i and N. Then we can approximate the aggregate distribution S using Monte-Carlo simulation as follows:

- **Simulate the number of claims, n, from the claim frequency distribution.**
- **For each simulation of n, generate $x_1, x_2, x_3, \ldots, x_n$ at random from f_k, where f_k is the claim size distribution from the ground up.**
- **Apply the policy terms and conditions to each x_i, for example, limits and deductibles.**
- **Sum the x_i s together.**
- **Apply further policy terms and conditions as appropriate, for example aggregate deductibles and reinstatements.**

We often use discrete distributions to simulate the number of claims, including the Poisson and negative binomial distributions. In many applications, we find the parameters by fitting them to historical claim data that have been adjusted to be representative of the aggregate claim experience S that we are trying to model. Adjustments include allowances for IBNR and claim inflation or other factors related to the class of insurance being modelled.

Example

You are given the following information:

(1) The claim number distribution is $Poisson(2)$.

(2) The claim amount distribution is Pareto with parameters $\alpha = 5$ and $\lambda = 1,000$.

(3) An excess of 50 applies to each individual claim.

Use simulation to generate two values from the aggregate loss amount distribution for the insurer, allowing for the excess. Use as many of the following values from a $U(0,1)$ distribution as you need.

0.27 0.82 0.44 0.66 0.81 0.88 0.51 0.02 0.66 0.98

Solution

First we need to generate a value for the number of claims. Using the distribution function method from Subject CT6, we need the probabilities for a $Poisson(2)$ distribution. These are:

$$P(0) = e^{-2} = 0.1353 \qquad P(1) = \frac{e^{-2}2^1}{1!} = 0.2707$$

$$P(2) = \frac{e^{-2}2^2}{2!} = 0.2707 \qquad P(3) = \frac{e^{-2}2^3}{3!} = 0.1804 \quad \text{and so on.}$$

So if we generate values from a $U(0,1)$ distribution, then we find the corresponding values from a $Poisson(2)$ distribution by assigning:

values between 0 and 0.1353 to the Poisson value zero

values between 0.1353 and 0.1353 + 0.2707 to the value 1

values between 0.1353 + 0.2707 and 0.1353 + 0.2707 + 0.2707 to the value 2

and so on.

Using the first random number, *ie* 0.27, we find that this corresponds to a Poisson value of 1. So we assume that we have one claim.

To find the simulated claim amounts we invert the distribution function of the Pareto distribution:

$$F(x) = 1 - \left(\frac{\lambda}{\lambda + x}\right)^{\alpha} = u$$

$$\Rightarrow x = \frac{\lambda}{(1-u)^{1/\alpha}} - \lambda = \frac{1{,}000}{(1-u)^{1/5}} - 1{,}000$$

Substituting in $u = 0.82$, we find that we get a claim amount of 409.11. Applying the excess, we have a net payment of 359.11.

Question 11.9

Repeat the process using the given random numbers to find the next simulated value of the aggregate claim distribution.

A wide range of distributions is available to model the ground-up claim size distribution. As this distribution is often positively skewed, we most commonly use distributions with this shape. These include Pareto, lognormal and gamma distributions.

It is common to parameterise the claim size distribution by using methods such as Maximum Likelihood Estimation to fit parameters to historical claims adjusted to be representative of S, the aggregate claim distribution being modelled. We should be careful if the dataset is too small to be fully credible or if we believe the data is censored in some way. A common consideration in this respect is how to fit large claims to the tail of the distribution of the data, where there will typically be fewer data points.

We should repeat this process a large number of times to reach satisfactory convergence. Simulation error will occur as we can only carry out a finite number of runs of the model. Therefore the distribution produced by the simulation will necessarily differ from the true underlying distribution but it can form a good approximation.

6.1 Reducing simulation error

We can reduce the simulation error by increasing the number of simulations. The number required will depend on the variance of the underlying distributions and the purpose for which the simulation is to be carried out. For example, we will require a larger number of simulations when we estimate extreme percentiles than when we estimate the mean or less extreme percentiles of the aggregate distribution. However, the number of simulations that we can run may be limited by the available computer power and time constraints. In general, as the model becomes more complex, the number of simulations required for convergence increases, as does the time each simulation takes.

An alternative method to reduce the simulation error is to adjust the method used for generating the realisations of the random variables. We can use low discrepancy points (LDPs), also known as the Latin Hypercube, to do this.

Low discrepancy points attempt to generate the random numbers in a systematic fashion such that the multi-dimensional space (hypercube) of uniform numbers is filled out with as little discrepancy as possible given the number of iterations. This is a method particularly useful in practical applications as the approach leads to a quicker convergence of the approximated distribution to the true underlying distribution.

Latin Hypercube is an example of what is often called a *stratified sampling* technique.

The general principle is to generate more targeted samples from across the range of the distribution. The result is that we require fewer iterations in order to bring about convergence.

Let's see how we might apply this to taking samples for the Pareto distribution given in the example above. Instead of just taking points at random from the distribution, we might proceed as follows:

(1) Use the 10th, 20th, 30th, *etc* percentiles to subdivide the distribution into 10 ranges of values, each of which contains one tenth of the probability distribution.

(2) Create a simulated sample value from each of these ranges, using the appropriate Pareto model within the range.

This will ensure that we have a sample that reflects the shape of the Pareto distribution, but which will create sample results that converge more quickly to the desired distribution.

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Chapter 11 Summary

Individual risk model

The individual risk model considers the payments made under each policy (risk) separately. The risks are assumed to be independent and the number of risks is fixed over the period of insurance cover. The aggregate claim random variable S may be written as

$$S = X_1 + X_2 + \cdots + X_n$$

where X_i denotes the claim amount under the j th risk and n denotes the number of risks.

The number of claims from the j th risk is either 0 or 1 and the probability of a claim arising from the j th risk is q_i. Hence each X_i has a compound binomial distribution.

Collective risk model

Under the collective risk model the total claim amount S payable during a specified period in respect of a block of policies is:

$$S = X_1 + X_2 + \cdots + X_N$$

where X_i is the claim amount payable during the period in respect of the i th claim and N is the random number of claims during the period.

We assume that the claim amounts are identically distributed and that they are independent from each other and from the number of claims.

The mean and variance of S under the collective risk model are given by:

$$E(S) = E(N)E(X) \qquad \operatorname{var}(S) = E(N)\operatorname{var}(X) + \operatorname{var}(N)[E(X)]^2.$$

The moment generating function of S under the collective risk model is given by:

$$M_S(t) = M_N[\log M_X(t)].$$

Recursive formula

If claim amounts take discrete values and there exist a and b such that:

$$P(N=n)=\left(a+\frac{b}{n}\right)P(N=n-1), \quad n=1,2,...$$

where N denotes the claim number random variable, then the following recursive formula can be used to calculate the probability function of the aggregate claim amount random variable S:

$$P(S=0)=P(N=0)$$

$$P(S=s)=\sum_{x=1}^{s}\left(a+\frac{bx}{s}\right)P(X=x)P(S=s-x), \quad s=1,2,\ldots$$

Normal approximation

If we know only the mean and variance of the random variables N and X, then we cannot use the recursive formula. However, if $E(N)$ is large, then we can approximate probabilities involving S assuming that it is normally distributed.

Translated gamma distribution

If we know the mean, variance and skewness of S, then we can approximate the distribution of S using a translated gamma distribution. We assume that there exist k, α and δ such that $S-k \sim Gamma(\alpha,\delta)$. The values of k, α and δ can be calculated using the equations:

$$E(S)=\frac{\alpha}{\delta}+k$$

$$\text{var}(S)=\frac{\alpha}{\delta^2}$$

$$\text{skew}(S)=\frac{2\alpha}{\delta^3}$$

In place of the third equation given above, we could use the fact that:

$$\text{Coefficient of skewness } = \frac{2}{\sqrt{\alpha}}$$

Gamma probabilities are not listed in the *Tables*. However, they can be calculated using a computer package or by using the relationship between the gamma and chi-squared distributions:

$$Y \sim Gamma(\alpha, \delta) \Leftrightarrow 2\delta Y \sim \chi^2_{2\alpha}$$

Stochastic simulation

Simulation can also be used to approximate aggregate claims distributions.

Although simulation error occurs due to the finite number of runs of a model, we can reduce this either by performing more simulations, or by using a Latin Hypercube approach.

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Chapter 11 Solutions

Solution 11.1

$P(S \leq 20{,}000)$ is equivalent to saying that the number of claims is 0, 1 or 2. But the claim number distribution is $Bin(100, 0.01)$. So:

$$P(N \leq 2) = 0.99^{100} + 100 \times 0.99^{99} \times 0.01 + \binom{100}{2} \times 0.99^{98} \times 0.01^2 = 0.9206$$

Solution 11.2

$$p_r \,/\, p_{r-1} = P(N = r) \,/\, P(N = r-1) = \frac{e^{-\lambda}\lambda^r}{r!} \Bigg/ \frac{e^{-\lambda}\lambda^{r-1}}{(r-1)!} = \frac{\lambda}{r}$$

So we can write $p_r = \dfrac{\lambda}{r} p_{r-1}$, which satisfies assumption (3.1) with $a = 0$ and $b = \lambda$.

Solution 11.3

We first need the values of a and b for the negative binomial distribution. Using the "type-2" version given on page 9 of the *Tables*, we have:

$$p_r / p_{r-1} = \frac{\binom{k+r-1}{r} p^k q^r}{\binom{k+r-2}{r-1} p^k q^{r-1}}$$

Expanding the binomial coefficients gives:

$$p_r / p_{r-1} = \frac{(k+r-1)!}{r!(k-1)!} \times \frac{(r-1)!(k-1)!}{(k+r-2)!} \times q = \frac{k+r-1}{r} \times q = q + \frac{(k-1)q}{r}$$

So for the negative binomial distribution the values are $a = q$ and $b = (k-1)q$.

So the recursive formulae are:

$$P(S=0) = p^k$$

$$P(S=s) = \sum_{x=1}^{s} q\left(1 + (k-1)\frac{x}{s}\right) P(X=x)\, P(S=s-x),\ s = 1, 2, \ldots$$

Solution 11.4

Using the recursive formula we have just found with $q = 1 - p = 0.6$:

$$P(S=0) = p^k = 0.4^2 = 0.16$$

$$P(S=1) = q\left(1+\frac{1}{1}\right)P(X=1)\,P(S=0)$$

$$= 0.6 \times 2 \times 0.4 \times 0.16$$

$$= 0.0768$$

$$P(S=2) = q\left(1+\frac{1}{2}\right)P(X=1)\,P(S=1) + q\left(1+\frac{2}{2}\right)P(X=2)\,P(S=0)$$

$$= 0.6 \times \frac{3}{2} \times 0.4 \times 0.0768 + 0.6 \times 2 \times 0.6 \times 0.16$$

$$= 0.1428$$

Solution 11.5

For a Pareto distribution with parameters $\alpha = 4$ and $\lambda = 3$, we have $E(X) = 1$ and $E\left(X^2\right) = 3$.

(i) $\mu = 10$

If $\mu = 10$, then:

$$E(S) = 10E(X) = 10$$

and:

$$\text{var}(S) = 10E\left(X^2\right) = 30$$

So:

$$P(S \le x) \approx P[N(10, 30) \le x] = P\left(N(0,1) \le \frac{x-10}{\sqrt{30}}\right) = \Phi\left(\frac{x-10}{\sqrt{30}}\right)$$

(a) From the *Tables* we know that $\Phi(1.645) = 0.95$. So:

$$\frac{x-10}{\sqrt{30}} = 1.645$$

which gives $x = 19.01$.

(b) Since $\Phi(2.326) = 0.99$, the same procedure gives:

$$\frac{x-10}{\sqrt{30}} = 2.326$$

So $x = 22.74$.

(ii) $\mu = 50$

For $\mu = 50$, exactly the same method can be used. The mean and variance of S are now 50 and 150, respectively.

(a) $\dfrac{x-50}{\sqrt{150}} = 1.645 \Rightarrow x = 70.15$

(b) $\dfrac{x-50}{\sqrt{150}} = 2.326 \Rightarrow x = 78.49$

We'll have some comments to make on these results when we look at the translated gamma distribution in the next section.

Solution 11.6

Because S cannot take negative values, which constrains the left hand side of the distribution, whereas the right hand side can extend a long way out if there are some large claims included, or if there is a large number of claims.

Solution 11.7

Direct method

The k th moment of the $Gamma(\alpha,\delta)$ distribution is:

$$E(X^k) = \int_0^\infty x^k \frac{\delta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\delta x} dx$$

$$= \frac{1}{\delta^k} \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} \int_0^\infty x^k \frac{\delta^{\alpha+k}}{\Gamma(\alpha+k)} x^{\alpha-1} e^{-\delta x} dx$$

$$= \frac{1}{\delta^k} \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} \times P\,[0 < Gamma(\alpha+k,\delta) < \infty]$$

$$= \frac{1}{\delta^k} \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)}$$

since the probability that a gamma random variable takes a value between 0 and ∞ is 1.

So:

$$\text{skew}(X) = E(X^3) - 3E\left(X^2\right)E(X) + 2\left[E(X)\right]^3$$

$$= \frac{1}{\delta^3}\frac{\Gamma(\alpha+3)}{\Gamma(\alpha)} - 3\frac{1}{\delta^2}\frac{\Gamma(\alpha+2)}{\Gamma(\alpha)}\frac{1}{\delta}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} + 2\left[\frac{1}{\delta}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)}\right]^3$$

$$= \frac{(\alpha+2)(\alpha+1)\alpha}{\delta^3} - 3\frac{(\alpha+1)\alpha}{\delta^2}\frac{\alpha}{\delta} + 2\left(\frac{\alpha}{\delta}\right)^3$$

$$= \frac{\alpha^3 + 3\alpha^2 + 2\alpha - 3\alpha^3 - 3\alpha^2 + 2\alpha^3}{\delta^3}$$

$$= \frac{2\alpha}{\delta^3}$$

Using CGFs

Recall from Subject CT3 that the cumulant generating function (CGF) of a random variable is defined as the natural log of its moment generating function. We can calculated the mean, variance and skewness of a random variable by evaluating the first, second and third derivative of its CGF at the point 0.

So we have:

$$K_X(t) = \log M_X(t) = -\alpha \log(1 - t/\delta)$$

Differentiating this three times with respect to t, we get:

$$K'(t) = \frac{\alpha}{\delta}(1 - t/\delta)^{-1}$$

$$K''(t) = \frac{\alpha}{\delta^2}(1 - t/\delta)^{-2}$$

$$K'''(t) = \frac{2\alpha}{\delta^3}(1 - t/\delta)^{-3}$$

Setting $t = 0$, we get:

$$\text{skew}(X) = K'''(0) = \frac{2\alpha}{\delta^3}$$

Solution 11.8

The first step is to calculate m_3, the third moment about zero of X_i. This is given by the formula:

$$E(X^3) = \int_0^\infty x^3 \frac{4 \times 3^4}{(3+x)^5} dx$$

This integral has the form of the PDF of a generalised Pareto distribution (*ie* the three-parameter version on page 15 of the *Tables*) with parameters $k = 4$, $\lambda = 3$ and $\alpha = 1$. But:

$$\int_0^\infty \frac{\Gamma(5)}{\Gamma(1)\Gamma(4)} \frac{3x^3}{(3+x)^5} dx = 1$$

So that:

$$E(X^3) = \frac{\Gamma(1)\Gamma(4)}{\Gamma(5) \times 3} \times 4 \times 3^4 = 27$$

(i) $\mu = 10$

If $\mu = 10$, the formulae for the parameters of the translated gamma distribution are:

$$\frac{2\alpha}{\delta^3} = 270 \qquad \frac{\alpha}{\delta^2} = 30 \qquad \frac{\alpha}{\delta} + k = 10$$

Solving these, we obtain:

$$\alpha = 1.481 \qquad \delta = 0.222 \qquad k = 3.333$$

Since $2\alpha = 2.962 \approx 3$, $2\delta(S-k)$ has approximately a χ_3^2 distribution. So:

$$\begin{aligned} P(S \le x) &= P\left[2\delta(S-k) \le 2\delta(x-k)\right] \\ &\approx P\left[\chi_3^2 \le 0.444(x-3.333)\right] \end{aligned}$$

(a) From the *Tables*, $P(\chi_3^2 \le 7.815) = 0.95$, so that $0.444(x-3.333) = 7.815$, which gives $x = 20.93$.

(b) Similarly:

$$P(\chi_3^2 < 11.34) = 0.99$$

so that $0.444(x-3.333) = 11.34$, or $x = 28.87$.

(ii) $\mu = 50$

If $\mu = 50$, the formulae for the parameters of the translated gamma distribution are:

$$\frac{2\alpha}{\delta^3} = 1350 \qquad \frac{\alpha}{\delta^2} = 150 \qquad \frac{\alpha}{\delta} + k = 50$$

Solving these, we obtain:

$$\alpha = 7.407 \qquad \delta = 0.2222 \qquad k = 16.6667$$

Since $2\alpha = 14.81 \approx 15$, $2\delta(S-k)$ has approximately a χ_{15}^2 distribution. So:

$$\begin{aligned} P(S \le x) &= P\left[2\delta(S-k) \le 2\delta(x-k)\right] \\ &\approx P\left[\chi_{15}^2 \le 0.4444(x-16.6667)\right] \end{aligned}$$

(a) From the *Tables*, $P(\chi_{15}^2 \le 25.00) = 0.95$, so that $0.4444(x-16.6667) = 25.00$, which gives $x = 72.92$.

(b) Similarly:

$$P(\chi_{15}^2 < 30.58) = 0.99$$

so that $0.4444(x-16.6667) = 30.58$, or $x = 85.47$.

For the 5% points, the answers are pretty similar to those using the normal approximation. In the 1% point case, the translated gamma gives bigger answers. This is because the Central Limit Theorem does not work well in the tails of the distribution. The actual distribution is skewed, and the normal distribution does not take account of this.

Solution 11.9

The next random number is 0.44. This corresponds to a number of claims equal to 2 from the $Poisson(2)$ distribution.

So, generating the two individual claim amounts, we have:

$$x = \frac{\lambda}{(1-u)^{1/\alpha}} - \lambda = \frac{1,000}{(1-u)^{1/5}} - 1,000$$

$$= \frac{1,000}{(1-0.66)^{1/5}} - 1,000 = 240.81$$

and:

$$x = \frac{\lambda}{(1-u)^{1/\alpha}} - \lambda = \frac{1,000}{(1-u)^{1/5}} - 1,000$$

$$= \frac{1,000}{(1-0.81)^{1/5}} - 1,000 = 393.96$$

Applying the individual excesses, we have an aggregate claim amount of $240.81 + 393.96 - 100 = 534.77$.

Chapter 12

Rating methodologies and bases

Syllabus objectives

(i) (i) Understand the various components of a general insurance premium.

(ii) Describe the basic methodology used in rating general insurance business.

(iii) Appreciate the various factors to consider when setting rates.

(j) Develop appropriate rating bases for general insurance contracts, having regard to:

- *return on capital*
- *underwriting considerations*
- *reinsurance considerations*
- *investment*
- *policy conditions such as self retention limits*
- *the renewal process*
- *expenses.*

The above objectives are covered in part in this chapter.

0 Introduction

An appropriate pricing model is critical to the long-term profitability and success of a general insurance company.

There are many possible ways of calculating premiums. These range from a simple approach, to sophisticated rating models dealing with many different parameters and variables for each rating group.

The methodology that we use will depend on:

- **the class of business being priced**
- **the availability of relevant data, and**
- **the market in which the company is operating.**

0.1 Categories of pricing

In 2005 the General Insurance Board of the UK Actuarial Profession established a working party to investigate issues relating to the actuarial aspects of pricing general insurance business.

The *General insurance premium Rating Issues working Party* (GRIP) outlined five broad categories of pricing:

- **Tariff – where the regulator has significant influence over the rates – for example, in Germany.**

  This may be through the regulator setting the premium rates, or through "rate filing" where insurance companies are required to provide details of their premium rates and may have to justify any changes in premium levels, to the regulator. Rate filing is common in the USA.

- **Qualitative – where we cannot determine the "correct" price purely by numerical analysis and take account of subjective factors.**

  This might be used where the numerical data is sparse, incomplete or inaccurate.

- **Cost plus – where we set the price based on a statistically driven analysis, using the expected cost of claims, appropriately loaded for expenses, profit and so on.**

  This is the approach we describe in detail in this chapter.

- **Distribution – where we set the price allowing for non-cost elements such as the customer's propensity to shop around. This occurs in markets where we manage the pricing strategy across multiple distribution channels.**

  This approach could be seen as an extension of the cost plus approach, where we additionally consider the price sensitivity of different customers. This may result in charging different premiums according to the distribution channel used, with the aim of maximising the insurer's profits.

- **Industrial – where we focus on operational efficiency and economies of scale. Currently this is done only by the very large personal lines insurers who are operating multiple brands across multiple channels, and possibly across many countries.**

Subject ST8 concentrates on the cost plus approach to pricing. However, some features of the distribution approach are also considered; for example, Chapter 13 discusses practical considerations such as profit optimisation.

If you are interested in the other approaches (particularly if you are studying Subject SA3), more detail is available from:

- the GRIP report, which is available from the Actuarial Profession (**www.actuaries.org.uk**)
- the pricing wiki website that was established by the working party (**wiki.ratemaking.org**).

0.2 Purpose of this chapter

In this chapter, we will concentrate on the basic principles of deriving the risk premium and adjustments made once we have calculated a theoretical risk premium.

The adjustments will include additions to the risk premium and explicit loadings to cover other items.

Question 12.1

(i) What do you think is allowed for within the pure risk premium?

(ii) What do you think are the "other items" that are covered by explicit loadings?

Section 1 of this chapter gives an overview of the cost plus approach, including the steps involved in determining the pure risk premium.

Section 2 covers the data used in pricing.

Sections 3 to 5 cover, in much more detail, the process of deriving the pure risk premium.

Section 6 then describes the various loadings that are added to arrive at the technical (or office) premium.

The following chapter will discuss rating factors and practical considerations.

We consider the principles of rating based on the cost plus approach and pricing for relativities between different policies.

We will also consider some practical issues.

Pricing for the relativities between different policies is all about deciding which rating factors should be used and what effect these should have on the policyholder's premium. This is covered in Chapter 13, as are practical issues.

1 Cost plus pricing overview

1.1 Components of the premium

A technical premium is one that reflects all of the expected costs and profits arising from the policy based on technical analysis. It consists of the following elements:

Risk premium

- **the pure risk rate**
- **a loading for catastrophe and/or large loss claims**

Office premium

- **a loading for the cost of reinsurance**
- **a loading for expenses including commission**
- **a capital charge to reflect the cost of capital**
- **investment income**

Any allowance for investment income would normally act to *reduce* the premium, *ie* it is a "negative" loading.

The capital charge to reflect the cost of capital is sometimes referred to as a profit loading. Loadings for tax and contingencies might also form part of the office premium. The loading for contingencies contributes to the insurer's capital whereas the profit loading contributes to the servicing of the insurer's capital, *ie* achieving the required return on capital. Many practitioners combine the profit and contingency loadings because normally there would be a high profit loading for a risky line of business.

Other considerations

- **rating factors**
- **practical considerations concerning policy conditions, underwriting process, competition and so on.**

1.2 *Steps involved in calculating the pure risk premium*

The pure risk premium is the amount of premium required to cover the expected cost of claims. We may express it as a nominal amount or as a rate per unit of exposure.

Deriving the expected claims cost is often the major part of the work in deriving the risk premium. In establishing the overall level of claims costs, we would use an estimation or modelling process such as are used for reserving purposes.

We have to:

- **collect the relevant data**
- **adjust and group the data**
- **select the most appropriate rating model or estimation process for the specific case**
- **analyse the data**
- **set the assumptions required by the model or process**
- **test the assumptions for goodness of fit or likelihood probability**
- **run the model or process to arrive at an estimate of future claims costs**
- **perform sensitivity and scenario testing, or apply other methods, to check the validity of the estimate.**

There is a variety of statistical approaches that we can use to derive a risk premium. We will discuss these in later chapters.

For example:

- The frequency-severity approach, where statistical distributions are fitted to the frequency and severity of claims separately. These are then combined to give risk premium. This approach is described in Chapter 14.
- The simple burning cost approach to premium rating, using aggregate claims data, which is also described in Chapter 14.
- The "original loss curve" approach to premium rating, which is covered in Chapter 15.
- Generalised Linear Models and multivariate models can be used to rate premiums. These are described in Chapters 16 and 17.

The detail of how to calculate the pure risk premium under each specific approach will be discussed in the relevant chapters. This chapter covers general points that apply to any approach.

2 Data for pricing

2.1 Data requirements

To price policies appropriately the company needs data that is reliable and relevant. Policy data is needed to calculate exposure and identify characteristics of each risk group. Claims data is required to estimate the ultimate cost of claims. In particular, data by type of peril is required to cost policy options and make further adjustments (*eg* to allow for atypical events or changes in cover). Some data items are required (*eg* claim reference, loss date, description and amount) and others are useful.

Data requirements for pricing were discussed in detail **in Section 6 of Chapter 10.**

It would be a good idea to go back and review Section 6 of Chapter 10 at this stage.

2.2 Choice of internal or external data

Internal data, where available, is generally more appropriate than external data. However, market claims data, publicly available information (*eg* areas prone to flood) and competitors' rates are also invaluable. Reinsurers may help provide this.

This was discussed in detail **in Section 7 of Chapter 10.**

It would be a good idea to go back and review Section 7 of Chapter 10 at this stage.

3 *Subdivision of data*

Where possible and statistically relevant, we split the data into risk cells; that is, we subdivide the total available data into homogeneous subsets.

This will enable us to understand better the risks being handled and will help us to avoid cross-subsidies.

Hence**, profitability will not depend on a particular cross section of risks,** and so the company will be less exposed to changes in the business mix.

However, we will need sufficient data in each risk cell for credible analysis.

It may be necessary to gain market acceptance when launching particular risk divisions.

For example:

- the market may not like or accept the use of genetic testing as an additional rating factor
- regulation may allow only a limited number of rating factors to be used (as is the case in South Africa).

The way in which we express premiums per risk should accommodate distributors' (sales channels / brokers) **interests.**

As a general rule, we analyse the data with as many subdivisions as possible and regroup with care when calculating premiums.

In the analysis, we should adjust subdivided data to allow for changes in insurer's practice or relevance of past data.

We should also assess the validity of other risk groupings by stochastic analyses to test for differential results.

Question 12.2

Explain in more detail the meaning of the last sentence of Core Reading.

We should adjust the theory for practicalities, including the availability of information and the applicability of systems.

If we can't get the data in a reliable easy-to-use format, we may need to compromise our calculations.

The claims data included in the subdivisions must be recent. It should include up-to-date case estimates or other accurate information, projected amounts and/or reserves. It should be based on consistent approaches to claims recording, claims payment and claims settlement. It should include both the number of claims and the amounts.

It is very important to match the historical claims cost with a suitable exposure measure for the in-force business, which might simply be the number of policies exposed or an alternative measure, *eg* sum insured. The exposure measure used must be consistent with the claim amounts. The basic exposed-to-risk principles introduced in earlier subjects will apply. An inappropriate calculation of the exposure could easily lead to an inaccurate risk premium being calculated.

Question 12.3

How would you expect the insurer to calculate projected:

(a) claims volumes (*ie* numbers of claims)

(b) total claim amounts

for the period that the new premium rates will apply?

We often split the expected claims cost for a policy between smaller (attritional) and larger claims. We often use separate cost estimates, frequency-severity models or curves of ILFs to price for these elements.

Frequency-severity models are covered in Chapter 14. The use of original loss curves to produce ILFs (Increased Limit Factors) is explained in Chapter 15.

We discuss a variety of approaches to dealing with large losses in Section 5.

We may apply different trends (including inflation) to smaller and larger claims.

We may use different methods of projection to ultimate for the two elements, because of the different claims development patterns they are likely to have.

Adjusting for trends and projecting to the ultimate position are discussed further in Section 4.

The attritional cost may well be easier to estimate because the volume of data available is likely to be higher and the claims experience less volatile.

4 Adjusting for trends, etc

4.1 Adjusting the base values

A key question that we must ask ourselves is:

"In what way might the base values not be appropriate as the basis for setting new premium rates?"

Many different factors may cause the base experience to be different from that expected during the new rating period. In each case, we will need to make a suitable adjustment to both the exposure and the claims data.

Identifying the reasons why the base values may need adjustment is the first half of the task; the second half is making the adjustments. However, the difficulty here is that there is rarely any uniquely correct solution. There will inevitably be a level of subjective judgement in the adjustments.

Unusually heavy / light experience

Claims experience tends to fluctuate over time. For certain classes, experience in a single year can be unusually heavy or light, particularly where the risk is affected by the climate.

Another source of unusually heavy / light experience on some classes of business is the presence of a claims experience cycle. A claims experience cycle isn't the same as the insurance cycle, which can be explained by movements in premium levels. An example of a claims cycle is the frequency of motor vehicle theft, which is linked to the economic cycle.

If the experience in the ideal base period does not appear to be typical, we should:

- **choose another base year that is more typical**
- **aggregate more years' experience or**
- **apply an adjustment factor to the affected base year. Such a factor will be rather subjective, although market figures may be available.**

To help assess what, if any, adjustment may be needed we could:

- gather information on the results of other insurers, to establish whether the deterioration was industry-wide
- establish whether there are any global climatic or economic factors that would explain the unexpected experience (and how they are expected to affect future experience)
- look at previous years' results to try to identify trends or cycles.

The experience of claims frequency and claim severity should be analysed separately for trends or distortions.

Depending on the results of these further investigations, the base values might be scaled up, left alone or scaled down through careful judgement.

Large or exceptional claims

We discuss this in detail in Section 5.

Trends in claims experience

If we detect trends in the base data for any of the components of the risk premium, we should give as much weight as possible to the latest year's experience or adjust the earlier years' experience.

We should investigate any trends that we have detected in the base data to see if they are likely to continue into the future or if they are a result of a one-off change – for example, in office or market practice. If we expect them to continue, we will need an assumption to allow for them in the projection of the risk premium.

As mentioned previously, ideally claim frequency, claim severity and exposure per policy should all be analysed separately in order to identify trends.

Adjusting for inflation

To project future claims costs, we should allow for the effect of claims inflation. We discuss how we do this in Section 4.5.

4.2 Adjusting for changes in risk and/or cover provided

Reasons for changes in risk

Changes in the risk over time can be very awkward to deal with. Changes in the risk may arise because of changes in:

- **the mix of underlying risks**
- **cover / policy conditions**
- **claims handling / underwriting strategy**
- **the method of distribution**
- **the level of reinsurance coverage.**

Dealing with changes in risk

The changes in risk over time may show up as trends in the overall claims experience. We could project the trend forward.

Alternatively, we may try to separate the major elements of risk in the base data, project them separately, and then combine them with explicit assumptions about the future mix of these risks. We may also do this for significantly different types of claim if the relative mix of claims arising is changing.

The policy conditions under which insurance or reinsurance is written will have implications for the premium rates to be charged. For example, the premium rate for a policy that excludes particular risks should, all other things being equal, be lower than that for a policy that covers those risks.

This should be obvious to you. A home contents policy that includes accidental damage should have a higher premium than a policy (from the same insurer) that excludes it but has no other differences.

If an insurer tightens up underwriting or claim settlement procedures, this will also have implications for the premium rates.

Relevance of the historical data

The insurer will use the historical claims data to estimate the expected claim amounts that will be incurred during the risk period for which the premiums are to apply.

It is important to ascertain the consistency or otherwise of historical data with the risk period ahead for which the premiums are to apply. Significant inconsistencies (which distort the risk premium if suitable adjustments are not made) may arise in such matters as:

- **policy acceptance – the basis on which proposals are accepted, the scrutiny or "harshness" of underwriters**

  For example, the data may come from a period during which more stringent rules (such as exclusions or high excesses) applied.

- **policy coverage – the risks covered under the contracts in question relative to the period ahead**

  For example, a new peril or claim type may have been introduced.

- **method of distribution – the influence of the selling process on the nature of risks insured or policyholders covered**

  For example, the internet may be used for more sales in the future, leading to different claims experience.

- **claims settlement procedures – the internal practices that may affect the timing and possibly the amount of claims paid to policyholders.**

The company must investigate how any of the above factors have changed or how they might change in the future and make suitable allowances when calculating the risk premium.

Changes in policy conditions

The major changes in the policy conditions are likely to be in the perils covered and limits or excesses applied to any claim.

Changes in the perils covered

In the case of perils that are no longer insured, we may be able to exclude from the base data all types of claim that would not be covered under the new rating series. However, if a new peril (or indeed any new aspect of cover) is to be introduced, we will need to use external data such as market statistics, consumer or manufacturers' statistics, scientific data or government statistics to approximate the likely cost of the claims for this additional cover.

Changes in self retention limits

If there are limits on the amount of cover provided by a policy – for example, a policy excess, a self retention limit or a maximum sum insured – then the required premium should be lower than that for a policy without any such limitations.

If the limit has been reduced or the excess point has been increased, we can normally truncate the past claims experience to approximate the future costs. However the insurer would need a detailed database, so that we can identify each individual claim separately, and allocate it to a policy.

However, it is more difficult to estimate the effect of lowering the excess point because many insureds will not inform the insurer of losses below the excess point. We have to estimate the increase in both the frequency and size of the future claims. Data may be available from other similar risks, or from external sources. Otherwise, we must use more approximate adjustments, based on any knowledge available regarding the claim cost distribution. Either way, the information is likely to be incomplete.

Question 12.4

A company introduces a £50 excess on a policy. What is the effect on the:

- claim frequency?
- aggregate claim amount?
- average claim amount?

Question 12.5

We are carrying out a premium rate review. Since the period from which the base values were derived, the levels of excess have been increased. Explain briefly how the base values might be adjusted to allow for the new, higher level of excess.

4.3 *Adjusting for environmental changes*

Claim experience can be affected by a variety of environmental factors. The new premium must reflect these expected changes in claims experience. A few examples of such factors include:

- legislative factors
- advances in technology
- medical advances
- changes in the construction of property.

Legislative factors

These can have a significant impact on claims cost, particularly liability claims that might be expected to increase in line with an inflation index such as a salary index. In practice the size of many claims awarded via the court process increase at a rate far in excess of salary inflation. This is a phenomenon known as "court award inflation".

This was discussed in detail in Chapter 8.

Advances in technology

Advances in technology can lead to safer measures and better practices used in certain industries. They can also cause property to be more complex and so more costly to fix.

Example

An example is in the motor industry over the last few years where the technology behind the construction of side-view mirrors has improved greatly. This could improve the view that drivers have of other vehicles and obstacles, and so reduce the number of vehicle collisions.

It is hoped that both the number and average size of claims will be reduced as a result of this safety feature.

Medical advances

Medical practice and knowledge can improve greatly over time, which may have a positive or negative effect on expected claims levels for medical-related claims. Improved practices enable some problems to be identified more quickly and treated before they become too serious. Alternatively, early diagnosis could lead to claim payments being made much earlier than anticipated.

New medical treatments can also be very costly. This could lead to increases in expected claim amounts, if medical expenses are included in the cover.

Changes in construction of property

In a similar way to advances in technology, changes to the construction of property can have an effect on expected claim cost.

Example

For example, there has been a trend in recent years for lightweight materials to be used in UK property building as opposed to the traditional heavyweight brick constructions. This is an attempt to counteract the effects of subsidence.

However, the use of such materials has made these buildings less able to withstand flood damage and has led to increased claims costs over recent years where there have been a number of severe floods.

4.4 Time periods involved in the projection

It is important to remember that there may be a significant time delay between the period of exposure on which the claims data is based and the time to which projections are made for the new period of exposure.

Hence we need to adjust for claims inflation and other trends to ensure the new premium charged is appropriate for the new exposure.

We should investigate any trends detected in the base data to see if they are likely to continue before incorporating an allowance into the risk premium.

Reasons for delay

Time delays that may result in adjustments to the data may occur because of:

- **time taken for sufficient claims experience to develop from the historical data**
- **time taken to analyse the claims experience**
- **time taken to reach and agree the new premium rates and premium structure**
- **time taken to administer and implement the new rates**
- **time delay between the risk period and the payment of claims**

  There is often a delay between occurrence and payment of a claim. This differs depending on the length of the tail of the portfolio.
- **time taken for any approval needed from a regulatory body.**

This projection is surprisingly tricky. Many students will get this projection wrong through not giving the process enough attention. To illustrate this trickiness, consider the following question.

Example

You have carried out an analysis of claims occurring in 2009. Your aim is to set premium rates for 2011. How many years' inflation must you apply to the base claims data?

Most quick answers to this question will be "2009 to 2011 is 2 years ... the answer is 2".

However, the answer is not 2.

The key to getting the right answer to these types of questions is to treat each question on its own merits and to think very carefully about the mechanics of what is going on. The development of the correct answer to this question is as follows.

Solution

The mid-point of 2009 is the average date at which claim events took place. (This assumes that claim events were evenly spread over the year.)

For the new premium rating series, the average date on which policies will start is the mid-point of 2011. (This assumes that new policies are written evenly over the year.)

If we assume that policies are annual, and that risk is evenly spread over the policy year, then the average date of any claim event will be six months into the policy year. So for the average policy, this is the end of 2011.

The projection period is therefore 2½ years (mid-2009 to end-2011).

In solving this question we have also assumed that the run-off of claims from occurrence to settlement can be ignored. This is reasonable if both the run-off pattern and the rate of claims inflation are the same for the two cohorts of claims.

Diagrammatically, this example question looks like this:

2007	2008	2009	2010
		← write →	
← base →		← exposure	→
↑		↑	
mid-point		mid-point	

Question 12.6

For policies that provide 6 months' cover, you have carried out an analysis of claims for accidents in the period 1 June 2010 to 30 November 2011. You plan to set new premium rates to apply for the period from 1 December 2012 to 1 August 2013.

For how many years will you need to project claims inflation? State your assumptions.

Most analyses of claims experience will be on an accident-year basis. If they are not, we will need to be careful that our projections for inflation are consistent. Again, if we treat each case on its own merits, we should be able to deduce the correct projection period.

Question 12.7

You have conducted a claims experience investigation using a policy-year analysis, *ie* you have analysed the claims emerging from policies written in 2011. Your new premium rating series is to apply to annual policies written throughout 2014 and 2015. For how many years do you need to inflate the claims?

4.5 *Projecting the base values*

Projecting claims for inflation

We will express the total claim cost and exposure values produced from the initial analyses in the money terms of the base period. Therefore, as well as allowing for future trends and any proposed risk or cover changes, we need to allow in our projections for the expected effect of inflation on claims between:

- **the mean payment date of claims in the base period and**
- **the mean payment date of claims arising during the new exposure period.**

If claims occurred evenly over each period and were not subject to any delays, we could take these mean dates as the mid-points of each exposure period. However, this will very rarely be the case in practice, and so we should allow for inflation during the run-off of those claims.

The above simplification may be appropriate for very short-tailed business, but in practice, we would usually allow for the settlement delay of claims.

As the future inflation is likely to be different from that experienced (or projected) in the base claims data, we should inflate the claims cost in the base period to current-day terms before the projection.

In other words, we should express our prior data in real terms before projecting.

We should consider the actual nature of the costs before we decide a suitable index of inflation. Ideally we will be able to adjust claim values by reference to an index specific to the particular loss. In the UK, the ABI maintains some such indices.

The rate of inflation to use will depend on the type of inflation that claims are subject to. For example, for damage to motor vehicles, we need to consider estimates of:

- motor spare parts inflation (a primary component of car repair costs)
- mechanics' charge-out rates inflation (another primary component of car repair costs)
- new car price inflation (relevant where a car is written off).

We can expect all these elements to inflate at different rates.

For liability claims, we need to consider:

- earnings inflation, because many liability settlements are based on loss of earnings
- the inflation of court awards over the appropriate period.

Court award inflation appears to be only very loosely correlated with other forms of inflation, and some practitioners believe that the rate is now a constant 10% *pa* regardless of economic conditions. It tends to increase in steps. One judicial decision may increase court awards by 20% but then the court award inflation may be zero for a couple of years.

Past and future

In practice, when revaluing base values for future premium rates, there are two parts to the calculation:

- inflating base values to the present day using (broadly) known inflation rates
- projecting from the present day to the future using estimated future inflation rates.

The published indices can be used for the past inflation, although some judgement must be applied in estimating, for example, future mechanics' wage inflation.

Projecting to the ultimate position

We discussed earlier in this chapter how the claims data used should be adjusted to include unsettled claims and IBNR claims. We need to make sure that this is done.

As some of the past claims may not be fully developed, we need to project current claims data to the ultimate position. We may need to include reserves for those claims reported but expected to increase in size or for additional claims not yet reported.

Alternatively, we could use only years where the data are believed to be fully developed for analysis but then the data are based on less recent experience.

Example

Previous examples have ignored the complications caused by the delay from claim event to ultimate settlement. At the same time as incorporating this feature in an example, we will build in allowance for investment income.

Let's say that we want to find the risk premium applicable for new annual policies written in 2013 for a particular insurance class where premiums are expressed as £x per policy.

Let's suppose that our starting point is an accident year investigation for claims occurring in 2011. Suppose also that:

(a) we are working with a homogeneous group of risks (so we won't get distracted by discussions about different risk groups)

(b) the claims data we are using incorporate best estimates of IBNR and outstanding reported claims

(c) all possible distortions have been eliminated.

Suppose that we have also produced the following data from our analysis of 2011 claims:

- average delay from occurrence to settlement was 10 months
- average claim payment per policy was £267

and we have made the following estimates:

- settlement delays will stay at 10 months on average
- there will be a deterioration in claims experience from the base period to the future exposure period, increasing claim frequency by 2% *pa* compound
- claims inflation from the mean payment date in the base period to now, and also in the future, averages 10% *pa*
- investment return will be 8% *pa*.

Question 12.8

Without looking over the page, try for yourself to derive a suitable risk premium.

For this sort of question there are often many acceptable ways in which a suitable answer could be calculated depending on how you interpret the data given, and depending on any further assumptions you make. One answer can be derived as follows.

Solution

The average claim amount as at the mean payment date of the base period is £267.

We know the projection period here is 2½ years (mid-2011 to end-2013), so we need to inflate the £267, allowing for the deterioration in claim frequency and also for claims inflation. Therefore, the monetary amount of the average claims payment in the new rating era will be:

$$267 \times 1.02^{2.5} \times 1.10^{2.5} = £356.$$

As our goal is to find the pure risk premium (allowing for investment return of 8% *pa*), we now need to discount back to the average date at which the premium would be received, *ie* for 16 months (10 months settlement delay and 6 months from inception to accident date).

This risk premium for the new rating series should therefore be:

$$£356 / \{1.08^{(16/12)}\} = £321.$$

This assumes that the insurer receives the risk premium the day that the policy starts. In practice there is often a delay of up to a few months.

4.6 Projecting exposure values

In order to arrive at a risk premium rate, we must divide the projected claim cost by a corresponding projected value of the exposure.

Previous examples in this chapter have been based on policies where the premium is stated as an amount per policy as, for example, with private motor insurance where the premium is quoted per vehicle-year.

Where these exposure units are expressed in terms of monetary units, we need to project the base exposure values at an appropriate rate of inflation. This may not be the same as that applied to the claim cost. Here, the projection is only to the mid-point of the exposure period arising under the new rates.

For example, in many forms of property insurance, the premium is quoted per £1,000 sum insured. You will sometimes see this is written as "per mille". In these cases, high inflation does not necessarily mean that the premium rate must be increased. If the exposure measure inflates as quickly as the average claim amounts, then premium rates might stay constant.

For example, a premium rate of £2 per mille set in 1980 might still be appropriate in 2012. But, we very much doubt whether a premium of £15 per vehicle-year in 1980 would still be acceptable in 2012!

Impact on calculations

We will look back at the previous worked example, and suppose on this occasion that the premiums for these policies are expressed as £y per mille.

Suppose that the average sum insured for the policies within the base period was £16,000. Because the sum insured is fixed at the inception date, we need to work from average inception date rather than average date of accident.

Now the average inception date for the policies giving claims in 2011 was 1 January 2011. The average inception date in the new rating period will be mid-2013, giving a 2½ year inflation period. If we assume that the inflation of the sum insured will be 8% *pa*, then the average sum insured in the new rating series will be £19,395.

> ***Question 12.9***
>
> Find the new risk premium rate and the original risk premium rate, stating any assumptions that you make. Comment on the difference between them.

The inflation rate to be used for the exposure measure might be the same as the claims inflation rate, although it would not necessarily be the same. For example, in household contents insurance, the inflation of the exposure measure will be based on changes in the prices of household goods. The inflation of claim amounts would be greater than this if burglars were gradually getting better at stealing more items or if the prices of "thievables" were rising faster than those of household goods in general. So, in this instance, the average cost per claim would be rising more rapidly than the sum insured.

5 Catastrophe and large claims loadings

In addition to the attritional (normal) **claims experience, the premium should also reflect the expected loss due to catastrophic and large claims.**

We often remove these claims from the base attritional data to enable a reliable analysis. However, we should still reflect the actual (expected) **cost of these claims in the premium. We can estimate the required loading from the insurer's own data, provided sufficient experience is available.**

Otherwise we use external data for such claims, or a catastrophe model.

Own data

Unless the particular class of business is new, we would have a "feel" for the relative frequency of unusually large claims or catastrophes. If we know that we generally experience a couple of freak claims each year, our premiums need to allow for these. Such claims should not be removed from the base values (although they should perhaps be truncated and the excess re-spread over the whole portfolio).

However, if there was an exceptional claims event, like an earthquake, we would need to decide how often that event would generally occur (say once every n years) and then include $1/n$th of the cost of claims from that exceptional event within our base values. In this case n years would be known as the *return period*.

Similarly, if there were *no* exceptional events in our base period, we might decide to load the base values to allow for the fact that such events do tend to occur every n years or so. Alternatively, if such claims will be covered by a reinsurance agreement, the reinsurance cost can be included within the premium calculation (see Section 6.2).

Catastrophe modelling

Catastrophe modelling has a significant advantage over historical data analysis in the case of low frequency, high severity risks. This is because the observed historical losses may not reflect the true underlying risks, as the period over which losses have been observed may be much shorter than the return period of the losses under consideration – for example, a ten-year burning cost model is unlikely to be a reliable method of pricing for an earthquake risk where a likely return period is 100 years.

Use of a burning cost model is a simple example of a cost plus approach to pricing. It uses historical aggregate claims, expressed as an annual rate per unit of exposure. It does not make use of the number of claims. It is covered in more detail in Chapter 14.

The models estimate the cost of numerous possible future events based on latest research in areas such as seismology, meteorology, hydrodynamics, structural and geotechnical engineering. They also allow for changes in the risk exposure and factors such as changes in building codes, construction types, engineering surveys and loss mitigation.

As a result, we usually omit catastrophe claims from the analysis.

We discuss this in more detail in Chapter 21 (and in Subject SA3)**.**

Large (non-catastrophe) losses

Several approaches are possible for large non-catastrophe losses:

- **We can omit them from the analysis and allow for them separately in the risk premium.**
- **We can truncate large claims at a set point and spread any cost above this level across the larger portfolio of risks.**
- **We can leave large claims in the data, although we rarely do this because this would implicitly assume that future occurrences will replicate those seen in the past.**

6 Calculation of the office premium from the risk premium

6.1 Loadings

The risk premium reflects the expected claims cost, including allowances for large and catastrophic losses. However, this is not the premium that the insurer will charge. We will make additions to cover some or all of the items below.

The additions are commonly called "loadings". The loadings that will be described in this section are:

- reinsurance premium
- expense loadings
- profit loading
- investment income.

The *office premium* is the risk premium adjusted for the above loadings. There are further considerations that should be made, such as competitor influences, before arriving at the actual premium charged. These more pragmatic considerations will be discussed in the next chapter.

6.2 Reinsurance premium

Allowing for the cost of reinsurance

We can incorporate the cost of reinsurance into a premium rate in one of two ways, either:

- **as the net cost of reinsurance in a premium rate based on a gross risk premium, or**
- **as the gross cost of reinsurance in a premium rate based on a net risk premium.**

When we talk about gross and net here, we mean gross and net of reinsurance recoveries.

Example

The gross risk premium for a policy is equal to £50.00. The cost of reinsurance applicable to this policy is £12.00 and the expected reinsurance recoveries from this cover are £10.00.

If we allow for the cost of reinsurance in the first way described above, then we have:

- **Net cost of reinsurance = £12 – £10 = £2**

  plus

- **Gross risk premium = £50**

  equals

- **Total premium = £50 + £2 = £52.**

If we allow for the cost of reinsurance in the second way described above, then we have:

- **Gross cost of reinsurance = £12**

  plus

- **Net risk premium = £50 – £10 = £40**

  equals

- **Total premium = £40 + £12 = £52.**

In situations where only a small proportion of the gross premium is passed to the reinsurer, it is normal for the first approach to be used. The net cost will arise because the reinsurer will want to cover its own expenses, profit margin and so on.

When there is a high level excess of loss or catastrophe reinsurance, we expect the claim frequency to be low but the claim amounts to be high. In these cases, it is common to calculate a net premium allowing for reinsurance recoveries and then add the cost of the reinsurance to the premium as an expense loading.

In other words, it is common in these situations for the second approach to be used.

In some short-tailed classes, the reinsurer's risk premium may be lower than the insurer's, for example where the reinsurer has a larger pool of experience. In these cases the "net" reinsurance cost will be negative. It would be unusual, except in circumstances of aggressive competition, for the insurer's risk premium to be reduced to reflect this.

Effect of trends in the cost of reinsurance

It may also be necessary to allow for the effect of trends in the cost of reinsurance. As with normal insurance, the cost of reinsurance is driven by the capacity and demand in the market. Reinsurance capacity in particular is sensitive to the recent claims experience and the sentiment of reinsurers. If we believe that the cost of reinsurance is likely to change in the future, we should incorporate this into the premium rates of the insurer.

Capacity is the insurance term used to describe the level of supply available in the market.

Question 12.10

Comment on the suggestion that insurers who use lots of excess of loss reinsurance should have to charge higher premiums because their reinsurance premiums will, on average, exceed the recoveries they make.

6.3 Expense loading

Expense loadings are required in the rating formula to ensure that sufficient premium is charged to cover not only the expected claim costs, but also the costs of expenses related to administration and claims handling for the business written under these rates and to provide a contribution to the general fixed costs of the company.

For most insurers, the expenses, in roughly descending order of amount, might be:

- staff salaries, pensions costs, national insurance contributions, *etc*
- commission payments
- office rent and related costs
- office equipment (*eg* computers)
- office consumables (*eg* stationery).

Analysis of expenses by different sources and types is required to ensure that suitable expense allowances are made in the rating calculation.

The assessment of expenses will be explored in detail in Chapter 19.

It was also covered in Subject CA1.

You may be able to cover some of the material in this section relatively quickly or use it for revision purposes.

The stages involved in determining the required expense loadings are:

- to allocate the expenses by class or product and, possibly, by major rating factor groups
- to allocate expenses by function
- to decide on how to allow for each expense type in the rating formula.

Types of expenses

The expenses of a general insurer can be split into groups of expenses that occur at different stages throughout the lifetime of an insurance policy. Other expenses called indirect expenses (or overheads) will not be directly attributable to any particular policy. Examples of overheads are general management costs and office rent.

Theoretically we may divide expenses into:

- **underwriting and brokers' commission**
- **policy administration**
- **claims handling**
- **overheads**
- **levies to insurance guarantee schemes, such as the Financial Services Compensation Scheme (FSCS) in the UK.**

The FSCS is the compensation fund of last resort for customers of financial service products in the UK. It is funded by annual levies on insurers and other financial service providers.

Each of the elements above, apart from overheads, will probably be directly allowed for in the premium rating formula. Methods may be used to allocate overheads between the other elements. Alternatively, overheads may be implicitly allowed for in the profit loading. This loading would then be to cover profit and a contribution to overheads.

Question 12.11

A general insurer that writes only one class of business wishes to split its general management costs and its office rent between the initial, alteration and claims handling expenses. How might the split be made?

Each element should be clearly identifiable within the rating basis, so that when expenses change, we can adjust the rates accordingly.

For example, the expenses associated with a particular product, say, and a particular function could be explicitly allowed for in the premium formula. We give some example premium rating formulae at the end of this section so that you can see this for yourself.

Direct / indirect expenses

Direct expenses are those that can be identified directly as belonging to a particular class of business.

However, a detailed breakdown of the expenses associated with a particular product in a particular class of business is often not available. In such instances, we might make broad assumptions about the general overall level of expense and incorporate these within the rate.

Indirect expenses, or overheads, will need to be apportioned to different classes of business or products using some sensible method.

Question 12.12

Suggest methods that can be used to allocate indirect expenses to different classes of business or products.

Fixed / variable expenses

Expenses may be further split into those that are fixed and those that vary depending on the volume of business or the size of the premium.

Examples

For a class of business, the amount of commission paid varies by the size of the premium and the type of risk covered.

The cost associated with printing policy documents tends to be a fixed amount.

In practice, all expenses can vary *in the long term*, so the concept of fixed expenses makes most sense if we confine it to the short term.

Examples of variable costs that might be related to one or more different measures of the amount of business are:

- amount of premium income (*eg* commission)
- number of policies sold (*eg* postal costs for policy documents)
- amount of claim payments (*eg* legal expenses)
- number of claims (*eg* postal costs for claim forms).

There is no fixed rule as to the boundary between fixed and variable costs. Some costs could easily fall into either category. For example, the cost of processing a new policy might be described as a fixed cost because you would not pay higher salary costs as a result of the new policy. On the other hand, if marginal sales of policies meant paying staff bigger bonuses or making overtime payments, then the cost of processing a new policy is a variable cost.

Allowing for expenses in the rating formula

Insurers have many different ways of charging for commissions, expenses and other margins:

- **Some do no more than add an overall percentage to the risk premium, based on the evidence of the previous year's underwriting results.**
- **Others go a little further and take into account the split between fixed and variable expenses and likely volumes of business in the future.**

The danger of using an overall percentage approach is that it may not accurately reflect the actual expenses if there are changes in the volume and nature of the business written. Some expenses, such as the commission, vary with the business volume. Other expenses, such as the company overheads, are relatively fixed and need to be shared in a reasonable distribution across the lines of business.

Sometimes we charge a policy fee explicitly rather than add loadings into the rating structure. This can impact smaller policies more than larger ones.

The idea here is that the policy fee would represent the per-policy expense costs. However, the policy fee may need to be reduced from its theoretical level in order to make the total premiums for small policies more competitive or, indeed, reasonable.

However, to avoid the risk of not covering expenses, the reduction will need to be compensated for elsewhere – for example, by an expense loading that is expressed as a percentage of premiums. The insurer would need to model future assumed business mix and volumes to ensure that the total expense contribution received by the company would be the same as what it would have been, had the theoretically correct premium rates and policy fee been used.

Question 12.13

What is the risk to the company in reducing the policy fee, even if compensated for elsewhere?

New business vs renewals

In theory, the loading for expenses should be higher for new business than for renewals. However, for competitive reasons, this is often not the case.

The cost of assessing a new proposal for cover and the subsequent processing of that risk if it is accepted is relatively high compared to the expenses associated with renewing an existing piece of business. Therefore there is at least a theoretical case for charging a higher premium for new business than for renewals, with a specific (per new policy) loading being made for this within the rating formula.

In practice, however, we rarely do this in the UK. The short-term nature of contracts and the relatively low levels of persistency are likely to cause the absolute level of the policy fee to appear unacceptably high to the policyholder, relative to the amount being paid for the risk element. We generally incorporate the additional cost of writing new business into the overall level of expenses, based on an assumed level of future new business and its average expected duration before lapsing.

One approach would be to make an assumption about the number of years for which new policies are expected to be renewed; the excess of the initial costs over the renewal costs is spread over this number of years. Adding this annual (amortised) initial expense to the expected renewal cost for the year gives the total per-policy administration expense assumption.

6.4 Profit loading and return on capital

An insurer will need to incorporate a profit loading in its premium rates to give a reasonable return on the capital required to support the risks underwritten. The size of economic capital allocated to support the particular class of business should reflect the underlying variability and uncertainty of the risks being covered, after the effect of reinsurance protection.

The required rate of return on this capital should be appropriate to the shareholders' risk/return appetite.

Insurers need a large amount of shareholders' capital to support the business. We should therefore make allowance within our premium formula for an appropriate return on shareholders' funds.

In the rating process the profit loading may be set as an agreed percentage of the gross office premium (or possibly the risk premium). This seems sensible for two reasons:

Firstly, the more business that is written, the greater the total variance of the risk and the greater the capital needed to support it. However, if more business is written, the variance in percentage terms reduces, which results in lower risk. This implies a lower return on capital (or percentage profit loading) may be needed.

The second reason for expressing the profit loading as a percentage of premium is that as free reserves (and hence shareholders' funds) reduce, the degree of risk of capital inadequacy increases. For a given percentage profit of office premium, the return on capital will be higher if the free reserves are lower, which makes sense.

For example, a company that has £100*m* free reserves sets a profit target of 10% of written premium. If it writes £100*m* of premiums then the expected return on capital is 10% of the shareholders' funds ((10% of 100) / 100 = 10). If its free reserves are reduced to £50*m*, then the expected return to shareholders is 20% ((10% of 100) / 50). The expected return has increased to reflect the extra risk.

In theory, the required return on capital should also vary between insurance classes. A higher return will be required for the higher risk classes.

Question 12.14

What makes an insurance class high risk?

Give an example of a class that would be described as low risk.

In practice, however, uncertainties are such that many companies combine products and lines of business and attempt to achieve a target return over all of the combined business.

Although the amount of capital (as a proportion of premium) held by the insurer varies according to the riskiness of the underlying business, it is common to have the same required return (built into the premium) across many or all classes of business, as, quite often, the capital allocation to classes is notional rather than explicit.

Question 12.15

A small general insurer writing only private motor insurance through independent brokers wishes to use a profit loading of 15% of gross office premium. Why might the company decide not to use this loading?

We discuss how to incorporate the return on capital into the premium calculation in more detail later, in Subject SA3.

6.5 Investment income

Investment is an integral part of the insurance process. Insurers will be able to invest the premiums for a period of time until the claims and expenses are paid out. We can allow for investment income on the premiums by discounting the projected claims and expense cashflows at a suitable rate of interest to the date at which the premium is paid.

We need to decide whether a gross or net rate is applicable.

By this the Core Reading means whether the investment return used should be gross or net of tax. This depends on how the insurance company is taxed.

Example

In the UK, mutual general insurers are taxed on the investment return, in which case the investment return assumption should be net of tax.

However, UK proprietary general insurers are taxed on profits, in which case an investment return assumption that is gross of tax would be appropriate.

The return will also depend on:

- **the duration for which the funds are invested and**
- **the form of the liabilities (whether they are fixed, increase in real terms and so on).**

In general, we will assume a conservative rate of return.

For long-tailed classes, the effect of discounting will be significant. For short-tailed classes, discounting will be a minor and in some cases a negligible factor in the rating process. Hence, for short-tailed classes and highly volatile long-tailed classes where the uncertainty is much more significant than the effect of discounting, it may be more suitable for the actuary to adjust the profit loading than to discount.

The premium formula should recognise, either explicitly or implicitly, that the insurer can expect to earn some investment income on the premiums received during the period from receipt of premium through to final settlement of claims.

In short-tailed classes, the assumption regarding investment income is not critical. It might be several months before the premiums are received for investment (because intermediaries like to hang on to the premiums for as long as possible). Also, for these classes, claims are reported and settled relatively quickly. Premiums might be invested for just six months on average.

However, the situation is very different for long-tailed classes, where premiums may be invested for many years before being needed to settle claims. The assumption regarding investment returns is then significant. (Note also that the inflation assumptions are far more significant for long-tailed classes.)

What rate is suitable?

The following factors will be considered when selecting an assumed rate of investment return:

- the likely investment conditions at the time when the premiums are receivable
- the assets likely to be held (which should be matched to the nature of the liabilities)
- consistency with the inflation assumption used to project expected claims and expenses.

Question 12.16

An insurance company sells a particular annual contract through brokers. The probability of a claim is 0.2 and all claims are for 400. There is no inflation but a return of 5% is earned on investments. Brokers hold premiums for an average of 2 months and all claims are settled 6 months after the end of the contract. Expenses and commission are 25% of the gross premium and are all paid at the start of the policy.

Calculate an office premium for this contract.

6.6 A sample premium rating formula

When calculating an office premium in an exam, or in practice, it is usually best to work from first principles, because there are many, many different ways in which the situation could be set up and loadings could be applied. You are much more likely to go wrong if you *learn* formulae.

The equation of value to start from is:

$$PV\ of\ gross\ premium\ =\ PV\ of\ claims\ costs + \ PV\ of\ expenses\ and\ loadings$$

(here, PV stands for present value.)

Example

Suppose that CF is our expected claim frequency per policy, CA is the expected average claim amount and we want to calculate a gross premium GP incorporating the following allowances:

- commission as a proportion, c, of gross premium
- a contribution to fixed expenses of FE per policy
- variable expenses, other than commission, as a proportion, ve, of gross premium
- other per policy variable expenses of VE
- claims handling expenses of CE for each claim
- profit as a proportion, p, of gross premium.

Assume, for the sake of clarity, that all amounts have been converted already to present values, so the equation of value becomes:

$$GP = CF \times CA\ + c \times GP\ +\ FE\ +\ ve \times GP\ +\ VE\ +\ CF \times CE\ +\ p \times GP$$

$$GP(1 - c - ve - p) = CF \times (CA + CE) + FE + VE$$

so $$GP = \frac{CF \times (CA + CE) + FE + VE}{1 - c - ve - p}$$

Question 12.17

Develop a premium formula where:

- CF' is the expected claim frequency net of reinsurance
- CA' is the expected average claim amount net of reinsurance
- commission is a proportion, c, of gross premium
- contribution to fixed expenses is a proportion, f, of gross premium
- non-commission variable expenses are a proportion, ve, of gross premium
- other per policy variable expenses of VE
- claims handling expenses are a proportion, h, of each net claim settlement
- gross profits are a proportion, p, of gross premium
- reinsurance premium is a proportion, r, of gross premium
- premium income is immediately invested in cash securities at i% per month gross
- ignore tax.

State any further assumptions that you make.

Question 12.18

Develop a formula for a gross premium per mille sum insured where:

- CF and FE relate to a whole class
- CF is the expected claim frequency net of reinsurance
- CA is the expected average claim amount net of reinsurance
- the contribution to fixed expenses per policy is FE
- c is commission expressed as a proportion of gross premium
- the expected sum insured for the class is SI (expressed in £000s)
- CA incorporates the cost of claim handling expenses
- variable expenses are ve expressed as a proportion of risk premium plus claim handling expenses
- all fixed expenses and the loadings for profit, contingencies and reinsurance are incorporated within FE and ve
- all amounts above are expressed in present value terms.

7 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Product pricing
- Profit testing
- Rating basis.

Chapter 12 Summary

An appropriate pricing model is critical to the long term profitability and success of a general insurance company. There are many possible ways of calculating premiums, ranging from a simple approach to sophisticated rating models dealing with many different parameters and variables for each rating group.

The methodology actually used will depend on:

- the class of business being priced
- the availability of relevant data
- the market in which the company is operating.

Five broad categories of pricing are: tariff, qualitative, cost plus, distribution and industrial.

The process may start with a calculation of the pure risk premium (*ie* the expected future claim amount), before loadings are added to give the office premium. This is the cost plus approach.

The pure risk premium can be expressed as a nominal amount, but usually as a rate per unit of exposure.

To price policies appropriately the company needs data that is reliable and relevant. Policy data is needed to calculate exposure and identify characteristics of each risk group. Claims data is required to estimate the ultimate cost of claims. In particular, data by type of peril is required to cost policy options and make further adjustments (*eg* to allow for atypical events or changes in cover). Some data items are required (*eg* claim reference, loss date, description and amount) and others are useful.

If the experience underlying the data includes anomalous events or untypical experience, we should remove claims arising from such sources. Alternatively, we can choose a more typical base period, collect more years' data or apply an adjustment factor.

Internal data, where available, is generally more appropriate than external data. However, market claims data, publicly available information (*eg* areas prone to flood) and competitors' rates are also invaluable. Reinsurers may help provide this.

The base period needs to be chosen to balance the conflicting requirements of relevance and credibility. Data from the most recent years may need adjusting for IBNR and unsettled claims.

Data should be split into homogeneous groups for analysis, usually considering claim frequency and average cost separately, over time. In general, the sub-division should be in as much detail as possible, subject to there being sufficient data in each risk cell to allow credible analysis. Small (attritional) and large claims may be considered separately.

The proposed premium rates must allow for the conditions appropriate to the dates of cover. In particular, unusual features in the base period data, trends that are expected to continue, changes in risk (*eg* mix, cover, underwriting or reinsurance), claims inflation and environmental changes must be allowed for.

A loading may be applied for catastrophe or large loss claims. This can be estimated using own data, external data or a sophisticated catastrophe model.

Further loadings would be applied for:

- the cost of reinsurance, including trends in this cost
- expenses (including commission), *eg* by applying a simple overall loading or a more detailed approach allowing for the different expense types (such as fixed or variable)
- the cost of capital (or profit loading), reflecting the underlying variability of the class of business (net of reinsurance) and shareholder risk appetite.

The premium may also be reduced to allow for investment income on the premiums for the period until claims and expenses are paid out. This is more important for long-tailed business.

Chapter 12 Solutions

Solution 12.1

(i) The amount of premium required to cover the expected claim amounts only.

(ii) The explicit loadings cover the (net) cost of reinsurance, expenses (including commission), the cost of capital (*ie* margin for profit), and investment income.

(There are other considerations to be made, such as the impact of competition, in determining the premium to be charged, but these are usually not allowed for through explicit loadings.)

Solution 12.2

We need to investigate whether the use of alternative rating factors would improve the risk classification and create more homogenous risk cells. Stochastic modelling using generalised linear models (introduced in Subject CT6) might be used.

Note that more detail is given about this topic later in the course.

Solution 12.3

(a) Claim volumes for each risk group could be calculated by applying the projection of expected claim frequency per policy (from the risk premium calculation) to the insurer's forecasts of the number of policies in force during the period that the new rates will apply, *ie*

$$\text{no. of claims} = \text{expected claim frequency per policy} \times \text{forecast no. of policies}$$

(b) Total claim amounts for each risk group could be calculated by applying the calculated future risk premium to the insurer's forecast exposure during the period that the new rates will apply, *ie*

$$\text{claim amounts} = \text{future risk premium} \times \text{forecast future exposure}$$

Solution 12.4

The claim frequency will reduce (since policyholders will not claim for losses less than £50 and may not even bother claiming for losses of, say, £60).

The aggregate claim amount will reduce. All claims will be £50 lower than before, or zero.

The average claim amount (net of the excess) is likely to fall because the vast majority of claims notified will be above the excess. If all claims notified were above the excess, the average claim amount would fall by £50. There will probably be a small number of claims below the excess which are notified, so the reduction will probably be a little less than £50.

Solution 12.5

The safest method is to convert the original claim payments into gross amounts by adding back the excess. Then we project the base values for inflation and subtract the new levels of excess.

Solution 12.6

The mid-point for claim events in the base period is 1 March 2011. The mid-point for new policies to start is 1 April 2013, and 1 July 2013 would be the mid-point for accidents.

Therefore the projection period is 2 years 4 months.

This assumes:

- claims spread uniformly over the base period
- policies incept uniformly over the new rating period
- risk is even over the calendar year.

Solution 12.7

Policies in the base period are written on average in the middle of 2011. Claims from these policies will occur, on average, 6 months' later, *ie* at the end of 2011. Policies under the new rates will be written on average, at the end of 2014. Claims on these policies will occur 6 months later, *ie* in the middle of 2015. The projection period is therefore 3½ years (end-2011 to mid-2015).

Solution 12.8

A possible solution is given after the Question in the chapter.

Solution 12.9

We calculated the risk premium in the new rating period to be £321. Dividing this by 19.395 gives the new risk premium rate as £16.57 per mille. Assuming that the investment return was always 8%, the original risk premium must have been:

$$£267/\{1.08^{(16/12)}\} = £241$$

corresponding to a risk premium rate of $£241/16 = £15.06$ per mille.

It makes sense that the new rate should be higher because the inflation of claims has exceeded the inflation of sum insureds over the period.

Solution 12.10

Although an insurer should charge premiums which reflect the risk, an insurer who charges a higher premium than competitors just because they use more reinsurance is likely to lose out on business volumes.

The reinsurer *will* aim to make a profit. Balancing this, the reinsurance should reduce the riskiness of the insurer's business, enabling it to reduce its contingency margins.

If two insurers charge the same premiums then, all other things being equal, the insurer using more reinsurance will have lower profit margins per policy (unless the reinsurer's risk premium is lower than the insurer's). However, higher reinsurance may require less capital to back the policies and allow higher volumes of policies to be written.

Solution 12.11

The general management costs may be split in proportion to the number of employees in the new business, servicing and claims departments.

The office rent may be split in proportion to the floor space occupied by the three departments.

Solution 12.12

Indirect expenses can be allocated to classes of business or product by:

- floor space occupied by staff
- charge out basis (*eg* for computing costs)
- in proportion to policies in force, premiums received, funds under management, *etc*
- in proportion to all other expenses.

Solution 12.13

The risk here is in selling more smaller-sized policies than anticipated (either more in absolute terms, or as a proportion of the new business portfolio). If this happens the company might not recoup the marginal expenses. In addition, the contribution to fixed expenses would be smaller than expected.

Solution 12.14

High risk classes usually have:

- long tails
- high variation in claim frequencies and claim amounts
- high exposure to catastrophes, accumulations and latent claims
- little past experience on which to base premium rates.

An example of a low risk class is household contents insurance.

Solution 12.15

Competitive forces are likely to influence the profit loading that can be achieved. The company may use the profit loading to come up with a preferable office premium, but will then consider competitors' rates when deciding whether to charge an actual premium that is higher or lower than the theoretical office premium.

The position in the insurance cycle may heavily influence the loading that is used.

Solution 12.16

Equating NPVs at the time the premium is received:

$$P = 0.25P \times 1.05^{2/12} + 0.2 \times 400 \times 1.05^{-16/12}$$

$$P = \frac{0.2 \times 400 \times 1.05^{-16/12}}{1 - 0.25 \times 1.05^{2/12}}$$

$$P = 100.22$$

Solution 12.17

Assume that:

- all policies are annual
- claim events occur and payments are made on average mid-term, *ie* after 6 months
- reinsurance recoveries are made instantaneously.

Equating present values gives:

$$GP = CF' \times CA' \times (1 + h) \times (1 + i)^{-6} + VE + GP \times (c + f + ve + r + p)$$

Rearranging the premium formula becomes:

$$GP = \frac{CF' \times CA' \times (1 + h) \times (1 + i)^{-6} + VE}{(1 - c - f - ve - r - p)} .$$

Solution 12.18

Let *GP* now be the premium rate per mille sum insured, then:

$$\textit{pv of gross premium} = \textit{pv of claims costs} + \textit{pv of expenses and loadings}$$

ie $\quad GP \times SI = CF \times CA + c \times GP \times SI + FE + ve \times (CF \times CA)$

$$GP \times SI \times (1 - c) = CF \times CA \times (1 + ve) + FE$$

so $\quad GP = \dfrac{CF \times CA \times (1 + ve) + FE}{SI \times (1 - c)} .$

Chapter 13

Further considerations when rating

Syllabus objectives

(i) (i) Understand the various components of a general insurance premium.

(ii) Describe the basic methodology used in rating general insurance business.

(iii) Appreciate the various factors to consider when setting rates.

(j) Develop appropriate rating bases for general insurance contracts, having regard to:

- *return on capital*
- *underwriting considerations*
- *reinsurance considerations*
- *investment*
- *policy conditions such as self retention limits*
- *the renewal process*
- *expenses.*

The above objectives are covered in part in this chapter.

0 Introduction

The last chapter discussed the rating process and the items that make up the office premium. This chapter considers some of other considerations that should be allowed for when calculating the premium to charge.

Section 1 covers underwriting and the selection of rating factors.

Section 2 discusses practical considerations.

Section 3 describes systems of experience rating.

1 Underwriting considerations / rating factors

The work involved in the underwriting and acceptance of risks varies greatly from one class of business to another and from insurer to insurer.

For personal lines business such as motor and household, risks are accepted and rated almost entirely by reference to rating factors determined directly from answers to questions on proposal forms. Reference to an underwriter is only made in exceptional cases or for very large risks.

This is because risks are relatively homogeneous in nature and the cost of individual underwriting is not usually justified.

For fire insurance on a large industrial complex, on the other hand, underwriting will be a skilled job based on detailed reports from brokers, surveyors, fire officers and so on. The underwriter might also visit the site and may make recommendations that help reduce the level of risk, for example, the introduction of sprinklers and burglar alarms. Therefore, the rating will be based on much more than a set of objective rating factors.

Recall that fire insurance is another term for commercial property insurance. The considerations for pricing risks on an individual basis are discussed further in Section 2.1.

1.1 Purpose of rating factors

We should establish the pricing structure to give equity between policyholders. The primary function of the rating structure is to categorise policyholders into broadly similar, homogeneous risk groups, so that premium rates are appropriate to the relative risk of each such sub-group. A fair structure will reduce the likelihood of anti-selection by policyholders where a higher proportion of policies are taken up within the risk cells that are relatively under-priced.

The choice of rating factors is heavily constrained by the rating factors currently used in the market and the reliability of data to support alternatives.

Let's suppose that a company wants to extend its portfolio to an additional class of business. Because the company is new to the class, it will not have its own data on which to base a rating structure. The dangers of selection would make the company very reluctant to move away from the rating structures already established within the market.

Even if a company believed that a different rating structure was appropriate, the company would be nervous about moving away from the market structure unless it was certain (*ie* from reliable data) that an alternative structure was appropriate.

However, we will require new rating structures from time to time. We may base these purely on subjective views or may follow a detailed analysis after a period of collecting relevant data.

(a) *Subjective views.* This process will start with an underwriter (or someone) having a hunch that a further rating factor will have a material impact on defining the risk more precisely. Because the company has a relaxed view on risk, it decides to rely upon the underwriter's judgement and whatever scant data it can put together to adopt the new rating factor immediately, without collecting the data as above. In the view of this company, the possibility of enhanced profits outweighs the risk of selection.

(b) *Detailed analysis*. This process could start in the same way, with a "feeling" that a particular factor is relevant. The company then starts to collect as much information as possible on this new factor. It is quite plausible that no reliable past data exists, and that the company will need to collect data gradually over the next two or three years. Assuming that the data is collectable, the company will eventually have enough data to test statistically whether the new factor helps define the risk.

Rating factors are often initially introduced with very small differentials in price between the new risk groups. The rating differentials could then be adjusted over time in line with the actual experience.

1.2 Requirements of rating factors

The most important requirement of a rating factor is that it helps to provide a good definition of the risk.

Good rating factors will satisfy the following requirements:

- **define the risk clearly**

  In quantitative terms, this means that the rating factor should be correlated with the expected claims.

- **do not correlate too closely with other rating factors**

  This ensures that they add value to the underwriting process.

- **are practical to obtain and record**

  We would not ask policyholders for their claims history over their entire lifetime as they may be unable to recall all the information.

- **are objective**

  This avoids disputes between the insurer and policyholder over the truth of the information provided.

- **are verifiable and preferably factual**

  This helps to prevent fraudulent behaviour.

- **are acceptable to the policyholder**

  Otherwise, the insurer may lose potential customers or renewals. For example, requiring genetic test results to be disclosed might be unacceptable to policyholders.

- are non-manipulable

  For example, we would not ask policyholders how many claims they expected to make!

- are acceptable to the market, *eg* brokers may object to a proposed new rating factor.

Example

For example, in car insurance, the insurer would really like to know whether the insured is a good driver, but answers would be subjective and hard to monitor. Therefore, we use objective proxies, such as the number of years since the last claim or the level of no-claim bonus that has been earned.

If the insurer cannot obtain and, if need be, verify the data, it would not be reliable as a rating factor. One exception regarding objectivity is for larger risks where underwriters make their own judgements. This is perfectly acceptable. However, it would not be acceptable to leave the judgements to the policyholders.

Hence, we should choose each additional rating factor to remove as much of the residual heterogeneity as possible.

1.3 *Selection of rating factors*

The rating factors chosen should have as much explanatory power as possible and remove the heterogeneity within each risk group. In selecting the rating factors to be used, we usually do an analysis of variance (ANOVA) exercise. This ANOVA can be a one-way analysis, two-way analysis or multivariate analysis.

ANOVA was introduced in Subject CT3. Each of the above types of analysis is explained in more detail in Chapter 17 (multivariate analysis) of this course.

In the case of one-way ANOVA, we investigate the amount of variability explained by each factor without taking into consideration the correlation between factors.

We may find that when we split policyholders into different age groups, for example, the variability of claims experience within each age group is small relative to the variability in the overall portfolio of risks. Hence the factor "age" helps to 'explain" the variability because, after grouping the policyholders by age, there is little residual variability left within the groups.

In a two-way analysis of variance, we investigate each factor and the correlations between any two of the factors. This can explain the variability better than a one-way analysis.

For example, the one-way analysis may show that the size of a household claim is highly related to both the number of bedrooms and the value of the content. A two-way analysis may reveal that these two factors are in fact highly correlated, so that only one should be included in the pricing factors.

Multivariate analysis, such as generalised linear models (GLMs), is an extension of this. It allows all factors, as well as the correlations between any numbers of the factors, to be investigated.

Generalised linear models are covered in detail in Chapter 16.

We need to achieve a balance between the number of rating factors and the homogeneity of the risks. We should choose each additional rating factor to remove as much of the residual heterogeneity as possible. If the factors do not sufficiently distinguish between different levels of risks, insurers are likely to attract the under-priced risks and lose the over-priced ones. However, if too many factors are chosen, insurers may experience difficulty due to high administrative costs and resistance of the market and brokers.

Question 13.1

State, with reasons, whether the following suggested rating factors could be used in private motor:

(i) Level of driving ability: excellent / good / OK / poor / dreadful

(ii) Colour of your car: white / black / red / blue / other

(iii) How often you drive on high risk roads: very often / sometimes / rarely

(iv) Car garaged at night: yes / no

There is little value in having a rating factor that is fully correlated with another rating factor. In case you find this concept hard to follow, we have set out a simple illustration below.

Illustration of how rating factors reduce heterogeneity

Suppose that for an imaginary class of insurance business for children, policies have one of ten different levels of risk. Grade 1 is the lowest risk and grade 10 is the highest. Suppose also that, on initial inspection of each individual policy, it is not possible to tell the grade of risk.

We happen to have a sample of thirty different policies (of which there are three at each grade, although of course, we don't know that).

For premium rating purposes, we believe that height is a primary rating factor. A division of our sample of thirty by height generates the following two cells:

Short	Tall
1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 7, 7, 9	2, 3, 4, 5, 6, 6, 7, 8, 8, 8, 9, 9, 10, 10, 10

By inspection, we can see that height has been very effective in defining the amount of risk. However, there is still some residual heterogeneity within each risk group (*eg* the grade 9 risk in the short cell and the grade 2 risk in the tall cell).

Let's try to make more homogeneous cells by introducing age as a further rating factor:

	Short	Tall
Young	1, 1, 1, 2, 2, 3, 4, 4, 5, 5, 7, 7, 9	4, 5, 8
Old	3, 6	2, 3, 6, 6, 7, 8, 8, 9, 9, 10, 10, 10

Age and height for children are closely correlated, so this additional rating factor has not helped us much. There is still some heterogeneity within each cell.

We therefore try another factor, which is less closely correlated with the original rating factor. Let's try gender:

	Short	Tall
Boy	1, 1, 1, 2, 2, 3, 3, 4	2, 3, 4, 5, 6, 6, 8
Girl	4, 5, 5, 6, 7, 7, 9	7, 8, 8, 9, 9, 10, 10, 10

With these two rating factors we have started to generate relatively homogeneous cells.

2 Practical considerations

There's an old joke that running a general insurance company can be likened to driving a car:

- the chief executive controls the steering wheel
- the actuary looks out of the back window shouting to the chief executive about which way to steer.

(In the original joke, there are other people in the car, like marketing and sales managers, but let's stick to our own discipline for now …)

So far, this and previous chapters have presented a very "actuarial" approach to pricing. That is, you get the past data, adjust it and project it, and as a result get a set of premium rates to charge going forward.

However, in the real world, things aren't always so pragmatic, and the price actually charged by an insurer can be very different from the theoretical office premium.

This section explains some additional considerations when pricing general insurance.

2.1 Individual risk rating

The methods that we have discussed (in this and the previous chapter) **may equally be applied at class or individual insured level. In both cases we need good quality and adequate data and we need to allow for trends and inflation. In reality data may be sparse at an individual insured level.**

An example here would be if you were asked to provide premium rates for a risk where there simply is no experience. This often arises when the risk is unique and there is no vast database of claims experience that the actuary can reliably analyse using the techniques we have discussed so far.

Here experienced underwriters can assist. Using their knowledge of similar risks, we can estimate appropriate assumptions for the particular perils (and amounts) that apply to the specific case in question.

This was mentioned in Section 1, where we discussed underwriting considerations. There will inevitably be a certain amount of judgement (or *gut feel*) involved.

2.2 Terms and conditions and other coverage changes

We will need to adjust premiums when an individual policyholder alters the basis of risk. We will assess the degree to which the risk potential has changed and its statistical impact on claims experience. We can then decide an appropriate premium adjustment.

An example of this is a commercial property policyholder upgrading the fire prevention system in its insured building. As the risk has probably changed, the policyholder would normally be obliged to tell the insurer, and the insurer then has to decide how to deal with it. In this example, notice that the adjustment in premium would be downward (assuming that the upgrade in fire prevention system reduces the likelihood of claims).

Not all changes in risk have to be notified to the insurer during the period of cover. The policy document will state which events *should* be notified. If a notifiable event is not reported, the insurer usually has the right to not pay *any* claims on that policy, or to cancel the policy retrospectively.

2.3 Market considerations

The prices actually charged by the insurer will reflect the company's market share objective and the competitive environment. There are a number of reasons why the rate charged by the underwriter might differ from the theoretically correct premium. These include:

Business objectives

For example, if the insurer is attempting to increase market share it may charge premiums that are lower than the technical premiums. This may be a short-term measure to satisfy the objectives of the management and shareholders.

Competitive pressures

Most general insurance policies are renewable annually. It follows that policyholders have the opportunity to review the market and move to another insurer if they feel that the insurer will sell them insurance on better terms. Therefore in setting its premium rates, the insurer must have regard to commercial market considerations. If a company tries to charge what it considers to be the theoretically correct premiums, and if these premiums are well above those its competitors charge, it may obtain or retain only a small amount of business.

On the other hand it may not want such loss-making business.

Some segments of the market are highly competitive. Insurers have to adopt market rates to maintain market share.

Competitors' rates are more important for classes with homogeneous risks and large volumes than those with very large and heterogeneous individual risks.

> ***Question 13.2***
>
> What factors other than the premium may influence a policyholder when deciding whether or not to renew the contract?

Difficulty in establishing the technical price

In some classes, experience is extremely volatile and significant judgement is required in pricing the business. In these classes, it is more likely for the price to be market supply and demand driven rather than precisely reflecting the underlying risks.

This is often the case for high layers of reinsurance, for example, or for marine and aviation insurance, or for insurance covering nuclear power stations.

In some circumstances, companies may be able to charge a premium for their brand or customer loyalty. Some customers may have a lower tendency to "shop around" and so, in these cases, the insurer may seek to maximise profit by charging the largest amount each policyholder is willing to accept.

In these cases, insurers might charge a higher premium to customers who renew compared to new customers (who are often much more price-sensitive). This is often called *inertia pricing*.

Once new business and loyalty has been won, it may be possible to increase premium rates in order to restore profitability, as long as persistency is not harmed.

Market conditions

Market conditions can be determined by the stage of the insurance cycle that the product being considered is at. The insurance cycle, also known as the underwriting cycle, was described in Chapter 8

> ***Question 13.3***
>
> What is the "soft phase" of the insurance cycle?

During the soft phase of the insurance cycle, some insurers are willing to follow the market downturn to some degree to maintain a minimum volume of business to support the fixed overhead costs. In addition, the cost of renewal business is lower than for new business, given the lower levels of costs associated with renewals, and so the company may be willing to reduce the premium to maintain its customer base.

As discussed above, the insurer may charge renewing customers a higher premium than that charged to new customers. If this is the case, it will make more profit from each renewal, compared to new policies. On the other hand, if premiums are getting cheaper due to increased competition, renewing customers may be more tempted to switch insurers. The insurer may decide to reduce premiums, particularly on renewal, in order to encourage customer loyalty.

Similarly, in the hard market, insurers will charge a higher rate than is required to cover the technical risks.

They can make more profit because there is less competition.

The importance of the insurance cycle in pricing is discussed more fully below, in Section 2.5.

Market acceptability

As mentioned above, the theoretical premium for renewal business may be lower than that for new business.

This was discussed in the previous chapter in the section on expense loadings.

For some classes of business, for example motor insurance, the claims experience for new business can also be significantly worse than for renewal business.

The theoretical premium should therefore be higher for new business than for renewals.

However, charging these different premiums in practice is unlikely to be acceptable to consumers. As a result, the insurer usually charges the same premium for both new business and renewals, with the total costs spread over all contracts.

It follows that an insurer with a stable portfolio and high rate of renewals can charge a lower premium rate than its competitors, whilst still making profits. The insurer will have to be aware of this situation, and may have to adjust its rates accordingly.

The renewal process

We are saying that the expenses associated with writing new business are generally much higher than those for renewing business. This means that, in most cases, established insurers will incur lower expenses than new insurers because a lower proportion of their business will be for new (as opposed to renewing) policies. Therefore established insurers could charge lower premiums. Whilst this is generally true, there are reasons why new insurers may not necessarily have higher expense levels than their more established competitors.

Because of the difference in costs between new and renewal business, it does not (necessarily) **follow that an established insurer with a stable portfolio and a high renewal rate can charge lower premiums than one with a low renewal rate. A new entrant to the market will have relatively high expenses until its business matures and its renewal rates stabilise.**

Question 13.4

A student having read the above paragraph told a friend:

"There won't ever be any new motor insurers. They wouldn't attract any business because the rates they would have to charge are higher than those charged by an established insurer."

Comment on this statement.

On the other hand, the newer entrant may have lower overheads to be spread per policy. Furthermore, the newer entrants are unlikely to be burdened with or benefit from possible prior years reserve deficiencies / surpluses; that is, the need to increase / release reserves for business already on the books.

Use of no-claims discounts

A no-claim discount (NCD) system is where an individual policyholder may be granted a discount from the relevant base premium depending on his or her claims experience. They are common on private motor insurance, but can be applied to other personal lines products, such as household insurance.

These are often set by market practice rather than being statistically justifiable.

2.4 Profit optimisation

Insurers can optimise the total profit by accepting a lower profit margin on individual policies in return for a higher total business volume. Although a lower profit is achieved by each policy, the business volume generated by the lower level of premium may result in a higher total profit to the company.

In other words, the profit per policy will be lower, but the total profit across all policies may be higher as a result of selling a greater number of policies.

Another element of profit optimisation is flexing the profit over different parts of the book and over time to maximise the present value of expected profit streams.

This often involves modelling the profitability of different segments of the book, together with their responses to premium rate changes (*ie* their price-elasticity). By adjusting the profit margins, and hence the premium rates, to different extents for different customers, the insurer will aim to maximise the overall profit.

2.5 Pricing and the insurance cycle

The insurance cycle is a phenomenon that affects all forms of insurance and reinsurance. It is driven by the changing levels of profitability in the market arising from changes in market capacity.

Stages of the insurance cycle

When premium rates are profitable (hard), new entrants are attracted to the market. This increased capacity causes rates to fall until they may even become loss making. At this point some market participants will leave the market, or reduce their market share. This effect may be exacerbated by catastrophes and economic factors. This reduction in market capacity leads to rates increasing once more until we reach our starting point of the cycle and it begins again.

This was also discussed in Chapter 8.

Reasons for insurance cycles

The cyclical nature of premium rate movements is a very real market challenge, perhaps the most challenging aspect of managing a portfolio of risks for many lines of business.

It is important that the insurer is aware of the stage of the insurance cycle that the market is in when deciding on the final premium.

There are many causes of underwriting cycles, including:

- **a delay in the understanding of the emergence of higher claims costs, which leads to under-pricing of current risks**
- **inflows and outflows of capacity to the market, often as a result of large-scale catastrophes**

  Catastrophes may jolt prices higher due to providers leaving the market hence causing a reduction in supply.

- **deliberate under-pricing (or the use of expense advantage) by key players in an attempt to drive out competitors**
- **an attempt to grow in volume in order to cover high fixed expenses** (or to achieve market share)
- **pricing strategy being determined by chasing market prices (upwards or downwards) rather than being based on sound technical prices, with no player in a given market willing to be the first to break the cycle.**

Different markets are impacted to a greater or lesser extent by these factors, and certain markets tend to exhibit more or deeper cyclical trends than others.

Importance of knowing the current stage of the insurance cycle

We should know the stage of the insurance cycle that the market has reached in order to assess how to allow for business objectives and competition. While we should evaluate every risk on its available information (or that pertaining to its class), the tide and optimism of the market can often affect judgement; and indeed the data itself may be less reliable at certain stages of the market.

Example

In a rapidly rising market, the insurer might give priority to the new business area in the allocation of systems and administration time, with the result that claims information might be less up-to-date.

In addition we might be less prudent in our claims estimation and reserving, if we believe that the business currently being written is more profitable than it really is.

Dealing with the insurance cycle

Market cycles are a feature of the insurance market, but arguably we can reduce the associated risk, if we base prices on a model with a sound technical cost base and communicate this clearly to the pricing decision-makers. Everyone then clearly recognises and understands departures from the technical price. Actuaries can have a key role to play in both of these areas.

Question 13.5

"There is no point in offering an actuarial theoretical premium rate if the rest of the market is undercutting you". Discuss whether you agree with this comment.

Further practical considerations in rating are discussed in the next chapter, when we consider the frequency-severity and burning cost approaches.

2.6 Other influences on rating

These will include the following issues:

- **Competition and the need to maintain or build market share.**

  For example, many new companies offer artificially low premium rates in order to build a customer base. This has been discussed above.

- **The availability of capital to support new business.**

  A large capital base means that riskier products or larger volumes of business can be written. The downside is that, in return for offering greater capital, the providers of that capital will probably require a greater return, which means higher loadings for profit are needed in the premium rates.

- **The impact of reinsurance capacity.**

  If reinsurance is not readily available at an acceptable price, the original insurer may have to increase its premium rates to compensate for the extra risk it retains. In the extreme, it may not be able to offer the product at all.

- **The sophistication of sales and quotes systems.**

  Complicated flexible sales and quotes systems cost money, which probably has to be recouped from premium rates eventually.

- **The demands of regulators in the rating area.**

  In some countries, there are regulatory restrictions on premium rates. This could simply be a requirement to file rates with the regulators (as in parts of the USA), or could be restrictions on rates (maxima or minima) or rating factors used (as in some parts of Africa).

- **Relationships with particular distributors or brokers.**

  For example, in order to appease a particular broker that brings in large volumes of profitable business, a company might wish to tweak prices so that they favour that broker's target market.

- **Differences between the "direct" approach and more traditional routes.**

  The direct approach refers to the relatively new approach of selling via the internet and via direct telesales, the traditional routes being other channels such as using brokers and company representatives.

  The method of sale, as discussed in Chapter 7 (General insurance markets), will impact premium rates. For example, selling over the internet should mean cheaper premium rates as the ongoing expenses should be lower (although the initial development costs will be high).

3 Experience rating systems

Although there is no specific Core Reading on this topic, experience rating systems are mentioned several times throughout the course and are defined in the Glossary. We explain how they work here, so that you can understand the material. This may also prove useful in the exam.

3.1 Description

An experience rating system is one in which the premium for each individual risk depends, at least in part, on the actual claims experience of that risk.

The prospective rating factors that are used to define a rating structure can go only so far in distinguishing between high and low risks. Thus, within any rating cell there will be a mixture of different levels of risk. The concept underlying experience rating systems is that high-risk policyholders tend to remain high-risk policyholders and low-risk policyholders tend to remain low-risk policyholders. It follows that the more claims a policyholder has made in the past the more likely they will be to make claims in the future. It therefore makes sense for an insurer to charge relatively higher premiums in the future to policyholders who have frequently submitted claims and relatively lower premiums in future to those who have been claim-free. Indeed, if they do not adjust their premiums in this way, they may find that the low-risk policyholders are identified and attracted away by competitors and they are left with the high-risk cases for which the premiums will be inadequate.

Question 13.6

This phenomenon is an example of what type of risk?

Some experience rating systems are based on simply the occurrence of claims regardless of their amount, whilst others take account of the cost of the claims. Either type of system may be applied prospectively or retrospectively. It may be more advisable to try to ascertain the risk factors that identify or cause the higher propensities to claim and rate the policy more accurately.

3.2 Prospective vs retrospective basis

With prospective rating, the premium at the renewal date depends on the experience of the risk prior to that renewal. The insurer takes on all the underwriting risk in such an arrangement.

Prospective rating uses the past experience as a rating factor. NCD in private motor is a prospective system of experience rating, because if you make a claim now it will affect your next premium. The underwriting risk is that poor risks may not renew their policies so that the insurer is unable to recoup its losses. Note how this contrasts with retrospective rating below.

With retrospective rating, the premium for the current policy period is adjusted, based on the experience of that period of risk. A deposit premium, paid at the inception of the policy, will usually be followed by an adjustment premium, or refund, at the end of the period.

Retrospective rating uses the experience during the exposure period as a rating factor. The underwriting risk to the insurer is reduced here because a further premium may be charged following poor experience by a risk during a particular year.

3.3 Number-based systems

This is where the premium adjustments (whether prospective or retrospective) are based on the number of claims paid in respect of the policyholder, and the amounts of the claims are ignored.

The most common examples of experience rating systems based solely on the occurrence or non-occurrence of claims, regardless of their amount, are the bonus-malus or NCD systems commonly used in motor insurance for policies covering a single vehicle. Relating the future premiums to the cost of claims would scarcely be practical in view of the time that may elapse before claims are settled, and it is doubtful whether the additional information would significantly enhance the predictive value.

You should have already come across NCD systems in Subject CT6. A bonus-malus system is a type of NCD system in which the premium paid by a policyholder after a claim has been made may be higher than the original premium paid when the policyholder first took out the policy. You could think of the system as having negative as well as positive discounts. The term is used throughout Continental Europe and elsewhere.

Systems based on claim frequency are generally used for small individual risks with relatively low expected numbers of claims, as the variability of the actual claim cost in any one policy year would be too great to judge the relative severity of the underlying risk.

3.4 Cost-based systems

This is where the premium adjustments (whether prospective or retrospective) are based on the total amount of claims incurred in respect of the policyholder over a defined period.

Systems based on the cost of claims tend to be used for larger risks or groups of risks where the aggregate cost of claims experienced within a year may be a more suitable indicator of the relative level of the underlying risk.

Examples of classes that might use cost-based systems include motor fleet (for larger fleets) and employers' liability.

Question 13.7

State whether or not the following arrangements are examples of experience rating.

(i) Profit sharing, where the insurer charges a higher initial premium, and returns some profit to policyholders whose claims are lower than expected.

(ii) The policyholder pays an end of year adjustment premium to reflect the amount of exposure during the year (*eg* as in employer's liability).

(iii) As for (i), but the adjustment is based on the insurer's overall experience for all policies of this type.

(iv) A system where a policyholder's next premium is the average of the insurer's book rates and the policyholder's average claims cost over the last year.

Question 13.8

What are the two main types of experience rating?

Question 13.9

Explain in layman's terms what is meant by a "retrospective system of experience rating".

4 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Bonus hunger
- Experience rating
- Fleet rating
- Loss sensitive
- Swing rated.

Chapter 13 Summary

The work involved in the underwriting and acceptance of risks varies greatly from one class of business to another, *eg* commercial property risks may be priced on an individual basis, based on the underwriter's subjective assessment of the risk.

Risks should be separated into homogeneous sub-groups for premium rating. An equitable pricing structure will reduce the likelihood of anti-selection. However, the choice of rating factors is constrained by those used in the market and the reliability of data to support alternative rating factors.

Rating factors should define the risk clearly, not correlate too closely with other rating factors, be practical, objective, verifiable and acceptable to the market.

An analysis of variance (ANOVA) can be undertaken to help determine the rating factors. This might be a one-way, two-way or multivariate analysis (*eg* GLMs) – the allowance for correlations between factors depends on the approach taken.

Each additional rating factor should be chosen to remove as much of the residual heterogeneity as possible. However, too many rating factors can be expensive and be unpopular with the customers and brokers.

The premium actually charged will differ from the office premium, as it should reflect the company's market share objective and the competitive environment. This might be the case:

- to meet business objectives, *eg* to increase market share
- to maintain market share in highly competitive markets or in certain market conditions (such as the "soft phase" of the insurance cycle)
- if it is difficult to establish the technical premium
- if insurers can charge certain loyal customers more (inertia pricing)
- if the market does not accept different premiums (*eg* between new business and renewal premiums)
- where no-claim discounts apply.

Theoretically, premiums for new business should be higher than those for renewals. However this is rarely the case due to market pressures. This means that an established insurer with a stable portfolio and a high renewal rate can usually charge lower premiums overall than one with a low renewal rate, although this might not always be the case.

Insurers can optimise the total profit by accepting a lower profit margin on individual policies in return for a higher total business volume, which may result in a higher total profit to the company.

It is important to know the stage of the insurance cycle the market is currently facing in order to understand how business and competition should be handled. We can try to reduce the risk of the insurance cycle by accurately modelling the theoretical price and communicating this to the pricing decision-makers.

There are other practical considerations affecting premium rates, which include:

- competition and market share
- capital availability
- reinsurance capacity
- sales and quotes systems
- regulations
- relationships with sellers
- the method of sale.

Adjustments to premiums will also be required when an individual policyholder alters the basis of risk.

Experience rating is where the premium a policyholder pays depends on their individual claims experience. It can be applied prospectively or retrospectively, using claim numbers or claim amounts.

Chapter 13 Solutions

Solution 13.1

(i)	Driving ability	No. It is subjective and not directly verifiable.
(ii)	Car colour	Possibly, although it might be argued to be irrelevant, and shades may not be easily measurable.
(iii)	Risky roads	No. It is subjective and impossible to verify.
(iv)	Night garage	Yes, although it would be impractical to check that an available garage was actually used.

Solution 13.2

Other factors that might influence a policyholder are:

- the level of service
- the policy conditions
- the reputation of the company
- the ability to pay out claims
- the reluctance to pay out claims
- any loyalty discounts
- any advertising campaign
- inertia (*ie* it's convenient not to have to shop around)
- payment method (*eg* if paying by direct debit, no effort is required to shop around).

Solution 13.3

The "soft phase" of the insurance cycle is the stage when there is more competition in the market, and so premium rates are generally low and therefore less profitable.

Solution 13.4

The statement is almost certainly not true.

A new company may be able to subsidise its premium rates until it has a mature portfolio.

The new company may accept a lower profit margin, especially if it entered the market at the top of the insurance cycle.

The brand name of the company may be attractive to potential customers.

The company may introduce innovative product designs, *eg* new scales of NCD.

A new company may employ good managers and use efficient new processes that result in lower expenses than the established companies.

Innovative product design or services may attract customers.

A new company will not need to have its profits depressed by the need to fund any prior year deficits which some established insurers might have.

Solution 13.5

There is certainly no point in offering higher premium rates than everybody else, if the higher price means that you attract no business and the business would have been profitable at the lower rate.

However, if the business would be unprofitable at the lower rate, then it may be best not to take on the business (although a view will have to be taken on the *long-term* impact of turning down business – for example, it may in the long run be more expensive not to sell business and suffer the later cost of trying to rebuild market share).

There are also differentiators other than price. A more attractive product may still win business despite being more expensive if the added benefits outweigh the extra cost to the policyholder.

Solution 13.6

Anti-selection.

Solution 13.7

(i) Yes. This is a typical retrospective arrangement based on claim amount.

(ii) No. This adjustment is nothing to do with claims experience.

(iii) No. The adjustment is not based on the claims experience of the individual risk.

(iv) Yes. This is a simple prospective arrangement based on claim amount.

Solution 13.8

Trick question: we don't think anyone can say which of the two splits is the main one: *ie* numbers *vs* amounts, or retrospective *vs* prospective.

If asked to give examples of the two types in practice you could use the same pair whichever your split, *ie* private motor NCD (prospective, numbers) *vs* a large motor fleet (retrospective, amounts).

Solution 13.9

Policyholders end up paying a premium for cover over a given period that depends somehow on the claims they make during that same period. They usually get a partial rebate or pay an extra premium at the end of the period, once everyone knows what claims, if any, they actually made.

Chapter 14

Rating using frequency-severity and burning cost approaches

Syllabus objectives

(k) (i) Describe the burning cost approach to rating.

(ii) Understand the assumptions required when using this approach.

(iii) Outline the practical considerations when using this approach.

(l) (i) Describe the frequency-severity approach to rating.

(ii) Understand the assumptions required when using this approach.

(iii) Outline the practical considerations when using this approach.

0 Introduction

So far, we have covered the general principles employed in rating general insurance business. We now discuss two specific techniques that are used under the cost plus approach to calculate the risk premium (*ie* the expected cost of claims in the period that the premiums will apply):

- the frequency-severity approach
- the burning cost approach.

The loadings described in Chapter 12 would then be applied to this risk premium, together with further considerations, such as rating factors and practical considerations (discussed in Chapter 13), to arrive at the premium to be charged.

Question 14.1

List the loadings that would be applied to the risk premium to derive the office premium.

Even though both of these approaches are commonly used in rating general insurance, the precise meaning of these terms can sometimes be a little ambiguous.

At a very high level, the term "*frequency-severity*" can be used to describe any rating technique that projects frequency and severity separately, and then combines the two in order to calculate the risk premium.

This contrasts with a *burning cost* approach, which (pretty much) ignores the number of claims, but rather projects $\dfrac{\text{total claims}}{\text{total exposure}}$ to obtain the risk premium directly.

In practice, there are variations within each method. The Core Reading in Section 1 of this chapter describes a specific frequency-severity approach in detail, which is mainly used for rating a single commercial risk. The discussion of the burning cost approach in Section 2 is more general, and covers various approaches.

We have split this chapter into two parts. In the first part (Section 1)**, we discuss the frequency-severity approach. In the second part** (Section 2)**, we discuss the burning cost approach.**

In each part, we cover some or all of the following topics:

- **a description of the method**
- advantages and disadvantages (relative to the other approach)
- **a description of the data required**
- **adjusting for trends**
- **developing loss values to their ultimate level**
- **fitting distributions to trended and developed losses (first part only)**
- **the use of simulation models (first part only)**
- **practical considerations.**

The premiums derived in this chapter are risk premiums. We would then convert these into office premiums as discussed in the previous two chapters.

1 Frequency-severity approach

1.1 Description

In general, we use the frequency-severity approach to calculate premiums for commercial risks.

When we use this approach, we assess the expected loss cost for a particular insurance (or reinsurance) structure by estimating the expected claim frequencies and severities for that structure and combining the results.

The material in this section mainly covers a specific statistical approach, involving fitting distributions to frequency and severity. We shall see later that this approach is particularly useful where excesses, deductibles or limits apply. This material is written mainly in the context of rating liability insurance on a single commercial risk, *eg* a large company, although it can be used for rating other forms of business, *eg* excess of loss reinsurance.

The key assumption is that the loss frequency and severity distributions are not correlated.

The analysis and projection of frequencies is separate from the analysis and projection of claim amounts.

Advantages and disadvantages

We list below some advantages and disadvantages of the approach that will be described in this section relative to the burning cost approach, which is described in Section 2.

Note that the burning cost approach is based on aggregate claims, and so does not make use of the number of claims or individual claim amounts.

Advantages of the frequency-severity approach are that:

- **This approach mirrors the underlying process – a number of losses are generated, each with its own ultimate value – and so is readily understood by underwriters.**

  In other words, the model tries to reflect what happens in practice, in that claims occur and each of these claims has a value. The burning cost approach simply looks at the total claims, so takes no account of the underlying *process*.

- **We can use the approach for complex insurance structures.**

  In particular, the impact of deductibles or limits can be allowed for appropriately.

- **By separately assessing information on loss frequency and severity, we gain additional insight into aggregate loss amounts.**

- **It helps us to identify trends.**

  Trends in frequency and severity can be allowed for separately. These trends may not be apparent if only the burning cost was considered. For example, an increasing frequency may be offset by a decreasing severity, and so no trend would be observed if we were only looking at the overall claims per exposure.

Disadvantages of the frequency-severity approach are that:

- **Assessing the compound frequency-severity loss distribution has more onerous data requirements than assessing aggregate amounts.**

  For example, you do not need information on claim numbers to use the burning cost approach.

- **This approach can be time-consuming for a single risk and requires a high level of expertise.**

  We will see that it is much more complex than the burning cost approach. For example, it often requires simulation techniques.

- **More data is required.**

  A sufficient volume of data is required to:

  - derive appropriate development factors with which to calculate ultimate claim amounts
  - fit statistical distributions to the data with an acceptable goodness of fit.

  These processes will be covered in Sections 1.4 and 1.5 respectively.

As a consequence, frequency-severity approaches are commonly restricted to commercial lines (liability) classes of business.

This is because single risks can give rise to many claims in commercial lines liability classes, unlike in personal motor for example.

1.2 Data

We require more data to parameterise separate frequency and severity distributions than we do to assess aggregate amounts.

Information required

Chapter 12 covered the data requirements for rating using any cost plus approach. We now look at the specific case of using the frequency-severity approach for rating liability insurance for a single commercial risk, *eg* a large multi-national company.

The information required for frequency-severity rating of commercial risks is usually provided by a broker, although for renewal business an insurer will usually have some information on its own systems.

The broker will usually provide the following items:

Submission document

This provides background information about the insured as well as details of the cover sought. This document is likely to be of a qualitative nature and written from an underwriter's viewpoint. However, information on the countries and industries in which the insured operates may be useful as rating factors where an insurer has compiled benchmark information.

Exposure information

This shows the insured's historical exposure as well as that projected for the prospective policy period. Common exposure measures are:

- **for pubic liability – turnover**
- **for employers' liability – employee numbers and wage-roll, sometimes split between clerical and manual.**

To allow for the actual future exposure turning out differently to that projected, we usually express the final premium as a rate per unit of exposure so that it is adjustable (in line with the exposure).

A deposit premium will be paid at the inception of the policy. This will usually be followed by an adjustment premium, or refund, at the end of the period. This is because the size of the exposure (*eg* number of staff employed during the policy year) is unlikely to be known with certainty at the start of the period.

Individual loss information

This provides details on each historical loss. Large insureds may provide information on several thousand past losses so should provide this information in electronic format rather than hard copy for it to be of use. The suggested data requirements can be found in Chapter 12.

So for a single commercial risk, the broker will provide qualitative background information (in a submission document), plus details of the risk's exposure (past and present) and claim history. The details of the actual data items required were given in Chapter 12.

Common data issues

Form of data

We should obtain individual loss information gross of reinsurance and ideally "from the ground up" (FGU) for all claims.

Question 14.2

What is meant by *from the ground up*?

If we only obtain losses above a certain threshold, the analysis may be complicated and we may need to make an assumption about the distribution of losses below this level. One possible source of data for this sort of adjustment may be the insurer's own databases.

Consistency between claims and exposure

The individual loss information must be consistent with the exposure information provided. In particular, where we have adjusted exposure information to reflect corporate acquisitions or disposals, we should adjust the treatment of individual loss information in a similar way.

Adjustments for changes in risk were described in Chapter 12.

Choice of base period

We should obtain loss and exposure information for as many historical years as is possible. Typically we require five years of data when we apply frequency-severity approaches to liability classes, but more is desirable. (This volume of information is generally available in the UK market but may not be in other countries.)

The period of historical data required for other classes would depend on the length of reporting delays and typical claims frequency.

Question 14.3

Explain whether the base period required would be longer or shorter if the class of business had:

- a shorter reporting delay
- a lower claim frequency.

1.3 Trending

When we use frequency-severity methods, we should apply separate frequency and severity trends to losses.

Chapter 12 described, in general terms, the factors that would be allowed for to make the data from the base period relevant for the period for which the new premium rates will apply. The process of determining and applying these adjustments to the base data is often called "trending". Note that trending does not include developing the claims to ultimate values. That part of the process is covered in Section 1.4.

Question 14.4

List some of the adjustments that would be made to the base data in the trending process.

We now explore in detail how these adjustments are actually applied when the frequency-severity approach is being used. Under this approach, frequency and severity are considered separately. For each, the combined impact of all the relevant adjustments is usually summarised by constructing a series of index figures.

General considerations

We should:

- **first project historical frequencies and severities in line with assumed trends to current values and**
- **then project them to the mid-point of the future exposure period.**

As such, the assumed trends will contain both past and future components.

Rather than applying a constant past annual trend rate, a more realistic approach is to apply an index, which can reflect periods of high and low trend and incorporate discontinuities caused by one-off changes in the legal environment.

For simplicity, some exam questions on rating have used a constant rate, *eg* 3% *pa*. Other questions have provided an index showing figures for each year, for example:

Year	2006	2007	2008	2009	2010
Index	100	105	110	108	114

Although trends are typically thought to increase over time, decreasing trends can occur as well. For example, a company that has strengthened its health and safety procedures may show a decreasing incident frequency trend.

Having obtained a trend, be it increasing or decreasing, you should ask yourself whether it is reasonable, and whether it is likely to continue in the future.

We usually consider catastrophic losses separately, as we have mentioned in Chapter 12. It is usual to use the output from one of the many catastrophe modelling systems rather than take the average of past experience.

Catastrophe modelling will be discussed further in Chapter 21 (and in Subject SA3).

Frequency

The causes of frequency trends include changes in:

- **accident frequency**
- **the propensity to make claims and other changes in the social and economic environment**
- **legislation**
- **the structure of the risk.**

Environmental influences, including social (which includes the propensity to make claims), economic and legislative effects were discussed fully in Chapter 8

Changes to the structure of the risk could include (in the case of employers' liability insurance) changes in the employee profile, working practices, policy conditions or underwriting.

The term "structure" is sometimes used to specifically refer to changes in the policy conditions and in particular, excesses/deductibles, limits or experience-rating arrangements. Examples of some complex structure features are given later, in Section 1.6.

Question 14.5

Which of the above four causes of frequency trends are likely to be "one-off" changes and which are likely to be continual trends?

For each historical policy year, we can calculate the frequency of losses as:

$$\textbf{frequency} = \frac{\textbf{ultimate number of losses}}{\textbf{exposure measure}}$$

The way in which the ultimate number of losses can be obtained from the number of losses reported to date will be explained in Section 1.4. However, for now, you just need to recognise that the number of claims divided by the appropriate exposure measure is calculated for each year of account. For example, for employers' liability insurance, this would be the number of claims per employee. These observed frequencies would be analysed year-on-year.

Although the pattern of historical frequencies by year for the individual risk provides an indication of the frequency trend to apply, we rarely rely on this. More often we apply a standard trend, which may be based on:

- **an analysis of all risks within an insurer's portfolio, or**
- **external information, such as industry surveys.**

However, if there have been changes *specific* to the risk involved, the standard trend index should be adjusted to allow for these if possible. Examples of such changes would be changes to the policy conditions, to the employee profile, or to the health and safety measures employed.

If the exposure measure is expressed in monetary terms – for example wage-roll or turnover – then it is important that, when we apply trends to the historical frequencies, we also allow for inflation of this exposure measure.

When we have projected the frequency for each historical year in line with trends to the prospective policy period, we can apply these to the projected exposure to produce a range of estimates of the number of losses for the future period.

So, for each year in the base period, an estimate of the expected number of losses during the rating period will be obtained.

Severity

The drivers of severity trends include:

- **economic inflation**
- **changes in court awards and legislation**
- **economic conditions**
- **changes to the structure of the risk.**

The first three of these were covered in Chapter 8.

We usually apply severity trending at the ground-up individual loss level (the whole loss without deductions for excesses or deductibles), whereas we apply frequency trending to the claim frequencies for each historical policy year.

This is an important difference. We end up with:

- an estimate of the number of claims for each year in the base period, so we would obtain, say five estimates if there were five years in the base period
- an estimated claim amount for *each* individual claim in the base period, and there could be lots of these!

As part of the process of deciding what trend should be applied to the claim amounts, the losses may be grouped, *eg* into years. However, the trend adjustments would apply to each individual loss. The adjusted claim amounts should then be relevant to the future rating period.

For each historical policy year, we can calculate the average severity of losses as:

$$\textbf{average severity} = \frac{\textbf{ultimate cost of losses}}{\textbf{ultimate number of losses}}$$

As with the frequency trend, we can use the observed pattern of historical severities for the risk as an indication of the severity trend to apply but it is more common to apply a standard trend.

For the UK, this may be based on the insurer's whole portfolio or publicly available sources such as the ABI or Office for National Statistics (ONS).

Although we apply trends at an individual loss level, it can be useful to review the pattern of past severity values by policy year both:

- **before we apply the trends – as an indication of the historical trend to apply, and**
- **after we apply the trends – if the projected severities are similar for each policy year, this indicates that an appropriate trend has been applied.**

Question 14.6

For employers' liability insurance, give examples of possible drivers of:

- frequency trends
- severity trends.

Split by size of claim

We may consider losses in aggregate, or banded into two or more size-based groupings. If we follow this approach, we will also need to split the frequency.

We usually apply the same severity trend to all claims irrespective of their size. It is generally believed, however, that inflation affects different sized claims differently and that the inflationary trend will be depend on the size of the claim. For example, small losses are likely to have a relatively small legal expense component and so be less exposed to legal cost inflation than large losses.

We can increase the accuracy by applying a severity trend that is a function of the size of loss. Against this, the approach is more complicated and is rarely used for primary or low excess layers.

For pricing non-proportional reinsurance contracts, the need to accurately estimate the individual claim amounts becomes greater at high levels of excess.

Large losses

We should consider the impact of large losses. If we do not adjust for the impact of large losses, we may obtain a misleading severity pattern and assume inappropriate severity trends. Approaches include:

- **capping large losses (although the selected capping level will itself affect the resulting trend)**
- **basing the trend on the historical median rather than mean values**

  A few very large losses may distort the mean significantly but will have little effect on the median if we have a large number of attritional claims. However, this approach might not give enough allowance for large losses.

- **assessing large losses separately from the smaller losses.**

Any large losses that are removed from the severity analysis must also be removed from the frequency analysis. They would then be allowed for separately, as discussed in Chapter 12.

Results of trending

Following trending, an estimate of the amount of each claim in the base period, had it occurred during the future rating period, will be obtained.

When we have projected each individual loss in line with trends to the prospective policy period, we can fit a claim size distribution to these observations.

After trending the losses, we would then develop them to their estimated ultimate values before fitting a claim size distribution. We discuss how to obtain ultimate claim amounts in the next section.

1.4 Developing losses

Adjustments required

The claims data collected at any one point in time will have some claims that:

- have been reported but have not been fully settled, *ie* outstanding reported claims
- have occurred but have not been reported yet – these will not be included in the database, but may arise on policies that were written during the base period.

Standard reserving techniques, such as the chain ladder or Bornhuetter-Ferguson methods, could be used to calculate the ultimate overall claim amounts for each development year. These methods are described in Subject CT6 and Subject ST7. However, for the frequency-severity approach, we need to consider the following two components separately:

- the ultimate cost of outstanding reported claims
- the number and ultimate cost of incurred but not reported claims.

Outstanding reported claims

For outstanding reported claims, the insurer will normally have data on any claim payments made plus estimates of each outstanding claim amount (known as the "case estimate"). The sum of the two is called the reported incurred claims, and so we have:

$$\text{reported incurred claims} = \text{claims paid} + \text{case estimates}.$$

The calculated risk premium should allow for expected differences between the current reported incurred claims and the ultimate claim amounts expected to be paid. This will reflect any over- or under-reserving in the case estimates. This adjustment is sometimes referred to as the "IBNER (incurred but not enough reported) development".

When we have obtained the data, we should increase individual loss amounts in line with development factors to their estimated ultimate level.

Methods of doing this for the frequency-severity approach are discussed in detail below.

Incurred but not reported (IBNR) claims

We should also develop the reported loss count for each historical policy period to its ultimate level. This will depend on the reporting delays experienced.

Methods for calculating the allowance for IBNR explicitly include:

- simple ratio methods, where the IBNR is taken to be a percentage of a figure in the accounts (*eg* annual premiums or claims)
- using a delay table of reporting delays to estimate the number of outstanding claims and multiplying this by the expected average IBNR claim amount
- a projection method, which uses the historical development of claims by origin year to project the IBNR.

A delay table is also known as a claims run-off analysis.

These methods are not discussed further here, but are described in Subject ST7.

Note that the term "incurred claims" is sometimes used to refer to the reported incurred claims and sometimes used to refer to the reported incurred claims plus IBNR and IBNER. It is therefore important to clarify the intended meaning of "incurred claims" before using data with this label.

Developing individual losses

When we apply frequency-severity methods it is important to develop *each* individual loss to its likely ultimate level, instead of only developing the aggregate loss amount for a cohort of claims. We tend to use the latter approach for burning cost methods.

Methods used

We may develop individual loss amounts in a variety of ways:

1. **We could apply an incurred development factor to each individual loss (that is, open and closed claims), reflecting its maturity, to estimate its ultimate settlement value.**

   The development factors could be obtained from a chain-ladder type projection of a development triangle of incurred claims.

2. **A more realistic approach is to only develop open claims using "case estimate" development factors. These case estimate development factors will usually be higher than incurred development factors at the same maturity to offset the effect of not developing closed claims.**

   A comparison of the above two methods is given in the example below.

3. **For losses not in the data set, we will need to assume an ultimate size, usually based on the known losses from this cohort.**

   This is referring to the IBNR claims. The number of IBNR claims could be derived using the delay table method mentioned above, for example.

4. **We could use stochastic development methods to allow for the variation that may occur in individual ultimate loss amounts around each of their expected values.**

   Stochastic claims reserving methods are covered in Subject ST7.

Excesses and limits

For commercial lines policies, it is common for the insured to retain the first part of the loss amount and for there to be a policy limit. In these situations it is critical to estimate accurately the loss amounts that are below the insured's retention and those that are above the policy limit. To do this, we need a good understanding of the likely distribution of ultimate losses, and the need for this accuracy is the reason for the increasingly complex methods listed above.

The following simple example illustrates why it is particularly important to accurately determine ultimate claims on an individual loss basis in the situation where excesses, deductibles and/or limits apply.

Example

The table below is a simple example and illustrates the difference in estimated ultimate FGU and layer loss costs as a result of developing case estimates (Method **2) rather than incurred claim amounts (**Method **1).**

We assume the four claims are of the same maturity so that the same development factors can be applied to each claim.

Under Method 1, an incurred development factor of 2 is assumed to apply to each claim, and so we have:

$$\text{estimate of ultimate claims} = \text{incurred claims } \times 2$$

Under Method 2, a case estimate development factor of 3 is assumed. This only applies to the open claims, and so we have:

$$\text{estimated ultimate claims} = \text{claims paid} + \text{case estimate} \times 3$$

Loss	Data			Estimated Ultimate		Loss Cost > 50,000	
	Paid	Case	Incurred	1. Incurred	2. Case	1. Incurred	2. Case
1	5,000	-	5,000	10,000	5,000	-	-
2	5,000	-	5,000	10,000	5,000	-	-
3	10,000	-	10,000	20,000	10,000	-	-
4	-	20,000	20,000	40,000	60,000	-	10,000
Total	20,000	20,000	40,000	80,000	80,000	-	10,000

(This table is part of the Core Reading.)

The case estimate development factor (of 3.00) is consistent with the incurred development factor (of 2.00) in that both methods project a total ultimate cost for the four losses of £80,000. However if the insured were to retain the first £50,000 of each loss, the case estimate development method would project an excess loss cost of £10,000 whereas no cost would be projected by the incurred development method.

Using case estimate development factors (Method 2) is more accurate than applying incurred claim development factors (Method 1). Of course, if data and available resources permit, a stochastic reserving method may be preferable.

If a deductible is being reduced then both the frequency and severity of losses to a contract may increase. This makes it especially important to use information on claims from the ground up.

Source of development pattern

Ideally we would base the development patterns on the insured's own experience and so take account of its particular mix of claims. However, development triangles may not be available for an individual insured, or may lack credibility, in which case we may apply benchmark patterns.

Even when a development triangle is available for the insured's own experience, it is important to be aware of the different insurers that have held the risk over the period being considered. The claims departments of different insurers may have quite different case reserving philosophies, for which we should allow.

The accuracy of the case estimates may therefore vary over time, and by type of claim.

It is important to apply a development pattern that is appropriate for the losses being developed. For example, it would not be appropriate to apply a ground-up development pattern to losses relating to an excess layer.

When pricing a layer of excess of loss reinsurance, the usual approach is to:

- trend each loss at each development period (using ground up data)
- apply the excess and limit to each claim at each development period
- aggregate the claims for each origin year to create a triangle of trended claims to the layer (*ie* no longer "ground up")
- develop the claims based on this triangle, using one of the above methods.

Question 14.7

Explain why it is more accurate to use a development pattern based on claims to the excess layer rather than one that applies to the ground up claims.

Further, we could apply different development patterns to losses within different size bands. However, as when we apply trends, the resulting increased accuracy is usually outweighed by the increased complexity of this approach and we rarely do this for primary or low excess layers.

1.5 *Fitting distributions*

If there is a sufficient number of historical claims, we may estimate the expected loss cost for the future policy period directly from the historical experience.

This would be the traditional approach to pricing, *ie* of multiplying the projected claim frequency and severity to obtain the risk premium. This may be appropriate for simple structures where there is sufficient data to calculate the risk premium with certainty.

However it is more usual to fit frequency and severity distributions to the losses after we have adjusted them for trends and development and then estimate the loss cost for the particular structure being considered from these.

This approach is now discussed in detail. The stages involved are:

- decide on an appropriate base period from which to collect the (trended and developed) data
- choose a suitable distribution (or density function) for each of frequency and severity
- estimate the required parameters for the chosen distributions
- check that the goodness of fit are acceptable and, if not, attempt a fit with different distributions.

Choice of base period

An important step in the fitting process is the choice of base period (that contains the historical claims and experience data to be analysed) on which the fit is to be based.

Estimated ultimate losses for older policy years will be more reliable. However, these will be less relevant to the prospective policy period than more recent years.

Question 14.8

Explain the above sentence.

Practical approaches include:

- **excluding information from the most recent historical year**
- **excluding information from any year less than a specific percentage developed**
- **including all years, but giving more weight to the more developed years (the Cape Cod approach is an example of this).**

The Cape Cod approach is an objective way of deriving a weighted average, where the weight applied to each year depends on the expected proportion of claims developed (based on chain ladder cumulative development factors) and the proximity of each year to the origin year concerned.

The approach chosen will possibly depend on:

- the extent of changes affecting the relevance of the data
- the development pattern (*ie* type of business and length of tail)
- the credibility of the data available (*ie* volatility and volume of data)
- the base period used in previous rating exercises
- subjective judgement.

Choice of distributions

One way to choose a suitable distribution for claim severity would be to plot each of the observed claims in a bar chart, or equivalent, by size of claim:

Now convert the left-hand scale so that the total area under the curve is 1, *ie* by dividing through by the total number of observed claims.

Then select a function, $y = f(x)$ which has a similar shape to our plotted data. This is the probability density function (pdf).

The following loss distributions are often used:

- **Frequency: Poisson, negative binomial**
- **Severity: log-normal, Weibull, Pareto, Gamma.**

Remember that key information on many statistical distributions is given in the *Tables*.

We would then select a method of fitting to find parameter values for our chosen distribution. We may use different methods (*eg* method of moments or method of maximum likelihood) and then select the one that gives the best fit to our data.

The method and parameters that have been fitted would be scrutinised using a number of statistical tests to determine how well the observed claims fit the modelled claims.

The fitting process

Proprietary software packages are available, which help to fit distributions to loss data. However, fitting routines can be developed in-house.

Whichever approach is used, it is important to be aware of the underlying fitting algorithm (to choose the parameters of the distribution). Common methods are:

- **maximum likelihood estimation**
- **method of least squares**
- **method of moments**.

By plotting the density functions of the observations and fitted distribution, we can quickly assess the goodness of fit by eye.

However, statistical goodness of fit tests are more robust. As with the fitting algorithm, it is helpful to be aware of the features of any statistical tests that are used.

The following tests are commonly used:

- **Chi-Squared statistic**

  A chi-squared test, also referred to as a chi-square test or χ^2 test, is any statistical hypothesis test in which the sampling distribution of the test statistic is a chi-squared distribution when the null hypothesis is true, or any in which this is *asymptotically* true, meaning that the sampling distribution (if the null hypothesis is true) can be made to approximate a chi-squared distribution as closely as desired by making the sample size large enough.

  Chi-squared tests were covered in Subject CT3.

- **Kolmogorov-Smirnov statistic**

  The Kolmogorov-Smirnov test (K-S test) is a nonparametric test for the equality of continuous, one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution (one-sample K-S test), or to compare two samples (two-sample K-S test). The Kolmogorov-Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples.

  The null distribution of this statistic is calculated under the null hypothesis that the samples are drawn from the same distribution (in the two-sample case) or that the sample is drawn from the reference distribution (in the one-sample case). In each case, the distributions considered under the null hypothesis are continuous distributions but are otherwise unrestricted.

  The mathematics behind the Kolmogorov-Smirnov statistic is beyond the scope of Subject ST8.

- **Anderson-Darling statistic.**

  This is a statistical test of whether there is enough evidence that a given sample of data did not arise from a given probability distribution. In its basic form, the test assumes that there are no parameters to be estimated in the distribution being tested, in which case the test and its set of critical values is distribution-free. However, the test is most often used in contexts where a family of distributions is being tested, in which case the parameters of that family need to be estimated and account must be taken of this in adjusting either the test statistic or its critical values.

When applied to testing if a normal distribution adequately describes a set of data, it is one of the most powerful statistical tools for detecting most departures from normality.

Indeed, various studies have found that this is more powerful for testing for normality than the Kolmogorov-Smirnov statistic.

In addition to its use as a test of fit for distributions, it can be used in parameter estimation as the basis for a form of minimum distance estimation procedure.

K-sample Anderson-Darling tests are available for testing whether several collections of observations can be modelled as coming from a single population, where the distribution function does not have to be specified.

The mathematics behind the Anderson-Darling statistic is beyond the scope of Subject ST8.

Proprietary software packages may fit a wide variety of distributions to the observed data. Although goodness of fit tests provide a strong indication of the distribution that should be used, it is also good practice to consider whether the chosen distribution makes theoretical sense.

For frequency, a negative binomial distribution is commonly used in practice, because it allows for dependencies between claims, whereas the Poisson distribution assumes successive claims are independent of each other.

Question 14.9

An excess of loss contract provides cover of £250,000 in excess of £50,000 for any one claim. Assuming that the underlying gross claims have a claim frequency of c and a claim size distribution of $f(x)$, derive the equations for calculating:

(a) the claim frequency, and

(b) the average cost per claim,

under the excess of loss contract.

Combining severity distributions

When an insured has a sufficient volume of losses, we may fit a number of different severity distributions to different parts of the overall loss range. For example, we may fit separate distributions to:

- **the smallest attritional losses, say those below £5,000**
- **losses greater than £5,000 up to a large loss threshold, say £500,000**
- **losses over the threshold.**

We can justify this approach in terms of the different nature of incidents that underlie the overall claim severity distribution. The overall distribution reflects the particular combination of different types of incidents (slips, falls, vehicle accidents, and so on) experienced by a particular company. Each type of incident may produce a quite distinct severity distribution and we would ideally fit a separate severity distribution to each incident type. However, data shortcomings often prevent this and we adopt the approach described in the bullet points above as a practical approximation.

1.6 Simulation approaches

Although we can derive a formula for the aggregate loss distribution for certain combinations of frequency and severity distributions, in practice we often derive the aggregate loss distribution through simulation.

You may recall from Subject CT6 various compound distributions, such as the compound Poisson and compound negative binomial. We can derive formulae for the moments of these compound distributions. This can give exact results but the calculations can become unwieldy (or impossible) if the method used, or the product structure, is complicated. For this reason, simulation approaches are often used.

Structure terminology

Before illustrating how simulation can be used, we define some features (or “components”) of a complex structure.

It is essential to use simulation approaches when we estimate the loss cost likely to be borne by the insurer under more complex structures.

Some of the more common components found in insurance arrangements are:

- **Aggregate deductible – The maximum amount that the insured can retain within their deductible when all losses are aggregated.**

- **Non-ranking deductibles – The non-ranking component of a deductible (applied to each individual loss) does not contribute to an insured's aggregate deductible.**
- **Ranking deductible – The ranking component of a deductible does contribute towards an insured's aggregate deductible.**
- **Trailing deductible – The amount that is retained by the insured for each individual loss once the aggregate deductible has been fully eroded.**
- **Per occurrence limit – The maximum amount that the insurer can retain for each individual loss.**
- **Annual aggregate limit – The maximum amount that the insurer can retain when all losses for an annual policy period are aggregated.**

Don't worry if you didn't get your head around all of these at the first reading! The example and question that follow demonstrate how these features apply. Working through these may help you to remember them.

Example: Simulation model

Simulation approaches were covered in Chapter 11.

The figure on the next page illustrates the layout of a simple frequency-severity spreadsheet simulation model.

For the particular iteration shown below:

- **17 losses have been simulated from a Poisson(10) frequency distribution.**
- **Each ground-up loss was generated from a log-normal (10,000, 30,000) distribution.**
- **The insured retains the first £1,000 of each loss within its deductible.**
- **The aggregate deductible limits the insured's total retention from £16,474 to £15,000.**
- **Of the original total ground-up loss of £264,912 an amount of £249,912 is borne by the insurer.**

Frequency Distribution		
	Mean	Simulated
Poisson	10	17

Structure	
Deductible	1,000
Agg Ded	15,000

Severity Distribution		
	μ	σ
LogNormal	10,000	30,000

Simulation						
	Ground Up	Within Deductible		Within Agg Ded		Loss to
Loss	Loss	Incremental	Cumulative	Cumulative	Incremental	Insurer
1	9,410	1,000	1,000	1,000	1,000	8,410
2	1,876	1,000	2,000	2,000	1,000	876
3	3,063	1,000	3,000	3,000	1,000	2,063
4	4,930	1,000	4,000	4,000	1,000	3,930
5	7,174	1,000	5,000	5,000	1,000	6,174
6	3,874	1,000	6,000	6,000	1,000	2,874
7	31,269	1,000	7,000	7,000	1,000	30,269
8	4,744	1,000	8,000	8,000	1,000	3,744
9	2,210	1,000	9,000	9,000	1,000	1,210
10	38,329	1,000	10,000	10,000	1,000	37,329
11	1,814	1,000	11,000	11,000	1,000	814
12	123,523	1,000	12,000	12,000	1,000	122,523
13	10,304	1,000	13,000	13,000	1,000	9,304
14	4,587	1,000	14,000	14,000	1,000	3,587
15	3,484	1,000	15,000	15,000	1,000	2,484
16	13,848	1,000	16,000	15,000	-	13,848
17	474	474	16,474	15,000	-	474
18						
19						
20						
…						
TOTAL	264,912	16,474	16,474	15,000	15,000	249,912

The tables are part of the Core Reading. The data shown represents just one simulation. Of course, thousands of simulations will be carried out, each producing its own estimate of the number of claims and a corresponding set of claim amounts.

As well as assessing the expected loss cost to the insurer, we can use simulation models to assess the distribution of other components of the particular structure being considered. In particular, actuaries often comment on the adequacy of aggregate deductibles.

In this case, the simulation model can be used to ascertain the impact on total expected claims of various levels of aggregate deductibles.

Simulation models are essential for assessing loss sensitive or swing-rated (or experience-rated) premiums.

Question 14.10

Define loss sensitive or swing rated premiums, and explain the form that such a premium usually takes.

Question 14.11

Consider a simulation of the following seven (trended and developed) claims that occur in the order shown:

Claim	Amount
1	£11,392
2	£4,976
3	£128,676
4	£9,381
5	£613
6	£34,815
7	£6,210

Calculate the expected loss to the insurer of *each* claim under the following structure:

- Aggregate deductible of £5,000
- Individual deductibles of £1,000 (ranking), £500 (non-ranking) and £500 (trailing)
- Per occurrence limit of £100,000
- Annual aggregate limit of £150,000.

1.7 *Practical considerations*

Where simulation is used, we should run through enough iterations of the simulation model to ensure that the results are stable:

- **We will require more simulations if investigating the tails of the resulting loss distribution than if investigating the mean.**

  Few simulations will result in values at, say, the 99.5^{th} percentile. If we are investigating such very large losses, more simulations will therefore be needed to ensure a sufficient sized sample of values in the tail.

- **We will require more simulations if assessing an excess layer than if assessing the underlying primary layer.**

  A very high excess layer might cover claim sizes rarely seen. Running too few simulations could result in no simulated claims observed hitting the layer or so few as to lack credibility.

Because of the speed of modern computers, the time required to run a simulation model for a sufficient number of iterations should rarely be a problem.

Other, more general, practical considerations to bear in mind when pricing were discussed in Chapter 13.

2 Burning cost approach

2.1 Description

The discussion in Section 1 on the frequency-severity approach has mainly considered a specific method where frequency and severity are considered separately.

The comments on the burning cost approach in this section are more general. These cover approaches that range from the very crude, such as ones that do not allow for inflation, to methods that make various adjustments. This section focuses on

- points additional to those covered in Chapters 12 and 13
- differences between this approach and a frequency-severity approach.

In general, the burning cost approach is based on aggregate claims, expressed as an annual rate per unit of exposure, and so does not make use of the number of claims.

One definition of the burning cost approach is:

This is an experience rating approach. We define the burning cost as the actual cost of claims during a past period of years expressed as an annual rate per unit of exposure. We use a simple regression model, based entirely on historical data.

We may use the burning cost approach to rate an individual risk (or insured) or a portfolio of similar risks.

In either case, we may adjust the historical data to allow for inflation, IBNR, and so on. We may calculate the burning cost using either unadjusted data (effective burning cost) or indexed data (indexed burning cost).

Notice the word "may" appears twice in the above paragraph. In some cases the burning cost approach can be very simple and crude, with no allowance for:

- future inflation
- development of losses to obtain ultimate claims (IBNR and outstanding claims)
- past inflation or other trending adjustments.

These adjustments have been covered in Chapter 12 (and also in Section 1 of this chapter in relation to the frequency-severity approach). Other considerations specific to the burning cost approach will be discussed later in this section.

This approach is sometimes used for pricing excess of loss reinsurance contracts. It can be particularly hard to assess changes in contract structure given the lack of claims data.

A frequency-severity approach would ideally be used for these contracts, but lack of data may prohibit this.

Advantages and disadvantages

The following advantages and disadvantages are relative to the frequency-severity approach. Hence, many of these were also discussed in Section 1.1.

Advantages of a burning cost approach include:

- **simplicity**
- **needs relatively little data**
- **quicker than other methods to perform**
- **allows for experience of individual risk or portfolios.**

Disadvantages of a burning cost approach include:

- **harder to spot trends so it provides less understanding of changes impacting the individual risks**
- **adjusting past data is hard**
- **adjusting for changes in cover, deductibles and so on may be hard as we often lack individual claims data**
- **it can be a very crude approach**

This final point partly depends on what adjustments are made (*eg* for trending) and how accurate these adjustments are.

The use of this method has been heavily criticised where it is applied to current figures without any adjustments. This is because:

- **we would ignore trends such as claims inflation, and**
- **more importantly, by taking current exposure (often premiums) and comparing this with current** (undeveloped) **claims, we will understate the ultimate position.**

As a result, if we price using this approach, we will often end up with loss ratios higher than the plan.

Losses will be higher than expected if our expectations are based on analysis which has not allowed for inflation, other trends, IBNER and IBNR.

2.2 Data

Data requirements

Chapter 12 covered the data required for rating using any cost plus approach. Section 1.2 of this chapter covered the requirements for rating a single commercial risk using a frequency-severity approach. We now discuss requirements specific to a burning cost approach.

We need policy data to calculate the overall exposure or the split within each risk group. We therefore need the following information for each policy:

- **dates on cover**
- **all rating factor and exposure measure details**
- **details of premiums charged, unless they can be calculated by reference to the details on rating factors and exposure. This may be required because premium is often used as the measure of exposure.**

We often use this method where little individual claims data are available. In theory, the requirements would be similar to those for the frequency-severity approach. However, we often use the burning cost method where less data is available. Often only aggregated claims data by policy year are available. Even if this is the case, it is often possible to get details of large and catastrophic claims included, so we can remove these and treat them differently.

We also need claims data. The claims data requirements can be found in Chapter 10.

Estimating outstanding claims

If aggregate claims data has been obtained, there will be occasions where individual case estimates of outstanding claims have not been made or are not available. For example, statistical methods (such as a chain ladder model) may have been used to calculate reserves, rather than individual case estimates.

Sometimes, for particular groups of data within a class of business, the data records may not contain estimates of amounts outstanding for individual claims.

Where this is the case, we must find a way of dividing the total amount outstanding for the class of business between groups. One method is to make case estimates for specimen policies within each group. We then extrapolate to find the total claims outstanding for the whole group. We should be careful to ensure that the total for the whole class of business obtained by this method is consistent with the total obtained by statistical methods.

2.3 *Calculation of premium*

Premiums are usually quoted in relation to a unit of exposure so we should calculate the risk premium in the same terms.

The burning cost premium

In terms of formulae, the burning cost is defined as:

The burning cost premium $\textbf{BCP} = \left(\sum \textbf{Claims}\right) / \textbf{Total Exposed to Risk}$

As discussed above, **we should increase the claims to allow for past inflation and for IBNR.**

Analysis of experience: frequency, severity and burning cost

We may lose much of the information on trends being experienced by not considering frequency and severity separately.

As discussed above, the advantages of these elements being analysed separately are so that:

- distortions in the data can be identified and allowed for
- trends in experience can be spotted and projected into the future.

In terms of equations:

Pure risk premium per unit of exposure = Expected claim amount per unit of exposure

The basic elements of the pure risk premium can be derived by expanding the claim amount per unit of exposure as follows:

$$\frac{Total\ claim\ amount}{Exposure} = \frac{No.\,of\ claims}{Exposure} \times \frac{Total\ claim\ amount}{No.\,of\ claims}$$

This gives the usual formula for the pure risk premium:

$$Pure\ risk\ premium = Expected\ claim\ frequency \times Expected\ cost\ per\ claim$$

For example, in private motor insurance, if the expected claim frequency is 25% per vehicle-year and the expected cost per claim is £1,200, then the risk premium per vehicle-year is £300 (*ie* 0.25 × £1,200).

For some classes (*eg* household contents) it does not make a lot of sense to express the claim frequency in terms of the exposure measure. For example, a claim frequency of 2% per £1,000 sum insured per year feels odd. In these cases we can break down the risk premium into three factors:

$$\frac{\textit{Total claim amount}}{\textit{Exposure}} = \frac{\textit{No. of policies}}{\textit{Exposure}} \times \frac{\textit{No. of claims}}{\textit{No. of policies}} \times \frac{\textit{Total claim amount}}{\textit{No. of claims}}$$

This could be expressed as:

$$\frac{\textit{Total claim amount}}{\textit{Exposure}} = \frac{\textit{No. of claims}}{\textit{No. of policies}} \div \frac{\textit{Exposure}}{\textit{No. of policies}} \times \frac{\textit{Total claim amount}}{\textit{No. of claims}}$$

On this basis, the three basic elements of the risk premium per unit of exposure are:

- claim frequency per policy
- average unit of exposure per policy
- average cost per claim.

and the pure risk premium is then:

$$\frac{\textit{Expected claim frequency per policy}}{\textit{Average exposure per policy}} \times \textit{Expected cost per claim}$$

Question 14.12

For a domestic household contents policy, what is the risk premium per £1,000 sum insured per year, given:

- expected claim frequency per policy = 15%
- average sum insured is £18,000
- expected cost per claim = £1,200?

2.4 Trending

Chapter 12 described general factors that would be allowed for in order to make the data from the base period relevant for the period for which the new premium rates will apply. Section 1.3 of this chapter discussed these adjustments in more detail, particularly for the frequency-severity approach. We now consider trending where we are unable to consider frequency and severity separately.

Different approaches can be taken to trending, and the level of accuracy will vary. In the extreme, no adjustments may be made at all!

Question 14.13

What are the two stages of trending?

We should adjust both exposure and claims to current values.

An index would apply to the historical *aggregate* claims, rather than having separate indices for frequency and severity (and possibly exposure per policy). Alternatively, a constant past annual trend might be assumed. For either approach, an index (or constant trend) may also be applied to the exposure if the latter is expressed in monetary terms.

As individual claims data may not exist, we may have to make more assumptions about type of loss, date of occurrence, date of payment and so on.

Under the frequency-severity approach, the precise date that a loss occurred could be used to project this loss to the mid-point of the exposure period. However, if only aggregate data is used, an average loss occurrence date will need to be assumed. For example, we could assume all claims for a particular accident year occurred mid-way through that year.

We have to decide whether to leave large and exceptional claims in the data, or truncate them at a cap and spread the excess over the cap, or remove them altogether. Our decision will depend on the extent to which we expect such claims to recur during the exposure of the new rating series.

Factors that can affect the trends in the amounts of historical claims for a policy are the same as for the frequency-severity approach, and include:

- **economic – inflation, economic conditions**
- **legal – legislation, court rulings**
- **technical changes – workplace hazards, structural changes**
- **insurance – policy conditions, policy structure, underwriting and claims controls**
- **changes in business – volumes, structure, splits of business.**

These have been discussed in previous chapters as well as earlier in this chapter.

2.5 Developing losses

Unless a very crude approach is being taken, the claims information should be developed to ultimate values. Standard reserving techniques, such as the chain ladder or Bornhuetter-Ferguson methods, could be used to calculate the ultimate overall claims for each development year.

As individual loss information is not available, any assumptions will be less detailed. We are likely to calculate IBNR factors from either the individual risk data, or the aggregated results of a book of business. We should also allow for inflation.

We might allow for inflation by using a reserving method such as the inflation-adjusted chain ladder method. The inflation assumptions used might be those derived from the trending analysis.

2.6 Practical considerations

Practical considerations when pricing were discussed in Chapter 13.

3 Finally

3.1 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Cape Cod method
- Claims run-off analysis
- Delay table
- Development factors
- From the ground up
- Grossing-up factor
- Incurred but not enough reported (IBNER) reserve
- Link ratios.

3.2 End of Part 3

You have now completed Part 3 of the Subject ST8 Notes.

Review

Before looking at the Question and Answer Bank we recommend that you briefly review the key areas of Part 3, or maybe re-read the summaries at the end of Chapters 11 to 14.

Question and Answer Bank

You should now be able to answer the questions in Part 3 of the Question and Answer Bank. We recommend that you work through several of these questions now and save the remainder for use as part of your revision.

Assignments

On completing this part, you should be able to attempt the questions in Assignment X3.

Chapter 14 Summary

This chapter describes the additional considerations to be made when deriving the risk premium using:

- the frequency-severity approach
- the burning cost approach.

The frequency-severity approach involves analysing and projecting frequency and severity separately, and then combining the two to calculate the risk premium.

A burning cost approach ignores the number of claims, but takes the actual total cost of claims during a past period of years, expressed as an annual rate per unit of exposure. This could apply to a single risk or to a portfolio of similar risks.

Advantages and disadvantages

The frequency-severity approach mirrors the underlying process, allows complex structures to be modelled more easily, provides a better understanding of the data, and enables frequency and severity trends to be identified separately. It can be more accurate than the burning cost approach. However, it requires more data (by volume and detail) and is more time consuming and complex.

Data

For frequency-severity rating of commercial risks, the broker will usually provide a submission document, plus details of the risk's exposure (past and present) and claim history.

Individual loss data is required for the frequency-severity approach. Less data is required for the burning cost approach, which uses aggregate claim data and may not have individual case estimates.

Data should be gross of reinsurance and from the ground up.

Trending

Historical data needs to be adjusted to make it relevant to the period for which future premium rates will apply. For the frequency-severity approach, separate frequency and severity trends will apply. For the burning cost approach, both the exposure and the claims data should be adjusted.

Standard indices will normally apply for trending, although analysis of past loss trends for the risk will also be useful. The severity trend might vary according to claim size. Large losses (and catastrophes) may require special treatment.

Developing losses

Claims need to be developed to their ultimate level and allowance made for IBNR.

For the frequency-severity approach, each individual loss should be developed, *eg* by applying development factors or by using stochastic methods. An accurate method is particularly important where there are deductibles or limits. The development factors used should be appropriate to the data and may vary by size of claim.

The burning cost approach should allow for trending (past and future) and/or claim development (including IBNR), but often this is not done in practice.

Fitting and simulation using frequency-severity

For the frequency-severity approach, frequency and severity distributions are usually fitted to the trended and developed loss data. The choice of base period should allow for the relevance and development of the data, and weights may be applied. It is important to be aware of the fitting algorithms and goodness of fit tests being used.

The following loss distributions are often used:

- Frequency : Poisson, negative binomial
- Severity : log-normal, Weibull, Pareto, Gamma.

Separate distributions may be fitted to different parts of the overall loss range.

Simulation techniques are commonly used to assess the expected loss to the insurer, and also to assess the impact of components of a particular structure being considered – for example, deductibles (aggregate, non-ranking, ranking and trailing) and limits (per occurrence and annual aggregate).

The number of simulations should be sufficient to ensure the results are stable, *eg* more simulations will be required when examining claim tails or claims to excess layers.

Chapter 14 Solutions

Solution 14.1

Loadings would be made to allow for the cost of reinsurance, expenses (including commission), profit and return on capital. Allowance may also be made for investment income (as a negative loading), tax and contingencies (to the extent not allowed for in the profit loading).

Solution 14.2

"From the ground up" claims data shows all claims, no matter how small they are, and shows the original claim amount. It is often used in reinsurance to refer to data which shows all claims, even though reinsurance is only required for large claims.

Solution 14.3

If the class of business had a shorter reporting delay, there will be more claims data available, and so the base period required could be reduced.

If the class of business had a lower claim frequency, there will be less claims data, and so a longer base period may be necessary.

Solution 14.4

The base data would be adjusted to allow for:

- unusual features in the base-period data
- large or exceptional claims
- trends and inflation
- changes in the risk and/or cover (*eg* mix, cover, underwriting or reinsurance)
- environmental changes.

Solution 14.5

Changes in legislation and changes to the structure of the risk are likely to be one-off changes.

Changes in accident frequency, the propensity to make claims and the social and economic environment are more likely to happen gradually over time, and so be continual trends.

Solution 14.6

Possible drivers of frequency trends include:

- increasing compensation culture
- propensity of no-win-no-fee arrangements
- growth of claims management companies (placing of claims)
- changes in health and safety regulations
- court decisions
- changes in economic conditions
- emergence of latent claims
- changes in policy terms, conditions, excesses, limits, *etc*.

Possible drivers of severity trends include:

- salary inflation
- court decisions / inflation, *eg* their assessment of compensation for pain and suffering
- medical advances / medical inflation
- inflation of legal costs
- legislative changes (*eg* structured settlements)
- interest rate changes (*eg* affects discount rate used for Ogden tables)
- changes in policy terms, conditions, excesses, limits, *etc*.

Solution 14.7

By definition, the excess layer will only consist of claims greater than the excess point. Larger claims may show different development patterns to claims in general. For example, larger liability claims may take longer to report and to settle than average. This should be allowed for when developing the claims.

However, care should be taken when using triangulated data of excess of loss business that the excess limits have remained constant over each origin year.

Solution 14.8

Estimated ultimate claims for older policy years will be more fully developed. The claims incurred figures will depend more on actual claims paid amounts rather than on estimates of outstanding claims, and so should be more reliable.

However, claims for older policy years will be less relevant as these are more likely to be based on different risks, external conditions, policy conditions, mixes of business, underwriting, *etc*.

Solution 14.9

(a) Claim frequency under the contract is given by the claims frequency multiplied by the proportion of claims in excess of £50,000,

ie overall frequency × proportion of claims over £50,000 = $c \times \int_{50,000}^{\infty} f(x)dx$

(b) The average cost per claim is given by the total payments divided by the number of claims. Considering each element in turn,

payments if the upper limit is not exceeded (A): $\int_{50,000}^{300,000} (x-50,000) f(x)dx$

payments where the upper limit is exceeded (B): $\int_{300,000}^{\infty} 250,000 f(x)dx$

proportion of claims over £50,000 (C): $\int_{50,000}^{\infty} f(x)dx$

thus average cost per claim = (A + B) / C

$$ie \qquad \frac{\displaystyle\int_{50,000}^{300,000} (x-50,000)\, f(x)dx + 250,000 \int_{300,000}^{\infty} f(x)dx}{\displaystyle\int_{50,000}^{\infty} f(x)dx}$$

Solution 14.10

Loss sensitive (or swing-rated) premiums are a form of experience rating. These are premiums that depend, at least in part, on the actual claims experience of that risk in the period covered. They will usually be applied in the form of a deposit and adjustment premium.

Solution 14.11

The loss to the insurer can be summarised in the following table:

Loss	Ground up loss	Ranking deductible	Non-ranking deductible	Trailing deduct-ible	Loss to insurer (max £100k each)	Loss to insurer net of annual agg. limit
1	11,392	1,000	500		9,892	9,892
2	4,976	1,000	500		3,476	3,476
3	128,676	1,000	500		100,000	100,000
4	9,381	1,000	500		7,881	7,881
5	613	613	0		0	0
6	34,815	387	500		33,929	28,751
7	6,210			500	5,710	0
Total	196,063				160,887	150,000

The ranking deductible of £1,000 per claim is limited to £5,000 in aggregate. Claim 5 is below £1,000, and so the ranking deductible is limited to the claim amount of £613. At this point the cumulative aggregate deductible is £4,613, and so only the remaining £387 applies to claim 6.

In addition, the non-ranking deductible applies to each claim (except claim 5, which is too small), until the aggregate deductible is reached (on claim 6). For subsequent claims (*ie* claim 7), the trailing deductible applies.

The loss to the insurer for each claim is then the ground up loss less the sum of the deductibles. However, each loss to the insurer is limited to £100,000 (the per occurrence limit), and so claim 3 is reduced accordingly.

Finally, we need to check to see if the annual aggregate limit is breached. As you would expect in an examination question, it is breached, and so we need to limit the total claims to £150,000. Hence, claim 6 is reduced so that the sum of the losses to the insurer up to this point is £150,000. The insurer will not pay any subsequent claims.

(Note that the trailing deductible has no impact in this question since the annual aggregate limit is breached before it has any effect.)

Solution 14.12

Expected claim cost per policy: $0.15 \times 1{,}200 = 180$.

The average policy has sum insured of £18,000, so the risk premium per mille sum insured is: $\dfrac{180}{18} = £10$.

Solution 14.13

The two stages required when trending are:

- restate historical claims to current values
- project them to the midpoint of the future exposure period.

Chapter 15

Rating using original loss curves

Syllabus objective

(m) (i) Describe how original loss curves can be used in rating.

(ii) Understand the assumptions required when using this approach.

(iii) Outline the practical considerations when using this approach.

For this topic, the Core Reading also gives some "wider objectives", which are as follows:

Use original loss curves to price policies with different limits and attachment points.

(i) Understand and use appropriate terminology.

(ii) Show how original loss curves are related to original loss distributions.

(iii) Understand and explain the key properties of original loss curves.

(iv) State and explain the main assumptions relating to the derivation and use of original loss curves.

(v) Derive prices for direct and facultative business and treaty excess of loss business.

(vi) Describe important considerations, limitations and uncertainties associated with the practical use of original loss curves.

(vii) Understand how original loss curves can be constructed from claims data and the practical difficulties associated with their construction.

0 Introduction

We commonly use original loss curves in general insurance pricing to infer prices for layers at which the data are too sparse to derive a credible experience rate. We frequently use them for excess of loss pricing, although we can also use them to obtain large loss loadings on full-value covers, or consistent prices for different primary limits (*ie* direct insurance which is subject to different limits)**.**

You may have come across original loss curves in other disguises (such as *increased limit factors*) already, as the terminology varies greatly amongst practitioners. This is discussed further in Section 1.

Question 15.1

List further circumstances for which you think using original loss curves would be useful.

The overall process used to obtain a loss cost for a high layer cover is to take the known loss cost at a lower layer cover (for which there is plenty of data) and then use the curve to estimate the losses at the higher layer.

Very broadly, the methodology is usually as follows:

(i) **We obtain a loss cost estimate for a full value cover (*ie* ground-up, limited only at the sum insured) or a primary layer with a relatively low limit. This will usually be from an exposure-related method but could be an experience rate.**

Note that:

- **in experience rating, we derive the premium rates from the data of the insured (internal data)**
- **in exposure rating, we derive the premium rates from benchmarks or data which are external to the insured.**

(ii) **We use original loss curves (which are derived from the probability distributions of the underlying losses) appropriate to the risk (or group of risks) concerned to infer from this loss cost what the cost should be for different layers of cover.**

The curves we use depend on the class of business we are pricing. In Section 2 we will look at curves for property business and in Section 3 we will look at curves for liability business (more often called *casualty* business in the USA).

We will see how the curves are derived (constructed) in Section 4, and in Section 5 we look at examples of curves used in practice.

(iii) We then load the cost for the required layer for expenses and risk / profit to obtain a premium estimate.

Frequently, premiums calculated for excess of loss layers using this method are credibility weighted with experience rates.

In Section 6, we will look at some more complicated uses of original loss curves, and Section 7 gives some closing remarks.

1 Terminology and different types of curves

1.1 Terminology

The term "original loss curves" is rarely used in general insurance markets.

Instead, we use the terms described below.

The curves used are related closely to loss severity distributions and the terminology that is used in practice:

- **depends on the precise definition of the curves**
- **varies from market to market, and even between practitioners within the same market.**

The most commonly used forms of original loss curves are as follows:

(i) ***First loss scales / exposure curves*: Usually seen in property business, these curves give the proportion of the full value premium allocated to primary layers limited at different values. An alternative way of looking at this is that they give us the value (expressed as a percentage of the full value premium) of imposing deductibles at different levels. We usually express the limits as a fraction of the sum insured, maximum probable loss (MPL) or EML. We also sometimes refer to these curves as *loss elimination functions*.**

In the Glossary, the Core Reading refers to the MPL as the PML (Probable Maximum Loss). EML is the estimated (or expected) maximum loss.

We will see an example of this curve in more detail in Section 4.1.

(ii) ***ILFs*** (increased limit factors)**: We choose a "basic limit"; this is usually a relatively low primary limit. We construct a table of multiplicative factors (increased limit factors or ILFs) giving the ratio of the premium for higher limits to the basic limit premium. We usually use this terminology and format for casualty business.**

Table 3.1 in Section 3.1 gives an example of a table of ILFs.

(iii) ***XL scales*** (excess of loss scales)**: Similar to a first loss scale except they give the proportion to be allocated to the excess layer rather than the primary layer.**

In a sense, excess of loss scales deal with the section of the curve beyond the limit, whereas first loss scales deal with the section of the curve below the limit.

The terminology is sometimes used loosely and some practitioners use the above terms interchangeably.

1.2 *What is allowed for in the curves?*

We mentioned above that the original loss curves describe the ratio of pure loss cost at different levels of deductible or excess.

In addition to the pure loss cost, we sometimes construct curves to allow for:

- **expenses; that is, allocated loss adjustment expenses (ALAE) and possibly unallocated loss adjustment expenses (ULAE) at each limit; and**

> ***Question 15.2***
>
> Explain what is meant by ALAE and ULAE.

- **a load for risk at each limit.**

  This load could allow for contingencies, and would normally be greater for higher limits.

If we already allow for such loadings in the curves, this will have implications for the approach followed (for example, what we should include in the "full value" or "basic limits" estimate; what loads we should add at the end).

How the curves are constructed and applied should depend on the coverage included in the policy. For simplicity, the theory that follows is on the assumption that the curves cover indemnity costs only.

> ***Question 15.3***
>
> What are indemnity costs?

An additional consideration is whether the curves are on a per-claim (paid) or per-occurrence basis. Clearly, any curves used should be appropriate for the cover being granted.

A per-claim basis means that curves are based on the amounts that will be paid to each individual claimant (*plaintiff*) for losses that arise from one incident. A per-occurrence basis means that curves are based on the total amounts paid to *all* plaintiffs for losses that arise from one incident.

1.3 Properties of the curves

This section explains the mathematics behind the curves' creation.

The curves, however they are expressed, are closely related to the original loss distribution. They are usually proportional to the *limited expected value* (LEV) function. So the properties of the LEV function carry forward to the curves. So we will start by looking at some of the properties of the LEV function. Let:

> ***X*** **be the random variable representing the loss severity,**
>
> $\boldsymbol{F_X(x)}$ **be the cdf (cumulative distribution function) of** ***X***
>
> $\boldsymbol{S_X(x) = 1 - F_X(x)}$ **be the survival function.**

If your memory of statistics is rusty, $F_X(x)$ is the cumulative distribution function of the claim size, and is related to the underlying probability function (noticing that losses cannot be negative). Hence:

$$F_x(x) = \int_0^x f(y)\,dy = \Pr(X \le x) \text{ and } S_x(x) = \Pr(X > x).$$

Then

$$\boldsymbol{E(X) = \int_0^\infty x\,dF_X(x) = \int_0^\infty S_X(x)dx} \quad \textbf{(1.1)}$$

The first equality follows from the fact that:

$$f(x) = \frac{dF_X(x)}{dx} \text{ and so } E(X) = \int_0^\infty xf(x)dx = \int_0^\infty x\frac{dF_X(x)}{dx}dx.$$

To prove the second equality in (1.1), use integration by parts to show that:

$$E(X) = \int_0^\infty S_X(x)dx$$

by choosing $u = S(x) = 1 - F(x)$ and $\dfrac{dv}{dx} = 1$.

We now define the LEV function to be the expected value of X but where X is limited to x. This is denoted by $LEV_x(x) = E[X \wedge x]$. In other words:

$$\boldsymbol{LEV_X(x) = E[X \wedge x] = \int_0^x y dF_X(y) + S_X(x)x = \int_0^x S_X(y)dy} \qquad \textbf{(1.2)}$$

where the last equality in each of (1.1) and (1.2) applies because loss distributions are non-negative.

The equalities mentioned here can be proven using integration by parts with the same choice of u and $\frac{dv}{dx}$ as suggested for the integration by parts above.

We can see from the definition of the LEV function that, in insurance terms, it represents the expected value of the losses limited to a primary layer of size *x*.

We will make an important assumption here.

The curves should actually represent the expected indemnity *cost* at the various limits. However, cost is made up of frequency as well as severity. So, if we assume that the underlying frequency of claims is independent of both the severity and the limit, then we can ignore frequency, and take the curve to represent just the limited expected severity.

This is why the first paragraph in this section said that "the curves are *closely related* to the original loss distribution".

Differentiating we get:

$$\boldsymbol{LEV'_X(x) = S_X(x)} \qquad \textbf{(1.3)}$$

From equations (1.2) and (1.3), two properties of the LEV function are clear:

- $\boldsymbol{LEV(x)}$ **is an increasing function (or, strictly speaking, non-decreasing)**
- **it increases at a decreasing rate (since $\boldsymbol{S(x)}$ is itself non-increasing), that is, it is a concave function.**

Both of these properties should be intuitively obvious because as the limit x increases, so will the expected value of X limited to x, but by a decreasing rate.

Hopefully these properties will also be obvious when we look at graphs of the curves, which we will see later in this chapter.

Since the different forms of original loss curves (*eg* ILFs) are really just expressions of the LEV function, they also share these properties (with the exception of XL scales that are related to $E(x) - LEV(x)$, so these are non-increasing and convex).

This exception is because the excess of loss scales are effectively the same curve but expressed from the "other end", and so the properties above are reversed.

Although the basic pricing methodology is the same for both property and casualty rating, some of the assumptions and considerations are specific so we will look at them separately.

In the next section we look at curves used in property business, and in Section 3 we will look at curves for casualty (liability) business.

2 Property XL rating using exposure curves

In this section, we look specifically at the use of original loss curves in property business. Remember from earlier that, in this case, the curves are often referred to as first loss scales or exposure curves.

2.1 Outline theory

Continuing with the definitions from Section 1.3, we now consider a single property risk. Let:

- ***M* be a measure of the size of the risk; this could be sum insured (SI), MPL and so on.**
- ***Y* be the random variable representing the size of loss as a proportion of *M* (so $Y = \dfrac{X}{M}$). We will call this the relative loss severity.**

  For example, if the loss X is half of the sum insured, then Y would take the value of ½.
- **$F_Y(x)$ be the cdf of *Y*.**
- **$S_Y(x) = 1 - F_Y(x)$ be the survival function of *Y*.**

Note that, depending on the definition of *M*, *Y* could take values greater than one.

> ***Question 15.4***
>
> How can this be?

We then define an exposure curve as:

$$G(x) = \frac{LEV_Y(x)}{E[Y]} \tag{2.1}$$

The exposure curve $G(y)$ can be thought of as a graphical representation of the ratio of the claims cost below a certain point y to the claims cost in total, where y is the chosen proportion of the maximum claim.

For example, let's suppose that for a household policy the sum insured is £10,000 and the pure risk premium to cover claims up to that level is £100. Let's also suppose that we're interesting in knowing the pure claims cost for claims up to £5,000. Then y is 0.5 and $G(0.5)$ will tell us the proportion of the £100 to use as our pure risk premium for the lower limit.

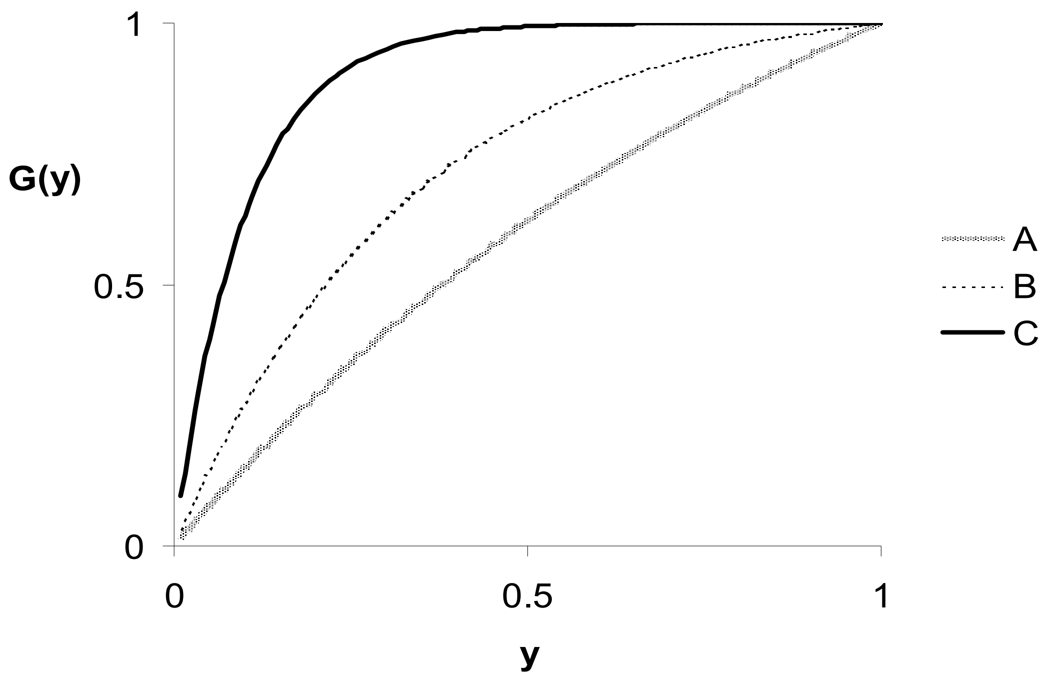
Notice that $G(1)=1$ because the pure risk premium will be the same if the sum insured is unchanged, and that $G(0)=0$ because the claims cost is zero if the sum insured is zero.

The more the risk is concentrated towards the lower end of the sum insured spectrum, the steeper the curve. This makes intuitive sense – we would normally expect, all else being equal, a policy with sum insured £1,000 to have a greater risk premium (per unit sum insured) than a policy with sum insured £10,000, since losses at the lower end are more likely (*ie* loss distributions are normally positively-skewed).

Some examples of what exposure curves look like are shown in the graph below. Note that the values given are arbitrary and should not be used for pricing anything.

The steepness of the curve is related to the severity of the loss distribution. The closer the curve is to the diagonal the greater the proportion of large losses (relative to *M*). So in the examples below, curve A is the most severe and curve C the least.

In the above, "severe" means that a large proportion of the losses are for large amounts.



Now we consider a simple pricing example:

- **Assume we have a risk size *M* and cover is required for a layer of *L* excess of *D,* where $0 < D < L + D < M$.**

  This could be, for example, an excess of loss reinsurance policy with limits D and $L + D$.

- **Let *N* be the random variable representing the annual number of losses to the risk (ground up) with *X* and *Y* as previously defined.**

  X is the original claim size distribution, and Y is the size of loss as a proportion of the maximum.

- **Let *S* be the random variable representing the total annual ground-up losses to the risk.**

  This will be the product of the frequency and the severity.

- **Let S_L be the random variable representing the total annual losses to the layer.**

Now we find the loss cost to the layer in terms of the exposure curve function. Remember that the $\wedge$ sign means "limited to …". Also, we introduce the $\vee$ sign which means the reverse, *ie* "of a minimum size …".

$$C = \text{Ground-up loss cost} = E[S] = E[N]E[X] \tag{2.2}$$

$$\begin{aligned}
C_L &= \text{Layer loss cost} = E[S_L] \\
&= E[N]E\left[\{(X-D)\wedge L\}\vee 0\right] \\
&= E[N]E\left[X\wedge(L+D) - X\wedge D\right] \\
&= \frac{E[N]E[X]E\left[X\wedge(L+D) - X\wedge D\right]}{E[X]} \\
&= \frac{C\times E\left[Y\wedge\left(L/M + D/M\right) - Y\wedge\left(D/M\right)\right]}{E[Y]} \\
&= \frac{C\times\left(E\left[Y\wedge\left(L/M + D/M\right)\right] - E\left[Y\wedge\left(D/M\right)\right]\right)}{E[Y]} \\
&= C\times\left(G\left(\frac{L+D}{M}\right) - G\left(\frac{D}{M}\right)\right)
\end{aligned} \tag{2.3}$$

Example

A policy has a sum insured of £10,000 for which the risk premium is £100.

You are about to purchase excess of loss reinsurance for £5,000 xs £2,500 and want to calculate the revised net risk premium.

Your exposure curve gives $G(0.75) = 0.9$ and $G(0.25) = 0.7$.

Then, using equation (2.3), the revised net risk premium is $£100 \times (0.9 - 0.7) = £20$.

So to estimate the loss cost to the layer we need to have an estimate of the full value loss cost. For direct and facultative business the company is likely to have analysed full value loss cost and have rates for different types of property for different perils and covers (treaty pricing is covered in Section 2.4).

There then remains the problem of deriving or selecting an appropriate exposure curve.

This is discussed in the following section.

2.2 *Selecting exposure curves*

Notice that the exposure curve works with the variable *Y*, which is the *relative* loss size distribution, and not the original loss size distribution *X*. This is because, if we used *X* directly, it would be more likely that the curve would depend on the size of the risks giving rise to the claims distribution *X,* and so we would need a different curve for each size of policy.

The key question when using exposure curves is:

> **"To what extent is the relative loss size distribution *Y*, and hence the exposure curve, independent of the individual characteristics of the risk?"**

The motivation behind switching to *Y* in (2.3), rather than staying with the original loss in absolute values *X*, is the belief that, in some circumstances at least, *Y* can be considered independent of the size of the risk.

Thankfully, research on *homogeneous* home insurance business in the USA has indicated that this is indeed the case:

Ruth Salzmann, in a 1960s study of fire losses (buildings only) on USA homeowners' policies, concluded that this assumption was a reasonable approximation within each construction type for her, reasonably homogenous, data. In this study, the buildings sum insured was used as the risk size measure. Other authors have supported this conclusion for reasonably homogenous sets of USA homeowners' data.

However, problems arise when the data is less homogenous (including wider ranges in risk size). Salzmann and Ludwig both showed that the distribution of *Y* is very much dependent on the peril, and Ludwig, when he extended the analysis to a set of data relating to small commercial business, showed that the assumption was not valid across the full range of sums insured for any of the causes of loss studied. He found that, for his data, smaller risks have a higher proportion of severe losses (relative to the sum insured) than larger risks. Other authors have supported this finding.

If you are interested, details of the above are given in the following Casualty Actuarial Society papers, which are not part of the Core Reading:

- Salzmann, R.E., Rating by Layer of Insurance, PCAS, Vol. L, 1963, pp. 15-26
- Ludwig, S., An Exposure Rating Approach to Pricing Property Excess of Loss Reinsurance, PCAS, Vol. LXXVIII, 1991, pp. 110-145.

Some other sources of heterogeneity that are highly likely to alter the distribution of *Y* are:

- **differences in jurisdiction and claims environment**
- **different sub-classes (*eg* retail *vs* manufacturing *vs* warehouse)**
- **different coverages (buildings only, contents and degree of business interruption cover).**

In theory, at least, we should derive separate exposure curves wherever these differences become significant, for each peril and by banding of risk size. We should then apply these to the ground-up rate for each grouping / peril. However, deriving these curves would require an enormous amount of quality data, which is unlikely to be available to most practitioners. In practice therefore, in many circumstances, we are likely to use judgement to reduce the number of groups or to adjust exposure curves for different groups.

In summary then, we can use published exposure curves for property business, but we need to be aware that the more specific our business is, the greater the need to look for specific curves or to treat the curves we are using with some caution or adjustment.

2.3 Effect of claims inflation

We now consider the effects of claims inflation on exposure curves. Practitioners often refer to claims inflation as "trend", and the practice of adjusting for claims inflation as "trending".

If we believe that the effect of claims trend is uniform across all loss sizes, and that the sums insured are being adjusted appropriately for trend, then we require no adjustment to the exposure curves for trend since the loss distribution *X* and risk size *M* increase in proportion, leaving *Y* (which is X/M) unchanged. Where this is not the case, we will need to adjust the exposure curves by considering the relative effects of trend on different loss sizes, or by reworking the entire analysis.

There are several reasons why the effect of inflation on different loss sizes may differ, as the components of the loss are likely to change as size increases.

For example, consider firstly a commercial property fire which results in total loss – the claim cost could include property reinstatement as well as consequential loss. Secondly, consider a small fire which resulted in minimal damage to stock – in this case, the claim cost would cover simply the replacement of stock. As the two claims represent quite different types of loss, they would be affected differently by inflation.

2.4 Using exposure curves in risk XL treaty rating

We now consider the use of exposure curves when pricing reinsurance treaties. In particular, we consider property reinsured under an excess of loss (XL) arrangement.

We frequently use exposure curves in property per risk excess of loss rating. We usually refer to the method in this context as "exposure rating".

To exposure rate a risk XL treaty the reinsurer requires detailed information on the sizes of risks written by the cedant (may be in the form of SIs or MPLs, and so on) and the premium income. The cedant often presents a risk profile in the form of a table of original premiums by sum insured band (see example below).

Alternatively, the cedant may provide an entire download of the risks written, with the risk size and premium for each one. Note that both the risk profile and the risk download will probably relate to the past (usually some time in the previous year) and will not necessarily be representative of the prospective period covered by the treaty.

The outline approach is illustrated by an example shown in Table 2.1. Please note that the values shown in the table are completely arbitrary and are only for the purposes of illustrating the general principles.

Table 2.1: Outline of treaty exposure rating **Arbitrary units and values**

A	B	C	D	E	F	G	H	I	J	K	L	M	O
Sum insured lower limit	Banding upper limit	Average in band	Original premium	Original loss ratio	Original loss cost	Treaty attachment	Treaty exit point	Selected curve	Treaty attachment %	Treaty exit %	Exp % attachment	Exp % exit	Loss cost to layer
-	5	2.5	10,000	60%	6,000	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
5	10	7.5	9,000	60%	5,400	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
10	20	15.0	8,000	60%	4,800	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
20	30	25.0	7,000	60%	4,200	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
30	40	35.0	6,000	60%	3,600	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
40	50	45.0	5,000	60%	3,000	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
50	100	75.0	4,000	60%	2,400	96.2	480.8	A	100.0%	100.0%	100.0%	100.0%	
100	150	125.0	3,000	60%	1,800	96.2	480.8	B	76.9%	100.0%	94.8%	100.0%	94
150	200	175.0	2,000	60%	1,200	96.2	480.8	B	54.9%	100.0%	85.0%	100.0%	180
200	250	225.0	1,000	60%	600	96.2	480.8	B	42.7%	100.0%	76.0%	100.0%	144
250	300	275.0	500	60%	300	96.2	480.8	B	35.0%	100.0%	68.4%	100.0%	95
300	400	350.0	600	60%	360	96.2	480.8	B	27.5%	100.0%	59.1%	100.0%	147
400	500	450.0	400	60%	240	96.2	480.8	B	21.4%	100.0%	49.8%	100.0%	120
500	1,000	750.0	1,000	60%	600	96.2	480.8	B	12.8%	64.1%	33.6%	89.9%	338
Sum			57,500										1,118
										Loss cost rate			1.9%

Referring to Table 2.1:

- **Columns A, B and D are data provided by the cedant.**

  A and B show the sum insured band, and D gives the cedant's premium income for policies sold within each band.

- **Column C is used where a banding is provided of sums insured. A single value is required to apply to the exposure curve. Here the midpoint of each interval has been chosen. The loss cost derived can be quite sensitive to the assumption here, which is discussed in a little more detail below.**

  For example, if the average loss cost is towards one end of each band, quite different results will be obtained.

- **A reinsurer is unlikely to have the detail to estimate ground-up rates from the original exposures so usually will have to estimate the full-value loss cost from the premium data provided by the cedant. To do this, the reinsurer estimates the cedant's (ground-up) loss ratio. Ground-up loss data may be available to estimate the loss ratio, otherwise an assumption may be required.**

The loss ratio would be expected to vary with the insurance cycle. It need not be the same for each banding although in Table 2.1 we have assumed it is (column E). If we use different curves for different perils and risk groupings, we should split the original loss cost between these perils and apply the different curve to each element of the loss cost. In some cases cedant data will be available to estimate this split.

We then derive the original full value loss cost estimate (column F) using the original premium and loss ratio.

So C is the cedant's loss ratio assumed for the band, and F is the product of D and E.

- **Columns G and H are derived from the treaty limits. The treaty here is 400 xs 100, but the limits have been adjusted for trend. This is because the profile is, in this example, a year out of date. Rather than inflate all the sums insured in the table for a year's trend it is simpler to deflate the treaty limits from the middle of the prospective period to the date of construction of the risk profile; the result is the same. In this example the data is from the middle of the current treaty-year so one year's inflation (at 4%) has been used. So column G = 100/1.04; H = (100 + 400)/1.04.**

- **In column I the reinsurer has selected the appropriate exposure curve to use for each banding. In this case a different curve has been selected for the larger risks.**

  The possible curves that are used in practice are discussed later in this chapter.

- **Columns J and K show the treaty attachment and exit points as a proportion of the sum insured for use with the exposure curve *ie* $J = \dfrac{(G \wedge C)}{C}$. Similarly for column K. Note that here we have assumed that the values given in the profile genuinely cannot be exceeded. If this is not the case the values in columns J and K should be allowed to exceed 100% and the loss cost associated with losses that are greater than *M* reflected in the exposure curve.**

  So, for example, in the first few rows, column J shows 100% because the attachment point in column G is greater than column C. Once column C reaches 125.0, then column J is $96.2/125.0 = 76.9\%$.

- **Columns L and M are read from the selected exposure curve and give the proportions of the loss cost estimated to relate to ground-up layers with the limits given in G and H. So using the notation from equation (2.1) we have $L = G(J)$, (and similarly for M).**

  We cannot work out the figures in columns L and M manually since we have not been given the exposure curve used.

- **In column O the loss cost to the layer is estimated using equation (2.3). Continuing to use the column references from the table we have $O = (M - L) \times F$.**

- **The final loss cost rate (as a percentage of the original premium) is 1.9% and is calculated by: $\text{Rate} = \dfrac{\sum O}{\sum D}$.**

As previously noted, in column C we have taken the arithmetic average of the edges of the band as a measure of the average sum insured within the band and allocated the entire band's premium to the calculated average risk size. If the banding widths are not small compared with the layer priced, this assumption can have a significant impact on the resulting loss cost estimate. For bandings that straddle the attachment point, an average value within the band is likely to be inappropriate.

In this example, simply taking 75.0 to represent the band 50–100 would be inappropriate, as this would assume that all these claims would be beneath the layer, whereas in fact any claims greater than 96.2 will contribute to the cost of the layer (once inflation has been applied).

If more information is not available on the distribution of risk size within the bandings, it is sensible to test this assumption for sensitivity (say by assuming that every risk is at the top of the band). It is not unheard of for risk profiles to be presented to reinsurers in such a way that a band that exposes the layer has its mid-point just below the attachment point of the treaty.

This would of course, look as if claims are unlikely to hit the layer of reinsurance, and so result in a lower premium for the cedant. It is important for a reinsurer to question the validity of any data presented by the cedant or broker.

If the cedant provides a complete download of risks, then this problem is alleviated. We can do the analysis as in Table 2.1 but with an individual line for every original risk. Alternatively, if this is computationally too intensive, the full risk profile at least allows us to investigate the banding and look at the distribution of risk sizes within important bands.

We have assumed that the risk profile provided will be representative of the one for the prospective treaty year (subject to a uniform adjustment of sums insured for trend). Material changes in the shape of the profile are likely to cause a material change in the premium. If material changes are likely and we cannot predict the impact of them, then this approach is not appropriate. Risk profiles can change quite dramatically during the insurance cycle.

Another point about the example in Table 2.1, which is really more a feature of treaty structure rather than of the pricing methodology itself, is how sensitive the rate can be to changes in the volume of premium exposing the treaty. In this case, the treaty premium is expressed as a rate on the entire original premium. But 85% of the subject premium (*ie* 49,000, which is the sum of the original premium from 10,000 down to 4,000) **is in respect of risks that are thought not to expose the treaty to losses. This can make the treaty loss cost rate sensitive to apparently small changes in the volume of business that exposes the treaty. Note that treaties can be structured to alleviate this problem (sometimes by removing blocks of business from the treaty base that cannot expose the treaty).**

2.5 Treaty exposure rating – additional considerations

When we use exposure curves for treaty pricing, there are a series of considerations and problems additional to those we encounter when we price direct and facultative business.

The main problem is lack of data.

Choice and availability of exposure curves

One major problem with using benchmarks such as exposure curves is simply their lack of availability, at least at a suitable level of detail or applicability.

The reinsurer is very unlikely to have the full range of exposure curves available (or even enough detail from the cedant) to allow division into completely homogenous subgroups. So we must exercise considerable judgement to select the best curves and groupings possible. Clearly this leads to greater uncertainty in the estimated loss cost.

Also, where commercial business is being reinsured, the original insurance is likely to cover properties at several locations. Sometimes the risk profile (against which the full premium for the whole risk is allocated) may show just the largest value property (called the top location). The relative loss distribution here will depend on the distribution of the values for all the locations. We can construct curves on this basis, but in using them, we rely on the assumption that the distribution of values for the reinsured risks is similar to those underlying the exposure curve selected. In recent years it has become more common for information detailing the distribution of insured values to be supplied for large multi-location risks.

The values shown in the risk profile could be insured values or could be MPLs (or EMLs and so on). We should derive the exposure curves used on a consistent basis.

Estimation of original loss ratio

As explained in Section 2.4, we usually require an estimate of the cedant's loss ratio. There may be limited data with which to do this, adding another level of uncertainty to the process.

Although each loss ratio will include an estimate of the outstanding claims and hence be subject to a particular reserving basis, this should not be too subjective for property business because it is largely short-tailed (although for some weather-related perils, loss ratios can vary considerably). However, it is more the level of detail to which loss ratios are recorded that may be lacking, *eg* for each location, sum insured band, *etc*.

Treatment of original deductibles

In many situations, the original business will be excess of underlying deductibles (or excesses)**. These may be small working deductibles** (*eg* £100 for each household contents claim)**, or they may be much larger (for example, where the original insured retains large primary layers via a captive, or where the cedant is writing excess business). We derive some exposure curves already allowing for deductibles, in which case the methodology is unchanged providing we are comfortable that the deductible structure underlying the treaty is sufficiently similar to that underlying the derivation of the exposure curves.**

Where the exposure curves are based on ground-up data, we should adjust the methodology.

Instead of:

$$C_L = \text{Layer loss cost } = C \times \left(G\left(\frac{L+D}{M} \right) - G\left(\frac{D}{M} \right) \right)$$

equation (2.3) then becomes:

$$\boldsymbol{C_L = \text{Layer loss cost} = C \times \frac{\left(G\left(\frac{L+D+d}{M+d} \right) - G\left(\frac{D+d}{M+d} \right) \right)}{\left(1 - G\left(\frac{d}{M+d} \right) \right)}} \qquad \textbf{(2.4)}$$

Here, the notation is the same as for equation (2.3) but with C representing the loss cost for the original risk (no longer ground-up) and *d* representing the original deductible.

Note that the introduction of original deductibles has a proportionately greater impact on the original insurer's loss cost than the reinsurer's. All other factors being equal therefore, we would expect the presence of original deductibles to increase the reinsurance rate when expressed as a percentage of the original premium.

This makes intuitive sense, since the insurer will not be liable for the original deductible.

Treatment of inuring reinsurances

Reinsurance X is said to *inure* to the benefit of another reinsurance arrangement Y if X acts before Y, and Y acts on the net retained amount after X has been applied.

There may be other (facultative or proportional) reinsurances that inure to the risk XL treaty.

For example, there may be a quota share treaty on a property account (which may have been arranged for reciprocity purposes) that acts to reduce all gross claims by a fixed percentage before the risk XL treaty is applied.

Similarly, some or all of the original business may have been written on a coinsurance basis.

We should adjust the risk profile and methodology to take these into account. The exact details of the adjustments required depend on the nature of the inuring covers.

Treatment of stacked limits

If a policyholder is insured over multiple periods, each with its own claim limit, or has multiple policies, then normally only one limit would apply to a particular claim. However, in some territories, policyholders have the option to add all the limits. This means that if a loss occurs that causes damage over more than one policy period or against more than one policy, the policy limits can be added up for the purposes of assessing the maximum claim, even though the loss is a single occurrence. This is known as *stacking*.

If reinsurance of a large commercial book is being priced, the cedant may have written shares on different layers. These layers may not be adjacent and the shares on the layers may be different. It is likely that losses to these layers from a single event will be summed for the purposes of applying the limits of the treaty. We should obtain details of stacked limits and adjust the methodology accordingly.

The recoveries from the reinsurance contract will differ depending on whether the different layers (of the same risk) written by the cedant are regarded as stacked or independent within the terms of the XL reinsurance treaty. For example, suppose that layers $400k$ *xs* $100k$ and $1m$ *xs* $500k$ are written by the cedant and a reinsurance contract for $500k$ *xs* $500k$ covers these contracts. If the layers are stacked, then the underlying contract is essentially $1.4m$ *xs* $100k$ and the reinsurance covers the range $[600k, 1.1m]$. If they are independent, the reinsurance only affects the $1m$ *xs* $500k$ contract, covering the range $[1m, 1.5m]$.

Catastrophe XL rating

Finally, it is worth pointing out that we could, in theory at least, apply this method to catastrophe XL pricing. In practice the curves would be very sensitive to the geographical distribution of the cedant's insured values. Also the insured events have long return periods, making the curves difficult to estimate. Usually we use different exposure-based methods for cat XL rating (for example, catastrophe models or, where a model is not available, we consider zonal aggregate exposures, *ie* the aggregate sum insured or number of policies within each geographical zone)**.**

Even within risk XL treaty rating, it is common to model the "nat-cat" element of the loss cost separately (for example, using a cat model) and add this to the cost of other perils modelled on the relevant exposure curves.

"Nat-cat" is short for *natural catastrophe*, such as earthquakes and floods. Catastrophe models will be covered in Chapter 21 (and further in Subject SA3).

3 Casualty XL rating using ILFs

So far we have looked at how original loss curves are used in property business, where they are called first loss curves or exposure curves. Now we look at how they can be used to rate casualty (*ie* liability) business, where are they often called increased limit factors (ILFs).

Many of the considerations in casualty rating are similar in nature to those outlined for property in Section 2. However, the terminology and some of the issues are different. These differences are outlined in this section.

3.1 Outline theory

The main difference arises from the assumption used in constructing property exposure curves that, within certain homogenous groups at least, the relative loss size distribution *Y* can be considered to be independent of the size of the risk. This assumption is based on the belief that the sum insured (or MPL and so on) gives genuine information about the ground-up unlimited loss severity distribution for the risk in question (that is, the degree of under-insurance is limited).

The last part of Core Reading in brackets just means we are assuming that the insureds choose appropriate sums insured for themselves, neither overestimating nor underestimating the amount required.

Remember that the relative loss size distribution Y represents the original loss X as a proportion of the size of the risk M, *ie* $Y = \dfrac{X}{M}$.

In casualty there is no real equivalence. There is no easily definable upper limit on the possible severity of loss to the original insured, which will often depend on court awards. Many factors influencing the limit actually purchased may not be connected with the potential loss severity (such as the insured's insurance budget, costs of cover, insured's risk aversion, market practice, cover purchased historically).

The two points in the paragraph above are connected. Just as it is not easy for the insurer to estimate the probable maximum loss on a particular liability policy, it is also not easy for the policyholder concerned. Hence their choice of limit may just be based on their insurance budget, risk aversion, what other companies do or what they did last year. If the limit purchased is too low, it does provide a "definable upper limit" on the possible severity of loss. The problem is that it will often be unrealistically high and so not reflective of the probable maximum loss.

So for casualty business, the limit purchased does not provide much information about the potential loss severity in the way that the insured values do for property treaties. Thus we gain nothing in the analysis by switching from the original loss distribution *X* to the relative loss size distribution *Y*, as there is no reason to believe that *Y* will be the same for different risks.

For casualty business, therefore, we select risk groups, and within these, we make the following assumptions:

- **the (ground-up) loss frequency is independent of the limit purchased**
- **the (ground-up) severity is independent of the number of losses and of the limit purchased.**

These assumptions will often be reasonable. However, we should not accept them without question. For instance, we could argue that those who buy higher limits have a greater propensity to higher losses (perhaps because they have "deeper pockets").

Question 15.5

What do you think "having deeper pockets" might mean?

Alternatively, we could argue that they have better risk control, leading to a reduced frequency (and possibly severity) of losses.

The argument is that wealthy companies tend to be large ones and larger companies tend to have better risk control systems in place, *eg* better enforcement of health and safety standards in the workplace.

Once again it is important that we select risk groupings for which these assumptions are, at the very least, plausible.

Increased limit factors (ILFs)

For casualty business, we use original loss curves in the form of ILFs.

We usually express ILFs with reference to the loss cost for a relatively low limit (the "basic limit").

So, for example, the ILF at level x, relative to basic limit b, is:

$$ILF(x) = \frac{\text{Expected loss cost}(x)}{\text{Expected loss cost}(b)}$$

$$= \frac{\text{Expected frequency}(x) \times \text{Expected severity}(x)}{\text{Expected frequency}(b) \times \text{Expected severity}(b)}$$

where the term in brackets indicates the limit.

However, earlier we assumed that the frequency is independent of the limit, so the first term on the numerator and denominator cancel.

Based on similar notation to the preceding sections (and considering, as per Section 1.2, only the loss cost) with basic limit *b*, we define:

$$\boldsymbol{ILF(x) = \frac{LEV_X(x)}{LEV_X(b)}} \tag{3.1}$$

Note that, for $x > b$, the ILF will be greater than one.

This makes intuitive sense, because the expected value of an amount limited to x must be greater than the expected value of an amount limited to an amount less than x (unless something *very* strange indeed is going on with the claim distribution!).

Also note that here we have not switched to a relative loss size distribution *Y*, but have remained with the original loss distribution. Therefore ILFs are usually functions of a monetary amount, represented by *x* in equation (3.1). It is easy to see from equation (3.1) that the ILF represents the ratio of the loss cost for a primary limit *x* to the loss cost for the basic limit *b*.

Example

For a particular line of business, losses are as follows:

Original loss (£)	*Limited at £1,000*	*Limited at £2,000*
100	100	100
500	500	500
1,500	1,000	1,500
3,000	1,000	2,000

The ILF taking the basic limit of £1,000 to the limit of £2,000 would be calculated as:

$$ILF = \frac{100 + 500 + 1,500 + 2,000}{100 + 500 + 1,000 + 1,000} = 1.6$$

ILFs are often presented in the form of a table. An example is shown in Table 3.1; the example is for illustration only (it is actually based on a log-normal distribution).

Table 3.1 – Example ILFs
Arbitrary units and values

Limit	**ILF**
100,000	**1.000**
200,000	**1.432**
300,000	**1.717**
400,000	**1.927**
500,000	**2.092**
1,000,000	**2.600**
1,500,000	**2.877**
2,000,000	**3.058**
3,000,000	**3.288**
4,000,000	**3.431**
5,000,000	**3.530**
10,000,000	**3.771**

Question 15.6

In the table, the ILFs are increasing as the limit increases, but at a decreasing rate. Why is this expected?

The treatment of ALAE is usually a much more important consideration for casualty business than for property business as this can often represent a very significant proportion of the claims cost. We can derive ILFs to include the ALAE component. However, it is important to understand the treatment of ALAE in the policy before deciding on the nature of the ILFs to use and how to allow for the ALAE cost. For simplicity we continue to ignore ALAE in the theory below.

The theory below could also be extended to add in ULAE and risk loads as discussed for property business in Section 2.

The following derivation is for a layer of L excess of D.

To derive XL prices using ILFs, similarly to Section 2, we have:

$$\boldsymbol{C_b} = \textbf{Basic limits loss cost} = \boldsymbol{E[N]E[X \wedge b]} \quad \textbf{(3.2)}$$

$$\begin{aligned}
\boldsymbol{C_L} &= \textbf{Layer loss cost} \\
&= \boldsymbol{E[N]E\big[\{(X - D) \wedge L\} \vee 0\big]} \\
&= \boldsymbol{E[N]E\big[X \wedge (L + D) - X \wedge D\big]} \\
&= \frac{\boldsymbol{E[N]E[X \wedge b]E\big[X \wedge (L + D) - X \wedge D\big]}}{\boldsymbol{E[X \wedge b]}} \qquad \textbf{(3.3)} \\
&= \frac{\boldsymbol{C_b \times E\big[X \wedge (L + D) - X \wedge D\big]}}{\boldsymbol{E[X \wedge b]}} \\
&= \frac{\boldsymbol{C_b \times \big(E\big[X \wedge (L + D)\big] - E[X \wedge D]\big)}}{\boldsymbol{E[X \wedge b]}} = \boldsymbol{C_b \times \big(ILF(L + D) - ILF(D)\big)}
\end{aligned}$$

The formula above makes intuitive sense. Let's denote the layer between x and y as $[x, y]$. The cost to the layer $[0, L+D]$ is C_b multiplied by $ILF(L+D)$ and the cost to the layer $[0, D]$ is C_b multiplied by $ILF(D)$. Hence the cost to the layer [D,L+D] is the difference between them.

So we require an estimate of the loss cost at the basic limit (analogously to the full-value loss cost in property rating). From this, we estimate the loss cost to the layer using ILFs.

> ***Question 15.7***
>
> The expected loss cost for a layer at a basic limit of £100,000 is £100. Use Table 3.1 to calculate the expected loss cost for a layer at a limit of £1,000,000.

3.2 *Selecting appropriate tables of ILFs*

As for property rating, the key consideration when pricing using ILFs is to select risk groupings such that the assumptions in Section 3.1 are valid. Clearly, given the dependence of liability claims on the legal and social environment, the jurisdiction and the nature of the coverage offered will be key considerations here.

Additional considerations are:

- **The treatment of ALAE in the coverage offered.**
- **The treatment of ULAE and loadings for risk.**
- **The nature of the limits offered. It is common in the USA for split limits to be offered (a per-claimant limit and per-occurrence limit). Insurers construct ILF tables taking these split limits into account (although the theory in Section 3.1 needs some modification).**
- **Effects of trend and secular changes in the claims environment are very important for casualty business and are discussed briefly in the following section.**

Remember that trend is another word for inflation. Here, secular means long-term.

3.3 *Effect of claims inflation*

If we expect claims inflation to affect the loss distribution uniformly across all loss sizes, the adjustment to ILFs is quite easy. If we expect losses to have increased by *a*% between time *t* and time t' then a little thought shows that:

$$ILF_{t'}(x) = ILF_t\left(\frac{x}{1+a}\right) \qquad \textbf{(3.4)}$$

However, it is often the case that trend will not impact the loss distribution uniformly in this way. (In many jurisdictions large liability losses have been shown to be subject to higher inflation than smaller ones.) In this case, we should consider the change in shape of the original loss distribution and re-work the ILF table.

Problems with the inflation allowance arise due to what is termed the "leveraged effect of inflation" on excess of loss rating. Inflation will not affect lower layers very much as most claims are capped at a low limit, but it affects higher layers in a big way because most (or all) of the claim is increased by the inflation factor.

Legal reform can affect different parts of the loss distribution in very different ways.

Example

For instance, a reform that reduces the overall ground-up loss cost may have a much greater impact on smaller claims than larger ones. In this case, we should rework the ILFs.

Note that, in this example, since the reform has a greater proportionate impact on ground-up loss costs than excess loss costs it is likely to lead to higher reinsurance rates (as expressed as a rate on original premium) as competitive pressures (and/or regulators) lead cedants to reduce original rates to reflect the reduced original loss cost.

3.4 Using ILFs in XL treaty rating

The approach for exposure rating XL treaties using ILFs is similar to the approach for property rating outlined in Section 2.4.

However, the theory in Section 3.1 starts with an estimate of the basic limits loss cost. For the treaty pricing, we start with an estimate of the loss cost for the original limit issued (the equivalent of column F in Table 2.1). Therefore, we need to make an adjustment in the final calculation (column O in Table 2.1). From our ILF tables we have (with *I* as the original limit):

$$C_I = \textbf{Original limit loss cost} = C_b \times ILF(I) \qquad \textbf{(3.5)}$$

Substituting into equation (3.3) we have:

$$C_L = \textbf{Layer loss cost} = C_I \times \left(\frac{ILF(L+D) - ILF(D)}{ILF(I)} \right) \qquad \textbf{(3.6)}$$

Therefore, in order to calculate the layer loss cost, you take the difference between the ILFs at the upper and lower limit, and "normalise" by dividing by the ILF at the original limit.

The treatment of the cedant's ALAE is an important consideration in much casualty treaty rating. We require different adjustments to the methodology and ILFs, depending on the coverage of defence expenses in the original policies and on the coverage of ALAE in the treaty (for example, costs inclusive, pro rata costs in addition, and so on).

Because ILFs are expressed in absolute values, we should be careful to adjust them appropriately for trend between the date of their derivation and the prospective treaty period.

This was discussed in Section 3.3.

Treaties on a cessions basis

In addition, the dangers of using an out-of-date risk profile are probably more pronounced for casualty rating, as original limits offered and purchased can vary very significantly with the insurance cycle. For this reason, it is common in some markets to structure treaties on a cessions (or ceded) basis.

A cessions basis is also known as a cessions-made basis, or a cessions schedule.

In these treaties, the premium ceded to the treaty depends directly on the limit for each original risk, and the premium is determined from ILF tables set out in the treaty.

ILF tables can be used because that's in effect what they do – transfer limits from original risks to higher layer risks.

In this case the ILFs must make full allowance for the treatment of ALAE, and for the reinsurer's expenses and risk / profit.

4 Constructing original loss curves from claims data

In this section, we outline a methodology that can be used to construct exposure curves and ILFs. Unfortunately, many practitioners will not have access to sufficient data of sufficient quality to derive curves from data with any confidence. Those working for very large insurers or with access to quality market data will have fewer problems. However, even with large data sets, credibility is a significant problem for high value losses.

4.1 Deriving exposure curves

In principle, the derivation of property exposure curves from claims data is fairly straightforward. A numerical example is shown in Table 4.1. (This example is not based on real data and should not be used in any genuine pricing exercise.)

Table 4.1 – Constructing exposure curves **Arbitrary units and values**

A % of risk MPL (*x*)	B Total value of losses ≤ *x*% of MPL	C First *x*% of losses > *x*% of MPL	D Total accumulated loss cost 1st *x*% (B+C)	E Empirical exposure curve
1%	1,094,420	3,558,600	4,653,020	18.3%
2%	2,761,098	4,821,600	7,582,698	29.8%
3%	4,205,991	5,475,600	9,681,591	38.1%
4%	5,501,414	5,805,600	11,307,014	44.5%
5%	6,700,305	5,919,000	12,619,305	49.6%
6%	7,784,211	5,918,400	13,702,611	53.9%
7%	8,712,070	5,896,800	14,608,870	57.4%
8%	9,536,582	5,860,800	15,397,382	60.5%
9%	10,291,052	5,788,800	16,079,852	63.2%
10%	10,974,633	5,712,000	16,686,633	65.6%
15%	13,656,782	5,292,000	18,948,782	74.5%
20%	15,727,450	4,668,000	20,395,450	80.2%
30%	18,601,541	3,474,000	22,075,541	86.8%
40%	19,830,064	3,216,000	23,046,064	90.6%
50%	21,131,685	2,550,000	23,681,685	93.1%
60%	21,679,543	2,448,000	24,127,543	94.9%
70%	22,144,739	2,352,000	24,496,739	96.3%
80%	22,501,070	2,304,000	24,805,070	97.5%
90%	23,061,721	1,998,000	25,059,721	98.5%
100%	23,519,291	1,740,000	25,259,291	99.3%
110%	25,429,757	-	25,429,757	100.0%
120%	25,429,757	-	25,429,757	100.0%

The process works as follows:

- **We collect large volumes of claims data. For each claim, we obtain details of the amounts paid, date(s) of payment, risk size (*M* = SI / MPL / EML), cause of loss, classification of risk and territory.**

- **We express each claim as a percentage of the risk size.**

- **We divide the data into risk classifications/groupings, different perils and bandings of risk size such that we think they are reasonably homogenous groups. There is trade-off here between the statistical credibility of the data and the homogeneity of the classification.**

- **We construct a table of accumulated loss costs (in monetary amounts) by percentage of risk size. We calculate the value at *x*%** (x is shown in column A) **by summing the full claim amount for those claims less than or equal to *x*% of *M* (column B). In column C, we include all claims above *x*% of *M*, but each taken at a value equal to *x*% of *M*. (Note that *M* here differs for different risks.) We sum over all claims within the selected risk grouping. We repeat for values of *x* at selected intervals.**

- **We construct empirical exposure curves (column E) for different groups by dividing accumulated loss cost at each value of *x* (column D = B + C) by the total value of the losses for the group (the maximum value in column B).**

- **We combine groups for which there is no significant difference in the empirical exposure curve (we need to use judgement here and consider carefully the credibility of each curve, particularly for larger loss amounts).**

- **We smooth the empirical curves ensuring that the resultant curves display the properties outlined in Section 1.3.**

So, for example, in Table 4.1, we can see that there are £4,653,020 of losses below 1% of the MPL, which accounts for 18.3% (*ie* divided by the last figure in column B) of the total losses. Of these, £1,094,420 is in respect of losses that are less than 1% of the MPL, and the other £3,558,600 is in respect of bigger losses but only the part that lies beneath 1% of the MPL.

We should consider the problem of open claims.

The problem here is that we do not know the ultimate claim cost for open claims. Usually, it would be simply a case of projecting our claims data to ultimate using, for example, standard triangulation methods on aggregate claims data. However, for exposure curves and ILFs this is not straightforward as we are more concerned with the growth of a *distribution* of losses. We therefore also need to address changes in the *shape* of the loss distribution.

We can usually minimise this by excluding data from the most recent accident years. Settlement delays are much more of a problem for liability business and are discussed further in Section 4.2.

Additional problems are caused where policies have significant deductibles or are limited at values below *M*.

Information on the total loss size will not be available if all we have is claims that have already been subject to a deductible or have been capped at a certain level.

These problems are discussed further in the following section.

4.2 Deriving ILFs

Constructing ILFs for casualty business also has its problems.

There are problems of volume and credibility of data, when we attempt to derive ILFs. There are also some additional problems:

- **Because of policy limits, we can lose information about large losses. This is a more acute problem for liability losses since the limit purchased is not closely related to an estimate of the maximum possible size of loss.**

  The impact of original limits will affect the *distribution* of losses to a greater extent than for property business.

- **Losses must be adjusted for trend (which itself needs to be estimated).**

- **Many claims will not be closed.**

These last two problems are more significant for casualty business (compared to property) due its long-tailed nature.

Note that the following two naïve approaches to the problem of open claims are not appropriate:

- **Ignore open claims and just perform the analysis from Section 4.1 on the claims that have closed. This is inappropriate because there is frequently a relationship between the loss size and the settlement delay (larger claims can take longer to settle).**

  Therefore ignoring the open claims will underestimate the losses and skew the ILF curve. If we thought that open claims were completely independent of size then it wouldn't be such a problem.

- **Use the current reported claim amount (paid + case reserve) for open claims, and proceed as in Section 4.1. This is inappropriate because claims are often settled for very different values from the case reserves. Even though the case reserves may be sufficient or even conservative, on average a small number of losses may be settled for values well in excess of the case reserve. Using the current reported value, we will tend to understate the volatility of the loss distribution and can underestimate the loss cost at higher layers (and overestimate the cost at lower layers). Note that applying average ground-up development factors to individual claim amounts does not solve this problem.**

These problems make the construction of ILFs from underlying data a difficult process. Here we will briefly outline a methodology designed by the Insurance Services Office (ISO) in the USA.

ISO is an organisation that provides a wide variety of insurance and risk-related services to the insurance industry in the USA. It employs many members of the Casualty Actuarial Society (the USA equivalent of the UK Profession).

Only a brief outline of the methodology will be given.

Further details, including numerical examples, can be found in the Casualty Actuarial Society study note by Joseph Palmer entitled *Increased Limits Ratemaking for Liability Insurance* (2006).

In essence, the method applies survival functions to the *closed* claims (*ie* settled amounts). The process is:

- **We trend** (*ie* adjust for inflation) **the individual losses from the experience period to the period for which the ILFs will be applied.**
- **We consider only closed claims (but from several accident years) and group these claims by payment lag (time in years from accident year to settlement). Where multiple payments are made, we use the average lag weighted by amount paid.**

- **For each lag, we construct an empirical survival function for the claim size (see below). In practice, lags beyond a certain period, say five years, are grouped. Beyond this the loss distributions are thought to be similar for all lags, and so grouping increases the credibility of the estimates.**
- **We estimate the proportion of the number of loss occurrences for an accident year that are settled at each lag.**
- **We combine the empirical survival functions at each lag using the estimates or proportions settled at each lag to estimate the combined survival function for all claims.**
- **We smooth the tail of the combined survival function (often by fitting a truncated Pareto distribution above a selected threshold).**
- **Fit a parametric distribution to the smoothed curve (ISO use a mixed exponential).**
- **We can derive limited average severities (and hence ILFs) from the fitted loss distribution.**

Estimating the empirical survival function (required above)

This section describes how the third bullet point above is done in practice. The process is as follows:

For each payment lag, we select discrete loss-size intervals (typically > 50 adjacent intervals).

For each interval, we estimate the conditional survival probability (CSP). This is the probability of a loss exceeding the upper bound of the interval, provided it exceeds the lower bound. So for the *i*th interval (with upper bound u_i and lower bound l_i) we have:

$$CSP(i) = P\left[X > u_i \middle| X > l_i\right] \tag{4.1}$$

To estimate this probability, we consider only those policies whose attachment point is less than or equal to l_i and exit point (attachment point + policy limit) is greater than u_i. This means we only consider policies where we can potentially observe losses at both the bottom and the top of the interval. Within this group we count the number of occurrences (Nl_i) with (ground-up) loss size greater than l_i and the number of occurrences (Nu_i) with (ground-up) loss size greater than u_i.

The estimate of the conditional survival probability is then given by:

$$C\hat{S}P(i) = \frac{Nu_i}{Nl_i} \tag{4.2}$$

We can then estimate discrete points on the survival function for the ground-up losses by multiplying the conditional survival probabilities together:

$$\hat{S}(u_n) = \prod_{i=1}^{n} C\hat{S}P(i) \tag{4.3}$$

Again, the Palmer paper describes this in good detail.

5 Some curves used in practice

We mentioned earlier that one of the major problems with original loss curves is that they are not particularly widely available, mainly due to data scarcity.

This section introduces some of the curves that *are* available throughout the insurance world, both in the UK and the USA.

The possibilities discussed here are:

- to fit statistical curves to data that we have
- to use Riebesell curves (described below), based on an assumption for the original loss cost
- to use a published curve available on the market.

5.1 Curve fitting

We often derive both exposure curves and ILFs and show them in the form of tables giving values at discrete intervals. We often use interpolation methods or parametric curve fits to find values at intervening limits. Parametric curve fits have the advantage that we can choose curves that have the properties from Section 1.3 so the fitted curve is automatically valid.

Question 15.8

Without looking back to Section 1.3, what properties are we talking about here?

If we use a different interpolation or graduation method, it is important that the final curves retain the properties derived in Section 1.3. Otherwise, pricing between layers can become inconsistent or irrational. We can then program the full (interpolated or graduated) curves into a pricing model for ease of use.

One family of parametric curves that has been shown to be a good fit to a wide range of exposure curves are the MBBEFD curves. These are so called because of their use within statistical mechanics; the letters stand for Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac. These curves have the advantage of having a small number of parameters (two) and the flexibility to fit a range of exposure curves used in practice.

MBBEFD distributions are defined on the interval $[0,1]$ and used to model losses relative to some MPL or SI, with a value of 1 corresponding to a total loss.

Whereas the MBBEFD curves are useful for constructing exposure curves, the following list of distributions is useful for constructing ILFs.

Distributions that have been used as a basis for ILFs are:

- **log-normal**
- **Pareto**

  Both the lognormal and Pareto distributions have two parameters. You will find the distribution functions in your *Tables*.
- **truncated Pareto**
- **mixed exponential (a weighting of two or more exponential distributions with different parameters).**

An advantage of representing curves in parametric form is that it is often possible to adjust the parameters to derive new curves with slightly different properties (for example, if the nature of the risks being priced requires a curve with properties somewhere between two standard curves, it may be possible to adjust parameters to give a curve with these properties).

Whichever curves are used, it will be important to test the goodness of fit to any data you might have. However, in the industry, certain curves are already well known to be a reasonable fit for particular types of claim.

5.2 The Riebesell curves

Often a reinsurer will not have sufficient data to derive ILFs for the original business that it is reinsuring. In this case, we must use judgement to select ILFs from other sources or appropriate distributional assumptions on which to base ILFs. We might assume one of the loss distributions from Section 5.1. An assumption quite frequently used for European casualty treaty business leads to a family of curves called Riebesell curves.

These curves are very popular in Germany (Paul Riebesell was a German mathematician).

The reason that the curves are more commonly used for casualty (liability) business rather than property business is as follows:

In property business, a common assumption when applying exposure curves is to assume that, for a particular group of risks, the loss when expressed as a proportion of the sum insured is independent of the risk.

However, in casualty business, the sum insured is not the maximum amount of a claim (since claims can effectively be unlimited), but instead is the limit of indemnity chosen by the policyholder, which varies with each risk. This messes up the maths when using ILFs. Riebesell curves successfully allow for this complexity.

More details are given in the following paper, which is not part of the Core Reading:

- Mack, T and Fackler, M, Exposure Rating in Liability Reinsurance, ASTIN Colloquium Berlin 2003.

These curves are not derived from underlying data, but are based on an assumption regarding the original loss cost. The assumption is that each time the sum insured doubles, the loss cost increases by a constant factor (*x*%). Thomas Mack and Michael Fackler showed that this assumption is consistent with an original loss distribution having a Pareto tail (parameter < 1).

The Pareto parameter we are referring to here is the shape parameter, called α in the *Tables*.

So if we believe that our casualty claims have a Pareto tail with a shape parameter less than 1, then we can assume that the Riebesell curves are suitable to use for ILF purposes.

Mack and Fackler also showed that the assumption can be consistent with the collective risk model provided that it is only used for sufficiently high loss thresholds (that is, the level at which the reinsurance cover starts).

For the original loss distributions they considered, a threshold of about $5 \times E[X]$ was appropriate (so that the reinsurance has to cover losses in excess of $5 \times E[X]$ or greater).

For many international casualty markets, where limits of liability are high, this condition is easily met and reinsurers use a family of Riebesell curves with the factor *x* varying depending on the nature and class of the original business. Note, though, that the threshold condition means that using the Riebesell curves to adjust for original deductibles (analogously to equation (2.4)) is often not appropriate.

Question 15.9

Explain why it is often not appropriate to use Riebesell curves to adjust for original deductibles.

The Riebesell curves are easy to use in practice because they and the ILFs derived from them are scale invariant. This means they and the ILFs do not have to be adjusted for inflation or changes in currency (provided the attachment points remain sufficiently high for the curves to be valid). This is a consequence of the assumption (explained above) on which the curves are based.

5.3 Some market curves

Many reinsurers will not have sufficient data to derive a whole suite of exposure curves. They will usually have a set of curves from different sources (both internal and external) that they consider appropriate for different types of business and use judgement to select the most appropriate curves for each treaty. Some curves have been published from time to time and have been used throughout the market.

For example:

- **the Swiss Re / Gasser curves based on an analysis by Peter Gasser of Swiss fire loss data for 1959-67**
- **so-called Lloyd's curves which originated within the London reinsurance market (though the origin and basis is uncertain)**
- **other exposure curves derived by Swiss Re.**

These are purely examples of some curves that have been used in the market. We should seriously consider whether a curve is appropriate before using it for any specific pricing exercise. Anecdotes abound within reinsurance markets of curves being lifted from actuarial papers and used in completely inappropriate contexts for years, or even decades.

The ISO in the USA derives exposure curves for USA property business classes and ILFs for USA casualty classes. These are available as part of computer packages sold by ISO.

6 Some other uses of curves in reinsurance pricing

In this section, we mention two further complications that arise when applying exposure curves. The interested student (particularly those studying Subject SA3) might wish to read further into these topics, using the references below.

6.1 Inferring treaty loss distributions from exposure rating

The methodology outlined in Sections 2.4 and 3.4 is appropriate when we estimate the annual average loss cost for a treaty with unlimited free reinstatements. Often there will be some features of a treaty that are affected by the aggregate losses to the layer.

These are often simply called *aggregate features*. The important thing about aggregate features is that they depend on the claims experience of the treaty itself.

In their simplest form this may be an annual aggregate limit or deductible. Paid reinstatements and swing-rated premiums are also common in some markets.

Question 15.10

Explain each of the following terms:

- annual aggregate limits and deductibles
- unlimited free reinstatements and paid reinstatements
- swing-rated premiums.

A full introduction to pricing these aggregate features is beyond the scope of this section.

Here, we just give a brief introduction.

However, we will require an estimate of the aggregate loss distribution (before any aggregate limit or deductible) to the layer. We will sometimes assume a parametric form of the aggregate distributions (either for aggregate losses or frequency and severity separately depending on the nature of the layer and the frequency of losses). We will choose the parameters so that the modelled loss cost matches the mean annual loss for the layer. We then make volatility assumptions based on a benchmark.

However, we can use the following approach to make sure that the severity distribution chosen is consistent with the exposure rating (and hence the underlying risk profile and assumed original loss severity distributions).

From equation (2.3) we have:

$$C_L = \text{Layer loss cost} = E[N]E\left[X \wedge (L+D) - X \wedge D\right] \tag{6.1}$$

Consider now a narrow layer of limit δl

$$\begin{aligned} C_{\delta l} &= E[N]E\left[X \wedge (\delta l + D) - X \wedge D\right] \\ &= E[N]\big(S(D)\delta l\big) \end{aligned} \tag{6.2}$$

where the second line follows from equation (1.2) and the fact that δl is small.

Note that $S(D)$ is the survival function defined in Section 2.1, *ie* $S(x) = 1 - F(x) = P[X > x]$**.**

Therefore:

$$E[N]S(D) \approx C_{\delta l} \big/ \delta l \tag{6.3}$$

$E[N]S(D)$ represents the expected frequency of a (ground-up) loss of size D or greater.

Suppose that we have an exposure rated estimate of the aggregate loss for each layer.

The process is as follows:

- **We can divide the layer into a series of narrow layers starting at the attachment point and ending at the limit of the treaty.**
- **We estimate the aggregate loss for each layer on the exposure rated basis.**
- **We use equation (6.3) to estimate a series of points on the severity distribution that is consistent with the exposure rated estimates.**
- **We fit a parametric curve to these points or derive a discrete estimate of the distribution function for the severities.**

To model the aggregate loss distribution it is still necessary to make an assumption about the distribution of the number of claims impacting the layer as the method (described above) only gives us an estimate of the mean. Often we will use a negative binomial distribution with benchmark volatility assumptions. Because of parameter error and possible dependence between loss occurrences, a Poisson assumption is unlikely to be appropriate.

We can estimate the aggregate distribution of losses to the layer (before allowance for aggregate features) using, for example, simulation, Panjer Recursion or a Fast Fourier Transform method.

You should remember the Panjer Recursion method from Chapter 11.

A Fourier Transform is an example of an *inversion method*, *ie* a statistical technique that can be used to obtain the probability function of a distribution given some other measure, such as the moment generating function. A *Fast* Fourier Transform is an algorithm that enables this to be performed very quickly. It is not covered in any of the CT Subjects, and so you should not need to know the detail behind it.

When we have estimated the aggregate distribution, we can estimate the impact of the aggregate features on the losses/premiums and adjust the proposed rate accordingly.

The method is explored in detail in:

Mata, AJ, Fannin, B and Verheyen, MA, Pricing Excess of Loss Treaty with Loss Sensitive Features: An Exposure Rating Approach, GIRO Convention Paris 2002.

6.2 *Exposure adjustment in treaty experience rating*

We can use exposure rates to help to experience rate XL (excess of loss) **treaties.**

When experience rating excess of loss treaties, it can be difficult to allow for changes to the risk profile over time.

A standard way to adjust for exposure changes is to use the on-level premiums (original premiums adjusted for rate changes). However, because of changes in the risk profile, it is possible that changes in exposure to the layer are very different from the ground-up exposure change given by on-level premiums.

One way around this is to use historical limits profiles to exposure rate the layer for previous years. This will give an estimate of how the exposure to the layer has changed over time. We can use this as the exposure measure for experience projections.

The method is explored in detail in:

Mata, AJ and Verheyen, MA, An Improved Method for Experience rating Excess of Loss Treaties using Exposure Rating Techniques, Casualty Actuarial Society Forum, Spring 2005 pp. 171-214.

7 Closing remarks

Exposure curves and ILFs are very useful tools to enable us to estimate consistent prices for different layers, provided we can estimate the full value loss cost (or a limited loss cost). They are particularly useful where historical data for a risk are sparse and/or lacking in relevance, so experience-based rates are not credible.

This chapter has given an introduction to the methodology. Like most actuarial methods the application in practice is difficult, mainly, in this case, because of uncertainties in estimating and/or selecting appropriate curves. It is important to note that the modelled loss cost to layers (particularly high ones) can be extremely sensitive to the selected curve. Practitioners in many markets will not have access to sufficient relevant data to model curves with any confidence so judgement will be key, and it is important to monitor closely the emerging results. It will be difficult to improve on this situation in many markets without improved market-wide data collection.

We have not considered loadings for expenses and risk / profit in this chapter, but we should consider the uncertainties associated with the methodology when we add the loadings. We should also consider the uncertainties when judging the credibility of the rates derived against those from another method (for example, experience rating).

After noting some of the difficulties and disadvantages of the methodology, it is worth reminding ourselves of some of the advantages:

- **it is relatively simple to implement**
- **it is relatively easy to explain to non-technical colleagues**
- **the loss costs obtained should be internally consistent**
- **it can be used where little or no credible loss data is available.**

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Chapter 15 Summary

Original loss curves are more often called:

- Property business: *first loss scales, exposure curves* or *loss elimination functions*. They show the proportion of the full value premium allocated to primary layers at different values. When showing the proportion to allocate to the excess layer rather than the primary layer, they are also called *excess of loss scales*.
- Casualty business: *increased limit factors*. They give the ratio of premium for higher limits to a basic limit.

Curves often only account for pure loss cost, but may be adapted to allow for ALAE, ULAE and a load for risk at each limit.

The curves are closely related to the limited expected value function, which is a non-decreasing function that increases at a decreasing rate.

Property business

Exposure curves are defined as $G(x)=\dfrac{LEV_Y(x)}{E[Y]}$, where Y is the relative loss severity.

The closer the curve is to the diagonal, the greater the proportion of large losses. To calculate the loss cost of a layer, we use the formula: $C_L = C\times\left(G\left(\dfrac{L+D}{M}\right)-G\left(\dfrac{D}{M}\right)\right)$.

When using exposure curves, we may need to make adjustments for:

- the heterogeneity of the underlying business, *eg* by using separate curves for each peril or risk size band
- the effect of claims inflation.

When using exposure curves in XL treaty rating, we should also be aware of:

- the lack of suitable curves to use
- the lack of credible data
- uncertainty of the original loss ratio
- the treatment of original deductibles
- the treatment of inuring reinsurances (which apply before the treaty does)
- stacked limits
- alternative methods to allow for the nat-cat element separately, *eg* cat models.

Casualty business

Here, severity can be unlimited (or an upper limit selected), and so we assume that within each risk group:

- the ground-up loss frequency is independent of the limit purchased
- the ground-up severity is independent of the number of losses and of the limit purchased.

Here, we use ILFs: $ILF(x) = \dfrac{LEV_X(x)}{LEV_X(b)}$, where the base limit is b.

For a layer we use: $C_L = C_l \times \left(\dfrac{ILF(L+D) - ILF(D)}{ILF(l)} \right)$, where l is the original limit.

Constructing curves

The following process could be used to construct exposure curves:

- collect claims data and express each claim as a percentage of the risk size.
- divide data into homogenous groups.
- construct table of accumulated loss cost by percentage of risk size.
- construct empirical exposure curve by dividing accumulated loss cost by total value of losses and combine groups where curve is similar.
- smooth the curve.

Also, allow for:

- open claims
- deductibles/limits on underlying policies
- inflation (trend).

Deriving ILFs for casualty business presents additional problems. The ISO methodology using closed claims, grouped by time lag to construct empirical survival functions might be used.

Curves used in practice

In practice, we might:

- fit statistical curves to the data, *eg* MBBEFD curves, log-normal, Pareto, truncated Pareto, mixed exponential
- use Riebesell curves, based on an assumption for the original loss cost
- use a market curve, *eg* Swiss Re, Gasser, Lloyd's.

Special treatment is required:

- where there are aggregate features in the data
- to allow for changes in risk features when experience rating XL treaties.

Advantages and disadvantages

The application of exposure curves and ILFs is difficult in practice, often due to uncertainty in estimating or selecting the appropriate curves.

However, they are relatively simple to implement, easy to explain, provide internally-consistent loss costs and can be used where there is little or no credible data.

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Chapter 15 Solutions

Solution 15.1

In addition to those mentioned in the Core Reading, original loss curves might be useful for:

- small companies or lines of business where data is sparse, non-existent or unreliable
- pricing when changing the limits within the policy conditions.

Solution 15.2

ALAE are claims handling expenses that are directly attributable to a particular claim, for example, legal defence costs.

ULAE are claims handling expenses that are not directly attributable to a particular claim. They may have resulted from an array of claims or from some other general process involved in handling claims. For example, if an insurer pays a consultancy to perform a review of case estimates and IBNR held, the fees charged would be part of the insurer's ULAE.

Solution 15.3

Indemnity costs are the pure claims costs.

Solution 15.4

A claim can be bigger than the PML. The PML (probable or possible maximum loss) is an estimate of the biggest loss, not the *absolute* maximum.

The cover could be on a "first loss" basis, which the Glossary says is:

"A form of insurance cover in which the sum insured is less than the full value of the insured property, so that the policyholder has to bear any loss in excess of the sum insured. It is appropriate in circumstances where the policyholder considers that a loss in excess of the sum insured is extremely unlikely or the item is effectively priceless. It is commonly used in fire business."

Solution 15.5

Having deep pockets means that the defendant is relatively wealthy, and so there's a lot of money to be had if someone decided to sue them, and succeeded. The Glossary says that deep pocket syndrome is:

"A situation where claims are made based on the ability of the defendant to pay rather than on share of blame. An injured party will try to blame the party with the greatest wealth (*ie* deepest pocket) where there is more than one potential defendant."

Solution 15.6

This is because the LEV is increasing at a decreasing rate.

Solution 15.7

Using the table:

$$\text{Cost} = £100 \times \frac{ILF_{1,000,000}}{ILF_{100,000}} = £100 \times \frac{2.600}{1.000} = £260 .$$

Solution 15.8

The two properties of the LEV function are:

- it is an increasing function (or strictly speaking non-decreasing)
- it increases at a decreasing rate, *ie* it is a concave function.

Solution 15.9

The deductible d on the original policy written by the direct writer is unlikely to exceed five times the expected losses. Hence we cannot use the Riebesell curve to estimate $ILF(d)$. This makes it difficult to adjust for original deductibles using Riebesell curves.

Solution 15.10

Annual aggregate limits and deductibles

These are limits and deductibles that only apply to the annual aggregate (otherwise recoverable) losses. For example, with an annual aggregate deductible of £1m on an excess of loss treaty, this means that if reinsurance recoveries on an excess of loss treaty total less than £1m (within a year), then no recovery will in fact be made. Once the recoveries exceed £1m, then the excess over £1m will be paid.

Reinstatements

Within excess of loss reinsurance, reinstatements are the restoration of full cover following a claim. Normally, the number of reinstatements, and the terms upon which they are made, will be agreed at the outset. Once agreed, they are automatic and obligatory on both parties.

Unlimited free reinstatements mean that reinstatements can continually be made, at no cost. *Paid* reinstatements mean that a reinstatement premium must be paid before the reinstatement(s) go ahead.

Swing-rated premiums

This is where the premium of each individual risk depends, at least in part, on the actual claims experience of that risk in the period covered.

Chapter 16

Generalised linear modelling

Syllabus objectives

(n) Understand the applications of Generalised Linear Models to the rating of personal lines business and small commercial risks.

0 Introduction

With the increasing power of computers, the use of generalised linear models (GLMs) is now widespread for personal lines pricing, and is increasingly used for some commercial lines of business. There are a number of commercially available software packages that enable actuaries and underwriters to calculate frequencies, average claim costs and burning costs to use as a basis for setting future premium rates.

This chapter considers some of the mathematics behind GLMs, which you will have met in Subject CT6, and then goes on to consider how we might use these models. If it has been a while since you studied Subject CT6, it is probably worth reviewing the material on GLMs there before tackling this chapter.

Section 1 explains the concepts of GLMs and looks at the exponential family of distributions in some detail.

In Section 2 we describe the principles of how a GLM is constructed and look at the types of factors that might be included within a model.

Section 3 deals with the techniques we can use to check the significance of the factors used in the model.

Section 4 describes techniques that can be used to check the appropriateness of the chosen model structure, *eg* by considering the residuals.

Finally, in Section 5, we consider how we might refine our model. We will look at how we can use interactions and offsets, and will discuss aliasing of potential rating factors.

1 Concepts of GLMs and exponential family

A generalised linear model (GLM) can be used to model the behaviour of a random variable that is believed to depend on the values of several other characteristics, *eg* age, sex, vehicle group. The Glossary defines a GLM as follows:

In statistics, the generalised linear model (GLM) is a flexible generalisation of ordinary least squares regression. The GLM generalises linear regression by allowing the linear model to be related to the response variable via a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value.

The link function is formally defined in Section 1.6.

1.1 Multiple linear regression and the normal linear model

A linear regression model with k parameters representing the relationship between a number of observations Y_i and independent variables X_{ik} takes the form:

$$Y_i = \beta_0 + \beta_1 f_1(X_{i1}) + \beta_2 f_2(X_{i2}) + \dots + \beta_k f_k(X_{ik}) + \varepsilon_i$$

where:

- Y_i **are the observed data values (also called the response variable)**
- $f_j()$ **may be non-linear functions**
- X_{ij} **is the value of the j th independent variable factor**
- β_j **is the j th parameter value, and**
- ε_i **are random variables representing errors in the relationship.**

This can be written as:

$$Y_i = \beta_0 + \sum_{j=1}^{k} \beta_j f_j(X_{ij}) + \varepsilon_i .$$

It can also be written in matrix form, by allowing appropriately for the constant term and treating the independent factors as applying the variables net of any non-linear functions, as:

$$\mathbf{Y} = \mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\varepsilon}.$$

One way to deal with the constant term would be to define a dummy variable X_{i0} which just takes the value 1 for all observations i and let f_0 be the identity function, so that $f_0(X_{i0})=1$. Then we would have $\mathbf{Y}=\mathbf{X}.\boldsymbol{\beta}+\boldsymbol{\varepsilon}$ where

$$\mathbf{Y}=\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix} \qquad \mathbf{X}=\begin{pmatrix} X_{10} & X_{11} & \cdots & X_{1k} \\ X_{20} & X_{21} & \cdots & X_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n0} & X_{n1} & \cdots & X_{nk} \end{pmatrix} \boldsymbol{\beta}=\begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} \qquad \boldsymbol{\varepsilon}=\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

Here $\boldsymbol{\varepsilon}$ is the error term for the model and $E(\mathbf{Y})=E(\mathbf{X}.\boldsymbol{\beta})$.

The expected value of the error term is zero.

As an illustrative example, the following table summarizes a simple dataset of the frequency of vehicle collisions in a year, which has two independent variables, age and gender. Note that, under proposed EU legislation, it will no longer be permissible to rate on gender from 22 December 2012.

	Young	Old
Male	90%	10%
Female	45%	5%

We assume a linear model structure and that the observations Y_i have the normal distribution with mean μ_i and variance σ^2. So we define the matrices as follows:

$$\mathbf{Y}=\begin{pmatrix} 90\% \\ 10\% \\ 45\% \\ 5\% \end{pmatrix}, \ \mathbf{X}=\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \ \boldsymbol{\beta}=\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_3 \end{pmatrix}$$

where the first column of the matrix X is a constant term, the second column means "Is male?", and the third means "Is old?". We do not include columns for "Is female?" or for "Is young?" because these are dependent on the "Is male?" and "Is old?" variables respectively. For instance, if the person is not male, then they will by definition be female. The columns which have been omitted from the design matrix construction are usually termed "base levels".

β_0, β_1 and β_3 are the constant, male parameter and old parameter respectively. So the "base levels" for this example are female, young.

The omitted "Is female" column would have corresponded to β_2 and the "Is young" column would have corresponded to β_4.

Here we have $n = 4$ and $k = 2$, and the model is:

$$\begin{pmatrix} 90\% \\ 10\% \\ 45\% \\ 5\% \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix}.$$

We can also write this in component form as:

$$90\% = \beta_0 + \beta_1 + \varepsilon_1$$

$$10\% = \beta_0 + \beta_1 + \beta_3 + \varepsilon_2$$

$$45\% = \beta_0 + \varepsilon_3$$

$$5\% = \beta_0 + \beta_3 + \varepsilon_4.$$

We now want to estimate the parameters β_0, β_1 and β_3. We can do this using the method of maximum likelihood. Since we have n observations, $y_1, y_2, ..., y_n$, from a continuous (in this case, normal) distribution, the likelihood function is of the form:

$$L = \prod_{i=1}^{n} f(y_i)$$

The density function for the normal distribution is:

$$f(y_i; \mu_i, \sigma^2) = \exp\left\{ -\frac{(y_i - \mu_i)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2) \right\}$$

This is a rearranged form of the expression given on Page 11 of the *Tables.*

So here **the likelihood function is:**

$$L(y; \mu, \sigma^2) = \prod_{i=1}^{n} \exp\left(-\frac{(y_i - \mu_i)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2) \right)$$

To calculate the maximum likelihood estimates of the parameters, we can first take the log of the likelihood function and then calculate its maximum. The log of the likelihood function is:

$$l(y;\mu,\sigma^2)=\sum_{i=1}^{n}-\frac{(y_i-\mu_i)^2}{2\sigma^2}-\frac{1}{2}\ln(2\pi\sigma^2).$$

It can be proved that maximising the log-likelihood is equivalent to minimising the sum of squared errors:

$$l^*(y;\mu)=\sum_{i=1}^{n}(y_i-\mu_i)^2.$$

The sum of squared errors for our example is:

$$l^*=(90\%-\beta_0-\beta_1)^2+(10\%-\beta_0-\beta_1-\beta_3)^2+(45\%-\beta_0)^2+(5\%-\beta_0-\beta_3)^2.$$

We want to calculate the values of the parameters β_0, β_1 and β_3 that make l^* as small as possible. So we differentiate l^* with respect to each of β_0, β_1 and β_3 and set the three partial derivatives equal to 0.

Solving this in the usual way by setting partial derivatives to zero gives:

$$\frac{\partial l^*}{\partial\beta_0}=0=-2(90\%-\beta_0-\beta_1)-2(10\%-\beta_0-\beta_1-\beta_3)-2(45\%-\beta_0)-2(5\%-\beta_0-\beta_3)$$

$$\frac{\partial l^*}{\partial\beta_1}=0=-2(90\%-\beta_0-\beta_1)-2(10\%-\beta_0-\beta_1-\beta_3)$$

$$\frac{\partial l^*}{\partial\beta_2}=0=-2(10\%-\beta_0-\beta_1-\beta_3)-2(5\%-\beta_0-\beta_3)$$

This gives:

$$\hat{\beta}_0=55\%,\qquad \hat{\beta}_1=25\%,\qquad \hat{\beta}_3=-60\%.$$

Using these model parameters we can readily calculate the predicted values for Y.

The predicted values for Y are:

$$E[\mathbf{Y}] = \begin{pmatrix} 80\% \\ 20\% \\ 55\% \\ -5\% \end{pmatrix}$$

So, for example, $E(male, old)$ = constant + male parameter + old parameter

= 55 + 25 – 60 = 20%

Clearly, with more than two factors, a manual solution becomes increasingly long-winded.

> ***Question 16.1***
>
> Show that the solution of the system of equations above is:
>
> $$\beta_0 = 55\%, \qquad \beta_1 = 25\%, \qquad \beta_3 = -60\%,$$
>
> and confirm the predicted values for Y.

The model that we have just seen is an example of a GLM. In this example, we assumed that the random variables Y_i were normally distributed. In any GLM, we have to assume that the distribution of the variable that we are trying to model is one that belongs to the exponential family, which we describe in the following section.

1.2 Definition of the exponential family

The exponential family is the set of distributions whose probability function or probability density function can be written in the form:

$$f(y;\theta,\varphi) = \exp\left\{\frac{y\theta - b(\theta)}{a(\varphi)} + c(y,\varphi)\right\}$$

where $a(\varphi)$, $b(\theta)$ and $c(y,\varphi)$ are specific functions. Different choices for $a(\varphi)$, $b(\theta)$ and $c(y,\varphi)$ define a different distribution and solution to the GLM problem. The parameter θ is termed the canonical (or natural) **parameter and φ is the scale parameter.**

This formula is given on Page 27 of the *Tables*.

The parameter θ is a function of the mean $\mu = E(Y)$. The mean and variance of Y satisfy the equations:

$$E(Y) = b'(\theta)$$

$$\text{var}(Y) = a(\varphi)b''(\theta)$$

We also introduce the variance function defined by:

$$V(\mu) = b''(\theta).$$

Note that this is not equal to the variance of Y. This function will be used further in section 1.4.

In Subject CT6 we saw that the binomial, Poisson, normal, exponential and gamma distributions all belong to the exponential family. Here we add another two distributions to that list, namely the inverse Gaussian (or Wald) distribution and the Tweedie distribution.

The pdf of the inverse Gaussian distribution is nasty! It is:

$$f(y) = \left(\frac{\lambda}{2\pi y^3}\right)^{\frac{1}{2}} \exp\left(-\frac{\lambda(y-\mu)^2}{2y\mu^2}\right) \qquad \text{for } y > 0$$

The pdf of the Tweedie distribution is even nastier! It is:

$$f_Y(y;\theta,\lambda,\alpha) = \sum_{n=1}^{\infty} \frac{\left\{(\lambda\omega)^{1-\alpha}\kappa_\alpha(-1/y)\right\}^n}{\Gamma(-n\alpha)n!\,y} \exp\left\{\lambda\omega[\theta_0 y - \kappa_\alpha(\theta_0)]\right\} \quad \text{for } y > 0$$

where:

$$\kappa_\alpha(\theta) = \frac{\alpha-1}{\alpha}\left(\frac{\theta}{\alpha-1}\right)^\alpha$$

$$\theta_0 = \theta\,\lambda^{1/(1-\alpha)}$$

and ω is the prior weight corresponding to the exposure of the observation in question. See Section 1.5 for more interesting facts about the Tweedie distribution.

The chart below summarises a number of familiar distributions in the exponential family:

	$a(\varphi)$	$b(\theta)$	$c(y,\varphi)$
Normal	φ/ω	$\theta^2/2$	$-\tfrac{1}{2}(\omega y^2/\varphi + \ln(2\pi\varphi/\omega))$
Poisson	φ/ω	e^{θ}	$-\ln y!$
Gamma	φ/ω	$-\ln(-\theta)$	$\left(\omega/\varphi - 1\right)\ln(\omega y/\varphi) - \ln\left(\Gamma(\omega/\varphi)\right)$
Binomial **(*m trials*)**	φ/ω	$\ln(1+e^{\theta})$	$\ln\binom{\varphi}{\varphi y}$
Inverse Gaussian	φ/ω	$-\sqrt{-2\theta}$	$-\tfrac{1}{2}\left\{\ln(2\pi\varphi y^3/\omega) + \omega/(\varphi y)\right\}$
Exponential	φ/ω	$-\ln(-\theta)$	0
Tweedie	$1\big/\lambda\omega\lambda^{1/(1-\alpha)}$	$\dfrac{\kappa_\alpha(\theta_0)}{\lambda^{1/(1-\alpha)}}$	$\ln\left(\displaystyle\sum_{n=1}^{\infty}\frac{\left((\lambda\omega)^{1-\alpha}\kappa_\alpha(-1/y_i)\right)^n}{\Gamma(-n\alpha)\,n!\,y_i}\right)$

In all but the Tweedie distribution it can be seen that the choice of $a(\varphi)$ is the same, containing ω.

ω is a prior weight, a constant that is specified in advance.

For insurance applications, common choices for the prior weight are equal to:

- **1 (for example, when modelling claim counts), or**
- **the number of exposures (for example, when modelling claim frequency), or**
- **the number of claims (for example, when modelling claim severity).**

Often the scale parameter φ is equal to 1 and plays no part in the modelling problem.

We now take a look at some of these distributions in more detail and show how these values are derived.

1.3 Deriving the specific functions of the exponential family

Given the above definition of the exponential family of distributions and the table of values for the functions $a(\cdot)$, $b(\cdot)$ and $c(\cdot)$, this section will demonstrate how to manipulate it into the exponential family form, beginning with the standard statistical notation for the distribution.

Poisson distribution

To show how these functions are derived, the Poisson is worked through below as an illustration. The probability function of the Poisson distribution (with mean λ) is:

$$P(Y=y)=\frac{\lambda^y e^{-\lambda}}{y!}$$

Hence:

$$f(k;\lambda)=\exp\left[k\ln(\lambda)-\lambda-\ln(k!)\right]$$

The Core Reading is using the notation $f(k;\lambda)$ instead of $P(Y=k)$ here.

Now replace y with y_i, and let $\theta_i=\ln(\lambda)$. Then:

$$P(Y=y_i)=\exp\left\{y_i\theta_i-\lambda-\ln(y_i!)\right\}$$

or equivalently:

$$P(Y=y_i)=\exp\left\{y_i\theta_i-e^{\theta_i}-\ln(y_i!)\right\}$$

This is of the form:

$$\exp\left\{\frac{y_i\theta_i-b(\theta_i)}{a_i(\varphi)}+c(y_i,\varphi)\right\}$$

where:

$$\varphi=1 \qquad c(y_i,\varphi)=-\ln(y_i!)$$

$$a_i(\varphi)=1 \qquad b(\theta_i)=e^{\theta_i}$$

The inclusion of $a_i(\varphi)$ just allows each observation to be weighted by the appropriate exposure.

Hence $a_i(\varphi)$ would be set to $\dfrac{\varphi}{\omega_i}$.

Question 16.2

Show that the exponential distribution belongs to the exponential family by deriving the corresponding functions a(), b() and c().

1.4 Mean, variance, and variance function

For the above distributions the mean and variance of Y_i can be shown to be:

$$E(Y_i) = \frac{db(\theta_i)}{d\theta_i} = \mu(\theta_i)$$

and:

$$\text{var}(Y_i) = \frac{d^2 b(\theta_i)}{d\theta_i^2} a(\varphi) = V(\mu) a(\varphi)$$

Recall that $V(\mu)$ is called the *variance function* and is defined to be equal to $b''(\theta)$.

For the distributions above:

Distribution	$\mu(\theta)$	$V(\mu)$
Normal	θ	1
Poisson	e^{θ}	μ
Gamma	$-1/\theta$	μ^2
Binomial	$e^{\theta}/(1+e^{\theta})$	$\mu(1-\mu)$
Inverse Gaussian	$(-2\theta)^{-1/2}$	μ^3
Exponential	$-1/\theta$	μ^2

Question 16.3

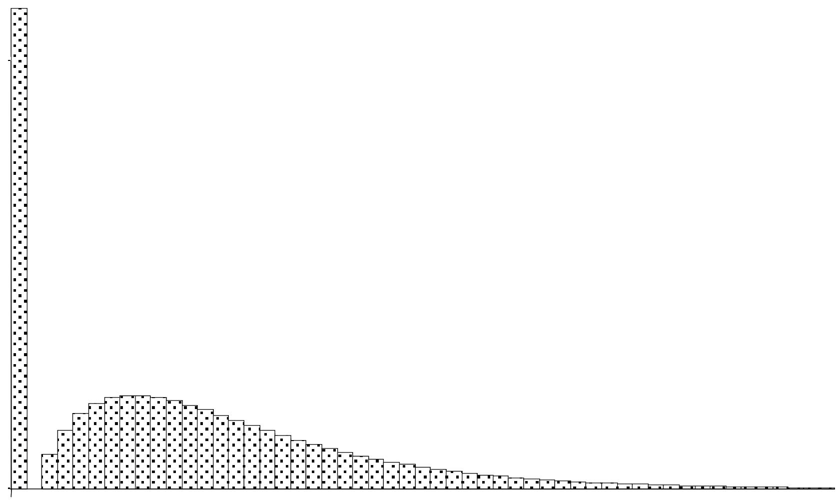
For the exponential distribution with parameter λ, we showed in Question 16.2 that:

$$b(\theta_i) = \ln\left(-\frac{1}{\theta_i}\right) = -\ln(-\theta_i)$$

where $\theta_i = -\lambda$. Use this to derive the formula for $V(\mu)$ given in the table above.

1.5 *The Tweedie distribution*

Direct modelling of pure premium or incurred loss data is problematic since a typical pure premium distribution will consist of a large spike (*ie* a point mass) **at zero (where policies have not had claims) and then a wide range of amounts (where policies have had claims). This is illustrated in the diagram below.**



> ***Question 16.4***
>
> In this diagram, how could you explain the "missing" data immediately to the right of the spike at zero?

Many of the traditional members of the exponential family of distributions are not appropriate for modelling claims experience from such a distribution since they do not have a point mass at zero combined with an appropriate spread across non-zero amounts.

The Tweedie distribution is a special member of the exponential family that has a variance function proportional to μ^p, with p being an additional parameter.

In the case of $1 < p < 2$ the Tweedie distribution has a point mass at zero and corresponds to the compound distribution of a Poisson claim number process and a gamma claim size distribution. The distribution can be Poisson-like (as $p \to 1$) or Gamma-like (as $p \to 2$).

Further information about the Tweedie distribution can be found in the paper "Fitting Tweedie's Compound Poisson Model to Insurance Claims Data" by Jørgenses, B and De Souza, M.C.P, Scand. Actuarial J. 1994 1:69-93.

1.6 The structure of a GLM

The linear model structure:

$$Y_i = \sum_{j=1}^{k} X_{ij}\beta_j + \varepsilon_i$$

generalises into the following structure:

$$Y_i = g^{-1}\left(\sum_{j=1}^{k} X_{ij}\beta_j + \xi_i\right) + \varepsilon_i.$$

This enables us to model a wider range of underlying data. Solutions are iterative and require significant computational power.

Again this can be written in matrix form as:

$$\mathbf{Y} = g^{-1}(\mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\xi}) + \boldsymbol{\varepsilon}$$

where:

- **$g()$ is known as the link function**
- **$\mathbf{X}$ is the design matrix of factors**
- **$\boldsymbol{\beta}$ is a vector of parameters to be estimated**
- **$\boldsymbol{\xi}$ is a vector of offsets or known effects – these are included when we know the effect of an explanatory variable and include this as a known effect**
- **$\boldsymbol{\varepsilon}$ is the error term appropriate to $\mathbf{Y}$.**

Recall that the distribution of the response variables Y_i should be a member of the exponential family.

A common link function is the log link, which essentially turns models from additive to multiplicative.

Offsets will be considered in more detail in Sections 5.2, 5.4 and 5.6.

Sometimes the linear predictor is written:

$$\boldsymbol{\eta} = \mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\xi}$$

and the expected value of Y shown as:

$$\boldsymbol{\mu} = g^{-1}(\boldsymbol{\eta})$$

This gives:

$$\mathbf{Y} = E[\mathbf{Y}] + \boldsymbol{\varepsilon} = \boldsymbol{\mu} + \boldsymbol{\varepsilon} = g^{-1}(\boldsymbol{\eta}) + \boldsymbol{\varepsilon} = g^{-1}(\mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\xi}) + \boldsymbol{\varepsilon}$$

Except for the special case of the Tweedie distribution, the variance of the i th observation is:

$$\text{var}[Y_i] = \frac{\varphi\, V(\mu_i)}{\omega_i}$$

where:

- $V[\mu_i]$ **is the variance function of the model**
- φ **is the scale parameter to adjust** $V[\mu_i]$ **to** $\text{var}[Y_i]$
- ω_i **are the weights assigned to each observation, usually defined as the exposure or credibility of the data.**

2 Constructing GLMs

In this section we describe the principles of constructing a GLM, with specific reference to its applications to general insurance. Before this, we look at the failings of using one-way analysis when analysing multiple variable distributions.

2.1 One-way analysis

The failings of using a one-way analysis when analysing multiple variable distributions are best examined using a simple counter-example.

This example relates to motor insurance.

First consider two factors in a dataset that are correlated, where each has an influence on the response variable. We will ignore any random variation in the response, and hence assume the predicted values are exact.

Let our first factor be age (young / old), and we will assume that old drivers are three times safer than young drivers.

Let our second factor be car group (low / high), and we will assume that the low car groups are twice as safe.

Risk relativities (assumed exact)		
	Young	Old
Age	3	1
	Low	High
Car Group	1	2

Relativities are just numbers that quantify the level of risk in one category compared to that in another. As their name implies, they do not describe an absolute level of risk. So, in the table above, we can see that drivers in the young category have three times the level of risk of those in the old category.

In this example, the relativities have been calculated in such a way that they apply multiplicatively. So the relativity for the combination of a young driver with a high car group is 6.

Assuming a simple multiplicative model gives the following predicted values. We also let the exposure be strongly skewed.

Age	Car Group	Predicted value	Exposure	Total response
Young	Low	3	10,000	30,000
Young	High	6	100	600
Old	Low	1	4,000	4,000
Old	High	2	6,000	12,000

The predicted values are calculated by multiplying the relativities together. Total response is given by:

$$\text{Predicted value} \times \text{Exposure}$$

Now let us calculate the one-way tables for these two factors and consider the outcome.

Age	Car Group	Predicted value	Exposure	Total response
Young		3.0	10,100	30,600
Old		1.6	10,000	16,000
	Low	2.4	14,000	34,000
	High	2.1	6,100	12,600

The exposure and total response figures are calculated by adding the appropriate values from the first table above. The predicted values are then calculated by dividing the total response by the exposure.

The one-way table for age understates the true relativities because the good experience for older drivers is masked by their tendency to choose high car groups.

The one-way table for car group is completely misleading because of the predominance of young drivers choosing the low car groups, and lack of them in the high car group.

Almost all the young drivers have cars in low car groups. The one-way analysis of car group predicts high claims experience for lower group cars. However, this may result not from the fact that these lower group cars are riskier, but from the fact that they are driven mainly by the higher risk young drivers.

A GLM is a way to unpick these relationships, and produce estimates of the true values of the relativities.

In other words, a generalised linear model can take account of correlations, and allow investigation of any interactions between the factors and/or variables present in the model. The terms *variable, factor* and *interaction* should be familiar from Subject CT6. In case you need a reminder, they are defined again in the next section.

2.2 Linear predictors

Variables

A GLM requires two kinds of variables to be defined.

1. Weight / exposure

These are the weights used in the model fit to attach an importance to each observation.

An observation could be an individual policy or claim, or it could be a combination of policies or claims that share certain defined characteristics.

For example, in a claim frequency model, exposure would be defined as the length of time the policy had been on risk. For an average claim size model, the exposure will be the number of claims for that observation.

2. Response

This is the value that the model is trying to predict. Hence, in the claim frequency model, it is the number of claims for that observation, and for the average claim size model, it is the total claims cost for that observation.

In general the "name" of the model corresponds to the meaning of the ratio $\dfrac{\textbf{Response}}{\textbf{Weight}}$**, *ie*:**

$$\textbf{Claim frequency} = \frac{\textbf{Number of claims}}{\textbf{Policy years}}$$

Also:

$$\textbf{Average claim size} = \frac{\textbf{Cost of claims}}{\textbf{Number of claims}}$$

The two types of variables described above always take numerical values.

Categorical factor

This is a factor to be used for modelling where the values of each level are distinct, and often cannot be given any natural ordering or score.

Any non-numerical input into a model, *eg* gender, would be classified as a categorical factor. A level of a factor is simply a distinct value that a factor can take. So if gender is a factor in a GLM and it can only take the values "male" or "female" then gender would be said to have two levels.

An example of this would be car manufacturer, which has various values, *eg* "Ford", "Vauxhall", "Toyota", "Lotus". These could be ordered in a number of ways: sorted alphabetically, sorted by exposure on risk, sorted by estimated risk. The ordering can help cosmetically when reviewing the results, but does not affect the calculations.

By contrast, a factor which is not categorical would be one that takes a naturally ordered value, *eg* "age" or "car value". These may need to be rounded at the input stage to reduce the number of levels to a convenient number (say, less than a hundred).

Interaction term

An interaction term is used where the pattern in the response variable is modelled better by including extra parameters for each combination of two or more factors. This combination adds predictive value over and above the separate single factors.

An interaction exists when the effect of one factor varies depending on the levels of another factor.

For example, male drivers may have an $x\%$ higher risk than female drivers. Young drivers may have a $y\%$ higher risk than older drivers. However, the combination of being young and male may result in a much higher risk than $\left(\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)-1\right)\times 100\%$. In this case, the effect of age depends on gender, and the effect of gender depends on age.

(Note that each factor has a base level that should not be included in the model, and for interactions, each base level row and column of the interaction parameter matrix should be removed. This will happen automatically in modelling software.)

The base level of each factor is the level to which every other level of that factor is compared and it will have a relativity of 1. The shaded areas in the table below represent the base levels of the age and car group factors and of the interaction term.

Using the earlier example, but adding in age.car group interactions from the base level:

Relativities

	Young	Old
Age	3	1

	Low	High
Car Group	1	2

Age.Car Group

	Low	High
Young	1	4
Old	1	1

This will be appropriate if the experience for young and high policies is sufficiently different from the value that would be predicted just by using (*ie* multiplying together) **the young relativity and the high relativity. Thus young people driving a high risk car may exhibit a different claim experience, higher or lower, than that estimated by the model that applies the effects of each factor independently and ignores the interaction. Note that the single factors also remain in the model.**

Age	Car Group	Predicted Value	Predicted Value without interaction	Exposure	Total Response
Young	Low	3	3	10,000	30,000
Young	High	24	6	100	2,400
Old	Low	1	1	4,000	4,000
Old	High	2	2	6,000	12,000

The predicted values in the table above are the products of the relativities of age, car group and age.car group.

The predicted value for the combination of a young driver in a high car group is 24 in the model containing the interaction term. This is very high compared to the value of 6 in the model that didn't have the interaction. This justifies the use of the interaction.

Linear predictor

This was defined earlier in the matrix form as $\boldsymbol{\eta} = \mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\xi}$.

ξ is the offset term, which we are taking to be $\mathbf{0}$ for the time being.

For the example above, which includes the interaction between age and car group, suppose that:

$i = 1$ for old, low car group
$i = 2$ for old, high car group
$i = 3$ for young, low car group
$i = 4$ for young, high car group

The linear predictor can be written in matrix form as follows:

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

where, for $j = 1, 2, 3, 4$:

- $X_{1j} = 1$
- $X_{2j} = \begin{cases} 1 & \text{if young} \\ 0 & \text{if old} \end{cases}$
- $X_{3j} = \begin{cases} 1 & \text{if high car group} \\ 0 & \text{if low car group} \end{cases}$
- $X_{4j} = \begin{cases} 1 & \text{if young and high car group} \\ 0 & \text{otherwise} \end{cases}$

With this formulation, old/low car group is the baseline group. Note that if we did not want to include the interaction term in the model, we would omit the fourth column of the matrix $\mathbf{X}$ and the parameter β_4. See Section 1.1 for an example of a two-factor model with no interaction term.

An example with categorical and polynomial terms

Let us expand the age/car group example above so that instead of taking the values "young" or "old", the age factor now takes individual values 17-99. Let us further assume that the age factor is to be fitted by a quadratic polynomial. We also add a base level parameter to scale to the right average.

So for the linear predictor we have:

$$\eta_i = X_{i0}\beta_0 + X_{i1}\beta_1 + \left(X_{i2}\beta_2 + X_{i3}\beta_3\right)$$

where:

- X_{i0} **always takes the value 1**
- X_{i1} **takes the value 1 if car group is high; 0 otherwise**
- X_{i2} **takes the value of age**
- X_{i3} **takes the value of** age^2.

So for each observation i **:**

$$\eta_i = \beta_0 + (\mathit{Group} = \mathit{High}\,?)\beta_1 + \left(\mathit{Age} \times \beta_2 + \mathit{Age}^2 \times \beta_3\right)$$

Suppose we have four data observations:

Observation	*Age*	*Car Group*
1	17	*Low*
2	21	*High*
3	35	*Low*
4	45	*High*

Then we get the matrix system:

$$\mathbf{X} = \begin{pmatrix} 1 & 0 & 17 & 17^2 \\ 1 & 1 & 21 & 21^2 \\ 1 & 0 & 35 & 35^2 \\ 1 & 1 & 45 & 45^2 \end{pmatrix} \qquad \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

This can then be solved by a modelling package using matrix inversion and iteration according to the error distribution and link function adopted.

2.3 *Estimating the parameters of a GLM*

In Section 1.1, we saw how the $\boldsymbol{\beta}$ values were calculated by solving simultaneous equations. In general, for a simple linear regression $\mathbf{Y} = (\mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\xi}) + \boldsymbol{\varepsilon}$ with identity link and normal distribution, the parameters can be estimated via matrix inversion:

$$\boldsymbol{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

Here $\boldsymbol{\varepsilon}$ is the error term for the model and $E(\mathbf{Y}) = E(\mathbf{X}.\boldsymbol{\beta} + \boldsymbol{\xi})$.

Note that the matrix $\mathbf{X}$ need not be a square matrix, *ie* it may not have the same number of rows as columns. Recall that only square matrices are invertible. Suppose that:

$$\mathbf{X} = \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1k} \\ X_{21} & X_{22} & \cdots & X_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nk} \end{pmatrix}$$

Then $\mathbf{X}$ is an $n \times k$ matrix and:

$$\mathbf{X}^T = \begin{pmatrix} X_{11} & X_{21} & \cdots & X_{n1} \\ X_{12} & X_{22} & \cdots & X_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1k} & X_{2k} & \cdots & X_{nk} \end{pmatrix}$$

is a $k \times n$ matrix. (Transposing a matrix swaps around its rows and columns.)

The product $\mathbf{X}^T\mathbf{X}$ is a $k \times k$ matrix:

$$\mathbf{X}^T\mathbf{X} = \begin{pmatrix} X_{11} & X_{21} & \cdots & X_{n1} \\ X_{12} & X_{22} & \cdots & X_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1k} & X_{2k} & \cdots & X_{nk} \end{pmatrix} \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1k} \\ X_{21} & X_{22} & \cdots & X_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nk} \end{pmatrix}$$

$$= \begin{pmatrix} \sum_{i=1}^{n} X_{i1}^2 & \sum_{i=1}^{n} X_{i1}X_{i2} & \cdots & \sum_{i=1}^{n} X_{i1}X_{ik} \\ \sum_{i=1}^{n} X_{i2}X_{i1} & \sum_{i=1}^{n} X_{i2}^2 & \cdots & \sum_{i=1}^{n} X_{i2}X_{ik} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} X_{ik}X_{i1} & \sum_{i=1}^{n} X_{ik}X_{i2} & \cdots & \sum_{i=1}^{n} X_{ik}^2 \end{pmatrix}$$

So it makes sense to talk about the inverse of $\mathbf{X}^T\mathbf{X}$ (assuming that this exists).

In cases where the distribution is more complex and other link functions are adopted, the solution is obtained by a variation of this formula that uses a matrix version of the Newton-Raphson formula to iterate towards a solution.

3 Analysis of the significance of factors

In this section we look at techniques used to analyse the significance of the factors used in the model. The more visual methods in Section 3.3 can be readily explained to non-technicians.

Note that all of the graphs given in this section form part of the Core Reading.

3.1 Chi-squared, F statistics and AIC

Deviance

Let d be each observation's contribution to the deviance defined by:

$$d\left(Y_i ; \mu_i\right) = 2\omega_i \int_{\mu_i}^{Y_i} \frac{\left(Y_i - \zeta\right)}{V\left(\zeta\right)} d\zeta$$

This compares the observed value Y_i to the fitted value μ_i with allowance for the weights and assigning higher importance to errors where the variance should be small.

Using the above definition of deviance residual, the total deviance for a model is defined as the sum of those:

$$D = \sum_{i=1}^{n} d\left(Y_i ; \mu_i\right)$$

Scaled deviance

The deviance can be adjusted by the scale parameter φ (see Section 1.6) to give a standardised measure that can be compared to other models:

$$D^* = \frac{D}{\varphi}$$

D^* is called the scaled deviance.

Question 16.5

A generalised linear model has independent Poisson responses $\{Y_i\}$ with:

$$E(Y_i) = \text{var}(Y_i) = \mu_i$$

The linear predictor η_i is linked to the mean response by the equation:

$$\eta_i = \ln \mu_i$$

Evaluate the expression:

$$d_i(y_i, \hat{\mu}_i) = 2\int_{\hat{\mu}_i}^{y_i} \frac{y_i - t}{V(t)}\, dt$$

to determine an expression for the general component $d_i(y_i, \hat{\mu}_i)$ of the deviance of this GLM in terms of the observed responses y_i and fitted values $\hat{\mu}_i$.

Chi-squared statistics

The number of degrees of freedom *df* for the model is defined as the number of observations less the number of parameters.

Two nested models (that is, one is a subset of the other) can be compared using a χ^2 test for the change in scaled deviance.

For example, if Model 1 has linear predictor:

$$\eta_1 = \alpha + \beta x$$

and Model 2 has linear predictor:

$$\eta_2 = \alpha + \beta x + \gamma x^2$$

then Model 1 is a subset of Model 2. (We have set $\gamma = 0$ in Model 1.)

If Models 1 and 2 are nested, then the change in scaled deviance follows a chi-squared distribution, *ie*:

$$D_1^* - D_2^* \sim \chi^2_{df_1 - df_2}$$

> ***Question 16.6***
>
> Suppose Model A and Model B are nested models with 6 and 10 parameters respectively. The scaled deviance of Model A is 17.80 and the scaled deviance for Model B is 11.08. Explain whether Model B is a significant improvement on Model A.

F statistics

In cases where the scale parameter for the model is not known, its estimator is distributed as a χ^2 distribution, and the ratio of two χ^2 distributions is the *F* distribution:

$$\frac{(D_1 - D_2)}{(df_1 - df_2)(D_2 / df_2)} \sim F_{df_1 - df_2, df_2}$$

Again we need the models to be nested for this result to be valid.

> ***Question 16.7***
>
> Suppose Model C and Model D are nested models with 8 and 16 parameters, respectively, and have been fitted to a set of 50 observations. The deviance for Model C is 40.89 and the deviance for Model D is 26.40. Explain whether Model D is a significant improvement on Model C.

Akaike Information Criteria (AIC)

The AIC (an abbreviation for the "Akaike information criterion") is a statistic primarily used for model selection (that is, comparing which of two models is a better fit).

If two models are nested, then the more usual chi-squared test is the most appropriate to use.

If the models are not nested, the AIC can be used. The AIC for a model is calculated as:

$$\textbf{AIC} = -\mathbf{2} \times \textbf{log-likelihood} + \mathbf{2} \times \textbf{number of parameters}$$

The AIC looks at the trade-off of the likelihood of a model against the number of parameters: the lower the AIC, the better the fit. For example, if two models fit the data equally well in terms of the log-likelihood, then the model with the fewer parameters is the more parsimonious (and therefore "better").

3.2 Measuring uncertainty in the estimators of the model parameters

One of the outputs of the matrix fitting process is the hat matrix.

The hat matrix is the matrix $\mathbf{H}$ such that:

$$\hat{\mathbf{y}} = \mathbf{H}\,\mathbf{y}$$

So the hat matrix puts a hat on $\mathbf{y}$!

For a model of the form $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, we have seen that the estimated parameters are:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

where $\mathbf{y}$ is the vector of observed values. So the fitted values are given by:

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

Hence the hat matrix is:

$$\mathbf{H} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$$

The diagonal entries of the hat matrix are called the *leverages*. These measure the influence that each observed value has on the fitted value for that observation.

The hat matrix gives the rate at which the log-likelihood falls off from the optimum solution in each direction.

The second derivative of the likelihood in the direction of each parameter has been shown by the Cramér-Rao lower bound theorem to be inversely proportional to the variance for the parameter. We can use this to calculate standard errors for each parameter estimator. A poorly defined parameter will have a large standard error.

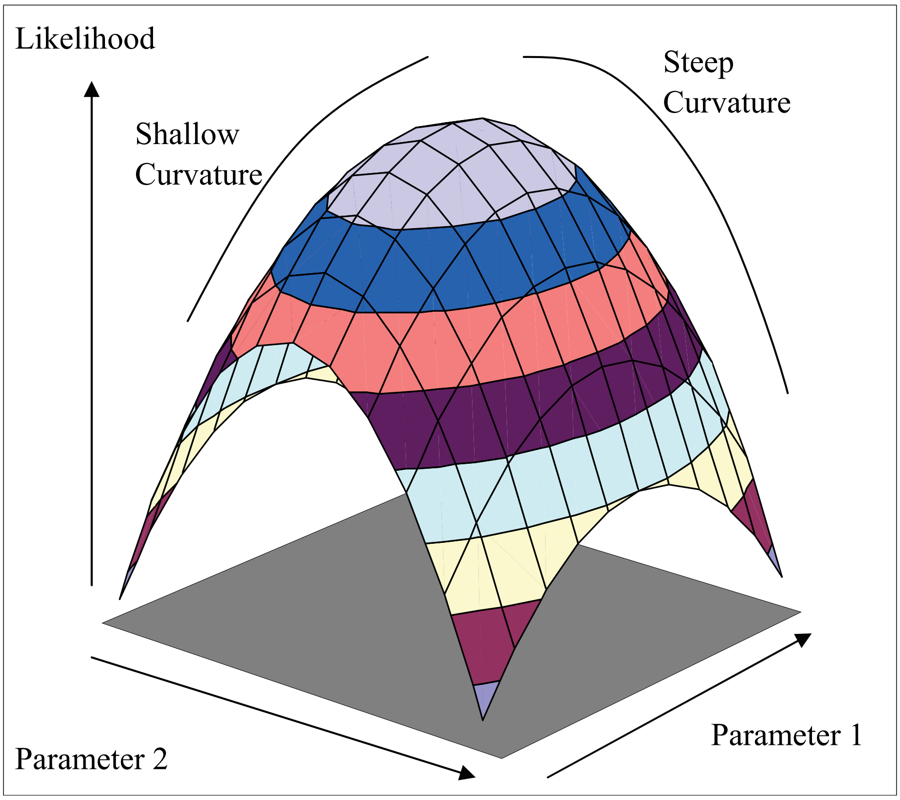
Recall from CT6 that the Cramér-Rao lower bound theorem is as follows:

Given a random sample of size n from a distribution with density (or probability function in the discrete case) $f(x;\theta)$, the maximum likelihood estimator $\hat{\theta}$ is such that, for large n, $\hat{\theta}$ is approximately normal, and is unbiased with variance given by the Cramér-Rao lower bound, that is:

$$\hat{\theta} \mathrel{\dot\sim} N(\theta, \text{CRLB})$$

where $\text{CRLB} = \dfrac{1}{nE\left\{\left[\dfrac{\partial}{\partial\theta}\log f\left(X;\theta\right)\right]^2\right\}} = \dfrac{1}{-E\left[\dfrac{\partial^2}{\partial\theta^2}\log L\left(\theta,\underline{X}\right)\right]}$.

In the diagram on the next page, Parameter 2 has a steep curvature indicating that the parameter is tightly defined, and Parameter 1 a shallow one. Hence Parameter 1 is poorly defined.



To see the difference in curvature, consider the arches upon which the whole shape is formed. The relative slopes are quite different in terms of the speed by which likelihood declines, depending on the direction by which we leave the optimal point.
For parameter 2, the curvature is steepest indicating that the associated second derivative is highly negative. From the formula for the CRLB above, this implies that the CRLB and hence variance is small.

3.3 *Other methods for checking the significance of a factor*

Comparison with expectations

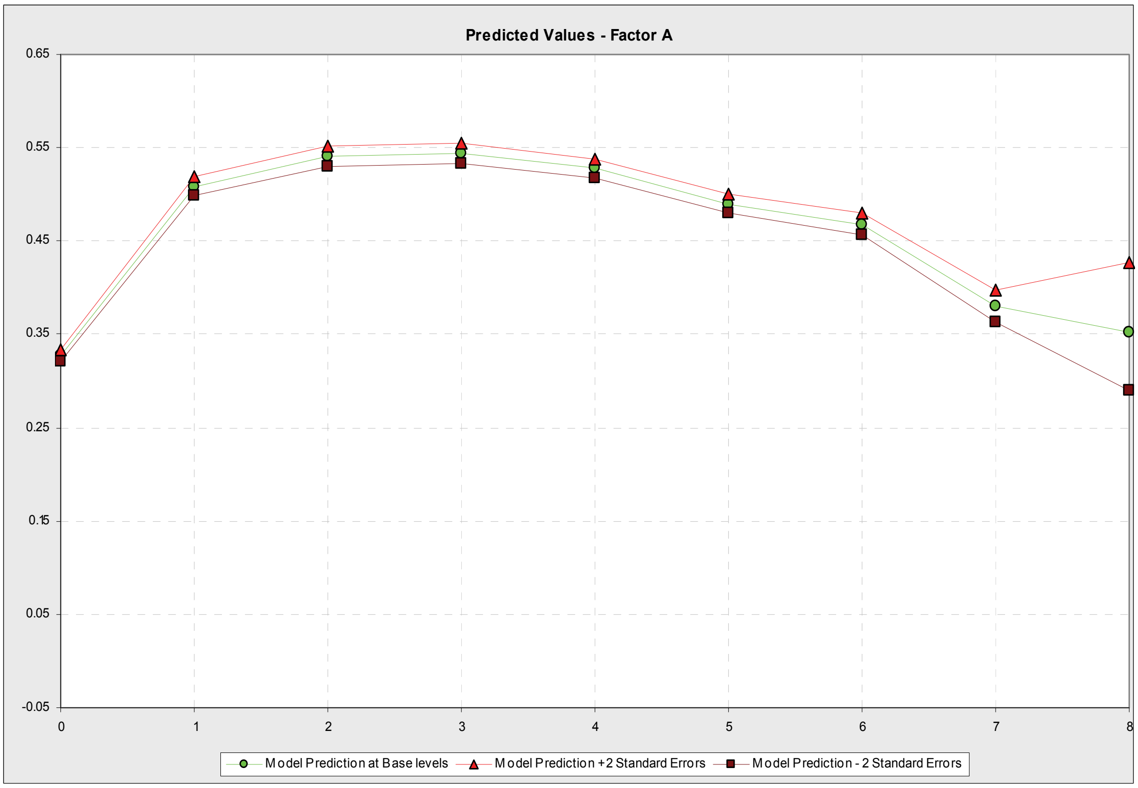
Graphs can provide an intuitive feel as to whether a factor is significant in a GLM, although this is more subjective than the statistical tests introduced in Section 3.1.

Factor significance is initially checked by considering the spread of relativity values for each level, combined with the standard errors at each level. In the graph below (on the next page), **the errors ± 2 standard deviations are plotted. If the relativities are being fitted individually (as in this case), then the pattern of their values should be consistent with the definition of the factor under consideration.**

For example, if the levels represent a continuous variable (such as vehicle age), then the relativity should vary smoothly as the factor value increases. The error ranges of these relativities are also distinct (they don't overlap too much), indicating that the response from the data underlying each level has a significantly different relativity value. Hence the factor should be accepted.

The graph below shows the fitted parameter values from a GLM for a particular factor that has 9 levels, along with two lines showing ± 2 standard deviations. We can be fairly confident (approximately 95%) that the true relativity for each level of the rating factor will lie between the two standard error lines. (At this stage you don't need to worry about what this factor is or what the scale on the *y*-axis represents.)

For this factor, the standard error lines are close together. This indicates that there is a high degree of certainty in the parameter estimates. This factor is therefore useful in predicting the risk.

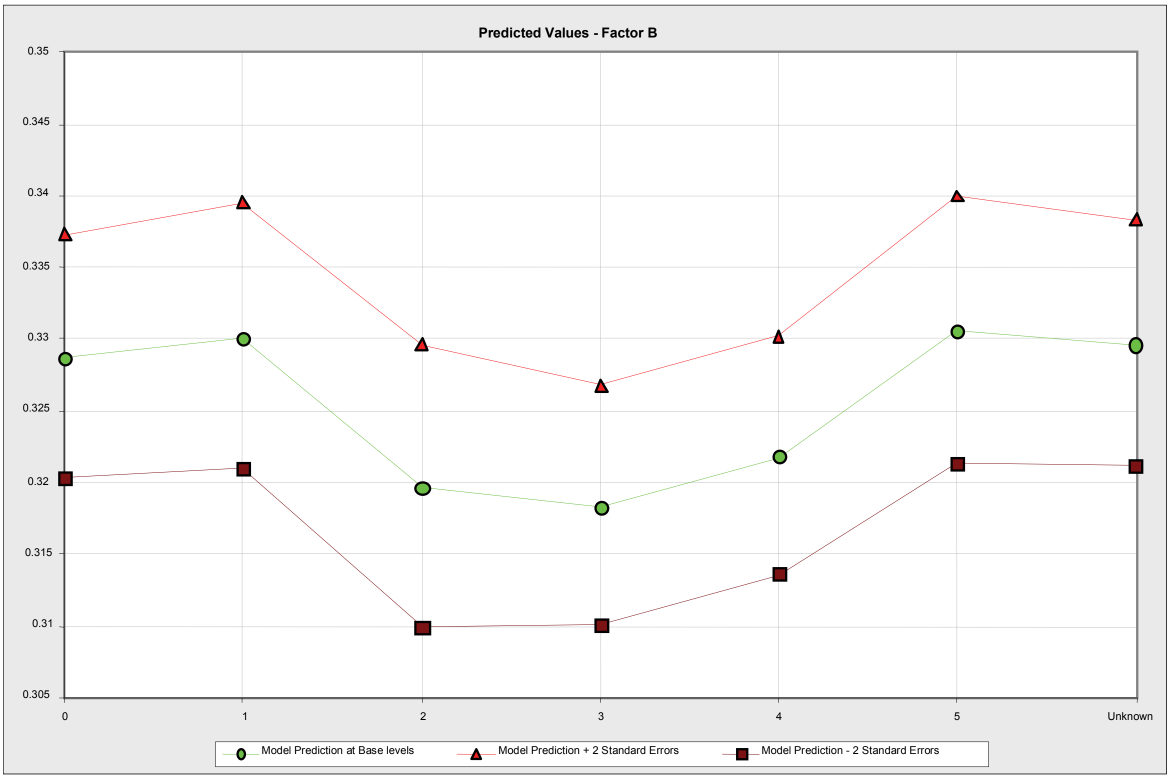


In the second example below, the opposite is true. In fact the error ranges overlap so much that it would be possible to draw a horizontal line of relativities that stayed within the errors shown. Hence the factor should be discarded.

Here we can see that the standard error lines are wide apart. This indicates that there is a lot of uncertainty in the parameter estimates.

For example, the parameter estimate for Level 2 of this factor is 0.32 but the standard error lines suggest that this estimate is likely to fall anywhere between 0.31 and 0.33. Similarly, for Level 3, the estimate could fall between 0.31 and 0.327. There is a high degree of overlap between the ranges for these two levels and therefore we would observe that the parameter estimates are not significantly different from each other. Extending this argument to the other levels of this same factor, we would conclude that none of the levels are really any different to any of the others and therefore this factor is not helping to differentiate the risk.

This could be due to low amounts of exposure or simply because this factor doesn't explain the underlying risk very well.



Comparisons over time

To test the consistency of parameter estimates over time, we can fit a model that includes the interaction of a single factor with a measure of time, *eg* a calendar year.

Recall from Section 2.2 that an interaction exists when the effect of one factor varies depending on the levels of another factor. In the case of time interactions, we are testing whether the effect of our factor varies depending on the year (or other specified time period) of the experience.

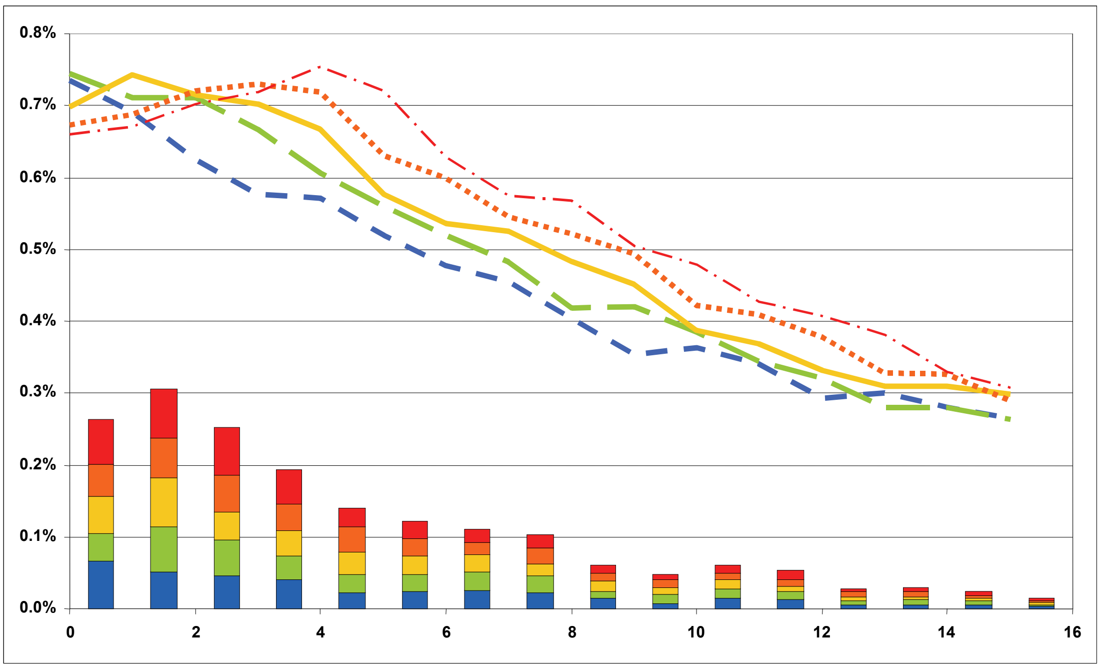
The time consistency check (derived by interacting each factor in turn with a time-related factor) is important for pricing work, because typically you will be analysing data from two to seven years ago, and then deploying rates for the next year. So if the pattern you select is moving rapidly over time, then the model average selected will be inappropriate for future periods.

The time consistency check is also used to determine more generally whether the effect of each factor is consistent from year to year. If it is consistent then it is likely to be a good predictor of future experience.

Let us consider two examples.

Example 1

The first example **is where there is a time consistency problem.**



This shape was generated on a car theft frequency chart a few years ago, during the period when motor manufacturers were rapidly improving their security measures. In this chart

- **the *x*-axis is vehicle age,**
- **the lines are the response rates for each year (newest is "– · – · – ·", oldest is "– – – –"), and**
- **the bars are the exposure by year.**

The response rates in this example are the car theft frequencies. The exposure bars on the graph represent the amount of data at each level of vehicle age in each year, *ie* a different shade is used for each year.

This rendered newer vehicles untouchable for a while, so thieves started to target older, less secure vehicles. Hence the responses appeared to translate year by year to the right.

New cars are the most valuable to thieves but, with improvements in security measures over time, they are becoming more difficult to steal or break into. Therefore the highest theft frequencies in any year are likely to relate to the newest cohort of cars that don't have high levels of security. Each year this cohort gets older by one year, *ie* vehicles that were new in Year n would be one year old in Year $n+1$. In terms of the graph above, we can see that the level of vehicle age with the highest claim frequency is increasing year on year and this is why the lines on the graph appear to be shifting to the right.

Question 16.8

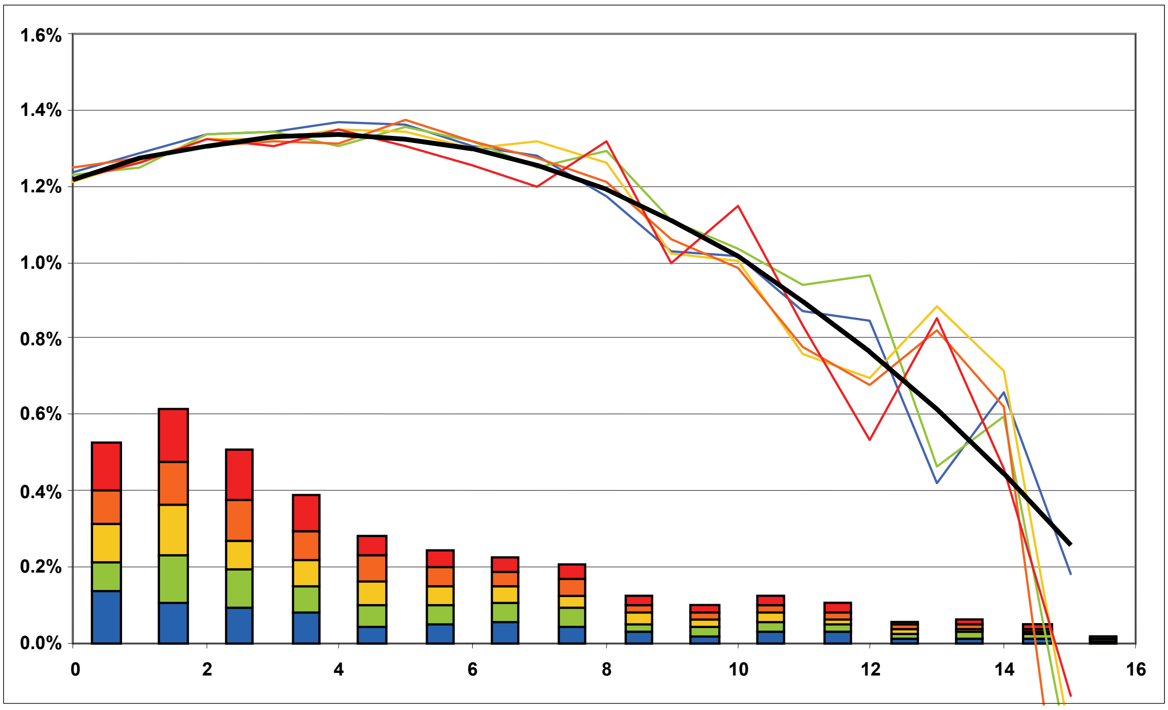
If there has been such a rapid improvement in manufacturer security features, why don't the new cars on the graph above show a zero theft claim frequency for the more recent years?

In this case the relativities that would be suitable to deploy for a future year should be something like the response for the latest year, translated by the projection period, and with an ad-hoc adjustment for how the modeller thinks this change may develop.

Example 2

The second example is more usual. We see the model fit line (thick bold curve) varying smoothly across the factor, and the time consistency responses varying randomly around this average. The amount of variation should decrease with larger exposure.

This is another example showing the effect of vehicle age (along the x-axis) although in this case it is more likely to be a graph of damage frequencies than theft frequencies. We can see that the relativities for the lower car ages are based on more exposure data than the higher car ages and therefore show less variation in the estimates.



Consistency checks with other factors

Note that time is not the only factor that can be used as a consistency check.

If you are producing a model for a multi-distribution channel business then it is particularly important that each factor is checked to ensure that it is valid for every channel.

For example, a factor such as the age of the vehicle would be expected to show the same pattern regardless of the distribution channel used. Fitting a model that includes an interaction between age of vehicle and distribution channel, and plotting a graph of the results, would highlight any distribution channels where the effect of age of vehicle was different from the others. If this is the case then you would need to do further investigations.

Differences in the data collection methods by channel can cause problems here.

Also a random factor could be created in the data, as a means to check consistency for a factor.

Each observation in the data could be randomly allocated to one of ten (say) groups, the idea being that each group is then a representative sample of the total data. Each of the ten randomly allocated groups would be expected to behave in a similar way to each of the other groups and to the whole data.

A new factor would be created, with ten different levels, representing the random group that each observation had been allocated to. Let's call this new factor *randomgroup.*

So if we wanted to test whether age of policyholder (for example) was showing a consistent pattern throughout the whole data, we could fit a model to the whole data that included an interaction term between policyholder age and randomgroup.

If this interaction term is insignificant in the model then we would conclude that the effect of policyholder age is the same for every level of randomgroup and that policyholder age is consistent throughout the whole data.

4 Testing the appropriateness of models

In this section we describe the techniques used to check the appropriateness of the model structure.

Before you start modelling, it is important to check that the model you are intending to use has the right error structure and is appropriate for your data. If it is not, then this could invalidate your results.

4.1 Residuals

Various measures of the residual can be derived to show, for each observation, how the fitted value differs from the actual observation.

Deviance residuals

One residual measure is the deviance residual:

$$\boldsymbol{r_i^D = sign(Y_i - \mu_i)\sqrt{d(Y_i;\mu_i)}}$$

where d is as given in Section 3.1.

Recall that $d(Y_i;\mu_i)$ is the ith observation's contribution to the deviance and is defined by:

$$d(Y_i;\mu_i) = 2\omega_i \int_{\mu_i}^{Y_i} \frac{(Y_i - \zeta)}{V(\zeta)} d\zeta$$

and we define:

$$sign(Y_i - \mu_i) = \begin{cases} +1 & \text{if } Y_i - \mu_i > 0 \\ -1 & \text{if } Y_i - \mu_i < 0 \end{cases}$$

The deviance residual is a measure of the distance between the actual observation and the fitted value.

In general they (the deviance residuals) **will be more closely normally distributed than raw residuals (the difference between actual and GLM expected values) as the deviance corrects the skewness of the distributions. For continuous distributions it is possible to test whether the residuals are normally distributed. Any large deviations indicate that the distributional assumptions are being violated.**

Pearson residuals

A Pearson residual is the difference between the observed response and the predicted value, scaled by the standard deviation of the predicted value:

$$r_i^P = \frac{Y_i - \mu_i}{\sqrt{\dfrac{\varphi V(\mu_i)(1-h_{ii})}{\omega_i}}} = \frac{Y_i - E[Y_i]}{SD[Y_i]\sqrt{1-h_{ii}}}$$

where h_{ii} is the i th diagonal element of the hat matrix:

$$\mathbf{H} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$$

derived from the design matrix $\mathbf{X}$. It can be viewed as a measure of how much influence an observation has over its own fitted value. It lies between 0 and 1. If it is close to 1, it is likely that the residual for that observation will be unusually small because of the high influence the observation has on the fitted value.

h_{ii} is also referred to as the i th leverage.

This measure allows observations with different means to be comparable, but does not adjust for the shape of the distribution.

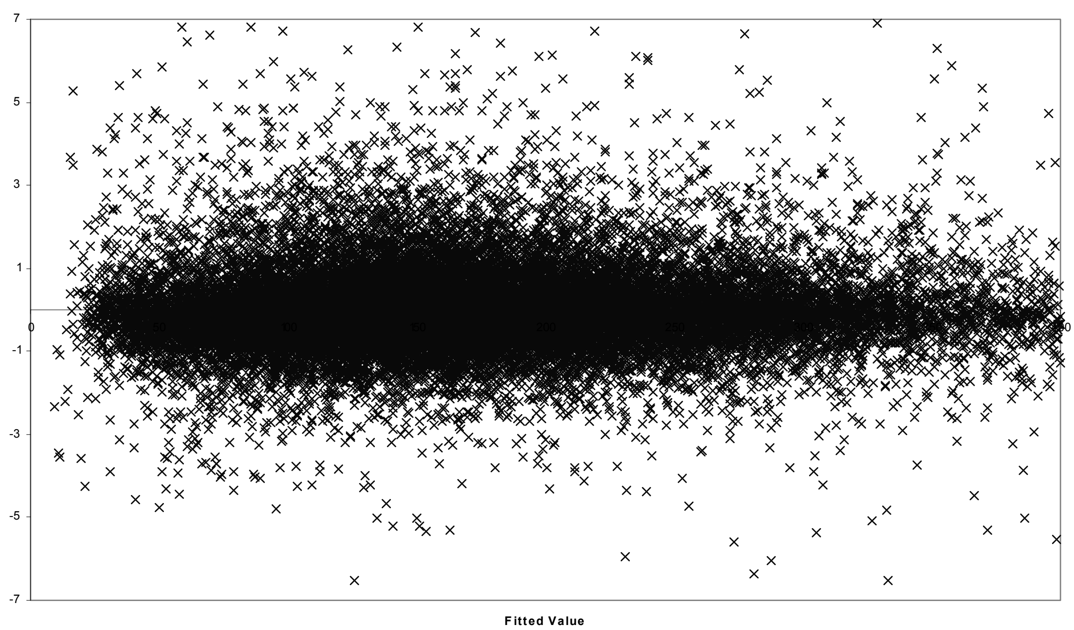
4.2 Residual plots

Here we look at typical residual patterns for models of average cost, frequency, burning costs and probabilities.

For a particular model (for example, average cost, frequency, burning costs and probabilities), if the chosen error structure is appropriate then the residuals chart such as the one below will have an average residual of zero, and the range of the residual values will be fairly constant across the width of the fitted values.

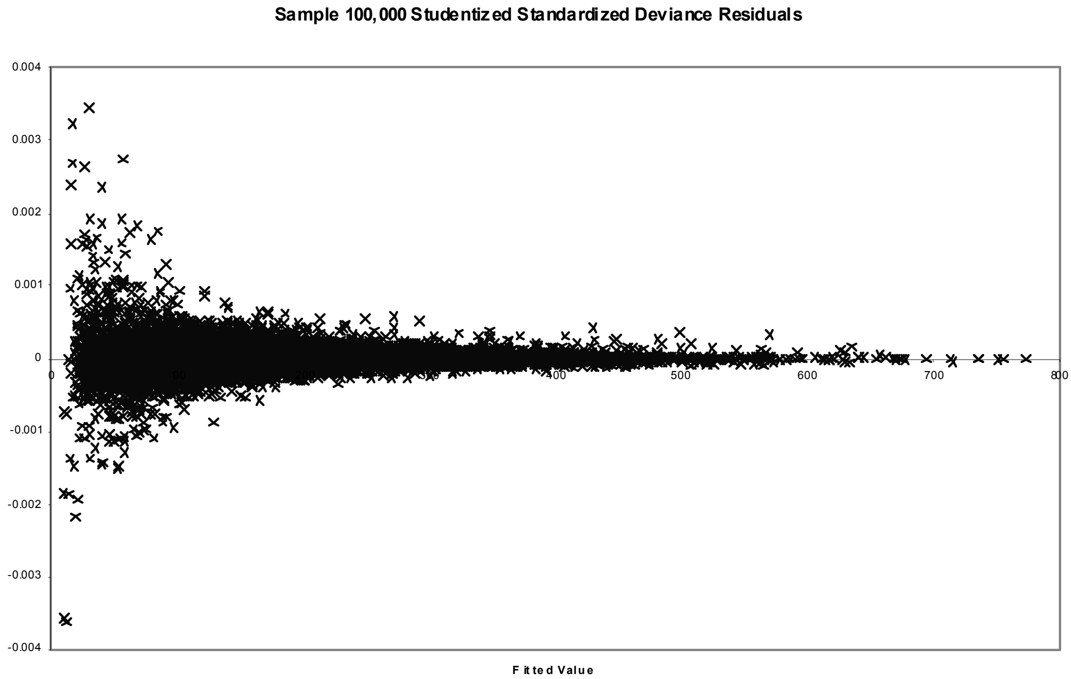
The *error structure* relates to the statistical distribution assumed for the error term ε. For example, where a Poisson error structure is used, the assumption is that the variance is proportional to the mean, μ. For a gamma error structure, the variance is assumed to be proportional to μ^2.

Sample 100,000 Studentized Standardized Deviance Residuals

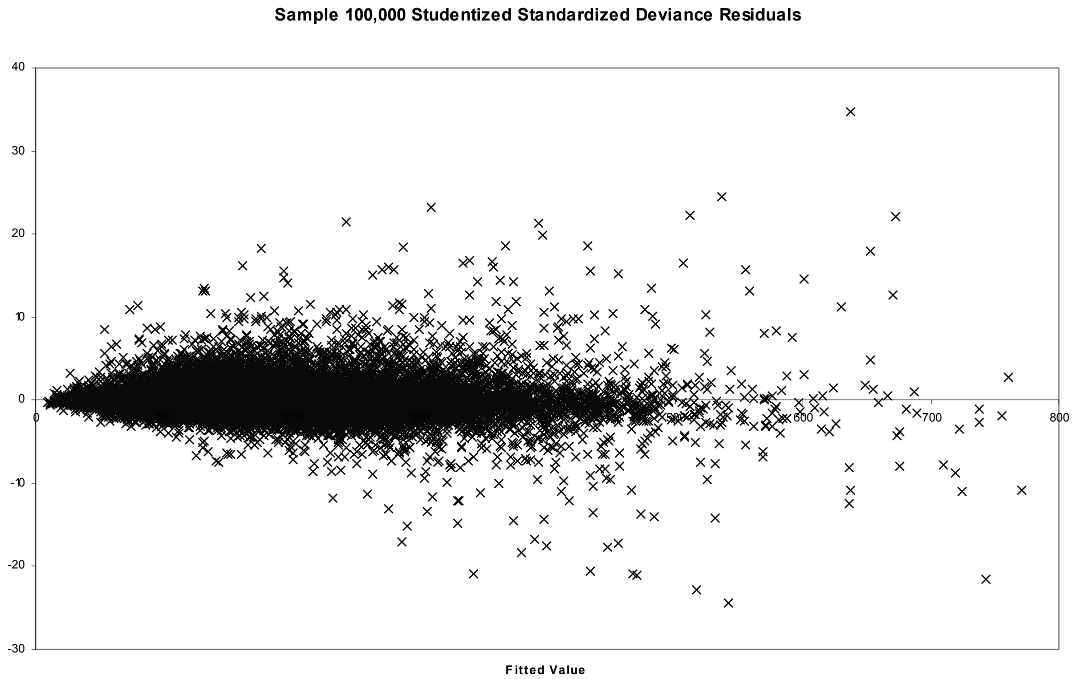


If the plot were to "fan out" at the right hand side, then this would tell us that the residuals were increasing as the fitted value increased, indicating that our chosen distributional assumptions were not correct.

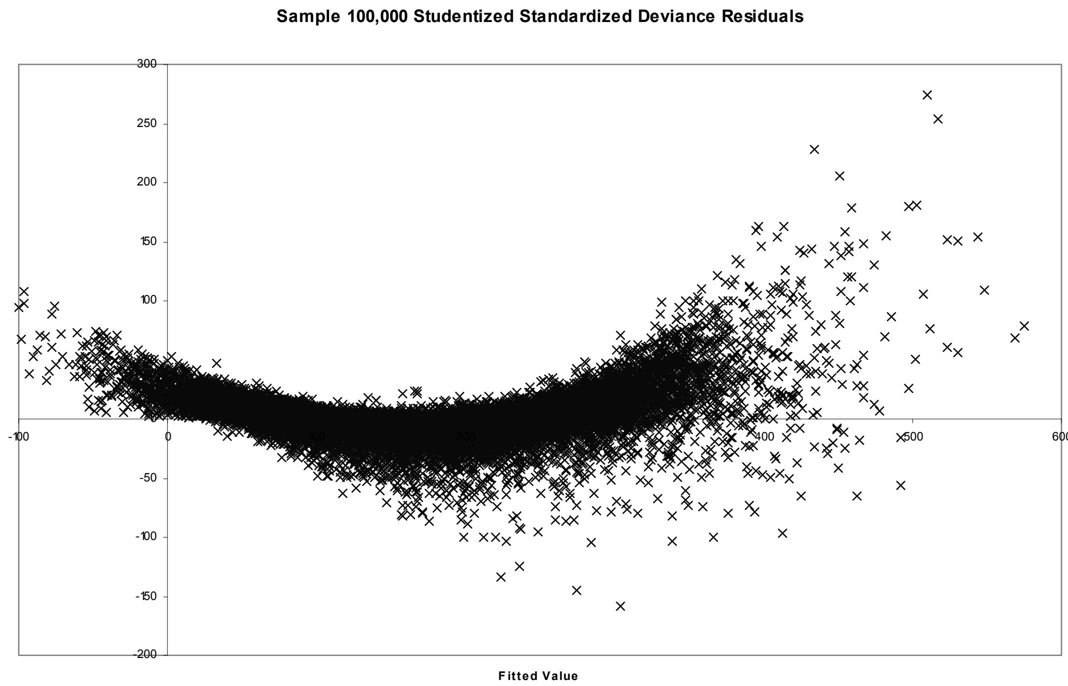
For the graph below, the error structure has been increased to μ^4, which is far higher than the power 2 gamma model from the first chart. The result is a set of residuals where the range narrows as the fitted value increases, indicating that the model is poor.



Likewise in this third graph, the error structure has been decreased to $\mu^{0.2}$, which is far too low. The result is a set of residuals where the range widens as the fitted value increases, indicating that the model is poor.



Finally in this graph, the residual structure for an identity link normal distribution model is shown. When the distribution used is inappropriate, the pattern of residuals will not be symmetrical about the x-axis, as can be clearly seen below.



So a good model should produce residuals that

- are symmetrical about the x-axis
- have an average residual of zero
- are fairly constant across the width of the fitted values.

Question 16.9
What would we do if the residual checks suggest that our model is not a good fit to the data?

4.3 *Cook's distance and leverage*

The term h_{ii} introduced in Section 3.2 is called the leverage.

Recall that the leverage is a measure of how much influence each observed value has on the fitted value for that observation.

Data points with large residuals (outliers) and/or high leverage may distort the outcome and accuracy of a regression model. Cook's distance is used to estimate the influence of a data point on the model results. Points with a Cook's distance of 1 or more are considered to merit closer examination in the analysis.

As a result of your investigations into any data points with a high Cook's distance, you might decide to cap the amounts or remove the observations altogether. If you do this, it is important to remember to allow for them later, to avoid understating claim frequencies or amounts.

Cook's distance for the i th data point is defined as:

$$c_i^P = \frac{h_{ii}}{(1-h_{ii})\left(\sum_i h_{ii}\right)}\left(r_i^P\right)^2$$

where the ith leverage h_{ii} is the ith diagonal element of the hat matrix $\mathbf{H} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ derived from the design matrix $\mathbf{X}$ and r_i^P is the Pearson residual, defined in Section 4.1.

5 Model refinement

In this section we examine the refining of models using interactions and restrictions.

We have chosen the structure of our model and checked that it is appropriate for our chosen factors by testing the residuals. Then we have removed any factors from the model that did not help to define the risk. We can now start to further refine the model. One way of doing this is to look for interactions.

Question 16.10

Explain the difference between correlations and interactions.

5.1 Complete and marginal interactions

Interactions can be expressed in different ways. For example, consider the case of two factors, each with four levels.

Complete interactions

One way of expressing an interaction is to consider a single factor representing every combination of the two factors (a "complete" interaction). A set of multipliers (in the case of a multiplicative model) could therefore be expressed as follows:

Factor 1:		**A**	**B**	**C**	**D**
Factor 2:	**W**	**0.72**	**0.80**	**0.88**	**0.96**
	X	**0.90**	**1.00**	**1.10**	**1.20**
	Y	**0.97**	**1.20**	**1.45**	**1.66**
	Z	**1.26**	**1.40**	**1.85**	**2.10**

In this case, the base level has been selected to be the level corresponding to Level B of Factor 1 and Level X of Factor 2, and the interaction term has 15 parameters.

The base level is the combination that has a multiplier of 1.00.

Marginal interactions

An alternative representation of this interaction is to consider the single factor effects of Factor 1 and Factor 2 *and* the additional effect of an interaction term over and above the single factor effects (or "marginal" interaction). A set of multipliers in this form can be set as follows:

Factor 1:			**A**	**B**	**C**	**D**
			0.90	**-**	**1.10**	**1.20**
Factor 2:	**W**	**0.80**	**1**	**-**	**1**	**1**
	X	**-**	**-**	**-**	**-**	**-**
	Y	**1.20**	**0.9**	**-**	**1.1**	**1.15**
	Z	**1.40**	**1**	**-**	**1.2**	**1.25**

In this case fewer parameters are present in the additional interaction term because the presence of the single factor effects makes some of the interaction terms redundant. When fitted in a GLM (assuming that the single factor effects were declared first) the redundant terms in the additional interaction term would be aliased.

Aliasing occurs when there is a linear dependency among the observed covariates, *ie* where one covariate is identical to some combination of other covariates. Aliasing is discussed in Section 5.3.

Overall, the three terms combined (Factor 1, Factor 2 and their interaction) **still have 15 parameters, and result in identical predicted values.**

For example, in the case of Factor 1 Level D and Factor 2 Level Z, the multiplier is:

$$\mathbf{1.2 \times 1.4 \times 1.25 = 2.10}$$

This is because we are assuming that this is a multiplicative model and:

- the multiplier for Factor 1 Level D is 1.20
- the multiplier for Factor 2 Level Z is 1.40
- and the multiplier for the D.Z interaction term is 1.25.

Note that we obtain the same multiplier from both representations of the model.

Question 16.11

How might you decide which interaction terms to test for inclusion in a GLM?

5.2 *Model restrictions*

The theoretical risk premium results from a GLM claims analysis will differ from the rates implemented in practice since consideration needs to be given to price demand elasticity and the competitive situation.

Price demand elasticity is a measure of how the demand for a product changes in response to a change in its price.

The competitive situation is relevant because there may be pockets of business where the theoretical risk premium rates would produce market premiums that are much higher or lower than those of your competitors. For example, if your premiums are much lower than those of your competitors then there may be an opportunity to increase your rates but still be the cheapest in the market, thereby increasing profit on this section of business without reducing volumes.

There are, however, some situations where legal or commercial considerations may also impose rigid restrictions on the way particular factors are used in practice. When the use of certain factors is restricted, if desired, the model may be able to compensate to an extent for this artificial restriction by adjusting the fitted relativities for correlated factors. This is achieved using the offset term in the GLM (see Section 5.6).

If a factor is removed from a GLM then the remaining factors will try to explain as much as possible of the risk that had previously been explained by the removed factor. This means that the parameter values for the remaining factors will change. The extent of the change will depend on the levels of correlation between each remaining factor and the removed factor. Similarly, whenever a factor is added to a model, or redefined in some way, the model compensates by changing all the parameter values.

Offsets were introduced in Section 1.6. Up until now, we have assumed that the offset term is equal to zero.

Specifically, the required parameter estimates (logs of multipliers in the case of a multiplicative model where specifically a log link function is used) are calculated for each record and added to the offset term ξ_i. The factor in question is then not included as an explanatory factor in the GLM. (This can intuitively be thought of as fixing some selected elements of β_i to be specified values.)

When we fix (offset) the parameter estimates for a particular factor at a level other than that calculated by the GLM, the remaining factors in the model will try to explain the difference in risk arising from differences between the calculated and the fixed parameter estimates for the factor that has been offset. Therefore the parameter estimates for these remaining factors are likely to change, if they are correlated with the factor that has been offset.

> ***Question 16.12***
>
> Suppose we have a GLM and we decide to offset the parameter values of one of the factors at the levels suggested by the GLM. Assuming we make no other changes to the model, what might the effect be on the parameter values for the remaining factors?

Although restrictions could be applied either to frequency or amounts models (or in part to both), generally it is more appropriate to impose the restriction on the model at the risk premium stage, since this allows a more complete and balanced compensation by the other factors. This can be achieved by calculating the expected cost of claims for each record, according to "unrestricted" GLMs, and then imposing the restriction in the final GLM, which is then fitted to the total expected cost of claims. For restricted risk premium models (*eg* where you fix your NCD levels and compensate for the "true" cost by varying values of other factors) this approach is necessary even in the case of a single claim type.

Amounts models are often referred to as severity models. The risk premium stage of the modelling process is the point at which the frequency and severity models are combined to give risk premiums.

In the UK, many personal lines policies contained discounts for factors like "No Claims Discount" (NCD) that were equal across all companies in the market.

NCD is an established practice within UK private motor insurance. The Core Reading says "contained" rather than "contain" here because it is no longer the case that equal NCD scales apply across all insurers. However, for most, the discount awarded to drivers with four or five claim-free years is far in excess of that which could be justified by theoretical risk models. Policyholders like the concept of big discounts and so, for competitive reasons, insurers are reluctant to move away from the established discount scales. Therefore, when using GLMs, the NCD scale is often incorporated into the risk premium models using the offset term.

There is more detail on how to apply offsets in Section 5.4.

Soon European regulation may impose restrictions on various factors, gender and age being examples mandated by regulation.

Under the Sex Discrimination Act 1975, UK insurers were prevented from treating men and women differently. However, there was an exemption for the provision of insurance services in cases where the discrimination was made with reference to actuarial or other data from a source on which it was reasonable to rely.

Current UK gender legislation (and the EU Gender Directive) allows the use of gender as a rating factor when based on actuarial and statistical data on gender-related risk differences. However, the European Court of Justice has ruled that this does not comply with the EU Treaty. As a result, from 22 December 2012, insurers will no longer be able to differentiate premiums by gender as a result of a judgment of the European Court of Justice (ECJ).

There have also been discussions around age discrimination and the Equality Bill in the UK.

It is possible that these (and other) issues will be reviewed further in the future and that insurers may become more restricted in their choice of rating factors.

Today's models may indicate that these discounts are not supported by the claims experience, or in many cases may even indicate a surcharge. If a company chooses to continue offering such discounts, it is important that these restrictions are incorporated into the modelling process, since such restrictions can affect the relativities that become appropriate for other correlated factors.

Counter-intuitive model results may occur on behavioural factors such as factors that policyholders self-select, *eg* limits and deductibles. These factors may require restriction if they are to be used directly in rate-making.

An example of a potentially counter-intuitive result in motor insurance is NCD protection. This allows a policyholder to retain his/her NCD level even if he has made one or more claims (where the number of claims he can make within a defined time period will be specified in the policy). There will be an extra premium payable for this protection because it is an extra benefit.

NCD protection will typically only be available to policyholders who have had four or five claim-free years. A policyholder who has managed to stay claim-free for this long, and who is now seeing the benefit of lower premiums as a result, is likely to be willing to pay a small extra premium to retain this advantage, particularly if he/she is risk averse. So the policyholders who choose to protect their NCD level are generally the subset of the better risks that are risk averse, *ie* careful.

In the context of a GLM, NCD protection will be a two-level factor (Yes/No) and NCD protection = Yes will become a proxy for lower risk. This could lead to the theoretical premium being lower for those choosing (self-selecting) NCD protection than for those who don’t, even though the market premium needs to be higher.

(There is, of course, the counter-argument that policyholders might only buy NCD protection if they expect to have a claim in the next year, but remember that they will need to have a number of claim-free years already and that there is a limit on the number of claims they can make without losing their NCD protection.)

Prior to incorporating restrictions, it is still important to assess the true effect of all factors upon the risk by initially including them in the analysis as if they were ordinary factors. In addition, a comparison of the fitted values of the theoretical model and the restricted models will demonstrate the degree to which other factors have compensated for the restriction.

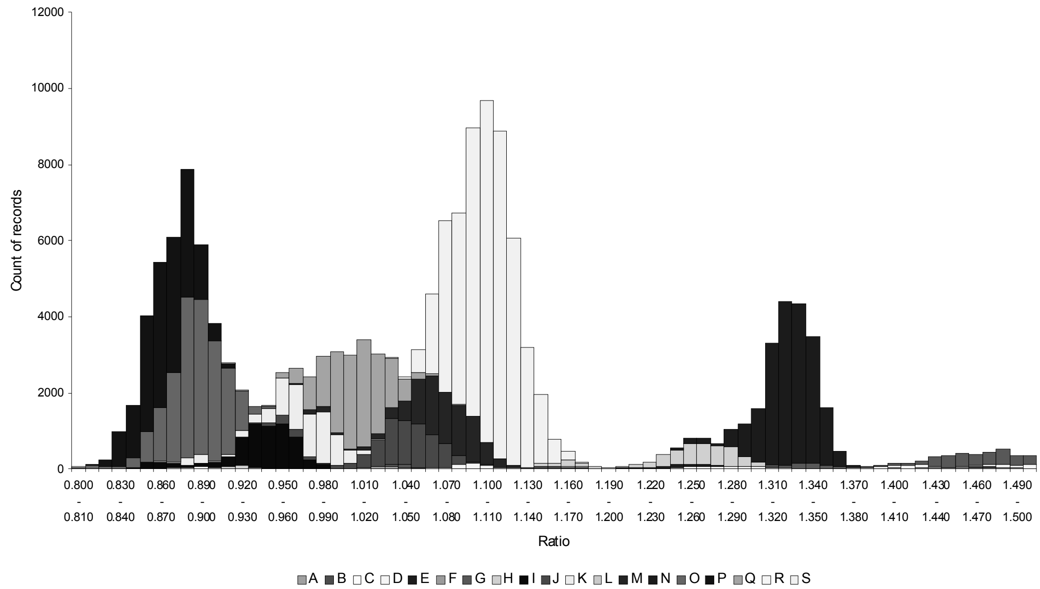
The examples below show two such comparisons. Each graph shows the number of policies (on the y-axis) that have different ratios of restricted to unrestricted fitted values (on the x-axis). The graph is subdivided by levels of the restricted factor (shown in different shading). If the GLM can compensate well for a factor restriction (because there are many other factors in the model correlated with the restricted factor) then this distribution will be narrow. Conversely, if the GLM cannot compensate well for the restriction, this distribution will be wider.

The ratio of restricted to unrestricted fitted values is a measure of how much the fitted values (*eg* frequencies or severities) change for each observation as a result of restricting a particular factor.

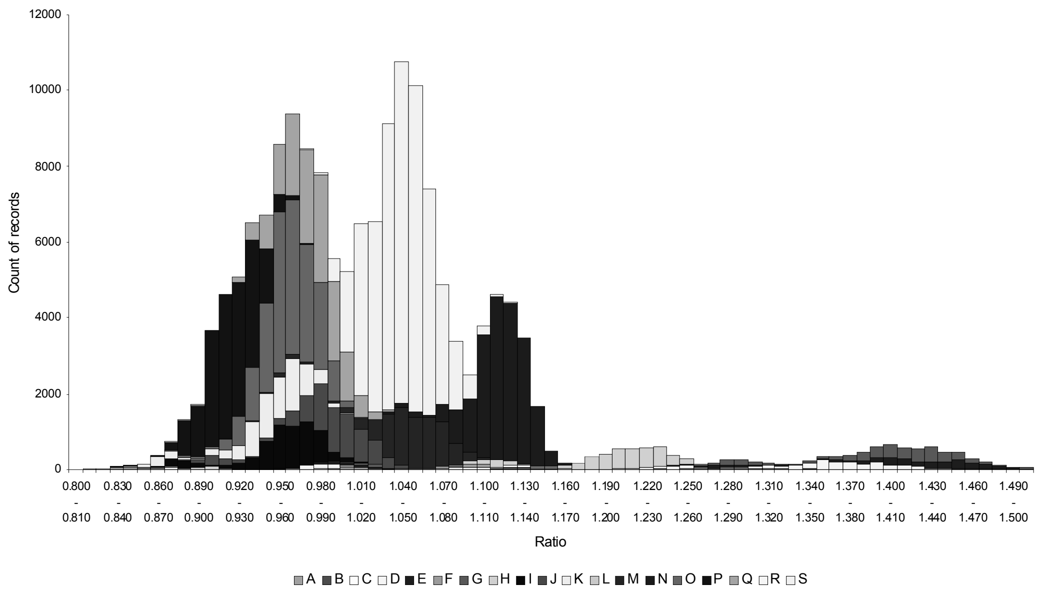
In this particular example, the factors in the upper graph have not compensated well for the restriction. The wide distribution of the restricted to unrestricted ratio implies that the restriction is moving the model away from the theoretical result. The lower graph, on the other hand, shows a model that contains factors that are more correlated with the restricted factor, and that have compensated better for the restriction.

These graphs are part of the Core Reading. Don't worry too much about the detail – they are just examples of the type of analysis that could be undertaken.

Distribution of ratio of fitted values between restricted and unrestricted models (showing little compensation from other factors)



Distribution of ratio of fitted values between restricted and unrestricted models (showing some compensation from other factors)



We discuss offsetting further in Section 5.6.

5.3 Aliasing

Aliasing occurs when there is a linear dependency among the observed covariates $X_1, ..., X_p$. That is, one covariate may be identical to some combination of other covariates. For example, it may be observed that:

$$X_3 = 4 + X_1 + 5X_2$$

Equivalently, aliasing can be defined as a linear dependency among the columns of the design matrix $\mathbf{X}$.

This means that one or more columns can be expressed as a linear combination of other columns. See Solution 16.1 for an example of this phenomenon.

There are two types of aliasing: intrinsic aliasing and extrinsic aliasing.

Intrinsic aliasing

Intrinsic aliasing occurs because of dependencies inherent in the definition of the covariates. These intrinsic dependencies arise most commonly whenever categorical factors are included in the model.

For example, suppose a private passenger automobile classification system includes the factor vehicle age, which has the four levels:

0-3 years (X_1), 4-7 years (X_2), 8-9 years (X_3), and 10+ years (X_4)

Clearly, if any of X_1, X_2, X_3 is equal to 1, then X_4 is equal to 0; and if all of X_1, X_2, X_3 are equal to 0 then X_4 must be equal to 1.

This is because each vehicle has to belong to exactly one of these categories.

Thus:

$$X_4 = 1 - X_1 - X_2 - X_3$$

The linear predictor:

$$\eta = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$$

(ignoring any other factors) can be uniquely expressed in terms of the first three levels:

$$\begin{aligned}\eta &= \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 (1 - X_1 - X_2 - X_3) \\ &= (\beta_1 - \beta_4) X_1 + (\beta_2 - \beta_4) X_2 + (\beta_3 - \beta_4) X_3 + \beta_4\end{aligned}$$

Upon renaming the coefficients this becomes:

$$\eta = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_0$$

The result is a linear predictor with an intercept term (if one did not already exist) and three covariates.

The intercept term is α_0. In a GLM with more than one factor, the intercept for the whole model is a combination of the intercept terms for each factor.

GLM software will remove parameters that are aliased. Which parameter is selected for exclusion depends on the software. The choice of which parameter to alias does not affect the fitted values. For example, in some cases the last level declared (*ie* the last alphabetically) is aliased. In other software the level with the maximum exposure is selected as the base level for each factor first, and then other levels are aliased dependent upon the order of declaration. (This latter approach is helpful since it minimises the standard errors associated with other parameter estimates.)

Extrinsic aliasing

This type of aliasing again arises from a dependency among the covariates, but when the dependency results from the nature of the data rather than inherent properties of the covariates themselves. This data characteristic arises if one level of a particular factor is perfectly correlated with a level of another factor.

For example, suppose a dataset is enriched with external data and two new factors are added to the dataset. The new factors are number of doors and colour of vehicle. Suppose further that, in a small number of cases, the external data could not be linked with the existing data, with the result that some records have an unknown colour and an unknown number of doors.

Exposures		No. of doors				
		2	3	~~4~~	5	Unknown
Colour	~~Red~~	13,234	12,343	15,432	13,432	0
	Green	4,543	4,543	13,243	2,345	0
	Blue	6,544	5,443	15,654	4,565	0
	Black	4,643	1,235	14,565	4,545	0
	~~Unknown~~	0	0	0	0	3,242

Selected Base: No. of Doors = 4: Colour = Red
Additional Aliasing: Colour = Unknown

In this case because of the way the new factors were derived, the level unknown for the factor colour happens to be perfectly correlated with the level unknown for the factor number of doors. The covariate associated with unknown colour is equal to 1 in every case for which the covariate for unknown number of doors is equal to 1, and *vice versa*.

The covariate associated with unknown colour is equal to 1 when colour is unknown and the covariate associated with unknown number of doors is 1 when number of doors is unknown. As we can see from the table above, if number of doors is unknown then colour is unknown and if number of doors is known then so is colour.

This example assumes that the model has been fitted with only two factors, colour and number of doors. There are five possible levels of colour and five possible levels of number of doors, giving a starting point of ten covariates.

Elimination of the base levels through intrinsic aliasing reduces the linear predictor from 10 covariates to 8, plus the introduction of an intercept term. In addition, in this example, one further covariate needs to be removed as a result of extrinsic aliasing. This could either be the unknown colour covariate or the unknown number of doors covariate. Assuming, in this case, the GLM routine aliases on the basis of order of declaration, and assuming that the number of doors factor is declared before colour, the GLM routine would alias unknown colour reducing the linear predictor to just 7 covariates.

"Near aliasing"

When modelling in practice, a common problem occurs when two or more factors contain levels that are almost, but not quite, perfectly correlated. For example, if the colour of vehicle was known for a small number of policies for which the number of doors was unknown, the two-way of exposure might appear as follows:

Exposures		No. of doors				
		2	3	~~4~~	5	Unknown
Colour	~~Red~~	13,234	12,343	15,432	13,432	0
	Green	4,543	4,543	13,243	2,345	0
	Blue	6,544	5,443	15,654	4,565	0
	Black	4,643	1,235	14,565	4,545	5
	Unknown	0	0	0	0	3,242

Selected Base: No. of Doors = 4: Colour = Red

In this case the unknown level of colour factor is not perfectly correlated to the unknown level of the number of doors factor, and so extrinsic aliasing will not occur.

When levels of two factors are "nearly aliased" in this way, convergence problems can occur.

For example, if there were no claims for the 5 exposures indicated in black colour level and unknown number of doors level, and if a log link model was fitted to claims frequency, the model would attempt to estimate a very large and negative parameter for unknown number of doors (for example, –20) and a very large parameter for unknown colour (for example, 20.2). The sum (0.2 in this example) would be an appropriate reflection of the claims frequency for the 3,242 exposures having unknown number of doors and unknown colour, while the value of the unknown number of doors parameter would be driven by the experience of the 5 rogue exposures having colour black with unknown number of doors. This can either give rise to convergence problems, or to results that can appear very confusing. The issue here is that the model will try to compensate for the rogue number by producing a parameter that will "work" for all real data and thus negative parameters will appear where there is no obvious negative relationship.

In order to understand the problem in such circumstances, it is helpful to examine two-way tables of exposure and claim counts for the factors that contain very large parameter estimates. From these, it should be possible to identify those factor combinations that cause the near-aliasing. The issue can then be resolved either by deleting or excluding those rogue records, or by reclassifying the rogue records into another, more appropriate, factor level.

5.4 *Aliasing – a practical example*

Consider the example where a factor called payment frequency is collected for the direct channel, but is unknown for all policies written by the broker channel.

This situation may be caused by variations in the data collection methods, or perhaps because the broker arranges his own premium credit loans, but these are not sourced through the insurance company.

The exposure data is likely to relate to numbers of policies and is as follows:

Exposure	Annual	Monthly	Unknown
Direct	17,774	8,262	0
Broker	0	0	12,046

Suppose that annual and direct are the two selected base levels for the model; some simplified data are given below.

Channel	Payment Frequency	Exposure	Response	Rate
Direct	Annual	17,774	1,777	10%
Direct	Monthly	8,262	1,074	13%
Broker	Unknown	12,046	2,048	17%

The response in this example might be the number of claims.

Note that:

$$\text{Rate} = \frac{\text{Response}}{\text{Exposure}}$$

Now a modelling package may spread the broker / unknown uplift of 1.7 between the two parameters "is broker" and "is unknown".

The uplift figure of 1.7 is the factor that the direct / annual (*ie* the selected base level) rate is multiplied by to get the broker / unknown rate. In the table below, the 1.7 is split into two factors of 1.3 for each of "is broker" and "is unknown". (All figures are rounded to 1 decimal place here.)

		Base	Monthly	Unknown	Broker	Predicted
Direct	Annual	10%	1.0	1.0	1.0	10%
Direct	Monthly	10%	1.3	1.0	1.0	13%
Broker	Unknown	10%	1.0	1.3	1.3	17%

Alternatively it may choose any manner of wild self-cancelling results:

		Base	Monthly	Unknown	Broker	Predicted
Direct	**Annual**	**10%**	**1.0**	**1.0**	**1.0**	**10%**
Direct	**Monthly**	**10%**	**1.3**	**1.0**	**1.0**	**13%**
Broker	**Unknown**	**10%**	**1.0**	**0.0010**	**1700**	**17%**

The point is that the "is unknown" factor and the "is broker" factor must have a product of 1.7.

This is an example of what is called "extrinsic aliasing", and is likely to be handled by the modelling package by eliminating one or other of the parameters automatically.

Automatic parameter removal will cause the remaining parameter to obtain all the uplift required to fit the response. In this case, let's assume that "is unknown" is eliminated and therefore that "is broker" takes on the value 1.7.

In other words, the "is unknown" factor takes the value 1.

Now consider a circumstance where this may not be appropriate. Perhaps the modelling task includes an objective to explain channel differences, or perhaps the future data collection for the broker channel may change and be expected to include valid annual and monthly values.

In this case we might no longer be happy with a seemingly arbitrary allocation of the 1.7 value between "is broker" and "is unknown".

New predicted values from these relativities will look like this:

		Base	Monthly	Unknown	Broker	Predicted
Broker	**Unknown**	**10%**	**1.0**	**1.0**	**1.7**	**17%**
Broker	**Annual**	**10%**	**1.0**	**1.0**	**1.7**	**17%**
Broker	**Monthly**	**10%**	**1.3**	**1.0**	**1.7**	**22%**

This is because the parameter estimates are:

- 1.0 for "is annual" because this is the selected base level for premium frequency
- 1.3 for "is monthly" from the model
- 1.0 for "is unknown" because this was automatically eliminated
- 1.0 for "is direct" because this is the selected base level for channel
- 1.7 for "is broker" because we've assumed that "is unknown" was eliminated and therefore this takes on the whole 1.7 value.

Hence the annual business is given the same prediction as the previous data collected with the unknown level. The monthly business will attract an extra loading resulting in a prediction of 22%.

The broker monthly predicted value = $10\% \times 1.3 \times 1.0 \times 1.7 = 22.1\%$

It is quite likely that these predictions are not suitable even as initial estimates before any new data can be analysed.

This is because "is unknown" was automatically eliminated by the modelling package and therefore given the value of 1.0. However, we know that the observations in this category are actually a mix of annual policies and monthly policies, in which case we would expect the predicted value for the unknown level to lie somewhere between the annual value of 1.0 and the monthly value of 1.3.

Suppose that a brief investigation estimates that the "unknown" broker data was likely to contain two-thirds annual business and one-third monthly. Then the best way to achieve a good set of parameters is to fix the "is unknown" parameter with an offset to the weighted average of the estimates values, *eg*:

$$\mathbf{1.0 \times 2/3 + 1.3 \times 1/3 = 1.1}$$

Then the "is broker" parameter will float down to:

$$\mathbf{1.7/1.1 = 1.545}$$

because we know that the product of "is unknown" and "is broker" must be 1.7.

Now the predictions will adjust as:

		Base	Monthly	Unknown	Broker	Predicted
Broker	**Unknown**	**10%**	**1.0**	**1.1**	**1.545**	**17%**
Broker	**Annual**	**10%**	**1.0**	**1.0**	**1.545**	**15%**
Broker	**Monthly**	**10%**	**1.3**	**1.0**	**1.545**	**20.1%**

which make more sense as 15% and 20% average to the original fitted value of 17%.

5.5 Parameter smoothing

Smoothing is one of the most important techniques to apply to current modelling exercises. This is because, over recent years, datasets have become much larger, and computer software and hardware have become capable of handling these larger datasets.

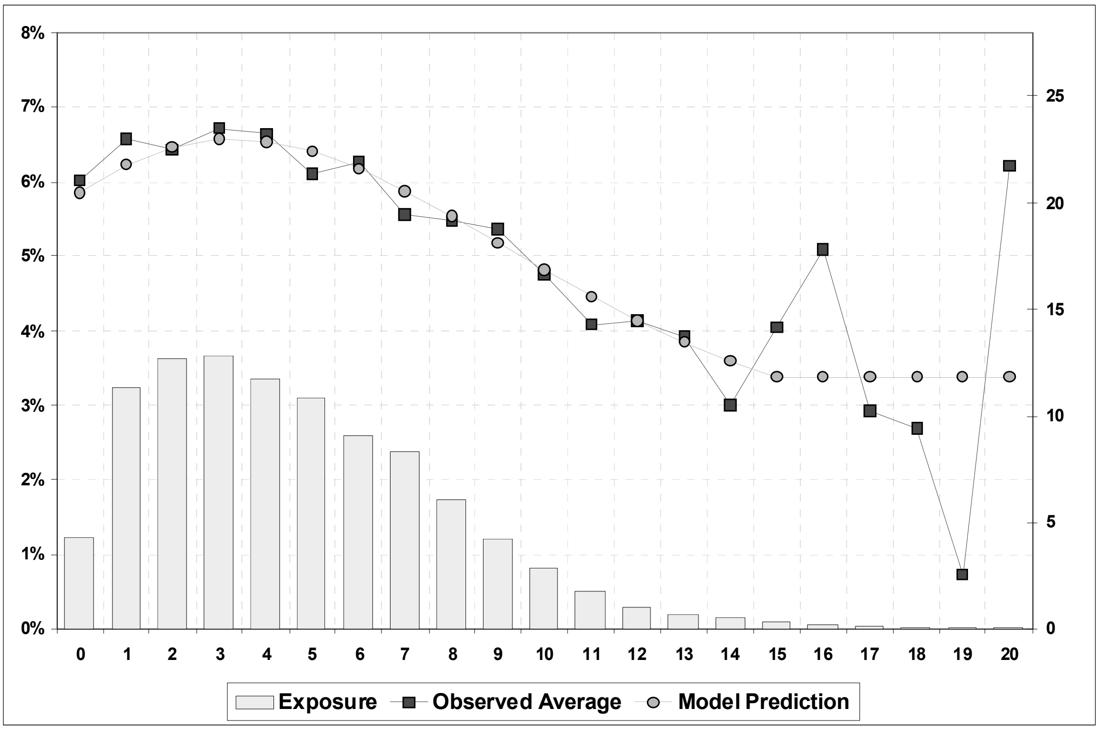
As a result, historical work to guess the best ways to group and summarise the input data prior to loading into a modelling package have become redundant. A much better job can be done by retaining most of the granularity in the data and then using the patterns in the data itself to help define the grouping and smoothing to apply.

In the past, it was necessary to group the data before loading it into the modelling software. For example, factors like policyholder age often had to be grouped into bands of five years to make the model more manageable. Such groupings tended to be based on the results of one-way and two-way tables as well as on the experience of the modeller, but these were subjective and potentially led to a reduction in the accuracy of the final models. With today's modelling packages and the increasing power of computers, pre-grouping is no longer necessary, giving much more flexibility and accuracy.

The word *granularity* refers to the level of detail in the data. A high level of granularity indicates a lot of detail. When we group levels of factors, or reduce the number of factors in a model, we reduce the number of possible combinations of levels of factors and reduce the level of granularity.

In the example below, a polynomial fit has been used for the levels 0 to 14, and the remaining levels grouped together as the data was too sparse to fit a pattern for levels 15 to 20. The polynomial and the grouping have been rendered piecewise continuous.

We explain the term "piecewise continuous" in the example that follows.



This graph is typical of private motor claim frequency by vehicle age.

Fitting a polynomial (a curve) to the data is one way of smoothing the raw parameter estimates. This is generally only used for factors where there is a natural order in the levels. A polynomial of order 1 is simply a straight line; a polynomial of order 2 is a quadratic equation, *etc*. The higher the order of the polynomial, the better it will fit the unsmoothed parameter estimates but the less smooth it will be. We are aiming to find the most parsimonious model and so we want to find the polynomial of the smallest order that still fits the data well.

Question 16.13

Explain the principle of parsimony.

You can see from the graph that there is a lot of variability in the parameter estimates for levels 15 and above, where there is very little data. It is quite common in practice to group together levels of a factor where the data are very sparse. This means that the smoothed parameter estimates for levels 15 to 20 are all equal and are a weighted average of the unsmoothed estimates.

So we now have a curve fitting one part of the range and a straight line fitting the rest, but we may also need to make sure that there is no discontinuity at the point where the curve meets the straight line, *ie* that it is piecewise continuous. This can be achieved within the modelling software.

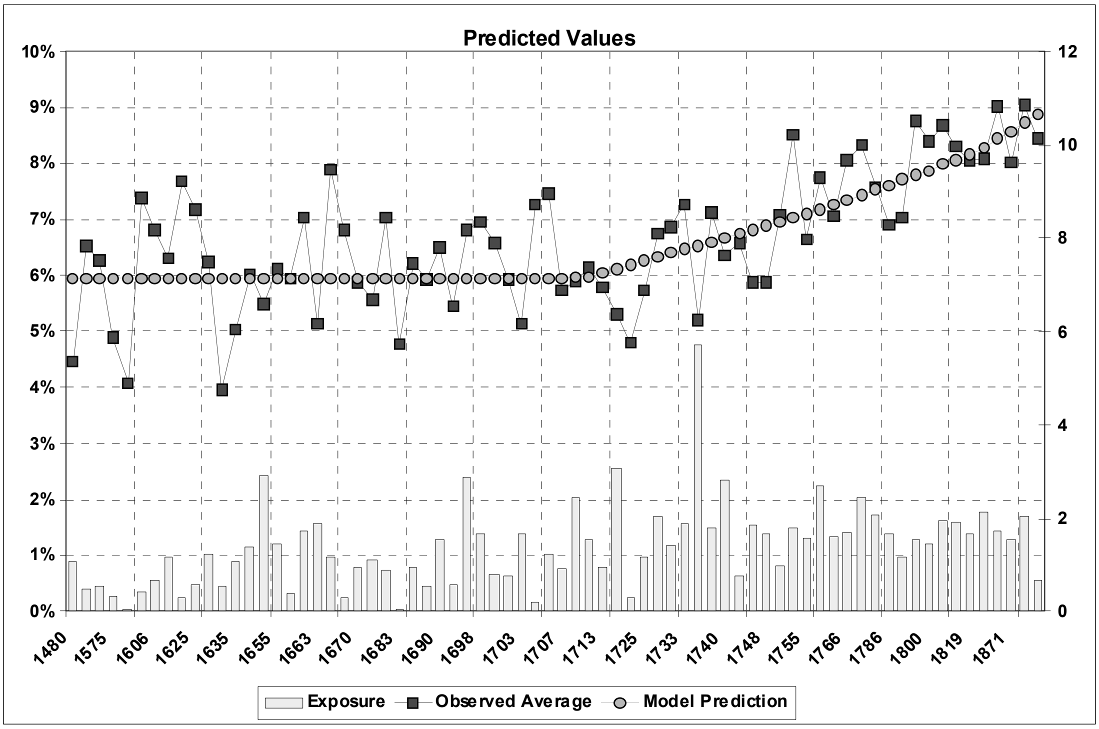
In the next example, the granular data shows significant noise, with no trend on the left-hand levels, and then a fairly steep trend on the right. The factor has been fitted by fitting a level group to the left, and a piecewise continuous straight line to the right.

You will notice that this factor has a large number of levels and so the amount of data in any individual level is likely to be much smaller than it would be for a factor with fewer levels. This is one reason why there is a lot of randomness (noise) in the parameter estimates.

Question 16.14

Why else might there be lots of "noise" in the parameter estimates for this factor?

Had the data been grouped prior to loading into the modelling package, this change in trend may not have been as easy to discern.



Think about one way in which you could have pre-grouped the levels of this factor and what the parameter estimates would look like. Now choose a different pre-grouping and estimate what the parameter estimates would look like. You will see that the estimates for the pre-determined groups will vary depending on your choice of grouping, and this could affect any modelling decisions you make. It is therefore better to avoid grouping levels of factors prior to loading into the modelling software.

Summary of methods for simplifying factors

To sum up, there are four methods for factor simplification:

1. Group and summarise the data prior to loading

This requires knowledge of the pattern that is expected. It is now mainly adopted as a method to thin out redundant codes from the data that has little exposure.

We saw in the previous example, where we didn't have any prior knowledge of the pattern expected, that the results can change depending on our choice of grouping.

However, this approach can still be useful at the data cleaning stage. For example, we may have policyholder gender as a factor, with exposure data as shown in the table below.

Gender	Number of policies
Male	64,932
Female	35,055
Unknown	13

We know that we don't want to use the 13 "unknown" policies to set future rates for unknown gender, and these represent such a small proportion of the total number of policies that there is little merit in keeping them separate. (Indeed, as we have seen in Section 5.3, if these are aliased with the unknown level of other factors then it could cause problems to keep them separate.)

One option is to remove these 13 policies completely from the data. But then we are throwing away the rest of the data attached to these policies. If there are lots of factors, each with a few policies in the unknown level, and we always remove these policies, then we might find that we end up with a much smaller (and potentially biased) dataset than we had expected.

A better option is to group the "unknown" policies with the "male" policies. This will have a negligible effect on the "male" parameter estimates because the number of unknowns is so small, and it is likely that 8 or 9 of the policies relate to male policyholders anyway. In this instance, it is perfectly acceptable to group the data prior to modelling.

2. Grouping in the modelling package

Often called a custom factor, this method simply assigns a single parameter to represent the relativity for multiple levels of the factor.

A *simple factor* is a factor where the levels of that factor represent the levels in the raw data. For example, policyholder age could be a simple factor, taking values between 17 and 100. A GLM would calculate parameter estimates for each individual age.

A *custom factor* is where two or more levels of a factor are grouped together in the model. Policyholder age-band might be a custom factor, with levels 17-50 and 51-100. The GLM would now only calculate parameter estimates for these two levels (one of which would be the base level).

Within a GLM, the custom factor would replace the simple factor in the model. The two models, *ie* one with the simple factor and the other with the custom factor, are nested models and so the modeller can, and should, use the appropriate statistical tests to check whether the simplified model is still valid. These are the same statistical tests that would be used to test for the significance of a rating factor.

Question 16.15

Name the three types of statistical test that can be used to test for the significance of a rating factor.

The grouping done for levels 15 to 20 in the car age example earlier in Section 5.5 is another example of a custom factor.

3. Curve fitting, or use of a variate

The levels of a factor are each assigned an x-value and a polynomial (in the examples above a cubic and linear were used) is fitted to the factor. In this case, the parameters in the model are just the parameters from the polynomial itself, excluding the constant term.

4. Piecewise curve fitting

The factor levels are broken into sections and a custom factor and/or curve from Methods 2 and 3 is applied to each section. By combining these in different ways, the join at each section boundary can be disjoint or piecewise continuous as the modeller thinks appropriate.

This method is really just a combination of Methods 2 and 3.

A good modelling package will make this process easy and fun to achieve with just a few mouse clicks, allowing the modeller to focus on producing quality results.

However, because it is so easy to produce a GLM using modelling software, it is important that the modeller has a full understanding of the data and of the product being analysed to ensure that any models produced are sensible and that observed patterns can be adequately explained.

5.6 Offsetting

We have already met offsets in Section 5.2. The offset applies to the linear predictor η_i, and not the fitted values μ_i. If a subset of β_j is fixed, then the sum of their contributions to η_i is called an offset ξ_i so that:

$$\eta_i = \sum_{j=1}^{k}\left(x_{ij}.\beta_j\right) + \sum_{j=k+1}^{n}\left(x_{ij}.\beta_j\right) = \sum_{j=1}^{k}\left(x_{ij}.\beta_j\right) + \xi_i$$

where the summation is over the terms for which the parameters β_j are not fixed. In other words, we subtract the offset from the linear predictor and the results can then be regressed on the remaining variables. Add the offset back again after the fit. Everything else is as before.

In this equation, the subset of β_j that is fixed (using the offset) is for values of j from $k+1$ to n.

The main purpose of offsetting is to fix the relativities of a factor to a set of values that would differ from the naturally fitted values. A classic example of this would be the discount provided on private car insurance for different levels of no claims discount. These discount percentages may be fixed by marketing and hence cannot be fitted, but need to be allowed for when fitting the other data.

A subtle use of offsetting is to "fix" a level of a factor as part of the process of removing aliasing from the data. See the direct / broker example above.

This example is in Section 5.4.

Offsetting is also commonly used in more complex models where a hierarchy of models is wanted. This is achieved by fitting the first model, offsetting all the values (*ie* fixing the parameters at the values obtained in fitting the first model)**, and then fitting a second model to explain the remaining patterns in the data.**

Offsetting can be useful when comparing a final model to a new "hold-out" sample of data. To do this, first divide the data into modelling and test sets. A division like ¾ and ¼ may be appropriate. Then fit the model on the main dataset, and then fully offset it. Offsetting fixes the parameters so that they can be reloaded and scored against the test set. The predictiveness of the model is judged by how well it performs against this test set, by comparing the observed values directly to the fitted values from the offset model.

A "hold-out" sample of data is a dataset that can be used to validate the results from a model. It should be a representative sample of the whole dataset and may be, *eg* 10% of the original data, chosen at random. This allows the modeller to check whether the parameter estimates derived from the model are useful in predicting the claims experience for a completely separate set of data.

Similarly, model results can be validated by applying the fitted values to data from a more recent policy year rather than from a hold-out sample.

6 Glossary items

Make sure you read the Glossary items relating to this chapter. They are:

- Generalised linear model
- Link function.

Chapter 16 Summary

Exponential family

The exponential family is the set of distributions whose probability function or probability density function can be written in the form:

$$f_i(y_i;\theta_i,\varphi)=\exp\left\{\frac{y_i\theta_i-b(\theta_i)}{a_i(\varphi)}+c(y_i,\varphi)\right\}$$

The parameter θ_i is a function of the mean $\mu_i=E(Y_i)$. The mean and variance of Y_i satisfy the equations:

$$E(Y_i)=b'(\theta_i)$$

$$\text{var}(Y_i)=a_i(\varphi)b''(\theta_i)$$

The variance function is given by:

$$V(\mu)=b''(\theta)$$

The binomial, Poisson, exponential, gamma, normal, inverse Gaussian and Tweedie distributions are all members of the exponential family.

Uses of GLMs

A GLM can be used to model the behaviour of a random variable that is believed to depend on the values of several other characteristics, *eg* age, sex, vehicle group.

Components of a GLM

A GLM consists of:

- a distribution for the data, which must be a member of the exponential family
- a linear predictor, η, and
- a link function, $g(\cdot)$.

Matrix form of a GLM

A GLM can be written in matrix form as:

$$\mathbf{Y} = g^{-1}\left(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}\right) + \boldsymbol{\varepsilon}$$

where:

- $\mathbf{X}$ is the design matrix of factors
- $\boldsymbol{\beta}$ is a vector of parameters to be estimated
- ξ is a vector of offsets or known effects
- $\boldsymbol{\varepsilon}$ is the error term appropriate to $\mathbf{Y}$.

The linear predictor is:

$$\boldsymbol{\eta} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}$$

Estimating the parameters of a GLM

For a normal linear model with identity link function, the parameters can be estimated using the matrix formula:

$$\boldsymbol{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y}$$

For more complicated models and other link functions, the parameters are obtained using iterative techniques.

Comparing models

The contribution of the ith observation to the deviance is:

$$d\left(Y_i;\mu_i\right) = 2\omega_i \int_{\mu_i}^{Y_i} \frac{\left(Y_i - \zeta\right)}{V\left(\zeta\right)} d\zeta$$

and the total deviance is given by:

$$D = \sum_{i=1}^{n} d\left(Y_i;\mu_i\right)$$

The scaled deviance is:

$$D^* = \frac{D}{\varphi}$$

It can be used to compare the goodness of fit of nested models. In particular, if Models 1 and 2 are nested, then:

$$D_1^* - D_2^* \sim \chi^2_{df_1 - df_2}$$

If φ is unknown, the goodness of fit of nested models can be compared using the result:

$$\frac{(D_1 - D_2)}{(df_1 - df_2)(D_2 / df_2)} \sim F_{df_1 - df_2, df_2}$$

For models that are not nested, the Akaike Information Criterion (AIC) can be used to compare the goodness of fit. The AIC is:

$$\text{AIC} = -2 \times \text{log-likelihood} + 2 \times \text{number of parameters}$$

The lower the AIC, the better the fit.

The hat matrix

The hat matrix $\mathbf{H}$ is one of the outputs of the model-fitting process. It is given by:

$$\mathbf{H} = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T$$

Leverages

The diagonal entries h_{ii} of the hat matrix are called the leverages. These measure the influence that each observed value has on the fitted value for that observation. Data points with high leverages or residuals may distort the outcome and accuracy of a model.

Pearson residuals

The i th Pearson residual is:

$$r_i^P = \frac{Y_i - \mu_i}{\sqrt{\dfrac{\varphi V(\mu_i)(1-h_{ii})}{\omega_i}}} = \frac{Y_i - E[Y_i]}{SD[Y_i]\sqrt{1-h_{ii}}}$$

Deviance residuals

The i th deviance residual is:

$$r_i^D = sign(Y_i - \mu_i)\sqrt{d(Y_i;\mu_i)}$$

where:

$$d_i = 2\omega_i \int_{\mu_i}^{Y_i} \frac{(Y_i - \zeta)}{V(\zeta)} d\zeta$$

Residuals should be small and have mean 0.

Cook's distance

Cook's distance may also be used as a measure of the influence of a data point on the model results. Cook's distance for the i th data point is given by:

$$c_i^P = \frac{h_{ii}}{(1-h_{ii})\left(\sum_i h_{ii}\right)}\left(r_i^P\right)^2$$

Points with a Cook's distance of 1 or more are considered to merit closer examination in the model analysis.

Model refinement

Interactions can be included in a model. These may be complete or marginal.

Offsets can be used to constrain or fix certain elements of a model in such a way that the fitted relativities for the other factors adjust to compensate.

Aliasing occurs within GLMs. Intrinsic aliasing occurs because of dependencies inherent within the definition of the covariates. This is dealt with by modelling software. Extrinsic aliasing occurs when two or more factors contain levels that are perfectly correlated. "Near" aliasing occurs when this correlation is almost, but not quite, perfect.

A GLM can be improved by smoothing the parameter values. This can be achieved by grouping levels of factors or by fitting curves to the values.

This page has been left blank so that you can keep the chapter summaries together for revision purposes.

Chapter 16 Solutions

Solution 16.1

We have:

$$\frac{\partial l^*}{\partial \beta_0} = 0 \Rightarrow 150\% - 4\beta_0 - 2\beta_1 - 2\beta_3 = 0 \tag{1}$$

$$\frac{\partial l^*}{\partial \beta_1} = 0 \Rightarrow 100\% - 2\beta_0 - 2\beta_1 - \beta_3 = 0 \tag{2}$$

$$\frac{\partial l^*}{\partial \beta_3} = 0 \Rightarrow 15\% - 2\beta_0 - \beta_1 - 2\beta_3 = 0 \tag{3}$$

From (1), it follows that:

$$2\beta_0 = 75\% - \beta_1 - \beta_3. \tag{*}$$

Substituting from (*) into (2) and (3) gives $\beta_1 = 25\%$ and $\beta_3 = -60\%$ respectively. Substituting these values back into (3) gives:

$$15\% - 2\beta_0 - 25\% + 120\% = 0$$

$$\Rightarrow \beta_0 = 55\%.$$

So:

$$E[\mathbf{Y}] = \mathbf{X}\hat{\boldsymbol{\beta}} = \begin{pmatrix} \hat{\beta}_0 + \hat{\beta}_1 \\ \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_3 \\ \hat{\beta}_0 \\ \hat{\beta}_0 + \hat{\beta}_3 \end{pmatrix} = \begin{pmatrix} 80\% \\ 20\% \\ 55\% \\ -5\% \end{pmatrix}.$$

Solution 16.2

For the exponential distribution:

$$f(y;\lambda) = \lambda e^{-\lambda y}$$

Now replace y with y_i:

$$f(y_i;\lambda) = \exp\left\{-\lambda y_i + \ln\left(\lambda\right)\right\}$$

Now set $\theta_i = -\lambda$ and define $\varphi = \omega$ so that $\dfrac{\varphi}{\omega} = 1$.

This gives:

$$f(y_i;\theta_i) = \exp\left\{\frac{y_i\theta_i - \ln\left(-\dfrac{1}{\theta_i}\right)}{\dfrac{\varphi}{\omega}}\right\}.$$

Comparing the expression above with the formula for the exponential family, we see that:

$$c(y_i,\varphi) = 0$$

$$a(\varphi) = \frac{\varphi}{\omega} = 1$$

$$b\left(\theta_i\right) = \ln\left(-\frac{1}{\theta_i}\right) = -\ln\left(-\theta_i\right)$$

Solution 16.3

If $b(\theta) = -\ln(-\theta)$, then:

$$b'(\theta) = -\left(\frac{-1}{-\theta}\right) = -\frac{1}{\theta}$$

and:

$$b''(\theta) = \frac{1}{\theta^2} = \frac{1}{\lambda^2}$$

since $\theta = -\lambda$. Now since $\mu = E(Y) = \dfrac{1}{\lambda}$ for $Y \sim Exp(\lambda)$, it follows that:

$$V(\mu) = b''(\theta) = \mu^2$$

Solution 16.4

It is likely that an excess has been applied to the incurred loss data, meaning that any losses between zero and the amount of the excess will be included in the zero category.

Solution 16.5

We saw in Section 1.4 that the variance function for the Poisson distribution is the identity function, *ie*:

$$V(t) = t$$

So:

$$d_i\left(y_i, \hat{\mu}_i\right) = 2\int_{\hat{\mu}_i}^{y_i} \frac{y_i - t}{V(t)}\, dt = 2\int_{\hat{\mu}_i}^{y_i} \frac{y_i - t}{t}\, dt = 2\int_{\hat{\mu}_i}^{y_i} \left(\frac{y_i}{t} - 1\right) dt$$

Integrating gives:

$$\begin{aligned} d_i\left(y_i, \hat{\mu}_i\right) &= 2\left[y_i \ln t - t\right]_{\hat{\mu}_i}^{y_i} \\ &= 2\left[y_i \ln y_i - y_i - y_i \ln \hat{\mu}_i + \hat{\mu}_i\right] \\ &= 2\left[y_i \ln\left(\frac{y_i}{\hat{\mu}_i}\right) - \left(y_i - \hat{\mu}_i\right)\right] \end{aligned}$$

Solution 16.6

The difference in the scaled deviance is $17.80 - 11.08 = 6.72$.

The difference in the number of degrees of freedom is the same as the difference in the numbers of parameters in the models, *ie* $10 - 6 = 4$.

Since $6.72 < 9.488$, the upper 5% point of χ_4^2, there is insufficient evidence at the 5% significance level to reject Model A in favour of Model B.

Percentage points of the χ^2 *distribution are given on Page 169 of the Tables.*

Solution 16.7

The difference in the deviance is $40.89 - 26.40 = 14.49$.

The difference in the number of degrees of freedom is $16 - 8 = 8$.

The number of degrees of freedom in Model D is $50 - 16 = 34$.

So the value of the test statistic is:

$$\frac{14.49}{8 \times \left(\frac{26.40}{34}\right)} = 2.333$$

From Page 172 of the *Tables*, the upper 5% point of $F_{8,34}$ is 2.225. Since our test statistic exceeds this value, we reject Model C in favour of Model D.

Solution 16.8

The data used to analyse the claim frequencies will contain a mix of different makes and models of vehicle. Some manufacturers will introduce higher levels of security than others and at different times. An individual manufacturer may also introduce different levels of security depending on the model of the vehicle.

The claim frequencies in this example are likely to relate to theft *of* vehicles but also to theft *from* vehicles. The security improvements, *eg* immobilisers, are likely to have a greater effect on "theft of" but will not necessarily stop "theft from".

New cars are still usually the most valuable in the eyes of thieves and over time they will find new ways of getting around the security features.

Some policyholders may not make use of the security measures on their vehicles, making these more susceptible to theft. Even if it is a condition of the insurance to use these, a claim is still likely to be paid if the insurer can't prove that the policyholder has been negligent.

Solution 16.9

We would need to re-specify the model by choosing a different statistical distribution.

Solution 16.10

Correlations occur when there is a relationship between the distribution of exposure between levels of two or more factors. GLMs automatically take account of correlations (unlike one-way tables).

Interactions relate to the effect that factors have upon the risk. To define the risk accurately, an interaction would be necessary where the effect of two (or more) factors depend on each other. GLMs can be specified to include interactions of our choice.

Solution 16.11

One option is to test every possible combination of pairs (or triplets) of factors and test each for statistical significance and reasonableness. However this is very time-consuming and unlikely to be done in practice.

You might look at the structure of your existing rating algorithms and see which interactions can be included without the need for IT support, *eg* by checking which interaction rate tables already exist. There is little point in coming up with a highly sophisticated rating structure if it is too complicated to actually be implemented.

You could use your experience of the product and the market in which it operates. For example, in private motor insurance it is commonplace to include an interaction between policyholder age and policyholder gender.

You could speak to underwriters and other experts to see whether there are any parts of the account where your rates might be out of line with the market. This could indicate that you haven't defined the risk correctly and that perhaps an interaction term might help.

Solution 16.12

There will be no effect on the remaining parameter values because this is effectively the same model.

Solution 16.13

Parsimony is the concept that "less is better". For GLMs, this means that the model with the smallest number of parameters is preferred, as long as we are not removing statistically significant parameters.

A model with more parameters than are needed might fit the past data extremely well, accounting for all the random "blips" observed but this may lead to a lower level of predictive power for the future.

Solution 16.14

There might be lots of "noise" simply because this factor does not explain the risk very well.

Solution 16.15

The three types of statistical test given in Section 3.1 are:

- Chi-squared
- F statistic
- Akaike information criteria (AIC).

Chapter 17

Use of multivariate models in pricing

Syllabus objectives

(o) (i) Understand the uses of multivariate models in pricing.

(ii) Outline the different types of multivariate models.

0 Introduction

Further to the above syllabus objectives, the Core Reading below describes in more detail the additional aims of this chapter.

1 Risk and rating factors

- **Explain in broad terms how typical risk and rating factors affect claims experience for UK motor and household insurance, and why.**
- **Explain in broad terms the categories of external data available to insurers, which may provide further information on individual risks, and explain how such external rating factors affect claims experience for UK motor and household insurance.**

This is covered in Section 1.

2 Types of multivariate and classification models

- **Describe in broad terms possible multivariate models that could be used, including generalised linear models.**
- **Describe in very broad terms possible approaches to classification, including:**
  - **spatial smoothing methods**
  - **vehicle classification techniques.**

This is covered in Section 2.

3 Forms of models

- **Describe typical forms for modelling: claim frequency, claim numbers, average claim amounts, burning cost and probabilities. Descriptions should include reference to the error distribution used, the link function, scale parameter, variance function, weights and offset term.**

This is covered in Section 3.

4 Initial analyses

- **Describe initial analyses typically performed prior to multivariate modelling, explaining the purpose of each, including:**
  - **one-way analyses**
  - **two-way analyses**
  - **correlation analyses**
  - **distribution analyses.**

This is covered in Section 4.

5 Claim types models

- **Explain what claim types are typically modelled for UK motor and household insurance and why.**

This is covered in Section 5.

6 Model combining

- **Describe how to combine models of claim types to produce risk premiums.**
- **Explain how to combine frequency and amounts models multiplicatively for one claim type.**
- **Explain how to combine frequency and amounts models across additive claim types by fitting a further model to the expected cost of claims.**

This is covered in Section 6.

7 Model validation

- **Describe how a random sample of data can be retained for model validation purposes, and describe how models can be tested (see D.46 to D.52 of GRIP report).**

The GRIP (General insurance premium Rating Issues working Party) report is available from the Actuarial Profession's website, **www.actuaries.org.uk**.

The information on model validation is in Appendix D of this report, in paragraphs 46 to 52, although all of this information is repeated in Section 7.

8 Implementation

- **Describe ways of representing the results of the analysis, including:**
  - **showing results of the GLM, existing relativities and competitor relativities simultaneously**
  - **showing the overall effect of all factors combined using graphs showing the distribution of the ratio of fitted values (see D.172 to D.185 of GRIP report).**

This is covered in Section 8.

The interested student might like to read the GRIP paper – it is a long paper but fairly easy to read, and full of information on various methods used in pricing of both insurance (personal lines and commercial lines) and reinsurance.

1 Risk and rating factors

In this section we describe the data that may be available when pricing a UK motor or home insurance policy.

1.1 Motor insurance

Details of risk and rating factors used in motor insurance are covered in more detail in Chapter 3.

A motor insurer might typically seek the following information when preparing a quote:

- **Information on the policy and coverage, *eg***
  - **type of cover** (such as comprehensive or third party, fire and theft)
  - **payment frequency** (such as monthly or annual)
  - **voluntary excess.**
- **Details of the proposer, *eg***
  - **age**
  - **gender**
  - **marital status**
  - **occupation.**
- **Details of each driver, *eg***
  - **driver experience**
  - **driver restrictions**
  - **age**
  - **relationship** (to the proposer).
- **Details of the vehicle and its use, *eg***
  - **registration number**
  - **make / model**
  - **parking location.**

Question 17.1

Why might it be better to record the date of birth for each driver rather than their age?

> ***Question 17.2***
>
> Insurers sometimes ask about the number of children under 16 that a proposer has. Why do you think this information is useful for rating?

There are a number of factors that will ultimately affect the actual claims cost of providing the insurance coverage. For example, some of the risk factors are:

- **drivers**
  - **their driving style, experience, level of skill, powers of observation, attitude to risk, ability to predict road hazards**
- **vehicle**
  - **the value of the vehicle / repair costs if it should be damaged**
  - **safety features available to protect passengers (airbags, *etc*), and to improve car control (ABS, traction control, *etc*)**

  ABS stands for Anti-lock Braking System. Such systems are designed to prevent vehicles skidding when braking. Traction control systems are designed to prevent vehicles skidding when accelerating.

  - **security features (to prevent theft)**
  - **performance, speed, size, weight**
- **environment**
  - **the road environment where the car is used**
  - **type of road**
  - **the time** (of day) **and hence level of congestion / pedestrian risks**
  - **natural hazards (rain, sun, ice, flooding, wind)**
- **exposure**
  - **the amount of driving (number of miles / minutes) incurred in each policy period**
- **third parties – the other people driving in the vicinity of the insured, who may well become involved in an accident or even prevent an accident, including their level of skill.**

Most of the questions asked at the point of sale do not directly measure the genuine risk factors from this second list. This is because many of those factors cannot readily be quantified, or they change over time (or even real-time, *ie* continuously**), and certainly cannot be defined by the customer at the point of sale.**

For example, if you asked a prospective policyholder to rate his level of driving ability, the chances are that he would rate himself as excellent. This is clearly very subjective and cannot be accurately defined by the (biased) driver, and so cannot be used as a basis for pricing the risk. Instead, we would have to use the questions asked at the point of sale to give us an idea of the level of risk because these are more objective.

The cost of claims is therefore predicted using point of sale questions that act as proxies for unknown genuine risk factors.

The effectiveness of such proxy rating factors depends upon:

- **how directly the factor measures a genuine risk factor**

  ("Vehicle value" is a good example of this since, although the customer will probably disclose the value they paid for the vehicle, it (a claim) **will directly relate to the residual value of the vehicle at the time of the accident** or theft.)

  Vehicle value would need to be updated at every renewal; otherwise it could become very out of date and be of little use in rating. There will be an element of subjectivity involved, since policyholders are likely to think their vehicle is worth more than it actually is.

- **if the proxy is a factual quantity that is known to the proposer**

  ("Postcode" is an example of a well-defined fact with good granularity. In the UK, postcode defines the location to around 15 houses: this can be used to look up a rating area and as a key for linking external data.)

  "Granularity" refers to the level of detail involved. The more granularity there is, the greater the level of detail (*ie* the information is broken down into a larger number of 'grains').

- **whether the fact has an obvious direction, which the proposer may be tempted to mis-state to obtain a cheaper quote**

  ("Estimated annual mileage" will often be under-reported by the proposer compared to the actual annual mileage, since it is clear that this is likely to decrease the price.)

  If every proposer underestimated their annual mileage by, say, 2,000 miles then this would not be a problem as the relative difference between groups of policyholders would be the same. However, if some did this to a greater or lesser extent than others then it could cause some inaccuracy when setting premium rates.

- **the extent to which the proxy overlaps with other existing factors.**

  (“Age of licence” is clearly strongly correlated with “age of driver” and “no claims discount” for drivers aged 17 to 22, so its value is diluted by these overlaps and it only has significant value when identifying novice drivers in older age.)

1.2 Home insurance

There are two broad types of cover for home insurance: buildings and contents. While customers are likely to be asked the same types of questions for both types of insurance, the rating will generally be done separately because different factors drive the claims experience for each type of cover.

A typical list of questions requested by a home (buildings and contents insurer) at the point of sale is as follows:

- **policy**
  - **date on risk**
  - **number of adults / children in household and whether they are family**
  - **number of claims (buildings / contents)**
  - **type of cover (buildings and/or contents)**
  - **accidental damage cover (buildings / contents)**
  - **excess (buildings / contents)**
  - **special items cover, garage / freezer / garden / away from home cover**
- **proposer**
  - **age (date of birth)**
  - **gender**
  - **marital status**
  - **smoker / non-smoker**
  - **employment status** (*eg* employed / self-employed / unemployed / retired, part-time / full-time)
  - **length of residence in UK**
  - **ever refused insurance?**
  - **convictions / bankruptcy**

- **house details**
  - **purchase year**
  - **type of property (detached / semi / bungalow), own front door**
  - **listed building status**
  - **state of repair**
  - **occupied during day / night**
  - **is property for sale?**
  - **age of property (year built)**
  - **construction type** (*eg* timber-framed / brick / solid wall / thatched roof)
  - **number of bedrooms / other rooms**
  - **rating area (postcode)**
  - **ownership (owned / mortgaged / rented)**
  - **property rebuild value (sum insured)**
  - **contents value (sum insured)**
  - **alarm / neighbourhood watch / smoke detector**
  - **business use**
  - **flood / subsidence experience**
  - **trees near house**
  - **door / window / patio lock types.**

For household policies, a wide range of different perils is covered by the standard policy. These include fire, storm, flood, subsidence, theft and, if purchased, accidental damage.

These factors (*ie* the questions asked) **usually relate to the claims experience as follows:**

- **Some of the questions at the point of sale relate to particular perils, for example:**
  - **smoker / non-smoker is related to the fire peril because many house fires are caused by the householder's own cigarettes**
  - **construction type identifies thatched cottages, which are particularly vulnerable to fire damage**
  - **the presence of an alarm or neighbourhood watch programme has an impact on theft experience.**

- **Questions relate to the risk exposure for various perils, for example:**
  - **number of children**
  - **number of bedrooms**
  - **sum insured.**

  Sum insured is the generally-accepted exposure measure for home insurance, where, for buildings cover, this relates to the rebuild cost of the property and for contents cover, to the value of the contents. However, many policyholders do not know the rebuild value of their house (which will generally be very different to the market value), and so some insurers now use the number of rooms or bedrooms as a proxy for the sum insured to make the application for insurance easier for their customers.

- **A significant feature of home insurance is the element of discretion involved in the claim. Many customers either do not realise that they could claim for a particular event, or decide that the amount is too small to be worth the effort. Employment status, number of prior claims** and **postcode act as significant proxies for this behaviour.**

  Many home insurance policies have followed the lead of motor insurance and now offer NCD for customers who have had no claims in the last few years. The discounts are generally quite a lot smaller than for motor, but are often sufficient to deter policyholders from making small claims.

1.3 External data

Motor and home insurance are extremely competitive classes of business in the UK and insurers are constantly trying to find additional factors that will help them to further refine their assessment of each individual risk. Rather than restricting themselves to the data available within the policy and claim records, insurers are increasingly testing and using rating factors that originate from external data.

In addition to the factors requested at the point of sale, many other factors from other data sources can be used to predict claims experience. To attach external data to assist in a modelling exercise the source data needs to include linking fields against which the external data can be referenced. These are typically:

- **person (identity of customer: name / address / date of birth)**
- **location (full address-point / postcode)**
- **insured asset (for motor insurance this could be registration number, or make / model code, as issued by the ABI in the UK).**

Then for each of these, various external data can be added. Note that this process can also include data external to the point of sale questions but internal to the company itself.

Examples of external data

Examples include:

- **proposer**
  - **previous insurance company and claims experience**
  - **other cross-product holdings**

  For example, a customer asking for a car insurance quote from an insurer may already have household insurance with that insurer. Information already held in respect of their household policy (*eg* claims experience) may be useful for determining the appropriate car insurance premium.

- **customer lifetime value models**

  Customer lifetime value models consider how profitable a customer will be over the entire period that they are expected to stay with an insurer. A customer who is only likely to stay with the insurer for one year may be charged a higher premium initially, to reflect the fact that this is likely to be the only opportunity to make a profit (and recuperate fixed expenses) on that policy. Conversely, a customer who is likely to be loyal to the company for many years may be charged a lower premium – profits each year will be lower, but with the expectation that these profits will arise for many years to come.

- **customer behaviour models**

  Customer behaviour models attempt to understand what drives each type of customer to buy and then renew their insurance. For example, some customers are very price-elastic and will be attracted to the policy with the lowest price, while others will be price-inelastic and may consider factors other than price (such as convenience and customer service) to be more important.

- **credit score / customer insurance score**

• **location (full address-point / postcode)**

  - **average wealth data / socio-economic categories**
  - **subsidence / geological soil type data (for household)**
  - **flood data (for household)**
  - **theft survey data**
  - **census data / electoral roll**

    The census data for the UK is updated every ten years and includes many data fields, grouped by full postcode, including the age structure of the residents, employment and occupation information, type and size of accommodation, amenities and household composition.

  - **household valuation data**

• **insured asset for motor** (*ie* data on the car itself)

  - **data from insurers' trade body, for example ABI in the UK (car group, engine capacity,** number of **seats, body shape, gearbox, fuel)**
  - **data from motor registration / licensing authority, for example the DVLA in the UK (ownership length, number of keepers, actual mileage)**
  - **additional vehicle data (make / model attributes, residual value)**

   - **data from inter-industry agreement to share claims and underwriting information, for example CUE in the UK (data that relates to prior claims).**

     CUE stands for the Claims and Underwriting Exchange. It is a central database of motor, home and personal injury / industrial illness incidents that have been reported to insurance companies, whether or not they gave rise to a claim. The aim of CUE is to prevent fraudulent activity either where multiple claims are made (in the hope of receiving payment for the same thing many times over) or where previous claims have not been disclosed.

How the external data relates to the claims experience

Some of the external data will potentially be relevant to the whole risk, while other data will relate only to very specific parts of the claims experience.

The factors relate to the claims experience in the following ways:

- **Questions that relate to particular perils, for example:**
  - **flood data (for household) provides height data, flood defence data, and prior flood experience in the area: these relate directly to the flood peril itself**
  - **theft survey data, although often sparse, and then modelled to smooth across a wider set of areas than originally collected: this data can provide significant uplift for the theft peril**
  - **data from insurers' trade body (for example, ABI in the UK) is a relevant indicator of performance and repair costs.**

- **Questions that relate to the risk exposure for various perils; for example:**
  - **household valuation data may help amend the customer's own estimate of the sum insured, particularly if rebuild value is required rather than current market value, which the customer will tend to provide**
  - **the national vehicle licensing body may collect actual mileage data.**

- **Questions that relate to customer discretionary behaviour:**
  - **wealth, prior claims, cross-product.**

2 Types of multivariate and classification models

2.1 Multivariate models – GLM

This is the most common method used today to model personal lines insurance data to assess claims costs. (See Chapter 16 for further details).

Often the link function $g()$ is chosen as the log function, as this creates the form:

$$Y_i = \exp\left(\sum_{j=1}^{k} X_{ij}\beta_j\right) + \varepsilon_i = \prod_{j=1}^{k} e^{X_{ij}\beta_j} + \varepsilon_i .$$

and results in a multiplicative relationship between the factors.

Remember that the linear predictor $\eta_i = \sum_{j=1}^{k} X_{ij}\beta_j$ is related to $\mu_i = E(Y_i)$ via the relationship:

$$\eta_i = g(\mu_i).$$

So if $g(\mu_i) = \log \mu_i$, it follows that:

$$E(Y_i) = \exp(\eta_i) = \exp\left(\sum_{j=1}^{k} X_{ij}\beta_j\right) = \prod_{j=1}^{k} e^{X_{ij}\beta_j} .$$

This form is convenient because the effects of different factors can simply be multiplied together (as you can see from the last term in the equation above).

2.2 Approaches to classification

Classification refers to the grouping of levels of factors. There will be many possible ways in which the levels of a particular factor can be grouped, and some groupings will produce better results than others. Where a factor has a very large number of levels (*eg* postcode in the UK), it is likely that there will not be enough data in every single one of these levels to enable the grouping to be done based only on an analysis of past experience. Here we consider other methods that can be used to help with the accuracy of such a classification.

Spatial smoothing methods

Spatial means "relating to space". The underlying assumption for spatial smoothing is that physically close areas are likely to have similar experience.

GLM models can be used effectively for factors where the number of levels is small (say, 100 or less), or where the level forms a naturally continuous variable that can be fitted as a function (say, a polynomial or a set of polynomial splines).

For a factor such as postcode where there are many levels (in the UK, the full postcode has 1.7 million), an alternative approach such as spatial smoothing is required. This allows the model to fit many values to the postcode factor, and then removes the noise from these predictions by adjusting the relativity to take into account neighbouring values.

The *noise* is just the random element of the past experience, which we do not want to project into the future.

This improves the predicted values by taking into account the credibility (or lack of) for the response in a single location.

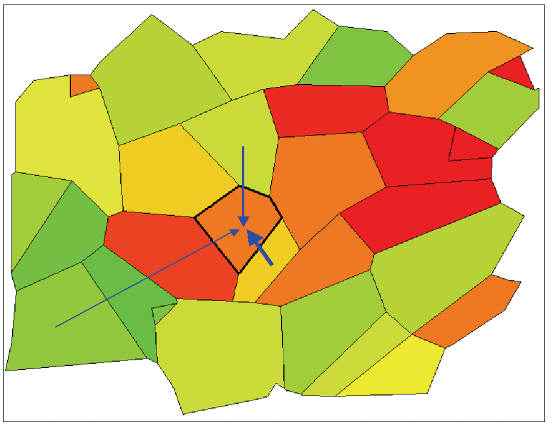
There are different ways in which we can take account of neighbouring values.

Two main forms of spatial smoothing are typically employed:

- **distance-based smoothing**
- **adjacency-based smoothing.**

The features of each form of smoothing make it more or less appropriate to use depending on the underlying processes behind the loss types being modelled.

Distance-based smoothing



Distance-based smoothing incorporates information about nearby location codes based on the distance between the location codes: the further away a location code, the less influence (or weight) is given to its experience.

So the past claims experience of an area 50 miles away from the area we are interested in will be given less weight than that of an area only 10 miles away.

This is true regardless of whether an area is urban or rural, and whether natural or artificial boundaries (such as rivers) exist between location codes.

If there is a wide river between two areas, with no bridge access to link those areas, then it would not necessarily be appropriate to link these areas when considering the theft risk in each. Even though the areas are physically close to each other, their relative theft experience could be very different, and so we would not want to use the experience of one to influence the other.

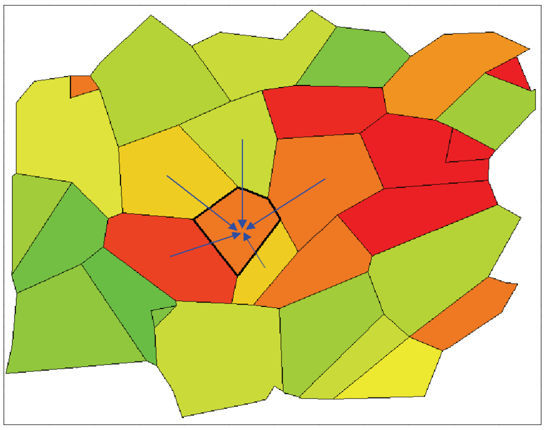
Urban postcodes tend to have smaller geographical areas than rural ones. Therefore, for any urban postcode, it is more likely that there will be a greater number of other postcodes that are close in terms of physical distance. Using this method might therefore influence the urban experience for a single postcode more than we might want.

As such, distance-based smoothing methods are often employed for weather-related perils where there is less danger of over- or under-smoothing urban and rural areas.

Distance-based smoothing methods have the advantage of being easy to understand and easy to implement, as no distributional assumptions are required in the algorithm.

Distance-based methods can also be enhanced by amending the distance metric to include "dimensions" other than latitude and longitude. For example, including urban density in the distance metric would allow urban areas to be more influenced by experience in nearby urban areas than by nearby rural areas, which may be appropriate.

Adjacency-based smoothing



Adjacency-based smoothing incorporates information about directly neighbouring location codes. Each location code is influenced by its direct neighbours, each of which is in turn influenced by its direct neighbours; distributional assumptions or prior knowledge of the claims processes can be incorporated in the technique. The algorithms are therefore iterative and complex to implement.

As this smoothing method relies on defining which location codes neighbour each other, natural or artificial boundaries (*eg* rivers or motorways) **can be reflected in the smoothing process.**

Location codes tend to be smaller in urban regions and larger in rural areas, so adjacency-based smoothing can sometimes handle urban and rural differences more appropriately for non-weather-related perils.

It is very rare for a very urban postcode to be adjacent to a very rural postcode and so, using this method, postcodes will be influenced primarily by other postcodes that are similar to them in terms of risk. Compare this with the distance-based methods where an urban postcode would be more likely to be influenced by a rural postcode.

Degree of smoothing

Employing too low a level of spatial smoothing would mean that near or neighbouring location codes have little influence on the location code in question. This can result in some of the random noise element being captured together with the true underlying residual variation. This causes distortions and reduces the predictiveness of the model.

Conversely, employing too high a level of spatial smoothing can result in the blurring of experience so that some of the true underlying residual variation is lost, again causing distortions.

Appropriate diagnostics (*eg* based on residual analyses) **should therefore be used to assess the level of smoothing required.**

Vehicle classification techniques

Vehicle classification may be an important input to multivariate analysis. We describe the UK system as one example.

The current system of vehicle grouping devised by the ABI has been around in various forms for many years. It classifies vehicles into one of 50 groups based on similarity of characteristics. The grouping of a particular vehicle is decided by a board who meet once a month to consider newly registered vehicles.

The factors used to establish the groupings are:

- **damage and parts costs**
- **repair times**
- **new car values**
- **body shells**
- **performance**
- **car security.**

Some of the factors used to establish the groupings will consider the likelihood of an incident (*eg* accident, theft) happening, while others look at the cost of repairing or replacing a vehicle given that an incident has occurred. So there are both frequency and severity considerations built in to the assessment.

These groupings can provide a useful indication of the likely relative cost of insurance when comparing one vehicle with another. However, insurers are not required to follow the ratings when setting premiums.

Many insurers assess the effect of vehicle on risk by using the ABI vehicle group as a starting basis for categorising vehicles, and then using additional adjustments (either via additional factors or via adjustments to the categorisation) based on their own claims experience.

An insurer that has a lot of past data relating to a particular make of car (perhaps because they underwrite an affinity scheme for a major car manufacturer) might be able to refine the ABI vehicle groups to make them more relevant to their expected future claims experience.

Question 17.3

List some additional vehicle-related factors that could be used to adjust the ABI vehicle grouping for a particular make and model of car.

3 Forms of models

To build a model that accurately describes the response data, the process for generating the response should be considered.

For a claims process there are likely to be several claim types or perils that may cause a claim, and the factors appropriate to each will differ.

In addition there may be benefit in considering the claim event itself (frequency) separately from the resulting size of claim. This is because different factors affect these two things in different ways, and the large variability in the size of claim may mask some of the patterns that could otherwise be identified accurately by modelling frequency in isolation.

It is very common when using GLMs or other multivariate models to analyse frequency separately to severity.

3.1 Claim frequency

By considering claim frequency in isolation from the claim size, the claim number may be well modelled by a Poisson process. A log link function is normally used since this results in a multiplicative model structure of factors, and these have been confirmed in practice as being the best reflection of the relationship between the variables.

Consider, for example, the motor collision peril. One factor in the model **will be the age of driver, as younger drivers with less experience tend to have more claims. Another factor will be vehicle group (or a measure of vehicle performance): a high performance vehicle will probably generate more claims.**

Intuitively, one would expect that the effect of putting the inexperienced driver in the high performance vehicle compounds both these effects in a multiplicative fashion, and the multiplicative effect of these and other factors can be demonstrated empirically by comparing the predictiveness of differing model forms (in particular by using the "Box-Cox" link function).

In this example, we expect a multiplicative effect when we combine these two factors, rather than an additive effect, and this is why a model with a log link is the most appropriate.

In fact, in practice, the combination of an inexperienced driver and a high performance vehicle is likely to present an even higher risk than that suggested by multiplying these two factor relativities together, and so we might also need to include an interaction factor. This would also act in a multiplicative manner.

Therefore, the typical form is a Poisson error function with a log link. The length of time the policy has been on risk acts as the exposure and becomes the weights within the model. No offset is required, although one can be used if, for example, the NCD levels are fixed by the policy wording. The scale parameter can be assumed equal to 1, or can be fitted.

Weights, offsets and scale parameters were all covered in more detail in Chapter 16.

Alternatively, the number of claims can be the *y*-variate, with each exposure observation having a weight of one, and with the log of the exposure being the offset term. In the particular case of the Poisson multiplicative model, this model form is identical.

3.2 *Claim severity*

Having modelled the claims frequencies in the model, another model is now needed to predict the claim size given a claim. Typically claim amounts are modelled with a Gamma error term, and, for the same reason as claim frequencies, with a log link function, which assumes that factors have a multiplicative effect on risk.

Because the Gamma distribution does not allow zero responses, zero-sized claims are normally removed from both the frequency and claims size models, and sometimes the frequency of such claims is modelled as a separate claim type for the purposes of attributing an element of expenses to the risk premium at a later stage.

The non-zero claim numbers act as the weights for this model.

The nil claims will normally be removed from the data prior to it being loaded into the modelling software. The overall claims amounts will be the same as if the nil claims hadn't been removed, but the balance between the frequency and the severity data will be different. If nil claims are removed then the frequency will appear lower but the average claims cost will be higher. The important thing to remember is that the treatment of nil claims must be consistent between the frequency and severity data.

3.3 Propensity

The word *propensity* means a tendency towards a particular way of behaving. In insurance, we would talk about, for example, the propensity for policyholders to renew their policies or the propensity for policyholders to make a claim.

Propensity to claim is binary in nature and is modelled using a binomial error distribution.

With a binary response, there will only be two possible outcomes. So, in the case of a claim, an individual policyholder either claims on his policy or he doesn't. There can be no in-between.

A multiplicative model is usually used for similar reasons to frequency and severity. However, we need the prediction to be in the range [0,1]. This is achieved using the logit link function shown in the table below.

However, because a prediction in the range [0,1] is required rather than $(0,\infty)$, the logit link function is used, and

$$\frac{e^{X.\beta}}{1+e^{X.\beta}}$$

returns a value in the range $(0,1)$.

(The logit function is a combination of logit link and binomial error, commonly referred to as a logistic model.)

The logit function is defined by the equation:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right) = \log p - \log(1-p)$$

So using the logit link function means:

$$\eta_i = \text{logit}(\mu_i)$$

and:

$$\mu_i = \frac{e^{\eta_i}}{1+e^{\eta_i}}$$

Question 17.4

Show that if $\eta_i = \text{logit}(\mu_i)$, then $\mu_i = \dfrac{e^{\eta_i}}{1+e^{\eta_i}}$.

Similar binary response variables include policy renewal or quoted policy acceptance.

A summary of typical model forms is set out in the below table.

Model	Error distribution	Link function	Scale parameter φ	Variance function $V(\mu)$	Weights ω_i	Offset ξ_i
Claim frequency	Poisson	$\ln(y)$	1	μ	Exposure	0
Claim numbers	Poisson	$\ln(y)$	1	μ	1	$\ln(\text{Exposure})$
Claim severity	Gamma	$\ln(y)$	Estimated	μ^2	Claim numbers	0
Total claim cost	Tweedie	$\ln(y)$	Estimated	$\mu^{1.5}$	Exposure	0
Propensity	Binomial	$\ln\left(\dfrac{y}{1-y}\right)$	1	$\mu(1-\mu)$	1	0

4 Initial analyses

This section describes typical initial analyses, which are typically performed prior to multivariate modelling. These include:

- one-way analyses
- two-way analyses
- correlation analyses
- distribution analyses.

Although GLMs are a multivariate method, there is generally benefit in reviewing some one-way and two-way analyses of the raw data prior to multivariate modelling.

4.1 One-way analyses

Firstly, the one-way distribution of exposure and claims across levels of each raw variable will indicate whether a variable contains enough information to be included in any models. (For example, if 99.5% of a variable's exposures are in one level, it may not be suitable for modelling.)

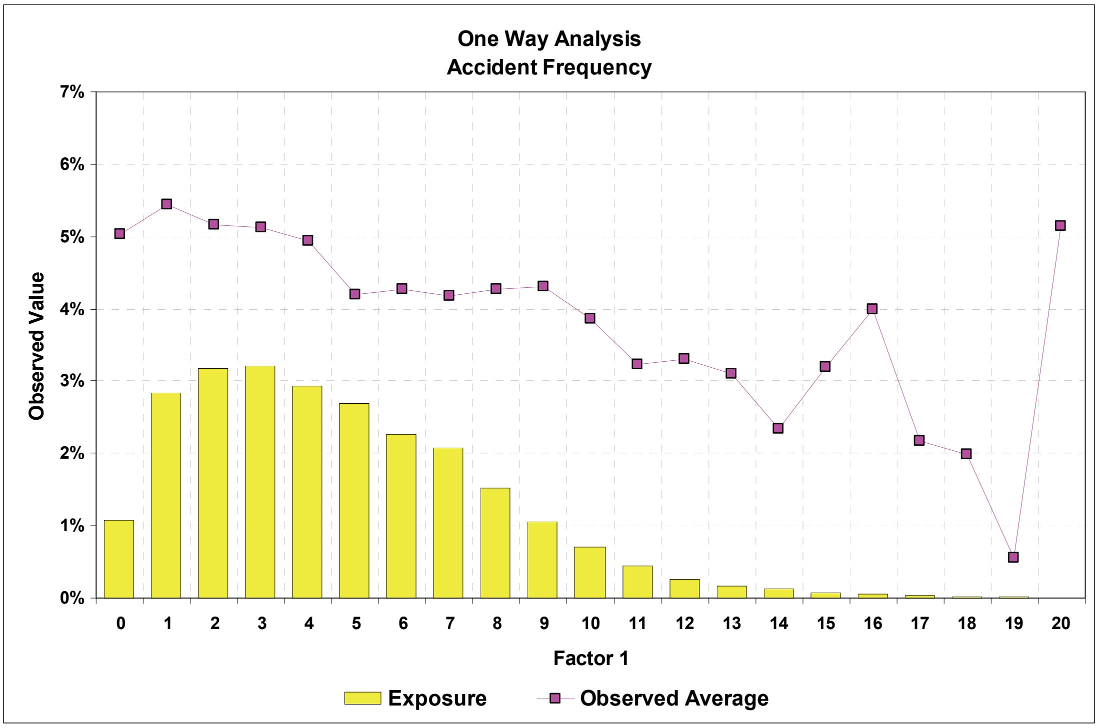
Similarly, if there is a high proportion of policies with an unknown level of a factor then we may decide (depending on what the factor is) that it will be of limited use for modelling.

Secondly, assuming there is some viable distribution by levels of the factor, consideration needs to be given to any individual levels containing very low exposure and claim count. If these levels are not ultimately combined with other levels, the GLM maximum likelihood algorithm may not converge. (If a factor level has zero claims and a multiplicative model is being fitted, the theoretically correct multiplier for that level will be close to zero, and the parameter estimate corresponding to the log of that multiplier may be so large and negative that the numerical algorithm seeking the maximum likelihood will not converge.)

In this situation, we might choose to combine the low exposure levels with another level of the same factor prior to loading the data into the modelling software. However, if we are not sure how best to combine these levels, we could test models with different groupings within the modelling software and choose the most appropriate at that stage.

In addition to investigating the exposure and claim distribution, a query of one-way statistics (for example, frequency, severity, loss ratio and pure premium) will give a preliminary indication of the effect of each factor.

The graph below shows a one-way analysis of motor accident frequency for a factor that has 20 levels, which could relate to vehicle age. Younger (higher claiming) drivers tend to drive older cars and therefore the one-way analysis is likely to overstate the true relativities for the older cars. However, this analysis does at least give an idea of the pattern we might expect from the GLM.

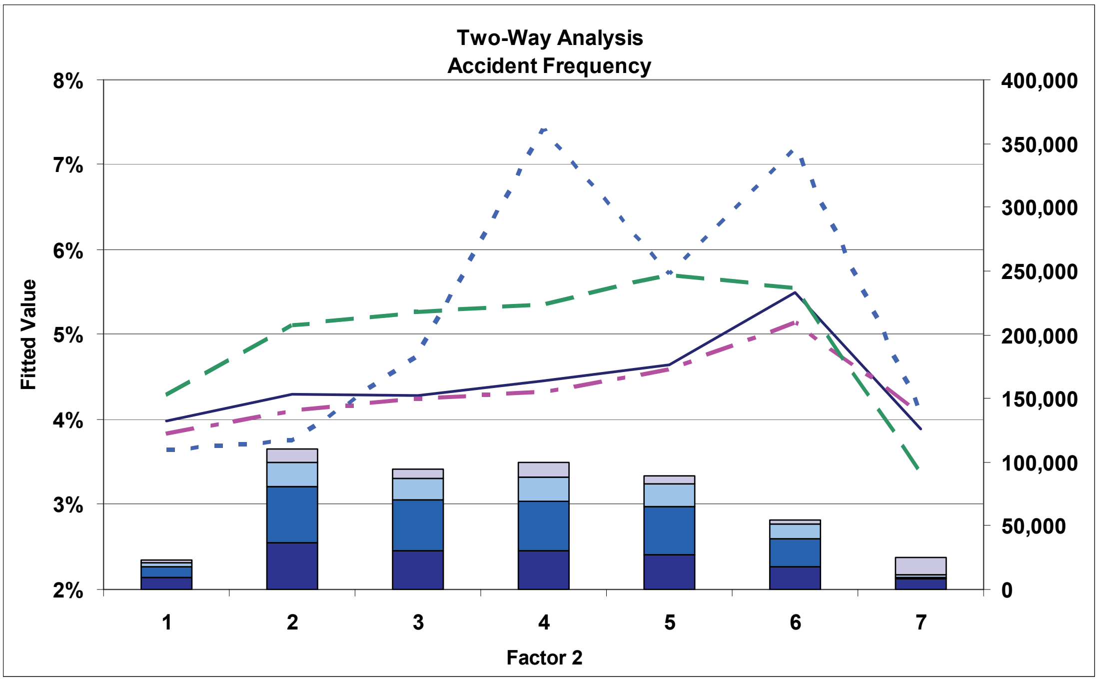


4.2 Two-way analyses

A two-way analysis considers key statistics summarised by each combination of a pair of factors. It can be represented graphically with multiple lines and stacked exposure bars as shown below.

In the graph below we have two rating factors. Factor 1 has four levels (shown as the stacks in each bar) and factor 2 has seven levels (shown on the x-axis). The dashed lines show the claims. There are four claims lines**, corresponding to the four different levels of factor 1. Hence we can see exposure and claims for each of the 4 x 7 = 28 combinations of the two rating factors.**

Two-way analyses can be particularly useful where we think there is some correlation between levels of two factors. These might be used to help us gain a better understanding of our data prior to performing a GLM.



Correlation analyses

Correlations occur when there is a relationship between the distribution of exposure between levels of two or more factors.

Although not used directly in the GLM process, an understanding of the correlations within a portfolio is helpful when interpreting the results of a GLM. In particular, it can explain why the multivariate results for a particular factor differ from the univariate results, and can indicate which factors may be affected by the removal or inclusion of any other factor in the GLM.

Recall, from Chapter 16, that a categorical factor is a factor to be used for modelling where the values of each level are distinct and often cannot be given any natural ordering or score.

One commonly-used correlation statistic for categorical factors is Cramer's V statistic, which is defined as:

$$\sqrt{\frac{\displaystyle\sum_{i,j}\frac{(n_{ij}-e_{ij})^2}{e_{ij}}}{\min((a-1),(b-1)).n}}$$

where:

- $a =$ **number of levels of factor one**
- $b =$ **number of levels of factor two**
- $n_{ij} =$ **amount of the exposure measure for the i th level of factor one and j th level of factor two**
- $n = \sum_i \sum_j n_{ij}$
- $e_{ij} = \dfrac{\sum_i n_{ij} \sum_j n_{ij}}{n}$

Note that e_{ij} represents the expected amount of exposure in the $i\,th$ level of factor one and $j\,th$ level of factor two. Hence the term:

$$\sum_{i,j}\frac{(n_{ij}-e_{ij})^2}{e_{ij}}$$

in the numerator of Cramer's V statistic is none other than the χ^2 test statistic.

Cramer's V statistic takes values between 0 and 1. A value of 0 means that knowledge of one of the two factors gives no knowledge of the value of the other. A value of 1 means that knowledge of one of the factors allows the value of the other factor to be deduced.

Categorical variables do not have a mean so we cannot calculate a covariance between two categorical variables. Instead, we measure the level of dependency between them by looking at Cramer's V statistic, a measure based on the number of risks in each cell of a frequency count table. Two such tables are shown on the next page.

The two tables below show possible two-way exposure distributions of two categorical factors, each with only two levels, A and B, expressed as either rows or columns. The top table shows a Cramer's V statistic of 0, and the bottom table gives an example of a Cramer's V of 1.

	A	B
A	**100**	**100**
B	**100**	**100**

	A	B
A	**100**	**0**
B	**0**	**100**

An example of the way in which this statistic can be considered for a range of factors is shown in the table below.

Factor (#Levels)	Class of Use (3)	Convictions (2)	Driving Experience (5)	Policyholder Sex (2)	Rating Area (18)	Vehicle Category (20)	Years Insured (12)
Class of Use (3)	0	0	0	0	0	0	0
Convictions (2)	0.026	0	0	0	0	0	0
Driving Experience (5)	0.105	0.098	0	0	0	0	0
Policyholder Sex (2)	0.04	-0.052	0.177	0	0	0	0
Rating Area (18)	0.029	0.023	0.041	0.027	0	0	0
Vehicle Category (20)	0.068	0.067	0.185	0.24	0.023	0	0
Years Insured (12)	0.033	0.047	0.124	0.032	0.015	0.026	0
Policyholder Age (42)	0.099	0.093	0.526	0.119	0.042	0.094	0.089
NCD (6)	0.078	0.017	0.264	0.104	0.03	0.145	0.165
Protected NCD (2)	0.065	0.025	0.516	0.082	0.077	0.205	0.119
Driving Restriction (5)	0.053	0.056	0.165	0.055	0.05	0.079	0.077
Vehicle Age (22)	0.068	0.038	0.029	0.037	0.024	0.05	0.035
Vehicle Value (15)	0.09	0.047	0.103	0.079	0.025	0.111	0.035
Voluntary Excess (5)	0.004	0.018	0.032	0.071	0.03	0.047	0.107

The entries within this table are the Cramer's V statistics. You can see that there is quite a high correlation between policyholder age and driving experience, which we would expect given that younger drivers can only have a low level of driving experience (measured as the number of years since they passed their driving test). Conversely, there is only a small correlation between rating area and the number of years insured, and indeed we wouldn't expect drivers in different rating areas to be significantly different to each other in terms of the length of time that insurance has been held for.

4.3 Distribution analyses

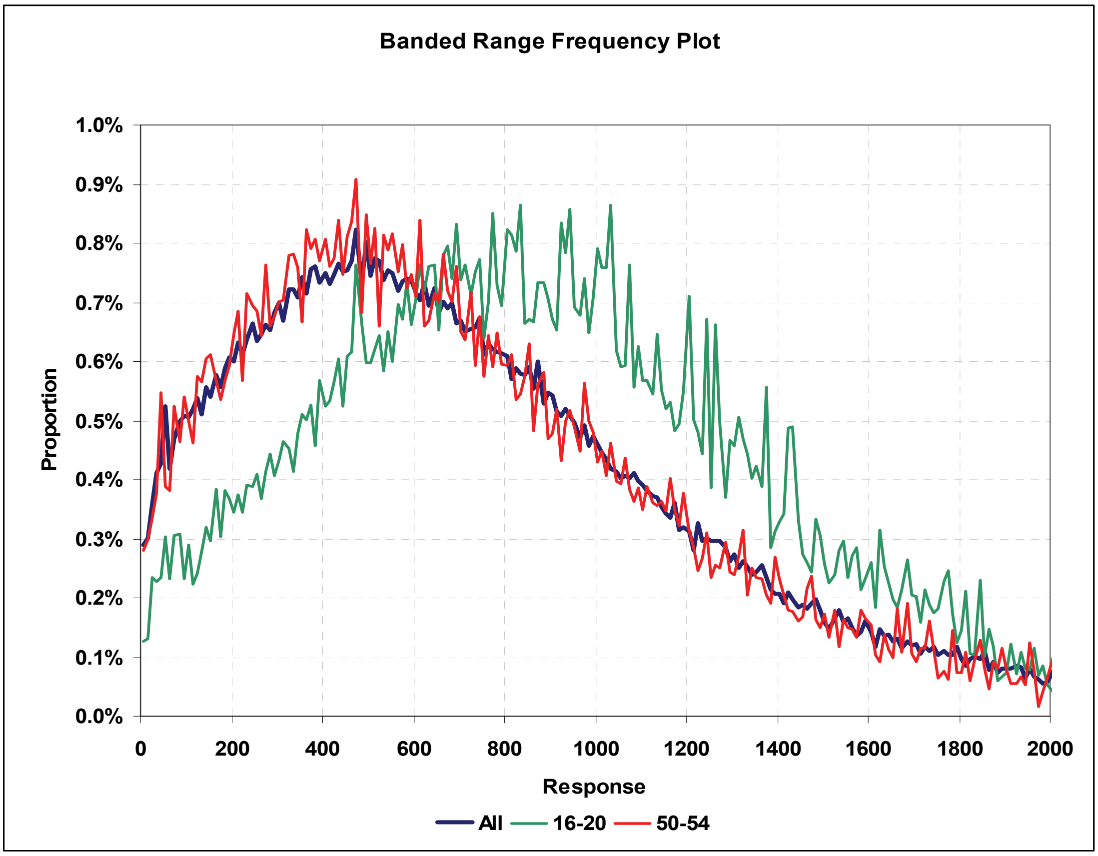
One helpful preliminary analysis is to consider the distribution of key data items for the purpose of identifying any unusual features or data problems that should be investigated prior to modelling. Mainly this concerns the distribution of claim amounts (that is, number of claim counts by average claim size), which are examined in order to identify features such as unusually large claims and distortions resulting from average reserves placed on newly reported claims.

This type of analysis can also highlight data problems or issues with the exposure data. For example, if all the exposure for a particular factor has ended up in just one level of that factor then it may be that there is an error in the code used to split that factor.

Understanding of the data is enhanced by investigating how the response distribution varies by different levels of a factor.

So, if we have data for a claim frequency model, we would look closely at the distribution of frequency by the different levels of each factor.

An example is shown below where the "50-54" line represents middle-aged driver responses and the "16-20" young drivers. The "All" line is the total of the dataset.



The "16-20" line is the slightly lighter line with its peak to the right of the other lines.

You will notice that the "All" line (the darker line) in the graph above is the smoothest line because it is based on a larger volume of data than the other two lines.

Distribution analyses can also highlight specific anomalies that might require addressing prior to modelling. For example, if many new claims have a standard average reserve allocated to them, it might be appropriate to adjust the amount of such an average reserve if it was felt that the average level was systematically below or above the ultimate average claims cost.

Remember that, for pricing, we want a best estimate of the outstanding claims amounts rather than a prudent or standard average estimate, so that we can calculate future premiums that don't contain implicit margins. Distribution analyses can therefore act as a check that this is what we've got.

5 Claim type models

Different claim types can be affected by different rating factors in different ways, and therefore (assuming appropriate volumes of data are available) a more accurate assessment of risk can be constructed by modelling different claim types separately.

Modelling different claim types separately is also helpful when considering different products that have different coverages corresponding to some of the claim types being modelled. Typical claim types modelled in the UK for motor and household insurance are given below:

Motor

Typical claim types are:

- **accidental damage**
- **third party property damage**
- **third party bodily injury**
- **fire**
- **theft**
- **windscreen.**

The low incidence of fire claims can lead to this claim type being combined with theft claims for convenience.

If there is a sufficient quantity of data for credibility and the data is recorded, it might also be possible to model theft *of* vehicles separately to theft *from* vehicles. Theft of vehicles is often going to result in a total loss payment (if the vehicle is not recovered) and older cars, with lower levels of security, will be more prone to this. Theft from vehicles will often result in much smaller claim payments (depending on what is stolen and on any limits of cover) and this might happen more equally across all ages of vehicle.

Bodily injury is increasing in importance as a proportion of the total losses. It has a long term to settlement, and a large variance in claims sizes. Hence this claim type is often treated with special care, often separating out the largest claims or capping the claim amount, and spreading these extra amounts in a more general way.

These extra claim amounts (above the cap) can be spread equally across all policies, perhaps as a percentage loading, or alternatively we may want to target the loading towards those groups of policyholders for whom the bodily injury claims tend to be the highest.

Household (separately for buildings and contents sections)

Typical claim types are:

- **accidental damage**
- **fire**
- **theft**
- **burst pipes / escape of water**
- **flood**
- **storm**
- **subsidence**
- **personal possessions**
- **liability.**

Increasingly, insurers are starting to split the flood peril between coastal flood and river flood so that they can model each separately.

6 Model combining

Once we have analysed the frequency and severity effects for each different type of claim or peril, we need to combine all these models together again if we want to calculate a total risk premium.

Fitting GLMs separately for frequency and severity experience can provide a better understanding of the way in which factors affect the cost of claims. This more easily allows the identification and removal (via smoothing) of certain random effects from one element of the experience. Ultimately, however, these underlying models generally need to be combined to give an indication of loss cost – or "risk premium" – relativities.

In the case of multiplicative models for a single claim type, the calculation is straightforward. The frequency multipliers for each factor can simply be multiplied by the severity multipliers for the same factors (which is analogous to adding the parameter estimates when using a log link function).

So, given frequency and severity models for each claim type, it is very easy to calculate a theoretical risk premium for any particular risk for each of those claim types. Then, for each claim type separately, we could add on the relevant loadings (expenses, commission, profit, *etc*) to give an office premium for each claim type. Adding across all the claim types covered by a particular policy would give an overall office premium for the policy.

Note that this method may be computationally intensive at the point of sale because, for a class of business with n claim types, the rating algorithm would need to go through n sets of frequency and severity multipliers and add on the loadings n times in order to calculate an overall premium.

Certain market conditions may warrant the development of a single theoretical risk premium model, even if different types of claim have been modelled separately. An example is the aggregation of private motor models by peril into a single rating algorithm at point of sale. Even if such an aggregation is not required, it can be helpful to understand the overall multivariate effect that a factor has on the overall level of claims.

If we have a single theoretical risk premium model (*ie* a combination of all the component risk premium models) then the point of sale rating algorithm becomes quicker and easier as it only needs to go through the calculation once. It is also easier (usually) to quantify the loadings for expenses *etc* at the overall policy level than at a claim type level.

However, there is more work to be done prior to the point of sale under this method, and this can be complex.

The derivation of a single model in this situation is not as straightforward since there is no direct way of combining the model results for the underlying claim types into a single overall cost of claims model.

In this situation, however, it is possible to approximate the overall effect of rating factors on the total cost of claims by using a further GLM to calculate a weighted average of the GLMs for each of the underlying frequency and severity models for each of the claim types.

Specifically this can be done by:

- **selecting a dataset that most accurately reflects the likely future mix of business**
- **calculating an expected claim frequency and severity by claim type for each record in the data**
- **combining these fitted values, for each record, to derive the expected cost of claims (according to the individual GLMs) for each record**
- **fitting a further GLM to this total expected cost of claims, with this final GLM containing the union of all factors (and interactions) in all of the underlying models.**

An illustrative example is shown on the next page.

Example

The first table represents the intercepts and multipliers from underlying frequency and severity models for claim types 1 and 2. The second table shows the calculation of the total risk premium, based on the underlying models, for the first four records in the data. The additional GLM is fitted to this last column in this dataset in order have a single theoretical risk premium model.

		Claim type 1		Claim type 2	
		Frequency	**Severity**	**Frequency**	**Severity**
Intercept		**0.32**	**1,000**	**0.12**	**4,860**
Sex:	**Male**	**1.00**	**1.00**	**1.00**	**1.00**
	Female	**0.75**	**1.20**	**0.67**	**0.90**
Area:	**Town**	**1.00**	**1.00**	**1.00**	**1.00**
	Country	**1.25**	**0.72**	**0.75**	**0.83**

Policy	Sex	Area	Fitted freq 1	Fitted sev 1	Fitted RP 1	Fitted freq 2	Fitted sev 2	Fitted RP 2	Total RP
82155654	M	T	32%	1000	320.00	12.00%	4860.00	583.20	903.20
82168746	F	T	24%	1200	288.00	8.04%	4374.00	351.67	639.67
82179481	M	C	40%	720	288.00	9.00%	4033.80	363.04	651.04
82186845	F	C	30%	864	259.20	6.03%	3630.42	218.91	478.11
…………	…	…….	…….	…….	…….	…….	…….	…….	…….

The fitted frequency is the frequency predicted by the model. This is calculated as the intercept term multiplied by the relativities for sex and area. So for the second policy in the table above, the fitted frequency for claim type 1 is $0.32 \times 0.75 \times 1.00 = 0.24$.

Similarly, the fitted (predicted) severity for the fourth policy in the table for claim type 2 is $4,860 \times 0.90 \times 0.83 = 3,630.42$.

The fitted risk premium for each claim type is the fitted frequency multiplied by the fitted severity. The total risk premium is the sum of the risk premiums for the two claim types.

Non-proportional expenses

Non-proportional expenses are those that are not a constant proportion for every policy. A fixed policy fee of £50 would be considered to be non-proportional because it will have a different relative impact on a policy with a premium of £200 than on one with a premium of £1,500.

In addition to combining frequency and severity across multiple claim types, the technique of fitting an overall GLM to fitted values of other GLMs can be used to incorporate non-proportional expense elements into the modelled relativities. For example, a constant pound amount could be added to each observation's expected risk premium and then a GLM re-fitted to this new field. The resulting "flattened" risk premium relativities will prevent high risk factor levels from being excessively loaded for expenses.

We can think of young drivers as being a high risk group within the "age" factor for a motor policy – the frequency and severity (and therefore risk premium) relativities are likely to be very high compared to other age groups. If we were to add a fixed percentage expense loading across all policies, this would have the effect of further increasing the differential between young and older drivers and would mean that the young drivers were subsidising the rest in terms of expenses.

A simple example will demonstrate how the premium relativities are "flattened" when a fixed amount is loaded onto the risk premium. The first table shows the situation where expenses are loaded as a fixed percentage (20%) for each policy.

	Risk premium (£)	Expenses (% of premium)	Expenses expressed as a £ amount	Office premium (£)
Young	1,500	20%	300	1,800
Other	300	20%	60	360

It is clear that the young drivers are contributing a lot more to expenses than the other drivers are. The implied risk premium relativities are 5:1 and the implied office premium relativities have remained at 5:1.

The next table shows the same risk premium relativities but with expenses loaded as a fixed pound amount for each policy.

	Risk premium (£)	Expenses (fixed £ amount)	Office premium (£)
Young	1,500	100	1,600
Other	300	100	400

With this treatment of expenses, all ages of driver are contributing the same amount but this is now a bigger proportion of the premium for the other drivers. The implied office premium relativities have reduced to 4:1. This is what is meant by a flattening of the relativities. In fact, if you imagine that the office premium relativities are plotted on a graph, you will see that this line is flatter than the one where the expenses were expressed as a fixed percentage.

Alternatively, the amount added to each observation's expected risk premium could be designed to vary according to the results of a separate retention study. This would allow risks with a high propensity to lapse to receive a higher proportion of fixed expense than those risks with a low propensity to lapse. As above, a further GLM is fitted to the sum of the expected risk premium and a (lapse-dependent) expense load.

7 Model validation

From a practical point of view, the whole reason for analysing claims data for premium rating is to get a model that can be used to predict future claims experience so that future premiums can be set accordingly. An over-fitted model, for example, might fit the past data (including some of the random noise) very well but be of little use for projecting into the future. It is therefore very important to check that our model is appropriate.

Whilst many individual aspects of the model selection can be tested using specific formal statistical tests during the modelling process, it can also be helpful to perform an overall validation of the effectiveness of a model by testing its predictiveness on out-of-sample experience.

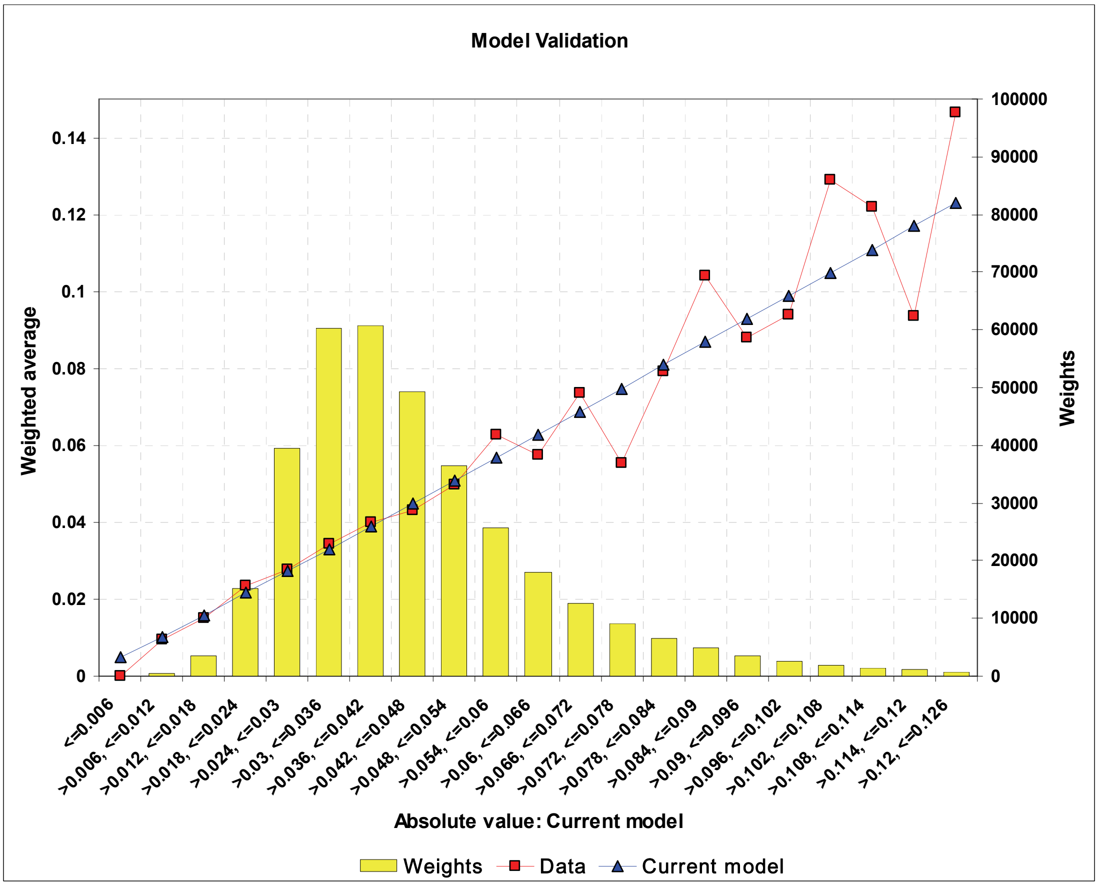
We met various methods of testing our model selection in Chapter 16.

Validation samples of, say, 20% of the total data can be withheld from the modelling process. A range of tests can then be undertaken on this validation sample comparing actual experience with that predicted by the selected model.

If we choose to withhold a random sample in this way, we are implicitly assuming that the future business, policy conditions, *etc* will be similar to those in the past data.

The validation data will need to contain both the actual claims data, and the amounts predicted by the model we want to test. There are various ways of displaying the relationship between the actual data and the predicted amounts.

One possible method of validating a model is to produce graphs such as the one below.



The above graph investigates actual versus predicted claims, grouped by predicted claim amount (bands on the x-axis). The bars show the exposure we have in each of these bands.

Unsurprisingly, we can see that, where we would expect a very low or very high claim cost (on the far left and right of the graph), then we have comparatively little exposure. This represents the more extreme insureds (for example, a young driver in a very powerful car).

The "triangle" line is the predicted values from the model. The "blocked" line represents the actual claims in these blocks. If the model were perfect, these lines would overlay each other. However, no model is perfect. Systematic differences can indicate poor model fitting; for example, if the line was consistently above or below the theoretical line. As can be seen, the fit is best where we have the most exposure.

A perfect fit of the model to the past data would not be appropriate here anyway as the pattern of random noise in the future would not be expected to follow precisely that of the past. In this example, the model appears to fit the validation data very well.

The distribution of exposure on the graph just provides information that is helpful when interpreting the results. Departures of actual from expected are more concerning when such departures apply to a significant proportion of the portfolio.

Students should note the reliance placed on internal model validation within the Solvency II directives.

Solvency II is a fundamental review of the capital adequacy regime for European insurers and reinsurers, planned to take effect in the next couple of years.

Insurers can choose to determine their capital requirements under Solvency II using an internal model, subject to this being approved by regulators. Demonstrating that the internal model is appropriate through validation is one of the criteria that must be met in order for the model to be approved. This will require its assumptions and output to be regularly checked for reasonableness against actual experience. This is covered further in Subject SA3.

7.1 Lift curves

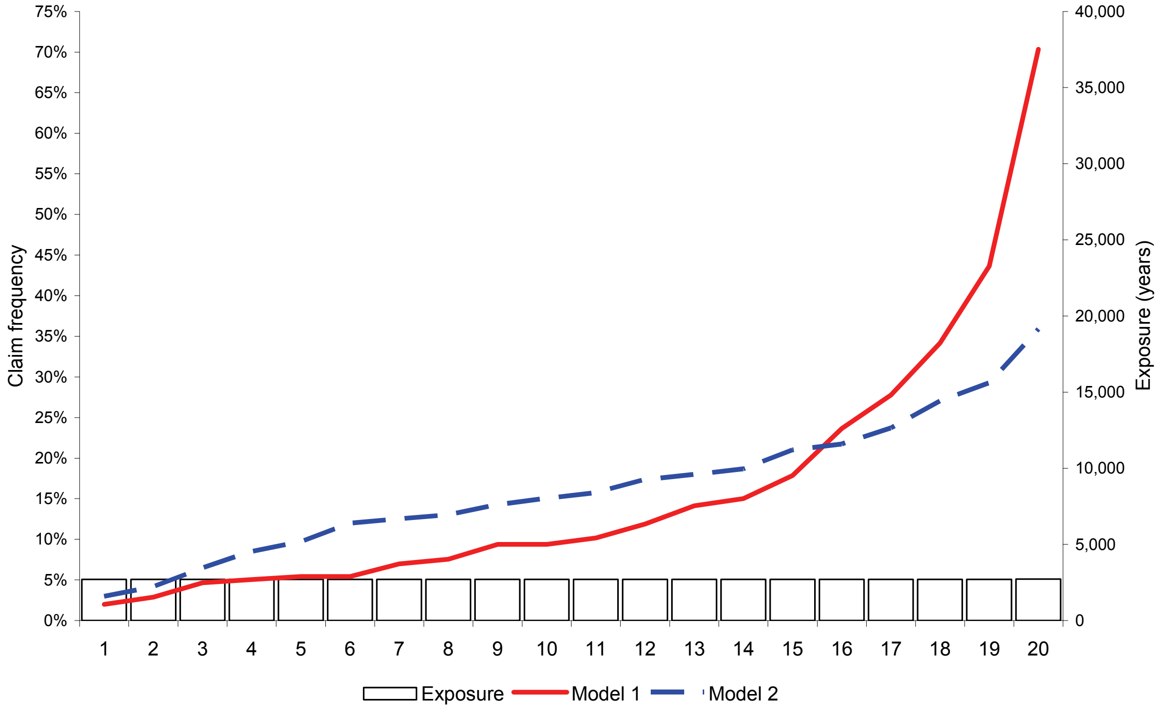
Another related way of assessing the predictiveness of a model is to calculate a "lift curve" on an out-of-sample model validation dataset.

As for the example above, this method uses an out-of-sample dataset for validation. This just means that the validation data has been taken as a random sample (say 20%) from the original data available for modelling. The multivariate models would then be constructed based on the remaining data.

This method is also useful for comparing two models of different forms.

One approach is to rank all policies in the validation dataset in order of expected experience (according to the model being tested), and then to group the policies into bands of equal exposure based on this ranking. The actual experience for each group can then be calculated and displayed as a curve. The steeper the curve, the more effective the model is at distinguishing between high and low experience because there is a greater differentiation between the good and bad risks.

The below graph (on the next page) **shows an example of such a comparison between two claim frequency models. It is clear from the graph that Model 1 is the more predictive of the two** because it is steeper.



Model 1 in this graph is shown by the solid line, while the dashed line relates to Model 2.

So, in this case, the validation data has been ranked by the expected (modelled) claim frequency and then split into 20 groups, each containing 5% of the total exposure. The lines on the graph show the actual claim frequency for each of these groups.

The difference between the frequency of the lowest group and the highest group is the lift. When comparing more than one model, the lift tells us which model is the most predictive of those tested.

Lift curves can be applied to claim severity or to burning cost as well as to frequency.

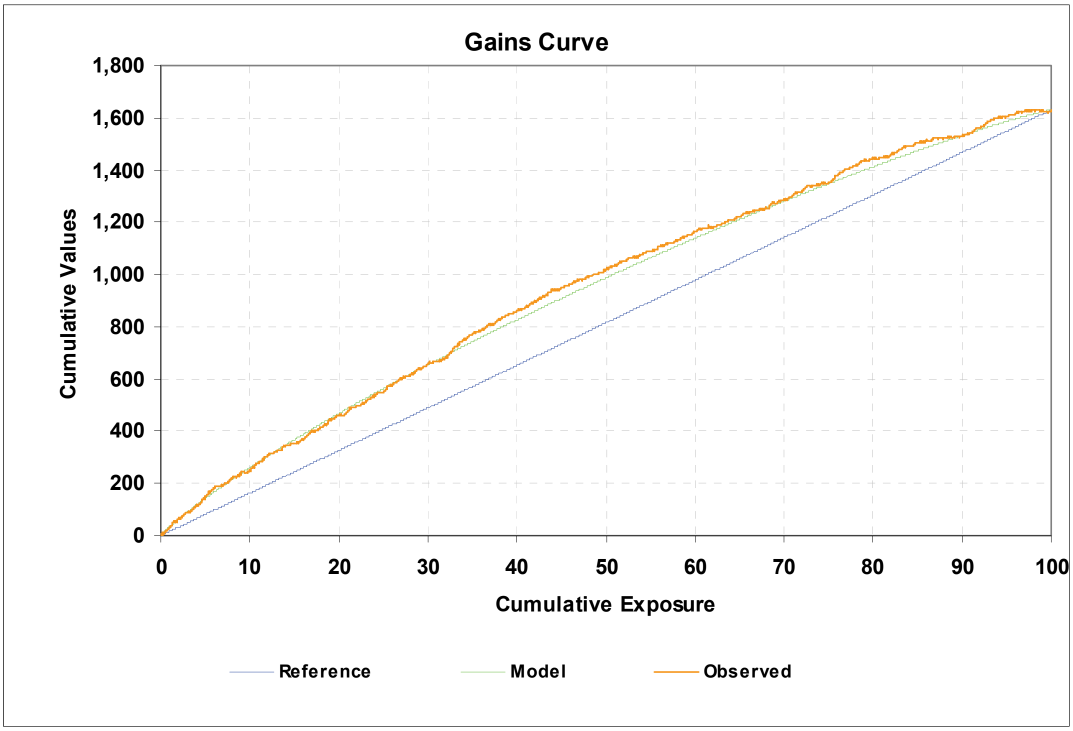
7.2 *Gains curve*

Another useful way of presenting the final fitted values from the model, and comparing them to the observed values, is the gains curve.

With this method the data are sorted high to low according to the fitted model values, and then the chart shows the cumulative values from the fitted model and the cumulative **observed values from the data. A statistical measure for the lift produced by the model is called the Gini coefficient. This can be thought of as the area enclosed by the model curve and the diagonal line as a ratio of the triangle above the diagonal.**

In this method, the data is not grouped into bands.

Because it is cumulative values that are plotted, the curve will always start at zero and end at the value relating to the sum of all the fitted values.



In the graph above, which forms part of the Core Reading, the straight diagonal line is the reference line. The smooth curved line shows the cumulative values for the fitted model. The area enclosed by these two lines is used to calculate the Gini coefficient.

The Gini coefficient is a measure of statistical dispersion that can range from 0 to 1. The higher the Gini coefficient, the more predictive the model.

The third line on the graph, which is not smooth but follows a similar path to the smooth line for the fitted model, shows the cumulative observed values.

8 Implementation

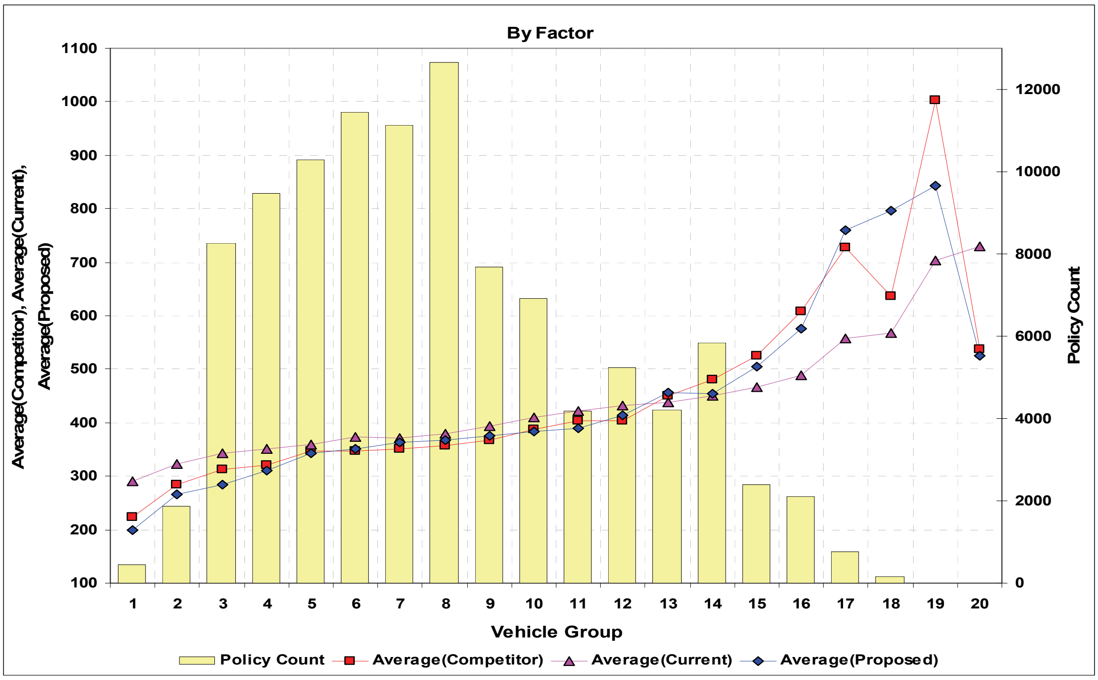
We have completed our multivariate modelling and are now happy that the theoretical model we've come up with is accurate and relevant. This can be thought of as a *ground-up* process because we have built up the proposed premium rates by putting together all the appropriate components.

But it is unlikely that we would use these theoretical rates without any further adjustments. For example, we also need to consider the impact that these rates would have in the market. This is particularly important for competitive classes of business such as personal lines motor.

Note that we need to use an *office* premium (including all loadings) rather than a *risk* premium for competitive comparisons, for consistency with our competitors' premiums.

Below is a chart showing how the impact of proposed changes may be evaluated. The *x*-axis is a single rating factor of interest, in this case vehicle group. The bars show the exposure weighting from the dataset.

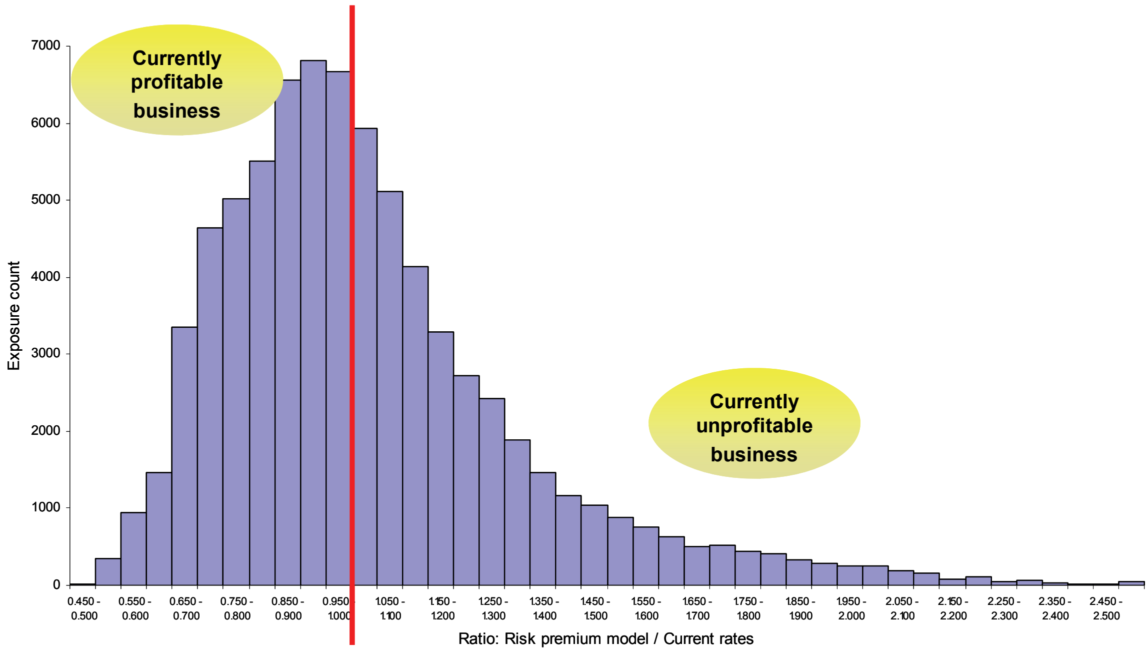
Current and proposed relativities can be compared and, in particular if estimated competitor rates are also available, these can be shown too. Competitor analysis has become increasingly important over the last few years, as new techniques have been introduced, but students should be alert to the restrictions imposed by any competition legislation. In particular, it is now possible to model the elasticity of customer demand and further to optimise prices subject to portfolio level constraints and stakeholder objectives. The reader should be aware that this example is only a high level introduction to the subject.



The results of a GLM analysis are interdependent and must be considered together. For example, while a GLM analysis might suggest that young driver relativities are too low, it may also suggest that relativities for inexperienced drivers (for example, less than two years licensed) are too high. Although the existing rating structure may be theoretically wrong, it might be the case that to a large extent these errors compensate each other. To understand the true "bottom line" difference between the existing rating structure and the theoretical claims cost, "impact" graphs such as the one below can be considered.

So we cannot restrict ourselves to looking at individual factors at this stage – we need to consider the impact on the *whole* book of business. Ideally, we would want to know the impact on the future business so, for the purposes of this impact analysis, we need to apply our model to a set of data that is representative of what we will write in the future.

Impact on portfolio of moving to theoretically correct relativities



The graph above shows the number of exposures in the existing portfolio that would experience different changes in premium if the rating structure were to move from its existing form to the theoretically correct form immediately.

The x-axis represents, as a ratio, the adjustment to current premiums that should be made to bring them in line with the theoretically correct ones. Those policies to the left of the thick vertical line should, in theory, be experiencing a reduction in premium, whereas those to the right of the line should have their premiums increased.

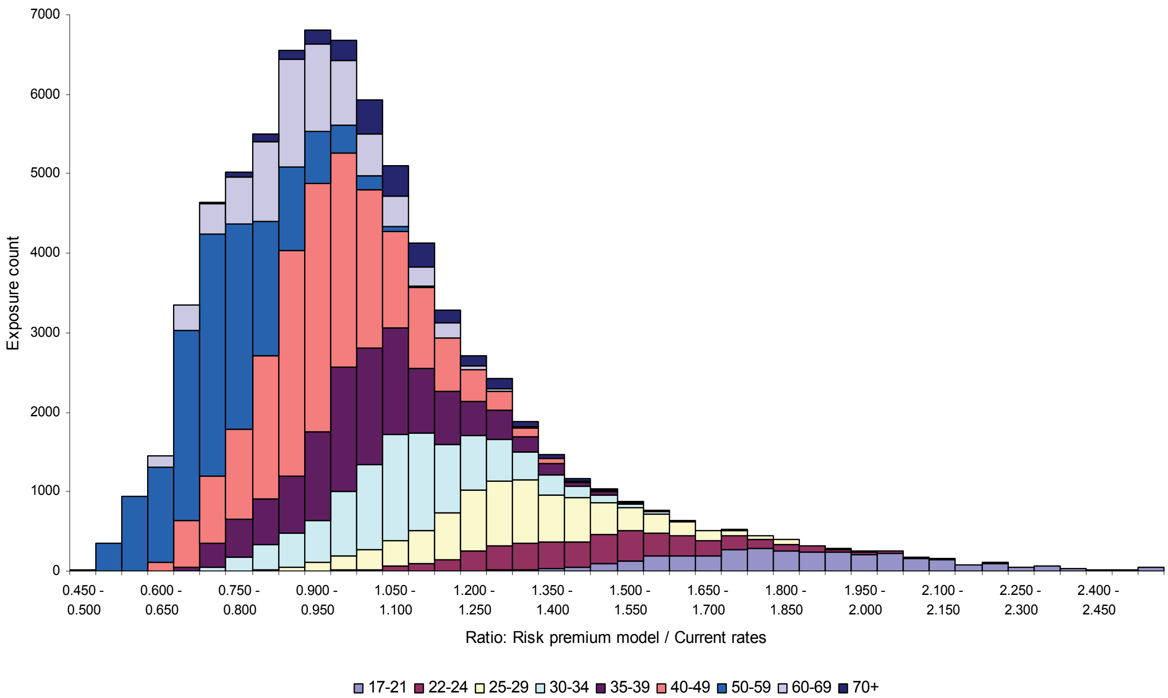
It is, of course, exceptionally unlikely that such dramatic change would be implemented in practice. The purpose of this analysis is to understand the magnitude of the existing cross-subsidies by considering the effect of all rating factors at the same time.

This graph can also be divided by levels of a particular rating factor. (Indeed one such graph can be produced for each rating factor.) This identifies which sectors of the business are currently profitable, and which are currently unprofitable, taking into account the correct theoretical model and considering the effect of all factors at the same time.

Note that we are measuring profitability here by reference to our newly-determined theoretical model as this is our most up-to-date view of the correct premium rates.

In the example below, the impact graph is segmented by age of driver (notice the shape does not change, only how the histogram is coloured).

Impact on portfolio of moving to theoretically correct relativities (segmented by age of driver)



The histogram shows the impact of all rating factor changes (not just the age of driver factor) by age of driver levels.

It is difficult to see what is going on in this graph because it is printed in black and white. The long flat band to the right of the graph relates to the 17-21 year olds; the darker band to the left and above of them relates to the 22-24 year olds; and the bands continue to move leftward and upward as the age bands increase.

It can be seen in this example that a large number of the exposures that would experience large increases in premium if the rating structure were moved immediately to the theoretically correct structure are young drivers.

Based on this graph, the 17-21 year old drivers would all experience increases to their current rates of between 135% and 245% if these new rates were implemented unadjusted.

It had already been seen from the GLM risk premium graphs (not included within this Chapter) **that young driver relativities were too low. This graph suggests there are no effects from other correlated factors that noticeably mitigate this effect; otherwise, young drivers would not be so strongly on the "unprofitable" side of the impact graph.**

An example may make interpretation of the graph above clearer.

Example

Assume the multiplicative claims model uses age, gender, marital status, territory and credit as rating factors.

Consider the following young driver profile with indicated rate change for each criterion in parentheses: age 17-21 (+60%), male (+15%), single (–5%), urban territory (+15%), high credit score (–20%).

All factors considered, the total indicated rate change for this risk profile is +61% and so this policy would contribute a count of one to the bar at 1.60-1.65.

This rate changed is calculated as:

$$1.60 \times 1.15 \times 0.95 \times 1.15 \times 0.80 = 1.61$$

There are roughly 600 total exposures in this band; roughly one-third of which correspond to drivers age 17-21.

9 End of Part 4

You have now completed Part 4 of the Subject ST8 Notes.

Review

Before looking at the Question and Answer Bank we recommend that you briefly review the key areas of Part 4, or maybe re-read the summaries at the end of Chapters 15 to 17.

Question and Answer Bank

You should now be able to answer the questions in Part 4 of the Question and Answer Bank. We recommend that you work through several of these questions now and save the remainder for use as part of your revision.

Assignments

On completing this part, you should be able to attempt the questions in Assignment X4.

Chapter 17 Summary

Data

A large range of questions is asked at the point of sale for personal lines motor and home insurance. These questions relate to the policy, the proposer and the vehicle or house. Many of these questions do not directly measure the genuine risk factors but act as proxies instead.

External data can be added, to help predict claims experience. It is necessary for linking fields to exist so that the external data can be attached to the internal data.

Multivariate models

Generalised linear models (GLMs), often with a log link function to create the form:

$$Y_i = \exp\left(\sum_{j=1}^{k} X_{ij}\beta_j\right) + \varepsilon_i = \prod_{j=1}^{k} e^{X_{ij}\beta_j} + \varepsilon_i$$

are the most common multivariate models used in personal lines pricing today. They are studied in Chapter 16.

There may be some factors (*eg* postcode or vehicle group) for which there are a large number of levels. To enable these to be included within a GLM, it is necessary to use some method of classification to produce a smaller number of groupings.

Approaches to classification include:

- spatial smoothing methods (distance-based or adjacency-based)
- vehicle classification techniques (*eg* based on ABI groups).

Forms of models

Separate models will be built for claim frequency and claim severity (average claims cost) and these will normally be done separately for each type of claim or peril.

A log link function will produce a multiplicative rating structure. Typical distributions for error term are Poisson for the frequency model and gamma for the severity model.

Claim propensity models tend to be multiplicative and have a binomial error distribution.

Initial analyses

Prior to fitting a GLM (or other multivariate model), we would normally look at one-way and two-way tables to check and familiarise ourselves with the data. Incorporating frequency, severity and loss ratio statistics will also give us an initial indication of the likely effect of each factor.

It may be helpful to analyse and understand the correlations within our portfolio and we can use Cramer's V statistic, defined as: $\sqrt{\dfrac{\sum_{i,j} \dfrac{(n_{ij}-e_{ij})^2}{e_{ij}}}{\min\left((a-1),(b-1)\right).n}}$.

Model combining

Once we have models for each peril or claim type and for both frequency and severity, we may need to combine these into a single risk premium model. This is not straightforward where there is more than one claim type.

Model validation

Validation samples can be withheld from the data used in modelling and then used to test how close the model predictions are to actual experience and therefore how accurate they are likely to be for future rates. A graph of actual *vs* predicted claims can be plotted.

Lift curves can be used to compare two models of different forms, to see which is the most predictive. Gains curves can show a comparison between fitted and observed values. The Gini coefficient can be calculated as a measure of statistical dispersion.

Implementation

Once the theoretical rates have been produced, they need to be compared with the current rates (to see what the effect would be on a particular book of business) and, if possible, with competitors' rates. There are various graphical representations that can help with this.

Chapter 17 Solutions

Solution 17.1

A driver's age will be as at a particular point in time – often the renewal date.

If we record date of birth instead, we will have much more flexibility as we can calculate the age as at any date we want.

For example, we would need the date of birth if we wanted to sell monthly renewable policies in the future. Also, if we wanted to use the data to assess the mix of business at a fixed point in time (*eg* to perform an impact analysis) then we might want to use the age as at that date rather than as at the previous renewal date.

Solution 17.2

The number of children under 16 might help in assessing the level of risk.

For example, people with children might tend to drive more carefully when their children are in the car. Alternatively, the children might cause the driver to be distracted from the road, thereby leading to a higher number of accidents.

In addition, if an accident does occur with children in the car, this could increase the amounts relating to third party bodily injury claims.

Solution 17.3

- security features (alarm / immobiliser / tracker)
- safety features (*eg* airbags)
- engine size (cc)
- top speed
- acceleration
- weight of car
- fuel type
- number of doors
- body type (*eg* cabriolet, estate, 4x4, hot hatch)
- transmission type (manual or automatic)

Solution 17.4

If:

$$\eta_i = \text{logit}(\mu_i) = \log\left(\frac{\mu_i}{1-\mu_i}\right)$$

then:

$$\frac{\mu_i}{1-\mu_i} = e^{\eta_i}$$

Multiplying by $1-\mu_i$ gives:

$$\mu_i = e^{\eta_i} - e^{\eta_i}\mu_i$$

Hence:

$$\mu_i\left(1+e^{\eta_i}\right) = e^{\eta_i}$$

and:

$$\mu_i = \frac{e^{\eta_i}}{1+e^{\eta_i}}$$

Chapter 18

Credibility theory

Syllabus objectives

(p)(i) Outline the fundamental concepts of credibility theory.

(p)(ii) Describe and compare the classical and Bayes credibility models.

(p)(iii) Describe the practical uses of credibility models in a general insurance environment.

0 Introduction

In this chapter we will revisit credibility theory, which is a technique that can be used to determine premiums or claim frequencies in general insurance. You will already have met Bayesian credibility in Subject CT6.

In Section 1 we provide a basic introduction to the motivation behind credibility theory.

Section 2 discusses classical credibility theory, and Section 3 discusses some of the models based on classical and Bayesian credibility.

In Section 5 we introduce the Bühlmann-Straub credibility model.

Sections 4, 6 and 7 discuss some of the issues surrounding the use of credibility models in practice.

1 Credibility

Credibility theory is an approach to rate setting that allows for consideration of actual experience as well as external information.

The problem with using past data to estimate future claims experience is that the past data will contain an element of randomness. It may, for example, include an unusually large loss.

The law of large numbers tells us, however, that the more data we have available, the lower the variation of the sample mean. It therefore seems logical to use as much data as possible when modelling future events. Why not, for example, use industry-wide data, based on the claims experience of all insurers in the country?

The problem is that, while this data seems more credible than just using claims information from the individual insurer (since it is based on a greater volume of data), it will not be as relevant to the specific business being modelled. For example, some of the claims may originate from risks written under very different conditions, or in very different geographic regions. They may therefore not be an appropriate predictor of claims for that particular insurer.

There are two extreme choices for deciding which dataset to use:

(i) we could choose an estimate based on the past data of the individual insurer (or even the individual policyholder) on the grounds that this is based on the most relevant data or,

(ii) we could choose an estimate based on market-wide data on the grounds that it is, in some sense, a more reliable figure.

The credibility approach to this problem is to take a weighted average of these two extreme answers.

For example, an insurance company uses past loss information from an insured or group of insureds to estimate the cost of providing future insurance coverage.

In other words, the insurer calculates its premium based on the past claims experience of the insured(s).

But, insurance losses arise from random occurrences. The average annual cost of paying insurance losses in the past few years may be a poor estimate of next year's costs. The expected accuracy of this estimate is a function of the variability in the losses. This data by itself may not be acceptable for calculating accurate insurance rates.

Rather than relying solely on recent observations, better estimates may be obtained by combining this data with other information.

For example, suppose that recent experience indicates that carpenters should be charged a rate of £5 (per £100 of payroll) for employers' liability insurance. Assume that the current rate is £10.

The rate of £5, based on recent claims experience is described as the "observation" in the formula below, while the current rate of £10 is described as "other information".

What should the new rate be? Should it be £5, £10, or somewhere in between? Credibility theory is used to weight together these two estimates.

The basic formula for calculating credibility-weighted estimates is:

$$\boldsymbol{Estimate = Z \times (Observation) + (1-Z) \times (Other\ \ information), \quad 0 \le Z \le 1}$$

The question lies in deciding what value to place on Z.

Z is called the credibility assigned to the observation. $1-Z$ is generally referred to as the complement of credibility.

If the body of observed data is large and not likely to vary much from one period to another, then Z will be closer to one. On the other hand, if the observation consists of limited data, then Z will be closer to zero and more weight will be given to other information.

In other words, the more reliable we believe our observed claims experience to be (as a predictor of future experience), the more we can weight our estimate towards that data.

The current rate of £10 in the above example is the "other information". It represents an estimate, or "prior hypothesis" of a rate to charge in the absence of the recent experience. As recent experience becomes available, then an updated estimate combining the recent experience and the prior hypothesis can be calculated.

This can be done, for example, by increasing the value of Z.

The carpenters' rate for employers' liability insurance under this model is:

$$\boldsymbol{Z \times £5 + (1-Z) \times £10}$$

In Subject CT6, a method for determining Z, known as greatest accuracy credibility or "Bayesian credibility" was introduced. In this chapter, an alternative approach for computing Z, known as the limit fluctuation approach or "classical credibility" is covered.

Bayesian credibility and its relationship to Bayesian statistics is revisited in the following example. A motivation for an alternative approach to credibility theory is also provided.

Example

In a population of motorists, the number of accidents caused by an individual motorist has a Poisson distribution with $X \mid \theta \sim Poisson(\theta)$, *ie*

$$f(x \mid \theta) = \frac{\theta^x e^{-\theta}}{x!}$$

The population of motorists is heterogeneous and it is believed to be made up of

- **"safe" motorists with $\theta = 0.2$, and**
- **"risky" motorists with $\theta = 0.4$**

where θ is the probability of an accident.

So, the number of accidents follows a Poisson distribution for each motorist, although the value of θ varies for different drivers.

For a randomly selected motorist where no further information is available, the expected number of claims in the next year is:

$$\mu = E[X] = E\left[E[X \mid \theta]\right] = E[\theta]$$
$$= 0.2 \times 0.5 + 0.4 \times 0.5$$
$$= 0.3$$

Because we don't know whether this randomly selected motorist is "safe" or "risky", an equal weighting (of 0.5) has been given to each category.

Now suppose that, for a particular motorist, we know that they have had one accident in the last year. We therefore have some prior knowledge. **The likelihood of this outcome is:**

$$f(x \mid \theta) = \frac{\theta^1 e^{-\theta}}{1!} = \theta e^{\theta}$$

This is just the probability of the motorist having 1 accident (x=1).

Using the formula for the posterior distribution, based on Bayes' Theorem as covered in Subject CT6, the posterior probability of the motorist being "safe" is:

$$P(\theta = 0.2 \mid \underline{x}) = \frac{f(\underline{x} \mid \theta = 0.2) P(\theta = 0.2)}{\left[f(\underline{x} \mid \theta = 0.2) P(\theta = 0.2) + f(\underline{x} \mid \theta = 0.4) P(\theta = 0.4)\right]}$$

$$= \frac{0.2e^{-0.2} \times 0.5}{\left[0.2e^{-0.2} \times 0.5 + 0.4e^{-0.4} \times 0.5\right]}$$

$$= 0.379$$

Since there are only two possible classifications in this example, **the posterior probability of the motorist being "risky" is $1 - 0.379 = 0.621$.**

The expected number of claims made in the next year by this motorist is thus:

$$E\left[X_{n+1} \mid \underline{x}\right] = E\left[E\left[X_{n+1} \mid \underline{x}, \theta\right]\right]$$

$$= E\left[\theta \mid \underline{x}\right]$$

$$= 0.2 \times 0.379 + 0.4 \times 0.621$$

$$= 0.324$$

Note that the expected number of claims in the next year by the motorist in question is larger than that of a randomly selected motorist as the sample data leads us to believe that the motorist is more likely to be "risky" than "safe". This estimate for the accident frequency is called the Bayesian premium and in Subject CT6, it is shown to be the estimate of the future number of claims with the smallest mean square error.

The problem with the Bayesian premium is that it requires the model distribution $f(x \mid \theta)$ and the prior distribution $\pi(\theta)$ to be completely specified. Bayesian credibility allows for a computation of the premium which only requires estimates of $var\left[m(\theta)\right]$ and $E\left[s^2(\theta)\right]$ where $m(\theta) = E\left[X \mid \theta\right]$ and $s^2(\theta) = var(X \mid \theta)$. In Subject CT6, the credibility premium is shown to be:

$$Z\overline{x} + (1 - Z)\mu$$

where

$$Z = \frac{n}{n + \frac{E\left[s^2(\theta)\right]}{var\left[m(\theta)\right]}}$$

For the example in question,

$$n = 1$$

$$E\left[s^2(\theta)\right] = E\left[var(X \mid \theta)\right] = E[\theta] = 0.3$$

$$\begin{aligned} var\left[m(\theta)\right] &= var\left[E[X \mid \theta]\right] = var[\theta] \\ &= E\left[\theta^2\right] - \left(E[\theta]\right)^2 \\ &= \left(0.2^2 \times 0.5\right) + \left(0.4^2 \times 0.5\right) - 0.3^2 \\ &= 0.01 \end{aligned}$$

The credibility assigned is thus $Z = \dfrac{1}{1+\frac{0.3}{0.01}} = \dfrac{1}{31}$

This is the credibility associated with the driver's own experience *ie* the one claim last year.

The estimate using Bayesian credibility for the expected number of claims made by the driver in question in the next year is:

$$\left(\frac{1}{31} \times 1\right) + \left(\frac{30}{31} \times 0.3\right) = 0.323$$

Although we have used the model and prior distributions for computing $E\left[s^2(\theta)\right]$ and $var\left[m(\theta)\right]$, a non-parametric approach has been covered in terms of the two Empirical Bayes Credibility Models in Subject CT6.

Recall from Subject CT6 that Empirical Bayes' Credibility Theory assumes that the claims for each risk are dependent on an underlying risk parameter θ. However, no assumptions are made about the form of the distribution of the claim amounts.

For the example presented, the Bayesian and Credibility premiums are very similar. In Subject CT6, the Bayesian Credibility premium is shown to be the linear estimate of the Bayesian premium with the lowest mean square error. For certain choices of the model and prior distribution, we have exact credibility in that the Bayesian and Credibility premiums are identical. This has been demonstrated for the Poisson-gamma and normal-normal models in Subject CT6.

Although the Bayesian Credibility does not require the prior distribution to be specified, estimates of $E\left[s^2(\theta)\right]$ and $var\left[m(\theta)\right]$ may not be available. Classical theory, which is introduced in Section 2 of **this chapter, can then be applied, as it allows for estimation of Z with even less information. Classical credibility can also be used to identify situations where it is acceptable to rely totally on sample data and ignore external information.**

2 Classical credibility

2.1 Introduction

Classical credibility considers how much data you need before you can statistically assign 100% credibility to a proposition.

In other words, we first ask ourselves "how big a sample size do we need for our answer to be reliable?" If we believe our sample size will give us reliable results, then we do not need to make any allowance for any other available data.

We refer to this amount of data as the "full credibility criterion" or the "standard for full credibility". If one has this much data or more, then $Z = 1$; if one has observed less than this amount of data then $0 \le Z < 1$.

In other words, if we think our sample size is too small to give reliable results, we can decide to assign less credibility to our data, by using a lower value of Z. This also means we will have to place some reliance on other information; our prior hypothesis.

In the example below, we first decide that $1,000$ observations are enough to give us credible results.

Example

If we observed 1,000 full-time carpenters for one year, we might assign 100% credibility to their data. (For employers' liability, that data would be pounds of loss and pounds of payroll).

It goes without saying that if we had more than 1,000 observations, we would still believe that the results were credible, *ie* **if we observed 2,000 full-time carpenters we would also assign them 100% credibility.**

However, for any size of data smaller than 1,000 observations, we would want to use a lower credibility rating. **One hundred full-time carpenters might be assigned 32% credibility. In this case the observation has been assigned partial credibility; that is, less than full credibility.**

Exactly how to determine the amount of credibility assigned to different amounts of data is discussed in the following sections. There are four basic concepts from classical credibility that will be covered:

1) **how to determine the criterion for full credibility when estimating frequencies**
2) **how to determine the criterion for full credibility when estimating severities**
3) **how to determine the criterion for full credibility when estimating aggregate losses**
4) **how to determine the amount of partial credibility to assign when one has less data than is needed for full credibility.**

2.2 Full credibility for frequency

We will start with a little revision of Subject CT6, and then introduce some more detail so that we can apply this to credibility theory.

Assume we have a Poisson process for claim frequency, with an average of 500 claims per year.

Recall from Subject CT6 that a Poisson process assumes that the number of claims occurring in a time interval $(0,t)$ are $Poisson(\lambda t)$, where λ is the Poisson parameter representing the mean number of claims per unit of time (in this case 500).

Then, the observed numbers of claims will vary from year to year around the mean of 500. The variance of a Poisson process is equal to its mean, in this case 500.

This Poisson process can be approximated by a normal distribution with a mean of 500 and a variance of 500.

The normal approximation can be used to estimate the spread of the observed results from the mean:

Example

How often can one expect to observe more than 550 claims?

Solution

Using the normal approximation, the probability of more than 550 claims is approximately:

$$\begin{aligned} \mathbf{1} - \Phi\left(\frac{\mathbf{550} - \mathbf{500}}{\sqrt{\mathbf{500}}}\right) &= \mathbf{1} - \Phi\mathbf{(2.24)} \\ &= \mathbf{1} - \mathbf{0.9875} \\ &= \mathbf{1.255\%} \end{aligned}$$

Thus, there is about a 1.26% chance that the observed number of claims will exceed the expected number of claims by 10% or more.

Or, equivalently, the standard deviation is $\sqrt{500} = 22.36$.

So 550 claims corresponds to about $\dfrac{50}{22.36} = 2.24$ standard deviations greater than average.

Since $\Phi(2.24) = 0.9875$, there is approximately a 1.25% chance of observing more than 550 claims.

More precisely, the probability should be calculated including the continuity correction. The probability of more than 550 claims is approximately:

$$\begin{aligned} 1 - \Phi\left(\frac{550.5 - 500}{\sqrt{500}}\right) &= 1 - \Phi(2.258) \\ &= 1 - 0.9880 \\ &= 1.20\% \end{aligned}$$

Similarly, (ignoring the continuity correction) **the chance of observing fewer than 450 claims is approximately 1.26%. So the chance of observing a number of claims that is more than $\pm$10% away from the mean number of claims is about 2.52%. In other words, the chance of observing within +10% or –10% of the expected number of claims is 97.48% in this case.**

We now derive a general formula for calculating this probability.

General formula for variability around the mean

More generally, one can write this algebraically:

The probability *P* that the observed value of a random quantity *X* is within a proportion $+k$ or $-k$ of the mean μ is:

$$\begin{aligned} P &= Prob(\mu - k\mu \le X \le \mu + k\mu) \\ &= Prob\left(-k\left(\frac{\mu}{\sigma}\right) \le \left(\frac{X-\mu}{\sigma}\right) \le +k\left(\frac{\mu}{\sigma}\right)\right) \end{aligned}$$

The last expression is derived by subtracting μ and then dividing through by the standard deviation, σ.

Assuming the normal approximation, the quantity $Z = \dfrac{(X-\mu)}{\sigma}$ is standard normally distributed. We thus have, in terms of the cumulative distribution for the unit normal $\Phi(Z)$,

$$\begin{aligned} P &= \Phi\left(k\left(\frac{\mu}{\sigma}\right)\right) - \Phi\left(-k\left(\frac{\mu}{\sigma}\right)\right) \\ &= \Phi\left(k\left(\frac{\mu}{\sigma}\right)\right) - \left(1 - \Phi\left(k\left(\frac{\mu}{\sigma}\right)\right)\right) \\ &= 2\Phi\left(k\left(\frac{\mu}{\sigma}\right)\right) - 1 \end{aligned}$$

(18.1)

The Core Reading here is using the term "unit normal" to refer to the standard normal distribution.

We can apply this to *numbers* of claims as follows:

For a Poisson distribution with expected number of claims *n*, $\mu = n$ and $\sigma = \sqrt{n}$.

This is just saying that for the number of claims X, where $X \sim Poisson(n)$, (*ie* $E(X) = \text{var}(X) = n$), we can assume that X is approximately normally distributed, with mean $\mu = n$ and standard deviation $\sigma = \sqrt{n}$, so that u is standard normal. This is exactly what we did in the example above.

The probability that the observed number of claims X is within a proportion $\pm k$ of the expected number $\mu = n$ using the normal approximation to the Poisson is:

$$P = 2\Phi\left(k\sqrt{n}\right) - 1 \qquad \textbf{(18.2)}$$

This just substitutes our approximation into equation (18.1) above, *ie* we substitute $\mu = n$, $\sigma = \sqrt{n}$ and $Z = \dfrac{(X-\mu)}{\sigma}$.

or, equivalently:

$$\Phi\left(k\sqrt{n}\right) = \frac{1+P}{2} \qquad \textbf{(18.3)}$$

This provides a general formula for the likelihood of the number of claims being a specified proportion k away from the mean.

Here is a table showing P for different numbers of claims n and for different values of proportion k:

Probability of being within a proportion $\pm k$ of the mean n

Expected number of claims	*k = 10%*	*k = 5%*	*k = 2.5%*	*k = 1%*	*k = 0.5%*
10	**24.82%**	**12.56%**	**6.30%**	**2.52%**	**1.26%**
50	**52.05%**	**27.63%**	**14.03%**	**5.64%**	**2.82%**
100	**68.27%**	**38.29%**	**19.74%**	**7.97%**	**3.99%**
500	**97.47%**	**73.64%**	**42.39%**	**17.69%**	**8.90%**
1,000	**99.84%**	**88.62%**	**57.08%**	**24.82%**	**12.56%**
5,000	**100.00%**	**99.96%**	**92.29%**	**52.05%**	**27.63%**
10,000	**100.00%**	**100.00%**	**98.76%**	**68.27%**	**38.29%**

Question 18.1

Verify the probability of 1,000 claims being within $\pm 5\%$ of the mean.

Application to credibility theory

This is all very well, but it doesn't yet tell us how much data we need in order to assign full credibility to our results.

Turning things around, given values of P and k, one can compute the expected number of claims n_N such that the chance of being within a proportion $\pm k$ of the mean is P.

In other words, for any given probability P and proportion k, we can find the required data volume (*ie* number of claims) n_N.

n_N can be calculated from the formula $\Phi\left(k\sqrt{n_N}\right)=\dfrac{1+P}{2}$. This is formula (18.3) above.

Let y be such that $\Phi(y)=\dfrac{1+P}{2}$. Then given P, y is determined from a normal table.

Solving for n_N in the relationship $y=k\sqrt{n_N}$ yields $n_N=\left(\dfrac{y}{k}\right)^2$.

If the goal is to be within a proportion $\pm k$ of the mean frequency with a probability at least P, then the standard for full credibility is:

$$n_N \geq \frac{y^2}{k^2} \tag{18.4}$$

where y is such that:

$$\Phi(y)=\frac{1+P}{2} \tag{18.5}$$

Here are values of *y* taken from a normal table corresponding to selected values of *P*:

Values of y as a function of P

P	$(1+P)/2$	y
80.00%	90.00%	1.282
90.00%	95.00%	1.645
95.00%	97.50%	1.960
99.00%	99.50%	2.576
99.90%	99.95%	3.291
99.99%	99.995%	3.891

This in itself doesn't yet help us a great deal; remember we're interested in the number of claims required in our dataset for our results to be credible, and y is just an interim step in obtaining this. However, we can use equation (18.5) above to convert these values into the required claim numbers.

Example

Let's say that we want to ensure that our observed claim frequency will not vary from the mean by more than 5%. If we have enough claims data to be 99% sure of this, then we will assign it full credibility.

In this example then, $k = 5\%$ and $P = 99\%$. So from the table above:

$$y = 2.576 \text{ and } n_N \geq \frac{y^2}{k^2} = \left(\frac{2.576}{0.05}\right)^2 = 2,654 .$$

In other words, given our criteria for k and P, we need to have an observed value of at least 2,654 claims, if we are to assign 100% credibility to our estimate of claim frequency.

Question 18.2

Without reading any further, how many claims would be required in order to assign full credibility to the data, with no more than a 10% probability of deviating from the mean by more than 5%?

Here is a table of values for the standard for full credibility for the frequency n_N, given various values of *P* and *k*:

Standards for full credibility for frequency (claim numbers)

Probability level P	***k = 30%***	***k = 20%***	***k = 10%***	***k = 7.5%***	***k = 5%***	***k = 2.5%***	***k = 1%***
80.00%	**18**	**41**	**164**	**292**	**657**	**2,628**	**16,424**
90.00%	**30**	**68**	**271**	**481**	**1,082**	**4,329**	**27,055**
95.00%	**43**	**96**	**384**	**683**	**1,537**	**6,146**	**38,415**
96.00%	**47**	**105**	**422**	**750**	**1,687**	**6,749**	**42,179**
97.00%	**52**	**118**	**471**	**837**	**1,884**	**7,535**	**47,093**
98.00%	**60**	**135**	**541**	**962**	**2,165**	**8,659**	**54,119**
99.00%	**74**	**166**	**664**	**1,180**	**2,654**	**10,616**	**66,349**
99.90%	**120**	**271**	**1,083**	**1,925**	**4,331**	**17,324**	**108,276**
99.99%	**168**	**378**	**1,514**	**2,691**	**6,055**	**24,219**	**151,367**

The value of 1,082 claims corresponding to $P = 90\%$ and $k = 5\%$ is commonly used as an acceptable standard for full credibility. For $P = 90\%$ we want to have a 90% chance of being within a proportion $\pm k$ of the mean, so we are willing to have a 5% probability outside on either tail, for a total of 10% probability of being outside the acceptable range.

Thus, $\Phi(y) = 0.95$ or $y = 1.645$.

Thus, $n_N \geq \dfrac{y^2}{k^2} = \dfrac{1.645^2}{0.05^2} = 1{,}082$ **claims.**

In practical applications appropriate values of *P* and *k* have to be selected. (For situations that come up repeatedly, the choice of *P* and *k* may have been made several decades ago, but nevertheless the choice was made at some point in time.)

While there is clearly judgment involved in the choice of *P* and *k*, the standards for full credibility for a given application are generally chosen within a similar range.

In other words, different practitioners have tended to use the same values of P and k, particularly for problems which arise quite frequently, and these have hardened into market practice over the years. Even where practitioners use different values of P and k depending on their subjective judgement, they are still likely to be reasonably similar.

This same type of judgment is involved in the choice α when forming a $100(1-\alpha)\%$ confidence interval around a statistical estimate of a quantity.

The Core Reading is here talking about the choice of k.

The standard for full credibility is not normally important in itself, but is important as a means of introducing consistency in the rate-making procedure and establishing proper relationships as regards reliability between different volumes of experience.

Often ± 2 standard deviations (corresponding to about a 95% confidence interval) will be chosen, but that is not necessarily better than choosing ± 1.5 or ± 2.5 standard deviations. So while classical credibility involves somewhat arbitrary judgements, this has not stood in the way of its being very useful for decades in many applications.

Variations from the Poisson assumptions

As we have seen, if one desires that the chance of being within a proportion $\pm k$ of the mean frequency be at least P, then the standard for full credibility is:

$$n_N \geq \frac{y^2}{k^2}$$

where y is such that

$$\Phi(y) = \frac{1+P}{2}.$$

However, this depends on the following assumptions:

- **we are trying to estimate frequency**
- **frequency is given by a Poisson process (so that the variance is equal to the mean)**
- **there are enough expected claims to use the normal approximation to the Poisson process.**

Occasionally, a binomial or negative binomial distribution will be substituted for a Poisson distribution, in which case the difference in the derivation is that the variance is not equal to the mean.

Example

Assume that claim frequency has a binomial distribution with parameters $n = 1{,}000$ and $p = 0.3$.

Recall that for $X \sim Bin(n, p)$ we have $E(X) = np$ and $\text{var}(X) = np(1-p)$. We now use the normal approximation, equating $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

The mean is 300 and the variance is $1{,}000 \times 0.3 \times 0.7 = 210$.

Let's say that we want a required proportion $k = 5\%$, *ie* the observed frequency should be within 5% of the mean.

Using equation (18.1), the chance of being within $\pm 5\%$ of the expected value can be calculated as:

$$P\left(0.95np \le X \le 1.05np\right) = P\left(\frac{-0.05np}{\sqrt{np(1-p)}} \le Z \le \frac{0.05np}{\sqrt{np(1-p)}}\right)$$
$$= P\left(\frac{-0.05 \times 300}{210^{0.5}} \le Z \le \frac{0.05 \times 300}{210^{0.5}}\right)$$

where Z is standard normal. Therefore:

$$\Phi\left(\frac{0.05 \times 300}{210^{0.5}}\right) - \Phi\left(\frac{-0.05 \times 300}{210^{0.5}}\right) = 2\Phi\left(\frac{0.05 \times 300}{210^{0.5}}\right) - 1$$
$$\approx 2\Phi(1.04) - 1$$
$$= (2 \times 0.85083) - 1$$
$$= 70.2\%$$

This is our required probability P (approximately 70%).

So, in the case of a binomial with parameters 1,000 and 0.3, the standard for full credibility with $P = 70\%$ and $k = \pm 5\%$ is about 1,000 exposures or 300 expected claims. This is a smaller expected number of claims than under the Poisson assumption.

Question 18.3

(i) Why would we expect the standard for full credibility to be lower under a binomial assumption of claim frequency than under a Poisson assumption?

(ii) What is the corresponding number of claims required for full credibility under the Poisson assumption?

The average policyholder can be expected to make 3 claims over a 10 year period.

(iii) How many policies need to be in force to be able to assign full credibility to the claims data?

(Standards for full credibility in terms of exposure numbers instead of claim numbers are discussed below.)

If, instead, a negative binomial distribution had been assumed, then the variance would have been greater than the mean. This would have resulted in a standard for full credibility greater than in the Poisson situation.

We can derive a more general formula when the Poisson assumption does not apply. The standard for full credibility for frequency is:

$$n_N \geq \left(\frac{y^2}{k^2}\right)\left(\frac{\sigma_N^2}{\mu_N}\right) \qquad \textbf{(18.6)}$$

There is an "extra" factor of the variance of the frequency divided by its mean. This reduces to the Poisson case when $\sigma_N^2 / \mu_N = 1$.

Question 18.4

Explain in words the impact of this extra factor on the volume of data required for full credibility.

Exposures v claims

Standards for full credibility are calculated in terms of the expected number of claims. It is common to translate these into a number of exposures by dividing by the (approximate) expected claim frequency. So for example, if the standard for full credibility is 1,082 claims (*P* = 90%, *k* = 5%) and the expected claim frequency in household insurance were 0.04 claims per house-year, then 1,082/0.04 ≈ 27,000 house-years would be a corresponding standard for full credibility in terms of exposures.

Subsequent sections deal with estimating severities or aggregate losses rather than frequencies. As will be seen, in order to calculate a standard for full credibility for severities or the aggregate losses, generally one first calculates a standard for full credibility for the frequency.

2.3 *Full credibility for severity*

Classical credibility can also be applied to estimating claim severity; the average size of a claim.

Suppose that we are given a sample of N claims, X_1, X_2, ... , X_N, which are each independently drawn from a loss distribution with mean μ_X and variance σ_X^2.

The suffix X above is used to denote claim severity.

The argument below is exactly analogous to that used in Section 2.2 above. We will:

1) first estimate the mean and variance of our claim size distribution, based on our observed claims data
2) then use these in our normal approximation
3) then rearrange the resulting equations to derive a formula for the number of claims needed to assign full credibility.

Throughout the discussion below, the term "severity" is used to denote the (unknown) mean of our underlying claim distribution, and "observed severity" is the mean of our sample claims data.

The severity (that is, the mean of the distribution) can be estimated by:

$$\overline{X} = \frac{X_1 + X_2 + ... + X_N}{N} = \frac{\sum X_i}{N}$$

This is the observed mean severity, which is used to give an estimate of μ_X.

The variance of the observed severity is:

$$\text{var}\left(\frac{\sum X_i}{N}\right) = \frac{1}{N^2}\sum \text{var}(X_i)$$

$$= \frac{\sigma_X^2}{N}$$

Therefore, the standard deviation for the observed severity is $\dfrac{\sigma_X}{\sqrt{N}}$**.**

These N 's are assumed to be known.

The probability that the observed severity $\overline{X}$ **is within a proportion** $\pm k$ **of the** (true) **mean** μ_X **is:**

$$P = Prob\left(\mu_X - k\mu_X \le \overline{X} \le \mu_X + k\mu_X\right)$$

According to the Central Limit Theorem, the distribution of $\overline{X} = \dfrac{(X_1 + X_2 + \ldots + X_N)}{N}$ **can be approximated by a normal distribution for large** ***N*****.**

We are saying that a reasonable approximation would be to assume that $\overline{X}$ follows a normal distribution with mean μ_X and variance $\dfrac{\sigma_X}{\sqrt{N}}$, so that $Z = \dfrac{\overline{X} - \mu_X}{\sigma_X / \sqrt{N}}$ is standard normal.

Assuming that the normal approximation applies, we have from equation (18.1) that

$$P = 2\Phi\left(k\left(\frac{\mu_X}{\frac{\sigma_X}{\sqrt{N}}}\right)\right) - 1$$

$$= 2\Phi\left(k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right) - 1$$

Equivalently, subtracting the mean μ_X, dividing by the standard deviation $\dfrac{\sigma_X}{\sqrt{N}}$ and substituting Z for $\dfrac{\overline{X} - \mu_X}{\sigma_X / \sqrt{N}}$ yields:

$$P = Prob\left(-k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right) \le Z \le k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right)$$

This formula is in fact the same as formula that precedes equation (18.1), except that every term is multiplied by $\sqrt{N}$. So the two results are analogous.

As in Section 2.2 when estimating claim frequencies, we define y such that $\Phi(y) = \frac{1+P}{2}$.

Question 18.5

Derive a formula for y in terms of N, k, μ_X and σ_X.

To have probability P that the observed severity will differ from the true severity by less than $\pm k\mu_X$, we want $y = k\sqrt{n_X}\left(\frac{\mu_X}{\sigma_X}\right)$ where n_X is the required sample size for full credibility for severity.

Solving for n_X:

$$n_X = \left(\frac{y}{k}\right)^2 \left(\frac{\sigma_X}{\mu_X}\right)^2 \qquad \textbf{(18.7)}$$

The ratio of the standard deviation to the mean, $\frac{\sigma_X}{\mu_X} = CV_X$, is the *coefficient of variation* of the claim size distribution.

Recall from Section 2.2 that the standard for full credibility for frequency is $n_N = \frac{y^2}{k^2}$.

Since n_N is the standard for full credibility of frequency, we have that:

$$n_X = n_N CV_X^2 \qquad \textbf{(18.8)}$$

This is the standard for full credibility for severity.

So any observed number of claims that is greater than this should be assigned full credibility.

This makes sense, since the reliability of our claim size estimate is dependent on both the volume of data (*ie* the number of claims) and the variability of each individual claim.

Question 18.6

Assume that claim frequencies follow a Poisson distribution, and that individual claim sizes follow a lognormal distribution with parameters $\mu = 3.615$ and $\sigma^2 = 1.980$. You want to be 90% sure that the observed mean claim size will not differ from the underlying mean claim size by more than 5%.

How many observed claims do you need in order to assign full credibility to your data?

2.4 *Full credibility for aggregate losses*

Since they depend on both the number of claims and the size of claims, aggregate losses, pure premiums and loss ratios have more reasons to vary than either frequency or severity. As a consequence, the standard for full credibility is larger than that for frequency or severity.

Suppose that *N* claims of sizes X_1, X_2, ... , X_N occur during the observation period.

Aggregate losses, *S*, can be represented using the collective risk model, where

$$\boldsymbol{S = X_1 + X_2 + \ldots + X_N}$$

Let μ_S and σ_S denote the mean and standard deviation of *S*. For large numbers of expected claims, aggregate losses are approximately normally distributed. (The more skewed the severity distribution, the higher the frequency has to be for the normal approximation to produce worthwhile results).

Assuming a normal approximation for *S*, full credibility standards can be calculated following the same steps as in Sections 2.2 and 2.3.

The probability that the observed aggregate loss is within a proportion $\pm k$ of the mean μ_S is:

$$\boldsymbol{P = \text{Prob}\left[\mu_S - k\mu_S \le S \le \mu_S + k\mu_S\right]}$$

$$= Prob\left(-k\left(\frac{\mu_S}{\sigma_S}\right) \le Z \le k\left(\frac{\mu_S}{\sigma_S}\right)\right)$$

where $Z = \dfrac{S - \mu_S}{\sigma_S}$ is a standard normal variable, assuming the normal approximation.

We now follow a very similar argument to Section 2.2, when we derived the standard for full credibility for frequency.

Applying equation (18.1), we have that

$$P = 2\Phi\left(k\left(\frac{\mu_S}{\sigma_S}\right)\right) - 1$$

Define y such that $\Phi(y) = \frac{(1+P)}{2}$ (see Section 2.2 for more details). Then, to have probability P that the observed pure premium will differ from the true pure premium by less than $\pm k\mu_S$:

$$y = k\left(\frac{\mu_S}{\sigma_S}\right) \tag{18.9}$$

Question 18.7

Derive this result.

We now consider the special case where *S* has a Compound Poisson distribution.

Suppose that the frequency is Poisson with mean *n*. *S* is then said to have a Compound Poisson distribution. The mean and variance of *S* were derived in Subject CT6:

$$\mu_S = n_S \mu_X \tag{18.10}$$

$$\sigma_S^2 = n_S\left(\sigma_X^2 + \mu_X^2\right) \tag{18.11}$$

These are the standard results for the mean and variance of a compound Poisson distribution.

Substituting for μ_S and σ_S in equation (18.9), the expected number of claims, n_S, required for full credibility for aggregate losses satisfies:

$$y = k\left(\frac{n_S \mu_X}{\left(n_S\left(\sigma_X^2 + \mu_X^2\right)\right)^{1/2}}\right)$$

Solving for n_S:

$$
\begin{aligned}
\boldsymbol{n_S} &= \left(\frac{\boldsymbol{y}}{\boldsymbol{k}}\right)^{\mathbf{2}}\left[\mathbf{1}+\left(\frac{\boldsymbol{\sigma_X^2}}{\boldsymbol{\mu_X^2}}\right)\right] \\
&= \boldsymbol{n_N}\left(\mathbf{1}+\boldsymbol{CV_X^2}\right) \qquad \textbf{(18.12)}
\end{aligned}
$$

This is the standard for full credibility of the aggregate loss.

Recall that $n_N = \left(\frac{y}{k}\right)^2$ is the standard for full credibility of frequency that was derived in Section 2.2, and $CV_X = \frac{\sigma_X}{\mu_X}$ is the coefficient of variation of the severity.

It is interesting to note that the standard for full credibility of the aggregate loss is the sum of the standards for frequency and severity:

$$
\begin{aligned}
\boldsymbol{n_S} &= \boldsymbol{n_N}\left(\mathbf{1}+\boldsymbol{CV_X^2}\right) \\
&= \boldsymbol{n_N} + \boldsymbol{n_N CV_X^2} \\
&= \boldsymbol{n_N} + \boldsymbol{n_X} \\
&= \textbf{Standard for full credibility of frequency} \\
&\quad + \textbf{Standard for full credibility of severity}
\end{aligned}
$$

Recall from equation (18.8) that $n_X = n_N CV_X^2$.

Note that if we limit the size of claims, then the coefficient of variation is smaller. Therefore, the criterion for full credibility for basic limits losses is less than that for total losses. It is a common practice in rate-making to cap losses in order to increase the credibility assigned to the data.

If claims amounts are limited then there will be more certainty attached to their values. Also, by capping large losses, the claim amounts below the cap limit can be grouped with the attritional (*ie* small) claims and modelled together. Of course, a loading should be included in the premium rates for that portion of large losses that exceeds the cap level.

Variations from the Poisson assumption

As with the standard for full credibility of frequency, one can derive a more general formula when the Poisson assumption does not apply. The standard for full credibility is:

$$n_S = \left(\frac{y}{k}\right)^2 \left(\frac{\sigma_N^2}{\mu_N} + \frac{\sigma_X^2}{\mu_X^2}\right) \qquad \textbf{(18.13)}$$

Question 18.8

Derive this result.

This reduces to the Poisson case when $\frac{\sigma_N^2}{\mu_N} = 1$. If the severity is constant then σ_X^2 is zero and (18.13) reduces to (18.6).

2.5 Partial credibility

When one has at least the number of claims needed for full credibility, then one assigns 100% credibility to the observations. However, when there is less data than is needed for full credibility, less than 100% credibility is assigned.

Recall that the formula for calculating the credibility-weighted estimates is:

$$Estimate = Z \times (Observation) + (1-Z) \times (Other\ information), \qquad 0 \le Z \le 1,$$

which we can write as:

$$Estimate = Z \times (\bar{X}_n) + (1-Z) \times (\mu_{other}), \qquad 0 \le Z \le 1$$

where $\bar{X}_n$ is the observed sample mean.

If the number of claims observations n in our data is less than the number required for full credibility n_N, *ie* $n \le n_N$ then the variance of our sample data will be higher than the variance under full credibility.

We can choose our partial credibility factor Z such that the variance of our credibility-weighted estimate is the same as it would be if we had fully credible data:

$$var\left(\bar{X}_{n_N}\right) = var\left(Z\left(\bar{X}_n\right) + (1-Z)\left(\mu_{other}\right)\right)$$

This argument leads to the "square root rule":

Square root rule

Let n be the (expected) number of claims for the volume of data, and n_F be the standard for full credibility. Then the partial credibility assigned is $Z = \sqrt{\dfrac{n}{n_F}}$.

Note that n_F is equivalent to the n_N encountered earlier in this chapter.

If $n \geq n_F$, then $Z = 1$. Use of the square root rule applies for partial credibility for either frequency, severity or aggregate losses.

The Core Reading now provides a derivation of the square root rule.

Derivation of square root rule

The square root rule for partial credibility is designed so that the standard deviation of the contribution of the data to the new estimate retains the value corresponding to the standard for full credibility.

In other words, we choose factor Z such that the standard deviation of our credibility-weighted estimate is the same as it would be if we had fully credible data.

We will demonstrate why the square root rule accomplishes that goal.

Let $X_{partial}$ be a value calculated from partially credible data; for example, $X_{partial}$ might be the claim frequency calculated from the data. Assume X_{full} is calculated from data that just meets the full credibility standard.

For the full credibility data, the estimate = X_{full}, while the partially credible data enters the estimate with a weight Z in front of it: *ie*

$$\boldsymbol{estimate = Z\, X_{partial} \;+\; (1-Z)(other\;\; information)}$$

The credibility factor Z is calculated so that the expected variation in $Z\,X_{partial}$ is limited to the variation allowed in a full credibility estimate X_{full}. The variance of $Z\,X_{partial}$ can be reduced by choosing a Z less than one.

Suppose you have estimates $X_{partial}$ and X_{full} based on different size samples of a population. Then they will have the same expected value. But, since it is based on a smaller sample size, $X_{partial}$ will have a larger standard deviation $\sigma_{partial}$ than the standard deviation σ_{full} of the full credibility estimate X_{full}. The goal is to limit the fluctuation in the term $Z\,X_{partial}$ to that allowed for X_{full}. This can be written as:

$$P = \text{Prob}(\mu - k\mu \le X_{full} \le \mu + k\mu) = \text{Prob}\left(Z\mu - k\mu \le ZX_{partial} \le Z\mu + k\mu\right)$$

Here, μ is the value of the true underlying mean.

(Note that in both cases fluctuations are limited to $\pm k\mu$ of the mean)

Taking the right hand side of this equation and subtracting through by the mean and dividing by the standard deviation gives:

$$P = \text{Prob}\left(\frac{-k\mu}{Z\sigma_{partial}} \le \frac{ZX_{partial} - Z\mu}{Z\sigma_{partial}} \le \frac{k\mu}{Z\sigma_{partial}}\right)$$

(Note that the mean of $Z\,X_{partial}$ is $Z\mu$ and the standard deviation is $Z\sigma_{partial}$.)

Assuming that $X_{partial}$ is approximately normally distributed (so that $\dfrac{ZX_{partial} - Z\mu}{Z\sigma_{partial}}$ is approximately a standard normal variable)**, we have:**

$$P = 2\Phi\left(\frac{k\mu}{Z\sigma_{partial}}\right) - 1$$

This is derived in exactly the same way as for equation (18.1).

Rearranging the equation above, we have:

$$\frac{1+P}{2} = \Phi\left(\frac{k\mu}{Z\sigma_{partial}}\right)$$

Since P is the acceptable proportion for full credibility, we have from equation (18.5) that

$$y = \frac{k\mu}{Z\sigma_{partial}}$$

Solving for Z yields:

$$Z = \frac{k\mu}{y\sigma_{partial}} \qquad \textbf{(18.14)}$$

Thus, the partial credibility Z will be inversely proportional to the standard deviation of the partially credible data.

Assuming that we are trying to estimate frequency where a Poisson distribution holds with the expected number of claims n being less than that required for full credibility (*ie* $n \le n_F = \left(\frac{y}{k}\right)^2$), we have:

$$Z = \frac{kn}{y\sqrt{n}} = \frac{\sqrt{n}}{\left(y/k\right)} = \sqrt{\frac{n}{n_F}} \qquad \textbf{(18.15)}$$

For a Poisson distribution, $\mu = n$ and $\sigma = \sqrt{n}$, and substituting these into equation (18.14) gives equation (18.15).

This proves the square root rule for frequency and a similar result is arrived at when estimating severity or the aggregate loss.

Note that this proof requires the assumptions of the normal approximation and a Poisson claims distribution.

3 Classical and Bayesian credibility models

In the previous section, we discussed the "classical" credibility formula, and saw how we can choose our partial credibility factor Z such that the variance of our credibility-weighted estimate is the same as it would be if we had fully credible data. This section compares the classical credibility formula with another method of choosing a value for Z; the "Bayesian" credibility formula.

3.1 Classical credibility formula

This chapter provides the motivation for the following formula for the "classical" credibility factor Z_c:

$$Z_C = \begin{cases} \left(n/n_F\right)^{1/2} & 0 \le n \le n_F \\ 1 & n \ge n_F \end{cases} \tag{18.16}$$

where n is the number of claims and n_F is the standard for full credibility.

This is the credibility factor derived in equation (18.15) above.

3.2 Bayesian credibility formula

The "Bayesian" credibility factor Z_B, based on the principle of minimising the mean square error, is given by:

$$Z_B = \frac{n}{n+k} \tag{18.17}$$

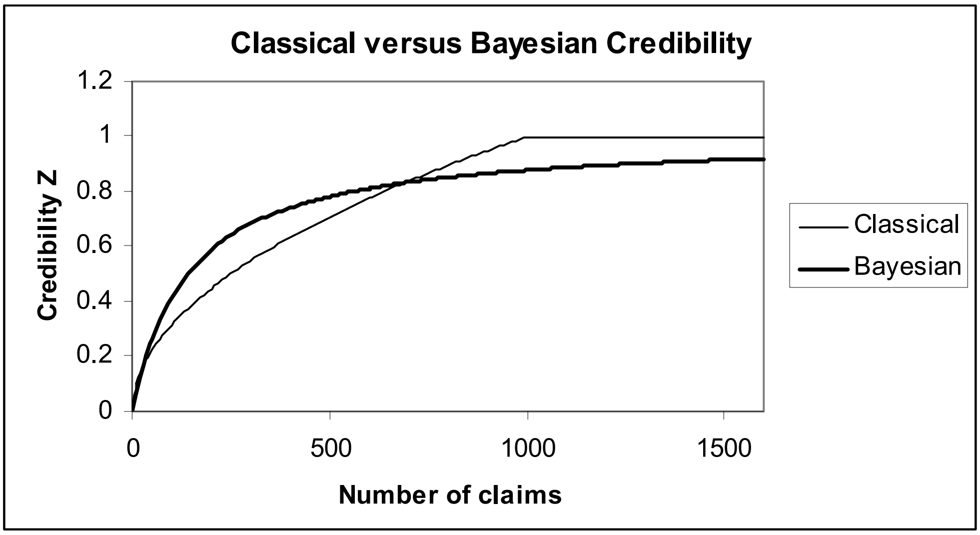
where $k = \dfrac{E\left[s^2(\theta)\right]}{var\left(m(\theta)\right)}$

k is called the Bayesian credibility parameter. You can think of $n+k$ as the standard for full credibility under Bayesian credibility, although at this point we do not necessarily know what value to assign to k.

3.3 Comparison of the two equations

We need to consider the practical impact of choosing between classical and Bayesian credibility. The answer depends on the parameters used in the two credibility formulas. For a certain simple relationship between the parameters, the choice between classical and Bayesian credibility makes only a relatively small difference and for many practical applications this difference is acceptable.

Although the formulae are very different, classical credibility and Bayesian credibility can produce very similar results as illustrated below for the case $n_F = 1000$ and $k = 140$:



Classical and Bayesian credibility formulae will produce approximately the same credibility weights if the full credibility standard for classical credibility n_F is about 7 to 8 times larger than the Bayesian credibility factor k.

So, for most practical applications, we could find the Bayesian credibility factor, k, and multiply by 8 to get the number of claims for full credibility, n_F. When estimating k, we do not need to be extremely precise as even an inaccurate estimate of k can still produce a fairly good estimate.

The most significant difference between the two models is that Bayesian credibility never reaches $Z = 1$, which is an asymptote of the curve. Either model can be effective at improving the stability and accuracy of estimates.

For a particular application, the actuary can choose the model appropriate to the data and goals.

Where estimates of $E\left[s^2(\theta)\right]$ and $var\left[m(\theta)\right]$ are available, Bayesian credibility generates the most accurate insurance rates and forms the basis of most experience rating plans.

Classical credibility can be used if these estimates are unknown or difficult to calculate and is often used in the calculation of overall rate increases.

As well as the consideration as to which model gives the most accurate results, there may be other reasons for choosing one model over another.

Larger insureds with favourable claim results usually want their future insurance premiums to be based solely on their own experience and can do so where standards for full credibility are met.

Classical credibility is simpler to work with and easier to explain to non-actuaries.

4 Practical uses of credibility models: complements of credibility

Many actuarial papers discuss credibility. Actuaries use credibility when data is sparse and lacks statistical credibility. Specifically actuaries use it when historical losses have a large error around the underlying expected losses (average of the distribution of potential loss costs) that the actuary is estimating.

For example, if the premium rating statistic varies around the true expected losses with a standard deviation equal to its mean, it will probably have a very low credibility. Therefore, the vast majority of the rate (in this context, expected loss estimate) will come from whatever statistic receives the complement of credibility. So it is important to use an effective statistic for the ancillary statistic (here after called the complement of credibility).

4.1 Fundamental principles – what should the actuary consider?

There are four types of issues that any actuary must consider when choosing the complement:

- **practical issues**
- **competitive market issues**
- **regulatory issues, and**
- **statistical issues.**

We now discuss each of these factors in more detail. We then finish this section by summarising the desirable qualities of a complement of credibility.

Practical issues

The easiest statistic to use is one that is readily available.

For example, the best possible statistic is next year's loss costs. Unfortunately, that statistic is not available (otherwise, companies would not need actuaries). The actuary must choose from the statistics that are available. Since some statistics require more complicated programming or expensive processing than others, some statistics are more readily available than others.

Ease of computation is another factor to consider. If a statistic is easy to compute, it is often easier to explain to management and customers. Since few actuaries have unlimited budgets, they usually weigh the time involved in computing a very accurate statistic against the accuracy improvement it generates. Also, when computations are easy to do there is less chance of error.

Competitive market issues

Rates are rarely made in a vacuum. Generally, whatever rate the actuary produces will be subject to market competition.

If the rate is too high, competitors can undercut the rate and still make a profit. That will cost the actuary's employer customers and profit opportunities. If the rate is too low, the employer will lose money.

So, in mathematical terms, the rate should be:

- **unbiased (neither too high nor too low over a large number of loss cost estimates) and**
- **accurate (the rate should have as low an error variance as possible around the future expected losses being estimated).**

Also the difference between lack of bias and accuracy is important. An unbiased statistic varies randomly about the following year's losses over many successive years, but it may not be close. An accurate statistic may average higher or lower than the following year's losses, but it is always close.

Ultimately, the complement of the credibility should help make the rate as unbiased and accurate as possible.

Regulatory issues

Sometimes, rates require some level of approval from insurance regulators. The classic rate regulatory law requires that rates be "not inadequate, not excessive, and not unfairly discriminatory".

The principles of adequacy and non-excessiveness imply that rates should be as unbiased as possible. For most purposes, actuaries interpret "not unfairly discriminatory" in the premium rating context as "unbiased". Many believe that if a rate truly reflects a class's probable loss experience, it is fair by definition.

Those principles could be stretched to imply that rates should be accurate. The argument goes as follows: inaccurate rates create a much greater risk of insolvency through random inadequacies. The law is concerned with inadequacy because it seeks to prevent insolvencies. So, law suggests rates should be as accurate as possible.

The actuary can mitigate regulatory concerns by choosing a complement that has some logical relationship to the loss costs of the class or individual being rated. That means that it is easier to explain a high rate for a class or individual in light of the related loss costs.

Statistical issues

Clearly, the actuary must attempt to produce the most accurate rate that is practical, but in doing so, must consider all the types of error that make up the prediction error.

(The prediction error is the squared difference between the credibility-weighted prediction and actual results, so it is a measure of the accuracy of the credibility–weighted estimate**.)**

There are of course, the natural year-to-year variations in losses about the true mean due to process variance. There may also be errors because the predictor has a different mean than the losses (bias).

The error of the predictor may stem from the error of its components. The historical losses (the usual base statistic), when trended and developed, will contain prediction errors because the factors used to bring losses to a fully developed current cost level are different to what will happen (loss development and trend variance). When mathematical models of losses are used as complements, there may be errors in both the type of model used (model error) and the specific parameters selected for the model (parameter error). All of these (including any process error and bias of the complement) contribute to prediction error and reduce the accuracy of the prediction.

Process uncertainty, model uncertainty and parameter uncertainty are discussed in Chapter 9 and in Subject ST7.

If the complement of the credibility is accurate in its own right and relatively independent of the base statistic (which receives the credibility), the resulting rate will be more accurate.

The rationale involves statistical properties of credibility-weighted estimates. The details of these statistical properties are beyond the scope of Subject ST8. However, interested students may like to read "The Complement of Credibility" by Joseph A. Boor, from which much of this section has been drawn, available at www.casact.org. We discuss the impact of these statistical properties below:

The error in the credibility estimate depends on the errors in the base statistic and the complement of credibility. In other words, the greater the inaccuracy of either the base statistic or the ancillary statistic, the greater the increase in the inaccuracy of the credibility-weighted estimate.

The accuracy of the complement of credibility is just as important as the accuracy of the base statistic. In other words, any inaccuracy in the ancillary statistic will have just as much effect as a similar inaccuracy in the base statistic.

The benefits of independence are more subtle. As it turns out, independence is most important when credibility is most important. That is, independence is most important for the intermediate credibilities (*Z* between 10% and 90%). In other words, the greater the independence between the base statistic and ancillary statistic, the greater the accuracy of the credibility-weighted estimate, particularly for $10\% \le Z \le 90\%$.

In fact, the prediction is best when there is a negative correlation between the errors of the two statistics (*ie* when the errors offset each other), but this is rarely the case in practice. In general the two statistics will be positively correlated to some extent.

So, a complement of credibility is best when it is statistically independent of (that is, not related to) the base statistic.

4.2 Summary of desirable qualities

The previous section showed six desirable qualities for a complement of credibility:

- **accuracy as a predictor of next year's mean loss costs (that is, low variance around next year's mean loss costs)**
- **unbiasedness as a predictor of next year's mean expected losses (that is, the differences between the predictor and the subsequent loss costs should average out near zero)**
- **independence from the base statistic**
- **availability of data**
- **ease of computation**
- **explainable relationship to the loss costs of the class or individual being rated.**

Other factors to consider when choosing the complement include:

- ease of explanation to managers and customers
- the time taken to calculate the complement (balanced against the need for accuracy)
- the chance of error in calculating the complement
- the sources of error in the complement.

5 Bühlmann-Straub credibility model

The Bühlmann-Straub model was developed in 1970 in the determination of claims ratios in reinsurance. It is still by far the most widely used and important credibility model in insurance practice. However, since then a variety of applications has been found for its use in both life and non-life insurance, in primary insurance as well as reinsurance.

5.1 Motivation

Let us assume that for a class of business *eg* motor third party liability insurance the actuary is tasked with estimating the "true" pure risk premium for a future period. This may be done on the basis of individual claim observations X_{ik} for risk i in year k or from benchmarking the risk i against other similar risks. The question becomes how much relevance should be given to the results of each method.

To solve this problem we are going to use the Bühlmann-Straub Model.

5.2 Bühlmann-Straub Model

Definitions

Let us define:

V_i = Volume measure (*eg* volume = premium income) for risk i
S_i = insurance claims for risk i
X_i = claims ratio (= S_i / V_i) for risk i .

$S_1, S_2, \ldots S_n$ are random variables representing the aggregate claims for each risk $1, 2, \ldots, n$.

The random variable X_i represents the aggregate claims, or the number of claims for risk *i*, standardised to remove the effect of the size of each risk.

Since S_i is a random variable then the corresponding claims ratio must also be a random variable.

It is assumed that the volume measure V_i and the claims ratio X_i are based upon the volume measures V_{ik} and claims ratios X_{ik} for risk i across all years k .

For example, V_i and S_i might be the sums of the previous *k* years of historical data for risk *i*, so that X_i would be the long-run average claims ratio, based on historical data.

We are trying to estimate next year's claims ratio for risk *i*, *ie* $X_{i,k+1}$.

We now assume that the distribution of each X_{ik} depends on the value of a parameter θ_i, whose value is the same for each year *k* but is unknown.

The Bühlmann-Straub assumptions are that there exists a latent parameter θ_i (which can be characterised as a risk profile and which in itself is a random variable) such that:

$E\left(X_{ik}\,|\theta_i\right)=\mu\left(\theta_i\right)$ **(that is, the underlying long-run claims ratio of risk *i*)**

$\text{var}\left(X_{ik}\,|\theta_i\right)=\dfrac{\sigma^2\left(\theta_i\right)}{V_{ik}}$ **(that is, the variance of the observed claims ratio of risk *i* is inversely proportional to the volume measure of risk *i*).**

Other assumptions are:

- **The i^{th} risk is described by the pair, $(\theta_i,(X_{ik})_{k\geq 1})$ where $(X_{ik})_{k\geq 1}$ is the sequence of claims ratios observed for risk i in years k.**
- **The pairs $(\theta_i,(X_{ik})_{k\geq 1})$ are mutually independent.**
- **The θ_i are independent and identically distributed.**
- **Conditionally on θ_i, the X_{ik}'s are independent (but not necessarily identically distributed).**

It then follows that:

- $E\left(X_{ik}\,|\theta_i\right)$ does not depend on k.
- $V_{ij}\times E\left(X_{ik}\,|\theta_i\right)$ does not depend on k.

The first of the two expressions above is saying that, for a given risk i, the expected value of the claims ratio (as a function of θ_i) does not vary from one year to the next.

Similarly, the volume-adjusted variance of the claims ratio (as a function of θ_i) for a given risk does not vary from year to year.

Now define β, ϕ and λ:

$$\boldsymbol{\beta = E\left(\mu\left(\theta_i\right)\right)}$$ **(that is, the benchmark claims ratio).**

Recall from your knowledge of conditional expectations that $E(X) = E\left(E\left(X|Y\right)\right)$, so that:

$$\begin{aligned}\beta &= E\left(\mu\left(\theta_i\right)\right) \\ &= E\left(E\left(X_{ik}|\theta_i\right)\right) \\ &= E(X_{ik}).\end{aligned}$$

So β is the average claims ratio across all risks. Recall that we are trying to estimate next year's claims ratio using a weighted-average of past data for the risk in question and ancillary data. Since β is derived from data not relating to the specific risk, it is the ancillary statistic, or as the Core Reading calls it, the benchmark claims ratio.

$$\boldsymbol{\phi = E\left(\sigma^2\left(\theta_i\right)\right)}$$ **(that is, the expected variance of the observed claims ratios per unit of V).**

By definition, $\sigma^2\left(\theta_i\right) = V_{ik} \times \text{var}\left(X_{ik}|\theta_i\right)$, so that $E\left(\sigma^2\left(\theta_i\right)\right) = E\left(V_{ik} \times \text{var}\left(X_{ik}|\theta_i\right)\right)$.

ϕ is therefore the average variability of the observed claims ratios, allowing for the average volume of data in each cell.

$$\boldsymbol{\lambda = \text{var}\left(\mu\left(\theta_i\right)\right)}$$ **(that is, the variance of the long-run claims ratios for all risks).**

By definition, $\text{var}\left(\mu\left(\theta_i\right)\right) = \text{var}\left(E\left(X_{ik}|\theta_i\right)\right)$.

Now that we have defined the model, we can use it to derive a formula for the credibility-weighted estimate for $X_{i,k+1}$. As with Section 0, this will minimise the variance of the estimate around the true result (*ie* the estimate given will be the least squares linear unbiased estimate).

You are not required to derive the formula.

Bühlmann-Straub Formula

For fixed values of β, ϕ and λ the best linear estimator C^{BE} of $\mu(\theta_i)$ (with respect to the mean squared error) is the credibility estimator:

$$C^{BE} = z_i X_i + (1 - z_i)\beta$$

where:

$$z_i = \frac{V_i}{V_i + \phi/\lambda} \text{ .}$$

Notes

The expression for c^{BE} takes the form of a credibility-weighted average of the observed (experience) claims ratio for risk *i* and the benchmark claims ratio.

The weight given to the observed **experience** for risk i **increases when:**

- **V_i increases (*ie* the credibility increases if the claims ratio X_i is based on a greater exposure period or a larger risk size)**
- **ϕ decreases (*ie* the credibility increases if the inherent variability of X_i per unit of V_i for each risk *i* is smaller)**
- **λ increases (*ie* the credibility increases if there is wide variation of the long run claims ratios in the benchmark portfolio).**

Note the similarities between this approach and a Bayesian credibility approach. Indeed, with certain distributions and parameters the two methods will yield the same result.

Note that the Bühlmann-Straub Formula tells us how to estimate the value of $X_{i,k+1}$, the claims ratio for risk i for the coming year. If we want to estimate $S_{i,k+1}$, the aggregate claim amount for the coming year for risk *i*, we have to multiply our estimate of $X_{i,k+1}$ by $V_{i,k+1}$, the risk volume for the coming year.

In other words:

Pure Premium = Claims Ratio (= Insurance Claims / Volume) × Volume. For example in motor insurance the volume measure would be the number of vehicle-years; or in fire insurance sums insured.

Let's look at an example.

Example

You manage a small, specialist portfolio, where the total losses were $Y_1 = £5,400$ in year one and $Y_2 = £3,780$ in year two. The number of policies sold were $V_1 = 6$ in year one and $V_2 = 24$ in year 2.

You believe that the distribution of your average losses in year k is $X_k|\theta \sim Gamma\left(V_k, \frac{V_k}{\theta}\right)$, where $\theta \sim U(50,500)$.

Calculate the Bühlmann-Straub risk premium for the total losses in year three, assuming that you sell $V_3 = 12$ policies next year.

Solution

Note that there is only one risk here so we do not need a subscript i corresponding to the risk.

Using the formulae for the mean and variance of the gamma distribution given in the *Tables*, we get:

$$\mu(\theta) = E(X_k|\theta) = \frac{V_k}{V_k/\theta} = \theta$$

$$\sigma^2(\theta) = V_k\left[Var(X_k|\theta)\right] = V_k \times \frac{V_k}{(V_k/\theta)^2} = \theta^2.$$

Similarly, using the formulae for the mean, variance and second moment of the uniform distribution given in the *Tables*, we get:

$$\beta = E(\mu(\theta)) = E(\theta) = 275$$

$$\phi = E\left(\sigma^2(\theta)\right) = E\left(\theta^2\right) = 92,500$$

$$\lambda = Var(\mu(\theta)) = Var(\theta) = 16,875.$$

We now use the Bühlmann-Straub formula for the credibility factor. The volume figure V is equal to the total exposure (*ie* total number of policies) over the previous two years (*ie* $V = V_1 + V_2 = 30$) and the base statistic is the average claims ratio over the previous two years.

$$Z = \frac{V}{V + \phi/\lambda} = \frac{30}{30 + 92{,}500/16{,}875} = 0.84551$$

$$X_3 = \frac{Y_1 + Y_2}{V_1 + V_2} = \frac{5{,}400 + 3{,}780}{30} = 306.$$

So the Bühlmann-Straub credibility premium per unit of volume is:

$$\begin{aligned} C^{BE} &= Z \times X + (1 - Z) \times \beta \\ &= Z \times 306 + (1 - Z) \times 275 \\ &= 301.211. \end{aligned}$$

So the risk premium for the total losses in year three is $301.211 \times 12 = £3{,}614.53$.

6 Practical uses of credibility models: experience rating plans

In practice, many different forms of credibility model are used in experience rating, and the choice of model will depend on the class of business concerned, market practice in the territory where the business is written and the preferences of the individual insurer.

This section discusses the factors to consider when deciding which credibility model to use in practice.

When designing experience rating plans (*ie* when deciding what form of experience rating to use)**, there are some administrative considerations that cannot be overlooked. The first is that experience ratings are done frequently and so simplicity is of paramount importance.**

A second consideration is that experience rating, as opposed to class rating, is very visible to the individual insured. A consequence of this is that the experience rating plans must give due consideration to what the insured perceives to be fair. Historically, these plans have done this by including the following two features:

1) **A single claim should change the premium by no more than a predetermined amount. This predetermined amount is known as the swing of the experience rating plan.**

2) **All insureds above a certain predetermined size are self-rated; that is they are rated entirely on the basis of their own experience.**

In addition to the administrative considerations mentioned above, there are some mathematical considerations. The mathematical foundations of experience rating come from Bayesian estimation and credibility theory. The success of the application of Bayesian estimation and credibility theory depends upon how closely the model represents reality. So the choice of model should be a good fit to the situation being modelled.

The experience rating formulae derived from administrative considerations may be different from those derived from the mathematical credibility considerations, often referred to as "theoretical" formulae. We would judge the formulae to be compatible if the accuracy of the "practical" formula (*ie* where a simplified model is used) **is near that of the "theoretical"** (*ie* unsimplified) **formula. While it is by no means certain that accuracy in simplified models implies accuracy in real life situations, inaccuracy in a simplified model should imply that something is wrong with the formula being tested.** So there is a balance to be made between simplicity and accuracy.

7 Practical uses of credibility models: credibility and data considerations

Another estimation problem concerns the credibility of the data. Since competition encourages insurers to refine their classification systems, refinement will generally continue to the point where the credibility of the data becomes minimal.

The competitive nature of insurance means that insurers need to charge as accurate a premium as possible. If the premium is too high they will lose business, but if it is too low then the business will be unprofitable.

In an effort to improve the accuracy of their premium rates, insurers will try to divide their risks into homogeneous sub-groups. This makes the data within each sub-group more characteristic of that group, removing any distortions in the results and enabling a more accurate premium to be charged. The problem is that if the data is grouped into too many sub-divisions, there will not be enough data in each group to allow a credible analysis.

When we estimate a group's future cost, we can use credibility to compare the relevance of the group's past cost to the relevance of the credibility complement's cost. Assume, for example, that the task is to estimate the cost of Group *A*. If Group *A* has a large body of data, that experience alone may be sufficient for estimating its cost. As Group *A* becomes smaller, at some point it will be useful to compare Group *A*'s empirical costs to the cost of some other group. This other group is the credibility complement. Group *A*'s empirical cost may be twice the cost of the complement. Since Group *A* has less data or less reliable data, the actuary may decide that Group *A*'s true cost is only 60% higher than the complement. This is just saying that we can estimate future values by using a mix of past data and ancillary (or credibility complement) data.

Thus, two credibility-related problems emerge:

(1) how to obtain more data or more reliable data

(2) what is the most appropriate credibility complement?

Obtaining more (or more reliable) data can be done in several ways. Most obviously, more years of data or, possibly, data from several areas of the country (or countrywide) can be used. Of course, the threshold question is whether the broader base actually applies, *ie* is the extra data relevant? **Has there been a change over time? Do national indications apply in each area?**

Another method is to give more weight to more stable phenomena. For example, relativities can be based primarily on frequency (by looking only at claim counts or by limiting the size of claims), instead of pure premiums.

No claims discount systems are a good example of this, where the premium depends in part on the number of previous claims on a policy but not on the amounts of those claims.

Partial pure premiums can be calculated. For example, property damage liability costs may be more stable than bodily injury liability; employers' liability medical costs may be more stable than deaths or permanent disabilities. In determining relativities, more emphasis (credibility) is given to the more stable phenomena.

In other words, the insurer could calculate the premium for each claim type under a policy and then calculate the final premium as a mix of these partial premiums. It will give more weighting to the more stable claim types.

The choice of credibility complement may be more difficult than obtaining more (or more reliable) data. It may not be clear which group is most nearly the same as the group in question. National or regional data may be applicable. Related industry group data may be applicable. In most of these cases, adjustments must be made because the level of costs can be quite different for the complement. Often, the percentage change in the complement is considered, rather than the actual value. As a last resort, the complement may be based on the prior year's analysis; this, in effect, takes more years of data into account.

Finally it should be noted that the application of credibility theory is never purely one of checking for adequacy of exposure and then employing formulae slavishly. The assessment of the premium requires considerable judgement, even to allow for everyday features in risk experience.

- **Large claims**

  To what extent should an individual risk be charged fully for its own experience, if it suffered from a single large and unusual claim in the recent past? In considering this matter, we need to think about **what is large, what is unusual and what is recent?**

  If the individual risk is not to be fully "charged" for its own claims, how is the surplus to be spread over the balance of the experience-rated portfolio (and indeed the risk itself)?

  If we expect to receive these large claims at any point in the future then it is important that we load for the cost of them somewhere, to avoid making losses. The unusual nature of these claims makes it difficult to know which policy might give rise to the next one and therefore difficult to know which policies to load.

- **Trends**

  To what extent should an individual risk's future premium be adjusted for claims trends that are an accepted feature of that line of business?

  Inflation is possibly an obvious candidate for adjustment. But equally, patterns in recent frequency and severity will need to be considered before accepting a risk premium based purely on past experience.

  We will be setting premium rates for future periods so it will almost always be necessary to adjust for trends and inflation.

  These issues have already been discussed in the burning cost approach to premium rating in Chapter 14.

- **Divergence of book rate and observed experience**

  What should be done in practice when the credibility proposition is significantly different from that suggested by normal underwriting, in either direction?

  Certainly the risk proposed must be examined very carefully before one premium is accepted ahead of the other. Certainly there may be business capture possibilities but, equally, the insurer could lose considerably if key features are overlooked in the chosen approach.

8 Accreditation

The Faculty and Institute of Actuaries would like to thank the numerous people who have helped in the development of the material contained in this Core Reading, and are grateful to the Casualty Actuarial Society for permission to use some of its educational material for sections within the Core Reading.

In particular various sections of this chapter have been used by permission of the Casualty Actuarial Society. Republished with minor modifications from *Foundations of Casualty Actuarial Science*, Fourth Edition, (Arlington, Virginia, USA: CAS, 2001).

Please note that the development of many of the numerical examples in Section 3 can be studied further if necessary in the appendices and exhibits of "An actuarial note on credibility parameters" by Howard C. Mahler, from which much of this section has been drawn.

http://www.casact.org/pubs/proceed/proceed86/86001.pdf

Chapter 18 Summary

Claim frequencies and aggregate losses can be estimated using a combination of direct data (*ie* data from the risk under consideration) and ancillary data (*ie* data from other similar, but not identical, risks).

$$\text{Credibility weighted estimate} = Z \times \text{Observation} + (1-Z) \times \text{Other information}, \quad 0 \le Z \le 1$$

Standards for full credibility

If the goal is to be within a proportion $\pm k$ of the mean, μ, with a probability of at least P, then for $\Phi(y) = \dfrac{1+P}{2}$, the standard for full credibility is:

Frequency (Poisson) $\quad n_N = \dfrac{y^2}{k^2}$

Frequency (general) $\quad n_N = \dfrac{y^2}{k^2}\left(\dfrac{\sigma_N^2}{\mu_N}\right)$

Severity $\quad n_X = n_N CV_X^2$ where $CV_X = \sigma_X / \mu_X$,the coefficient of variation of the claim size

Aggregate losses (Poisson frequency) $\quad n_S = n_N + n_X$

Aggregate losses (general) $\quad n_S = \left(\dfrac{y}{k}\right)^2 \left(\dfrac{\sigma_N^2}{\mu_N} + \dfrac{\sigma_X^2}{\mu_X^2}\right).$

Partial credibility

If n is the (expected) number of claims for the volume of data and n_F is the standard for full credibility, then for $n > n_F$, $Z = 1$ and for $n \le n_F$:

Classical formula (square root rule) $\quad Z_C = \left(n/n_F \right)^{1/2}$

Bayesian formula $\quad Z_B = \dfrac{n}{n+k}$ (expressed in units of claims).

For $Z_B \approx Z_C$, choose n_F to be about 7 to 8 times larger than the Bayesian factor k.

Bühlmann-Straub model

Definitions:

S_i represents the insurance claims for risk i

V_i represents the volume measure for risk i

$X_i = S_i / V_i$ represents the claims ratio for risk i

$\mu(\theta_i) = E\left(X_i \middle| \theta_i\right) \qquad \sigma^2(\theta_i) = V_i \times Var\left(X_i \middle| \theta_i\right)$

$\beta = E\left[\mu(\theta_i)\right] \qquad \phi = E\left[\sigma^2(\theta_i)\right] \qquad \lambda = Var\left[\mu(\theta_i)\right]$

Credibility factor: $z_i = \dfrac{V_i}{V_i + \phi/\lambda}$

Credibility premium: $C^{BE} = z_i X_i + (1 - z_i)\beta$

Practical considerations

Issues to consider when using credibility theory in practice include:

- simplicity
- visibility – consider imposing a maximum swing, or self-rating
- goodness of fit – *ie* accuracy *versus* simplicity
- level of grouping *versus* accuracy
- source of data – more years / more locations / national data *etc*
- stability of data – *eg* weightings based on numbers, not amounts
- use of partial premiums
- choice of credibility complement – accuracy / bias / independence from base data / availability / ease of calculation / relationship to risk
- the need to use considerable judgement when considering how to allow for large claims, trends and differing opinions of the correct rate.

Chapter 18 Solutions

Solution 18.1

We are given that

$$k = 5\% \text{ and } n = 1,000,$$

so:

$$\begin{aligned} P &= 2\Phi\left(k\sqrt{n}\right) - 1 \\ &= 2\Phi\left(0.05 \times \sqrt{1,000}\right) - 1 \\ &= 2 \times 0.9431 - 1 \\ &= 88.62\% \end{aligned}$$

Solution 18.2

$$\Phi(y) = \frac{1+P}{2} = \frac{1+0.9}{2} = 0.95.$$

Therefore:

$$y = 1.645, \text{ and } n_N = \frac{y^2}{k^2} = \left(\frac{1.645}{0.05}\right)^2 = 1,082.$$

This is consistent with the value given in the table of Core Reading that follows.

Solution 18.3

(i) If claim frequencies follow a binomial distribution, the variance will be lower than the mean. We will therefore require fewer data points in order to be reasonably sure that our observed frequency falls within a specified range around the true mean.

(ii) For $P = 70\%$ and $k = \pm 5\%$:

$$\Phi(y) = \frac{1+P}{2} = 0.85$$

So:

$$y = 1.0364$$

and:

$$n_N \geq \frac{y^2}{k^2}$$

$$= \frac{1.0364^2}{0.05^2}$$

$$= 429.65$$

In other words, at least 430 claims observations are required to assign full credibility.

(iii) The underlying claim frequency is believed to be 0.3 claims per year.

This is consistent with the claim frequency assumed under the previous binomial model. An insurer will naturally want to test the appropriateness of different models to its data, as we are doing here.

Our standard for full credibility, in terms of number of claims is 430, and:

$$\text{Exposure} \times 0.3 = 430.$$

Rearranging gives:

$$\text{Exposure} = \frac{430}{0.3} = 1,433\,.$$

Solution 18.4

The standard for credibility is a function of the ratio between the mean and the variance of the underlying frequency distribution. The greater the variance (as a proportion of the mean), the higher the volume of data required for full credibility.

Solution 18.5

So far, we know that the probability that our observed average claim size is within a proportion $\pm k$ of the true mean μ_X is:

$$P = Prob\left(-k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right) \le Z \le k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right)$$

where Z is standard normal.

Therefore:

$$P = \Phi\left(k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right) - \left[1 - \Phi\left(k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right)\right]$$

$$= 2\Phi\left(k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right) - 1$$

which is exactly analogous to equation (18.1).

Rearranging gives:

$$\Phi\left(k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)\right) = \frac{1+P}{2}$$

which is exactly analogous to equation (18.3).

In other words:

$$y = k\sqrt{N}\left(\frac{\mu_X}{\sigma_X}\right)$$

Solution 18.6

The full credibility standard for claim frequency is:

$$n_N = \frac{y^2}{k^2} \text{ where } y = 1.645 .$$

Hence:

$$n_N = \frac{1.645^2}{0.05^2} = 1,082 .$$

Alternatively, you could have looked at the table in Section 2.2.

We find the mean μ_X and variance σ_X^2 of the lognormal distribution:

$$\mu_X = e^{\mu + \frac{1}{2}\sigma^2} = e^{3.615 + \frac{1}{2} \times 1.980} = 99.983 \text{, and}$$

$$\sigma_X^2 = \left(e^{2\mu + \sigma^2}\right)\left(e^{\sigma^2} - 1\right) = \left(e^{2 \times 3.615 + 1.980}\right)\left(e^{1.980} - 1\right) = 62,406 .$$

So the coefficient of variation is:

$$\frac{\sigma_X}{\mu_X} = \frac{\sqrt{62,406}}{99.983} = 2.499$$

And the required credibility standard is:

$$\begin{aligned} N &= n_N CV_X^2 \\ &= 1,082 \times 2.499^2 \\ &= 6,757. \end{aligned}$$

Solution 18.7

We know the probability that the observed aggregate loss is within a proportion $\pm k$ of the mean μ_S is:

$$\begin{aligned}P &= \text{Prob}\left[\mu_S - k\mu_S \le S \le \mu_S + k\mu_S\right] \\ &= \text{Prob}\left[-k\left(\frac{\mu_S}{\sigma_S}\right) \le z \le k\left(\frac{\mu_S}{\sigma_S}\right)\right]\end{aligned}$$

where $z = \dfrac{(S-\mu_S)}{\sigma_S}$ is a standard normal variable. (This assumes that the aggregate loss follows a normal distribution with mean μ_S and standard deviation σ_S.)

Then:

$$\begin{aligned}P &= \Phi\left(k\left(\frac{\mu_S}{\sigma_S}\right)\right) - \Phi\left(-k\left(\frac{\mu_S}{\sigma_S}\right)\right) \\ &= \Phi\left(k\left(\frac{\mu_S}{\sigma_S}\right)\right) - \left(1 - \Phi\left(k\left(\frac{\mu_S}{\sigma_S}\right)\right)\right) \\ &= 2\Phi\left(k\left(\frac{\mu_S}{\sigma_S}\right)\right) - 1\end{aligned}$$

Rearranging:

$$\Phi\left(k\left(\frac{\mu_S}{\sigma_S}\right)\right) = \frac{1+P}{2}$$

so that:

$$\Phi(y) = \frac{1+P}{2} \text{ where } y = k\left(\frac{\mu_S}{\sigma_S}\right).$$

Solution 18.8

We use the standard results for the mean and variance of a compound Poisson distribution.
Recall from Subject CT6 that the mean and variance of a compound distribution are:

$$E(S) = E(N)\mathrm{E}(X)$$

and

$$\mathrm{var}(S) = E(N)\mathrm{var}(X) + \mathrm{var}(N)[E(X)]^2$$

(We are assuming that claim numbers and amounts are independent.)

Given $\mu_N = n_N$ is the expected number of claims, then:

$$\mu_S = \mu_N \mu_X = n_N \mu_X$$

and:

$$\sigma_S^2 = \mu_N \sigma_X^2 + \sigma_N^2 \mu_X^2$$

Substituting for μ_S and σ_S in equation (18.9) gives:

$$y = k\left(\frac{n_N \mu_X}{\left(\mu_N \sigma_X^2 + \sigma_N^2 \mu_X^2\right)^{1/2}}\right)$$

Solving for n_N (and using $\mu_N = n_N$):

$$n_N = \left(\frac{y}{k}\right)^2 \left[\frac{\mu_N \sigma_X^2 + \sigma_N^2 \mu_X^2}{\mu_N \mu_X^2}\right]$$

$$= \left(\frac{y}{k}\right)^2 \left(\frac{\sigma_N^2}{\mu_N} + \frac{\sigma_X^2}{\mu_X^2}\right)$$

Chapter 19

Actuarial investigations

Syllabus objective

(g) Outline the major actuarial investigations and analyses of experience undertaken with regard to pricing for general insurers, including the monitoring of business being written.

0 Introduction

In much actuarial work, we analyse data and draw conclusions from the results. We do most analyses on a regular basis. Often in an investigation, we will compare the expected forecast from the previous analysis with the actual observed experience since the analysis was performed.

You will often hear this referred to as an "actual *versus* expected" analysis.

Some of the investigations described in this chapter are covered in greater detail elsewhere in the course, so you should use this chapter to complement those other areas of the course.

This chapter sets out the key investigations that are regularly performed and others that may be needed reasonably frequently.

Section 1 describes the steps involved in performing a rating analysis.

Section 2 discusses the analysis of expenses and considers how the expenses can be sub-divided into different types. This may be familiar to you from Subject CA1.

Section 3 looks at monitoring the business that is sold, in terms of business volumes and persistency, allowing for any changes to premium rates that have occurred.

Finally, Section 4 suggests how these types of investigation might be used in practice.

We note that there is no limit to the investigations that a general insurance actuary can be called on to perform. It is important for the actuary to be clear on the question being asked and the data available. The analysis that can be undertaken will depend on the available data and should be appropriate to the question being asked.

1 Rating analyses

We need to determine the premiums to be charged over a forthcoming period both for an established class of business for which there is an existing rating structure and existing underwriting guidelines, and for a new class of business being underwritten for the first time. In the former case, we need to allow for changes to policy conditions over the period of the data being used as a base; for example, policy excesses may have increased.

In other words, if we are reviewing the premium rates for an existing class of business, we will use policy and claims data relating to our existing business, but may need to adjust this data for past and future changes in policy conditions.

We will discuss results with management and underwriters, so we should document and explain the key features of the results, including key assumptions and areas of uncertainty.

In these circumstances, when we estimate the premium rates needed to meet the insurer's profit objective, we will go through the following steps:

- **Estimate the cost of claims incurred in recent periods as an intermediate step in the rating process.**

  The new premium rate for a policy will be based on the expected cost of claims for that policy. We estimate this using recent claims experience from the existing portfolio

  We will base any pricing analysis on past data, and we will have to project claims to their ultimate level in order to get a realistic view of recent past claims experience.

  For example, if we are setting premium rates for business to be sold in 2012 based on the claims experience in the years 2009–2011, some of the claims will not yet have been settled, or even reported. An estimate of the ultimate claim amount will therefore be required.

- **Estimate the profitability of the existing premium rates by reference to the recent claims experience, adjusted if necessary for any abnormal features.**

  In this analysis, we will look at the claims corresponding to the premium and compare actual and expected experience. We should analyse the reasons for any differences between the two.

This is important for understanding how our assumptions can be revised to make them more accurate, and is an application of the actuarial control cycle, covered in Subject CA1.

We are likely to express the results not only in money terms, but as a return on capital employed in that class of business.

The company will have a target level of profitability that it hopes to achieve, relative to the capital resources it uses. It will therefore want to know whether or not the existing rating structure is achieving that level of profitability.

In excluding abnormal experience, we should bear in mind the limitations of statistical models in explaining the real world.

The existence of process uncertainty, model uncertainty and parameter uncertainty means that we should take care when excluding *abnormal* claims in case these turn out to be *normal* claims after all.

Process uncertainty, model uncertainty and parameter uncertainty were discussed in Chapter 9 and also in Subject ST7.

- **Project forward to the period over which the new rates will be charged and the corresponding claims will be settled.**

  In doing this, we make assumptions about future claims trends. This introduces a further element of uncertainty into the investigation.

  Our projections will need to allow for claims inflation as well as trends.

- **Review the suitability of the existing rating structure, perhaps with the aid of a sophisticated rating model, and consider possible changes.**

  The term *rating structure* refers to the relative levels of premium charged to different policyholders, depending on their particular risk profile. For example, in motor insurance, a young male driver will usually be charged a very different premium to a middle-aged woman.

  If the rating structure is inappropriate, it may lead to anti-selection.

  This is an in-depth investigation, and will be the most difficult to explain to non-actuaries / non-statisticians. However, it will have the most impact on the final prices charged. We may use one-way, two-way or multivariate analyses for this.

Many sophisticated multivariate rating analysis models have been developed by general insurance consultants, and their software packages can be purchased in order to analyse a company's data.

Multivariate analysis is covered in Chapter 17.

- **Compare final rates with those of competitors, using external data.**

  In making the final pricing decision, we will be strongly influenced by what the market is doing and will aim for an overall profit level, with some parts of an account expected to make more money than others where we believe the market is overpricing risks.

  In other words, our premium rates must make allowance for our competitors' rates and for the potential for cross-subsidies to exist between different areas of the account.

  There is a pure risk cost (including direct expenses) that we must always keep in mind, in particular when comparing actual and expected results.

  The pure risk cost is the amount of premium required to cover the expected cost of claims. Sometimes the pure risk cost will also include expenses directly attributable to the contract. This was covered in more detail in Chapter 12. Note that the final premium will also need to allow for other items such as indirect expenses, profit, tax, reinsurance costs, *etc.*

- **Apply adjustment for lifetime value.**

  Adjustments might be applied to the technical premium in consideration for the value added by the ongoing loyalty of particular policyholders.

  It is normally more cost-effective for an insurer to renew an existing policy than to write a new one and so loyal customers (who may stay with the insurer for many years) are valuable to the insurer. These customers may therefore be rewarded by a reduction in their premium.

- **Analyse profitability of old years on new rates.**

  The new rates should give us our most up-to-date view of risk and so applying this to old data will give us an up-to-date view of profitability.

Other more detailed aspects of the steps in rating are considered in Chapter 13.

The scope for detailed modelling is usually greatest in the case of personal lines business, where the number of policies in force is often sufficient to support such analyses. However this is not always the case; for example, an insurer embarking on personal lines for the first time may have considerably more experience and data for commercial lines.

So far, we have considered the situation where an insurer already has premium rates and data from existing business, which it can use for its rating analysis.

When an insurer is about to enter a new market or introduce a new type of policy, it will often have no suitable data from its own experience. In these circumstances, we may be able to use some relevant external data or internal data from related accounts, but in general, we will be obliged to adhere closely to existing market practice where we can identify that. This is an area where we need to seek out external and internal data sources, and adjust as necessary to make them relevant.

If an insurer doesn't have any suitable data from its past experience, it will generally not want to deviate far from the rating structures or levels of rates used by other insurers in the same market.

As alternative options, we could:

- **ask reinsurers**
- **partner with another insurer on a quota-share basis and use their data**
- **use market data such as that compiled by Lloyd's of London**.

Question 19.1

List the possible sources of the "relevant external data".

The rating process was covered in much more detail in earlier chapters of the course.

2 Expense analyses

Owing to the short-term nature of contracts and the relatively high volume of claims and endorsements, expenses form a very significant element of an insurer's total outgo, especially if the business written suffers from low persistency rates.

An endorsement is an amendment to a policy during the term of the policy. Endorsements are sometimes called "mid-term adjustments".

It generally costs more to write a new policy than to renew an existing policy. Therefore, if persistency rates are low, the insurer will be writing a high proportion of new business, leading to higher expenses overall.

In an analysis of expenses, we are mainly concerned with allocating the insurer's expenses correctly between the different classes and rating groups in the portfolio. This enables us to measure the past performance of each class and determine a level of expense allowance in any future premium-rating exercise, after adjusting for inflation.

An expense analysis is also an important part of an insurer's financial plans.

If management know where and how expenses are being incurred then they can introduce effective measures to control the expenses.

In statutory returns, we are required to show specific details of expenses, including an allowance within the technical liability calculations for deferred acquisition costs and claims handling expenses. An expense analysis, therefore, should include sufficient detail for this purpose.

Question 19.2

Define deferred acquisition costs.

For the deferred acquisition cost calculation, we should analyse administration expenses between:

- **those incurred on inception of the policy**
- **those incurred throughout the policy year in general administration support**
- **claims** handling **costs.**

This enables us to express those expenses incurred at inception as a percentage of the corresponding written premiums.

We will compare the expenses incurred in writing the class of business, in its administration and in the payment of claims with the assumptions in the pricing basis, and make revisions to the pricing assumptions **where appropriate.**

2.1 Subdividing the data

Direct vs indirect expenses

The concept of direct and indirect expenses should be familiar to you from Subject CA1.

In theory, we can divide the expenses of an insurance company into:

- **direct – the expenses that we can allocate accurately to individual policies, whether new business acquisition or the administration of business on the books**

  Direct expenses are those that are incurred directly as a result of providing insurance cover and that can therefore be directly allocated to a class of business or to an individual policy (*eg* underwriting costs, commission and claims settlement expenses).

- **indirect or overheads – the balance of the expenses; that is, those that relate to general management and service departments that are not directly involved in new business acquisition or policy maintenance activities, and that are insensitive to either the volume of new business or the level of business on the books**.

  Indirect expenses relate to support functions and therefore cannot be directly attributed to any one class of business or policy (*eg* computing costs, human resources department and general management costs).

In practice, there is not a clear dividing line between these two categories.

To begin with, we normally exclude commission from the expenses because its format is known and we can add it in later by a formula approach.

Commission is often expressed as a percentage of written premium, although it can be expressed as a fixed absolute amount per policy, or as a combination of the two.

For the purpose of an expense analysis, we can split the non-commission expenses into:

- **initial expenses, which arise when business is being acquired and written into the books of the insurer**
- **administration expenses, which arise at times during the policy year (or other such policy term)**
- **renewal expenses, which are incurred in the recosting, renewing or lapsing of a policy at the end of the policy period; these are often markedly different to initial expenses**
- **claims expenses, which are incurred in the assessment and payment of policyholder compensation**
- **investment expenses, which relate to the management of the company's assets.**

Question 19.3

State whether each of these five types of expense is direct or indirect.

We can split each of the first three further according to whether the expense is proportional to:

- **the number of contracts being acquired or on the books**
- **the amount of sum insured or Estimated Maximum Loss**
- **the amount of premiums being acquired or on the books.**

So the initial, administration and renewal expenses can be expressed as an amount per policy or as a percentage of either the sum insured or the premium.

We find in practice that most of these expenses are proportional to the number of contracts in force. Exceptions include:

- **underwriting expenses – mainly related to the amounts of new business premium**
- **claims expenses – typically related to the level of claims payout.**

Fixed vs variable expenses

We can also divide the expenses of an insurance company into:

- fixed – the expenses that remain relatively fixed, regardless of how many policies we sell; for example, rental costs for the building or the costs associated with building maintenance
- variable – expenses that vary according to the amount of insurance business being handled. These expenses may be linked to the number of policies or claims or to the amount of premiums or claims. Isolating variable expenses is particularly important for premium rating purposes.

Note that all variable expenses are direct, but fixed expenses can be direct or indirect.

Question 19.4

Why is it important to be able to isolate variable expenses for the purposes of premium rating?

In practice, all expenses can vary in the long term. For example, the salaries of administrative staff are generally considered to be a fixed cost but, if the company wants to scale back its operations, it could implement redundancies to reduce its staffing levels and costs over time. So the concept of fixed costs makes most sense if we confine it to the short term. Property costs and salary costs do not generally vary with the amount of business in the short term.

Question 19.5

Give an example of a variable cost that might be related to each of the following:

- number of policies
- number of claims
- premium amount
- claim amount.

There are no fixed rules as to the boundary between fixed and variable costs. Some costs could easily fall into either category. For example, the cost of processing a new policy might be described as a fixed cost because you would not hire an extra member of staff or pay higher salary costs as a result of the new policy. On the other hand, if marginal sales of policies meant paying staff bigger bonuses or making overtime payments, then the cost of processing a new policy is a variable cost.

The insurer will want to analyse which costs are attributable to each class of business, *eg* in order to include an accurate allowance for expenses in its premium rates. The problem is that indirect expenses cannot be easily attributed to any one class and so approximations will be required. These are discussed in Section 2.2 below.

2.2 The process

We will need to split the expenses down and analyse them into the required "cells".

Typically the cells may be:

- **the whole business of the insurer**
- **the whole business of a particular accounting fund**
- **each main product line of the insurer.**

> ***Question 19.6***
>
> Why would a general insurer want to allocate expenses to different product lines?

The choice of cells will vary across offices. It depends upon the types and volumes of business written, and the requirements of the analysis. The cells chosen should not be so small that the analysis becomes unreliable.

Equally, the cells should not be so large that the analysis becomes of little use for the required purpose.

The main items of expense are:

- **salaries and salary-related expenses**
- **property costs (rent, property taxes, heating, lighting and cleaning)**
- **computer costs**
- **investment costs (investment department, stamp duty, commission and so on)**
- one-off capital costs
- claim handling costs.

We now discuss how to apportion each of these items to individual product lines.

There is no definitive approach to splitting these. We describe one possible approach:

Salaries and salary-related expenses

These expenses cover staff salaries, and also related items such as pensions costs and national insurance contributions.

A large part of the expenses are staff-related, owing to the labour-intensive nature of administering the business. In the short term, much of this may remain fixed in real terms. In the longer term, staff costs (and accommodation costs) will vary to meet changing levels of business being written, changes in services provided and the degree of automation used to provide those services.

We can split staff into three groups:

(i) staff whose work falls entirely within a single cell of the analysis

(ii) staff whose work falls within more than one cell

(iii) other staff.

The other staff in (iii) are likely to be working wholly or partly in support functions that do not relate directly to providing insurance cover (*eg* in IT, HR or finance departments).

We can directly allocate the salaries and related costs of staff in (i) to the appropriate cell.

For group (ii), we can use staff timesheets to split their salaries and related costs between the appropriate cells.

The work of the group (iii) staff will straddle both overheads and direct expenses.

For example, some of the IT staff might be assigned to working solely on computer projects within specific lines of business or within specific departments, whereas others might work on projects that are on a company-wide basis.

We are likely to make a pragmatic split between the two. We can split the direct part further in proportion to the overall split of the group (i) and (ii) staff.

In other words, we first split the group (iii) costs into direct and indirect expenses, and then further split the direct portion of these into each cell, in proportion to the overall split of groups (i) and (ii).

Property costs

We should charge an actual rent (if the insurer rents office space) or a notional rent (if the insurer owns any of the buildings that it occupies) to the relevant departments.

> ***Question 19.7***
>
> If a company owns the property, why would we charge a notional rent rather than allowing for the actual purchase costs?

Of course, if the company actually pays rent to a third party then we would use this amount in our expense allocation.

We can split this rent, plus property taxes, heating costs and so on, for example, by floor space occupied, between departments. We can then allocate the departments' property costs between cells in the same proportions as the departments' salaries.

Computer costs

We could amortise the cost of purchasing a new computer over its useful lifetime and then add it to the ongoing computer costs. We can then allocate these according to computer usage.

The term "amortise" is used here to mean "depreciate" although, strictly speaking, it should be used in reference to intangible assets.

Investment costs

These are the investment expenses that would be deducted from any gross rate of return achieved on assets before deciding what rates of return might be used in pricing and setting liabilities and in any comparative analysis of investment or asset class return.

So the investment return used for pricing or other analyses would be reduced by the amount of the investment expenses.

One-off capital costs (other than purchase of a new computer)

The purchase of a new computer was dealt with above.

The expenses analysed exclude large one-off capital costs that we should amortise over the expected useful lifetime of the item purchased. We may then treat the amortised cost as part of the overheads.

If we can treat the item as an asset of the general insurance fund – for example, a new head office building – we would not amortise the cost. Instead we would usually make a charge, for example, a notional rent.

We would exclude exceptional items (which are not likely to recur) from the analysis.

Remember that we will be setting premium rates for the future, so we would only want to include likely future expenses and not exceptional past expenses.

Claim handling costs

We should analyse claim handling expenses between direct internal expenses, indirect internal expenses and those directly related to specific claims. We can then calculate the direct claim expenses as a percentage of the corresponding cost (or number) of claims handled in the period as required.

Direct claim handling expenses, and those related to specific claims, can be easily allocated to product lines. We can apportion all the other (indirect) claim handling expenses according to the number or amount of claims arising from each product line.

3 Analysis of policy experience

General insurance organisations typically invest much time and resource into analysing the business they have recently written. The aim of this section is to outline the reasons why these organisations do this, what key statistics are typically monitored, and how they might do it.

3.1 Reasons for monitoring business written

The main reasons for monitoring the business written, covered in this section, are to:

- assess performance against goals
- manage risk
- gain market intelligence
- satisfy the regulators
- influence the market
- assist with reserving
- validate assumptions as part of the actuarial control cycle.

These reasons are now covered in more detail.

Assessing performance against the organisation's goals

The ultimate goal for most general insurance companies is to exceed a minimum level of profit or return on equity for a given level of risk. However, companies will break this objective down into more specific targets. The hope is that if these individual targets are met then so will the overall company objective.

Breaking the objective down into specific targets enables different parts of the business to be monitored more effectively. If one target is not met, this can be investigated and rectified, whereas an overall profit analysis would hide any underperforming areas behind other areas where targets are being exceeded.

Examples of targets might include:

- a minimum level of new business, by policies or premium income
- a minimum level of persistency
- a maximum loss ratio or expense ratio.

A general insurance company will monitor the business it has written to gauge its performance against these targets. This enables informed planning and decision making.

The contents of a financial plan are discussed in more detail in Subject SA3.

Managing risk

Monitoring written business allows the company to assess how much risk is inherent in the portfolio (for example, accumulations). The amount of risk will be a factor in determining how much capital the company should hold and what its reinsurance purchasing strategy should be.

A risky portfolio will generally require more capital and more reinsurance, although to a certain extent capital and reinsurance are interchangeable (but not for some purposes, *eg* legislative reasons).

Determining appropriate reinsurance for a particular portfolio is covered in Subject ST7. Capital modelling is also covered in Subject ST7.

Gaining market intelligence

Monitoring written business can provide useful information about competitors' strategies. It can also allow the company to compare itself with the market and assess the underwriting cycle.

For example, a sharp decline in volumes written may indicate that competitors have reduced their premium rates or made their coverage terms more generous.

Satisfying regulators

Market regulators may require periodic monitoring and reporting of written business.

Influencing the market

A company may be able to influence the market by publishing the results of its monitoring exercises.

Question 19.8

Give an example of how an insurer's results might have an impact on market behaviour.

Reserving

The outputs of any monitoring exercise can be used for other purposes such as an input into the reserving process. Considered in isolation this would not necessarily be a reason to monitor written business. The most common example is the use of rate indices (derived from the monitoring exercise) to adjust *a priori* loss ratios (often called initial expected loss ratios) in Bornhuetter-Ferguson reserving methods.

Reserving is covered in detail in Subject ST7. However, you might remember from Subject CT6 that the Bornhuetter-Ferguson method includes an independent assumption of the ultimate loss ratio, which is used with the development pattern derived from a chain-ladder method in order to project claims to date to ultimate.

The choice of the *a priori* loss ratio is important, and it may be that a premium rate index (discussed later in this section) can assist with this assumption.

Part of the actuarial control cycle

Another reason for monitoring would be to validate assumptions in a model.

You should remember the actuarial control cycle from Subject CA1 if you have studied it.

By comparing actual experience with that expected, we can assess the need to adjust our assumptions, for example, in our premium model.

3.2 Key factors that are monitored

In this section, we describe how we can monitor:

- premium rate changes
- portfolio movements:
  - lapses at renewal / renewal rates
  - new business volumes
  - quotes that result in written business (called the *strike* or *conversion rate*)
  - mid-term cancellations
  - policy endorsements
  - mix of business
- volumes of quotations
- persistency and profitability by source.

Premium rate changes

In broad terms, the premium rate is a measure of how profitable a policy or segment of business is expected to be. In principle, if premium rates and business volumes are high enough then the insurance company can expect to make a profit. Therefore, the premium rate is a key statistic to monitor.

If we can be clear about how we're defining a premium *rate* (*eg* as a percentage of claims) then monitoring becomes more meaningful and should be a better indication of profitability.

Definition

There are many definitions of premium rate used in the general insurance market. Examples are:

- **premium income per unit of expected loss**
- **premium per unit of limit**
- **premium per unit of exposure**
- **premium per unit of risk-adjusted exposure.**

The suitability of each definition depends largely on the type of business being monitored. For example, liability classes often do not have a limit and so the second example above would not be appropriate.

Question 19.9

Discuss which of the above definitions might be best for monitoring domestic household business.

Even *premium* within the above examples can have many definitions.

Premium can be gross or net of commission. It may or may not include the effects of trends on claims. The definition used often depends on

- **the purpose of the analysis (for example, to monitor profitability we would usually require premium net of commission; for Bornhuetter-Ferguson reserving we would require premium gross of commission)**
- **the available data (for example, exposure may not be recorded at a policy level).**

Question 19.10

Explain the choice of net / gross in the two examples given in the first bullet point above.

Premium rates can be calculated at a policy level or at an aggregated portfolio level.

The choice would depend on the purpose of the analysis. For example, for a rough approximation of relative overall profitability year-on-year, we might use premium rates at an aggregated portfolio level.

In order to analyse the change in premium rate over time, we need to construct a premium rate index.

The change in premium rate from time t_1 to t_2 is defined as:

$$\text{Rate change}_{t_1 \to t_2} = \frac{\text{Premium Rate}_{t_2}}{\text{Premium Rate}_{t_1}} - 1$$

A series of premium rate changes can easily be expressed as an index.

The index can start at an arbitrary point, say, 100. The subsequent rate changes can then be expressed in terms of this starting point.

Calculating premium rate changes

There are many ways to calculate rate changes. Some examples are described below.

We will describe four such methods, namely:

- direct calculation for each risk separately
- direct calculation using a "standard" risk
- measuring rate changes on individual renewals
- using underwriters' views.

Direct calculation of the premium rate

This is the most obvious method – that of simply calculating the premium rate for every policy, then comparing the results from one time period to the next.

This method requires the use of actuarial techniques to assess the expected loss and hence the premium rate at different points in time.

The advantages of this method are that many factors affecting the expected loss can be taken into account and the absolute (*ie* not relative) premium rate is calculated in addition to the rate change.

Quantifying the effect of softer factors may be difficult (for example, subtle changes in terms and conditions, risk management changes).

An example of a "subtle" change in terms and conditions might be a requirement for all policyholders to make all claims over the internet rather than by telephone. This may affect the reporting delays and/or likelihood of claiming in an unquantifiable way.

An example of a change in risk management would be the use of television advertising to encourage safer driving.

Price a standard risk

This is a similar method to the direct calculation method described above. However, the premium rate is calculated for a specified "standard" risk or sample of risks which reflect the business mix of the portfolio as a whole.

This method is simpler, quicker and less data onerous than calculating the expected loss for every risk written. However, the method requires the additional assumption that the rate change for the standard risk is equal to the rate change for the entire portfolio.

Measure rate changes on individual renewals

For some segments of the general insurance industry it may be very difficult to determine accurately the absolute (*ie* not relative) level of the premium rate. This is because of the technical problems in assessing the expected loss for a heterogeneous book.

This might be the case, for example, with a book of large unique commercial property risks. In a heterogeneous book of this kind, it is difficult to estimate the expected losses for a particular risk because there is not a sufficient number of similar risks. In other words, there are a lack of credible data for rating.

We address this problem by considering only the renewing policies and estimating the *change* in expected losses for these policies without determining the expected losses themselves.

The aim of this method is to express the premium rate at t_2 as a proportion of the premium rate at t_1 for every renewed policy. If a rate change is calculated in this way then we do not need to know the absolute level of the premium rate.

We can express this mathematically by identifying factors that are proportional to the expected loss. In the following example we assume that the expected loss for a policy depends on the limit / attachment, the coinsurance share and the exposure measure. All other factors affecting the expected loss are assumed to be constant from t_1 to t_2.

$$E\left(Loss_t\right) \propto \left(ILF\,@\,Lim_t - ILF\,@\,Attach_t\right) \times Share_t \times Exp_t$$

where:

- $ILF\,@\,Lim$ **is the increased limit factor at the upper policy limit**
- $ILF\,@\,Attach$ **is the increased limit factor at the policy attachment point**
- Exp **is the exposure of the policy.**

You should remember the concept of increased limit factors from Chapter 15.

The ratio of expected losses for a risk which originally incepted at time t_1 and renews at t_2 is therefore:

$$\frac{E\left(Loss_{t_2}\right)}{E\left(Loss_{t_1}\right)} = \frac{\left(ILF\,@\,Lim_{t_2} - ILF\,@\,Attach_{t_2}\right)}{\left(ILF\,@\,Lim_{t_1} - ILF\,@\,Attach_{t_1}\right)} \times \frac{Share_{t_2}}{Share_{t_1}} \times \frac{Exp_{t_2}}{Exp_{t_1}}$$

and:

$$\frac{Prem\ Rate_{t_2}}{Prem\ Rate_{t_1}} = \frac{Prem_{t_2}}{Prem_{t_1} \times \dfrac{\left(ILF\,@\,Lim_{t_2} - ILF\,@\,Attach_{t_2}\right)}{\left(ILF\,@\,Lim_{t_1} - ILF\,@\,Attach_{t_1}\right)} \times \dfrac{Share_{t_2}}{Share_{t_1}} \times \dfrac{Exp_{t_2}}{Exp_{t_1}}}$$

The denominator on the right-hand side is sometimes known as the "as-if" premium.

This is because it is *as if* the premiums from time t_1 were being applied at time t_2. An as-if basis is also commonly called an "on-level" basis.

It represents the premium that would have been charged for the renewal at t_2 if the premium rates at t_1 had applied.

This model can be easily extended to allow for other factors (for example, claim inflation in excess of exposure change, change in rating factors and change in policy duration).

The rate change for a group of renewed policies can be expressed as:

$$\textit{Rate Change}_{t_1 \to t_2} = \frac{\sum \textit{Prem}_{t_2}}{\sum \textit{As - if Prem}_{t_1}} - 1$$

The main disadvantage with this method is that the impact of new and lost business written at different premium rates is ignored. It can also be difficult to quantify some of the "soft" factors.

Underwriter's view of the rate change

This method involves recording how the underwriters perceive premium rates to be changing.

The main advantage of this method is that it can allow for more of the soft factors mentioned above that would otherwise be unquantifiable.

The main disadvantage is that it is very subjective and therefore difficult to ensure consistency. It is also difficult to assess across companies and verify in detail analytically. There may also be confusion around pure rate change and mis-pricing. For example, an underwriter may say that premiums have reduced because there have been no claims, thus a zero rate change. We would define this as a rate decrease.

Portfolio movements

A "movement" refers to a policy going on risk, off risk, or moving between risk groups. Movements can be viewed for a whole portfolio, or split by:

- class of business
- policy cover level, *eg* third party, comprehensive
- risk group.

We usually monitor the movements in respect of:

- **lapses or renewals** failure to renew, or renewal of, an existing policy
- **new business** inception of a new policy
- **quotes not taken up** where a quote is given but the policy is not incepted
- **mid-term cancellations** where a policy is stopped mid-term
- **endorsements** where a policy is amended during the policy year, also called a "mid-term adjustment".

An insurer will also wish to monitor the mix of business in its portfolio.

Changes in the total number of policies within a portfolio can be explained by adding and subtracting the movements:

number at end = number at start + new business – lapses – cancellations.

If we were analysing the number of policies within a risk group, the right-hand side would include adjustments for policies moving by endorsements into or out of that group.

Movement rates can therefore be used as a check that all the data are accounted for and can indicate errors in the data.

Question 19.11

List some endorsements that might take place on a private motor policy.

By studying the movements of the business, and their trends, we can:

- **measure the extent to which different parts of the portfolio are growing or contracting**
- **get an early indication of undue losses or gains in business that might indicate that rates are out of line with the market**
- **assess the effects of a new set of rates or marketing campaign on the business and, hence, the sensitivity of the portfolio to market influences.**

The movements give an important early warning on adverse changes. They might indicate that we need to review premium rates for certain risk groups. Hence we will do a movement analysis frequently.

Since movement rates relate to rates of change, they are good to look at to help spot trends at the earliest opportunity. This can give an initial guide as to where the premium rates might be out of line with the rest of the market. In a competitive class (such as personal lines motor), a high lapse rate for a particular rating cell suggests that the rates for that cell are high. Conversely, a high new business rate indicates very competitive rates.

We usually calculate movement rates by reference to a suitable measure of exposure, using the principle of correspondence; for example, by number of policies or by currency unit of premium.

We now describe each of the movement rates in turn:

Lapses at renewal / renewal rates

A company may not want to renew all of its in-force business. Lapses at renewal of a policy can (by definition) only stem from those policies actually invited for renewal in that period. The best way to measure lapses is therefore to express them as a percentage of the renewals invited in that period.

$$\text{Lapse rate} = \frac{\text{no. of lapses for period } x}{\text{no. of renewals invited for period } x}$$

So we would use the number of renewals invited rather than the number of policies coming up for renewal, to ensure correspondence between the numerator and the denominator of the calculation.

The renewal rate is usually defined as the number of renewals divided by the number of expiring policies in a given period (although this can be adjusted for cancellations).

Question 19.12

How would you adjust this for cancellations?

The renewal rate is the complement of the *lapse rate* because all policies invited for renewal either lapse or renew (although this depends on how you treat those upgrading or reducing cover).

The company will want to monitor its loss experience because it will be keen to renew its more profitable policies. If policyholders are more likely to shop around, the company will need to ensure that it offers competitive rates on renewal to those policyholders that it wants to keep.

Issues to consider when analysing lapse rates include:

- **We may do the analysis at a total level and on specific subgroups.**
- **We will normally monitor lapses on a monthly basis. We should be careful to allow for the time lag between the renewal date and the point at which it becomes apparent that a policy has lapsed. We might estimate this lag from recent past experience, but we cannot necessarily assume that such an estimate is accurate for current experience.**
- **If there are processing delays, we may not know the final number of lapses stemming from a particular month of renewal until some months later. For this reason, insurers usually impose a deadline for acceptance of renewals. We can estimate the ultimate number of lapses from a particular cohort of policies using standard chain ladder techniques.**
- **We can obtain lapse rates by projecting the number of lapses processed to date for each month's renewal cohort to their ultimate value, using normal chain ladder techniques.**

New business

We have the same problems in estimating new business rates as we have for lapse rates. Delays occur because of internal processing delays or, more significantly, intermediary delay. The delays are likely to be shorter than for lapses, but we can still do a projection by month of notification.

We can measure new business rates in similar ways to those for lapse rates.

The major difficulty in this case, however, is in relating new business to a relevant measure of exposure. Normally, the best solution will be to relate new business incepting in a particular month to the corresponding number of renewals invited in that month (that is, the same base as we use to measure lapses).

$$\text{New business rate} = \frac{\text{no. of new policies for period x}}{\text{no. of renewals invited for period x}}$$

In this way, we can screen out normal seasonal variations in the volumes of business transacted and we can compare the new business rates more readily with the corresponding lapse rates.

If we define the new business rate in this way then we can add it to the renewal rate to give a quick indication of whether the business is growing or contracting.

We can also relate the number of new policies to the number of quotes given. However there will often be problems in defining the number of quotes (even if these have been recorded and the information is available). For example, some prospective policyholders will obtain multiple quotes, maybe for different cover levels or excess levels. For others, it may not be possible to receive a quote if their risk details fall outside the underwriting criteria set by the insurer, particularly where automated quote systems are used.

Not taken up / strike rates

The strike rate is usually defined as the number of written policies divided by the number of quoted policies in a given period (although this can be adjusted for declinatures).

The strike rate is often called the *conversion rate*.

The strike rate is the complement of the *not-taken-up* (NTU) rate, because all quotes will either be taken up or not.

Any quotes are usually only valid for around one month. This limits the delay problem we discussed above for renewals.

A declinature is where the insurer refuses to provide cover.

An insurance company will monitor renewal rates, strike rates and new business volumes because a change in any of these statistics may indicate that the company's pricing is out of line with the rest of the market.

This could result from a change in the company's or competitors' pricing strategy. Lower renewal rates, strike rates and new business volumes indicate that premium rates are generally higher than that of competitors and *vice versa*.

It is important to consider a company's targets for renewal rates, strike rates and new business volumes in conjunction with targets for premium rates.

Mid-term cancellations

Mid-term cancellations can, in theory, occur at any time to any policy in the portfolio, irrespective of renewal date, although in reality they tend to arise earlier in the policy year. Hence, the best exposure measure we can relate these to is the mean in-force portfolio over (say) the last 12 months.

$$\text{Cancellation rate} = \frac{\text{no. of cancellations during period } x}{\text{no. of policies exposed for period } x}$$

This gives a central rate, *ie* analogous to an m-type mortality rate.

Alternatively, as a rough approximation, we may relate them to the renewals invited in the last 12 months. This may be preferable, if we want to compare the cancellation rate more directly with the lapse and new business rates.

$$\text{Cancellation rate} = \frac{\text{no. of cancellations during period } x}{\text{no. of renewals invited for period } x}$$

This gives an initial rate, *ie* analogous to a q-type mortality rate.

When we use this method, we should be careful in the case of an expanding portfolio, because mid-term cancellations can also arise from new business and not just from renewals invited.

If we use the number of renewals invited in the last 12 months as the denominator then this will not allow for new business. However, the numerator *will* include mid-term cancellations on new business. Since an expanding portfolio will include a relatively high proportion of new business, the cancellation rate will appear to be correspondingly high.

Cancellation rates are normally relatively low in comparison with new business and lapses, so we can use simpler calculation methods than those for lapses and new business.

Policy endorsements

Endorsements occur internally within a class of business whenever the risk covered by a policy is changed.

Examples of endorsements include:

- **a change of address under a motor insurance policy**
- **an increase to sum insured for a domestic home contents policy following purchase of a valuable antique to be kept on the premises**
- **the installation of a market-leading water sprinkler system in a warehouse covered by a commercial fire policy, which may reduce the risk of a total loss.**

Some endorsements may leave the premium unchanged. Others may result in an increase or decrease in the premium.

There will not normally be any change in the total numbers of policies in force. However, there will be a change in the number of policies when we analyse movements by risk factor. Perhaps the easiest way of allowing for this is on the same lines as for mid-term cancellations. We have to create both an exit from the old sub-group and an entry into the new sub-group, with exits and entries measured separately or netted off.

So, for example, if a policyholder changes his 6 year old group 10 car for a brand new group 12 car mid-way through the policy year, the policy would earn half a year's exposure in the "group 10, age 6" risk group and half a year in the "group 12, age 0" risk group.

As for mid-term cancellations, the best measure of exposure is likely to be the mean in-force policy count.

$$\text{Endorsement rate} = \frac{\text{no. of endorsements during period } x}{\text{no. of policies exposed for period } x}$$

> ***Question 19.13***
>
> Check that you can define the various movement rates and explain why they are as you have defined them.

Business mix

For practical, legal and commercial considerations, policyholders are seldom charged the theoretically correct premium rate. This results in cross subsidy between groups of policyholders and exposes the insurer to the possibility of selection. A change in the mix of business between groups can result in a change in the overall level of profitability.

A change in business mix can also result in a change in the risk profile of the portfolio, so that a different risk management solution may be required.

For example, the reinsurance arrangements may need to be reviewed if the business mix becomes more concentrated.

An insurer will monitor the mix of business in its portfolio for both of these reasons.

Quote volumes

An insurance company will monitor the change in the number of quoted policies in a given period. This is especially useful when assessing the effectiveness of marketing campaigns, the level of support from different brokers and the impact of new distribution channels.

When monitoring the number of quotes, this will naturally lead to a need to monitor the strike rate, as above. For example, a rapid increase in quote volumes but a simultaneous drop in strike rates might indicate a successful marketing campaign combined with uncompetitive prices.

Analysis of persistency and profitability by source

An insurance company may sell business through different sources, *eg*:

- sales direct to the policyholder
- salaried sales staff
- brokers
- banks or other organisations
- other insurers (*eg* reciprocal arrangements).

The form of an analysis by business source will depend to some extent on the structure of the insurer. However, we should try to measure persistency and profitability by branch, broker / agency and direct business (telephone, internet, post and so on).

This should reveal from where the better quality, longer-lasting business comes, helping the insurer to avoid the less profitable sources of business or to devise incentive schemes or commission terms to retain or attract the more profitable business and reward better providers.

Question 19.14

Give examples of how analysing business by source could help an insurance company detect and avoid adverse selection.

We can analyse profitability by source in the same way that we analyse it by class, taking note of claims and expenses, provided the policy records contain a source identifier.

There is a close link between profitability and persistency as the expenses are usually greater for new business than for renewals. It may therefore be worthwhile to include the length of time that a policy has been on the books as a factor in the profitability analysis by source. This gives a further indication as to why a particular source is profitable or not and will help further in devising commission or incentive schemes.

The insurer may also be able to negotiate reductions to commission payments on business which proves to be less profitable, or concentrate on selling business through sources with lower commission amounts payable if the quality of such business is not markedly different.

Problems

In practice there are a number of problems to overcome when analysing the profitability of business from different sources. To form sensible comparisons between two sales outlets the data will need to be standardised to remove distortions caused by the mix of business.

The premium rating structure will usually result in some types of policy being more profitable than others. For example, premium rates may have proved to be inadequate in one geographical area or for one type of risk, *eg* sports cars in a motor portfolio. If so, business from a broker introducing lots of this type of policy would appear to have a relatively bad claims experience, through no fault of the broker. If the insurance company can "correct" its premium rating structure, then that broker's future business could be as profitable as any other.

Different intermediaries could be paid different levels of commission. The overall expense the company incurs may be reduced, though, where intermediaries do much of the work themselves. Building societies, for example, typically do much of the administration for the block policies they operate, and hence receive a higher rate of commission than other intermediaries. If possible, the insurance company should calculate the total net expense (including commission) to itself of the business from a given source.

To assess the profitability of a given block of business it is important to take a long-term view. Initial expenses are typically much higher than renewal expenses. Hence we would expect policies with a high degree of persistency to result in larger profits in the long term.

To analyse fully the profitability by source would require a profit testing assessment of the present value of future profits of the business from that source (in the same way as used by life insurance companies). In practice, however, few general insurers carry out analyses that are as sophisticated as that.

3.3 Key features of a good system for monitoring business

This section outlines some of the important IT and data issues for a monitoring system.

Systems for monitoring business typically include a data capture process, calculations and/or manipulations on the data, and a process for reporting the results. Desirable features for such a system are set out below.

Tailored output

Output should generally be concise ("information not data"), and tailored to the strategic goals of the organisation. Output should aid decision making.

Accuracy

- **Data used should be reliable and validated.**
- **Calculations should take into account all the key drivers.**
- **Results should be validated (for example, historical premium rate changes could be compared with changes in emerging loss ratios). This is part of the actuarial control cycle.**

Ease of use

- **Data should be easy to collect.**
- **Calculations should not be overly complex.**
- **System should be documented, extendable and low maintenance.**
- **Results should be clear and easy for users to interpret.**

Consistent

- **Output should be consistent over time. If, for example, methods for calculating rates change, then indices should be restated.**
- **Inputs should be consistent with other data sources. If two quantities are the same (for example, premium) then it should be the same regardless of data source. Otherwise the user will lose confidence.**
- **Outputs should be consistent with other analyses (for example, results should be shown split into the same business segments).**

Timely production of results

The delay between the data cut-off date and the production of results should be kept to a minimum.

4 Utilising actuarial investigations

We may use the results of the above types of investigation to:

- **Carry out a profit testing exercise – to do this, we model positive and negative income streams under sample policies, to assess both the timing and impact of cashflows. We can use the net income streams by month or by year to assess, for example, profitability for a given premium level, or the premium to be charged for a given profit criterion.**

  Profit testing involves projecting future income (from premiums and investments) and future outgo (expenses and claims) to give the expected profitability of a set of premium rates. The projection may also allow for any statutory solvency margin that might be required and for the strength of the statutory reserving basis. Profit testing is often used when setting premium rates.

- **Estimate price elasticity curves – to do this, we model customer behaviour in the light of potential premium rate changes and we may include an allowance for competitor reaction.**

  We can use price elasticity curves to estimate changes in business volumes as a result of changes to our premium rates. These are particularly useful for highly competitive classes of business, such as private motor insurance.

  One way in which insurers can investigate new business price elasticity is by increasing or reducing the premium quoted to a random sample of potential policyholders and looking at the effect this has on uptake.

- **Create lifetime pricing models – to do this, we estimate customer lifetime value. This is the inherent added worth of an existing customer to whom a policy has already been sold. It will take account of the policyholder's likelihood to renew, the policyholder's acceptance of future rate increases and the potential for cross-selling other products.**

- **Redesign rating tariffs – we estimate and recalculate premium rates on behalf of markets where the regulator decides the rates for certain segments and classes of business.**

- **Help to make other pricing and reinsurance decisions.**

- **Feed into other processes, such as capital modelling.**

General insurers, like other companies, prefer to make decisions based on information and data. Some will be regular decisions, such as what prices to charge or what reinsurance to buy. Others will be one-off decisions, such as whether to start writing a new class of business.

Before starting any investigation, we should be clear on the question being asked and the limitations of the data. Management will base decisions on these analyses, and it is important that they understand all relevant information, particularly in situations where the person preparing the analysis is not part of the discussion. Otherwise results can be taken out of context and used in ways that were not envisaged when the analysis was undertaken.

5 Finally

5.1 Glossary items

Having studied this chapter you should now read the following Glossary items:

- Allocated loss adjustment expenses (ALAE)
- Claims handling expenses
- Commutation
- Commutation account
- Commutation clause
- Endorsement
- Functional costing
- Fund (or funded) accounting
- Lapse
- Lapse rate
- Persistency
- Unallocated loss adjustment expenses (ULAE).

5.2 End of Part 5

You have now completed Part 5 of the Subject ST8 Notes.

Review

Before looking at the Question and Answer Bank we recommend that you briefly review the key areas of Part 5, or maybe re-read the summaries at the end of Chapters 18 and 19.

Question and Answer Bank

You should now be able to answer the questions in Part 5 of the Question and Answer Bank. We recommend that you work through several of these questions now and save the remainder for use as part of your revision.

Assignments

On completing this part, you should be able to attempt the questions in Assignment X5.

Chapter 19 Summary

There are a number of actuarial investigations that a general insurer should perform, including:

Rating analyses

The steps involved in a rating analysis include:

- estimating ultimate claims
- estimating profitability of existing rates
- projection forward to a new rating period
- reviewing the suitability of the existing rating structure
- comparing rates with those of competitors
- applying adjustments for lifetime value
- analysing the profitability of old years on new rates.

Expense analyses

In expense analysis, we:

- split between direct and indirect expenses
- split between types of expense (*eg* initial, admin, renewal, claims, investment)
- allocate by class and rating group
- express the expenses as a proportion of numbers of policies or claims, or of amounts of premium, sum insured or claim.

Direct expenses are those that can be directly allocated to a class of business, while indirect expenses (overheads) relate to support functions and therefore cannot be directly attributed to any one class of business or policy.

The main items of expense for a general insurer are salaries (and related costs), property costs, computer costs and investment costs.

Monitoring business written

The main reasons for monitoring business written are to:

- assess performance against goals
- manage risk
- gain market intelligence
- satisfy the regulators
- influence the market
- assist with reserving
- validate assumptions as part of the actuarial control cycle.

The key factors monitored are:

- premium rate changes
- portfolio movements:
  - lapses at renewal / renewal rates
  - new business volumes
  - quotes that result in written business (called the *strike* or *conversion rate*)
  - mid-term cancellations
  - policy endorsements
  - mix of business
- volumes of quotations
- persistency and profitability by source.

The four main methods of calculating premium rate changes are:

- direct calculation for each risk separately
- direct calculation using a standard risk
- measuring rate changes on individual renewals
- using underwriters' views.

A good system for monitoring business has the following features:

- tailored output
- accurate and validated data / results
- easy to use and well documented
- consistent over time and with other data sources and analyses
- minimal delay between data cut-off and production of results.

Actuarial investigations can be used to:

- carry out profit testing
- estimate price elasticity
- create lifetime pricing models
- redesign rating tariffs
- help with other pricing and reinsurance decisions
- feed into other processes, *eg* capital modelling.

This page has been left blank so that you can keep the chapter summaries together to use as a revision tool.

Chapter 19 Solutions

Solution 19.1

- reinsurers' data
- industry data, *eg* a motor organisation such as the Association of British Insurers (ABI) in the UK
- other insurers' data
- relevant organisations or government bodies.

Solution 19.2

Deferred acquisition costs are defined in the Glossary as acquisition costs relating to contracts in force at the balance sheet date. They are carried forward as an asset from one accounting period to subsequent accounting periods in the expectation that they will be recoverable out of future margins within insurance contracts, after providing for future liabilities.

Solution 19.3

- initial expenses – direct
- administration expenses – direct
- renewal expenses – direct
- claims handling expenses – direct
- investment expenses – indirect

Solution 19.4

The premium charged for a particular policy should cover not only the expected claims costs but also the variable expenses incurred by that policy – otherwise it will cost more to write the policy than will be covered by its premium. So it is important to know how much the variable expenses are.

Note that the company will want the premium to cover the expected claims cost, the variable expenses, and a bit more, to cover the fixed costs and profit. However, even if a policy makes only a small contribution to the company's overall fixed expenses, it may still be worthwhile writing that policy as long as it has covered its own variable expenses.

Solution 19.5

- number of policies – postal costs for policy documents
- number of claims – postal costs for claim forms
- premium amount – commission, levies
- claim amount – legal expenses

Solution 19.6

The insurer will want to allocate the expenses as accurately as possible to ensure that the premium charged to each class (and therefore to each policy) is correct and that the profitability of each class is assessed correctly.

Solution 19.7

It would not be practical to charge a premium that includes an allowance for the actual purchase cost of the property, because the premium in the year of purchase would be excessively high. A notional rent is charged instead. This provides an approximate annual cost of the property, which can then be allocated by product line.

Solution 19.8

A company might choose to publish its persistency rates. If the company has been retaining its business, this may attract prospective policyholders in the belief that they must be getting "a good deal" since the company is popular with existing policyholders.

Solution 19.9

For domestic household business, the limits will be different for each peril, so per limit would not be ideal.

For either buildings or contents, the sum insured is readily available as the exposure measure, so we could use this, either risk-adjusted or not. Alternatively, per unit of expected loss might be more accurate, as very few claims under household insurance are for the full sum insured.

Solution 19.10

To monitor profitability we would usually require premium net of commission. This would be the case if we were analysing the profitability of the business from a pure claims point of view. If we also wanted to build in the effects of commission on profitability, we would probably use premium gross of commission (unless we analyse this separately).

For Bornhuetter-Ferguson reserving we would require premium gross of commission when the *a priori* loss ratio is also based on premiums gross of commission, and *vice versa*.

Solution 19.11

- change of cover
- change of car
- addition or removal of a driver
- change of address
- change of use
- enhancement of a car, *eg* change of engine

Solution 19.12

The denominator would simply be the number of expiring policies minus the number of mid-term cancellation. In other words it would be the number of renewals *invited*.

Solution 19.13

$$\text{Lapse rate} = \frac{\text{no. of lapses for period } x}{\text{no. of renewals invited for period } x}$$

$$\text{New busines rate} = \frac{\text{no. of new policies for period x}}{\text{no. of renewals invited for period x}}$$

$$\text{Cancellation rate} = \frac{\text{no. of cancellations during period x}}{\text{no. of policies exposed for period x}}$$

Or, as an approximation:

$$\text{Cancellation rate} = \frac{\text{no. of cancellations during period x}}{\text{no. of renewals invited for period x}}$$

$$\text{Endorsement rate} = \frac{\text{no. of endorsements during period x}}{\text{no. of policies exposed for period x}}$$

The key point in each of these definitions is that the numerator and denominator must correspond to each other. The new business rate is defined like this for comparison with the lapse rate.

Solution 19.14

Some brokers might favour other insurers in preference to us. This might become evident if our results from those brokers were worse than from other, similar brokers. (We won't usually be able to check competitors' results for business from those brokers directly.)

Reciprocal business from a particular company could be poor quality overall. This would show up if our business from that source suffered much worse performance than the other company had on its retained business.

Chapter 20

Reinsurance pricing

Syllabus objectives

(q)(i) Outline the similarities and differences between pricing direct and reinsurance business.

(q)(ii) Describe how to determine appropriate premiums for each of the following types of reinsurance:

- *proportional reinsurance*
- *non-proportional reinsurance*
- *property catastrophe reinsurance*
- *stop losses.*

(q)(iii) Describe the data required to determine appropriate premiums for each of the above types of reinsurance.

0 Introduction

0.1 Objectives

The aim of this chapter is to provide a grounding in the methodology and techniques for pricing property and casualty reinsurance.

If you are studying Subject ST7, you will find that there are many common themes between this chapter and the Subject ST7 chapter on reinsurance reserving.

After reading this chapter **the candidate should be able to:**

- **outline the data required to price a reinsurance contract**
- **understand and explain why the data requirements might differ for different types of reinsurance and original business**
- **identify the key assumptions that might need to be made when pricing a reinsurance contract**
- **carry out a simple rating exercise, given appropriate information**

  The later sections in this chapter will describe how to price the main reinsurance contracts:

  – pricing excess of loss (Section 3)
  – pricing proportional reinsurance (Section 4)
  – pricing stop loss reinsurance (Section 5).

  You should note that the pricing methods described in this chapter are by no means definitive. Other common methods exist that are not described by the Core Reading. In the exam, you will need to choose a suitable method, which will largely depend on the data you have been given and the product you are pricing.

- **describe how to deal with some of the more complex features of certain reinsurance contracts.**

  This is discussed in Section 6.

0.2 Overview

The basic principles behind reinsurance pricing are no different to those for direct insurance pricing, particularly the pricing of excess layers.

These common principles will be outlined in Section 1.

The key differences are:

- **the volume and nature of data available to inform the pricing analyses and decision**
- **there are very few standard contracts – it is always necessary to read the slip to clarify how the contract works and what it covers**
- **the individual nature of most pricing exercises**
- **reinsurers often price stochastically.**

These key differences will be discussed further in Section 2.

The emphasis of the analytical work will also differ slightly depending on whether:

- **the task is to quote terms (that is, price the contract from scratch) or**
- **to act as a following underwriter, where we are being asked to write a line on the contract at an already quoted rate.**

For example, if we are the lead underwriter, then it will be our objective to determine the price (or the commission for a proportional treaty) that we would be willing to offer in order that our profit criteria can be met. Hence the profit criteria will be an *input* to our model and the price/commission will be the *output*.

However, if we are a following underwriter, then the price and/or commission will already have been set by the lead underwriter, and hence be an *input* to our model. In this case, our objective will be to assess whether the price/commission generates sufficient profit, which will be the *output* of our model.

The methods described in the following sections are illustrative and they are by no means exhaustive. Many different methods are used in practice.

1 *Elements of pricing essentially identical to pricing direct business*

This section outlines some of the similarities in technique between pricing direct business and pricing reinsurance business. The overall technique – that of finding an expected loss cost then loading for various other items – is exactly as for direct insurance.

As a reminder, the normal loadings for direct insurance are:

- expenses
- commission and/or brokerage
- reinsurance
- profit and contingencies, *ie* return on capital, or return on equity (ROE)
- investment income (as a deduction rather than loading)
- insurance premium tax, if applicable.

Question 20.1

Which of these loadings will *not* apply when pricing reinsurance?

1.1 *Expense loadings*

Aside from commission and brokerage, a reinsurer will load for own expenses using the same approaches and techniques as the direct insurer. This includes allowances for:

- **operational expenses** (*eg* general management expenses or the cost of HR)
- **expenses associated with the administrative maintenance of the policy (which may be partly fixed and partly variable**). For example, the expenses relating to handling claims from the cedant.
- **the costs of the reinsurer's own reinsurance (retrocessional) protections.**

However, good practice dictates that the reinsurer should always be aware of the gross risk premium.

As the pricing is usually individual to the risk, the final form of the loading may tend in practice to be simpler than for direct pricing (a percentage loading to the risk premium for instance) even where the loading was originally derived on a more complex basis.

You will recall from earlier chapters that expense analyses and allocation for pricing direct insurance can become quite involved.

1.2 Profit/ROE loadings

As for direct insurers, reinsurers will build a profit loading into the reinsurance price.

This can be derived in a number of ways:

- **from a profit target, perhaps expressed as a percentage of gross or net premiums**
- **from a return on capital target**
- **from a target loss or combined ratio.**

Normally, we talk of the profit and contingency loadings as one combined loading. This is because an insurer (or reinsurer) will not necessarily have the same profit loading for all contracts and then add a contingency margin for riskier business – rather, it will have higher profit margins for riskier business.

1.3 Commission and brokerage loadings

As for direct insurance, the expected brokerage to reinsurance brokers, and commission to be paid to the ceding company, will be loaded into the reinsurance price.

The level of both can vary by:

- **line of business**
- **type of reinsurance**
- **broker**
- **territory from which the reinsurance placement is being driven.**

Brokerage is usually priced in as a percentage load to the net of brokerage reinsurance cost. It varies to reflect the administrative cost to the reinsurance broker of supporting the placement of the cover and the servicing of the contract thereafter. In the case of reinsurance of long-tailed lines in particular, the broking support provided in making and processing the reinsurance recoveries associated with reinsurance cover for a single 12-month period continues over many years. The costs of this can be very significant, and the broker may have an additional service agreement with the cedant under which explicit fees are charged related to the number of claims transactions being processed.

As quota share reinsurances tend to involve higher reinsurance premiums than excess of loss, the percentage rate of brokerage tends to be relatively low, of the order of 1% to 3%.

> ***Question 20.2***
>
> Why do quota share reinsurances usually involve higher reinsurance premiums than excess of loss?

For other covers the standard brokerage would tend to be around 10% (though can be up to 20%).

The rate of brokerage will be stated on the slip circulated to reinsurers.

The broker may in practice agree to rebate part of this to the cedant (they may have agreed a fixed fee for instance), but this is handled as exactly that – a refund of some of the brokerage received by the broker to the cedant. The reinsurer would not be party to this agreement and is unaffected by it and should load for the level of brokerage stated on the slip.

1.4 *In practice*

In practice, however, although the various loadings may have been derived separately, for simplicity and clarity they will often be combined into a single loading, *eg* one total percentage loading or into a target loss ratio that is to be applied to the premium.

For example, if R is the risk premium, then you might apply a total 30% loading in order to get a total office premium of $1.3R$.

2 Elements of pricing where approach differs from that for pricing direct business

In this section, we will see that whilst the broad techniques behind pricing reinsurance are as for direct business, there are a few differences that need consideration, stemming mainly from the different claims characteristics of the business.

2.1 Derivation of expected risk premium – overview

The fundamental idea behind assessing the risk premium – based on the expected loss cost plus a loading that reflects the risk that the actual loss cost turns out to be worse than expected – is just as it is for direct insurance.

Notice here that risk premium is being defined as including a contingency margin or "risk loading", *ie* a prudent estimate of the expected loss cost. Elsewhere throughout this course, the Core Reading has taken risk premium to be the expected loss cost without any loading.

The form of the risk loading being used in practice can also vary from one reinsurer to the next, just as it can for a direct insurer, and might be as follows:

- **a proportion of the standard deviation of the expected loss cost to the contract**

  The more variable the expected loss cost, the higher the risk loading. Remember that much reinsurance operates at the extremes of distributions, and so the variance can be substantial.

- **based on a required return on capital**

- **investment-equivalent pricing**

  In other words, the loading is designed so that the extra premium charged provides the extra return required by shareholders for the extra risk.

- **the marginal impact on capital of writing a risk and load for the required rate of return on the additional capital required to write that risk.**

The main difference for reinsurance pricing (as opposed to direct insurance pricing) is the amount of and type of data and information available to assess the expected loss cost and the distribution (or volatility) of that loss cost.

In addition, the purchaser (*ie* the cedant) **is arguably as knowledgeable as the seller** (*ie* the reinsurer)**, unlike the situation for many direct insurance transactions.**

For example, when pricing high-layer excess of loss reinsurance for which there is limited (or no) historical claims experience, the expected future claims experience is pretty much anybody's guess!

This makes the process more of a negotiation – more akin to insuring a major corporation.

The exact approach also differs materially by the line of business – in particular between property catastrophe covers and all other covers.

These differences will now be discussed further.

2.2 Derivation of risk premium – property catastrophe business

Catastrophe models

The claims experience of property catastrophe reinsurance business is by its nature very volatile. Hence, little emphasis can be placed on the historic claims data available (if any).

The pricing of property catastrophe reinsurance is heavily dependent on the proprietary catastrophe models such as RMS, AIR and EQE.

These models are discussed further in Chapter 21.

These models do differ in the way in which they work and also in what is included in the information that they output. It is therefore important to have a good understanding of their relative merits. It is important to consider:

- **which models are viewed as more robust for which perils and in which geographical locations. This can change over time, so reinsurers need to keep up to date.**

  This can be done by reading the insurance press and having regular discussions with the catastrophe model providers. Selling such models is a commercial enterprise, and so it will be important to read between the lines of "marketing hype".

- **how the assumptions behind the models differ and how often they are updated**

- **how the input data requirements differ – this may have an impact on results**

- **how the output differs – the primary outputs are files containing large numbers of simulated events (earthquakes, windstorms, floods) along with the probability of that event happening and the expected loss for the set of risks in the input data. There is also a measure of the uncertainty in that expected loss. Some models include two sources of uncertainty in that measure, some only one. Hence the output from some models would imply a lower amount of volatility or a smaller range of possible outcomes, and hence a smaller risk load than others. The two sources of uncertainty are:**

  (i) uncertainty about which events will happen

  (ii) uncertainty about, for a given event, the exact amount of insured loss it will cause.

  The latter source is often referred to as secondary uncertainty, and is not accounted for in the output of some of the catastrophe models.

  Where catastrophe models do allow for this secondary uncertainty, it forms part of the "vulnerability" and "financial analysis" modules of the catastrophe model. This will be discussed further in Chapter 21.

When pricing property catastrophe reinsurance, it is important:

(i) to read the catastrophe model manuals – they are more informative and interesting than might be thought

(ii) to keep abreast of model developments and changes

(iii) to talk to the catastrophe modellers – both about the models in general and for a particular risk about the quality of the data and any concerns that they may have about the reliability of the results.

Many students will not work for a company that has direct access to catastrophe models or their developers. In those circumstances, we suggest you look in more detail at the websites and literature produced by the model providers mentioned above.

Note that sometimes the reinsurer will receive not the original data itself, but the model output, either as run by the cedant or by their reinsurance broker. In this case, the reinsurer will need to speak to the catastrophe modellers at the cedant or broker, and involve the reinsurer's own catastrophe modellers if possible.

From catastrophe model output to risk premium

This section describes just one approach used by some catastrophe models to convert the output into a suitable risk premium. Other approaches and output formats do exist.

The catastrophe model output would usually be the distribution of events. There are two bases for these files:

- **OEPs – an occurrence exceedance probability file, which considers the probability that the largest individual event loss in a year exceeds a particular threshold.**

  The OEP file for exposures under a reinsurance contract would be used mainly for evaluating per-event reinsurance; for example, losses likely to arise from a covered event seen only once in 200 years. The problem with this file is that it may ignore the possibility of multiple events.

- **AEPs – an aggregate exceedance probability file, which considers the probability that the aggregate losses from all loss events in a year exceeds a particular threshold.**

  The output of this file is often displayed as a graph (the AEP curve) with loss amounts on the x-axis and the corresponding probability of exceeding that loss amount on the y-axis. The area under the AEP curve is equal to the annual expected (gross) losses. The AEP outputs will often be used for capital or reserving projections.

The reinsurer can use these event files in a stochastic frequency-severity model to simulate catastrophe loss experience from the cedant in an annual period. Next the reinsurance contract terms can be applied to these simulated losses to calculate the resulting recoveries. Then the distribution of the annual reinsurance recoveries can be derived, along with the expected annual recoveries and the volatility measure being used in the risk loading (for example, standard deviation, 1 in 250 largest).

3 *Derivation of risk premium – property and casualty per-risk non-proportional covers*

This section describes some methods of how to price excess of loss reinsurance. Other methods do exist in practice. The decision as to which method to use largely depends on the data available and any cost / time restrictions.

There are currently few equivalents of the catastrophe models for non-natural catastrophe property and casualty reinsurance pricing.

As we will see in Chapter 21, some catastrophe models have been developed more recently to cover non-natural catastrophes such as terrorism risk and diseases.

As a result, the pricing methodology used has more in common with approaches used for pricing direct excess layer insurance.

This will consist of blending an assessment of the risk premium based on:

(i) benchmarks for the appropriate line of business / territory applied to the cedant's current risk profile ("exposure rating")

(ii) the cedant's own historical loss experience ("experience rating").

Section 3.1 will look at exposure rating, whilst Section 3.2 looks at experience rating.

3.1 *Exposure rating*

Introduction

The main principle of exposure rating is to not use historic claims experience at all, but instead to base premium rates on the amount of risk (*ie* exposure) that policies bring to the portfolio.

In exposure rating, reinsurers are using a benchmark to represent (or may be directly derived from) a market severity distribution for the line of business and territory being covered.

There are two common forms of benchmark: ILFs and first loss scales. The former tend to be more common in liability (re)insurance and the latter elsewhere.

ILFs and first loss scales were covered in detail in Chapter 15.

Ideally the cedant will have provided a list in electronic form of all the risks that they have written in a recent complete 12-month period. Ideally the 12 months should be the most recent 12 months that would be completely processed and entered on the cedant's systems.

If an older set of data was used, it would be less relevant to the risk environment that the proposed rates will be covering, and so would need more adjustment before it could be used.

The detail of what should be on the list will vary by the line of business.

The tables in Section 7 show what would be required for some of the more common areas of business.

It should be noted that large commercial risks are often shared between insurers ("subscription market" business). It is important to understand whether the limits in the risk data provided are the insurer's share or 100% of the limit for that risk – the total for all insurers involved. Reinsurers will need to know or be able to derive both. In addition, where a risk has been placed in a series of layers (for example, \$5*m* xs \$5*m*, \$15*m* xs \$10*m*, \$25*m* xs \$25*m*) the reinsurer will need to be able to identify and connect the programme layers on which the cedant has participated (which may not be all of them).

Basic exposure rating using ILFs (casualty reinsurance)

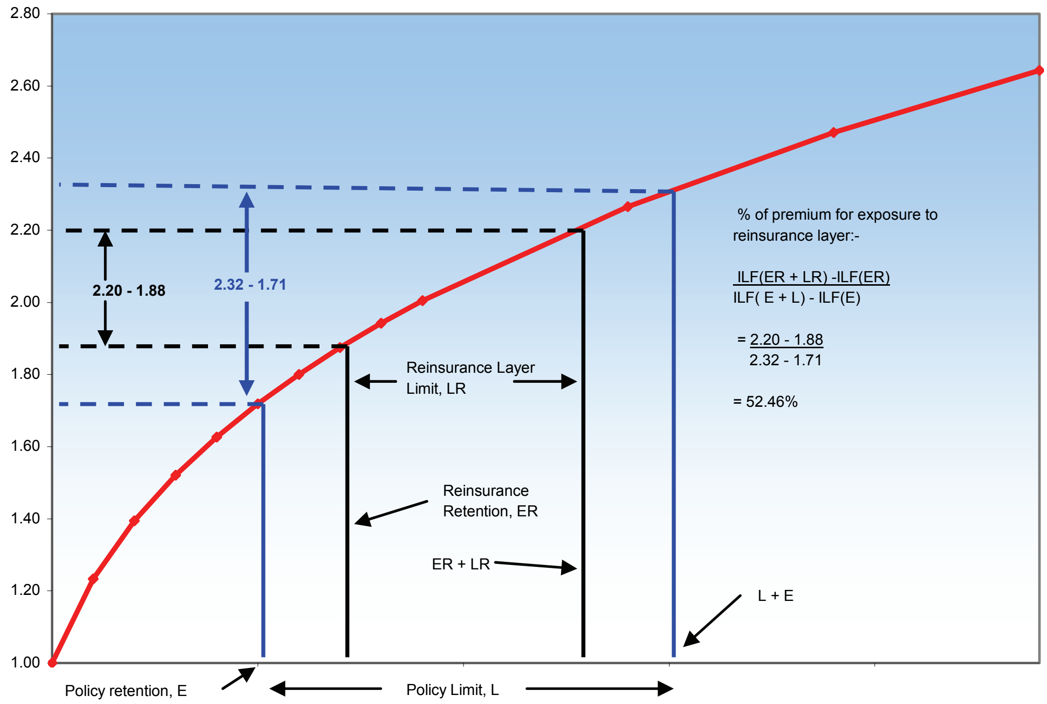
If a basic exposure rating exercise using ILFs is being performed, the expected loss cost only (and not the volatility or distribution of the loss cost) will be assessed.

On the assumption that 100% of the risk is being written, for each risk in the list of risks in the cedant's data, the reinsurer will need to use (calculate) the 100% limit (L) and the excess (E). Suppose that $ILF(n)$ is the ILF value corresponding to limit n in the ILF table. Suppose also that a reinsurance layer with limit LR and excess ER is being priced.

The ILF table will be used to calculate what proportion of the expected losses from the ceded risk would fall in the reinsured layer.

$$\text{Proportion} = \frac{ILF(LR+ER) - ILF(ER)}{ILF(L+E) - ILF(E)}$$

The diagram below helps to illustrate this. The curve shows the $ILF(n)$ (y axis) corresponding to each loss limit n (x axis).



This needs some modification:

- **where the reinsurance retention is below the bottom of the cover on the risk, *ILF(ER)* is replaced with *ILF(E)***
- **where the top of the reinsured layer is beyond the top of the cover on the risk, *ILF(LR+ER)* becomes *ILF(E+L)***
- **where the reinsurance layer finishes below the risk (0% reinsured)**
- **where the reinsurance layer starts above the cover on the risk (0% reinsured).**

This gives the expected losses to the reinsurance layer as a proportion of the expected losses for the reinsured (ceded) risk. Now the expected losses for the ceded risk are needed. For this, the premium charged for each risk is needed and, with a loss ratio for the risk, the expected losses could be calculated as the loss ratio times that premium.

Note that the denominator of the loss ratio and the premium must be consistent. For example, given gross premiums, a loss ratio will be needed as a percentage of gross premiums. Given premiums net of original brokerage and commissions, a loss ratio will need to be calculated on the same basis.

Sometimes the cedant will provide an expected loss ratio; at other times the historic development triangles and rate changes for the line of business, or the cedant may provide both or neither. All the available information should be used – which may include information for the market or other cedants – to determine a reasonable loss ratio. It is possible that this is not the same as the loss ratio the cedant provided. The reinsurer can apply the selected loss ratio to each risk in the risk data to calculate risk level estimates of expected losses.

So for each risk in the data, the reinsurer can calculate the expected reinsurance losses and then sum them over all the risks. This gives the undiscounted expected loss cost for the reinsurance contract. The reinsurer can then discount this to reflect the expected payment pattern of the reinsurance losses (that is, the payment of recoveries to the cedant from the reinsurer). This payment pattern could be based on the reinsurer's own experience (for example, from the latest reserve review) or from the historical large loss experience of the cedant, or on a blend of the two, reflecting the credibility of the cedant's historical loss experience.

So far, it has been assumed that there is no limit on the total amount of reinsurance recoveries available (called **"sideways cover") on the contract, and that there are no reinstatement premiums (that is, that all the reinsurance cost is paid up front).**

Reinstatements were discussed in Chapter 6 but we will meet them again at the end of this chapter. Following a reinsurance claim, a reinsurer will often require an additional payment (a *reinstatement premium*) to be paid in order to restore full cover. There will usually be a limit to the number of reinstatements that can be made before the reinsurance cover is exhausted.

The best way of handling this is to use the ILFs in a more sophisticated way.

There are some approximate methods that can be used to adjust for reinstatements and limited sideways cover (see later) where only basic exposure rating is being used (as may well be the case at busy periods).

Exposure-based rating – practice

In practice, even reinsurers have limited historical loss data for some lines of non-property catastrophe business, especially for very large loss events. Pricing for high excess layer reinsurance is often driven by minimum rate requirements and by expenses and cost of capital rather than expected loss cost, which may well be close to zero.

In other words, rather than using expected loss cost to derive the price for high layer business, there is a tendency to charge a price simply to cover expense costs and the extra capital needed to write such a volatile layer (which may come from regulatory or internal capital requirements).

In some instances, the cedant does not provide individual risk information. What the cedant normally provides in such cases is a limit profile – a table showing number of risks and written / earned premiums by limit bands and also by excess bands (where the cedant writes excess business). Sometimes this information is split into two separate tables, one for limit bands and one for excess bands. Occasionally only limit bands are provided.

Where this happens, it is necessary to make an assumption about where in the band the limits actually lie, bearing in mind commonly-purchased limits for the line of business. The reinsurer can then use the data in blocks. This is more approximate than using individually-listed data. It is even more approximate if the excess information is provided separately. Finally, the reinsurer will need to make assumptions about how the excess and limit information interact as well.

Let's explain this last paragraph a little further.

Suppose that risk profile data tells us that a total amount of premium income has been received in respect of several risks, and each risk has limits somewhere between £2.5*m* and £5*m*. We need to decide what value in that band we will use for the calculations.

We could, for example, assume that the limit for each risk is just the midpoint, *ie* £3.75*m*. However, certain limit sizes are more commonly purchased than others. For example we may be aware that for this line of business £5*m* is a very common limit, so we might choose £5*m* as being the most representative value to use instead.

Alternatively, we may know that for this line of business, the number of risks with a particular limit tends to reduce as the limit gets bigger. We may therefore pick a value below the midpoint, *eg* £3*m*, as being a better representation of the typical risk in that band. If we are being conservative, we would choose the highest limit value.

Often the profile given is a two-way table with limit bands down the side and excess bands across the top (or vice versa). Similarly then, we would need to select a representative excess value for risks in a band as well as a limit value.

Sometimes we may be given separate profiles for limit and for excess (*ie* two one-way tables). In this case, we would have to decide how to combine these into a two-way table and then select single representative limit / excess values for the calculations.

We could also split the band further (for example if there are two common limits falling in that limit band) but then we have to make even more assumptions about how the income splits between each subdivision.

All these calculations are really just an attempt to adapt grouped data so as to obtain approximate data on each individual risk. Once we have estimated data on an individual risk level we can estimate the reinsurance premium using the ILF method.

3.2 *Experience-based rating*

Introduction

This section looks at two possible methods of pricing excess of loss reinsurance using actual past claims experience. These methods are more commonly used than exposure rating techniques when there is credible and reliable data, for example on working layers.

There are two main approaches to assessing the cost of non-proportional reinsurance using the cedant's loss experience. The first is a basic burning cost calculation. The second is to construct a stochastic frequency / severity model.

Both approaches were described in Chapter 14.

The choice of approach is influenced by:

- **the approach to loading for profit (does it require a volatility measure or a distribution of outcomes?)**

  If distributions are required, the stochastic approach would clearly be preferable.

- **volume of loss data**

  For example, if you have plenty of reliable claims data, the variability is already captured and so you might rely on a burning cost approach.

- **time / resource constraints (especially close to peak renewal dates such as 1 January).**

Burning cost calculation

In the burning cost calculation, the reinsurer will apply the reinsurance terms to the historical losses after adjusting for claims inflation (trend) and expected future development.

Below is found a common method of calculation:

First the reinsurer will apply trends to the loss data.

We discussed "trending" in Chapter 14. It is a popular name given to the technique of adjusting the data for inflation (and sometimes other factors such as frequency trends). You will also hear the technique referred to as putting the data "on level" or putting it on an "as-if" basis (in other words, *as if* the claims were occurring today).

Note that the form of data will vary depending on the basis of cover. For "losses occurring during" cover, loss data should be capable of aggregation by year of loss and exposure (premium) data should be on a calendar year (earned) basis. For "risks attaching during" cover, loss data should be capable of aggregation by underwriting / policy year and exposure (premium) on a written year basis.

Losses occurring during (LOD) is another term for an accident year approach. Risks attaching during, also called "policies incepting", is another term for an underwriting year approach. Note that complications would arise if the cover basis changed during the period of investigation.

The loss data should include the paid and case reserve positions for each loss at regular (usually annual) points in time. The loss data should be from the ground up before application of the reinsurance contract excess and uncapped (that is, before application of the cedant's policy limit or reinsurance contract limit) if possible.

However, as mentioned earlier, often the reinsurer will get fully ground-up data but only for claims breaching a certain census point (often half the lowest retention). You need to make sure you look closely at the data you are working with, and if you're in any doubt about what it represents, you should make a suitable assumption.

The reinsurer will apply trends to the historical loss payments and corresponding case reserves, using appropriate inflation assumptions to a common fixed point in time.

It may instead be that the case reserves have already been set based on the expected settlement amount in the future, *ie* already allowing for future inflation. In this case, you would only need to trend the payment data.

Then the reinsurance contract terms will be applied to each trended loss, to give the trended losses to the layer.

Further complications arise if stability clauses apply to the layer, which would increase the effective limits. This is covered further in Section 6.3.

These amounts will be aggregated by year of loss or underwriting year (as appropriate) to produce triangulations of paid and incurred loss development for losses to the layer. The reinsurer can then develop these to ultimate, using as far as is reasonable development patterns derived from the triangles themselves.

At this stage you need to make sure that the development pattern you have is appropriate for your needs. The ideal development pattern to apply would be:

- applicable to the reinsurance layer you are pricing
- applicable for that cedant's experience
- already adjusted for inflation
- include allowance for both IBNR and IBNER.

Question 20.3

How would the method described above change if you were only given the development pattern for the fully ground-up cedant experience?

Where the data is sparse (for example, high excess layers), benchmarks will be used for the class / territory instead. These are usually based on aggregated data for all a reinsurer's cedants or may be based on industry standards (most easily available in the USA – RAA and ISO being the two main sources).

RAA is the Reinsurance Association of America. It is a trade association that influences and guides various bodies as they consider legislation affecting the reinsurance industry. ISO is the Insurance Services Office, a private US firm that deals with information about risk.

Benchmarks have also been developed in the UK by various bodies, *eg* actuarial consultants. If using benchmarks, you should take the usual precautions that the business they have been based on will be suitable for your own use.

This provides an estimate of losses to the layer for each historic year. However, so far, exposure changes have not been considered during the historical period covered by the loss data.

We need the exposure measure in order to calculate a burning cost.

The commonest exposure measure is premium – although for motor insurance, vehicle-years is also likely to be available at least for the private car element of the cedant's business.

Quite often, vehicle-year data is not available so readily for motor fleet business.

If reinsurers use premium as the exposure measure, information is required about historical rate changes plus an estimate of rate changes for the period of cover so that all the historical premiums can be adjusted to be as if they are based on rates for the contract year being priced (hence the term "as-if" that we mentioned earlier)**.**

In addition, if the rates for the underlying direct business are applied to an exposure base that is affected by inflation (for example, wages for direct employers' liability (EL), turnover for direct general liability (GL)), then premiums should change to the same extent as the exposure base without any change in the real exposure to risk. For example, in GL, turnover may increase purely as a result of the insured company increasing its prices to cover wage / expense inflation without any change in the amount of product it is selling.

What we are doing here is making exposure consistent with claims. We have already adjusted the claims data for inflation, to put them on an as-if basis, and now we are going to adjust the premiums too, to make them as if we are selling the business in the proposed period of cover. Only then will we be able to calculate a burning cost consistently.

A common mistake would be to use claims inflation to adjust the historic premiums, instead of using actual historic premium rate changes.

Question 20.4

What happens if you adjust both premiums and claims for claims inflation?

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As an example, consider the following table.

Year	Premium	Rate change	Rate index	Wage inflation	Wage index	Adjusted premium
1998	25,433,227		100.00		100.00	43,505,962
1999	27,566,903	-10%	90.00	4.82%	104.82	49,986,372
2000	31,298,563	-5%	85.50	4.49%	109.53	57,171,077
2001	32,096,751	5%	89.78	4.50%	114.46	53,432,742
2002	41,679,236	25%	112.22	3.54%	118.51	53,609,919
2003	45,905,285	15%	129.05	3.42%	122.56	49,646,361
2004	51,239,374	10%	141.96	4.38%	127.93	48,263,978
2005	56,289,106	5%	149.05	4.02%	133.08	48,542,363
2006	57,616,193	-5%	141.60	4.12%	138.55	50,234,637
2007	55,250,000	-10%	127.44	3.50%	143.40	51,714,000
2008	54,000,000	-10%	114.70	4.00%	149.14	54,000,000

Here, the *rate index* and *wage index* are just the yearly rate changes or wage inflation compounded up. The adjusted (to 2008) premium is then the original premium multiplied up by the ratio of indexes at 2008 to the indexes at the original year.

So, for example: $43,505,962 = 25,433,227 \times \dfrac{114.70}{100.00} \times \dfrac{149.14}{100.00}$

You may notice small rounding differences, as it appears that the above Core Reading table was produced by a spreadsheet.

Note that the real exposure for 2008 is actually less than that for 2000; not more as might initially have been thought.

This is simply because rates and wages have increased substantially since 2000, and so the as-if-2008 exposure in 2000 was actually quite sizeable.

By scaling the losses to the layer (trended and developed) for each historic year to 2008 exposure levels (for example, for 2000, multiply by 54*m*/57.17*m*), a set of exposure-adjusted losses to the layer can be obtained which in turn can be averaged (say) to arrive at an estimate of loss cost for 2008.

Alternatively we could divide each year's trended and developed losses to the layer by the corresponding exposure-adjusted premiums. This would give a set of loss cost rates, which we could average (say) to give a rate for 2008, to apply to the 2008 premium estimate.

It is advisable to examine the individual years' exposure-adjusted losses to the layer / rates, in case there are upward or downward trends still remaining. (Here, we mean trends in the traditional sense.) **These may be real (so the reinsurer will need to allow for them in the final selection of expected loss cost) or they may indicate that the trend (say) used is inappropriate.** (Here, we mean trend as in inflation adjustment.) **By discussing this with the underwriters (reinsurer and cedant) the most likely outcome can be determined.**

In the above example, there are further complications to be considered such as a change a in the basis of cover, any allowance for claims inflation already built into the case reserves, or indexation clauses.

Frequency / severity model

As an alternative to a burning cost analysis, the reinsurer may construct a frequency / severity model in which are fitted statistical distributions to the cedant's historical loss data.

Firstly, trends are applied to the individual losses, as for the burning cost calculation.

There may be times when the reinsurer will use this method, even though material development for open claims in the more recent years is expected – usually the case for liability lines, for example EL (employers' liability)**, GL** (general liability)**, E&O** (errors and omissions)**, motor TPL** (third party liability)**. In this case, either:**

(i) losses for these years can be ignored, or

(ii) losses on an individual basis can be developed.

The first of these approaches would be rather foolish, however.

The second approach is not entirely satisfactory. This is because, when development factors for aggregated experience are applied to each individual loss, the variability in development between losses is understated and possibly also overdeveloping claims with large current case reserves.

A better approach would be to use a stochastic model for the development factors. Using this, the reinsurer generates a large number of sets of trended and developed large losses, fits severity distributions to each set and then selects the distribution type with the best overall fit. The reinsurer will then use the distribution of the corresponding parameters to determine the mean fit and the parameter variability.

However, this is a far from trivial exercise and there may be occasions where following approach (ii) above may be the only practical option.

In either approach, the development should allow for incurred but not enough reported (IBNER) – *ie* the development of known claims, not development due to claims not yet reported. The reinsurer should apply the development factors to open, not settled, claims.

The IBNER development from the cedant's large loss experience can be derived by arranging it into development triangles aggregated by year and then comparing the (trended) incurred at time *t* for losses for year *n* reported at time *t*, with the (trended) incurred at time *t* + 1 for losses for year *n* reported at time *t*.

The Core Reading example on the next page illustrates the following steps:

- triangulate the loss experience for each claim (in the first table)
- aggregate the data by year of loss (in the second table)
- find the incurred amounts at time t+1, but only for losses notified at time t (in the third table).

  For example, for 2002 the incurred amount of 900k at time 3 (t+1) for claims notified at time 2 includes figures relating only to losses 6, 7 and 8 (*ie* 225k, 250k and 425k).

This gives us the development profile of claims already notified and thus helps us analyse IBNER.

Then the final table (headed *factors*) calculates the ratio of the figures in the third table to those in the second table, and takes the average.

For example:

Large Losses - Incurred Development History

Year	Claim	Development Year 1	2	3	4	5	6	7
2000	Loss 1	100,000	200,000	250,000	250,000	250,000	250,000	250,000
2000	Loss 2		260,000	275,000	280,000	295,000	307,000	306,000
2000	Loss 3		350,000	390,000	410,000	410,000	410,000	410,000
2001	Loss 4	200,000	240,000	250,000	255,000	255,000	255,000	
2001	Loss 5							
2002	Loss 6	175,000	210,000	225,000	240,000	238,000		
2002	Loss 7		225,000	250,000	275,000	280,000		
2002	Loss 8		400,000	425,000	450,000	445,000		
2002	Loss 9			310,000	350,000	356,000		
2003	Loss 10	125,000	145,000	165,000	165,000			
2003	Loss 11		200,000	245,000	250,000			
2004	Loss 12	150,000	175,000	180,000				
2004	Loss 13	300,000	350,000	370,000				
2004	Loss 14		110,000	140,000				
2005	Loss 15	130,000	150,000					
2005	Loss 16	110,000	125,000					
2006	Loss 17	245,000						
2006	Loss 18	210,000						

Year	Incurred at time t, for losses notified at time t 1	2	3	4	5	6	7
2000	100,000	810,000	915,000	940,000	955,000	967,000	966,000
2001	200,000	240,000	250,000	255,000	255,000	255,000	
2002	175,000	835,000	1,210,000	1,315,000	1,319,000		
2003	125,000	345,000	410,000	415,000			
2004	450,000	635,000	690,000				
2005	240,000	275,000					
2006	455,000						

Year	Incurred at time t+1, for losses notified at time t 1	2	3	4	5	6	7
2000	200,000	915,000	940,000	955,000	967,000	966,000	
2001	240,000	250,000	255,000	255,000	255,000		
2002	210,000	900,000	1,315,000	1,319,000			
2003	145,000	410,000	415,000				
2004	525,000	690,000					
2005	275,000						
2006							

Factors

Year	1 : 2	2 : 3	3 : 4	4 : 5	5 : 6	6 : 7
2000	2.00	1.13	1.03	1.02	1.01	1.00
2001	1.20	1.04	1.02	1.00	1.00	
2002	1.20	1.08	1.09	1.00		
2003	1.16	1.19	1.01			
2004	1.17	1.09				
2005	1.15					
2006						
Average	1.31	1.10	1.04	1.01	1.01	1.00
Weighted Average	1.24	1.10	1.05	1.01	1.01	1.00

So far, we have only calculated IBNER development. We also need to calculate IBNR.

The large loss count development can be used to determine IBNR development and so estimate the ultimate large claim count for each historic year. Next the reinsurer should adjust this for exposure changes in the same way as for the burning cost calculation. The final adjusted ultimate claim counts can be used to fit a frequency distribution.

Finally the frequency and severity distributions can be combined to produce a stochastic model for the cedant's large losses and so to model the corresponding reinsurance recoveries for the contract being priced.

This was all covered in more detail in Chapter 14.

Practical issues

Reporting threshold for large loss data

Typically, cedant historical loss data is provided for all losses that exceed a particular size.

This threshold is often called a *census point*.

The size is often dictated by the reporting requirements of their past reinsurers, and may be a fraction (often half) of their current lowest retention.

If we use a census point that is too high, we will not see many historical claims in the data provided, because of inflation. It would also mean that we would not be provided with any data relating to claims that have been notified but which have not yet developed beyond the retention limit.

Question 20.5

An insurer writes liability business that has a 10-year tail. Inflation in the past has averaged 10% per annum. Estimate a reasonable census point for claims in order to price an excess of loss reinsurance treaty that attaches at £1*m*.

In most countries, and with most lines of business, half the retention is reasonable as a census point given that inflation may act whilst the claim develops. For longer-tailed classes, or in countries where severe inflation is common-place, it is not unusual to have far lower reporting points (or even fully-ground-up data).

Lower reporting points are useful even if some smaller claims will never breach the retention, because more data is then available to the reinsurer for claims analysis purposes.

There can be two practical issues with this.

Firstly, if the losses reported are those that are currently over the reporting threshold, then the dataset will be missing losses that have exceeded the threshold in the past, but have developed downwards since and now have an incurred value below the threshold. Development factors assessed from such a dataset will tend to overstate the actual future development. The impact on frequency needs to be considered also. Ideally loss data should include all losses that have *ever* exceeded the threshold.

Secondly, if the reporting threshold is the same for all years then, without additional adjustment, the volume of data for the different years will actually be inconsistent.

Example

Consider a claim that occurred in 1997 and settled for £250,000. Because of claims inflation (whether economic, for example, wage inflation or social, for example, additional heads of damage) the same event in 2007 would settle for a larger amount.

Conversely a claim for £250,000 from 2007 would have settled for less if it had occurred in 1997. If the large loss data from the cedant is for losses over £250,000 (for example) for all years, then the reinsurer will be missing losses from all but the most recent year, that would be over £250,000 if trended to the most recent year.

In other words, there will be a number of losses from previous years that will exceed £250k once trended to today's terms, but which the reinsurer will not know about.

Hence any assessment of frequency of losses over £250,000 will be too low for all but the most recent year, and any assessment of severity will overstate average severity for all but the most recent year.

This is because we are ignoring some of the smaller losses.

To overcome this, the reinsurer will need to restate the threshold for all but the most recent year in the data. Using the same claims inflation rates as were used to trend the losses, the reinsurer will also inflate the threshold year by year, so that its real value is preserved and any untrended losses that never exceeded the inflated threshold for the appropriate year are discarded.

Equivalently, the trended losses can be compared to the original threshold trended to the same point in time. In practice this is often the easier approach.

Choice of inflation rates

The choice of inflation rate depends on a number of factors:

- **Whether the loss data to be inflated represents ground-up original losses or losses to an excess layer. Inflation assumptions for losses to excess layers will be higher – often much higher – than those for ground-up losses. However, the difference may be difficult to estimate.**

  For very low layers (if they exist), where most claims are for damage to a third party's property, claims may increase in line with a mix of price and labour inflation, for example 5–10% *pa* in the UK at the time of writing.

  Wage index inflation (say, 10% *pa*) would normally be appropriate for most liability excess of loss layers since most liability claims that are of this level are generally based on compensation for loss of earnings.

  At the very highest layers, judicial inflation and the general increase in the litigiousness of society can produce even higher claims cost increases.

- **The drivers of inflation. For example, for injury losses a large element of inflation is wages and care costs but there will be additional inflation over and above this covering trends such as increased tendency to claim, claiming compensation for a wider range of heads of damage, changes in society's view of the appropriate level of compensation for the injury itself (general damages – as opposed to the cost of its consequences, such as loss of earnings and care costs) and so on.**

  These were discussed in Chapters 7 and 8.

- **The territory or territories in which business is written (wage inflation, for example, varies by country).**

- **The size of loss – inflation rates will not necessarily be the same for small and large losses, although often this assumption is made because of insufficient data to establish any difference.**

Ideally a reinsurer will use all the data available to it from cedants and/or market sources (most common in the USA) to make its own inflation study.

In the UK, such studies also exist, for example those published by the International Underwriting Association (IUA).

There are other factors that are more dependent on payment or settlement dates rather than notification dates. For example, landmark legal decisions which affect all future claim payments but which will not be replicated again in the future. These factors must be considered on an individual basis to make sure that they receive appropriate action.

Treatment of shock losses

In many instances, a cedant's loss history may contain an exceptionally large loss. Depending on how many years of history the cedant is providing, it may be excessive to allow for a loss of that size to occur once in that many years. The reinsurer could instead remove the loss, do the loss cost analysis without it, and then add a loading for large losses at the end. The reinsurer could base this loading on an analysis of the reinsurer's entire book, or allow for it by spreading the cedant's own large loss over a more reasonable period of time.

Discounting

Theoretically, the reinsurer should base pricing on discounted values (for example, discounting the expected loss cost to reflect the expected payment pattern of the losses to the contract).

For high layer contracts with little claims experience, the payment pattern will have to be derived from benchmarks.

Some reinsurers do not explicitly discount, but may compensate for this by a reduction in other elements of the pricing basis. The choice of discount rate will reflect the investment return expected on the assets in which the reinsurer holds the reserves and capital required for the risk.

For liability lines the assets may be held for a very long time – the payment pattern being very long – and so the pricing can be very sensitive to the discount rate selected.

Changes in cedant business mix

Where the mix of business written by the cedant has changed materially, care must be taken. If it is possible, the business should be analysed in different segments, pricing each segment separately and then combining the segments in the expected future proportions.

Often this is not possible because there is an insufficient number of claims for this to provide reliable rates, or because lack of detail in the claims data does not allow the reinsurer to make the segmentation. In the latter circumstance, the reinsurer may need to discard data for years prior to the material change. If this leaves insufficient data, the reinsurer should make some subjective adjustment or reduce the credibility given to the experience-based pricing assessment.

4 *Pricing of property and casualty proportional covers*

Whereas in excess of loss contracts a price is paid for the possibility of future recoveries, in proportional reinsurance the prices of the direct business are already set by (or in conjunction with) the cedant, and the reinsurance premium is simply a percentage of the cedant's premium.

Determining a risk premium is not actually a major consideration of a proportional cover. The key considerations are:

- **assessing the likely overall loss ratio for the account**
- **determining what level of ceding commission and/or profit commission can be offered whilst still retaining an acceptable profit for the reinsurer.**

However, it is worth bearing in mind that many reinsurance programmes combine proportional and non-proportional reinsurance. If the non-proportional arrangement inures to the benefit of the proportional arrangement, then some adjustments will need to be made to the methodology described in this section.

Question 20.6

What does "inures to the benefit of" mean?

4.1 *Assessment of overall loss ratio*

This is essentially a best estimate reserving exercise, and so the data needed will closely resemble that for a reserve review.

The aim is to estimate the loss ratio for the period of cover. The data needed will be as follows:

- **As many years as possible (ideally at least 10) of triangulated premium, paid and incurred data. Usually, proportional covers are written on a risks incepting basis, so the triangles should if possible be on an underwriting year basis. Where more than one line of business is to be covered, separate triangles should be obtained for each line.**

  The ideal number of years of data will depend heavily on the line of business. For shorter-tailed lines such as household, it may be sufficient to look at perhaps five years of historic data.

- **Information on rate changes over the period covered by the data triangles, by the same line of business split as the data triangles.**

- **Estimates of premium income to be written and rate changes for the period of cover being priced.**

  The rate changes will be used to put the premium on-level, as we did in Section 3.2.

- **Information on changes in mix of business, and policy terms and conditions, over the period.**

- **Information on catastrophe exposure.**

- Inflation, historic and future, in order to put the claims on-level.

Next, an appropriate set of actuarial methods can be used to project the triangulated data to the ultimate settled position for each historical year. Then the resulting loss ratio for each year can be calculated. Finally, trends should be applied to these loss ratios (for the effect of claims inflation) to put them "on-level" to reflect the level of premium rates for the period of cover being priced.

Now that we have inflated claims to the proposed cover period, and adjusted premiums for rate changes to the proposed cover period, we have a series of possible ultimate on-level loss ratios that the policy we are pricing might result in.

The set of resulting adjusted loss ratios (say) could be averaged to give an estimate of the expected loss ratio for the period of cover. If they are considered as a set of observations from a statistical distribution, their mean and standard deviation could be used to parameterise a distribution. The log-normal distribution is commonly used for this purpose.

At this stage it is important to consider the effects of any other trends. If we averaged all the loss ratios blindly as suggested above, we may miss any trends and under/overestimate the loss ratio for the forthcoming year.

Example

Suppose the historic on-level ultimate loss ratios are 70%, 75%, 80%, 85%, 80%, 75% for the previous six years.

It would be tempting to say that the average loss ratio is 77.5% and to build this into the profit-test model to calculate a commission. However, there is clearly a trend in these ratios and, without doing any calculations, you may validly expect the loss ratio to be 70% in the forthcoming year.

Whether you use 77.5% or 70% will affect your calculated commission considerably and will undoubtedly affect your decision as to whether to write the business or not.

In addition to this, we need to consider the short-term and long-term goals. We may decide, for example, that a 70% loss ratio is acceptable for the forthcoming year, but a 77.5% long-term ratio is not acceptable. However, if we start making decisions in this way, we must realise that it is not always practical to accept and decline the same treaty at different points in time, as we need to consider relationships with the broker and cedant.

4.2 Determining acceptable ceding and/or profit commission terms for quota share contracts

If there is no profit commission and ceding commission is flat (that is, has the same percentage value regardless of the profitability of the underlying business), then this is a fairly simple exercise. It only remains to check that:

100 – loss ratio% – ceding commission%

leaves enough to cover the reinsurers' expenses and profit.

We may also need to make sure there is enough left to pay for any retrocession cover and brokerage.

In addition, we should allow for investment income. One way of doing this is to calculate the discounted values of all the cashflows.

Question 20.7

Suggest a reasonable discount period for each of the following cashflows:

- premiums
- claims
- expenses
- brokerage
- ceding commission.

It may also be instructive to use the loss ratio distribution to identify the probability of the reinsurer making a loss. If this is too high, then the reinsurer should reduce the ceding commission even though the mean outcome is satisfactory.

An alternative method is to add a cost of capital loading, or extra contingency loading, in addition to the cashflows mentioned above.

Allowing for loss-sensitive features

The analysis above gets slightly more complicated if we have cashflows within the quota share contract that depend on the loss experience itself. Examples of this include sliding scale and profit commissions.

Where the ceding commission percentage varies with the loss ratio on the underlying business ("sliding scale ceding commission") or there is a profit commission as well as (flat) ceding commission, then the loss ratio distribution should be used to calculate the distribution of the reinsurer's financial position.

The distribution is necessary because we need to estimate the probability that the loss-sensitive feature will bite, and if it does, by how much.

In such cases the reinsurer will often have requirements related to:

- **a probability of making a loss of no more than *x*%**
- **a probability of making a loss of *y*% or more of no more than *z*%**
- **a combination of the two.**

The reinsurer can adjust the ceding commission scale or the profit commission scale / percentage until these requirements are met.

Using the loss ratio distribution, a stochastic method will be needed to calculate a suitable allowance for the anticipated extra commission.

It is normal for the cedant or their broker to indicate the terms they would prefer, and if these meet the reinsurers'requirements no adjustment will be needed. If not, then the reinsurer has to negotiate this with the cedant or broker.

4.3 Surplus share contracts

Elsewhere in the Core Reading, these are called surplus treaty contracts. The term *surplus share* is more common in the US.

Surplus share contracts can be approached in a similar way to quota shares, but they are more complicated to assess.

Unlike quota share contracts, the differing cession rates between risks imply that the reinsurer's loss ratio can be materially different to that for the cedant; higher or lower. Two examples below illustrate this. Both cases cover the same set of risks with the same surplus retention and hence the same premium cession.

Surplus threshold 2,000,000

	Limit	Ceded %	Gross Premium	Ceded Premium	Gross Loss	Ceded Loss
Risk 1	1,000,000	0.0%	25,000	0	24,766	0
Risk 2	2,000,000	0.0%	34,500	0	1,856	0
Risk 3	3,000,000	33.3%	125,000	41,667	17,807	5,936
Risk 4	5,000,000	60.0%	250,000	150,000	59,212	35,527
Risk 5	5,000,000	60.0%	135,000	81,000	18,357	11,014
Risk 6	6,000,000	66.7%	320,000	213,333	66,262	44,175
Risk 7	7,500,000	73.3%	200,000	146,667	8,045	5,900
Risk 8	8,000,000	75.0%	210,000	157,500	2,283	1,712
Risk 9	10,000,000	80.0%	333,000	266,400	3,978	3,182
Risk 10	10,000,000	80.0%	400,000	320,000	917,724	734,179
TOTAL	57,500,000		2,032,500	1,376,567	1,120,289	841,625

ceded premium %	67.73%	gross loss ratio	55.12%
ceded loss %	75.13%	ceded loss ratio	61.14%

Surplus threshold 2,000,000

	Limit	Ceded %	Gross Premium	Ceded Premium	Gross Loss	Ceded Loss
Risk 1	1,000,000	0.0%	25,000	0	660,015	0
Risk 2	2,000,000	0.0%	34,500	0	28,267	0
Risk 3	3,000,000	33.3%	125,000	41,667	56,155	18,718
Risk 4	5,000,000	60.0%	250,000	150,000	2,785	1,671
Risk 5	5,000,000	60.0%	135,000	81,000	33,015	19,809
Risk 6	6,000,000	66.7%	320,000	213,333	4,296	2,864
Risk 7	7,500,000	73.3%	200,000	146,667	43,677	32,030
Risk 8	8,000,000	75.0%	210,000	157,500	15,601	11,700
Risk 9	10,000,000	80.0%	333,000	266,400	6,533	5,227
Risk 10	10,000,000	80.0%	400,000	320,000	6,013	4,811
TOTAL	57,500,000		2,032,500	1,376,567	856,358	96,830

ceded premium %	67.73%	gross loss ratio	42.13%
ceded loss %	11.31%	ceded loss ratio	7.03%

In the first table, the risks with high limits – and so high cession rates – have much larger losses than those with lower or no cession and so the ceded loss ratio is higher than the original gross loss ratio. In the second, the opposite is the case, and so the ceded loss ratio is lower than the original gross loss ratio. The reinsurer's experience is dependent on the way in which the large losses are distributed.

Hence it is important for the reinsurer to be confident that:

- **large limit risks do not have disproportionately heavy large loss experience**

  In fact, the reinsurer will want to make sure that the cedant is not simply passing larger proportions of the "bad" risks to the reinsurer, and keeping the "good" risks for itself. It can do this by simply keeping a close eye on the difference between the cedant's loss ratios and the reinsurer's loss ratios.

Question 20.8

Why, generally, don't cedants pass large proportions of "bad risks" to the reinsurers?

- **the contract terms restrict the choices that a cedant has to determine the amount ceded (the more choice afforded to the cedant over the cession rate the more potential there is for anti-selection, and the reinsurer will need to bear that in mind when assessing the potential claims experience).**

Many surplus treaties allow the cedant to pick and choose exactly how much of each risk to cede to the reinsurer (this is the flexibility that quota share does not give). In this case, it will be important for the reinsurer to keep an eye on the ceded experience.

Other surplus treaties have very closely-defined rules for cession, and some allow no choice at all. The more flexible the cession can be, the more cautious the reinsurer's assumptions will be when calculating the commission.

In the case of a surplus share, the risk data can be used – the same as would be used for non-proportional per-risk reinsurances – to assess the likely distribution of cession rates.

The reinsurer could then use the cedant's own loss data, or exposure rating, to parameterise the cedant's gross loss experience. Then, each time a gross loss is generated, the distribution of limits could be used and so cession rates to select randomly a cession rate (from those risks with a limit large enough to have a loss of that size) to apply to the loss and so calculate the ceded loss.

This should permit the calculation of the distribution of the reinsurer's financial outcomes, much as for a quota share.

In practice, however, if you are assessing the likely future ceded loss ratio for a surplus treaty, it is simpler just to use the historic loss ratios of the ceded business and project these when assessing a suitable commission. Of course, there is an implicit assumption here that the business ceded in the future will be similar to that ceded in the past, and a deterministic approach tells you nothing about the *variability* of the results.

Once you have a projected ceded loss ratio for the surplus treaty, the calculations are exactly as for quota share business.

Again there are likely to be ceding and profit commissions involved. As for a quota share, it is an important part of the pricing process to determine whether these are too generous or not.

In summary then, the pricing of a surplus treaty is similar to that of a quota share, except that the reinsurer will want to ensure that there is no adverse selection from the cedant in terms of the types and amounts of risk being ceded compared to that retained.

5 Pricing stop loss reinsurance

A stop loss contract is priced using similar methods to those already discussed for other types of excess of loss reinsurance, but where the excess point and limit can be expressed as a loss ratio rather than a monetary amount. The main difference with a stop loss is that it will respond to the aggregate of losses in the year (rather than just, say, individual large losses or large catastrophic losses).

A view of the loss ratio can be derived from historical experience, suitably adjusted, in a similar way to that described in Section 4.1.

This loss ratio would need to be modelled stochastically, to assess the volatility and hence the likelihood (and extent) of large aggregate losses.

Alternatively, the losses may be modelled separately – for example, by splitting out attritional, large and catastrophe losses (or attritional and large/catastrophe).

- **The catastrophe loss could come from a vendor model (where applicable),** such as those discussed in Chapter 21**.**
- **The large losses** could come **from a frequency and severity approach (see Section 3.2).**
- **The attritional losses** could be assessed **using past historical attritional experience, suitably adjusted (see Section 4.1).**

This may produce a better model of the overall volatility.

When pricing a stop loss, important considerations include:

- **meeting risk transfer criteria (regulatory minima)** *ie* any regulatory minimum transfer of risk
- **the particular terms of the stop loss in question**
- **any inuring reinsurance**
- the **amount of** the **recoverable layer returned by** the **cedant.**

6 Complications

This section deals with some of the more unusual elements of reinsurance arrangements.

6.1 Limited and/or paid reinstatements

In most lines of business, reinsurers will want to understand their maximum exposure, and so will limit the total cover offered under a reinsurance contract. They usually do this by imposing a maximum amount of recovery that can be made.

So, for example, a motor excess of loss contract might be £10*m* xs £1*m*.

If the maximum is large – essentially there just to ensure that there is a maximum – and unlikely to be needed in practice, then this would have little impact on the pricing.

If, however, the maximum is not that large and has a non-negligible probability of being applied, then the pricing needs to account for that.

The impact of the maximum will encompass

- **multi-class risk losses**
- **largest risk exposures**
- **RDS (realistic disaster scenarios)**
- **catastrophe losses.**

This is relatively simple to do for experience rating. For a burning cost, the maximum is applied to each year's losses. For a frequency / severity stochastic model, the same is done in each stochastic simulation of the loss experience and recoveries. Note that the impact on pricing is likely to be greater where loadings based on volatility are used. The maximum will affect the standard deviation as well as the expected loss, and probably to a greater extent.

It is less straightforward for a basic exposure rate. Reinsurers are likely to have created (and kept updated) tables of discounts, based on analyses using their benchmark severity distributions, to apply to exposure-based rates.

Often cover is limited and second and subsequent limits of cover are not paid for up front, but as cover is used up.

As reinsurance recoveries are made, "reinstatement premiums" are paid. For example, if a recovery of £100,000 is made on a £1*m* xs £1*m* reinsurance contract, 10% (100,000/1,000,000, *ie* the recovery as a percentage of the limit) of the original ("upfront") premium would be paid to the reinsurer as a reinstatement premium.

This calculation has assumed that the reinstatement premium is "at 100%" of original premium, as will be explained below.

This process stops once the maximum number of reinstatements has been paid. For example, if there are two reinstatements, a cedant continues to pay the reinstatement premiums until it has collected two limits of cover and then it collects the third and final limit of cover with no more premiums to pay.

Typically reinstatements are "at 100%" of original premium. So in the above example, recovering 10% of the limit implies a reinstatement premium of 10% (of 100%) of the original premium. Sometimes, they are at other percentages. For example, if in the above example the reinstatements are at 50%, then recovering 10% of the limit incurs a reinstatement premium of $10\% \times 50\% = 5\%$ of original premium.

Note that "original premium" refers to the reinsurance premium and *not* the original gross premium income (OGPI) written by the cedant.

It is relatively straightforward to adjust pricing to allow for paid reinstatements into a burning cost calculation or frequency / severity stochastic model calculation. It is again not as easy to allow for in a basic exposure rating exercise. Reinsurers may have constructed tables for common reinstatement terms (for example, 2 @ 100% for marine reinsurance) using their benchmarks in a stochastic model.

There is also an approximation that can be used (with more validity in some classes than others).

If it is assumed that losses are always limit (*ie* full) **losses (this works best for lines like property catastrophe and D&O, and not so well for EL, GL or motor), and that frequency is a Poisson variate and that the timing of the cashflows is ignored, then the discount for paid reinstatements is estimated from the frequency and cost for the same number of free reinstatements.**

For example, if P is the cost of a reinsurance contract with one free reinstatement, and P' is the upfront cost for one reinstatement at 50% then:

$$P' + 0.5P' \times \text{probability of at least one loss} = P$$

If frequency is $\text{Poisson}(\lambda)$ then $\Pr(\text{at least one loss}) = 1 - \Pr(\text{no loss}) = 1 - e^{-\lambda}$

So $P' = \dfrac{P}{1.5 - 0.5e^{-\lambda}}$**.**

6.2 *Aggregate deductibles*

Sometimes a contract will have an aggregate deductible. This means that the first £*X* of losses that would otherwise be recoverable under the contract terms is not recovered. For example, suppose a cedant has purchased a £1*m* xs £1*m* contract with a £1*m* aggregate deductible.

Here, we have assumed free reinstatements.

If the cedant has four losses of £1.5*m*, only £0.5*m* of the third and fourth losses can be recovered, as the £0.5*m* of each of the first two that falls in the £1*m* x £1*m* layer is used to exhaust the aggregate deductible.

Gross loss	Loss to £1*m* x £1*m* layer	Cumulative loss to layer	Cumulative recoverable loss	Incremental recoverable loss
£1.5*m*	£0.5*m*	£0.5*m*	£0.0*m*	£0.0*m*
£1.5*m*	£0.5*m*	£1.0*m*	£0.0*m*	£0.0*m*
£1.5*m*	£0.5*m*	£1.5*m*	£0.5*m*	£0.5*m*
£1.5*m*	£0.5*m*	£2.0*m*	£1.0*m*	£0.5*m*

Again, pricing this theoretically is relatively straightforward for a burning cost exercise or for a frequency / severity experience or exposure model. The reinsurer can reduce the losses to the layer for each year / simulation by the aggregate deductible (subject to a minimum of zero for each recoverable loss)**.**

Where basic burning cost analysis is being used, a reinsurer will again have tabulated discounts for common aggregate deductibles (probably tabulated by layer retention and aggregate deductibles as a multiple of limit). These will have been derived from stochastic modelling using the reinsurer's benchmark severity curves.

6.3 *Indexation clauses*

These are covered in numerous other parts of this course. Most of the rest of the Core Reading calls them *stability clauses*.

Liability claims can take a long time to settle, and bodily injury claims in particular are prone to inflationary effects during the delay between incident and settlement. As a result of this it is common to find "indexation clauses" applying to injury claims (only) covered by a contract.

Often the clause applies to *all* claims, but since large long-tailed claims *are* usually in respect of bodily injury, it makes little difference.

The aim of the clause is to try to maintain the real value of the limit and retention for the reinsurer.

Question 20.9

What would happen if no clause is applied?

There are different forms of indexation clause depending on where the contract was placed. There is a standard clause in the London Market (LMIC94) and this in effect indexes the limit and retention in line with average earnings up to the time the claim is settled. The earnings index to use is specified in the clause (for example, in the UK it is a particular index produced by the National Statistics Office; for other countries it is the index produced by the IMF, *ie* International Monetary Fund**).**

For contracts placed outside London, the reinsurer should read any indexation clause carefully to understand how it works. For example, many European clauses index each individual payment for a claim separately to its payment date and then apply the overall weighted average indexation for the sum of the payments under the claim to determine the indexed limit and retention. Indexation clauses are not common in the USA (nor in some other countries, *eg* Ireland)**.**

In addition to all this, indexation clauses come in three variants:

- **Fully indexed – index the limits and retentions for the full effect of inflation between the contract commencement date and claim settlement date (London clause) or loss date and payment date(s) (European clauses). That is, the product of the published applicable earnings inflation rates is calculated for the period required.**

- **Severe indexation – calculate the effect of indexation as above, but only apply it if the cumulative indexation increase is above a particular threshold (for example, 25%) and then only index for the excess of indexation over the threshold. So, if the cumulative indexation factor is 1.275 and the threshold was 25%, multiply the limit and retention by 1.02 (1.275/1.25) in effect.**

- **Franchise indexation clause – calculate the effect of the index as above, but only apply if the cumulative indexation is above a particular threshold. Once it exceeds the threshold apply the full cumulative indexation. So in the previous example we would multiply the limit and retention by the full 1.275.**

You may come across other types of indexation clause in your work.

The reinsurer can allow exactly for indexation in a burning cost calculation, but this can become very complex where multiple countries (so indices) and payment date indexation are involved.

The reinsurer can make a simpler, if approximate, allowance by estimating the average delay to settlement / payment (whichever is appropriate). By making an assumption about the average future rate of earnings inflation (which may be a weighted average if more than one country is covered) the average effect of indexation can be calculated. This can be used to calculate to what on average the limit and retention should be indexed, and thus to price a layer with this limit and retention.

The reinsurer can do this just for injury claims and price the non-injury claims, where there are any, separately. Or the reinsurer can estimate the proportion of the claims that the reinsurer believes will be injury and can use this to weight between the unindexed and indexed layer prices.

6.4 Swing rates

Sometimes – often in situations where there is a marked difference in opinion about the future loss experience between reinsurer and cedant – a contract may be swing rated. This means the final rate (premium) paid will depend on the loss experience very directly.

Question 20.10

Therefore, what is swing rating a form of?

There are two forms of swing rate:

- **In the first version, sometimes called "minimum plus", the final premium that the cedant pays consists of a minimum (paid upfront) plus a factor times the actual losses to the layer, subject to an overall maximum amount.**
- **In the second version, the cedant pays an upfront premium and then the final premium is determined as a factor times the losses subject to a minimum and maximum. Usually the deposit is between the minimum and maximum.**

The factors may be different depending on the chosen form. In the second form, the cedant might end up with a refund of some premium.

Common factors for the second version are 100/80 or 100/75. For the first version, the factors tend to be more variable, as they depend more on the minimum, but still tend to be greater than 1.

> ***Question 20.11***
>
> Why would insurers want this reinsurance if the factors used are greater than 1?

To price a swing-rated contract, the approach is rather similar to that for proportional contracts.

- **Start with an aggregate distribution.**
- **Check the terms of the swing (and again the cedant and/or their broker may suggest terms to start with) to see if they meet the required profit criteria.**
- **If not, the reinsurer can vary the terms to find combinations that do.**
- **The reinsurer then has to negotiate the terms.**

The reason these contracts work in situations where there is a disagreement about the expected loss experience is that if the reinsurer turns out to be right, the swing will ensure more premium is collected, and if the cedant turns out to be right, the swing ensures less premium is collected. In both cases, the actual losses determine the outturn (outcome).

6.5 *Loss ratio caps*

These commonly appear in conjunction with swing rating. The reinsurer applies a maximum limit on cover which prevents the loss ratio to the contract exceeding a certain limit. Caps of 250% and 300% are common.

These are awkward to price as the amount of limit depends on the price, *ie* the value that is being determined.

This can be done iteratively.

- **Set the price without a limit.**
- **Work out what amount of limit corresponds to the loss ratio using the unlimited price.**
- **Reset the price with that maximum limit.**
- **Repeat until the premium stabilises.**

It is worth checking from the aggregate loss distribution what the likelihood of the cap being breached on an unlimited price basis would be first. If it is very low, then it is probably not unreasonable to assume the unlimited price.

7 Tables of data items

This table shows some typical data items of interest for different types of reinsurance coverage.

Property (re)insurance	***Casualty***	***Marine***
Insured name	**Insured name**	**Insured name**
Policy inception date	**Policy inception date**	**Policy inception date**
Policy expiry date	**Policy expiry date**	**Policy expiry date**
Programme identifier (“stacking code”)	**Programme identifier (“stacking code”)**	**Programme identifier (“stacking code”)**
Property identifier	**Line of business (*eg* public liability and employers’ liability)**	**Line of business (*eg* hull, cargo, specie and P&I)**
Postal code/zip code/cresta zone	**Insured industry type/code**	**Insured flag (for hull, Protection & indemnity (P&I))**
Property TIV (total insured value)	**Limit**	**Limit**
Premium charged for property	**Deductible/excess**	**Deductible/excess**
Occupancy type	**Premium**	**Premium**
Insured activity (commercial property)	**Original brokerage/commission**	**Original brokerage/commission**
Buildings sum insured	**Cost of any fac/ri purchased**	**Cost of any fac/ri purchased**
Original brokerage/commission	**Limit of any fac/ri purchased**	**Limit of any fac/ri purchased**
Cost of any fac/ri purchased	**Deductible/excess for any fac/ri purchased**	**Deductible/excess for any fac/ri purchased**
Limit of any fac/ri purchased	**Cedant’s line on the insured risk**	**Cedant’s line on the insured risk**
Deductible/excess for any fac/ri purchased		
Cedant’s line on the insured risk		

The programme identifier (often also referred to as the “stacking code” or “link code”) should enable the reinsurer to identify individual policies that are all part of a programme of layers of insurance for a particular insured that stack one on top of another and all cover the same perils / risks / events. This is important because for this insured the cedant is exposed to a single claim equal to the sum of the limits the cedant writes on these stacking layers. It increases the maximum loss size that is possible. This is also important for recovering outward reinsurance.

The cedant's line is required as it caters for the situation where larger commercial risks do not / cannot place all their cover with one insurer, but instead share it between a number of insurers. The reinsurer will need to know whether the limit provided is the 100% original limit or just the part of the policy that this cedant wrote.

For example, suppose the insured wanted a $100*m* limit, and had placed 10% of that with the cedant. The 100% limit would be $100*m* and the cedant's line would be $10*m*.

This is important when exposure rating, because the benchmarks used are almost invariably derived from 100% claim and limit values. So the reinsurer will need to be able to apply the 100% limit to the benchmark and then scale down the resulting amount by the cedant's line. Applying the cedant's limit to the benchmark will produce an incorrect (because too high) value.

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Chapter 20 Summary

Pricing reinsurance differs from pricing direct business mainly in the amount and type of data available to assess expected claims and their volatility.

Property catastrophe reinsurance

Catastrophe models are the key tool to price these.

Non-proportional reinsurance

Here, we can use:

- exposure rating
- experience rating.

For ***exposure rating***, claims costs can be derived from benchmarks – either ILFs (for casualty business) or first loss scales (for property business). In practice, pricing for high layers is driven by factors other than expected loss costs.

For ***experience rating***, we use either a burning cost calculation or a stochastic frequency / severity model, depending on whether we need a volatility measure, the volume of data and any time / resource constraints.

For the burning cost calculation, the process is:

- trend the claims data
- apply the reinsurance contract terms to give the loss to the layer
- aggregate by year to give triangles of paid / incurred losses
- develop to ultimate using benchmarks if necessary
- adjust exposure (often premium) for past rate and exposure changes
- divide losses by exposure to get burning cost.

A stochastic (frequency/severity) approach is suitable if volatility information is needed, as long as there is enough historic loss data and adequate time/resource.

For either approach, take care with:

- the census point
- the choice of inflation rate
- shock losses
- discounting (particularly for long-tailed business)
- changes in the cedant's business mix.

For ***stop loss*** contracts, we model the loss ratios then use a stochastic model.

Proportional reinsurance

The process for a ***quota share*** is:

- adjust claims for inflation and premiums for rate / exposure changes
- use triangulations to get ultimate historic loss ratios
- decide on an estimated loss ratio for the period in question
- calculate a suitable commission, bearing in mind other outgo, *eg* expenses
- use a stochastic model if there is profit or sliding scale commission.

For ***surplus treaties***, we use the same approach but we may need an algorithm to anticipate the ceded proportion on each risk as the cedant and reinsurer's experience will now differ (which is worth monitoring).

Complications

These can arise due to:

- reinstatements
- aggregate deductibles
- indexation clauses
- swing-rated contracts
- loss ratio caps.

Chapter 20 Solutions

Solution 20.1

Commission will only apply if commission is payable to the cedant, which normally applies only for proportional reinsurance.

Reinsurance will not apply, but *retrocession* (when a reinsurer buys its own reinsurance) will.

Insurance premium tax is not necessarily payable on reinsurance premiums (for example, in the UK).

Solution 20.2

Simply because the cedant is passing a proportion of each and every premium to the reinsurer, whereas for excess of loss reinsurance, the cedant only pays a premium to reflect the expected large claims that breach the retention of the cover.

Solution 20.3

In this case, you would have to develop your claims *before* calculating the amounts hitting the reinsurance layer.

Solution 20.4

The adjustments are the same for both premiums and claims, and so when we calculate a loss ratio or burning cost, they cancel out and effectively we haven't adjusted the data at all.

Solution 20.5

10 years of inflation at 10% per annum means that the census point would ideally be no greater than $£1,000,000v^{10} = £385,543$. However, for prudence, perhaps a census point of, say, £350,000 would be more suitable. More complex approaches are possible, for example by allowing for the fact that not all claims would take 10 years to settle.

Solution 20.6

Recall from Chapter 15 that reinsurance X is said to *inure* to the benefit of another reinsurance arrangement Y if X acts before Y, and Y acts on the net retained amount after X has been applied.

Solution 20.7

Premiums – average time to premium receipt can be derived from the premium triangulations we used earlier.

Claims – average time to claim payment can be derived from the claims paid triangles we used earlier.

Expenses – half a year, if we assume that they are incurred evenly over each year.

Brokerage – depends on the terms of payment. For example, if brokerage is paid on the inception date of the contract, then no discount is needed.

Ceding commission – probably in line with premium discount period, if that is when commission is paid.

Solution 20.8

Because at renewal, the reinsurer will either decline the business, or at the very least reduce the commission dramatically, to compensate. These cedants may also find it difficult to get reinsurance elsewhere, as word gets around the reinsurance market that they don't "behave nicely".

Solution 20.9

More claims would eventually hit the layer, and so the price of the reinsurance would creep up over time. Also, the layer would effectively be targeting a different level of claims, which may no longer meet the insurer's needs – this may necessitate a manual change in the limits rather than automatic indexation.

Solution 20.10

Swing rating is a form of experience rating, as the premium paid depends on the loss experience in the period of cover.

Solution 20.11

If the cedant's experience is worse than expected (by a sufficient degree), the maximum will apply thus capping their losses.

Otherwise, the cedant will pay a bit more in premium than they would have paid in claims had they not reinsured, but they accept this as the price of removing some of the downside risk.

Chapter 21

Use of catastrophe models

Syllabus objectives

(r)(i) Outline the basic structure of a catastrophe model.

(r)(ii) Describe the key perils that can be modelled.

0 Introduction

This short chapter introduces the concepts and usage of catastrophe models, such as those used to model flood and earthquake events. Modelling infrequent extreme events will demand a significantly different approach to modelling attritional claims, due primarily to the uncertainty surrounding their frequency and severity.

Section 1 introduces the background to these models and their basic structure.

Section 2 looks at the main perils that are analysed within catastrophe models, both natural and man-made.

This topic is discussed further in Subject SA3.

1 Overview of catastrophe models

This section looks at why we use catastrophe modelling and the generic structure of catastrophe models.

1.1 Catastrophe modelling versus traditional rating approaches

Traditional rating approaches such as burning cost and frequency / severity (described in Chapter 14) **work well for a high frequency, low severity risk. However, they are much less appropriate for a low frequency, high severity risk. This is because the observed losses may not reflect the true underlying risks, as the period over which losses have been observed may be much shorter than the return period of the losses under consideration. In some cases certain event scenarios under consideration may not have occurred in history.**

The main actuarial assumption that is used when modelling high frequency, low severity risks is that the past is a reasonable guide (with some adjustment) to the future. This assumption is not so justifiable when it comes to events that are rare or uncertain in terms of either frequency or severity.

Typical such low frequency, high severity risks are naturally occurring hazards (or natural catastrophes) such as hurricanes and earthquakes, although some events are man-made (such as the terror attacks on the World Trade Center).

Question 21.1

Can you think of any other catastrophes or events (either naturally occurring or man-made) that are typically low frequency, high severity?

In simple terms, a ten-year burning cost model (or a frequency / severity approach based on ten years of observed losses) is unlikely to be a reliable method of pricing for earthquake risk on a fault with a 250-year return period.

Since the late 1980s a new approach has been developed for the assessment of such perils: catastrophe modelling.

Early catastrophe models were simplistic and largely theoretical. However, modern models were developed largely in response to a series of catastrophes (mainly hurricanes and earthquakes, particularly Hurricane Andrew) in the early 1990s.

> ***Question 21.2***
>
> Another new product was introduced in the early 1990s that accelerated the development of catastrophe models. From your knowledge of general insurance and reinsurance products, can you think what this might be?

This development has been made possible by increased computing power (and in particular the development of Geographical Information Systems (GIS) software) and an increased scientific understanding of the natural hazards themselves.

A GIS integrates hardware, software and data for capturing, managing, analysing and displaying all forms of geographically-referenced information.

A catastrophe modelling approach may start with historical events, but may apply over a much longer timescale (decades or centuries). The model uses past experience and a scientific understanding of the underlying causes of the natural hazards as a basis to create other possible future events – including ones that have never been observed historically – known as the stochastic event set. The model then calculates the effect of these events on the insured portfolio, providing a detailed understanding and representation of the actual locations insured.

In addition, the catastrophe modelling approach can allow for:

- changes in demographics
- changes in the vulnerability of buildings
- changing frequencies of events over time
- changing severity of impact of events
- changes in portfolio.

These allowances can be based on the latest research in areas such as:

- seismology (the study of earthquakes and their effects, *eg* tsunamis)
- meteorology (the study of the atmosphere, and weather in particular)
- hydrodynamics (the study of liquids in motion)
- structural and geotechnical engineering (the behaviour of earth materials).

As a result of the variety and complexity of the inputs to a catastrophe model, proprietary models – produced by a small number of specialist catastrophe modelling firms – predominate in the industry.

In the UK, two of the largest proprietary catastrophe model providers, AIR and RMS, were established in the late 1980s. A small number of other providers entered the market in the 1990s, most notably EQECAT. These providers are discussed further in Subject SA3.

The raw data behind many commercially available models is to an extent "hidden" (arguably to avoid inquisition and accusations of inappropriateness or unreliability, but also to avoid plagiarism).

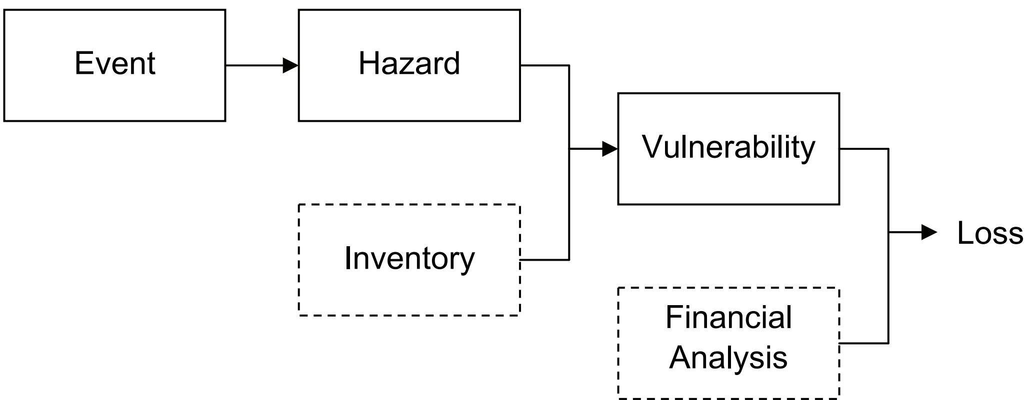
1.2 *Basic structure of catastrophe models*

Catastrophe models all tend to follow the same generic structure, consisting of a number of interlinked modules:

- **event module**
- **hazard module**
- **inventory (or exposure) module**
- **vulnerability module**
- **financial analysis module.**

Diagrammatic structure

The basic structure of a catastrophe model is as follows:



Explanation of modules

- **Event module**

  A database of stochastic events (the event set) with each event defined by its physical parameters, location and annual probability / frequency of occurrence.

  This module will critically contain details of the sorts of events that can occur, and their likelihood: for example, a storm akin to that in the UK in 1990 might be a 1-in-50 year event.

- **Hazard module**

  This module determines the hazard of each event at each location. The hazard is the consequence of the event that causes damage. For example, in the case of a hurricane, wind speed is the primary cause; for an earthquake it is ground shaking.

  So this module will specify, for example, that if a storm happens again, then it is likely to result in some wind damage, some floods, *etc*.

- **Inventory (or exposure) module**

  A detailed exposure database of the insured systems and structures. As well as location this will include further details such as age, occupancy and construction.

  This module is the detail of all the insured's risks, and associated risk factors such as location, *etc*.

- **Vulnerability module**

  Vulnerability can be defined as the degree of loss to a particular system or structure resulting from exposure to a given hazard (often expressed as a percentage of sum insured).

  This module specifies how much damage each insured item (*eg* property) is likely to sustain given a certain peril. For example, a detached house on a flood plain will be more vulnerable to flood than a third-floor flat. The degree of damage will be expressed in monetary terms.

- **Financial analysis module**

  Uses a database of policy conditions (limits, excess, sublimits, coverage terms) to translate the total ground-up loss into an insured loss.

  This module converts the physical damage loss from the vulnerability module to an insured recovery.

Of these modules, two – the inventory and financial analysis modules – rely primarily on data input by the user (an insurer or reinsurer) of the models. The data will be specific to the user. The other three modules represent the engine of the catastrophe model. The event and hazard modules are based on seismological and meteorological assessment, and the vulnerability module is based on engineering assessment.

The inventory and financial analysis module data is based on the insurer's own data, and so it will be important to control the quality of this data.

Catastrophe models tend to fall into one of two categories:

- Aggregate models.

  Here, detailed information on the exposed risks (*eg* properties) is not known. Instead, aggregate exposures (*eg* sums insured) in an area are used in conjunction with industry average losses to estimate the likely losses. This works well as long as the actual risks insured are representative of industry averages, for example in terms of size and construction.

- Detailed models.

  Here, individual insured risk information is used, and the likely loss for each insured risk is calculated, before summing to get aggregate losses.

The primary factors to consider when deciding whether to use an aggregate model or a detailed model are those of cost and time.

2 Key perils modelled

This section discusses some of the perils that are frequently covered by catastrophe models. As such, it is less actuarial and more about geographical hazards.

The key perils discussed are:

- hurricanes
- earthquakes
- tornadoes
- hailstorms
- winter storms
- floods
- disease
- terrorism.

2.1 Hurricane modelling

Hurricane and windstorm models are widely used for the USA, Caribbean, Northern Europe and Japan, and are increasingly used for other territories in the Far East and Latin America. As one might expect, the quality of the historical data upon which the models are based varies by territory. This, in turn, reflects the relative accuracy of the hurricane models for each territory.

For each of the hurricanes within the event set, the model will include a number of parameters such as:

- **track (the path the hurricane will travel)**
- **maximum sustained wind speed**
- **storm radius**
- **forward speed (the speed at which the hurricane is following its track)**
- **the rate of decay of the wind field (wind speed reduction as a function of distance from the storm centre).**

2.2 Earthquake modelling

Earthquake models are well established for the USA and Japan, and are also used for a number of other territories such as Canada, Southern Europe, Mexico and Indonesia. Whilst each year there may be a large number of tremors and minor earthquakes, major earthquakes are rare. Much of the historical information on major earthquakes has been derived from paleoseismic studies, where prehistoric seismic records are inferred from geologic sediments.

For each of the earthquakes within the event set, the model will include a number of parameters such as:

- **the moment magnitude (measuring the energy release)**
- **focal depth (shallow fault ruptures are more damaging for a given value of moment magnitude)**
- **total area of fault rupture**
- **fault type (convergent plate boundaries are highest hazard).**

Event recurrence along a fault line is affected by the elapsed time since the last rupture on that fault and by ruptures on other faults (these may increase or decrease the energy stored within a given fault).

In addition to the conventional "property damage" catastrophe models, for some territories there are also earthquake models available that can be used for assessing the risk to workers' compensation and group life business.

Workers' compensation is similar to employers' liability insurance, although there are slight differences in what exactly is covered, depending on the territory.

2.3 Other natural perils

There are a number of catastrophe models available for other weather-related natural perils, such as tornado, hail, winter storm, and river flood. Many of these models may be available for only a small number of territories.

More recently, infectious disease models have been created. Given the very low frequency of genuine epidemics these models necessarily include a large number of assumptions on how factors such as changes in demographics, ease and frequency of travel and improved medical care will impact epidemics. At present these models are not commonly used.

2.4 Non-natural perils

Terrorism models differ slightly from the models for natural perils both in the way they are constructed and the way in which output is used. These models may offer separate deterministic and probabilistic modules. The deterministic module will enable the insurer to assess its portfolio's maximum expected loss from each of a number of different types of terrorist attacks. The stochastic module includes a statistical model of the annual frequency for each of the types of attack considered within the model. Given the very volatile nature of the probability of attack, insurers and reinsurers typically rely more upon the results of the deterministic module.

The types of terrorist attacks include small scale, large scale and extreme attacks, for example:

- **explosive devices carried by an individual (small scale)**
- **vehicle bombs** (large scale)
- **civilian aircraft hijacking and crashing into targets (large scale)**
- **weapons of mass destruction (extreme scale).**

Relative to hurricane and windstorm, terrorism modelling is in its infancy. It is likely that the sophistication of these models will increase within a relatively short time.

Non-natural perils are commonly referred to as "man-made perils" in the industry.

Question 21.3

Now that you know what catastrophe models consist of, list the main uses to which you think they could be put.

Subject SA3 develops this topic further by discussing the shortcomings of catastrophe models.

3 And finally…

3.1 End of Part 6

You have now completed Part 6 of the Subject ST8 Notes.

Review

Before looking at the Question and Answer Bank we recommend that you briefly review the key areas of Part 6, or maybe re-read the summaries at the end of Chapters 20 and 21.

Question and Answer Bank

You should now be able to answer the questions in Part 6 of the Question and Answer Bank. We recommend that you work through several of these questions now and save the remainder for use as part of your revision.

Assignments

On completing this part, you should be able to attempt the questions in Assignment X6.

3.2 End of the course

This is also the end of the course. You should therefore now start your revision. This should include all of the following:

- re-reading all chapter summaries
- re-reading all the Core Reading
- looking at each Syllabus item
- doing all the questions in the Q&A Bank
- doing the Mock Exam(s) and having it(them) marked
- reviewing old assignments
- looking at past Subject ST8, ST3 and 303 exam questions, and the Subject ST8 specimen exam paper.

You'll be pleased to know that there are many sources of very useful information for students wishing to find out more about general insurance. In theory, you do not *need* to read any additional material to cover the Subject ST8 Syllabus but it may further develop your knowledge and understanding, which of course could be useful for the exam.

The general insurance section of the Institute and Faculty of Actuaries' website is an excellent starting point and has an extensive list of further references.

Good luck!

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Chapter 21 Summary

Catastrophe models can be used to replace traditional rating methods for low frequency, high severity risks. The range of possible future events modelled is called the stochastic event set.

Catastrophe models incorporate knowledge of seismology, meteorology, hydrodynamics and structural and geotechnical engineering to build models that consists of five inter-linked modules:

- event
- hazard
- inventory
- vulnerability
- financial analysis.

The perils modelled can include, for example:

- hurricanes
- earthquakes
- tornadoes
- hailstorms
- winter storms
- floods
- disease
- terrorism.

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Chapter 21 Solutions

Solution 21.1

Possibilities include:

- flood, both riverine and coastal
- hailstorm
- freeze
- disease
- nuclear disaster
- marine (tidal waves)
- financial
- tornado.

Solution 21.2

Catastrophe bonds.

Solution 21.3

Catastrophe models can be used for:

- monitoring aggregate insured losses as part of risk management
- structuring, pricing and purchasing insurance and reinsurance
- capital allocation and assessment, for both internal and regulatory purposes
- financial planning
- setting underwriting guidelines
- disaster recovery planning
- reserving assessment of major catastrophe events
- designing insurance-linked derivates such as catastrophe bonds.

Glossary

Syllabus objective

(a) Define the principal terms in use in general insurance.

Introduction

In general insurance many terms vary by company, class of business, market and country. An important part of any actuarial investigation is to verify the exact meaning of any important terms used. This glossary gives the definitions mainly used in practice and an appendix at the end provides a list of abbreviations used.

The glossary includes terms used in Subjects ST7, ST8 and SA3. An asterisk denotes terms applicable to Subject SA3 only. Some terms are defined that are not used elsewhere in the Core Reading – these are included because the candidate may come across them in background reading.

A potential source of confusion is the term used to denote the value assigned to the liabilities. It has been the practice of accountants to use the word "provision" to denote the value of a liability that is known or assumed to exist at the accounting date, and to confine the term "reserve" to an estimate of additional liabilities, over and above the provisions, either in respect of future events or in respect of past events for which the provisions may prove to be inadequate. However, among insurers, and also among actuaries, there has been a long-established practice of applying the term "reserve" to both categories.

In the European Union, following the adoption of the Insurance Accounts Directive and its enactment in the legislation of each of the Member States, it has become the practice to distinguish between provisions and reserves in insurance companies' shareholders' accounts and also in the accounts that form part of the statutory returns to the insurance supervisory authorities. However, in North America and to some extent in the Lloyd's market, the practice of applying the term "reserve" to both categories continues. It seems likely that among actuaries and others the habit of using the term "reserves" for what are often called provisions will persist for some time even in the UK, notwithstanding the legislative changes – an example being the Claims Reserving Manual published by the Faculty and Institute of Actuaries.

This glossary generally uses "reserve" in text, and in headings uses the word "reserve" followed by "provision" in brackets, for example:

Additional reserve (provision) for unexpired risk

It should be noted that the precise form of words may vary. Candidates may, for example, find references to an additional unexpired risk reserve (or simply an unexpired risk reserve).

24ths method

A method of estimating unearned premium reserve, based on the assumption that annual policies are written evenly over each month and risk is spread evenly over the year. For example, policies written in the first month of the year are assumed to contribute 1/24th of the month's written premium to the unearned premium reserve at the end of the year.

365ths method

A method of estimating unearned premium reserve, based on the assumption that the risk is spread evenly over the 365 days of a year of cover. For example, where a policy was written 100 days ago, 265/365ths of the premium is taken as being unearned.

Accident year

An accident year grouping of claims means that all the claims relating to loss events that occurred in a 12-month period (usually a calendar year) are grouped together, irrespective of when they are actually reported or paid and irrespective of the year in which the period of cover commenced. See *underwriting year, reporting year*.

Accounting classes*

The different classes of insurance business for the purpose of UK statutory returns.

There are currently ten different accounting classes (for example, accident and health, motor vehicle and general liability) that cover the 18 different classes of business for which insurers may be authorised.

The classes of business for which a company may be authorised are set out in Schedule 1 Part 1 (Contracts of General Insurance) of the Financial Services and Markets Act 2000 (Regulated Activities) Order 2001 (SI 2001 No 544).

Accumulation of risk

An accumulation of risk occurs when a single event can give rise to claims under several different policies. Such an accumulation might occur by location (property insurance) or occupation (employers' liability insurance), for example.

Acquisition costs

Costs arising from the writing of insurance contracts, such as commission.

Act of God

An event, such as a storm or flood, that is unexpected and outside human control. From the perspective of insurers, it is a cause of insurance losses.

*Actual total loss**

A form of total loss, defined by the Marine Insurance Act 1906.

Actual total loss is deemed to occur in one of three ways:

- **The insured item is totally destroyed.**
- **The item is so damaged that it can no longer be classed as the type of object originally insured.**
- **The insured is irretrievably deprived of the insured item.**

See also *constructive total loss.*

Additional reserve (provision) for unexpired risk

The reserve held in excess of the unearned premium reserve, which allows for any expectation that the unearned premium reserve will be insufficient to cover the cost of claims and expenses incurred during the period of unexpired risk.

Adjustment premium

The adjustment premium is an additional premium payable at the end of a period of cover. This may result from the use of retrospective experience rating or from a situation where the exposure cannot be adequately determined at the start of the period of cover.

Adverse development cover

A reinsurance arrangement whereby a reinsurer agrees, in return for a premium, to cover the ultimate settled amount of a specified block of business above a certain pre-agreed amount.

Agents' balances

Moneys (typically premiums) that belong to an insurer but are held by an agent.

Aggregate excess of loss reinsurance

A form of excess of loss reinsurance that covers the aggregate of losses, above an excess point and subject to an upper limit, sustained from a single event or from a defined peril (or perils) over a defined period, usually one year.

All risks

Cover that is not restricted to specific perils such as fire, storm, flood and so on. The cover is for loss, destruction or damage by any peril not specifically excluded. The exclusions will often be inevitabilities like wear and tear. The term is sometimes loosely used to describe a policy that covers a number of specified risks, though not all.

Allocated loss adjustment expenses (ALAE)

See *claims handling expenses*.

Annual basis of accounting

Annual accounting is one of two statutory bases of accounting in the UK, the other being fund accounting. Annual accounting is based on the cover provided during the accounting period, regardless of when the contracts of insurance start and end. Fund accounting is based on the contracts starting during the accounting period, regardless of the periods of cover provided.

This is also known as one-year (accident-year) accounting.

Anti-selection

The preference of some insurance applicants for policies whose underwriting requirements are less stringent than others. Thus anti-selection refers to an asymmetry of information between policyholder and insurer where the former has more knowledge of the negative aspects of the risks presented than has the latter.

Asset liability modelling

A form of actuarial projection which analyses future flows of investment income against liability outgo.

Atafs – Age to age factors

Used by the CAS to refer to link ratios or development factors.

Atufs – Age to ultimate factors

Used by the CAS in triangulation reserving methods to refer to the grossing-up factor to get from an intermediate period of development to ultimate.

Average

- **In non-marine insurance, the term relates to the practice of reducing the amount of a claim in proportion to the extent of underinsurance.**
- **In marine insurance, the term is generally used to describe damage or loss.**

Average cost per claim method

A method of reserving which relies on the average cost of claims paid or incurred.

Balance of a reinsurance treaty

The ratio of the total premiums receivable by a reinsurer under a surplus treaty to the reinsurer's maximum liability for any one claim, based on estimated (or expected) maximum loss (EML).

*Base Capital Resources Requirement (BCRR)**

An absolute minimum amount on the capital resources that an insurer must hold as set out in GENPRU 2.1.30 R (Table: Base capital resources requirement for an insurer). The amount depends on the type of insurer and the classes of business underwritten. The BCRR is subject to an indexation procedure in line with consumer price inflation at European level.

Also known as and referred to in Solvency I Directives as the Minimum Guarantee Fund.

Benchmark

A benchmark is any statistic derived from external sources; for example, loss ratio, expense-related measure, claim reporting or claim payment development pattern.

Binding authorities

Contractual agreements setting out the scope of delegated authority, allowing cover holders to enter into contracts of insurance and to issue insurance documents on behalf of Lloyd's managing agents.

Bonus hunger

The reluctance of policyholders under a no-claim discount (NCD) or bonus-malus system to notify claims or claim amounts when faced with a potential increase in premiums. Also known as hunger for bonus.

Bonus-malus

A rating system in which the base premium level can be discounted or loaded in response to the policyholder's claims experience.

Bordereau

A detailed list of premiums, claims and other important statistics provided by ceding insurers to reinsurers, so that payments due under a reinsurance treaty (or delegated authority schemes in direct insurance) can be calculated.

Bornhuetter Ferguson method

A reserving method which uses weights based on an a priori loss ratio and claim development.

Break-up basis

A valuation basis that assumes that the writing of new business ceases and cover on current policies is terminated. Current policyholders would normally be entitled to a proportionate return of the original gross premium and deferred acquisition costs would probably have to be written off. Also known as a wind-up basis. See also *going-concern basis*.

Broker

An intermediary between the seller and buyer of a particular insurance contract who is not tied to either party. A reinsurance broker is similarly defined where reinsurance contracts are bought and sold. See also *Lloyd's broker*.

Burning cost

The actual cost of claims paid or incurred during a past period of years expressed as an annual rate per unit of exposure. This is sometimes used (after adjustment for inflation, incurred but not reported (IBNR) and so on) as a method of calculating premiums for certain types of risks or monitoring experience, for example, motor fleets and non-proportional reinsurance.

Business interruption insurance

Insurance cover for financial losses arising following damage (for example, by a fire) to business premises. Also called loss of profits or consequential loss insurance.

Cancellation

A mid-term cessation of a policy that may involve a partial return of premium.

Capacity

The amount of premium income that an insurer is permitted to write or the maximum exposure that could be accepted. It could refer to an insurance company, a Lloyd's Name, a Lloyd's syndicate or a whole market.

Cape Cod method

A reserving method, similar to the Bornhuetter Ferguson method where, instead of an a priori loss ratio, it uses weights proportional to a measure of exposure and inversely proportional to claims development.

Captive

An insurer wholly owned by an industrial or commercial enterprise and set up with the primary purpose of insuring the parent or associated group companies, and retaining premiums and risk within the enterprise. Some insurers are set up with the primary purpose of selling insurance to the customers of the parent. These are often known as captives but, as they write third-party business, should not properly be so called. If the word "captive" is used without qualification it precludes this interpretation. Lighter regulatory capital requirements for captive reinsurers only apply if the purpose of the captive is to provide cover exclusively for the risks of the undertaking or group to which it belongs and so does not provide cover for third parties.

Case by case estimation

A method of determining the reserve for outstanding reported claims. Each outstanding claim is individually assessed to arrive at an estimate of the total payments to be made. The shorter term "case estimation" is often used and the estimates are referred to as case estimates.

Casualty insurance

Specifically the term is used in the USA, and to a lesser extent in the UK, as an alternative to liability insurance. In a wider context “casualty insurance” may be used as a phrase to cover all non-life insurance.

Catastrophe

In the context of general insurance a catastrophe is a single event that gives rise to an exceptionally large aggregation of losses.

Catastrophe reinsurance

This is a form of aggregate excess of loss reinsurance providing coverage for very high aggregate losses arising from a single event, that may be spread over a number of hours; 24 or 72 hour periods are commonly used. See *hours clause.*

Catastrophe reserve

A reserve built up over periods between catastrophes to smooth the reported results over a number of years. The purpose of a catastrophe reserve is smoothing not solvency.

Ceding company (cedant)

An insurer or reinsurer that passes (or cedes) a risk to a reinsurer. The insurer or reinsurer may be a company or a Lloyd’s syndicate.

Central fund (Lloyd’s)

A contingency reserve built up from contributions by Lloyd’s Names and held by Lloyd’s as a layer of protection for policyholders. A central fund held by Lloyd’s to demonstrate overall solvency to the regulator. This capital is in addition to members’ capital resources held as Funds at Lloyd’s. See *Funds at Lloyd’s.*

Chain ladder method

A statistical method of estimating outstanding claims, whereby the weighted average of past claim development is projected into the future. The projection is based on the ratios of cumulative past claims, usually paid or incurred, for successive years of development. It requires the earliest year of origin to be fully run-off or at least that the final outcome for that year can be estimated with confidence. If appropriate, the method can be applied to past claims data that have been explicitly adjusted for past inflation. Further variations on this original theme are now in wide usage.

Claim

The word "claim" has a variety of meanings. The most common ones are:

- **as a noun: an assertion by a policyholder that an insurer is liable to make a payment in accordance with the terms of a policy**
- **as a verb: to make a request for payment from an insurer.**

Care is often needed to discover the precise meaning in a given context; for example, whether a reference to "claims" is to the number of claims or their cost.

Claim amount distribution

A statistical frequency distribution describing the total amount of claims.

Claim cohort

A group of claims with a common period of origin. The period is usually a month, a quarter or a calendar year. The origin varies but is usually defined by the date of a claim, the date of reporting of a claim, the date of payment of a claim, or the date when the period of cover to which a claim attaches commenced.

Claim cost inflation

The rate of increase in the cost of claim payments.

Claim frequency

The number of claims in a period per unit of exposure, such as the number of claims per vehicle year for a calendar year or per policy over a period.

Claim frequency distribution

A statistical frequency distribution for claim occurrence.

Claim ratio

The ratio of the cost of claims to the corresponding premiums, either gross or net of reinsurance. An alternative term, especially in the USA, is loss ratio.

Claim size distribution

A statistical distribution describing the size of individual claims.

Claims equalisation reserve

See *equalisation reserve and equalisation provision*.

Claims handling expenses

The expenses incurred in handling and settling claims are known in some countries, including the UK, as claims handling expenses, the equivalent term in the USA (and increasingly elsewhere) being "loss adjustment expenses".

In the USA the terms allocated loss adjustment expenses (ALAE) and unallocated loss adjustment expenses (ULAE) are used.

Claims incurred

See *incurred claims*.

Claims made policy

A policy that covers all claims reported to an insurer within the policy period irrespective of when the incident occurred. The type of cover provided by such a policy is known as claims made cover.

Claims reported

Claims incurred that have been reported to the insurer. The term is often used in relation to those claims reported during the accounting period. It may refer to the number of claims themselves or the cost of claims that have been reported.

Claims run-off analysis

A tabulation showing the speed of reporting or settlement for cohorts of claims. Also called a delay table or, since it is usually triangular in form, a run-off triangle. The analysis may be in terms of claim numbers or claim amounts. It is often presented as an intermediate step in a chain ladder projection.

Clash cover

Excess of loss cover for liability business, limiting insurers' exposure to the risk that one event gives rise to claims on more than one policy.

Closed year

In the case of fund accounting a closed year is an underwriting year that is older than the prescribed limit for the class in question. In the Lloyd's market, a closed year is one that has been closed by reinsurance to close (RITC).

Coinsurance

An arrangement whereby two or more insurers enter into a single contract with the insured to cover a risk in agreed proportions at a specified premium. Each insurer is liable only for its own proportion of the total risk. It is frequently applied to individual "slip" business in the London Market where a lead insurer takes a major share of the risk and manages the outturn, while others subscribe on fixed terms. See *slip system*.

The term is also used in direct insurance and reinsurance to describe an arrangement in which the insured or cedant retains a proportion of their own risk.

Combined ratio

The sum of the claim ratio and the expense ratio (and thus not a ratio itself, unless the two separate ratios have the same denominator). Also called the operating ratio or underwriting ratio. The fact that the denominators for the claim and expense ratio may be different can give rise to anomalies.

Commercial lines

Classes of insurance for businesses. Those for individuals are usually referred to as personal lines.

Committee of Lloyd's

A committee that is responsible for administrative matters within Lloyd's under delegation from the Council of Lloyd's. Prior to the establishment of the Council of Lloyd's by the Lloyd's Act 1982, the Committee had sole responsibility for the overall direction of Lloyd's.

Commutation

The process of prematurely terminating a reinsurance contract by agreeing an amount to settle all current and future claims.

Commutation account

A register of the inflows and outflows to the treaty after the commutation has taken place.

Commutation clause

A clause in an insurance or reinsurance contract that allows the contract to be commuted under certain conditions. The clause works in conjunction with commutation accounts, which are used to calculate the relevant numbers.

Composite insurer

A single insurance company that writes both life and non-life business.

Consequential loss insurance

See *business interruption insurance*.

*Constructive total loss**

An expression defined by the Marine Insurance Act. Constructive total loss is where the insured abandons the insured item because an "actual total loss" is unavoidable or because the costs of preventing a total loss exceed the value saved.

Co-reinsurance

Similar to co-insurance, but referring to reinsurance of a risk rather than insurance. The phrases are used interchangeably.

Council of Lloyd's

The governing body responsible for the overall direction of Lloyd's. It was established as a result of the Lloyd's Act 1982 and consists of six working members, six external members and six nominated members whose appointment must be confirmed by the Governor of the Bank of England. One of the nominated members is the Chief Executive.

Cover note

A note issued by an insurance company to confirm the existence of insurance cover pending the issue of formal policy documentation.

Credibility

A statistical measure of the weight to be given to a statistic.

CRESTA zones

The Global Catastrophe Risk Evaluating and Standardising Target Accumulations (CRESTA) zone data set helps brokers and reinsurers assess and present risk, based on the zoning system established by the world's leading reinsurers. Based primarily on the observed or expected seismic activity (although drought, flood and wind storms are also considered) within a country, CRESTA zones consider the distribution of insured values within a country as well as administrative or political boundaries for easier assessment of risks.

Deductible

The amount which, in accordance with the terms of the policy, is deducted from the claim amount that would otherwise have been payable and will therefore be borne by the policyholder. See also *excess*.

Deep pocket syndrome

A situation where claims are made based on the ability of the defendant to pay rather than on share of blame. An injured party will try to blame the party with the greatest wealth (that is, deepest pocket) where there is more than one potential defendant.

Deferred acquisition costs (DAC)

Acquisition costs relating to contracts in force at the balance sheet date. They are carried forward as an asset from one accounting period to subsequent accounting periods in the expectation that they will be recoverable out of future margins within insurance contracts after providing for future liabilities.

Delay table

See *claims run-off analysis*.

Deposit premium

This occurs in cases where all relevant exposure or rating information is not known at the start of the period of cover, or the premium to be paid is dependent on the claims experience during the policy term. An initial premium is paid at the start of the period of cover, followed by an adjustment at the end when the information required is known.

Where this latter adjustment is stipulated at the outset as being upwards only, the term "Minimum and Deposit Premium" applies.

Where it is found in cases relating to retrospective experience rating, the term "swing rated premium" is often applied.

Development factors

The factors emerging from a chain ladder calculation that are the ratios of claims in successive development periods. Sometimes known as link ratios.

Direct business

This term has two meanings:

- **Business acquired without the intervention of an intermediary.**
- **The cover provided by an insurer to an original policyholder, as opposed to any reinsurance cover provided for the insurer.**

The meaning intended is usually clear from the context in which the term is used.

Discovery period

A time limit, usually defined in the policy wording or through legislative precedent, placed on the period within which claims must be reported. It generally applies to classes of business where several years may elapse between the occurrence of the event or the awareness of the condition that may give rise to a claim and the reporting of the claim to the insurer, for example, employers' liability or professional indemnity.

Dynamic financial analysis (DFA)

A phrase given to any form of actuarial modelling in financial services.

Earned premiums

The total premiums attributable to the exposure to risk in an accounting period; they can be gross or net of adjustment for acquisition expenses and gross or net of reinsurance.

Eighths method

A method of estimating unearned premium reserve, based on the assumption that annual policies are written evenly over each quarter and the risk is spread evenly over the year.

Endorsement

Some change to the policy wording, usually following a change in the risk covered, that takes effect during the original period of insurance and is usually, but not necessarily, accompanied by an alteration in the original premium.

Equalisation reserve (provision)

An equalisation reserve, sometimes called a claims equalisation reserve, is a reserve built up (generally from profitable years) as a cushion against periods with worse than average claims experience.

*** Insurance companies in the UK are required by INSPRU 1.4 to establish equalisation reserves for certain classes of business regarded as potentially volatile. These provisions, which are in addition to the provisions required to meet the expected ultimate cost of settlement of claims outstanding at the balance sheet date, are required to be included within technical provisions even though they do not represent known liabilities at the balance sheet date.**

Escalation clause

A policy clause that permits the insurer to raise automatically the level of benefits or sum insured (and therefore the premium) in line with some form of inflation index.

Estimated (or expected) maximum loss (EML)

The largest loss that is reasonably expected to arise from a single event in respect of an insured property. This may well be less than either the market value or the replacement value of the insured property and is used as an exposure measure in rating certain classes of business.

See also *probable (possible) maximum loss (PML)*.

Event

An occurrence that may lead to one or more claims.

Excess

The amount of a claim, specified in the policy, that the insured must bear before any liability falls upon the insurer.

Excess and surplus lines insurance

Excess and surplus lines insurance is a segment of the insurance market that allows consumers to buy property and casualty insurance through the state-regulated insurance market, where policyholders, agents, brokers and insurance companies all have the ability to design specific insurance coverages and negotiate pricing based on the risks to be secured.

Excess of loss (XL or XOL) reinsurance

A form of reinsurance whereby the reinsurer indemnifies the cedant for the amount of a loss above a stated excess point, usually up to an upper limit. The excess point and upper limit may be fixed, or indexed as specified in a stability clause. Usually this type of reinsurance relates to individual losses, but it can be a form of aggregate excess of loss reinsurance covering the total of all losses in a period and subject to a total aggregate claim limit. See also *risk excess of loss* and *aggregate excess of loss*.

Exclusion

An event, peril or cause defined within the policy document as being beyond the scope of the insurance cover.

Expected maximum loss (EML)

See *estimated (or expected) maximum loss (EML)*.

Expense ratio

The ratio of management expenses plus commission to premium (usually calendar accounted expenses to written premium, or sometimes to earned premium).

Experience account

Often a feature of multi-year financial engineering contracts, this is an account that tracks the performance of the business reinsured by the treaty so that the profitability or otherwise of the treaty can be determined.

Experience rating

A system by which the premium of each individual risk depends, at least in part, on the actual claims experience of that risk (usually in an earlier period, but sometimes in the period covered). The latter case is sometimes referred to as swing rated or loss sensitive, and there are often upper and lower limits defining a "collar".

Experience rating also has a more general meaning; for example, in the context of London Market rating. In this context, it is a rating based purely on the experience of the historic risk presented, as opposed to "exposure rating".

Expiry date

The date on which the insurance cover for a risk ceases.

Exposure

This term can be used in three senses:

- **the state of being subject to the possibility of loss**
- **a measure of extent of risk**
- **the possibility of loss to insured property caused by its surroundings.**

***Exposure statistics* are usually shown in one of three common bases: written exposures, earned exposures and in-force exposures.**

Exposure rating

A method of calculating the premium that is based on external data or benchmarks. The risk profile (exposure) of every insured from the products in question is examined. Scenarios of losses of various sizes are analysed and the impact on the policies is determined. The premium of each individual insured does not depend on the actual claims experience of that insured. Instead, the amount of exposure that the insured brings to the insurer and the experience for comparable risks is used to calculate a premium rate.

Exposure unit/measure

The basic unit used by the insurer to measure the amount of risk insured, usually over a given period and usually used directly in rating, with premiums expressed as the rate per exposure unit times the number of units for the risk.

Facultative-obligatory reinsurance

A reinsurance facility with an obligation placed on the reinsurer to accept.

Facultative reinsurance

A reinsurance arrangement covering a single risk as opposed to a treaty arrangement; commonly used for very large risks or portions of risk written by a single insurer.

Fidelity guarantee insurance

Insurance covering the insured against financial losses caused by dishonest actions of its employees (fraud or embezzlement).

Financial engineering

Financial engineering contracts can generally be characterised as ones that attempt to improve a company's balance sheet but with little or no transfer of risk.

Financial risk reinsurance, finite risk insurance or reinsurance

This is a form of reinsurance (or insurance) involving less underwriting risk transfer and more investment or timing risk transfer from the cedant than is customary in reinsurance.

*Financial Services and Markets Act 2000 (FSMA)**

The UK Act that, among other things, gives the Financial Services Authority its supervisory powers over the insurance industry.

*Financial Services Authority (FSA)**

The body responsible for, among other things, the supervision of insurance activity in the UK.

*Financial Services Compensation Scheme (FSCS)**

A scheme established by the Financial Services and Markets Act 2000 to compensate policyholders of insolvent insurance companies in defined circumstances. It is funded by a levy on insurance companies.

First loss

A form of insurance cover in which it is agreed that the sum insured is less than the full value of the insured property, and average will not be applied.

Fleet

A group of vehicles, ships or aircraft that are insured together under one policy.

Fleet rating

The process of determining premium rates for fleets.

Franchise

A minimum percentage or amount of loss that must be attained before insurers are liable to meet a claim. Once it is attained the insurers must pay the full amount of the loss. This feature distinguishes a franchise from a deductible or excess. Note that franchise is also a term to describe the permission given to syndicates to operate within the Lloyd's market.

Free reserves

The excess of the value of an insurer's assets over its technical reserves and current liabilities. Also known as the solvency margin and sometimes, in the case of a proprietary insurer, referred to as shareholders' funds or net asset value.

From the ground up

A statement of an original insurer's experience of a class of business offered for reinsurance is said to be from the ground up when it shows the number and distribution by amount of all claims however small, even though reinsurance is required for large claims only (above retention levels or other contract lower limits).

Functional costing

A process used within an expense analysis to split the expenses of each line department between the different classes of business covered by that department. The process usually relies upon fixing relative unit costs for each of the processes carried out by the department and counting the number of times that each of the processes is carried out over the period in question.

Fund (or funded) accounting

A method of accounting whereby premiums, claims and associated expenses are related to the underwriting year in which the policies start. The recognition of any underwriting profit is deferred until a subsequent accounting period but provision is made for losses as soon as they are foreseen. See also *annual basis of accounting*.

Funds at Lloyd's

Each member of Lloyd's is required to provide capital as security to support their total Lloyd's underwriting business. This is known as Funds at Lloyd's. The level of Funds at Lloyd's determines the amount of insurance business that a member can underwrite.

*General Insurance Capital Requirement (GICR)**

See *minimum capital requirement (MCR)*.

Generalised linear model (GLM)

In statistics, the generalised linear model (GLM) is a flexible generalisation of ordinary least squares regression. The GLM generalises linear regression by allowing the linear model to be related to the response variable via a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value.

Going-concern basis

The accounting basis normally required for an insurer's published accounts, based on the assumption that the insurer will continue to trade as normal for the long term future. See also *wind-up basis*.

Grossing-up factor

A factor used to adjust an immature or incomplete figure to an ultimate or complete one.

*Guarantee fund**

The higher of:

- **One third of the General Insurance Capital Requirement (GICR)**

and

- **Base Capital Resources Requirement (BCRR) (the Minimum Guarantee Fund).**

Hard premium rates

High, profitable premium rates. See also *soft premium rates*.

*Home foreign business**

General insurance business written in the UK relating to risks that originate or are located outside the UK.

Hours clause

A clause within a catastrophe reinsurance treaty that specifies the limited period during which claims can be aggregated for the purpose of one claim on the reinsurance contract. Commonly 24 or 72 hours are used.

Hunger for bonus

See *bonus hunger*.

Inception date

This is the date from which the insurer assumes cover for a risk. This may or may not coincide with the premium collection date.

Increased limit factors

These are factors which estimate the cost for a new limit as a multiple of the basic (original) limit.

Incurred but not enough reported (IBNER) reserve

A reserve reflecting expected changes (increases and decreases) in estimates for reported claims only (that is, excluding any "true" or "pure" IBNR claims). The abbreviation is sometimes stated as "incurred but not enough reserved". The two terms can be regarded as identical in meaning.

Incurred but not reported (IBNR) reserve

A reserve to provide for claims in respect of claim events that have occurred before the accounting date but still to be reported to the insurer by that date. In the case of a reinsurer, the reserve also needs to provide for claims that, although already known to the cedant, have not yet been reported to the reinsurer as being liable to involve the reinsurer.

In some types of work, especially in reinsurance and in the London Market, IBNR provisions include any IBNER provisions. Sometimes the provision for claims incurred on or before the valuation date and reported after the valuation date is referred to as the True IBNR or the Pure IBNR.

Incurred claims (or claims incurred)

Incurred claims refers to the total amount paid on a cohort of claims up to a specified valuation date plus the total of all case estimates on these claims as at the valuation date. In this sense it is used in contrast to paid claims, which refers only to the amounts paid up to the valuation date. Incurred claims usually include ALAE.

The term is also used to refer to the estimate of ultimate claims in annual accounting, which is defined as the total amount paid in the year plus the total claims reserve at the end of the year less the corresponding figure at the start of the year.

This term can be used in different senses and it is essential to confirm the intended meaning in every case.

Indemnity, principle of

The principle whereby the insured is restored to the same financial position after a loss as before the loss. This is typical of most types of insurance. This contrasts with the new-for-old basis of settlement, often used in home contents insurance, under which the insured is entitled to the full replacement value of the property without any deduction for depreciation or wear and tear.

Insurance certificate

A certificate provided by an insurer to confirm that the policyholder has insurance cover.

Insurance cycle

The observed tendency of insurance prices and hence profitability to vary over a period of several years.

Insured

The person, group or property for which an insurance policy is issued.

Integrated Lloyd's Vehicles (ILVs)

Syndicates, the entire participation in which has been bought by insurance groups.

Inwards reinsurance

Reinsurance business accepted or written by an insurer or reinsurer, as opposed to outwards reinsurance which is ceded to a reinsurer.

Knock-for-knock agreement

An agreement between two insurers specifying how claims costs are shared between them when vehicles insured by each of them are involved in the same accident. It specifies that each insurer meets the cost of the damage to the vehicle it has insured without any investigation or allocation of legal liability.

Lapse

When a policyholder, having been invited to renew the policy, does not do so, the policy is said to lapse.

Lapse rate

Usually defined as the ratio of the number of lapses in a defined period to the corresponding number of renewal invitations, but could be another ratio associated with lapses.

Latent claims

Strictly, latent claims are those claims that result from perils or causes that the insurer is unaware of at the time of writing a policy, and for which the potential for claims to be made many years later has not been appreciated.

In common parlance, latent claims are also those that generally take many years to be reported.

Lead underwriter

An underwriter who takes the lead in setting premium rates and agreeing policy conditions under a system of coinsurance (for example, in the Lloyd's market). A lead underwriter may, or may not, be the lead claims handler depending on market practice and agreements for the class of business.

Letter of credit

A financial guarantee issued by a bank that permits the party to which it is issued to draw funds from the bank in the event of a valid unpaid claim against another party.

Liability insurance

Insurance against the risk of being held legally liable to pay compensation to a third party.

Line

Three different meanings arise – the context usually makes it clear which is intended:

- **The ceding office's retention under a surplus reinsurance treaty.**
- **The percentage allocated to an insurer under coinsurance arrangements.**
- **An alternative name for a class of business.**

Line slip

A facility under which underwriters delegate authority to accept a pre-determined share of certain coinsured risks on their companies' behalf. The authority may be exercised by the leading underwriter on behalf of the following underwriters; or it may extend to the broker or some other agent authorised to act for all the underwriters.

Link function

A link function provides the relationship between the linear predictor and the mean of the distribution function.

Link ratios

See *development factors*.

Lloyd’s

A society, incorporated by the Lloyd’s Act 1871, that provides a market place and regulatory framework within which individual and corporate members may participate in the underwriting of insurance risks on their own account.

Lloyd’s broker

An agent approved by the Committee of Lloyd’s to place business with Lloyd’s underwriters.

Lloyd’s deposit

Wholly owned, non-assigned assets that must be lodged in trust with the Committee of Lloyd’s before a member can write any business. The amount of the Lloyd’s deposit, together with the Names’ deposit, when added to individual Names’ deposits or to incorporated Names’ capital, determines the maximum limit of premium income that may be written on their behalf. See also *Funds at Lloyd’s.*

Lloyd’s managing agent

A company appointed to manage the affairs of an underwriting syndicate, appoint an underwriter, and provide technical and administrative services.

Also known as an underwriting agent.

Lloyd’s members' agent

A company that looks after the interests of individual Lloyd's Names. The members’ agent will introduce Names to syndicates, and advise Names on how to spread their capital between different syndicates. The members’ agent will also be responsible for the regular audit of a Name’s wealth, and for submitting all financial statements to Lloyd’s.

Lloyd's special reserve fund

A contingency reserve of limited size that may be built up by individual Lloyd's Names out of pre-tax income.

LMX on LMX

Excess of loss reinsurance provided for syndicates or companies operating in the London Market in respect of LMX business written by them. This is a form of retrocession business.

LMX spiral

The concentration of risk that occurred prior to the mid-1990s when, through the writing of retrocession business (particularly LMX on LMX business), insurers unwittingly ended up reinsuring themselves.

London Market

The part of the insurance market in which insurance and reinsurance business is carried out on a face-to-face basis in the City of London. Sometimes known as the London Reinsurance Market although not all business transacted is reinsurance.

London Market excess of loss (LMX)

Excess of loss reinsurance provided for syndicates or companies operating in the London Market.

Long-tailed business

Types of insurance in which a substantial number of claims take several years from the date of exposure and/or occurrence to be notified and/or settled.

Loss

This may signify:

- **the financial loss suffered by a policyholder, as distinct from the amount of any insurance claim that may be payable in respect of that financial loss**
- **the amount of the insurance claim, as in the expression "loss reserve" which means the same as the reserve (or provision) for outstanding claims**
- **the opposite of "profit" in relation to accounts. In this case, the word needs to be appropriately qualified; for example, underwriting loss or operating loss.**

Loss expense reserve

Another expression for any type of claims handling expense provision.

Loss of profits

See *business interruption insurance*.

Loss portfolio transfer

An arrangement whereby the total liabilities in respect of a specified book of business is passed in its entirety from one insurance entity to another. Policyholders will be informed of this "novation".

Loss ratio

Another expression for claim ratio.

Loss reserve

Another name for claims reserve. The expression is also often used in association with the reserve deposited by a reinsurer with the cedant to cover in part outstanding claims (exact terms would indicate which party received the investment income on associated assets).

Loss sensitive

See *experience rating*.

Losses-occurring (losses occurring during or LOD) policy

A reinsurance policy providing cover for losses occurring in the defined period no matter when they are reported, as opposed to a claims made policy or a risk attaching policy.

Managing agent

See *Lloyd's managing agent*.

Members' agent

See *Lloyd's members' agent*.

Minimum and deposit premiums

See *deposit premium*.

*Minimum capital requirement (MCR)**

The MCR for insurers reflects the requirements of the Solvency I EU directives.

MCR (Required Minimum Margin) is the greater of:

- **the General Insurance Capital Requirement (GICR), which is the greatest of:**
  - **the premiums amount**
  - **the claims amount**
  - **the brought forward amount**

and

- **the Base Capital Resources Requirement (BCRR) (See *Base Capital Resources Requirement*); that is, the minimum guarantee fund (MGF).**

Model uncertainty

When modelling data for the purpose of reserving, the risk that an inappropriate model has been used is known as model uncertainty. The quantification of model uncertainty is difficult to assess, but by using alternative models the risk can be minimised and hence the level of uncertainty can be assessed by comparing the outputs of alternative models.

Moral hazard

Moral hazard refers to the action of a party who behaves differently from the way that they would behave if they were fully exposed to the circumstances of that action. The party behaves inappropriately or less carefully than they would otherwise, leaving the organisation to bear some of the consequences of the action. Moral hazard is related to information asymmetry, with the party causing the action generally having more information than the organisation that bears the consequences.

*Motor Insurers Bureau (MIB)**

An association of motor insurers in the UK that meets the cost of claims under compulsory motor insurance where the driver is uninsured or untraced.

Mutual insurer

Owned by policyholders to whom all profits (ultimately) belong.

See also *proprietary insurer.*

Names (Lloyd's)

The members of Lloyd's who accept the liability for (and profits from) the risks underwritten in their name. Names may be individuals or corporate entities.

Net premium

Usually, the premium net of the cost of reinsurance, although it could mean net of premium tax, or net of acquisition expenses and/or commission.

New for old

See *replacement*; also *indemnity, principle of*.

Nil claim

A claim that results in no payment by the insurer, because, for example:

- **the claim is found not to be valid**
- **the amount of the loss turns out to be no greater than the excess**
- **the policyholder has reported a claim in order to comply with the conditions of the policy but has elected to meet the cost in order to preserve any entitlement to no-claim discount.**

Also known as *zero claim*.

No-claim discount (NCD)

A form of experience rating in which an individual policyholder may be granted a discount from the relevant base premium depending on his or her claims experience. See also *protected NCD*.

Non-proportional reinsurance

Reinsurance arrangements, where the claims are not shared proportionately between the cedant and reinsurer. An excess of loss contract is an example.

Non-technical account

The non-technical account is a feature of accounts of insurance companies in the EU. It is an account made up from the balance on the technical account plus the balance of the investment income and gains not included in the technical account, plus profits on any other activities less tax, dividends and any other charges.

*Novation**

The transfer of the rights and obligations under a contract from one party to another.

Office premium

This is the total premium charged for the period of cover. This premium will contain the risk premium, commission, an allowance to cover all other types of expenses, an allowance for any premium tax and a profit loading.

Ogden tables

These are a set of tables used to help in the calculation of special damages and the present value of loss of earnings or annual expenses in personal injury and fatal accident cases. The tables provide multipliers which take account of life expectancies and a range of discount rates and are prepared by the Government Actuaries' Department. The discount rate is set by the Government.

One-year (accident year) accounting

A basis of accounting that presents, at the end of each year of account, the estimated technical account for business exposed during the year. See also *annual basis of accounting.*

Open year

Under fund accounting an open year is one that has not yet reached the stipulated period for closure. In the Lloyd's market, an open year is one that has not yet been closed by RITC. See also *closed year*.

Operating ratio

See *combined ratio*.

Original gross premium income (OGPI)

The gross premium income received by an insurer in relation to business that is covered by a non-proportional reinsurance treaty. The reinsurance premium is calculated as a percentage of this OGPI. Similar abbreviations, such as OGWP, OGEP, GWPI and GEPI are also used (see the list of abbreviations at the end of this Glossary**).**

Outstanding claims reserve (OCR) or outstanding claims provision

- **The reserve set up in respect of the liability for all outstanding claims, whether reported or not, including reserves for future payments on claims that are currently regarded as settled but may be reopened.**

- **The reserve set up in respect of the liability for all reported outstanding claims, including reserves for future payments on claims that are currently regarded as settled but may be reopened.**

Outwards reinsurance

Reinsurance ceded by an insurer or reinsurer, as opposed to inwards reinsurance, which is reinsurance accepted.

Over-riding commission

Additional commission paid by a reinsurer to an insurer ceding proportional business, as a contribution towards expenses and profit. The term is often used on primary business written through agents or brokers and refers to any addition to basic commission rates either for volume or for profitable business.

Parameter uncertainty

When a model is fitted based upon historic data, certain parameters are selected, for example, development factors and associated tail distributions or average cost assumptions. The goodness of fit represented by these parameters can be tested to identify this element of uncertainty.

*Part VII transfer**

A court-approved transfer of business from one insurer to another, under Part VII of The Financial Services and Markets Act (FSMA). See also *novation*.

Partial payment

- **Partial claim settlement paid on account, before a claim is finalised or closed.**
- **Any claim for less than the full sum insured.**

Peril

A type of event that may cause a loss that may or may not be covered by an insurance policy. An insured peril is one for which insurance cover is provided.

Period of unexpired risk

For a policy in force on an accounting date, the period from the accounting date to the expiry date.

Persistency

A measure of the probability that a policy will remain in force at renewal, rather than lapse.

Personal lines

Types of insurance products offered to individuals, rather than to groups or business entities. Products include private motor, domestic household, private medical, personal accident and travel insurance.

Points rating system

A system for calculating the office premium by relating it to points associated with each cell within a rating factor; the higher the risk associated with the cell, the higher the points and the higher the premium. For example, a driver aged 20 would be associated with many more points and, all other things being equal, a higher premium, than a driver aged 40.

Pooling

Arrangements where parties agree to share premiums and losses for specific types of class or cover in agreed proportions. To some extent all insurance is pooling but specific pooling arrangements often apply particularly where the risks have very large unit size (for example, atomic energy risks) or via mutual associations, such as P&I clubs, catering for an industry.

*Portfolio claims**

Used in proportional and other forms of reinsurance. The outstanding claims that, together with the portfolio premiums, make up the reinsurance premium required for a portfolio transfer; usually used to transfer obligations from one year of account to the next and hence enable a result for the year to be struck. Can also apply to the body of claims transferred in a portfolio transfer.

*Portfolio premiums**

The premiums that together with the portfolio claims make up the reinsurance premium required for a portfolio transfer.

*Portfolio transfer**

The reinsurance of an entire portfolio at a premium relating to the estimated outstanding claims (including IBNR) and unexpired risk under that portfolio. Usually used when an insurer has decided to discontinue writing a particular class, or by a reinsurer wanting to close a treaty year and pass on the liability to the following year for administrative reasons.

Possible maximum loss (PML)

See *probable (possible) maximum loss (PML)*.

Premium income limit

The amount of premium that a Lloyd's Name may write in a given year, determined by the size of the Name's wealth, the Name's Lloyd's deposit and whether or not incorporated.

Premiums trust fund (PTF)

A fund into which all premiums for a Lloyd's syndicate in a given underwriting year are paid. No moneys may be released from the fund other than any profit on closure and on-going claims and expenses.

Primary insurer

An insurer providing cover directly to the insured policyholder, as distinct from a reinsurer.

Probable (possible) maximum loss (PML)

The term "probable maximum loss" represents an attempt to quantify exposure, used in rating or to judge requirements for outwards reinsurance. It may be used as another term for estimated maximum loss, depending on the class of business. The term "possible maximum loss" implies the consideration of more remote scenarios than those for probable or estimated maximum loss and therefore carries a higher value. The fact that the same abbreviation, PML, may be used for both is a source of possible (and, indeed, probable) confusion.

Process uncertainty

Process uncertainty is the risk inherent in writing business and settling claims in general insurance. The modelling of number and amount of claims will vary from the true value owing to random variation.

Product costing

Product costing is the calculation of the theoretical office premium to be charged for a particular class of business.

Product pricing

Product pricing is the determination of the actual office premium. This will take account of current market conditions.

Profit commission

Commission paid by a reinsurer to a cedant under a proportional reinsurance treaty that is dependent upon the profitability of the total business ceded during each accounting period. Also used, in other arrangements, as any commission contingent on the claims experience.

Profit testing

A term used for estimating the economic value of contracts using net present value techniques; that is, proposed premium rates are tested by projecting possible levels of future business, claims, expenses, investment experience and profit. The process may be extended to include all business and so form a model office akin to those used in life companies.

Proportional reinsurance

A reinsurance arrangement where the reinsurer and cedant share the claims proportionally. Usually, premiums follow the same proportions but commission rates may differ. Two types commonly arise: quota share and surplus.

Proprietary insurer

An insurance company owned by shareholders; that is, not a mutual insurer. See also *mutual insurer*.

Protected NCD

A modification to an NCD system whereby a policyholder who has attained a high level of NCD may elect to pay an extra premium in order to be able to make claims without losing future entitlement to discount. There may be a specified limit to the number of claims that can be made without affecting the discount, or the insurer may simply reserve the right to withdraw the policyholder's option to continue on protected NCD.

Protection and Indemnity (P & I) Clubs

Mutual insurers of marine risks.

Quota share reinsurance

A form of proportional reinsurance where the proportions used in apportioning claims and premiums between the insurer and reinsurer are constant for all risks covered by the treaty.

Rate on line

For non-proportional reinsurance, the total premium charged (ignoring reinstatement premiums) for the reinsurance divided by the width of the layer covered.

Rating

The process of arriving at a suitable premium for an insurance risk. The term is sometimes synonymous with underwriting, though rating is strictly just one part of the underwriting process.

Rating basis

The collection of assumptions used to associate the risk premium with the characteristics of the risk being insured.

Rating factor

A factor used to determine the premium rate for a policy, which is measurable in an objective way and relates to the intensity of the risk. It must, therefore, be a risk factor or a proxy for a risk factor or risk factors.

Reciprocity

An arrangement between two insurers who agree to reinsure risks with each other. Commonly used with quota share reinsurance to diversify the insurers' overall portfolios.

Recoveries

Amounts received by insurers to offset directly part of the cost of a claim. Recoveries may be made from several different sources, for example, reinsurers, other insurers, salvage and liable third parties.

Reinstatement

The restoration of full cover following a claim.

Normally, the number of reinstatements, and the terms upon which they are made, will be agreed at the outset. Once agreed, they are automatic and obligatory on both parties.

Reinsurance

An arrangement whereby one party (the reinsurer), in consideration for a premium, agrees to indemnify another party (the cedant) against part or all of the liability assumed by the cedant under one or more insurance policies, or under one or more reinsurance contracts.

Reinsurance to close (RITC)

An agreement under the Lloyd's system of three-year accounting. Underwriting members (the reinsured members) for one year of account (the closing year) of a syndicate agree with another party (the reinsuring party) that the reinsuring party will assume responsibility for handling and paying all known and unknown liabilities of the reinsured members arising out of insurance business underwritten by the syndicate and allocated to the closing year.

The reinsuring party will usually be the subsequent open year of the same syndicate but could also be a later open year, an open year of another syndicate or a reinsurer outside Lloyd's.

The term is also sometimes used to refer to the premium paid to the reinsuring party by the reinsured members.

Reinsurer

An insurer providing reinsurance cover. Some reinsurers do not write any direct or primary insurance business.

Re-opened claim

A claim formerly deemed settled, but subsequently re-opened because further payments may be required.

Replacement

A basis of cover under which the insurer pays the cost of replacing the insured item with a similar but new item. Also referred to as "replacement as new" or "new for old" and contrasts with "the principle of indemnity".

Reporting year

A reporting year grouping of claims will combine all the claims that are reported within a given calendar year, irrespective of the date on which the relevant policy commenced, irrespective of when the claims are actually paid and irrespective of the year in which the incident actually arose. See also *underwriting year*, *accident year*.

*Required Solvency Margin**

The minimum level by which an insurance company's assets should exceed its liabilities according to EC Solvency I Directives. Referred to as the Capital Resources Requirement (CRR) in the FSA Handbook.

Retention

The amount (or proportion) of risk retained by the cedant under a reinsurance arrangement or the insured under an insurance arrangement.

Although, in the case of non-proportional insurance covering a band from R (retention) to U (upper limit), the cedant may be said to retain not only the risk from 0 to R but also the risk above U, it is R that would be termed the retention.

Retroactive date

Used for claims made cover. It is the date after which claims must have occurred in order to be covered.

Retrocession

Reinsurance purchased by a reinsurer in relation to its inwards reinsurance liabilities (that is, reinsurance of reinsurance).

Retrocessionaire

A reinsurer that accepts reinsurance from another reinsurer.

Return commission

Commission paid by a reinsurer to an insurer ceding proportional business, as a contribution towards expenses and profit. Also called overriding commission.

Return period

A return period also known as a recurrence interval is an estimate of the interval of time between events like an earthquake, flood or river discharge flow of a certain intensity or size. It is a statistical measurement denoting the average recurrence interval over an extended period of time, and is usually required for risk analysis (*ie* whether a project should be allowed to go forward in a zone of a certain risk) and also to dimension structures so that they are capable of withstanding an event of a certain return period (with its associated intensity).

Risk-attaching basis

A basis under which reinsurance is provided for claims arising from policies commencing during the period to which the reinsurance relates.

Risk-based capital (RBC)

The assessment of the capital requirement for a general insurer by considering the risk profile of the insurance business written and of any other operations. In the USA, the required minimum margins of solvency are determined after considering RBC requirements.

Risk excess of loss reinsurance

Excess of loss reinsurance that relates to individual losses affecting only one insured risk at any one time.

Risk factor

A factor that is expected, possibly with the support of statistical evidence, to have an influence on the intensity of risk in an insurance cover. See also *rating factor*.

Risk group

The rating cell or risk segment into which particular policies are categorised, within a type of insurance cover. The objective is to achieve a group of policies or risks that have homogeneous characteristics.

Risk premium

The amount of premium required to cover claims expected for a risk; that is, average claim amount times average claim frequency. It may alternatively be expressed as a rate per unit of exposure.

*Road Traffic Act**

The UK legislation that, inter alia, requires anyone using a motor vehicle on the road to have insurance to cover their legal liabilities to third parties (including passengers) in respect of personal injury and property damage.

Run-off basis

A valuation basis that assumes an insurer will cease to write new business, and continue in operation purely to pay claims for previously written policies. Typically expenses and reinsurance arrangements change after an insurer ceases to write new business.

Run-off triangle

See *claims run-off analysis*. The development or run-off triangle may be of paid or incurred claims by amount or number, or of premiums.

Salvage

Amounts recovered by insurers from the sale of insured items that had become the property of the insurer by virtue of the settling of a claim.

Self-insurance

The retention of risk by an individual or organisation, as distinct from obtaining insurance cover.

Short-tailed business

Types of insurance in which most claims are usually notified and/or settled in a short period from the date of exposure and/or occurrence.

Signing down

The process of reducing the proportion of risk that each coinsurer has accepted for a given risk where the slip has been more than 100% subscribed.

Slip system

The face-to-face system used within the London Market to coinsure risks. Proposed risks are described by a broker on a standard form (slip); terms and the premium rate are added after negotiation with a lead underwriter (who also signs for a certain proportion of the risk), before the slip is circulated by the broker amongst other underwriters who sign the slip to confirm the proportion of risk that they will accept.

Soft premium rates

Premium rates with significantly reduced margins due to the competitive state of the market. See also *hard premium rates*.

Solvency margin

Another term for free reserves.

Solvency ratio

The free reserves divided by the net (of reinsurance) written premiums.

Stability clause

A clause that may be included in a non-proportional reinsurance treaty, providing for the indexation of monetary limits (that is, the excess point and/or the upper limit) in line with a specified index of inflation.

*Statutory returns**

Annual statements and accounts that an insurance company is obliged to file under the UK Insurance Companies Acts and Regulations. The purpose is to enable the supervisory authorities to monitor the financial condition of the insurer and decide what action, if any, may be needed to prevent insolvency.

Stop loss reinsurance

An aggregate excess of loss reinsurance that provides protection based on the total claims, from all perils, arising in a class or classes over a period. The excess point and the upper limit are often expressed as a percentage of the cedant's premium income rather than in monetary terms; for example, cover might be for a claims ratio in excess of 110% up to a limit of 140%.

Subrogation

The substitution of one party for another as creditor, with a transfer of rights and responsibilities. It applies within insurance when an insurer accepts a claim by an insured, thus assuming the responsibility for any liabilities or recoveries relating to the claim. For example, the insurer will be responsible for defending legal disputes and will be entitled to the proceeds from the sale of damaged or recovered property.

Sunset clause

Clause defining the time limit within which a claim must be notified, if it is to be valid.

Suretyship

Insurance to provide a guarantee of performance or for the financial commitments of the insured. In the UK this is known as financial guarantee insurance.

Surplus lines insurance

Specialised property or liability coverage in the USA provided by an unlicensed insurer in instances where it is unavailable from insurers licensed in the state in question. See also *surplus reinsurance*.

Surplus reinsurance

A form of proportional reinsurance where the proportions are determined by the cedant for each individual risk covered by the treaty, subject to limits defined in the treaty.

Sometimes known as surplus lines insurance, but should not be confused with the USA definition above.

Swing rated

See *experience rating*.

Syndicate (Lloyd's)

A group of Lloyd's Names who collectively coinsure risks. The syndicates often specialise in particular types of insurance.

Tail VaR

Tail Value-at-Risk (TailVaR) is the expected value of the loss in those cases where it exceeds the predefined confidence level. It is sometimes also called conditional tail expectation (CTE), expected shortfall (ES) or expected tail loss. Thus the TailVaR is equal to the average loss a company will suffer in case of (extreme) situations where losses exceed a predefined confidence level (*eg* 99.5%).

*Technical Account – General Business**

For insurance companies in the UK, the Technical Account – General Business is part of the Profit and Loss Account. It is made up of earned premiums less incurred claims (both adjusted for reinsurance as appropriate), less expenses (with an allowance for deferred acquisition costs as appropriate), plus any part of the investment income that may be allocated to the technical account.

Technical reserves (provisions)

The accounting entries in the balance sheet that represent the insurer's liabilities from the business that has been written.

Three-year accounting

The usual form of funded accounting, in which the underwriting profits are first recognised at the end of the third accounting year from the start of the underwriting year.

Time and distance reinsurance

A type of financial reinsurance that had widespread use in the London Market and Lloyd's, whereby an insurer pays a single premium in return for a fixed schedule of future payments matched to the estimated dates and amounts of the insurer's claim outgo. The purpose of such contracts was to achieve the effect of discounting in arriving at the reserves for outstanding claims. Since Lloyd's changed its rules so that the credit allowed for time and distance policies in a syndicate's accounts was limited to the present value, such policies have become less popular.

Treaty reinsurance

Reinsurance that a reinsurer is obliged to accept, subject to conditions set out in a treaty.

Uberrima fides

Latin for "utmost good faith". This honesty principle is assumed to be observed by the parties to an insurance, or reinsurance, contract. An alternative form is uberrimae fidei: "of the utmost good faith".

UK Guarantee Fund

The UK guarantee fund is a fund operated by the Motor Insurers Bureau to compensate victims of negligent uninsured or untraced drivers who have no other source of compensation.

Unallocated loss adjustment expenses

See *claims handling expenses*.

Underinsurance

When the sum insured is less than that required under the terms of the contract. Depending on the policy conditions, where underinsurance is proved to exist, insurers may be able to claim that the policy is null and void. Alternatively, average may be applied to claim amounts.

Underwriter

An individual who assesses risks and decides the premiums, terms and conditions on which they can be accepted by the insurer.

Underwriting

The process of consideration of an insurance risk. This includes assessing whether the risk is acceptable and, if so, the appropriate premium together with terms and conditions of the cover. It may also include assessing the risk in the context of the other risks in the portfolio. The more individual the risk (for example, most commercial lines), the more detailed the consideration.

The term is also used to denote the acceptance of reinsurance and, by extension, the transacting of insurance business.

Underwriting agent

An organisation at Lloyd's providing management services for syndicates and/or advice for Names. See *Lloyd's managing agent.*

Underwriting factor

Any factor that is used to determine the premium, terms and conditions for a policy. It may be a rating factor or some other risk factor that is accounted for in a subjective manner by the underwriter.

Underwriting ratio

See *combined ratio*.

Underwriting year

An underwriting year grouping of claims will combine all the claims relating to loss events that can be attributed to all policies that commenced cover within a given calendar year, irrespective of when they are actually reported or paid and irrespective of the year in which the incident actually arose. See also *reporting year*, *accident year*.

Unearned premium reserve (UPR) or provision for unearned premiums

The amount set aside from premiums written before the accounting date to cover risks incurred after that date.

Unearned premiums

The portion of premium written in an accounting period that is deemed to relate to cover in one or more subsequent accounting periods. It can be calculated in at least two ways:

- **Net of deferred acquisition costs (DAC); that is, by deducting acquisition expenses before proportioning the written premium.**
- **Gross of DAC; that is, by proportioning the full written premium without any deduction for DAC.**

The first approach is consistent with a going-concern basis, whilst the second is consistent with a break-up basis. However, the second approach can also be used for a going-concern basis by including DAC as an asset in the balance sheet.

A typical balance sheet includes values gross and net of reinsurance also.

Unexpired risks reserve (URR) or provision for unexpired risks

This term is often used in two ways:

(1) The reserve required to cover the claims and expenses that are expected to emerge from an unexpired period of cover.

(2) The reserve required to cover the excess of (1) over the UPR. This is sometimes known as the additional reserve for unexpired risk (AURR).

Value at Risk (VaR)

In financial mathematics and financial risk management, Value at Risk (VaR) is a widely used measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading) is the given probability level.

Wind-up basis

See *break-up basis*.

Working layer

A layer of excess of loss reinsurance at a level where there is likely to be a fairly regular flow of claims.

Written premiums

The amount of premium, either gross or net of reinsurance, for which cover commenced in an accounting period.

Zero claim

Another term for *nil claim*.

APPENDIX – List of Abbreviations

ABI	**Association of British Insurers**
ACPC	**Average cost per claim**
AEP	**Aggregate exceedance probability**
AIC	**Akaike information criteria**
ALAE	**Allocated loss adjustment expenses**
ALM	**Asset liability modelling or management**
ALM	**Association of Lloyd's members**
ANOVA	**Analysis of variance**
AP	**Adjustment premium**
APH	**Asbestos, pollution, health hazard**
ART	**Alternative risk transfer**
ASTIN	**Actuarial STudies In Non-life insurance**
AURR	**Additional unexpired risk reserve**
AV	**Annual venture (Lloyd's)**
BAS	**Board for Actuarial Standards**
BCRR	**Base Capital Resources Requirement**
BI	**Bodily injury**
CAR	**Contractors' all risks (insurance)**
CAS	**Casualty Actuarial Society**
CDF	**Cumulative density function**
CGL	**Comprehensive general liability (policies)**
CIL	**Coming into line (Lloyd's)**
CRTF	**Credit for reinsurance trust fund (Lloyd's)**
CUE	**Claims and underwriting exchange (in UK)**
D&O	**Directors and officers (insurance)**
DAC	**Deferred acquisition costs**
DFA	**Dynamic financial analysis**
E&O	**Errors and omissions (insurance)**
E&S	**Excess and surplus (lines insurance)**
ECA	**Economic capital assessment (Lloyd's)**
ECR	**Enhanced capital requirement**
EIOPA	**European Insurance and Occupational Pensions Authority**

EL	**Employers' liability**
ELC	**Expected loss cost**
EML	**Estimated (or expected) maximum loss**
EPD	**Expected policyholder default**
ERM	**Enterprise risk management**
ESG	**Economic scenario generator**
FAL	**Funds at Lloyd's**
FATF	**Financial Action Task Force**
FGU	**From the ground up**
FPD	**Franchise performance directorate (Lloyd's)**
FSA	**Financial Services Authority**
FSCS	**Financial Services Compensation Scheme**
FSMA	**The Financial Services and Markets Act**
GAAP	**Generally accepted accounting principles**
GCR	**Group Capital Resources**
GDP	**Gross domestic product**
GEPI	**Gross earned premium income**
GEV	**Generalised extreme value (distribution)**
GICR	**General insurance capital requirement**
GIRO	**General Insurance Research Organisation**
GIS	**Geographical Information Systems**
GLM	**Generalised linear model**
GPD	**Generalised Pareto Distribution**
GRIP	**General insurance premium Rating Issues working Party**
GRIT	**General insurance Reserving Issues Taskforce**
GWPI	**Gross written premium income**
HSE	**Healthy and Safety Executive**
IAA	**International Actuarial Association**
IAIS	**International Association of Insurance Supervisors**
IASB	**International Accounting Standards Board**
IBNR	**Incurred but not reported**
IBNER	**Incurred but not enough reported**
ICA	**Individual capital assessment**
ICAS	**Individual capital adequacy standard**

ICG	**Individual capital guidance**
IFRS	**International Financial Reporting Standards**
IID	**International Insurers Division**
ILFs	**Increased limit factors**
ILS	**Insurance-linked securities**
ILVs	**Integrated Lloyd's Vehicles**
ILWs	**Industry loss warranties**
IMF	**International Monetary Fund**
IPO	**Initial public offering**
IUA	**Insurance Underwriting Association**
LDF	**Loss development factor**
LDP	**Low discrepancy point**
LMA	**Lloyd's Market Association**
LMX	**London Market excess of loss**
LoC	**Letter of credit**
LOD	**Losses occurring during**
LPT	**Loss portfolio transfer**
LSICA	**Lloyd's Society individual capital assessment**
LTA	**Long term agreement**
M&A	**Mergers and acquisitions**
MAT	**Marine, aviation and transport**
MCR	**Minimum capital requirement**
MGA	**Managing general agent**
MGF	**Minimum guarantee fund**
MGF	**Moment generating function**
MIB	**Motor Insurers Bureau**
MIG	**Mortgage indemnity guarantee (insurance)**
MLE	**Maximum likelihood estimate or estimation**
MoM	**Method of moments**
MPL	**Maximum probable loss**
MSARC	**Market supervision and review committee (Lloyd's)**
NAIC	**National Association of Insurance Commissioners**
NCD	**No-claim discount**
NCF	**New Central Fund (at Lloyd's)**

NYID	**New York Insurance Department**
OCR	**Outstanding claims reserve**
ODP	**Overdispersed Poisson**
OEP	**Occurrence exceedance probability**
OGEP	**Original gross earned premium**
OGPI	**Original gross premium income**
OGWP	**Original gross written premium**
ONS	**Office for National Statistics (UK)**
OPIL	**Overall premium income limit (Lloyd's)**
P&I	**Protection & Indemnity**
PCS	**Professional Conduct Standards**
PD	**Property damage**
PML	**Probable (possible) maximum loss**
PTF	**Premiums trust fund**
RBC	**Risk-based capital**
RDS	**Realistic disaster scenarios**
RITC	**Reinsurance to close**
RMM	**Required minimum margin**
RMS	**Required margin of solvency**
RMS	**Risk Management Solutions**
ROC	**Reserving Oversight Committee**
ROE	**Return on equity**
SAO	**Statement of actuarial opinion**
SLTF	**Surplus lines trust fund (Lloyd's)**
SME	**Small to medium (sized) enterprises**
SMR	**Solvency Margin Requirement**
SORP	**Statement of recommended practice**
TPL	**Third party liability**
TVaR	**Tail value at risk or Tail VaR**
ULAE	**Unallocated loss adjustment expenses**
ULR	**Ultimate loss ratio**
UPR	**Unearned premium reserve**
URR	**Unexpired risks reserve**
VaR	**Value at risk**

XL	**Excess of loss**
XoL	**Excess of loss**
YOA	**Year of account**

Part 1 – Questions

Introduction

The Question and Answer Bank is divided into 7 parts. The first 6 parts of the Question and Answer Bank include a range of short and long questions to test your understanding of the corresponding part of the Course Notes, whilst the last part contains a set of exam-style questions covering the whole course. For each part the questions may require knowledge from earlier parts of the course.

Each Question and Answer Bank part is split into two sections:

- Section 1 – Development Questions. The aim of these questions is to build on your understanding, test key Core Reading and bring your knowledge and skills to the level required to tackle Exam-style Questions.

- Section 2 – Exam-style Questions. These questions are of the level of difficulty you are likely to face in the examination. It is very important that you focus on these questions as preparation for the exam.

We strongly recommend that you use these questions to practice the techniques necessary to pass the exam. Do not use them as a set of material to learn but attempt the questions for yourself under strict exam style conditions, before looking at the solutions provided.

This last point highlights the difference between active studying and passive studying. Given that the examiners will be aiming to set questions to make you think (and in doing so they will be devising questions you have not seen before) it is much better if you practise the skills that they will be testing.

It may also be useful to you if you group a number of the questions together to attempt under exam time conditions. Ideally three hours would be set aside, but anything from one hour (*ie* 35 marks) upwards will help your time management.

> Note that the split between Development Questions and Exam-style Questions is somewhat subjective. For example, there have been past exam questions that test knowledge of the Core Reading, and so are similar to those we've included here as Development Questions. The Exam-style Questions involve more application and a wider range of ideas and are typically the more challenging questions in the exam.

1 Development questions

Question 1.1

Explain whether each of the following statements about motor insurance is true or false.

A Insurers rarely ask proposers where their car will be driven, although it is one of the main determinants of the risk underlying a policy.

B No-claims discount level can be used as a rating factor even though the premium that is finally charged depends on it.

C A large majority of motor liability claims are for large amounts.

D There is no sum insured for bodily injury in a third party only motor insurance policy. [2]

Question 1.2

State the usual measures of exposure for:

- personal accident insurance
- industrial fire insurance
- employers' liability insurance
- extended warranty
- creditor insurance
- legal expenses
- goods in transit
- mortgage indemnity guarantee
- business interruption cover
- motor insurance. [5]

Question 1.3

Define reciprocity and describe briefly the advantages and disadvantages for the parties involved. [7]

Question 1.4

Describe the main characteristics of Director and Officers (D&O) insurance. [6]

Question 1.5

(i) List the desirable properties of a measure of exposure. [2]

(ii) What measure of exposure might you use for a professional indemnity policy, and why? [2]

(iii) What are the disadvantages of your measure? [1]
[Total 5]

Question 1.6

(i) List the reasons why an insurer might wish to become the fronting insurer in a fronting arrangement. [3]

(ii) What is the main risk associated with being a fronting insurer and how can this risk be reduced? [2]
[Total 5]

Question 1.7

Explain the *principle of average*. [3]

Question 1.8

For commercial motor insurance, list the risk factors that would be different to the risk factors for private motor insurance. [2]

Question 1.9

Give four examples of possible fraudulent claims within household insurance. [2]

Question 1.10

(i) Explain what is meant by *commercial lines* of general insurance. [1]

(ii) Price competition is less important in many commercial lines than it is for many personal lines. Give two factors, other than price, that are also important to a business, in comparison to an individual, when choosing its insurer. [2]
[Total 3]

Question 1.11

Describe the features of travel insurance under the following headings:

- cover available and perils insured
- common exclusions
- measure of exposure
- risk factors
- rating factors used in practice
- usual forms of underwriting
- major characteristics of claims. [20]

Question 1.12

(a) Explain what is meant by *indemnity*.

(b) Give four distinct examples of general insurance policies that do not provide indemnity for a policyholder in the event of a claim. [5]

Question 1.13

Explain why rating factors are needed. [2]

Question 1.14

Outline the characteristics of property damage claims for the following classes of business:

(i) motor [3]

(ii) marine & aviation [3]

(iii) household. [3]
[Total 9]

Question 1.15

Describe briefly the purposes for which a medium-sized insurance company, writing all lines of general insurance business, might use reinsurance. [6]

Question 1.16

List two possible measures of exposure, excluding "premiums", which could be used for each of:

(a) motor

(b) employers' liability

(c) industrial fire

(d) household contents. [2]

Question 1.17

Describe the main features of surplus reinsurance. [2]

Question 1.18

Describe briefly:

(a) spread loss covers

(b) structured finance arrangements

(c) industry loss warranties (ILWs)

(d) committed capital arrangements. [5]

Question 1.19

(i) List the items that may be included in an excess of loss reinsurance treaty. [6]

(ii) Define a *stability clause* and explain why one is often included in an excess of loss reinsurance treaty. [4]
[Total 10]

Question 1.20

State two different measures of exposure that might be used when rating industrial fire business and outline the weaknesses of each one. [4]

Question 1.21

Give four examples of risks that may be excluded from personal accident policies. [2]

Question 1.22

(i) List eight underwriting factors that might be used for employers' liability insurance if the exposure measure is payroll. [2]

(ii) List possible stages of delay in an employers' liability claim. [4]
[Total 6]

2 Exam-style questions

Question 1.23

(i) Give two distinct reasons why there may be an end-of-year adjustment to the premium for motor fleet policies. [2]

(ii) State what the first basis for splitting claims would be when analysing the claims from a portfolio of motor fleet policies. [1]

(iii) State, with reasons, whether motor fleet business would be described as short tail or long tail. [2]

(iv) State the main determinant of the extent to which the premiums for a motor fleet are based on the fleet's own experience (*ie* "experience rated")? [1]

[Total 6]

Question 1.24

A general insurance company A writes only commercial property business. One risk which it coinsures with three other insurers B, C and D has a sum insured of $10 million, but an expected maximum loss (EML) of $500,000. Company A accepts 40% of this risk, with B, C and D accepting 20% each.

Company A reinsures with company X 5% of every risk under a quota share treaty. It is agreed that A will not write business for which its gross share of the EML exceeds $250,000.

Company A also has a three line surplus treaty with companies Y and Z, each taking 50%, which operates after the quota share, and is based on company X taking 5% of company A's gross business. The surplus treaty has a maximum retention of $50,000.

A single large claim gives rise to a loss of $750,000.

(i) Calculate the amount of the claim which Company A will pay, net of all reinsurance recoveries due. State any assumptions you make. [4]

(ii) Explain how your answer to (i) would differ if, immediately prior to this claim, companies B and Y were declared insolvent. [3]

(iii) State the information you would expect Company A to provide to Company Z during the handling of this claim. [3]

[Total 10]

Question 1.25

(i) State the ideal features of a risk that make it insurable for a general insurance company. [4]

It has been suggested that an insurance company should start selling people insurance to cover them against being late for work due to delays on public transport.

(ii) In the light of the features set out in (i), comment on whether you agree with this proposal or not. [4]

[Total 8]

Question 1.26

A large hairdressing boutique is considering offering its clients "haircut insurance". The clients would be entitled to a compensation payment if the resulting cut was unsatisfactory. You are an insurer who has been asked to quote for a premium, which will be paid to you by the hairdresser.

Discuss how the insurance arrangement might be made to work sensibly in practice and whether or not the premium should, at any given time, be the same in all cases. [7]

Part 1 – Solutions

1 Solutions to development questions

Solution 1.1

A is true. Asking where the car will be driven would be impractical. Proposers could lie, or might genuinely not know. Instead insurers use the policyholder's address as an indication of where the car may be driven. [½]

B is true. The NCD level helps determine the premium charged, therefore it is a rating factor. [½]

C is false. Most motor liability claims are for property damage. These do not cost a large amount to settle. [½]

D is true because no cover is given for bodily injury to the proposer and third party bodily injury cover is usually unlimited. [½]

[Total 2]

Solution 1.2

personal accident insurance:	person year or sum insured per year
industrial fire insurance:	sum insured per year
employers' liability insurance:	annual payroll
extended warranty:	number of appliances or appliance years
creditor insurance:	amount of the loan or total amount repayable
legal expenses:	number of policyholders or policyholder-years
goods in transit:	value of consignment
mortgage indemnity guarantee:	excess of the amount of the loan over a certain percentage of the value of the property
business interruption cover:	annual profit or turnover
motor insurance:	vehicle year [½ each, total 5]

Solution 1.3

Reciprocity is an arrangement between two insurers who agree to reinsure risks with each other. [1]

Advantages and disadvantages

+ Reciprocity increases net premiums, assuming that the alternative is to cede business to a reinsurer and not to accept any business in return. [½]

+ It increases the insurer's market share. [½]

+ Furthermore, it may improve the insurer's reputation in the market. [½]

+ Reciprocity should increase diversification … [½]

+ … which will increase the stability of the results. [½]

+ The financial strength of the insurers may be improved, since they are effectively merging resources. [½]

+/– The effect on the commission paid will depend upon the ceding commission arrangements with the other insurer. [½]

+/– It may reduce the solvency margin requirement on one or both of the insurers, depending on the supervisory regime and the insurer's current financial strength. [½]

– If the insurers operate in the same field then they may not get a better spread of business and the catastrophe position may not be improved. [½]

+/– The business ceded may be more profitable than the business received (or vice versa). [½]

– The exchange of business may result in disclosure of market knowledge. [½]

– The insurers may need to spend time and money underwriting the business received. [½]

– This may place unnecessary demands on management time, especially if there are several agreements in place. [½]

– There may be new types of management problems, *eg* if insurers are dealing with insurers in other currencies. [½]

– It will be necessary to consider the security of the other insurer and the standard of its underwriting. [½]

[Maximum 7]

Solution 1.4

Directors and Officers (D&O) insurance is purchased by companies to protect against the directors and officers of the company being sued for acts they have performed in their capacity as directors and officers of the company. [1]

Deliberate fraud by directors and officers will be excluded. [½]

The perils include the following:

- allowing a company to continue operating in circumstances when it should have been declared insolvent [½]
- any act resulting in the insured being declared unfit for his or her role [½]
- allowing false financial statements to be published. [½]

Cover is likely to be on a claims-made basis. [½]

Typical exposure measures are turnover and net assets and liabilities of the company. [½]

Reporting of claims tends to be fairly quick. [½]

However settlement may take time, due to the high incidence of litigation. [½]

Therefore this tends to be a relatively long-tailed class of business. [½]

Typical risk / rating factors are:

- nature of business [¼]
- past experience [¼]
- state of the economy. [¼]

This type of cover tends to be distributed through Lloyd's. [½]

[Maximum 6]

Solution 1.5

(i) ***Desirable properties of a measure of exposure***

- principle measure of the risk [½]
- practical
  - obtainable / measurable [¼]
  - verifiable [¼]
  - objective / not manipulable [¼]
  - acceptable to policyholder [¼]
- helps avoid selection [½]

[Total 2]

(ii) ***Suitable measure for professional indemnity***

A measure of exposure for a professional indemnity policy might be the number of professionals (or staff by category of work) covered by the policy. [1]

Why? It's a measure of the amount of work being done, and hence related to the amount of risk. [½]

It also satisfies the other criteria in part (i). (Total fee income would also be appropriate, as may staff salaries.) [½]

[Total 2]

(iii) ***Disadvantages of that measure***

It makes no allowance for the type of work being done and the likely financial implications of the work. (Many other possible risk factors should also be incorporated.) Thus, it is only a very crude measure of the amount of risk. [1]

Solution 1.6

(i) ***Reasons for becoming a fronting insurer***

The fronting insurer:

- will typically receive a fee to cover its expenses and profit [½]
- may wish to retain a presence in a particular market while having limited risk there [½]
- may benefit from a fronting arrangement with a well-known insurer, and so may build itself a presence in the market [½]
- may be able to keep a small amount of the risks taken on, which may be profitable [½]
- may be able to build up expertise on a particular product or territory through the fronting arrangement [½]
- may benefit from tax advantages. [½]

[Total 3]

(ii) ***Main risk and ways of reducing it***

The main risk to the fronting insurer is that the assuming insurer fails to meet its obligations to the policyholder, in which case the fronting insurer is legally liable to the insured. [1]

In order to reduce this risk, the fronting insurer should assess the credit risk of the assuming insurer before entering into the fronting agreement. [½]

The fronting company may require the risk-bearing party to provide some form of collateral to help mitigate the credit risk; for example a letter of credit. [½]

Additionally, the fronting insurer could front for a number of different insurers to reduce the concentration of credit risk with any one assuming insurer. [½]

[Maximum 2]

Solution 1.7

Principle of average

Possibly used with commercial fire insurance and other property insurance. [½]

A policyholder may insure an agreed proportion of the value of a property. Claim payments for losses suffered by the policyholder are then scaled down in that proportion. [1]

It is not used in domestic property insurance, other than where the insurer discovers (after a claim has been made) that the policyholder was underinsured. [1]

The insurer could refuse to pay the claim, but instead would normally scale down the claim payment (which is effectively an ex-gratia payment). [½]
[Total 3]

Solution 1.8

Alternative risk factors for commercial motor insurance:

- type of vehicle (*eg* lorry, as well as vehicle rating group)
- nature of goods transported
- vehicle weight and carrying capacity
- area of operation (*eg* local, national, international *etc*)
- vehicle maintenance procedures. [½ each, maximum 2]

Solution 1.9

Four possible fraudulent claims in household insurance:

- claiming for more than the value of goods lost / stolen
- claiming for fictitious goods that have not been lost / stolen
- engineering "accidents" to old / broken items and claiming full replacement
- arson. [½ each, total 2]

Solution 1.10

(i) ***Commercial lines***

Insurance sold to businesses, rather than to individuals. [1]

(ii) ***Two important factors other than price***

The quality of the service is a more important form of competition on commercial lines, *eg* a fleet manager with a large fleet will want efficient handling of the many claims likely to be made in each year. [1]

Also, businesses should pay more attention to the financial strength of the insurer to ensure that claims will be paid. [1]

[Total 2]

Solution 1.11

Cover available and perils insured

Policies cover losses arising from incidents occurring during holidays or periods of business travel, or from the expense involved in cancelling travel arrangements for certain specific reasons, *eg* illness, bereavement. [1]

A typical policy might have sections for:

- loss of or damage to luggage
- loss of personal money
- medical expenses while travelling
- scale of benefits for delays
- cancellation costs
- personal accident
- personal liability. [½ each]

Policies will usually specify monetary limits for claims. [½]

Note that these policies are generally non-annual although many insurers do offer annual policies. Cover will be limited to the duration of a trip, *eg* fifteen days. Vending machines in some airports allow individuals to buy instant cover for a single journey. [1]

Common exclusions

Policies may exclude some foreign destinations, *eg* those which are particularly politically unstable, and exclude claims arising from circumstances that could reasonably have been anticipated. [1]

Other exclusions might be the accident benefits and/or the medical expenses resulting from:

- drugs (unless medically prescribed)
- intoxicants
- attempted suicide
- taking part in dangerous pursuits, *eg* pot-holing, motor racing. [½ each]

Policies for high-risk sporting holidays will extend cover to the specified sports. Premiums are correspondingly higher for these policies. [½]

There may be a small excess on some sections of the policy, *eg* loss of or damage to luggage. [½]

Measure of exposure

Person-journey or person-week or, for annual policies, person-year. [1]

Risk factors

The main factors affecting the risk are:

- destination
- level of cover
- number, age and gender of persons in the party (if gender is allowed as a rating factor)
- duration of stay
- reason for travel
- current health
- quality of the tour operator. [½ each]

Rating factors used in practice

As policies are usually purchased "off-the-shelf" through travel agents, they need to be quite standardised. Premium rating is therefore driven by practical considerations. [1]

Premiums may also be split by the time periods for the cover, *eg* 0–3 days, 4–7 days, 8−14 days and so on. Most policies have a maximum of three months, although, as mentioned above, many insurers are now offering annual travel insurance cover. [1]

Age is also used for rating. Reductions are available for children in the party and some insurers also impose a loading for older travellers, *eg* those over 60. [1]

Health insurance is often a major part of the cover. Hence, the type of holiday is an important factor if the holiday includes, for example, winter sports. The destination, *eg* USA, is another factor that could lead to expensive medical treatment. [1]

Usual forms of underwriting

No direct underwriting takes place on standard travel policies. [½]

Major characteristics of claims

Delays

This is a short-tail class. Reporting delays are not usually significant, although claims may not be reported until the policyholder has returned from abroad. Insurers with a 24-hour telephone helpline are likely to have fewer reporting delays. Claims will in general be paid quickly. [1]

Claim frequency and severity

Often quite a high proportion of policies give rise to claims, *eg* 10%. Claim amounts are usually very small. However, the tail of the claim amount distribution can be quite long, because the insurer is exposed to the risk of the occasional large complex liability or medical expenses claim. [1]

Accumulations of risk

Concentrations of risk are quite likely. For example, where an insurer is providing cover for all passengers in a single coach. This is quite possible where a particular tour operator offers insurance policies from a single insurer. [1]

[Maximum 20]

Solution 1.12

Indemnity is compensation for a loss that restores the insured to the same position as before the loss occurred. [1]

Four distinct examples are required, eg:

- individual personal accident, where the insured is paid a fixed sum
- new for old policies under domestic contents, where the insured has a more valuable property than before the loss occurred
- creditor insurance, where the benefit may be say 80% of the insured's monthly mortgage payment due
- any insurance policy with an element of self-insurance, *eg* where there's an excess (or a maximum level of cover). [1 mark for each good example]

[Total 5]

Solution 1.13

Rating factors are needed because the exposure measure alone cannot generally account for all of the factors determining the risk from an individual policy. [½]

They help to classify risks into more homogenous groups. [½]

Failure to accurately account for all of these factors could mean that:

- potential policyholders will take advantage of the insurer's low inappropriate premium rates leading to large amounts of unprofitable business *(ie* anti-selection) [½]
- competitors setting a lower more appropriate premium rate will attract the other potential policyholders. [½]

[Total 2]

Solution 1.14

Comment

This question requires you to apply your basic product knowledge to claims characteristics. Characteristics that you might have considered for each are:

- *Size of claims*
  - *large or small*
  - *variability in size*
- *Number of claims*
  - *high or low frequency*
  - *variability in numbers*
- *Possibility of accumulations*
- *Length of tail*
  - *reporting delays*
  - *settlement delays*

We now translate out initial non-specific thoughts into specific comments about the three classes given.

(i) ***Motor***

- Average claim size is quite small and normally maximum limited by value of car. However other cars or property can also be included increasing size dramatically, *eg* motorway pile-up. [1]
- Claim frequency high but fairly stable for reasonably sized portfolio. [1]
- Accumulations possible, but only usually significant for fleet motor insurers. [½]
- Delays usually short, weeks rather than years. [½]

[Total 3]

(ii) ***Marine and Aviation***

- Claim amounts highly skew towards larger claims even though total loss is uncommon. [½]
- Claim frequency variable from year to year, although big disaster claims are rare. [½]
- Geographical accumulations lead to possibility of many claims from one incident. [½]

- Reporting delays are usually short, weeks rather than months. May be slightly longer than (a) and (c) though if minor incidents not reported until next port reached. [1]
- Settlement delays can be longer than (a) and (c). Whilst major repairs may be required immediately, minor repairs may be left until the ship returns to its home port or until its next major refit. Hence, some may take a year or two to settle.[1]

[Maximum 3]

(iii) ***Household***

- Average claim size quite small, similar to motor. SI usually much greater than motor. Total loss possible but much less common than smaller claims. [1]
- Claim frequency similar to motor. Can be quite variable from year to year due to weather patterns. [½]
- Possibility of many small claims from the same event, *eg* windstorm, flood. [½]
- Accumulations sometimes enhanced by sale of many policies through a regional building society. [½]
- Delays usually short in terms of weeks rather than years. Exception to this is subsidence claims that can take years to recognise, stabilise and settle cost of repair. [1]

[Maximum 3]

Solution 1.15

Reinsurance – purposes

Reinsurance may be used to allow the company to write business that could lead to large claims. [½]

Reinsurance may reduce or truncate these large claims and protect the solvency margin of the insurer. [½]

Protection may be obtained for individual large claims or an accumulation of claims. [½]

Reinsurance may be used to help to diversify the portfolio of risk ... [½]

… perhaps with the use of a reciprocal arrangement. [½]

Reinsurance may be used to control or fine tune the risk exposure without reducing market share. [½]

This includes writing more risks through better use of capital. [½]

Reinsurance can reduce claim fluctuations and hence smooth profits and perhaps dividends. [½]

It may also be used to reduce any statutory required minimum margin. [½]

The company may wish to benefit from technical assistance offered by the reinsurance company or broker. This may include help with setting premium rates, policy terms and conditions and claims control. [1]

Reinsurance may be used to improve the financial position of the company, including the cashflow position and the balance sheet. [½]

It may be a statutory requirement to use a minimum level of reinsurance. [½]
[Maximum 6]

Solution 1.16

Measures of exposure for:

(a) motor: policy year, vehicle year, driver year

(b) EL: wage-roll, man-hours, turnover

(c) industrial fire: sum insured per year, floor space, turnover

(d) contents: policy year, sum insured, number of rooms.
[¼ mark for each measure, maximum 2]

Solution 1.17

Surplus reinsurance is a proportional reinsurance, *ie* premiums and claims are split in proportion (although the premium paid to the reinsurer is net of reinsurance commission). [1]

The proportion can vary from risk to risk with the choice made by the cedant within limits set out in the treaty. [1]

Solution 1.18

(a) ***Spread loss covers***

Spread loss covers involve the insurer paying annual or single premiums to the reinsurer for coverage of specified claims. These accumulate with interest (contractually agreed) in an experience account; the balance of which is settled at the end of the multi-year period. [1]

These types of contracts involve very limited underwriting risk (limited practical risk transfer), but provide the insurer with the liquidity and security of the reinsurer. [½]

(b) ***Structured finance arrangements***

Reinsurers became involved in structured finance through their finite reinsurance business and the increasing need of financial guarantee insurers and investment banks for additional capacity. [½]

The typical financing solution provided by the reinsurer is a credit enhancement in which the reinsurer provides a financial guarantee or credit insurance wrap to the institution borrowing from the capital market. [½]

Credit enhancement or financial guarantees lower the cost of borrowing. [½]

(c) ***Industry loss warranties (ILWs)***

Industry loss warranties (ILWs) are a type of reinsurance where the basis of cover is not indemnity, *ie* repayment of actual losses suffered. [½]

Here one party will purchase protection based on the total loss arising from an event to the entire insurance industry rather than their own losses. [½]

The original size of the industry loss is used as a trigger for a recovery. [½]

(d) ***Committed capital arrangements***

Committed capital is based on a contractual commitment to provide capital to an insurer after a specific adverse event occurs that causes financial distress. [½]

The insurer purchases an option to issue its securities at a predetermined price in the case that the defined situation occurs, on the understanding that the price would be much higher after such an event. [½]
[Maximum 5]

Solution 1.19

(i) ***Items in an XL treaty***

Eighteen items that should be included in an XL reinsurance treaty, from:

- names of parties involved
- type of treaty, *eg* oblig/oblig
- cover period
- class(es) of business
- territories covered
- exclusions, including restrictions on sizes of risks
- precise interaction with any other reinsurance arrangements
- excess point, size of layer, and proportion of layer covered
- stability clause, dealing with indexation of excess point
- definition of an event (depends on type of XL)
- basis for determining premiums
- basis for reinsurance commission
- brokerage terms
- basis for keeping and checking records
- basis for paying premiums and commission and settling claims
- sunset clause and any other claim notification arrangements
- exchange rate basis, for foreign currency payments
- reinstatement terms
- arbitration basis
- termination terms. [6 marks less ½ for each point less than 18, minimum 0]

(ii) ***Stability clause***

A stability clause is a provision in an excess of loss treaty whereby the excess point and (usually) the width of the band are automatically increased in line with inflation. [1]

It is used so that if claim amounts increase in line with the inflation index then the split of claim costs between the cedant and reinsurer would remain constant over time. [1]

Alternatively, if limits are fixed in monetary terms, then inflation, or an increase in inflation over that assumed in the premium for excess of loss cover, hits the reinsurer much harder than the cedant. This is because: [1]

- some claims previously under the limit will come within the reinsurer's band [½]
- claims that enter, but do not extend to the upper limit will increase by the amount of inflation, but the increase will all fall on the reinsurer (unless the upper limit is exceeded). [½]

[Total 4]

Solution 1.20

Two different measures of exposure for rating industrial fire, with two weaknesses of each:

- Sum insured year [1]
  - Value of stocks unknown in advance [½]
  - Changes over time to stocks [½]
  - Uniqueness of properties means that there is no unique way of allowing for the effect of inflation on the exposure. [½]
- Unit year [1]
  - The large variety of properties, by size, use, location and construction makes this a poor indicator of the exposure. [½]

Another possible weakness for both (and every exposure measure for industrial fire):

- Poor measure of risk, so many rating factors needed. [½]

[Maximum 4]

Solution 1.21

- Dangerous pastimes or occupations [½]
- Aviation other than as a passenger [½]
- Suicide or self-inflicted injury such as alcohol abuse [½]
- Pre-existing conditions such as pregnancy [½]
- AIDS [½]
- Illegal activities such as drug abuse [½]

[Maximum 2]

Solution 1.22

(i) ***Underwriting factors***

Eight possible underwriting factors for EL if exposure is payroll:

- trade and occupations [¼]
- industrial processes and machinery used [¼]
- materials handled [¼]
- safety equipment installed [¼]
- medical facilities [¼]
- management practices, *eg* in regard to safety and training [¼]
- past experience of employer [¼]
- location. [¼]

[Total 2]

(ii) ***Delays***

Possible stages of delay in an EL claim include:

- condition emerging [½]
- condition being noticed [½]
- reporting condition to insurer [½]
- insurer processing the claim [½]
- establishing when the claim was incurred, if change of insurer [½]
- establishing liability (*ie* negligence by employer) [½]
- condition stabilising [½]
- assessing amount of settlement [½]
- processing the settlement. [½]

[Maximum 4]

2 Solutions to exam-style questions

Solution 1.23

(i) ***Reasons for end-of-year adjustment***

There may be an end-of-year adjustment for the actual exposure over the year, *ie* number of vehicles covered. [1]

There may be a profit sharing type of arrangement, reflecting the actual claims experience over the year. [1]

(ii) ***First basis for splitting claims***

As with private motor, *ie* by property damage and bodily injury. [1]

(iii) ***Motor fleet – short tail or long tail?***

It depends on the type of claim! The property damage claims are short tail, and the bodily injury claims are long tail. The overall result depends on the mix of claims from the particular portfolio. [2]

(iv) ***Main determinant of level of experience rating***

The size of the fleet (*ie* the credibility of the experience). [1]

Solution 1.24

(i) ***Company A's share of the claim***

Initially, A takes 40% of the gross EML. B, C and D each take 20%. [½]

A then reinsures 5% with X. Therefore A is left with 38% of the gross EML (95% of 40%) and X takes 2% of the gross EML. [1]

A's share of the EML at this stage is 38% of $500,000 which is $190,000. [½]

There are then two options:

(1) Assume that A holds the maximum retention, $50,000, and only uses 2.8 lines in the surplus treaty.

(2) Assume that A uses all three lines and retains less than the maximum retention.

Students were not expected to carry out both calculations, but to select an option and state clearly the assumption made. [1]

Under option (1), A would retain $\frac{1}{3.8} \times 38\% = 10\%$ of the original gross EML.
X and Y would each take 14%.

Under option (2), A would retain $\frac{1}{4} \times 38\% = 9.5\%$ of the original gross EML.
X and Y would each take 14.25%. [½ for either]

A single large claim of $750,000 would result in a payment by A of:

- $10\% \times 750{,}000 = \$75{,}000$ under option (1).
- $9.5\% \times 750{,}000 = \$71{,}250$ under option (2). [½ for either]

[Total 4]

(ii) ***B and Y declared insolvent***

A would not have to pay B's share of the claim because the risk is coinsured with B. [½]

However, as company Y is a reinsurer of A the insolvency will affect the recoveries A can make. [½]

In the most extreme case, A will have to pay Y's share. This is 14% under option (1) or 14.25% under option (2). [½]

Total payment $= 24\% \times 750{,}000 = \$180{,}000$ under option (1).

$= 23.75\% \times 750{,}000 = \$178{,}125$ under option (2). [½]

In practice, there will almost certainly be a partial recovery from the liquidators of Y, and so the answer will lie between the answer in (a) and the revised amount above. [1]
[Total 3]

(iii) ***Information provided to Company Z***

- description of the insured property
- date of claim event and date reported
- type(s) of claim
- peril
- amount claimed
- up to date estimates of total claim amount

- dates and amounts of claim payments
- any explanation why the claim is greater than the EML.

[½ each, maximum 3]

Solution 1.25

(i) ***Ideal features***

The policyholder must have an interest in the risk being insured and an interest in its consequences being minimised. [1]

A risk must be of a financial and reasonably quantifiable nature. [½]

The claim amount payable must be commensurate with the size of the financial loss. [½]

Individual risk events should be independent of each other. [½]

The probability of the event should be relatively small. [½]

Large numbers of potentially similar risks should be pooled in order to reduce the variance and hence achieve more certainty. [½]

There should be an ultimate limit on the liability undertaken by the insurer. [½]

Moral hazards should be eliminated as far as possible. [½]

There should be sufficient existing statistical data / information to enable the insurer to estimate the extent of the risk and its likelihood of occurrence. [½]

[Maximum 4]

(ii) ***Transport delay insurance***

The policyholder does have an interest in the risk, although how much interest and in what form will depend on the individual. [½]

The loss is not financially quantifiable. This rules out indemnity cover but a fixed benefit cover would be possible. [½]

It will be difficult to ensure that claim payments made are commensurate with the size of the financial loss because this loss is not quantifiable. [½]

There will not be independent risks, *eg* bad weather affects many policyholders. [½]

The claim frequency could be very high, depending upon the definition of "late", exclusions and quality of transport system. [½]

Pooling to reduce variability is limited by the non-independence of risks. [½]

To limit outgo use fixed benefits, but hard due to non-independent risks. [½]

Avoiding moral hazard will be very difficult, since hard to verify claims. [½]

There will probably be a limited amount of data available giving information about delays on public transport but the data is unlikely to be sufficiently detailed to be of any use. [½]

Therefore probably not practical due to non-independent risks, difficulty of verifying claims and potentially high claim frequency. [1]

[Maximum 4]

Solution 1.26

Comment

We realise that this situation might seem a bit strange. However the examiners do ask questions on unusual classes from time to time, to test your ability to apply your knowledge of other classes and apply that to the unusual class with some common sense.

The discussion could take various directions. Some points that could be covered are:

Level of cover

- One possibility is a refund of the charge; anything less would be too small. However, the customer could still sue the hairdresser so more cover may be required. [1]

- Compensating for "grief caused" could end up with large claims and/or be hard to quantify. For example, "I would have passed that job interview if my hair hadn't looked so silly". [½]

- An alternative would be to offer immediate re-cut or re-style by someone else (same hairdressers or different). [½]

- This is fine as long as there's enough hair left to play with after the first cut! [½]

- Another alternative may be to offer use of a wig / toupee. [½]

Decision on "unsatisfactory"

- Needs an independent arbitrator, acceptable to both sides
  - not hairdressing employee / manager
  - not client's friend / mum. [1]

- May need different arbitrator for certain styles, *eg* punk, mohican, gothic
  - video-phone link to suitable style guru? [½]

Same premium

- Simpler, but different ...
  - difficulty of styles
  - skill of individual hairdresser
  - awkwardness of individual customers
  - (there could be experience rating, *eg* NCD "no complaint discount") [2]

- Should collect data over time to confirm what rating factors would be best, if wanting premiums to vary by risk. [½]

[Total 7]

Part 2 – Questions

The split of the questions is described in the introduction to the Q&A Bank Part 1.

Note that the split between development questions and exam-style questions is somewhat subjective. For example, there have been past exam questions that test knowledge of the Core Reading, and so are similar to what we've included here as development questions. The exam-style questions involve more application and a wider range of ideas and are typically the more challenging questions in the exam.

1 Development questions

Question 2.1

State the principal differences in the risk features of a general insurance policy compared with a life assurance policy. [4]

Question 2.2

List the factors affecting the number of rating factors selected when modelling experience for pricing purposes. [5]

Question 2.3

Explain why some classes of business are considered more risky than other classes of business. [3]

Question 2.4

List the problems for an insurance company that may be associated with rapid growth in premium income. [2]

Question 2.5

Give examples of two parties, external to a general insurance company, whose failure might cause the insurer to suffer financial losses.

Explain for both why their failure might cause the insurer losses, and what actions the managers of the insurer could take to minimise the risk to their company. [2]

Question 2.6

Explain why it is very important to reduce heterogeneity in data for premium rating purposes. [2]

Question 2.7

Explain what a *pooling arrangement* is, and give one example. [2]

Question 2.8

Describe P and I Clubs. [3]

Question 2.9

State the factors limiting the usefulness of industry-wide data. [4]

Question 2.10

List the controls an insurance company might implement to ensure that policyholders' personal data is protected. [5]

Question 2.11

Describe the problems that might prevent an ideal data system being maintained. [4]

Question 2.12

State the factors that have the biggest influence on the quality of data that is likely to be available from different insurance companies. [3]

Question 2.13

Explain why the policy and claims data for a commercial fire account may not be fully captured in an automated system. [2]

Question 2.14

List the steps that could be taken to ensure that an insurance company's policy data is accurate. [6]

Question 2.15

Describe briefly the effects on an insurer of being at the bottom of the underwriting cycle. [4]

Question 2.16

List the areas of greatest uncertainty for a general insurer when producing estimates of its claims. [2]

Question 2.17

Outline the main risk and uncertainty features of property damage and liability classes of business. [4]

Question 2.18

A rich friend of yours has just become a Lloyd's Name. He has joined a syndicate that writes only marine insurance. List sources of risk and uncertainty which will affect the return that he makes from his capital outlay. [6]

2 Exam-style questions

Question 2.19

(i) State the issues that data protection laws might cover. [4]

(ii) Suggest the possible consequences to an insurance company of contravening such laws. [3]
[Total 7]

Question 2.20

Describe the impact of different types of inflation on private motor insurance claims and expenses. [7]

Question 2.21

A new general insurance company is being set up to specialise in private motor insurance.

(i) Outline the factors that should be considered by the company when establishing a central computer system. [10]

(ii) List the data you would expect to see held on the individual policy files, explaining why the data would be maintained. [8]
[Total 18]

Question 2.22

You have taken over responsibility for a poorly designed and administered claims database. State the types of errors or distortions that you may find in that database, and give examples of how they might have arisen. [10]

Question 2.23

A general insurance company has an established book of commercial property insurance policies.

(i) State, giving examples, the three broad categories of claim size that the insurer might face. [3]

(ii) Describe how each type of claim might be modelled. [5]

(iii) Suggest how the risks posed by each type of risk may be mitigated. [5]
[Total 13]

Part 2 – Solutions

1 Solutions to development questions

Solution 2.1

Comment

You may feel this question is unfair if you are from a pensions background and/or have not studied life assurance. However, you should be able to get all the points below using a combination of:

- *applied risk and uncertainty knowledge from Subject ST8*
- *understanding of mortality risk from earlier subjects*
- *common sense!*

- General insurance (GI) policy terms are usually one year, whereas life assurance (LA) terms are typically much longer. (This leads to greater competitive pressure to attract new business every year and greater importance of recovering fixed expenses each year.) [1]

- GI claims verification is usually more difficult than LA (*ie* dead or alive) … [½]

  … hence settlement delays tend to be longer for GI. [½]

- Similarly we can generally only claim once on a LA policy, whereas multiple claims are possible (and often expected) on commercial GI policies. [½]

- Also the payout for a LA policy is fixed upon the event, whereas GI claim amounts are often highly uncertain. [½]

- GI polices are subject to more policy endorsements (alterations) than LA. [½]

- For most LA policies the level of risk, *ie* mortality, increases over time. This is not usually true of GI policies which may have level, increasing, decreasing or seasonal risk incidence. [½]

- There tends to be better quality data in LA than GI and it tends to be easier to quantify the risks in LA. [½]

[Maximum 4]

Solution 2.2

- the exact purpose of the exercise, *eg* actually setting prices, repricing, or comparing prices with competitors' prices [½]
- the number of rating factors currently being used in the rating [½]
- the number of rating factors used for pricing by competitors [½]
- the capacity of the model being used [½]
- the time available to perform the investigation [½]
- IT (or other) restrictions on how many rating factors can be incorporated into rating algorithms [½]
- the quantity of data available [½]
- the level of detail of the data available [½]
- the judgement applied by the modeller [½]
- sales channel (may affect the level of complexity required) [½]
- level of competition in the market for the class of business being modelled [½]
- adjustments made to data either in cleaning, trending or developing. [½]

[Maximum 5]

Solution 2.3

Classes that are highly uncertain have high levels of risk. [½]

This uncertainty may be caused by:

- large variance in claim amount distribution *(ie* possibility of very large claims) [½]
- large variance in claim frequency *(ie* volatile claim numbers from year to year) [½]
- possibility of accumulations or multiple claims from one event. [½]

For some classes, there is great uncertainty regarding the claims, based on lack of knowledge or data. [½]

Long-tail liability classes are generally considered to have high uncertainty. Short-tail property classes are generally less "risky". [½]

[Total 3]

Solution 2.4

Problems that may be associated with very rapid growth in premium income include:

- if the increase is due to low premium rates, the increase may indicate future losses [½]
- it could indicate deteriorating experience because of anti-selection or reduced quality underwriting [½]
- administration strains could cause service standards to brokers and policyholders to fall, leading to bad publicity [½]
- the solvency margin could be reduced to close to the statutory minimum level if the minimum is based on premium income [½]
- internal controls may be weakened, *eg* risk management, expenses. [½]

[Maximum 2]

Solution 2.5

Two examples of parties whose failure might cause the insurer to suffer financial losses:

- A reinsurer, because the direct writer is fully liable for any claim payments due, even if it is unable to make the anticipated reinsurance recoveries. [½]
- The insurer could minimise the risk by spreading reinsurance business over a number of different financially sound reinsurers. [½]
- An intermediary, because they may be holding premiums due to the insurer. [½]
- The insurer could minimise the risk by insisting that brokers' balances are kept to a minimum (especially if a broker is not soundly financed) and diversifying, *ie* not relying too heavily on a small number of large brokers. [½]

[Total 2]

Solution 2.6

If heterogeneity exists in the data, then the premium rates set for each group will be an average of premium rates for each group, rather than an exact rate for each risk. [½]

This could lead to the company setting rates that are too low for some risk groups (so that the company would make losses) … [½]

… and too high for others (so that they might be uncompetitive). [½]

On average, premium rates may be acceptable, but the portfolio will be more sensitive to the mix of business written. [½]
[Total 2]

Solution 2.7

A pooling arrangement is where a number of similar organisations group together and agree to share premiums and losses for specific insurance classes or types of cover in agreed proportions. [1]

One example would be the members of a Protection and Indemnity Club in which ship owners pool their risks. [1]
[Total 2]

Solution 2.8

P and I Clubs are Protection and Indemnity Clubs. These are mutual associations of ship owners. [½]

Such clubs provide a pooling arrangement which covers risks not traditionally insured by a commercial marine hull policy, *eg* damage to harbours, removal of wrecks, pollution, loss of life and personal injury. [1]

They also provide ship owners with technical assistance in the marine market and advise on issues coming before the shipping industry. [1]

They can provide access to the International Group of P&I Clubs, which can be used to pool larger claims and to arrange reinsurance for very large claims. [1]
[Maximum 3]

Solution 2.9

Main problem is the potential for distortions within the data (heterogeneity). [½]

The data supplied by different companies may not be directly comparable because:

- it relates to different socio-economic or geographical areas of the market [½]
- policy conditions may differ [½]
- other practices may differ, *eg* underwriting, claims settlement [½]
- the nature of the data stored may differ [½]
- the coding used for the risk factors might vary. [½]

Other potential problems are:

- data usually less detailed and less flexible than that available internally to a company [½]
- often more out of date than internal data [½]
- quality may be suspect if some contributors have poor quality data systems or supply incorrect information [½]
- not all companies contribute. [½]

[Maximum 4]

Solution 2.10

- regular training of staff on: [¼]
  - data protection regulation [¼]
  - procedures relating to handling data [¼]
  - consequences of breaching data protection rules [¼]
- employing specialists for roles relating to: [¼]
  - data gathering [¼]
  - data processing [¼]
  - data use [¼]
- limiting the use of certain items of data to specified people through password protection [½]
- providing guidelines to third parties (*eg* consultancies) that have access to the data, *eg*: [½]
  - on use of the data [¼]
  - requirement to destroy data after use [¼]

- employing a compliance department, that will monitor data protection legislation [½]
- following rules on storage and transmission of data [½]
- allowing the subject access to the data and then the right to have it corrected [½]
- monitoring of data handling procedures [½]

[Maximum 5]

Solution 2.11

- age – over time, the system will gradually become out-of-date [½]
- mergers and acquisitions could lead to the existence of legacy systems … [½]

  … or data being migrated on to one of the systems, leading to potentially empty data fields [½]
- new rating factors being added, leading to potentially empty data fields for historic data [½]
- a reorganisation of class structures, with the insurer being unable to assemble the historic data into the new classes [½]
- human error, which is inevitable (eventually at least!) [½]
- conflicts between users, which could lead to a lack of data for some users … [½]

  … and too much data for others, resulting in a slower and more unwieldy system [½]

[Total 4]

Solution 2.12

Biggest influences on data quality from different companies:

- age of company [¼]
- size of company [¼]
- age of computer and information system [¼]
- existence of legacy systems [¼]
- integrity of systems, *eg* automated checks [¼]
- quality of managers [¼]
- quality and training of staff [¼]
- nature of organisation, *eg* direct insurer *vs* reinsurer [¼]

- sales outlet(s) used [¼]
- classes of insurance sold [¼]
- type of insured, *eg* large commercial risks *vs* small personal risks [¼]
- product design [¼]
- rate of change in products [¼]
- simplicity and ease of use of computer systems. [¼]

[Maximum 3]

Solution 2.13

Commercial fire insurance data may not be fully captured in an automated system because for this class policies often vary a great deal in terms of:

- the cover provided [½]
- the risk factors. [½]

Claims data may also be variable and depend on subjective reports by specialists. [½]

It may not be practical to have a sufficiently flexible system to deal with all the variations. [½]

An automated system may deal with most "mainstream" cases but, even then, data may not be *fully* computerised. Special codes may be needed to refer to paper records for individual non-standard cases. [½]

[Maximum 2]

Solution 2.14

Steps to *try* to ensure that an insurance company's policy data is accurate (but you can never *ensure* total accuracy):

- original form design: clear design that collects the correct information from policyholders
- original form design: clear and unambiguous questions
- data entry: should be typed straight from forms and input only once
- automatic validity checks on data entry
- use of unique policy numbers with check digits
- spot checks on sample data to trace any global errors
- spot checks that procedures are being correctly followed
- check key distributions of data for consistency
- subsequent monitoring and cross-checking
- checking totals are consistent with movements of policies
- individual checks on extreme data
- individual checks on empty fields
- ensure consistency with data from other sources, *eg* accounts
- give staff incentives to keep data accurate. [½ mark each, maximum 6]

Solution 2.15

The effects on an insurer of being at the bottom of the underwriting cycle are:

- premiums are low, and so profitability is low
- some business may even be loss making
- insurers may leave the market or reduce the amount of business that they write
- loss of business affects insurers' ability to cover fixed expenses
- loss of business also affects insurers' future growth prospects
- solvency margin may reduce, with some companies becoming insolvent
- insurers will require additional capital support from other activities
- reinsurance may be less readily available
- reinsurance terms will be less attractive.

[½ each, maximum 4]

Solution 2.16

Areas of greatest uncertainty for a general insurer when producing estimates of its claims are:

- claims emerging from the latest period of exposure, *ie* IBNR claims [½]
- the possibility of new types of latent claims emerging from liability classes [½]
- inflation of the longest-tail liabilities [½]
- catastrophe claims, *eg* the uncertainty surrounding huge floods due to climate change [½]
- claims arising from unexpired risks. [½]

[Maximum 2]

Solution 2.17

Property damage

- Short tail, so shorter time period, hence less time for events to go against the insurer. [½]
- Geographic accumulations possible. Potentially lots of claims from weather-related incidents. [½]
- Reasonably homogeneous risks. [½]
- Amounts limited by SI or value of building. [¼]

Liability classes

- Claims frequency lower on average but more variable. [½]
- Amounts much more uncertain too, possibly unlimited cover. [½]
- Long tail so more exposure to unexpected inflation, asset inadequacies and reinsurer failure. [½]
- Varied heterogeneous risks. [½]
- Possible accumulations due to concentrations of products or employees. [½]
- Exposure to court awards and judicial inflation. [¼]
- Exposure to latent claims. [¼]

` [Maximum 4]

Solution 2.18

Premiums may not be enough to cover the expected claim cost. [¼]

This may be due to:

- the underwriting cycle leading to lower rates [¼]
- anti-selection if the rating structure is incorrect [¼]
- inadequate data upon which to price the risks. [¼]

Premiums could be too high, leading to lower than expected volumes of business and a failure to adequately cover fixed expenses. [½]

Claims experience may be higher than expected. [½]

This could be due to:

- catastrophes
- latent claims
- aggregations of risk
- poor policy wording
- higher than expected claims inflation
- unfavourable judicial decisions. [¼ each, total 1½]

Higher than expected expenses, commission or expense inflation. [1]

Low investment income, falls in assets values or increased taxation of returns. [1]

Other possibilities:

- adverse currency movements
- failure of third parties, *eg* reinsurers
- changes in legislation
- mis-management of the syndicate. [¼ each, total 1]

[Maximum 6]

2 Solutions to exam-style questions

Solution 2.19

(i) ***Data protection laws***

Data protection laws may cover:

- what data a company may hold on its customers … [½]

  … both personal and commercial [½]
- what a company may use its data for [½]
- rights of access by a customer to its own personal data [½]
- rights of a customer to have incorrect data corrected [½]
- which parties are authorised to handle data at different stages, *eg*:
  - data gathering stage [¼]
  - data processing stage [¼]
  - subsequently [¼]
- guidelines for third parties (such as consultancies) to adhere to when dealing with data [½]
- requirements to destroy personal data after use [½]
- requirements on the security of data when it is being:
  - stored [¼]
  - transferred to other parties. [¼]

[Maximum 4]

(ii) ***Consequences of contravening laws***

There may be the following consequences:

- criminal prosecution of the responsible parties [½]
- legal actions against the company [½]
- fines to the company [½]
- requirements to compensate the customer [½]
- restrictions on the future activities of the company [½]
- more frequent monitoring of the company [½]
- negative publicity for the company (and the individuals involved) … [½]

  … which could impact business volumes. [½]

[Maximum 3]

Solution 2.20

In general, the importance of inflation depends on when and for how long the policies are in force, and the type of claim that arises. [½]

Property damage claims

These claims are usually reported and settled quickly, and so inflation is not so important. [½]

The appropriate inflation measures affecting such claims would be:

- inflation of motor mechanics' wages (*ie* related to standard earnings inflation), being a major part of repair costs [½]
- inflation of car component costs (to replace damaged spare parts) [½]
- car price inflation, as an indication of the cost of total replacement (*eg* in a major crash resulting in a write-off) [½]
- for cars where there is a large element of imports (either of parts or of the whole car), the local currency strength would also be relevant. [½]

Bodily injury claims

These claims may have significant reporting and settlement delays, so inflation can have a big impact. [½]

Court award inflation is important since settlements are often guided by court awards, even if the litigation does not go through the court process. [½]

Earnings inflation is also important, since bodily injury claims are usually related to loss of earnings, since the injury can prevent the ability to work. [½]

Medical expense inflation may also be relevant since the cost of rehabilitation will be accounted for in bodily injury claims. [½]

Bodily injury claims will also be affected by judicial decisions or legislation that change the method used to calculate compensation amounts (*eg* changes to the Ogden tables in the UK). [½]

General office expenses

The main influence is salary inflation, since salaries normally form the bulk of a general insurance company's expenses. [½]

There will also be some effects due to price inflation, for example inflation appropriate to the cost of office equipment (*eg* stationery and furniture). [½]
The inflation of expenses may be offset by improvements in efficiency over time. [½]

Claim expenses

For expenses associated with claims, again salary inflation will be the most important. [½]

Also, legal fee inflation will also have an influence, particularly on larger claims. [½]
[Maximum 7]

Solution 2.21

(i) ***Factors considered in establishing central computer system***

- Function of the system. Need input from all potential users:
  - administration
  - accounting
  - actuarial / statistical
  - management
  - marketing
  - other (underwriters, reinsurers, claims, investment, computing) [1]

- For private motor, there is potentially a lot of data to be processed. All of the above users will rely heavily upon the system. Their input initially is important. [½]
- Structure or outline of the system. How should the system operate? Will the branches and agents have direct access to the system? For private motor, the company will probably want to be able to give on-line premium quotes to brokers. [1]
- Which computer? What type of software? [½]
- Cost of implementation. [½]

- Security. How will the system be monitored? Who will have access to the system? Data protection laws should be considered, particularly if the system is to be used by third parties. [1]
- The data that will need to be held. For private motor, we should retain all the data for individual policyholders on the system. [1]
- How files will be structured. For example, should the policy and claims records be held on separate files? [½]
- How long the data will be stored. Statistical analyses will be necessary (in due course) using the data that develops. Every effort should be made to store data indefinitely. [½]
- Capability. The system will need to be able to capture claim features such as multiple payments, nil claims and reopened claims. [½]
- The output from the system is important. Again, ask all potential users of the system. Examples of required output are: premium quotations (linked to brokers' systems), policy documents, renewal notices, endorsement notices, management reports, marketing statistics, run-off tables, statutory returns, accounting reports. [1½]
- Ease of use. Need to consider the computer literacy of the end users. [½]
- Implementation of the system. How the system will be developed, and by whom. How it will be tested. [1]
- When the system will be ready. This needs to be linked to the proposed launch of the new policies. [½]
- Error aversion. What steps will be taken to ensure data accuracy? [½]
- Changes to the system. The system must be sufficiently robust so that future developments can be incorporated. [½]
- New company. There should be few constraints (other than cost), so the system design need not be too concerned with existing practices. [½]

[Maximum 10]

(ii) ***List of data (policy file) and explanation***

Administration data

Policy number	to define policy uniquely and provide link with related files, *eg* to claims file [½]
Name and address of policyholder	for correspondence, *etc* [½]
Broker / branch / commission	for accounting use, and also for management and marketing information [½]

Dates on risk	defines exposure period, and when renewal notices needed	[½]
Original year of issue	for statistical and management information	[½]
Endorsement data	indicates a change to the policy; prior data needed so that claim analyses are accurate	[½]
Premiums (amounts / timing)	needed for reserving, pricing and monitoring	[½]
Commission	to check that correct payments are made	[½]

Rating data

The following data items will be needed so that premiums can be determined. This data will also be necessary for subsequent claims experience or rating exercises. [½]

Cover	comprehensive, third party fire and theft, third party only	[½]
Type of use	SDP or business or domestic	[½]
Details on driver(s)	name date of birth (if age is allowed as a rating factor) sex (if allowed as a rating factor) accidents convictions other policies held by principal driver occupation	[¼ each]
NCD for this policy	No. years no claims, or protected NCD	[½]
Excess	compulsory, or voluntary	[½]
Details on car	make, model and engine capacity vehicle rating group area where kept year of manufacture / registration value garaged / off-street parking / street parking modifications	[¼ each]

Claims data is not required here, but students should mention that they assume that the claims data is stored in a separate file, with file references / links from the policy file. [1]

[Maximum 8]

Solution 2.22

Types of errors

Policy / claim numbers and linking

- wrong claim number – claim details allocated to wrong claim number initially [¼]
- wrong claim number – claim details allocated to wrong claim number reopened [¼]
- no policy number to link to [¼]
- link to non-existent policy number [¼]
- claim allocated to wrong policy number [¼]
- reinsurance linking wrong [¼]

Dates

- wrong claim date [¼]
- wrong payment dates [¼]
- wrong (or no) reported date [¼]
- wrong processing date [¼]
- reopened date not recorded [¼]
- settled date not completed [¼]

Amounts

- case estimates not systematically updated [¼]
- case estimates not consistently updated [¼]
- case estimate history not recorded – gives current position only [¼]
- incorrect amounts recorded [¼]
- amounts in wrong currency [¼]
- wrong exchange rates used [¼]
- inconsistent treatment of nil claims [¼]
- catastrophe claims missing [¼]
- recoveries information missing [¼]
- type of payment not recorded [¼]

Header fields

- wrong insured name [¼]
- wrong policy year [¼]
- wrong branch [¼]

Detail fields

- wrong risk details [¼]
- wrong peril code [¼]
- claim description wrong / insufficient detail [¼]
- poor description of large claims [¼]
- poor / missing catastrophe code [¼]
- no claim status flag [¼]

How errors may have arisen

Processing delays

- paper records not yet input [¼]
- claim settled but not input on the system [¼]
- reinsurance collected but not booked [¼]

Control errors

- no use of check digits [¼]
- no use of validity limits [¼]
- no consistency checks [¼]
- no use of compulsory fields [¼]
- no warning messages [¼]

Others

- poor training [¼]
- high staff turnover [¼]
- poor management / supervision [¼]
- changes in claim handling processes [¼]
- changes to basis for calculating case estimates [¼]

[Maximum 10]

Solution 2.23

(i) ***Types of claim***

The general insurance company is likely to face:

- attritional claims – these are typically unremarkable claims, which make up the majority of the claims by number (although not necessarily by cost) [½]
- large claims – these will be larger and less frequent than attritional claims [½]
- catastrophe claims – these are rare by nature, but could be very large. [½]

Examples:

- attritional claims may include minor damage to buildings (or contents), *eg* as a result of a fire in part of the building, or theft [½]
- large claims may include the complete destruction of a large building (and its contents), *eg* a large fire [½]
- catastrophe claims may include an earthquake or a bomb, which destroys many insured buildings in one particular area. [½]

[Total 3]

(ii) ***Modelling the claims***

Attritional claims

Attritional claims are likely to be modelled using the general insurance company's own past data. This may be sufficient in quantity, since the insurer is established in the market, and attritional claims (by their nature) are *relatively* homogeneous. [½]

It may be necessary to adjust the data before modelling to remove large and catastrophe claims. [½]

From the past data, the insurer should be able to find reasonable estimates of both the expected claim frequency and the expected claim severity … [½]

… and also variances around these rates. [½]

Large claims

Large claims are harder to model, as they tend to occur less frequently. Therefore there may not be a sufficient number of them to model in the same way as attritional claims. [½]

If there is sufficient volume to model large claims based on past experience, it might still be necessary to adjust the historic frequency to reflect anticipated experience. [½]

If not, then the insurer may have to use a more broad brush approach, *eg* using loss ratios. [½]

If excess of loss reinsurance has been purchased, then an alternative approach would be to cap large claims at the excess of loss threshold and allocate the reinsurance cost back to individual policies. [½]

In order to do this, it is necessary to make an assumption about the relative propensity for a claim to be large between different groups of policies. [½]

Catastrophe claims

Catastrophes present similar problems to large claims. By their nature, they are rare and there is unlikely to be adequate data volume to enable us to price or reserve accurately for the future risk. [½]

The insurer could use a proprietary catastrophe model to understand the impact of extreme events on a portfolio. [½]

However, the actual effect of a catastrophe often differs from that predicted and uncertainties will always remain about the frequency, timing, size and location of such events. [½]

[Maximum 5]

(iii) ***Mitigating the risks***

Attritional claims

Pooling of a large number of similar risks can help to mitigate the risks for attritional claims. [½]

Effective underwriting can also help to ensure that the premium is sufficient to cover the risks being written. [½]

Reinsurance, *eg* surplus treaty and/or aggregate XL may also be effective. [½]

Large claims

The insurer may not have a sufficient enough volume of large risks for risk pooling to be effective. In this case, the company is at risk of there being an unusually high number of large claims in a single year, or the (expected) large claims could be for unusually large amounts. [1]

This risk can be mitigated by the purchase of reinsurance. [½]

Surplus treaty or risk excess of loss reinsurance will be appropriate for this purpose. [½]

In addition, the reinsurer may be able to assist the insurer in modelling the large claims. [½]

Catastrophe claims

The insurer may be able to mitigate catastrophe risk through the purchase of appropriate catastrophe reinsurance contracts. [½]

Alternatively / additionally, it could mitigate catastrophe risk using:

- a spread loss treaty, which may be effective at spreading the cost of a catastrophe over a number of years [½]
- an industry loss warranty, which will be effective if the experience suffered by this insurer is in line with market experience [½]
- a securitisation of the book of business (*ie* issuing catastrophe bonds), which will transfer (some of) the catastrophe risk to the capital markets [½]
- weather derivatives, which may be effective for weather-related catastrophes. [½]

[Maximum 5]

Part 3 – Questions

The split of the questions is described in the introduction to the Q&A Bank Part 1.

Note that the split between development questions and exam-style questions is somewhat subjective. For example, there have been past exam questions that test knowledge of the Core Reading, and so are similar to what we've included here as development questions. The exam-style questions involve more application and a wider range of ideas and are typically the more challenging questions in the exam.

1 Development questions

Question 3.1

An investigation has been made of claims and exposure for a portfolio of policies over the period 1st January 2010 to 31st December 2011. The data from this base period is to be used to produce premium rates applicable for policies sold in the new rating period 1st November 2012 to 30th June 2013.

You are given that:

- the risk is not seasonal with reference to the calendar year
- the level of risk is constant over the policy year
- the claim frequency has been found to increase at a steady 0.25% simple per month for many years and is expected to continue to do so.

Calculate how much the average claim frequency observed during the base period should be increased to give the best estimate of the claim frequency for policies sold in the new rating period. [2]

Question 3.2

When choosing the base statistics from which to calculate the risk premium, what are the advantages and disadvantages of using:

(a) the most recent year's experience
(b) older years' experience? [4]

Question 3.3

Define experience rating and briefly describe the various systems used. [4]

Question 3.4

You are about to assess the premium rates for a class of business. List, under appropriate headings where possible, the information you would require to do this. [7]

Question 3.5

The claim frequency (*cf*) and the average cost per claim (*acpc*) have been determined for claims occurring in 2011 for a homogeneous group of policies. Both figures have been adjusted to allow for outstanding claims and IBNR. Give a formula for an office premium for annual policies sold to this group in 2013, defining the terms you use. [4]

Question 3.6

Give arguments supporting the use of experience rating in general insurance. [2]

Question 3.7

You are an actuary working for a general insurance company. You have been asked to review the premium rates for a class of business. An underwriter has provided you with the total claims paid in each of the last three years and the amount of premiums written in each of those years. Suggest reasons why this information is unlikely to be sufficient for determining the revised risk premiums. [6]

Question 3.8

Each policy in a certain class covers two types of peril. Peril I occurs for 5% of policies and results in losses which are uniformly distributed between £250 and £850. Peril II occurs for 15% of policies and results in losses that average £40 and are all under £100.

Calculate the reduction in the risk premium for these policies if an excess of £100 is introduced.

(You should assume that the two perils are independent and that introducing an excess would have no effect on loss frequencies or distributions.) [2]

Question 3.9

Find the parameters of the translated gamma distribution that would be used to approximate a compound Poisson distribution with Poisson parameter 50, where the individual claim sizes have a lognormal distribution with parameters $\mu = 6$ and $\sigma^2 = 1$. [5]

Question 3.10

The random variable S represents the annual aggregate claim amount from a risk. S has a compound Poisson distribution with Poisson parameter $\lambda = 4$. Individual claim amounts are assumed to be independent and identically distributed. The claim amount distribution is as follows:

x	1	2	3	4
$P(X = x)$	0.35	0.30	0.20	0.15

The insurer of this risk arranges individual excess of loss reinsurance with a retention limit of 2. The random variable S_R denotes the annual aggregate claim amount paid by the reinsurer.

Calculate $P(S_R = s)$ for $s = 0, 1, 2$. [7]

Question 3.11

S has a compound Poisson distribution with Poisson parameter 4. The individual claim amounts are either 1, with probability 0.3, or 3, with probability 0.7. Calculate the probability that $S = 4$. [4]

2 Exam-style questions

Question 3.12

A general insurance actuary is modelling the size of individual claims X arising from shipping accidents.

From past data he has estimated the mean, variance and skewness of the individual claims distribution to be \$5m, $(\$3m)^2$ and $(\$4m)^3$.

He has also assumed that claims arise as a Poisson process with a rate of 2 per annum.

(i) Calculate estimates of the mean, variance and skewness of the aggregate annual claims S. [4]

(ii) The actuary wishes to approximate the aggregate claims distribution by assuming that $\dfrac{S-a}{b}$ has a chi square distribution with n degrees of freedom, for some constants a, b and n.

(a) Find appropriate values for the parameters a, b and n.

(b) Hence estimate the probability that the insurer will experience total losses on its shipping portfolio exceeding \$30m in any given year. [6]

You are given that the mean, variance and skewness of the χ_n^2 distribution are n, $2n$ and $8n$, respectively.

(iii) The actuary has also applied the method described in (ii) to two other types of claims. For these, the estimated values of n (based on similar volumes of past data) were found to be 2 and 50. State briefly what this tells you about the loss distributions in these cases. [2]

[Total 12]

Question 3.13

A firm advising on the design and construction of commercial and industrial properties has asked your company to provide professional indemnity insurance. The firm wants to pay a single premium to your company for each property it designs.

Discuss:

(i) the principal characteristics and potential difficulties of the proposed scheme which you should consider [9]

(ii) possible measures of exposure and rating factors that might be appropriate [7]

(iii) any other factors that will affect the premiums you propose. [3]
[Total 19]

Question 3.14

You are given the following estimates of an insurer's claim size and frequency distribution for a class of business:

Average claim amount:	£500
Claim frequency:	20%
Total claims, by amount, from claims less than £100:	5% of the total claims

(i) Estimate the revised risk premium if an excess of £100 is introduced for this class of business. [4]

(ii) Based upon your initial estimate, a company director has asked for a detailed investigation into how much the risk premium would reduce by as a result of introducing this excess. Describe how you would investigate this, assuming you had full access to the company's claims data. [6]

(iii) The director is considering offering the policyholder a choice of two policies, one with the excess and one without the excess. If the policyholder opts for the policy with the excess then the director intends to offer the same discount as you have calculated above. Give reasons why, in practice, the discount would not be the same. [4]
[Total 14]

Question 3.15

An insurer has a portfolio that contains policies of two different types, Type I and Type II. The characteristics of the claims arising from these policies are as follows.

The number of claims arising from Type I policies has a Poisson distribution with parameter 5. The claim amounts have a $Gamma(15, 2)$ distribution.

The number of claims arising from Type II policies has a Poisson distribution with parameter 10. The claim amounts have a $Gamma(20, 4)$ distribution.

(i) Assuming independence between claim amounts and claim numbers, find the mean, variance and third central moment of the distribution of the aggregate claims from this portfolio. State any assumptions that you make. [7]

(ii) Determine the parameters of the translated gamma distribution that would be used to find an approximation to this claim distribution, and estimate the aggregate claim amount that will be exceeded with probability 2.5%. [6]
[Total 13]

Part 3 – Solutions

1 Solutions to development questions

Solution 3.1

The mid-point of exposure for the base period is 1 January 2011. [½]

The mid-point of sales for the new rating period is 1 March 2013. [½]

The mid-point of exposure from the new rating period is 1 September 2013. [½]

Hence, increase is for 2 years 8 months at 0.25% per month, *ie* 8%. [½]
[Total 2]

Solution 3.2

(a) The most recent year's experience is most likely to represent the current situation, and so be the most relevant. [1]

However, since the experience is likely to be incomplete, you will probably have to make adjustments for outstanding claims and IBNR, particularly for long-tailed business. [1]

(b) The experience of older years has the advantage that it will be more complete. [1]

However, it may not be appropriate to the current situation due to trends in frequency / severity, and changes in risk, cover, type of claim, *etc*. [1]
[Total 4]

Solution 3.3

Definition

Experience rating is a system whereby the premium of each individual risk depends, at least in part, on the actual claims experience of that individual risk. [1]

Common systems

These can be applied either retrospectively or prospectively. [½]

Prospective rating: premium at renewal depends on the experience prior to the current renewal date. [½]

Retrospective rating: an initial premium is adjusted at the end of the period of cover to reflect claims experience in the year of cover. [½]

The system can be based either on the number of claims or on amounts. [½]

For amounts-based systems a credibility rating is used. [½]

The credibility of claims experience is the measure of the weight to be attached to the experience of the particular risk compared with the experience of the insurer's portfolio of similar risks. [½]
[Total 4]

Solution 3.4

Market

- existing rates
- usual rating structure
- trends in premium rates

Risk premium

- exposure measure and rating factors
- full analysis of recent claims experience by rating factor
- estimates of outstanding claims
- likely delays in reporting / settlement
- inflation assumption

Expenses

- claim frequency and handling costs
- commission rates
- per policy costs
- contribution required to fixed costs (dependent on the number of policies)
- expense inflation

Profit

- required profit (or return on capital)
- contingency margin
- expected investment return
- tax rates on investment and profit [½ mark per point, maximum 7]

Solution 3.5

The formula given here assumes that *acpc* includes the expected payments for outstanding claims in nominal amounts (*ie* allowing for any inflation). The office premium per policy, *OP*, is given by:

$$OP = EXP1 + EXP2 \times OP + PROF \times OP + \frac{cf \times (1+t)^{2.5} \times acpc \times (1+inf)^{2.5}}{(1+i)^{(0.5+d)}} \qquad [1½]$$

t	the annual expected change in claim frequency from 2011 to 2013
infl	the expected annual rate of claims inflation
i	the assumed rate of investment income on technical reserves
d	the expected delay from accident to settlement
EXP1	the assumed amount of expenses per policy
EXP2	the expenses per premium (*eg* commission plus a bit)
PROF	the loading for profit and contingencies [1½]

We are given 2011 claims information and we assume that claims in our base data occur mid-year, on average. We also assume that policies are written evenly over 2013 and that claims occur on average after 6 months. Claims from policies written in 2013 are therefore expected to occur on average at the end of 2013. Hence we project from 01/07/11 to 31/12/13, *ie* for 2½ years. [1]

[Total 4]

Solution 3.6

Arguments supporting the use of experience rating in general insurance include:

- premiums better reflect the risk [½]
- encourages better behaviour by policyholders [½]
- discourages small claims and associated admin costs [½]
- might be wanted by policyholders. [½]

[Total 2]

Solution 3.7

The claims data should be incurred as well as paid, in order to be able to analyse outstanding amounts separately. [½]

There is no allowance for IBNR, outstanding claims, reopened claims and partially settled claims. [½]

Even claims incurred are unlikely to be sufficient; a triangulation of claims by year or month of accident would be preferable. [½]

The claims data and the premiums data would not correspond. We need earned premiums not written premiums and claims on an accident year basis. [½]

We cannot adjust the data for large or exceptional claims or one-offs, such as catastrophes. [½]

There is no allowance for inflation of past claims. Also we need to allow for different claim types (*eg* property damage and liability) and maybe also different claim sizes having different inflation indices. [½]

The past might not be a good guide to the future. [½]

There might be changes in policy conditions, levels of cover, underwriting, target market, product mix, perils covered *etc*. [½]

We would need exposure data as well as premium data. [½]

There is no information from which to judge the premium adjustments for different rating factors. [½]

We also must consider interactions and correlations between different rating factors. [½]

We cannot allow for trends in claim amount and claim frequency separately. [½]

Data is not split between smaller (attritional) and larger claims. This split may be useful as different trends might apply. [½]

We would also require information on environmental factors affecting claims experience, such as economic conditions, legal influences and technical changes. [½]

We need information regarding the reinsurance arrangements. Also, we need to know whether the amounts are net or gross of reinsurance. [½]

[Maximum 6]

Solution 3.8

Peril I has an average loss size of £550, so the risk premium without excess is given by:

$$\text{RP} = 0.05 \times 550 + 0.15 \times 40 = 27.50 + 6 = 33.50 \qquad [1]$$

With the excess, claims are for lower amounts: £450 on average for Peril I, £0 for Peril II, so:

$$\text{RP} = 0.05 \times 450 + 0.15 \times 0 = 22.50$$

Hence the reduction in risk premium is £11. [1]

Alternatively, you can also use: 0.05 × 100 + 0.15 × 40 = 11. If you use this method then make sure that you explain yourself clearly.

Solution 3.9

First we find the moments (mean, variance and skewness) of the compound Poisson distribution:

$$\lambda m_1 = 50 \times e^{\mu + \frac{1}{2}\sigma^2} = 50e^{6.5}$$
$$\lambda m_2 = 50 \times e^{2\mu + 2\sigma^2} = 50e^{14}$$
$$\lambda m_3 = 50 \times e^{3\mu + 4\frac{1}{2}\sigma^2} = 50e^{22.5} \qquad [1]$$

So, equating these expressions to the formulae for the moments of the translated gamma distribution, we have:

$$\frac{\alpha}{\delta} + k = 50e^{6.5} \qquad \frac{\alpha}{\delta^2} = 50e^{14} \qquad \frac{2\alpha}{\delta^3} = 50e^{22.5} \qquad [1]$$

Solving these equations, we obtain:

$$\alpha = 9.9574 \qquad \delta = 4.069 \times 10^{-4} \qquad k = 8{,}788 \qquad [3]$$

and these are the parameter values we require. [Total 5]

Solution 3.10

Let Z denote the amount paid by the reinsurer on an individual claim. Then:

$$Z = \begin{cases} 0 & \text{with probability } 0.65 \\ 1 & \text{with probability } 0.20 \\ 2 & \text{with probability } 0.15 \end{cases}$$

and $P(Z > 0) = 0.35$

Now let $W = Z \mid Z > 0$. The distribution of W is:

w	1	2
$P(W = w)$	$\frac{4}{7}$	$\frac{3}{7}$

[1]

Furthermore, if N_R denotes the number of annual claims involving the reinsurer, then N_R is Poisson with parameter $4 \times 0.35 = 1.4$ [1]

So:

$$P(S_R = 0) = P(N_R = 0) = e^{-1.4} = 0.24660 \qquad [1]$$

We can calculate the other probabilities using the recursive formula (with $a = 0$ and $b = 1.4$):

$$P(S_R = s) = \sum_{w=1}^{s} \frac{1.4w}{s} P(W = w) P(S = s - w), \quad s = 1, 2, \ldots \qquad [1]$$

This gives:

$$P(S_R = 1) = \frac{1.4}{1} P(W = 1) P(S_R = 0)$$

$$= 1.4 \times \tfrac{4}{7} \times e^{-1.4} = 0.8e^{-1.4} = 0.19728 \qquad [1]$$

Also:

$$P(S_R = 2) = \sum_{w=1}^{2} \frac{1.4w}{2} P(W = w) P(S_R = 2 - w)$$

$$= 0.7P(W = 1)P(S_R = 1) + 1.4P(W = 2)P(S_R = 0)$$

$$= \left(0.7 \times \tfrac{4}{7} \times 0.8e^{-1.4}\right) + \left(1.4 \times \tfrac{3}{7} \times e^{-1.4}\right)$$

$$= 0.92e^{-1.4} = 0.22687$$ [2]

[Total 7]

Alternatively, you could work from first principles as follows:

$$P(S_R = 0) = P(N = 0) + P(N = 1, \text{ claim amount is 1 or 2})$$
$$+P(N = 2, \text{ both claim amounts are 1 or 2})$$
$$+P(N = 3, \text{ all claim amounts are 1 or 2}) + \cdots$$

where N denotes the number of claims from the risk in a year, which is a Poi(4) random variable.

So:

$$P(S_R = 0) = e^{-4} + \left(4e^{-4} \times 0.65\right) + \left(\frac{4^2 e^{-4}}{2!} \times 0.65^2\right) + \left(\frac{4^3 e^{-4}}{3!} \times 0.65^3\right) + \cdots$$

$$= e^{-4}\left[1 + (4 \times 0.65) + \frac{(4 \times 0.65)^2}{2!} + \frac{(4 \times 0.65)^3}{3!} + \cdots\right]$$

$$= e^{-4} \times e^{4 \times 0.65}$$

$$= e^{-1.4} = 0.24660$$

Also:

$$P(S_R = 1) = P(N = 1, \text{ claim amount is 3})$$
$$+P(N = 2, \text{ one claim amount is 3 and the other is 1 or 2})$$
$$+P(N = 3, \text{ one claim amount is 3 and the others are 1 or 2}) + \cdots$$

Using the given claim number and claim size distributions, we obtain:

$$P(S_R = 1) = \left(4e^{-4} \times 0.2\right) + \left(\frac{4^2 e^{-4}}{2!} \times \binom{2}{1} \times 0.2 \times 0.65\right)$$

$$+\left(\frac{4^3 e^{-4}}{3!} \times \binom{3}{1} \times 0.2 \times 0.65^2\right) + \cdots$$

$$= 0.8e^{-4}\left[1 + (4 \times 0.65) + \left(\frac{4^2 \times 0.65^2}{2!}\right) + \cdots\right]$$

$$= 0.8e^{-4}e^{4\times 0.65}$$

$$= 0.8e^{-1.4} = 0.19728$$

Finally:

$$P(S_R = 2) = P(N = 1, \text{ claim amount is } 4)$$

$$+P(N = 2, \text{ one claim amount is 4 and the other is 1 or 2})$$

$$+P(N = 2, \text{ both claim amounts are 3})$$

$$+P(N = 3, \text{ one claim amount is 4 and the others are 1 or 2})$$

$$+P(N = 3, \text{ two claim amounts are 3 and the other is 1 or 2})$$

$$+P(N = 4, \text{ one claim amount is 4 and the others are 1 or 2})$$

$$+P(N = 4, \text{ two claim amounts are 3 and the others are 1 or 2})$$

$$+\cdots$$

Using the given distributions we have:

$$P(S_R = 2) = \left[4e^{-4} \times 0.15\right] + \frac{4^2 e^{-4}}{2!}\left[\binom{2}{1} \times 0.15 \times 0.65 + 0.2^2\right]$$

$$+\frac{4^3 e^{-4}}{3!}\left[\binom{3}{1} \times 0.15 \times 0.65^2 + \binom{3}{2} \times 0.2^2 \times 0.65\right]$$

$$+\frac{4^4 e^{-4}}{4!}\left[\binom{4}{1} \times 0.15 \times 0.65^3 + \binom{4}{2} \times 0.2^2 \times 0.65^2\right]$$

$$+\cdots$$

Summing the first terms in the square brackets:

$$4e^{-4}\times 0.15+\frac{4^2e^{-4}}{2!}\times\binom{2}{1}\times 0.15\times 0.65+\frac{4^3e^{-4}}{3!}\binom{3}{1}\times 0.15\times 0.65^2+\cdots$$

$$=0.6e^{-4}\left[1+(4\times 0.65)+\frac{(4\times 0.65)^2}{2!}+\cdots\right]$$

$$=0.6e^{-4}e^{4\times 0.65}$$

$$=0.6e^{-1.4}$$

Summing the second terms in the square brackets:

$$\left[\frac{4^2e^{-4}}{2!}\times 0.2^2\right]+\left[\frac{4^3e^{-4}}{3!}\times\binom{3}{2}\times 0.2^2\times 0.65\right]$$

$$+\left[\frac{4^4e^{-4}}{4!}\times\binom{4}{2}\times 0.2^2\times 0.65^2\right]+\cdots$$

$$=0.8^2e^{-4}\left[\frac{1}{2!}+\left(\frac{4}{3!}\times\frac{3!}{2!1!}\times 0.65\right)+\left(\frac{4^2}{4!}\times\frac{4!}{2!2!}\times 0.65^2\right)+\cdots\right]$$

$$=\frac{0.8^2e^{-4}}{2!}\left[1+(4\times 0.65)+\frac{(4\times 0.65)^2}{2!}+\cdots\right]$$

$$=0.32e^{-4}e^{4\times 0.65}$$

$$=0.32e^{-1.4}$$

So:

$$P(S_R=2)=0.6e^{-1.4}+0.32e^{-1.4}=0.92e^{-1.4}=0.22687$$

Solution 3.11

Using the recursive formula:

$$p_S(0) = e^{-4}$$
$$p_S(1) = \frac{4}{1} \times [p_X(1) p_S(0)] = \frac{4}{1} \times [0.3 e^{-4}] = 1.2 e^{-4}$$
$$p_S(2) = \frac{4}{2} \times [p_X(1) p_S(1)] = 0.72 e^{-4}$$
$$p_S(3) = \frac{4}{3} \times [p_X(1) p_S(2) + 3 p_X(3) p_S(0)] = 3.088 e^{-4}$$
$$p_S(4) = \frac{4}{4} \times [p_X(1) p_S(3) + 3 p_X(3) p_S(1)] = 3.4464 e^{-4} = 0.0631$$ [4]

(Alternatively, you could calculate the probability directly by summing the different possibilities.)

2 Solutions to exam-style questions

Solution 3.12

(i) The mean, variance and skewness of the aggregate annual claims are (working in millions of dollars):

$$E(S) = \lambda E(X) = 2 \times 5 = 10 \quad [1]$$

$$\text{var}(S) = \lambda E(X^2) = 2 \times [3^2 + 5^2] = 2 \times 34 = 68 \quad [1]$$

$$\text{Skew}(S) = \lambda E(X^3) = 2 \times [4^3 + 3 \times 5 \times 34 - 2 \times 5^3] = 648 \quad [2]$$

[Total 4]

(ii)(a) We are assuming that $\dfrac{S-a}{b} \sim \chi_n^2$. If we equate the first three moments, we get the simultaneous equations:

$$\frac{E(S)-a}{b} = n \qquad ie \quad \frac{10-a}{b} = n \qquad \text{Equation (1)}$$

$$\frac{\text{var}(S)}{b^2} = 2n \qquad ie \quad \frac{68}{b^2} = 2n \qquad \text{Equation (2)}$$

$$\frac{\text{Skew}(S)}{b^3} = 8n \qquad ie \quad \frac{648}{b^3} = 8n \qquad \text{Equation (3)}$$

[2]

By dividing Equation (2) by Equation (3), we get:

$$b = 2.382 \quad \Rightarrow n = 5.991 \approx 6 \quad [1]$$

Equation (1) then tells us that:

$$a = -4.272 \quad [1]$$

(ii)(b) We are assuming that $\dfrac{S-a}{b} \sim \chi_n^2$ *ie* $\dfrac{S+4.272}{2.382} \sim \chi_6^2$.

So the required probability is:

$$P(S > 30) = P\left(\frac{S+4.272}{2.382} > 14.39\right) = P(\chi_6^2 > 14.39) \approx 0.025 \quad [2]$$

[Total 6]

(iii) The χ_n^2 distribution is the same as $Gamma(\frac{1}{2}n, \frac{1}{2})$. When $n = 2$, this is just an exponential distribution, indicating that the individual claims distribution is very skew. For large values of n, such as $n = 50$, the gamma distribution (being the sum of n independent exponentials) approximates to a normal distribution, indicating that the individual claims distribution is almost symmetrical. [2]

Solution 3.13

> ***Comment***
>
> *A question about an unusual class of insurance may well come up in the exam. So this question is included to get you thinking about exposure measures and rating factors outside the more usual sphere of motor, employers' liability etc.*
>
> *A good way to generate the necessary ideas is to think about similar classes of insurance and adapt the ideas to this class.*
>
> *Also in the first part of the course, the Core Reading sets out the ideal features of a risk (to make it insurable). Intelligent application of that list could be useful here.*

(i) ***Characteristics***

- Cover not fully defined in the question: presumably the firm wants to be indemnified in the event of any claim, irrespective of when the claim is made. [1]
- Claims would be against inappropriate advice on design and construction. [½]
- Infrequent claims. [½]
- Large average amounts and high variability of claim amount. [1]
- Possibility of latent design faults with lengthy notification and settlement delays. [1]
- Claim inflation and trends in court awards might be significant. [1]

Difficulties

- Very little data from which to establish appropriate premiums. [½]
- Difficulty in establishing liability: possibility of protracted legal battles. [½]
- Have to calculate lots of single premiums (rather than one amount for the whole year), which may be onerous. [½]
- Building experts required to assess each risk. [½]

- Each building is unique, and will require a detailed report to assess the risks involved. The individual nature of buildings makes if difficult to assess the quality of the advice given on each. [1]
- Trends in building design and construction may make it difficult to assess the risks involved. [½]
- It will be difficult to assess the competence of the firm. [½]
- Obtaining a suitable spread of risks. There is a danger that we are exposed to only one firm, which could give poor advice. [½]
- Reinsurers will be wary of providing cover on such uncertain business (except at a high price). [½]

[Maximum 9]

(ii) ***Possible exposure measures***

- fees charged
- cost of building property
- sum insured (value of building)
- size of building (floor area) [½ each]

Rating factors

- general experience of the firm and experience of employees, *eg* number of years trading, payroll, number and qualifications of staff, *etc* [1]
- type of building [½]
- construction materials [½]
- use of building [½]
- occupation of occupier [½]
- type of design [½]
- deductibles, if any [½]
- number of floors or size of floor space [½]
- past claims experience, if any [½]
- location (*eg* UK, Europe, US) [½]

[Maximum 7]

(iii) ***Other factors affecting premiums***

- expenses (high initial and high claim settlement costs)
- investment income (long periods before claims)
- profit (return on capital) required

- competitors' premiums
- desire to gain business
- possibility of anti-selection (less likely if it is a professional requirement to have such insurance cover)
- regulatory constraints, *eg* maximum or minimum premiums
- currency risk, if business is located overseas. [½ each, maximum 3]

Solution 3.14

> ***Comment***
>
> *The first part of this question is quite tricky and a bit unusual. Therefore it is important that:*
>
> - *You take the question slowly and steadily and state the obvious.*
> - *You look at the data given and ask yourself why.*
> - *State any assumptions.*
> - *You score well on the later parts of the question, which are quite easy.*

(i) ***Calculation***

Three marks are available for producing an appropriate numerical answer using an appropriate method.

The revised risk premium is equal to:

the probability that a claim is above £100 × (the average claim size given that the claim is above £100 minus the £100 excess) × original claim frequency. [½]

Now, the total number of claims below £100 represents 5% of the total claims amounts.

So if there is a claim:

$$0.05 = \frac{\text{(Prob that the claim is less than £100 } \times \text{Ave amt of those less than £100)}}{\text{Ave amount of all claims}}$$

Assume that the average claim size of those less than £100 is £80.

Therefore if there is a claim, the probability that it is less than 100 is 31.25% [1½]

The probability that a claim is less than £100 × Average claim for those less than £100 *plus p*robability a claim is more than £100 × Average of those above £100 = £500.

So:

$$500 = 0.3125 \times 80 + (1 - 0.3125) \times \text{Average amt of those above £100.} \qquad [½]$$

So the average of those above £100 is £691.

The revised risk premium is 0.6875 × 591 × 0.2 = £81.25. [1]

Reasonableness check: we would expect the reduction in premium to be a little less than 20% (*ie* £100 for each £500) because we do not save £100 on all claims. [½]

This calculation assumes that the introduction of an excess will have no impact on the types of policyholder and the claims experience. [½]

[Maximum 4]

(ii) ***Collect and project data***

We need to collect a number of years' past data to calculate the probability of a claim and the claim amount distribution. [½]

Hence we collect claim numbers, claim amounts and exposure. This should allow for IBNR, partially settled claims and reopened claims. [½]

The data should be projected from the date of claim to the current date. The projection should allow for factors such as changes in policy conditions, underwriting, target market, changes in perils covered and possibly very large claims and seasonality. [1]

The claim amounts should be inflated to the current date using suitable inflation indices. Claims might be split into different claim types and size bands, since different claim types / sizes might have different inflation indices. [1]

Decide upon an excess

A suitable excess should be chosen, which should reflect the excess that is expected to be charged. This has initially been set at £100. However we might vary or sensitivity test this later. [½]

Consider future inflation and trends

Attention should be paid to the length of time for which the rates will be in force and a projection must be made to the mid point of the period of exposure. This is because claims will continue to inflate in the future, but the excess will be a fixed amount. [1]

Trends in claim frequency should also be projected up to the mid-point of the period of exposure. [½]

Fit distributions for frequency and amount

Distributions should be fitted for both the claim frequency and claim amount, without any excess. [¼]

For claim frequency, a Poisson model might be used. [¼]

For claim amount, a distribution that best fits the data will be used – this might be a gamma or a log-normal distribution. [¼]

We then combine the claim frequency and claim amount distributions to obtain the insurer's expected payout. This is most easily done using simulation. [1]

The process is then repeated having allowed for the excess, the ratio of the two figures will give the required reduction in risk premium. [½]
[Maximum 6]

Credit should also be given if a stochastic approach has been described.

(iii) ***Why this would not apply in practice***

The reduction, in practice, would apply to the office premium not the risk premium. Therefore, the allowance for expenses should also be considered. [1]

Consider how policyholders' behaviour might change as a result of introducing the excess. For example the policyholders might inflate claims if an excess is applied. Also the policyholder may not bother claiming for amounts just above the excess. [1]

Also the average sum insured or target market might be different for those with the excess. [1]

Consider the price sensitivity of the market and also competition and how much we need to reduce the premium by, if we offer the excess. [1]
[Total 4]

Solution 3.15

(i) ***Moments***

The aggregate claims arising from each type of policy have a compound Poisson distribution. Let S_1 be the aggregate claims arising from Type I policies, and let S_2 be the aggregate claims arising from Type II policies.

$$S_1 = X_1 + X_2 + \cdots + X_N \text{ where } N \text{ is } Poisson(5) \text{ and } X \text{ is } Gamma(15,2).$$

$$S_2 = Y_1 + Y_2 + \cdots + Y_M \text{ where } M \text{ is } Poisson(10) \text{ and } Y \text{ is } Gamma(20,4).$$

The aggregate claim amount from all policies is $S = S_1 + S_2$, which will have a compound Poisson distribution with Poisson parameter 15. [½]

From the *Tables*, the mean, second and third non-central moment for the gamma distribution are given by:

$$E(X) = \frac{\alpha}{\lambda} \text{ and } E(X^2) = \frac{\alpha(\alpha+1)}{\lambda^2} \text{ and } E(X^3) = \frac{\alpha(\alpha+1)(\alpha+2)}{\lambda^3}.$$

So for S_1 we have:

$$E(S_1) = \lambda E(X) = 5 \times \frac{15}{2} = 37.5$$

$$\text{var}(S_1) = \lambda E(X^2) = 5 \times \frac{15 \times 16}{2^2} = 300$$

$$\text{skew}(S_1) = \lambda\ E(X^3) = 5 \times \frac{15 \times 16 \times 17}{2^3} = 2,550 \qquad [2]$$

where $\text{skew}(S_1)$ denotes the third central moment of S_1. Similarly, for S_2 we have:

$$E(S_2) = \lambda E(X) = 10 \times \frac{20}{4} = 50$$

$$\text{var}(S_2) = \lambda E(X^2) = 10 \times \frac{20 \times 21}{4^2} = 262.5$$

$$\text{skew}(S_2) = \lambda\ E(X^3) = 10 \times \frac{20 \times 21 \times 22}{4^3} = 1,443.75 \qquad [2]$$

If we assume that all the risks are independent, then the means, variances and skewnesses of S_1 and S_2 are additive, and we can now find the first three moments of S: [1 for assumptions]

$$E(S) = E(S_1) + E(S_2) = 87.5$$

$$\text{var}(S) = \text{var}(S_1) + \text{var}(S_2) = 562.5$$

$$\text{skew}(S) = \text{skew}(S_1) + \text{skew}(S_2) = 3{,}993.75 \qquad [2]$$

So the mean, variance and third central moment of the aggregate claims distribution are £87,500, $(£23{,}717)^2$ and $(£15{,}866)^3$.

[Maximum 7]

(ii) ***Parameters***

To find the parameters of the translated gamma distribution, we need to equate means, variances and skewnesses. The mean, variance and skewness of a translated gamma distribution with parameters α, λ and k are $k + \alpha / \lambda$, α / λ^2 and $2\alpha / \lambda^3$ respectively. So the equations are:

$$k + \alpha / \lambda = 87.5$$
$$\alpha / \lambda^2 = 562.5$$
$$2\alpha / \lambda^3 = 3{,}993.75 \qquad [1]$$

Solving these simultaneous equations, we find that $\lambda = 0.281690$, $\alpha = 44.634$ and $k = -70.9507$. So these are the parameter values we require. [1]

We now need to find the value of β such that:

$$P(S > \beta) = 0.025$$

This is equivalent to the following gamma distribution probability:

$$P\left(Gamma(44.634,\ 0.281690) > \beta + 70.9507\right) = 0.025 \qquad [1]$$

We now use the result that if X is $Gamma(\alpha,\lambda)$, then $2\lambda X$ is $\chi^2_{2\alpha}$. So the probability can be written:

$$P\left(2\lambda\ Gamma(44.634, 0.281690) > 2\times 0.281690\times(\beta+70.9507)\right) = 0.025$$

$$\Rightarrow P(\chi^2_{89} > 2\times 0.28169(\beta+70.9507)) = 0.025 \qquad [1]$$

Using tables of the chi-square distribution (and using χ^2_{90} as the closest tabulated function), we find that the upper 2½% point is 118.1. So the equation for β is:

$$2\times 0.28169(\beta+70.7507) = 118.1 \qquad [1]$$

Solving this, we find that $\beta = 138.677$. So the aggregate claim amount that will be exceeded with probability 0.025 is £139,000. [1]

[Total 6]

Part 4 – Questions

The split of the questions is described in the introduction to the Q&A Bank Part 1.

Note that the split between development questions and exam-style questions is somewhat subjective. For example, there have been past exam questions that test knowledge of the Core Reading, and so are similar to what we've included here as development questions. The exam-style questions involve more application and a wider range of ideas and are typically the more challenging questions in the exam.

1 Development questions

Question 4.1

A generalised linear model has design matrix:

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

and hat matrix:

$$H = \frac{1}{4}\begin{pmatrix} 3 & 1 & 1 & -1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ -1 & 1 & 1 & 3 \end{pmatrix}$$

Given that $r_1^P = 0.9$, calculate Cook's distance for the first data point. [2]

Question 4.2

Describe the methodology used to estimate an expected loss cost using original loss curves for each of:

- property XL treaty rating
- liability XL treaty rating. [12]

Question 4.3

You are trying to model the claim rates for car insurance policies using a GLM. You have fitted the following GLMs to a particular data set of 40 drivers, and have calculated the scaled deviances given below:

Model	Linear predictor	Scaled deviance
A	$\eta_i = \alpha$	80.95
B	$\eta_i = \begin{cases} \alpha & \text{for } i = 1,2,\dots,20 \\ \beta & \text{for } i = 21,22,\dots,40 \end{cases}$	74.38
C	$\eta_i = \alpha_i$	55.07

Explain these models and assess whether or not:

(a) Model B is a significant improvement over Model A

(b) Model C is a significant improvement over Model B. [5]

Question 4.4

For two nested GLMs, you are given only the following information:

Model	Deviance	Degrees of freedom
1	360.6	50
2	225.8	40

Assess whether or not Model 2 is a significant improvement over Model 1, using a 5% significance level. [3]

2 Exam-style questions

Question 4.5

You are using generalised linear models (GLMs) to help set the premium rates for a book of motor business. You mentioned to a colleague that the model might be better if it included some interaction terms. He has never heard of these before and is interested to find out more.

(i) Define an interaction term. [1]

For a particular multiplicative GLM, we believe there is an interaction between two factors, each with three levels. Factor 1 can take values A, B and C, while Factor 2 can take values X, Y and Z.

(ii) Explain, using a numerical example, the difference between a complete interaction and a marginal interaction for these two factors. [7]

(iii) Explain why you would want to check consistency of the model over time and describe how you would use interactions to do this. [5]
[Total 13]

Question 4.6

(i) Define, using formulae where necessary, the total deviance and the scaled deviance in the context of a generalised linear model. [5]

(ii) Define each of the following tests for significance and state the circumstances in which each might be used:

(a) chi-squared statistic

(b) F-statistic

(c) Akaike Information Criteria (AIC). [7]
[Total 12]

Question 4.7

You work for a large general insurance company and specialise in generalised linear models (GLMs). A non-actuarial colleague in the pricing department has overheard you talking about residuals and is interested to learn a bit more about them.

(i) Describe the following measures that can be used to check that a GLM is appropriate for the data given. You are not required to produce mathematical formulae.

(a) Deviance residuals

(b) Pearson residuals [4]

You have plotted the deviance residuals from your model, to check that the error structure is appropriate.

(ii) Explain how you will determine from the residual plot whether or not your model is likely to be a good fit. [4]

[Total 8]

Question 4.8

You are the actuary of an established general insurer that began underwriting a portfolio of non-comprehensively insured motorcycle business five years ago. The account has grown slowly and now consists of approximately 45,000 policies, although the mix of business has remained consistent during this period. The current premium rates are based on five principal rating factors, namely, age of policyholder, engine capacity, make and model of motorcycle, district and no claims discount. There are good quality historical policy and claims data available.

(i) Describe how you would carry out a multivariate analysis of the historical experience in order to assess how the principal rating factors explain the variations in claim frequency across risk categories. [12]

(ii) State the assumptions you need to make and explain how you can test their validity. [6]

(iii) You are considering applying similar techniques to the average claim cost. Describe briefly the further problems that must be overcome. [6]

[Total 24]

Question 4.9

You are an actuarial trainee working for a UK general insurer and you have recently been seconded to the household pricing department. Your manager is considering whether it might be possible to use data other than that provided by the customer at point of sale to help in pricing, but he needs to know what types of data might be useful before deciding whether to go ahead.

State examples of the types of data that he could consider using, giving an explanation as to why each could be of benefit. [22]

Question 4.10

You work in the pricing department of a general insurance company that writes a wide range of personal lines business. You have collected and adjusted some premium and claims data for one of the classes of business written, with a view to doing a multivariate analysis using this data. Your next step is to do a one-way analysis of the data, but your manager is keen for you to start the multivariate analysis straight away and not to waste any time doing a one-way analysis.

Explain the points you would make to your manager in support of the need to do one-way analyses. [11]

Question 4.11

(i) Outline the main problems that can arise when attempting to derive increased limit factors (ILFs) for casualty (liability) business. [3]

(ii) Describe a methodology that could be used to construct ILFs that, in particular, attempts to deal with the problem of closed claims. [6]
[Total 9]

Question 4.12

You are the pricing actuary for a reinsurance company. A property insurer wishes to purchase £300,000 xs £200,000 risk XL treaty reinsurance and has provided you with the following information for Year 1:

- Original ultimate loss ratio for Year 1 (based on ground-up data): 70%
- Expected future claims inflation: 5% *pa*.

Original policy information for Year 1	
Sum insured band (£s)	Original premium (£000s)
0 to 50,000	8,500
50,001 to 100,000	9,000
100,001 to 150,000	8,500
150,001 to 200,000	8,000
200,001 to 300,000	14,500
300,001 to 400,000	12,000
400,001 to 500,000	10,000
500,001 to 750,000	15,000
750,001 to 1,000,000	8,000
Total	93,500

Exposure curve values	
y	$G(y)$
0%	0.0%
10%	44.7%
20%	56.9%
30%	65.6%
40%	72.6%
50%	78.5%
60%	83.6%
70%	88.3%
80%	92.5%
90%	96.4%
100%	100.0%

(i) Calculate the expected loss cost to the layer for this business in Year 2, expressed as a proportion of original premiums. State any assumptions that you make. [10]

(ii) Comment on the appropriateness of the assumptions that you made in part (i). [8]
[Total 18]

Question 4.13

Discuss reasons why premium rates implemented in practice might differ from the theoretical risk premium rates derived from a generalised linear model based on past data. Where appropriate, use examples and explain how these differences would be allowed for. [27]

Part 4 – Solutions

1 Solutions to development questions

Solution 4.1

Cook's distance for the first data point is:

$$c_1^P = \frac{h_{11}}{(1-h_{11})\sum_{i=1}^{4} h_{ii}}\left(r_1^P\right)^2 = \frac{3/4}{1/4\left(4\times 3/4\right)}\times 0.9^2 = 0.81\,. \qquad [2]$$

Solution 4.2

Property XL rating

For property business, we consider the *relative* loss severity. [½]

That is, $Y = \dfrac{X}{M}$

where:

- X is the random variable representing the loss severity
- M is a measure of the size of the risk, which could be sum insured, probable maximum loss, *etc*. [1]

We define the *exposure curve*, $G(x)$, as:

$$G(x) = \frac{LEV_Y(x)}{E[Y]}$$

where the LEV (limited expected value) function represents the expected value of the (relative) losses limited to a primary layer of size x. [1]

More formally,

$$LEV_x(x) = E[X \wedge x] = \int_0^x S_x(y)\,dy$$

where

$$S_x(x) = 1 - F_x(x) = P[X > x]. \qquad [1]$$

To calculate the cost to the layer, for a layer L excess of D, we use:

$$C \times \left(G\left(\frac{L+D}{M}\right) - G\left(\frac{D}{M}\right) \right)$$

where C is the ground-up expected loss. [1]

The underlying assumption in using the relative loss is that Y is independent of the size of the risk. [½]

This is generally true for reasonably homogenous data, *eg* for residential buildings fire claims. [½]

However, when the data becomes more heterogeneous, we may need different exposure curves for different risk groups. [½]

When allowing for claims inflation (trend), if its effect is uniform across all loss sizes and the sums insured M are being adjusted appropriately for inflation, no adjustment would be required (because the distribution of Y would be unchanged). [½]

Casualty XL rating

For casualty business, we consider the loss severity in monetary terms, X, rather than dividing by a risk size. [½]

This is because there is no easily definable limit on the loss severity – the limit of insurance actually purchased may bear no relation to the potential claim amount (it's just a limit on how much the insurer pays out on that particular policy). [½]

Hence, we select risk groups, and within these, we make the following assumptions:

- the (ground-up) loss frequency is independent of the limit purchased
- the (ground-up) severity is independent of the number of losses and of the limit purchased. [1]

The "curve" is expressed in terms of a table of *increased limit factors* (ILFs) at each limit x, which are defined as:

$$ILF(x) = \frac{LEV_X(x)}{LEV_X(b)}$$, where the base limit is b. [1]

To calculate the cost to the layer, for a layer L excess of D, we then use:

$$C_l \times \left(\frac{ILF(L+D) - ILF(D)}{ILF(l)} \right)$$, where l is the original limit. [1]

The treatment of allocated loss adjustment expenses (ALAE) is usually a much more important consideration for casualty business than for property business as this can often represent a very significant proportion of the claims cost. Hence, it is more likely that the ILFs include the ALAE component. [1]

Split limits, *ie* a per-claimant limit and per-occurrence limit, may apply, and so the ILFs (and method for constructing them) may need to be modified accordingly. [½]

The effect of inflation (trend, including secular changes) may be more significant for casualty business, and may be more likely to change the shape of the underlying loss distribution. [½]

If claims inflation is expected to affect the loss distribution uniformly, say by increasing losses by a% between time t and time t', then:

$$ILF_{t'}(x) = ILF_t\left(\frac{x}{1+a}\right).$$ [½]

[Maximum 12]

Solution 4.3

Model A assumes that the claim rate is the same for each observation because the linear predictor η_i is always equal to α. [½]

Model B assumes that Drivers 1 to 20 have the same claim rate and that Drivers 21 to 40 have the same claim rate. [½]

Model C assumes that each driver claims at a different rate. [½]

The models are nested (because one is a subset of the other) and the scaled deviances are given (*ie* the scale parameter is known) so we can use a χ^2 test. [½]

We will use a 5% significance level in the tests below.

The difference in the scaled deviance between Model A and Model B is 6.57. Model A has 1 parameter and Model B has 2. So we compare the difference in the scaled deviance with χ_1^2. [½]

From page 169 of the *Tables*, the upper 5% point of χ_1^2 is 3.841. Since $6.57 > 3.841$, we conclude that Model B is a significant improvement over Model A. [1]

The difference in the scaled deviance between Model B and Model C is 19.31. Model B has 2 parameters and Model C has 40. So we compare the difference in the scaled deviance with χ_{38}^2. [1]

From page 169 of the *Tables*, the upper 5% point of χ_{38}^2 is 53.38. Since $19.31 < 53.38$, we conclude that Model C is ***not*** a significant improvement over Model B. [1]

[Maximum 5]

Solution 4.4

The two models are nested but only the deviance is given, so the scale parameter is not known. We must therefore compare deviances using the *F*-statistic. [½]

We compare:

$$\frac{\left(D_1 - D_2\right)}{\left(df_1 - df_2\right)\left(D_2 / df_2\right)} = \frac{\left(360.6 - 225.8\right)}{\left(50 - 40\right)\left(225.8 / 40\right)} = 2.388$$

with $F_{df_1 - df_2, df_2} = F_{10,40}$. [2]

From page 172 of the *Tables*, the upper 5% point of $F_{10,40}$ is 2.077. So we conclude that Model 2 is a significant improvement over Model 1 (at the 5% significance level).
[1]
[Maximum 3]

2 Solutions to exam-style questions

Solution 4.5

(i) ***Definition***

An interaction exists when the effect of one factor varies, depending on the levels of another factor. [½]

Interactions would be used where the pattern in the response variable (*eg* frequency or severity) is better modelled by including extra parameters for each combination of two of more factors. [½]

[Total 1]

(ii) ***Complete and marginal interactions***

Complete and marginal interactions are alternative representations of the same thing. [½]

A complete interaction is expressed as a single factor that represents every combination of the factors involved. [1]

For example, for the two factors given, we would have a new single factor, representing the interaction, which would have 9 levels, *ie:*

AX, AY, AZ, BX, BY, BZ, CX, CY, CZ. [1]

Each of these levels would have a multiplier attached (since this is a multiplicative model). These could be written in the form of either a one-way or a two-way table. For example:

Factor 1:		A	B	C
Factor 2:	X	0.90	1.00	1.10
	Y	0.97	1.20	1.45
	Z	1.26	1.40	1.85

[1]

In this case, the base level has been selected to be the level corresponding to Level B of Factor 1 and Level X of Factor 2, and the interaction term has 8 parameters. [1]

A marginal interaction considers the *additional* effect of the interaction term over and above the single factor effects. [1]

In this case, the single factor effects will be observed separately from the marginal interaction term effects. [½]

Using the same example as above, the multipliers would look as follows:

Factor 1:			A	B	C
			0.90	-	1.10
Factor 2:	X	-	-	-	-
	Y	1.20	0.90	-	1.10
	Z	1.40	1.00	-	1.20

[2]

So the overall relativity for Factor 1 Level A and Factor 2 Level Y would be $0.90 \times 1.20 \times 0.90 = 0.97$ and this agrees with the figure given in the table of complete interactions.

We would therefore calculate this marginal interaction as $\dfrac{0.97}{1.20 \times 0.90} = 0.90$.

[Maximum 7]

(iii) ***Consistency over time***

When pricing, it is important to check that the patterns of relativities observed in a GLM are not changing too much over time. [½]

If a trend emerges over time then it is important to identify it, so that we can project the patterns to the period over which the rates will apply. [½]

The time consistency check is also used to determine whether the effect of each factor is consistent from year to year. If a factor is consistent then it is likely to be a good predictor of future experience. [1]

To test the consistency of parameter estimates over time, we can fit a GLM that includes the interaction of a single factor with a measure of time, *eg* a calendar year. [1]

Ideally we would do this for every factor in the model and would test the interaction for statistical significance. [½]

We would also look at graphs of the interaction results to see what patterns are being produced. [½]

These graphs would plot the response variable (*eg* frequency or severity) by the factor of interest, with different lines on the graph representing different time periods. [1]

If an interaction with time proves to be significant then this suggests that the pattern observed for that factor does change over time. We would need to interpret these results carefully in order to estimate the relativities that will be suitable for future years. [1]

[Maximum 5]

Solution 4.6

(i) ***Deviance and scaled deviance***

The deviance compares the observed value for each observation Y_i, to the fitted value μ_i, with allowance for weights ω_i … [1]

… and with higher importance assigned to errors where the variance should be small. [½]

Each observation's contribution to the deviance is defined as:

$$d\left(Y_i;\mu_i\right)=2\omega_i\int_{\mu_i}^{Y_i}\frac{\left(Y_i-\zeta\right)}{V\left(\zeta\right)}d\zeta\ . \qquad [1½]$$

The sum of the contributions to deviance, over all observations, is the total deviance for a model. [½]

The *total deviance* is defined as:

$$D=\sum_{i=1}^{n}d\left(Y_i;\mu_i\right). \qquad [½]$$

The *scaled deviance* is the model deviance adjusted by the scale parameter φ. [½]

This standardises the deviance so that it can be used when comparing different models. [½]

In formula terms, $D^*=\dfrac{D}{\varphi}$, where D^* is the scaled deviance. [½]

[Maximum 5]

(ii) ***Significance test definitions***

(a) *Chi-squared statistic*

Chi-squared statistics can be used for comparing two models that are nested (*ie* where one is a subset of the other) and where the scale parameter is known. [½]

A χ^2 test is applied to the change in scaled deviance between the two models:

$$D_1^* - D_2^* \sim \chi^2_{df_1 - df_2} \quad [1]$$

where df is the number of degrees of freedom, defined as the number of observations minus the number of parameters. [½]

(b) *F-statistic*

F-tests are used in cases where we want to compare two nested models but where the scale parameter is not known, *ie* where we have the deviance but not the scaled deviance. [1]

The estimator of the scale parameter is distributed as a χ^2 distribution, with the F-distribution being the ratio of two χ^2 random variables as follows: [½]

$$\frac{(D_1 - D_2)}{(df_1 - df_2)(D_2 / df_2)} \sim F_{df_1 - df_2, df_2} \quad [1]$$

(c) *Akaike Information Criteria (AIC)*

The AIC is also used to compare which of two models is a better fit. However, unlike the above two tests, the AIC can be used when the two models are not nested. [1]

The AIC for a model is calculated as:

$$\text{AIC} = -2 \times \text{log-likelihood} + 2 \times \text{number of parameters}. \quad [1]$$

It considers the trade-off between the likelihood of the model and the number of parameters. [½]

A model with a lower AIC has a better fit than one with a higher AIC. [½]

[Maximum 7]

Solution 4.7

(i) ***Residuals***

Deviance residuals

A deviance residual, for a given observation, is a measure of the difference between the observed value and the value fitted by the model. [½]

The deviance residual considers the square root of each observation's contribution to the deviance, adjusted for the direction in which the raw residual (the difference between the observed value and the fitted value) acts. [1]

The deviance measure corrects for the skewness of the distributions used, meaning that we would expect the deviance residuals to be more closely normally distributed than the raw residuals. [1]

Pearson residuals

A Pearson residual, for an individual observation, is the difference between the observed value and the fitted value (*ie* the raw residual), scaled by the standard deviation of the predicted value. [1]

This measure does not adjust for the shape of the distribution. [½]
[Total 4]

(ii) ***Residual plot***

The residual plot could be a scatter plot of deviance residuals against the fitted values. [½]

If the chosen error structure is appropriate for the data and response variable that we are modelling, the residual plot will have the following characteristics:

- the average residual will be zero, so there should be an equal number of points above zero and below zero on the graph [1]
- the pattern of residuals will be symmetrical about the x-axis [1]
- the range of residual values will be fairly constant across the width (the x-axis) of the fitted values. [1]

A residual plot where the range of residuals narrows or widens as the fitted value increases, or where the range of residuals is not symmetrical about the x-axis, indicates that the model specification is poor. [1]
[Maximum 4]

Solution 4.8

> ### *Comment*
>
> *This question is based on an old exam question. According to the examiners' report, it was poorly answered in the exam.*
>
> *It should have been an "easy" question for a well-prepared student. Whether you are describing an experience rating, statistical rating or more general rating approach you must show the examiner that you understand the need to analyse data in homogeneous groupings, by amounts and numbers separately, to remove distortions, and to look for, explain, and project trends.*

(i) ***Multivariate analysis to establish principal rating factors***

Past claims and exposure data is required. Given the small size of the portfolio it should cover the whole five years of existence. [1]

Prepare details of claim numbers and exposure for all combinations of rating factors. [½]

Given the lack of data, grouping levels of rating factors may be necessary. [½]

Spatial smoothing or vehicle classification methods could be used for grouping levels of factors such as postcode and make / model of motorcycle. [1]

One-way tables of each rating factor in isolation can be used to check the reasonableness of the prepared data and will give a preliminary indication of the effect of each factor in terms of claim frequency. [1]

Two-way tables can be used to help understand correlations between pairs of rating factors and to indicate which factors may be affected by the removal or inclusion of any other factor in the multivariate analysis. [1]

May need to adjust data for:

- changes in policy conditions [½]
- mid-year endorsements (allocate proportion to each cell) [½]
- unreported claims, especially latest year's data. [½]

Investigate frequency for each major claim type separately – for example, fire, theft, accidental damage, third party property damage and third party bodily injury. [1]

This is to avoid heterogeneity being introduced by changing policy conditions, and to develop a better understanding of the factors behind changes in overall experience. [1]

Look for trends over time in the claim data and try to explain them. [½]

Use a statistical rating model (*eg* a GLM) to produce expected claim frequencies for all combinations of rating cells chosen. The exposure from each year's data can be used to weight the claims frequencies. [1]

Test a range of models to see which gives the best fit to the actual data values. By comparing actual versus expected claim frequencies for different models, we can assess the error terms. [1]

We can then decide which model gives the best fit to the data. [½]

See part (ii) for how.

We can now study how claim frequency varies across rating factors and different levels of the factors. Need to consider if the results are reasonable or not. [1]
[Maximum 12]

(ii) ***Assumptions and validity***

Assumptions

- Choice of model – multivariate, GLM, generalised non-linear model, generalised additive model. [1]
- The link function (which determines whether it's an additive or multiplicative model, for example). [½]
- Grouping within rating factors. [½]
- Type of error structure. [½]

GLMs are commonly used as they are not too complex to run but are based on a sound statistical framework. [½]

A common choice for claim frequency error structure is a Poisson assumption. [½]

This is because it provides a good fit to claim frequency data and ensures that model frequencies stay positive. [1]

A log link is often chosen as this will result in a multiplicative model, as we would intuitively expect this to be the best reflection of the relationship between the rating variables. [1]

The residual errors can be analysed using simple residual plots and by best fit statistical tests. [1]

We can then assess if a particular model gives a good fit. [½]
[Maximum 6]

(iii) ***Further problems with ACPC models***

One important point to check is whether the data is complete. Another relates to the small size of the account, ie credibility, variability, large claims. Note that these problems are not unique to statistical rating techniques. They also apply to rating more generally.

Because we are using the latest data, we will need to rely upon estimates of IBNR and outstanding reported claims to project paid to date amounts to ultimate. [1]

Large claims can distort the figures by their presence or absence in particular cells. Bodily injury claim amounts especially can vary a great deal in amount. In this class they could form a very significant part of the claim amount. [1]

We could identify this effect by splitting out these types of claim, or mitigate it by using less detailed splits of rating factors. But we would then lose our ability to study the effects of the rating factors. In practice, a compromise will be necessary. [1]

We could truncate large claims at a suitable level and gross up across all cells for the occasional large claim. However, large claims may be associated with particular rating cells (*eg* low ages, high engine capacities, living in large cities) which have several past claims. [1]

We could therefore spread the large claims loading across only those cells from which we expect the large claims to come. [½]

We would need to allow for inflation of claim amounts, by different type of claim. [½]

We may need to adjust for reinsurance recoveries and salvage. [½]

The treatment of nil claims needs to be consistent over the period and consistent with the frequency model. [½]

Nil claims may be removed from both models or otherwise treated as a separate claim category. [½]

Need to use a different error structure. A gamma assumption is often found to be appropriate for modelling claim severity. [½]

[Maximum 6]

Solution 4.9

Extra information about the proposer

Other product holdings with our company – if a customer has a range of different types of insurance with us then it may indicate a degree of loyalty to our company. This may lead to lower levels of claims. This might also indicate that a customer is less sensitive to price and/or is likely to have better persistency. [2]

Previous household claims experience with our company – this would be available from the claims data but would need to be matched using a linking field such as policy number. [1]

A poor previous claims history may suggest that future claims may also be higher than average, but this must be interpreted in relation to the number of years each policyholder has been with us. [1]

We might also find that previous claims experience for other classes of business is predictive of future household claims. [½]

Customer lifetime value models. These might tell us which types of customer are likely to contribute the highest level of profit over their lifetime with our company. We might want to charge different amounts, or provide incentives, to these profitable customers. [1½]

Customer behaviour models. These would give information about price elasticity of different types of customers (*ie* how they would respond to a given change in price) and other aspects of customer behaviour. This information would enable us to better maximise profits over a whole book of business. [2]

Credit score / customer insurance score – a customer with a favourable score is likely to be relatively wealthy and more honest, and so may decide not to claim for small amounts, making their overall claims experience lower. Although they may also have more expensive possessions, this should be taken into account in the sum insured. [2]

Extra information about the location / postcode of the risk

Socio-economic data – there are companies that provide databases of socio-economic and wealth information by full postcode. [1]

As for the credit score above, those customers who live in wealthy areas are less likely to claim for small amounts and may be more cautious, leading to lower claims experience. [1]

Using this information, we may also be able to look at the claims experience (in particular for theft) of wealthy areas that are right next to poor areas. For example, a wealthy area next to a poor area may exhibit much worse theft claims experience than a wealthy area bordered by other wealthy areas. [1]

Subsidence and soil type data – the risk of subsidence depends primarily on the soil type in an area, together with the amount of rainfall, with clay soils being the most risky. Subsidence claims can be of relatively high amounts for household buildings insurance and so we need to ensure that we are pricing this part of the risk correctly to avoid unintentional cross-subsidies between policies. [2]

Flood data – in many countries, including the UK, flood is becoming a bigger risk than in the past. This, combined with the low frequency of flood events, means that past flood claims data is not likely to be sufficient for rating future policies. [1]

We could buy flood data which might include information such as:

- flood height
- flood defence data
- prior flood experience by area. [½ each, max 1]

Theft survey data – theft is the most significant insured peril for household contents and so it is important to be able to quantify this risk as accurately as possible. Although this data is often sparse (because it may not be collected for every area and theft frequency is fairly low in many areas), the external agencies that provide the data may have smoothed it to cover a wider set of areas. [1½]

Household valuation data – this may be used to help amend the customers' own estimates of their sum insured. This is particularly useful because, for buildings cover, rebuild value is required rather than current market value, which is what the customer will tend to provide. [1½]

Census data and/or electoral roll data – this contains a wealth of information about the types of people living in an area, and so can be linked to postcode. For example, it will give information on employment levels and types, levels of overcrowding, marital status, numbers of cars owned, ethnicity, levels of health, *etc*. [2]

The census data is produced once every ten years so will always be a bit out of date but it may be able to provide extra information that helps when trying to predict future claims experience. [1]

[Maximum 22]

Solution 4.10

The one-way distribution of exposure and claims across each level of each raw variable will indicate whether a particular *variable* contains enough useful information to justify its inclusion in the multivariate models. [1]

For example, if 99% of a variable's exposure is in one level then it may not be suitable for modelling. [½]

Assuming that there is a viable distribution by levels of a variable, a one-way table would highlight whether there is enough exposure and/or claims information in each *level* of the variable. [1]

If not, then we may need to do some grouping of levels prior to loading the data into the modelling software to ensure that the multivariate models can converge. [1]

We can calculate some simple one-way statistics for each level of each factor. [½]

Examples of these statistics are:

- frequency
- severity
- burning cost
- loss ratio
- pure premium. [¼ each, maximum 1]

This will give us a preliminary indication of the likely effect of each factor ... [½]

… and provide a reference to help with the interpretation of any results that we may observe in the multivariate analysis. [½]

The one-way tables will provide totals, maybe by distribution channel or account, which we can use for validating the data against other sources. [1]

We may also pick up some useful information by comparing these one-way tables with the ones produced for previous rate reviews (if these exist). [1]

One-way tables will also allow us to check whether the distribution of exposure by each level of each variable is as expected. [½]

For example, it might show that we had been selling higher volumes of business in particular segments than we would have liked. [½]

If the distribution was not as expected then this could also suggest that there might be errors in the raw data … [½]

… and/or programming errors introduced when cleaning the data. [½]

The production of one-way tables is likely to be relatively quick, especially if the process can be automated or we can use an existing program to do this. [1]

In summary, creating and checking the outputs of one-way tables is not a waste of time because it has the potential to highlight problems at an early stage, which will save time in the long run. [1]

[Maximum 11]

Solution 4.11

(i) ***Main problems when deriving ILFs***

- Lack of volume and of credibility of data. [½]
- Information about large losses may be lost, due to the impact of policy limits. [½]
- Adjusting for trend: [¼]
  - working out what the adjustment should be (depends on court inflation, legislation, economy, *etc*) [½]
  - adjusting the ILFs appropriately (as the impact of trend may vary by size of loss). [½]
- Many claims will still be open: [¼]
  - these may take a long time to settle [¼]
  - the final claim amount may be very different from the current case estimate. [½]

[Maximum 3]

(ii) ***Methodology used to construct ILFs***

Firstly, adjust the individual losses for inflation, secular changes, *etc* from the experience period to the period for which the ILFs will be applied. [½]

Consider only closed claims, but from several accident years. [½]

Group these claims by payment lag, *ie* the time in years from accident to settlement. Where multiple payments are made, use the average lag weighted by amount paid. [1]

For each payment lag, construct an empirical survival function for the claim size: [½]

- split data into (hopefully more than 50) discrete loss-size intervals [½]
- for each interval, estimate the probability that the loss will exceed the upper bound, given it exceeds the lower bound (*ie* the conditional survival probabilities) [½]
- estimate discrete points on the survival probability by multiplying these conditional survival probabilities together. [½]

In practice, lags beyond a certain period, say five years, are grouped and the same loss distribution assumed. [½]

Estimate the proportion of the number of loss occurrences for an accident year that are settled at each payment lag. [½]

Combine these proportions with the empirical survival functions at each lag to estimate the combined survival function for all claims. [½]

Smooth the tail of the combined survival function (*eg* by fitting a truncated Pareto distribution above a selected threshold) and then fit a parametric distribution to the smoothed curve (*eg* using a mixed exponential). [1]

We can then derive limited average severities (and hence ILFs) from the fitted loss distribution. [½]

[Maximum 6]

Solution 4.12

(i) ***Calculation of cost to the layer***

Assumptions:

(1) sums insured (SIs) are distributed evenly within each sum insured band, so that each band is represented by the average sum insured for the band

(2) the same loss ratio applies to each band

(3) the same risk profile (*ie* distribution of SIs) will apply in Year 2

(4) the data given is for the middle of Year 1, and hence exactly one year's inflation can be applied

(5) the (single) exposure curve given can be applied to all the business to be reinsured

(6) the same claim distribution (as represented by the exposure curve) will apply in Year 2

(7) the same inflation figure (5%) can be applied to all policies (*eg* irrespective of size)

(8) there are no features of the original business, such as deductibles or stacked limits, that would complicate the calculation

(9) there are no features of the reinsurance treaty, such as reinstatements or inuring reinsurance, that would complicate the calculation

(10) intermediate points on the exposure curve can be obtained by linear interpolation.

[½ each]

Working in £000s throughout.

To allow for 5% inflation, we can reduce the attachment point and exit point of the layer by 5%, *ie*

Adjusted attachment point: $\dfrac{200}{1.05} = 190.48$

Adjusted exit point: $\dfrac{500}{1.05} = 476.19$ [1]

Calculations for each band that falls within the treaty are as follows:

Average SI (£000s)	Original premium (£000s)	Attachment point % of SI	Exit point % of SI	Attachment point % of loss cost	Exit point % of loss cost	Cost to the layer (£000s)
250	14,500	76.2%		90.9%		923.7
350	12,000	54.4%		80.7%		1,621.2
450	10,000	42.3%		74.0%		1,820.0
625	15,000	30.5%	76.2%	66.0%	90.9%	2,614.5
875	8,000	21.8%	54.4%	58.5%	80.7%	1,243.2

Calculate a SI value to represent each SI band, such as the average (*Marks awarded for correct averages as above or any sensible assumption, eg taking upper points, where justified*; *deduct ¼ mark for each error*). [1]

For layers below £190*k*, there is no cost to the layer. Hence, if the layer £150*k* to £200*k* is represented by the average (£175*k*), this will not contribute to the layer. [½]

For higher layers, we need to calculate the attachment point as a proportion of the assumed sum insured. For example, for the layer £200*k* to £300*k*, this is:

$$\frac{190.48}{250} = 76.2\% .$$ [½]

Similarly, for all other layers (*see table above, deduct ¼ mark for each error*). [1]

Also, for layers above £500k, calculate the exit point as a proportion of the assumed sum insured. For example, for the layer £500*k* to £750*k*, this is:

$$\frac{476.19}{625} = 76.2\%$$ [½]

For the layer £750*k* to £1,000*k*, this is:

$$\frac{476.19}{875} = 54.4\%$$ [½]

Use the values from the exposure curve to convert the exposure proportions into the proportions of the loss cost for each layer (related to the ground-up losses).

For example, to calculate the cost of the attachment point as a proportion of the loss cost for the layer £200*k* to £300*k*, we could use interpolation to estimate $G(76.2\%)$, *ie*:

$$88.3+\frac{6.2}{10}\times(92.5-88.3)=90.9\%$$ [½]

Similarly, for all other attachment points and exit points (*See table above, deduct ¼ mark for each error, but any reasonable approximations permitted.*) [1]

For each band (for which the reinsurance will apply), the original loss cost can be estimated by applying the loss ratio (70%) to the original premiums.

For layers below £500*k*, the cost to the layer is the proportion $(100\%-G(L))$ applied to the original loss cost.

So, for example, the cost of the layer £200*k* to £300*k* is calculated as:

$$(100\%-90.9\%)\times 70\%\times 14{,}500=923.65$$ [1]

Similarly for the next two layers. (S*ee table above, deduct ¼ mark for each error.*) [½]

For layers above £500k, we take use the proportion $(G(U)-G(L))$, where L and U are the lower and upper limits respectively.

Hence, for example, the cost of the layer £500*k* to £750*k* is calculated as:

$$(90.9\%-66.0\%)\times 70\%\times 15{,}000=2{,}614.5$$

Total cost to the layer is the sum of this final column, *ie* £8,222.6*k*. [½]

As a percentage of the original premium this is:

$$\frac{8{,}222.6}{93{,}500}=8.8\%$$ [½]

[Maximum 10]

(ii) ***Appropriateness of the assumptions***

Assumption (1) may not be realistic. For example, for most bands, we might expect more policies with lower than average SIs, because the volume of business in each (equal width) band is decreasing with size of SI. [½]

On the other hand, there may be a significant number of policies at the very top of each band, *eg* 200*k* or 300*k*, because "round" SI amounts may be popular. [½]

A particular issue here is that we have excluded the £150*k* to £200*k* band in the calculation, whereas some policies above £190*k* will contribute to the cost of the layer. [½]

Also, the result may be very sensitive to this assumption, particularly as the band widths here are quite large. [½]

Ideally, we should obtain exposure data in more detail to investigate this assumption in more detail. At the very least, sensitivity testing should be carried out. [½]

Assumption (2) may not be true if:

- claims experience varies by sum insured (*eg* SI may be a proxy to socio-economic group and hence the tendency to claim) [½]
- the insurer's pricing basis has cross-subsidies by policy size (*eg* large policies subsidising smaller ones). [½]

For assumption (3), the distribution of sums insured by premium may be affected by the insurance cycle. [½]

For assumption (4), even if data is not mid-year, it is probably reasonable to assume one year's inflation, but we should check how the dates of the treaty compare with the data we have. [½]

Assumption (5) is unlikely to be realistic, unless the business being ceded is quite homogeneous. Ideally, we should split the data by risk groups and apply different exposure curves (and loss ratios) to each. However, the approach we've made may be the best we can do with the (limited) data we have. [1]

Assumption (6) may be rather simplistic. We should investigate whether large claims are affected by inflation differently from small claims. This may be the case, because different types of claims will be affected by different types of inflation to varying extents. [1]

Similarly, for assumption (7), large policies may be affected by inflation to a different extent than small ones. [½]

However, the inflation adjustment may have a relatively small impact as it is only for one year. [½]

If there are features of the business or reinsurance that have not been allowed for this would invalidate assumptions (8) or (9), and so adjustments would have to be made. [½]

Assumption (10) is probably an oversimplification and using straight interpolation could make a difference to the estimated cost because we only have a few values on the exposure curve table. It may be better to fit a parametric curve to the points (or obtain values in more detail). [1]

[Maximum 8]

Solution 4.13

Loadings

The biggest reason why implemented premiums would differ from theoretical risk premiums is that the risk premiums only cover the claims element of the premium whereas the implemented premium would include allowance for other loadings. [1]

These other loadings include:

- expenses [¼]
- commission [¼]
- profit and contingencies [¼]
- cost of capital [¼]
- investment income (a discount to the premium) [¼]
- reinsurance [¼]
- insurance premium tax. [¼]

Expenses could be allowed for in the final premium by a percentage of premium, a fixed monetary amount or combination of the two. [½]

Changes

The GLM is based upon past data. There are many reasons why claims experience in the future might be expected to be different from that seen in the past. [½]

Examples of changes that may need to be taken into account include:

- changes in the risk environment, *eg* climate change could affect weather-related household claims experience [½]
- changes in the level of cover provided, *eg* the addition or removal of perils or changes in limits or excesses [1]
- changes in underwriting or claims processes that could affect the overall level of claims experience [½]
- changes in the legal or regulatory environment, *eg* for motor insurance, there have been changes to the Ogden tables and to NHS costs that can be reclaimed [1]
- trends in the claims experience (either frequency or severity or both); judgement will need to be used to decide whether trends seen in the base data are likely to continue into the future and, if so, to what extent [1]
- an adjustment to allow for large or exceptional claims [½]
- unusually heavy or light experience in the base data that is not expected to continue into the future. [½]

For each of these changes, we would need to try to quantify the level of change and project this to future rating periods. [½]

Systems issues

A theoretical risk premium model is likely to include all factors that could help to explain the risk. If there is a risk factor within the GLM that has not been used in the past then it is possible that no rating table currently exists for that factor. [1]

If it is not easy or quick to change the rating algorithm and tables, it may be necessary to accept that one or more factors may need to be dropped from the model, at least in the short-term. [½]

The theoretical GLM could then be re-run without any such factors so that the relativities relating to other factors in the model can allow partially for the effects of the dropped factor(s). [1]

The ability to add new factors may depend upon the distribution channel used. For example, the broker system might be less flexible than the insurer's own direct sales system, and so it may be difficult to introduce new rating factors for business sold through brokers. [1]

Customer behaviour models

An insurer may take account of the price elasticity of various groups of customers in order to maximise profits and volumes for a whole book of business. The elasticity models would typically be run after the GLMs and used to adjust the theoretical rates.[1]

For example, an increase in rates might be applied to a customer who is perceived to be inelastic (*ie* likely to renew even if the price increases) in order to fund a reduction in rates for a more elastic customer. [1]

Similarly, an insurer may apply models of customer lifetime value to the theoretical rates so that the premiums can be flexed according to the insurer's view of how loyal and profitable each type of customer is likely to be in the long term. [1]

Legal and commercial considerations

There may be legal or commercial restrictions on the way factors can be, or are, used in practice. [½]

For example, there may be factors (such as age and sex) that, in some countries, are not allowed to be used as they are perceived to be discriminatory. [½]

NCD in motor insurance is an example of a factor that is likely to be implemented using a very different set of relativities to those that would be determined in a theoretically correct model. [½]

The NCD scales seen in the market for motor insurance are very steep, for commercial reasons, with the theoretically correct discounts being much lower. For household insurance, the NCD scales are much less steep and are a better reflection of the theoretical rates. [1]

We could allow for these restrictions within a GLM by offsetting the restricted factors and re-running the model. [1]

Company strategy

The insurer may have decided that it does not really want to target certain groups of potential policyholder, and therefore it may charge more to those groups than what is suggested by the theoretical models. [1]

Similarly, discounts may be given to other groups of policyholders to encourage them to insure with our company. [½]

Alternatively, the insurer may have a strategy to increase volumes of business by x% this year for a particular class of business, and so it may price at a level lower than the theoretical rate for a period of time for this class. [1]

Any such cross-subsidies need to be done with care to ensure that, allowing for the expected mix of business, the total premiums across the whole book are sufficient. [1]

The company may want to price-test some policies in order to calculate the price elasticity of demand for certain groups of customers (*ie* as a trial), and this will involve moving away from the theoretical prices. [1]

Comparison with other rates

Before implementing a new set of rates, the insurer will want to check how the new rates compare to the previous ones because customers generally do not like big changes in premium from one year to the next. [1]

The results of such an impact analysis might lead to the new theoretical rates needing to be adjusted. [½]

This could be achieved by capping or moderating any large changes from one year to the next … [½]

… or by identifying the groups of policyholders affected and changing their premium rate relativities, … [½]

… perhaps with a view to achieving the theoretical rates over the course of two or three years rather than immediately. [½]

Again, reductions to certain premiums need to be done in a way that ensures that the total premiums across the whole book are sufficient. [½]

The insurer will also try to check how the new rates compare to those offered by other insurers in the same market at that time. They might then adjust their own theoretical rates accordingly. [1]

Other considerations

The theoretical risk premiums from the GLM should already have been smoothed to an extent, *eg* over policyholder age. However, there might be further smoothing required before the rates are ready for the market. [½]

This further smoothing could be done as underwriter adjustments or to make the rates acceptable to brokers. [½]

For some lower risk policyholders, the theoretical rate could be very low, meaning that the insurer would not receive enough premium to cover their expenses. Therefore a minimum premium could be applied to prevent this from happening. [1]

[Maximum 27]

Part 5 – Questions

The split of the questions is described in the introduction to the Q&A Bank Part 1.

Note that the split between development questions and exam-style questions is somewhat subjective. For example, there have been past exam questions that test knowledge of the Core Reading, and so are similar to what we've included here as development questions. The exam-style questions involve more application and a wider range of ideas and are typically the more challenging questions in the exam.

1 Development questions

Question 5.1

Claim frequency per policy is expected to follow a Poisson distribution with parameter $\lambda = 0.05$. Claim sizes are expected to follow a lognormal distribution with mean $£1,200$ and variance $£^2 2,245,000$. Calculate the number of policies required for full credibility, if you want a 98% probability of being within plus or minus 6% of the true pure premium. [5]

Question 5.2

You are reading an old library book on credibility theory and you come across the following result:

"To obtain a probability P of the observed claims being within $\pm k\%$ of the underlying mean, then the standard for full credibility is:

$$n_N \geq \frac{y^2}{k^2}, \text{ where } \Phi(y) = \frac{1+P}{2}\text{."}$$

Unfortunately, a vandal has ripped out the following page, so you cannot read any further. State the assumptions underlying the above result. [2]

Question 5.3

You require a 98% chance that your estimated claim frequency for your motor book is within $\pm 7.5\%$ of the true underlying value. Calculate the number of claims required for full credibility. (You may assume that claim frequency follows a Poisson distribution.) [2]

Question 5.4

The number of claims from one group of drivers in a year has a Poisson distribution with mean λ, and the number of claims from a second group of drivers has a Poisson distribution with mean 2λ. In one year, there are n_1 claims from group 1 and n_2 claims from group 2.

(i) Derive the maximum likelihood estimator, $\hat{\lambda}$, of λ. [3]

(ii) Suppose that past experience shows that λ has an exponential distribution with mean $\dfrac{1}{\nu}$.

(a) Derive the posterior distribution of λ.

(b) Show that the Bayesian estimate of λ under quadratic loss may be written in the form of a credibility estimate combining the prior mean of λ with the maximum likelihood estimate $\hat{\lambda}$ in (i) and state the credibility factor. [4]

[Total 7]

Question 5.5

Claim amounts follow a distribution with mean 800 and variance 6,400,000. You want to be 93% sure that your estimate of claim severity is correct to within $\pm 5\%$. Find the number of claims required for full credibility. [3]

Question 5.6

(i) Explain why an insurer analyses its movement statistics. [2]

(ii) Explain why a general insurer analyses the profitability of its business by source. [2]

[Total 4]

Question 5.7

The full credibility standard for an insurer is set so that the total number of claims is to be within 2% of the true value with probability P. This full credibility standard is calculated to be 8,000 claims.

(i) Calculate P, assuming that claim frequency follows a Poisson distribution. [2]

The pricing actuary then decides to take into account the claim severity as well as claim frequency, so the standard is altered so that the total cost of claims is to be within 5% of the true value with probability P.

Claim severity has the following distribution:

$$f(x) = c(100 - x) \text{, for } 0 \leq x \leq 100 .$$

(ii) Calculate the expected number of claims necessary to obtain full credibility under the new standard. [7]

An analysis of your claims data indicates that the number of claims has a negative binomial distribution.

(iii) State how your answer to parts (i) and (ii) would differ if the number of claims has a variance that is twice as large as the mean. [2]

(iv) You have observed 1,230 claims. Use the square root rule to determine the credibility weighting you should assign to your data, and state the advantage of using this rule. [2]

[Total 13]

Question 5.8

The standard for full credibility for claim severity is 2,100 claims, and your observed data set consists of 1,096 claims totaling £10,132,000.

To supplement your data, you have analysed the claims arising on similar classes of business and you believe the average cost per claim on these classes is £8,678.

Calculate a credibility-weighted estimate of the average cost per claim for the particular class of business under consideration. State any assumptions you make. [3]

Question 5.9

You are the actuary for a general insurance company. The company's financial director has been concerned about the method of accounting for lapses in the private motor insurance account. The company currently includes premiums for all policies falling due for renewal in the premium income shown in its management accounts at the renewal date, even though it does not know whether or not the policies have been renewed. This is because the policies are sold through brokers who may not confirm whether a policy has renewed or lapsed until some weeks after the renewal date. If a policy lapses then the premium income is reduced at the date the lapse is notified.

You have been asked for your views on two issues:

- At the end of the financial year there will be a number of lapses outstanding which relate to renewals in the year just completed, but which will not be reported until the following year. As the renewal premiums for these policies have been included in the previous year's income, this will overstate the premium income for that year. Should a provision be set up for unnotified lapses?

- When a lapse is notified, even if it is before the end of the year, it will be recorded later than the renewal date. This will cause a distortion in the statement of earned premium. What could be done to correct these distortions?

Outline the contents of your reply. [10]

Question 5.10

Explain why persistency of business may be an important issue to a general insurer. [2]

Question 5.11

Define, giving formulae, four different types of movement rate. [4]

Question 5.12

State the main purposes for which an insurer analyses expenses. [3]

Question 5.13

Property damage claims amongst your company's insured drivers follow a claim size distribution X. The standard for full credibility when estimating the average property damage claim size is $n_X = 9,000$ claims for a given probability P and tolerance k. Liability claims follow a distribution Y, where Y has a standard deviation that is twice as large as that for distribution X, and a mean that is larger than X's by a factor of 3.

Assuming that you require the same P and k for both claim types, calculate the standard for full credibility for severity for liability claims. [4]

Question 5.14

You have just been recruited to work for a brand new general insurance company selling household business.

Suggest reasons why claims experience might be worse than assumed for such a company. [7]

2 Exam-style questions

Question 5.15

Aggregate claims are assumed to follow a Poisson process, with claim severity $X \sim Gamma(\alpha, \beta)$ with $\alpha = 1.75$ and $\beta = 6$. Your standard for full credibility requires a 95% probability of being within $\pm 5\%$ of the true pure premium.

Calculate the credibility to be assigned to 263 claims. State any assumptions you make.
[6]

Question 5.16

You have based your estimate of claim frequency on a data set of 900 claims, and you assume that claim frequency follows a Poisson distribution. The pricing actuary has asked you for a measure of the uncertainty surrounding your work. Calculate the percentage probability that your estimate will be within $\pm 6\%$ of the underlying value.
[3]

Question 5.17

You work for a general insurer who has always assumed that claim numbers follow a Poisson distribution. However, your analysis suggests that the number of claims is more likely to be negative binomial with parameters $k = 15$ and $p = 0.6$.

Calculate the percentage increase in the number of claims required for full credibility.
[4]

Question 5.18

In any year j ($j = 1, ..., n$), there are N_j claims from m_j policies. The number of claims on an individual policy is Poisson with parameter $\theta \sim Gamma\left(\alpha, \delta\right)$.

(i) Describe the assumptions underlying the Bühlmann-Straub model. [4]

(ii) Find the Bühlmann-Straub estimate of the total number of claims in year $n+1$ if the number of policies next year is m_{n+1}. [9]

[Total 13]

Question 5.19

State the three primary processes that should be included within a system for monitoring business, and list the key features of a good system. [7]

Question 5.20

A general insurance company is re-pricing its domestic household business. The company has an estimate of how many policies it will sell during the next year and it knows what the expenses of the "new business processing" department were last year. Discuss the factors that the insurer should take into account when loading the premium for initial expenses. [8]

Question 5.21

You are the pricing actuary for an established but rather old-fashioned general insurer selling only motor business. You have been asked to carry out an investigation into the appropriateness of the current premiums using a straightforward approach without GLMs.

(i) Describe the steps to follow when determining a new risk premium. [9]

The insurer has decided to expand its operations and to start marketing its business abroad.

(ii) Describe how you might determine the risk premium for the new business. [4]
[Total 13]

Question 5.22

The motor portfolio of a medium-sized insurance company (which has been operating for almost a decade) accounts for 80% of its general insurance portfolio. The motor business premium income in Year 9 was substantially lower than in Year 8. The general manager has requested a report on the motor account, highlighting the areas for concern and recommending measures to stop the decline in business.

(i) Outline the points you would expect to include in your report, describing possible reasons for the fall in premium income from Year 8 to Year 9. Suggest corresponding actions that the insurer could take to stop the decline. [13]

You have been given the following information on the private motor account.

		Comprehensive	Non-comp	Total
Premiums (£millions)	Year 9	44.1	15.8	59.9
	Year 8	49.6	19.0	68.6
Vehicle years (000's)	Year 9	402	197	599
	Year 8	447	219	666
Claim frequency	Year 9	14.3%	10.3%	13.0%
	Year 8	14.1%	10.8%	13.0%
Year 8 claim ratio estimated at 31/12 of Year 8		72%	62.3%	69.2%
Year 8 claim ratio estimated at 31/12 of Year 9		67.2%	58.6%	64.8%
Year 9 claim ratio estimated at 31/12 of Year 9		70.1%	64.5%	68.6%

In the above table, premiums and vehicle years are on a written basis and claim frequency and ratios on an earned basis.

(ii) Outline your main observations on the data above. [5]

The general manager is asking for an estimate of the average premium rate increase or decrease to apply to future premiums, as indicated by the data.

(iii) List any points of clarification you need. [3]

(iv) Describe the further details required before you would attempt to calculate the estimate. [9]
[Total 30]

Part 5 – Solutions

1 Solutions to development questions

Solution 5.1

The minimum expected number of claims is $n_S = n_N + n_X$, where $n_N = \dfrac{y^2}{k^2}$ and $n_X = n_N \left(CV_X\right)^2$.

So:

$$\Phi(y) = \frac{1+P}{2} = \frac{1+0.98}{2} = 0.99 \text{, so that } y = 2.3263\,. \qquad [1]$$

Hence,

$$n_N = \frac{y^2}{k^2} = \left(\frac{2.3263}{0.06}\right)^2 = 1503.242\,. \qquad [1]$$

And

$$n_X = n_N \left(CV_X\right)^2 = n_N \left(\frac{\sigma_X}{\mu_X}\right)^2 \text{ where}$$

$$n_X = 1,503.242 \times \frac{2,245,000}{1,200^2} = 2,343.596 \qquad [1]$$

So $n_S = 1,503.304 + 2,343.596 = 3,846.900$ [1]

The expected claim frequency is $\lambda = 0.05$ so that the required number of policies is $n_S/\lambda = 76,938$. [1]

[Total 5]

Solution 5.2

We are trying to estimate claim frequency. [½]

Frequency is given by a Poisson process (so that the variance is equal to the mean). [½]

There are enough expected claims to use the normal approximation to the Poisson process. [½]

We do not need to use a continuity correction when applying the normal approximation. [½]

[Total 2]

Solution 5.3

$$n_N \geq \frac{y^2}{k^2} \text{ where } k = 0.075 \text{ and}$$

$$\Phi(y) = \frac{1+P}{2} = \frac{1+0.98}{2} = 0.99 \text{, so that } y = 2.326.$$ [1]

So:

$$n_N \geq \frac{2.326^2}{0.075^2} = 962 \text{ claims are required for full credibility.}$$ [1]

[Total 2]

Solution 5.4

This question is revision of the material covered in Subject CT6. The Core Reading for Subject ST8 does not go into this much depth. However, since Subject ST8 does cover the Bayesian formula for the credibility factor Z, this question is still useful preparation for the exam.

(i) ***Maximum likelihood estimate***

In our sample we have obtained a value of n_1 from a Poisson distribution with parameter λ, and a value of n_2 from a Poisson distribution with parameter 2λ. By definition, the likelihood is the probability of observing the sample obtained:

$$L(\lambda) = \frac{e^{-\lambda}\lambda^{n_1}}{n_1!} \times \frac{e^{-2\lambda}(2\lambda)^{n_2}}{n_2!} = Ce^{-3\lambda}\lambda^{n_1+n_2} \text{, where } C \text{ is a constant.}$$ [1]

Taking logs gives:

$$\ln L(\lambda) = \ln C - 3\lambda + \ln \lambda^{n_1+n_2} = \ln C - 3\lambda + (n_1+n_2)\ln \lambda \qquad [1]$$

Differentiating and setting the derivative equal to zero gives:

$$\frac{d}{d\lambda}\ln L(\lambda) = -3 + \frac{n_1+n_2}{\hat{\lambda}} = 0 \quad \Rightarrow \quad \hat{\lambda} = \frac{n_1+n_2}{3} \qquad [1]$$

Checking whether we have a maximum

$$\frac{d^2 \ln L}{d\lambda^2} = -\frac{(n_1+n_2)}{\lambda^2} < 0 \quad \Rightarrow \quad \max \qquad [½]$$

[Maximum 3]

(ii)(a) ***Posterior distribution***

Since the prior distribution for λ is exponential with mean $\frac{1}{\nu}$, the prior pdf is $\nu e^{-\nu\lambda}$. So we have:

$$post_{pdf} \propto e^{-\nu\lambda} \times e^{-3\lambda}\lambda^{n_1+n_2} = \lambda^{n_1+n_2}e^{-(3+\nu)\lambda}. \qquad [½]$$

Comparing this to the standard distributions given in the *Tables*, we can see that this is the PDF of a gamma distribution with parameters $\alpha = n_1+n_2+1$ and $\delta = 3+\nu$. Hence the posterior distribution is $gamma(n_1+n_2+1, 3+\nu)$. [1]

(ii)(b) ***Credibility estimate***

The mean of the posterior distribution is:

$$\frac{n_1+n_2+1}{3+\nu} \qquad [1]$$

This can be written in the form:

$$\frac{3}{3+\nu}\times\frac{n_1+n_2}{3} + \frac{\nu}{3+\nu}\times\frac{1}{\nu} = \frac{3}{3+\nu}\times\frac{n_1+n_2}{3} + \left(1-\frac{3}{3+\nu}\right)\times\frac{1}{\nu} \qquad [1]$$

This is in the form of a credibility estimate with credibility factor $Z = \dfrac{3}{3+\nu}$. [½]

[Total 4]

Solution 5.5

$$\Phi(y) = \frac{1+P}{2} = \frac{1.93}{2} = 0.965 .$$

So $y = 1.81195$. [1]

The standard for full credibility for severity is $n_X = n_N CV_X^2$ where $n_N = \dfrac{y^2}{k^2}$ and where $CV_X = \dfrac{\sigma_X}{\mu_X}$, giving:

$$\begin{aligned} n_X &= \frac{y^2}{k^2} \times \left(\frac{\sigma_X}{\mu_X}\right)^2 \\ &= \frac{1.81195^2}{0.05^2} \times \frac{6,400,000}{800^2} \\ &= 13,133 \end{aligned}$$

[2]

[Total 3]

Solution 5.6

(i) ***Analysis of movement statistics***

As an early warning system to spot trends or adverse changes in business written. [½]

In particular, this will allow the insurer to:

- Measure the growth or contraction of parts of the portfolio. This may indicate that the premium structure is out of line with the rest of the market. [½]
- Get early notice of the growth or contraction of the whole portfolio. This may indicate that the insurer's level of premium rates is out of line with the rest of the market. [½]
- Assess the effects of a new set of rates or a marketing campaign on the business … [½]

- … and hence the sensitivity of the portfolio to market influences. [½]

[Maximum 2]

(ii) ***Analysis of profitability of business by source***

This will help the insurer in its financial planning by:

- showing where the better quality, longer-lasting business comes from, ... [½]
- … helping the insurer to avoid the less profitable (or unprofitable) sources of business [½]
- devising incentive schemes or commission terms to retain or attract the more profitable business and reward better providers [1]
- helping it to negotiate reductions in commission payments on less profitable business [½]
- enabling it to concentrate on selling business through sources with lower commission if the quality of such business is not markedly different. [½]

[Maximum 2]

Solution 5.7

(i) ***Probability P that the total number of claims falls within 2% of the true value***

We have

$$n_N = 8,000 = \frac{y^2}{0.02^2}\text{, so that } y = 1.7889\,. \qquad [1]$$

Hence

$$\frac{1+P}{2} = \Phi(y) = 0.96318\text{, and } P = 0.92636\,. \qquad [1]$$

[Total 2]

(ii) ***Expected number of claims with Poisson claim frequency***

Under the new credibility standard, y (as a function of constant P) will remain unchanged, so that:

$$n_N = \frac{y^2}{0.05^2} = \frac{1.7889^2}{0.05^2} = 1{,}280\,. \qquad [1]$$

We now need to find $n_X = n_N\left(CV_X\right)^2 = n_N\left(\frac{\sigma_X}{\mu_X}\right)^2$.

$$\int_0^{100} c\left(100-x\right)dx = 1 \text{ gives } c = 0.0002\,. \qquad [1]$$

$$\begin{aligned} E\left(X\right) &= \int_0^{100} 0.0002x\left(100-x\right)dx \\ &= \left[0.01x^2 - \frac{0.0002}{3}x^3\right]_0^{100} \\ &= 100 - 66.67 \\ &= 33.33 \end{aligned}$$

[1]

and:

$$\begin{aligned} E\left(X^2\right) &= \int_0^{100} 0.0002x^2\left(100-x\right)dx \\ &= \left[\frac{0.02}{3}x^3 - 0.00005x^4\right]_0^{100} \\ &= 6{,}666.67 - 5{,}000 \\ &= 1{,}666.67 \end{aligned}$$

[1]

Hence

$$\text{var}\left(X\right) = 1{,}666.67 - 33.33^2 = 555.56\,. \qquad [1]$$

$$n_X = 1,280 \times \frac{555.55}{33.33^2} = 640\,.$$ [1]

So

$$n_S = 1,280 + 640 = 1,920\,.$$ [1]

[Total 7]

(iii) ***Expected number of claims with negative binomial claim frequency and variance twice as large as mean***

$n_N = 8,000 = \frac{y^2}{k^2}\left(\frac{\sigma_N^2}{\mu_N}\right) = \frac{y^2}{0.02^2} \times 2$, so that $y = 1.2649$ and $P = 0.79410$. [1]

Under the new standard y will again remain unchanged, but we now need:

$$n_S = \frac{y^2}{k^2}\left(\frac{\sigma_N^2}{\mu_N} + \frac{\sigma_X^2}{\mu_X^2}\right) = \frac{1.2649^2}{0.05^2} \times \left(2 + \frac{555.56}{33.33^2}\right) = 1,600.$$ [1]

[Total 2]

(iv) ***Square root rule for partial credibility***

Using the observed number of claims as an estimate for the expected number of claims, we have:

$$Z = \sqrt{\frac{n}{n_F}} = \sqrt{\frac{1,230}{1,600}} = 0.8768\,.$$ [1]

The advantage of the square root rule is that we can choose our partial credibility factor Z such that the variance of the credibility-weighted estimate is the same as it would be if we had fully credible data. [1]

[Total 2]

Solution 5.8

We assume that the observed number of claims is an appropriate estimate of the expected number of claims on the portfolio. [½]

Using the square root rule, the partial credibility factor Z is:

$$Z = \sqrt{\frac{n}{n_F}} = \sqrt{\frac{1{,}096}{2{,}100}} = 0.7224\,. \qquad [1]$$

The average claim size of the observed data is $10{,}132{,}000/1{,}096 = 9{,}245$. [½]

So the credibility-weighted average claim size is:

$$\left(0.7224 \times 9{,}245\right) + \left(1 - 0.7224\right) \times 8{,}678 = £9{,}087\,. \qquad [1]$$

[Total 3]

Solution 5.9

The stated effects are likely to be present in the accounts but are likely to be smaller than expected because of a factor that acts in the opposite direction to lapses:

- Reporting of new business is also delayed … [½]
- … resulting in an understatement of written premium and earned premium. [½]

If delays and volumes of lapses and new business are similar, they will tend to cancel each other out. [1]

If there is a tendency for either lapses or new business to predominate then a similar effect will occur at the end of each year, which will tend to cancel out … [1]

… and unless conditions are changing rapidly, the distortions will be small. [½]

If the business is growing then the value of unreported new business is likely to be larger than the value of unreported lapses and therefore the profitability of the business may actually be understated. [1]

Will need to consider if the company is prepared to allow for this (*eg* have a negative provision) in its accounts). [½]

In order to measure the effect of the delays:

- analyse new business and lapses reported in any one month, into months of inception or renewal [½]
- repeat on a monthly basis for, say, two years, in order to get an idea of any seasonality [1]
- may also be sensitive to the number of working days in that month, especially December [½]
- possible that larger policies are reported more quickly or more slowly. [1]

There should be a consistency in the delay patterns, which can be used to project unreported new business and lapses. [½]

It should then be a simple exercise to project numbers and average premiums to calculate the total value of unreported new business and lapses. [1]

These estimates could then be used in place of the reported figures to assess written premium and earned premium. [½]

The accounting principle of consistency implies that accounting practices ought not to change unless there are clear improvements to be made. [½]

[Maximum 10]

Solution 5.10

Persistency of business may be an important issue to a general insurer because:

- with falling business volumes, it becomes more expensive (per policy) to spread fixed costs over each policy [½]
- a large proportion of the costs may be fixed, so this could have a material impact on profits [½]
- it is usually more expensive to acquire / regain new business than it is to process renewals … [½]

  … but the same premium is usually charged on both new and renewed business [½]
- it is a sign of customer satisfaction, and so low persistency may imply a poor level of service and/or that premium rates are not competitive. [½]

[Maximum 2]

Solution 5.11

Lapse rate is the rate at which potential renewals do not renew their insurance,
ie Number of lapses / Number of invitations to renew [1]

New business rate is the rate at which new policies start, as a proportion of renewals invited,
ie Number of new policies / Number of invitations to renew [1]

Cancellation rate is the rate at which policies cancel prior to the end of the period of cover,
ie Number of cancellations / Average number of policies in force
or Number of cancellations / Number of invitations to renew [1]

Endorsement rate is the rate at which policies are altered,
ie Number of endorsements / Average number of policies in force [1]

[Total 4]

Solution 5.12

An insurer analyses expenses in order to allocate its expense costs correctly between the different classes and rating groups in the portfolio. [1]

This enables the insurer to:

- measure the past performance of each class [½]
- determine the expense loadings for premium rating. [½]

An expense analysis is an important part of an insurer's financial planning process and helps to identify inefficiencies or areas where expenses might need to be controlled. [1]

Expenses need to be analysed so that the information can be included within the insurer's statutory returns. [½]

[Maximum 3]

Solution 5.13

We require $n_Y = n_{N_Y}\left(CV_Y\right)^2$.

$n_{N_Y} = n_{N_X}$ since both depend only on P and k, which are the same for both sections of the portfolio. [1]

$$CV_Y = \frac{\sigma_Y}{\mu_Y} = \frac{2\sigma_X}{3\mu_X} = \frac{2}{3}CV_X \,. \qquad [1]$$

Therefore:

$$\begin{aligned} n_Y &= n_{N_Y}\left(CV_Y\right)^2 \\ &= n_{N_X}\left(\frac{2}{3}CV_X\right)^2 \\ &= \frac{4}{9}n_{N_X}\left(CV_X\right)^2 \\ &= \frac{4}{9}n_X \\ &= 4{,}000. \end{aligned}$$

[2]
[Total 4]

(Note that since the ratio of the standard deviations ÷ the ratio of the means is $\frac{2}{3} < 1$*, the standard for credibility is actually lower for liability claims than it is for property damage claims.)*

Solution 5.14

- lack of reliable data upon which to base your premium rates / determine your reserves [½]
- misinterpretation of the data that you have [½]
- experience is volatile from year to year [½]
- anti-selection, *eg* if you use number of bedrooms rather than sum insured as an exposure measure, or don't rate by location (and other insurers do) [½]
- under-insurance leading to higher than expected claims per unit of exposure [½]
- worsening trend in crime [½]
- policyholders submitting fraudulent or inflated claims *eg* during recession [½]
- an increase in the propensity to submit claims [½]
- unusual weather patterns leading to increased subsidence claims [½]
- accumulations of risk, especially if regionally based [½]
- a storm or flood leading to a larger number of property damage claims than expected [½]
- an unexpected catastrophe, such as an earthquake, or terrorist attack [½]
- a demand surge (*eg* on builders, following a catastrophe) that pushes up the cost of claims [½]
- higher than expected claims inflation [½]
- a court award leading to high liability claims [½]
- poor control of claim payments [½]
- a change in regulation, *eg* a requirement to provide unlimited cover on certain claims [½]
- failure of a reinsurer, so that we make lower than expected recoveries [½]

[Maximum 7]

2 Solutions to exam-style questions

Solution 5.15

Claim frequency and severity are assumed to be independent. [½]

$$\Phi(y) = \frac{1.95}{2} = 0.975 \text{, giving } y = 1.960 .$$ [1]

$$n_N = \frac{1.960^2}{0.05^2} = 1,536.64 .$$ [1]

For $X \sim Gamma(1.75, 6)$ we have $\mu_X = \frac{1.75}{6}$ and $\sigma_X^2 = \frac{1.75}{36}$. [1]

$$\begin{aligned} n_S &= n_N + n_X = n_N\left(1 + CV_X^2\right) \\ &= 1536.64\left(1 + \frac{1.75/36}{(1.75/6)^2}\right) \\ &= 2,414.72 \end{aligned}$$

[1]

Using the square root rule, and assuming that the observed number of claims is an adequate approximation of the expected number of claims: [½]

$$Z = \sqrt{\frac{263}{2,414.72}} = 0.330 .$$ [1]

[Total 6]

Solution 5.16

$$n_N \geq \frac{y^2}{k^2} = 900 \text{ where } k = 0.06 .$$ [1]

So:

$$n_N \geq \frac{y^2}{k^2} = 900$$

$$y = \sqrt{900 \times 0.06^2} = 1.8$$ [1]

And:

$$\Phi(1.8) = 0.96407 = \frac{1+P}{2}$$ [1]

$$P = 0.96407 \times 2 - 1 = 0.92814$$

So you are 92.8% sure that your estimate lies within 6% of the true underlying claim frequency.

[Total 3]

Solution 5.17

For Poisson claim frequency, the standard for full credibility is: $n_P \geq \frac{y^2}{k^2}$. In the general case however, the standard for full credibility is $n_N = \frac{y^2}{k^2}\left(\frac{\sigma_N^2}{\mu_N}\right)$.

So the percentage increase in the number of claims required is:

$$\frac{n_N}{n_P} - 1 = \frac{\frac{y^2}{k^2}\left(\frac{\sigma_N^2}{\mu_N}\right)}{y^2 / k^2} - 1 = \frac{\sigma_N^2}{\mu_N} - 1$$ [1]

For a negative binomial distribution, we have:

$$\sigma_N^2 = \frac{kq}{p^2} = \frac{15 \times 0.4}{0.6^2} \text{ and } \mu_N = \frac{kq}{p} = \frac{15 \times 0.4}{0.6},$$ [2]

so that:

$$\frac{\sigma_N^2}{\mu_N} - 1 = \frac{15 \times 0.4}{0.6^2} \times \frac{0.6}{15 \times 0.4} = \frac{1}{0.6} - 1$$ [½]

So the percentage increase in the number of claims required for full credibility is 66.67%. [½]

(Note that it makes sense in this case to use the Type 2 format for the negative binomial distribution, since this gives a minimum number of claims = 0.)

[Total 4]

Solution 5.18

(i) ***Assumptions***

For each risk i, there exists a parameter θ_i, whose value is the same for each year j but is unknown (*ie* $\theta_{ij} = \theta_i$ for all j)… [1]

… and there exist functions μ and σ^2 such that:

$$\mu(\theta_i) = E\left(X_{ij} | \theta_i\right) \quad [½]$$

$$\text{var}\left(X_{ij} | \theta_i\right) = \frac{\sigma^2(\theta_i)}{m_{ij}} \quad [½]$$

where m_{ij} denotes the volume measure for risk i in year j. [½]

Other assumptions are: [½ mark for each]

- The ith risk is described by the pair, $(\theta_{i,}(X_{ik})_{k\geq1})$ where $(X_{ik})_{k\geq1}$ is the sequence of claims ratios observed for risk i in years k.
- The pairs $(\theta_{i,}(X_{ik})_{k\geq1})$ are mutually independent.
- The θ_i are independent and identically distributed.
- Conditionally on θ_i, the X_{ik}'s are independent (but not necessarily identically distributed). [Maximum 4]

(ii) ***Bühlmann-Straub estimate***

We only have one risk here so we can drop the double subscripts and just use the subscript j in place of $1j$. We can also write θ in place of θ_1. Hence $X_j = N_j / m_j$ is the average number of claims per policy, in year j.

If the number of claims on an individual policy is $Poisson(\theta)$ then the total number of claims from the portfolio of m_j policies is $N_j \sim Poisson\left(m_j\theta\right)$. [½]

$$\mu(\theta) = E\left(X_j | \theta\right) = E\left[\left(N_j / m_j\right) | \theta\right] = \frac{1}{m_j} E\left[N_j | \theta\right] = \frac{1}{m_j} \times m_j\theta = \theta \quad [1]$$

$$\sigma^2(\theta) = m_j \times Var\left(X_j \middle| \theta\right) = m_j \times Var\left(\frac{N_j}{m_j} \middle| \theta\right) = \frac{1}{m_j} Var\left(N_j \middle| \theta\right) = \frac{1}{m_j} \times m_j \theta = \theta$$

[1]

So :

$$\beta = E\left(\mu(\theta)\right) = E(\theta) = \frac{\alpha}{\delta}$$ [1]

$$\phi = E\left[\sigma^2(\theta)\right] = E(\theta) = \frac{\alpha}{\delta}$$ [1]

$$\lambda = \text{var}\left[\mu(\theta)\right] = \text{var}(\theta) = \frac{\alpha}{\delta^2}.$$ [1]

So

$$Z = \frac{m_j}{m_j + \phi/\lambda} = \frac{m_j}{m_j + \dfrac{\alpha/\delta}{\alpha/\delta^2}} = \frac{m_j}{m_j + \delta}$$ [1]

So the credibility estimate for the number of claims on an individual policy is:

$$\begin{aligned} C^{BE} &= Z\bar{X} + (1-Z)\beta \\ &= \frac{m}{m+\delta}\bar{X} + \frac{\delta}{m+\delta}\left(\frac{\alpha}{\delta}\right) \\ &= \frac{m}{m+\delta}\bar{X} + \frac{\alpha}{m+\delta} \end{aligned}$$

[1]

where $m = \sum_{j=1}^{n} m_j$ the total number of policy-years to date, and $\bar{X} = \dfrac{\sum_{j=1}^{n} N_j}{\sum_{j=1}^{n} m_j}$, the overall average loss per policy over the n years. [1]

Therefore, the Bühlmann-Straub estimate of the total number of claims in year $n+1$ is $m_{n+1}C^{BE} = m_{n+1}\left(\dfrac{m}{m+\delta}\bar{X} + \dfrac{\alpha}{m+\delta}\right)$. [½]

[Total 9]

Solution 5.19

The three processes are:

- a data capture process
- calculations and/or manipulations on the data
- a results reporting process. [½ each]

The ideal features of such a system are:

- the delay between data cut-off date and results production should be minimal
- data used should be reliable and validated
- data should be easy to collect
- inputs should be consistent with other data sources
- calculations should take into account all key drivers
- calculations should not be overly complex
- output should be consistent over time
- output should be concise and tailored
- outputs should be consistent with other analyses
- results should be validated
- results should be clear and easy for users to interpret
- the system should be documented, extendable and low maintenance. [½ each]

[Total 7]

Solution 5.20

Consider the appropriate amount to load

Consider all initial expenses, such as marketing, underwriting and premium collection. Take account of all other relevant expenses not covered by this department. [1]

Consider whether the new business processing department does any work for other classes. If so, only include the expenses that relate to domestic household business. [½]

Consider separate expense allowances for building and contents cover since expenses may differ, *eg* different underwriting costs. [1]

Consider the impact of changes in volumes on the per policy expenses. [½]

Consider the allowance for overheads. Have they been allowed for? Should they be allowed for? If so, consider how they should be allowed for. [1]

Allow for expense inflation to the mid-point of the period for which the rates will be in force. [½]

How to load into premium rates

Each element should be matched with the corresponding element in the formula. [½]

Consider the cross-subsidy between renewals and new policies. Do you want to have the same loading for each? [½]

You could charge a fixed expense amount for each policy. However this charges the same amount to small and large policies. Policyholders with small premium sizes may not take out the policy, resulting in lower sales of small policies. [1]

It may be better to charge the expense as a percentage of premium, especially since you might argue that larger policies do constitute more work (to some extent anyway). [1]

If you load as a percentage of premium, you must make sure that the smallest policies cover their marginal costs. You could allow for this by using a small fixed charge or a minimum premium size. [1]

In practice, you may use a mixture of these approaches, *ie* partly a fixed amount and partly a percentage of premium. [½]

You can allow for commission explicitly as a percentage of premium. [½]
[Maximum 8]

Solution 5.21

(i) ***Proposing a new risk premium based on past data***

You are reviewing the premium rates for an established motor book, so there should be sufficient data available for a detailed analysis. [½]

Past policy and claims data will be needed, but may need to be adjusted for past and future changes in policy conditions. [½]

Data should be grouped into homogeneous groups and in particular by claim type. [½]

The appropriate risk premium rates will be based on the expected cost of claims for the policy. This should be estimated using recent claims experience from the existing portfolio. [½]

The most recent claims data will not yet be complete, for example some claims will not yet be settled or even reported… [½]

… so we should project claims to their ultimate level using techniques such as the chain ladder method. [½]

The analyses should examine claim frequency and severity separately. This should help identify trends in the data. [½]

The estimated ultimate claims cost should be adjusted for any abnormal experience. For example, if recent wide-scale flooding resulted in abnormally poor claim experience, then these claims should be truncated and an average annual amount should be included in the estimate of future risk-premiums. [1]

However, we should be careful when truncating abnormally severe claims since it is difficult to judge what level of claim is "abnormal" and what is "normal". This is due to the potential for process uncertainty, model uncertainty and parameter uncertainty. [1]

We should compare the estimated ultimate claims to the risk premiums for the corresponding policies. The reasons for any differences between the two should be investigated so that our assumptions about future claims experience can be updated. [1]

The estimated ultimate claims should be projected forward to the mid-point of the period over which the new rates will apply and the corresponding claims will be settled. [½]

This will involve allowing for future inflation, and projecting forward any trends in the claims. For example, allowance should be made for expectations of future seasonal fluctuations or any expected future increases in claim severity *eg* due to court award inflation relating to liability. [1]

A decision should be made as to whether the above analysis should include direct expenses. If not, an appropriate allowance for this will need to be made when loading the premium for expenses. [½]

The key features of the results should be documented and explained, including key assumptions and areas of uncertainty. [½]

The results should be discussed with management and underwriters. [½]
[Maximum 9]

(ii) ***Proposing a risk premium for a new business***

The claim experience under the new portfolio is likely to be very different to the insurer's existing portfolio. [½]

For example, risks are likely to be subject to very different:

- legal, political and social factors *eg* court award inflation and policyholders' propensity to claim
- environmental factors
- drivers of claim inflation *eg* medical expenses
- economic conditions. [¼ each, total 1]

The insurer's current data is therefore unlikely to be relevant to the new target market. [½]

It may be possible for the insurer to use its existing data, if it believes it can adjust this for the expected differences in claim frequency / severity arising from the factors above. [½]

However, it is likely that its initial rating structure will follow market office rates quite closely. Hence the assumed risk premium will implicitly be governed by these less the value of required loadings for profit, expenses *etc*. [1]

The insurer should collect external data to help with this, for example from:

- reinsurers and brokers
- from other insurers, *eg* by arranging a reciprocal quota share agreement
- market sources. [1]

The insurer will be able to adjust its rates at a later date, when it has collected a sufficient amount of data to enable a credible analysis. [½]

[Maximum 4]

Solution 5.22

(i) ***Points to include in the report***

Investigate where the reduction has occurred. Analyse premium income by source and category of business to see if the fall is widespread or focused in particular areas. [1]

Consider and investigate possible reasons for the fall. These could include:

- market competitiveness overall: need to consider figures against motor insurance capacity in the market as a whole: [½]
  - perhaps rates are at the soft stage of the insurance cycle [½]
  - competition from other, non-traditional, providers [½]
  - competition from new sales methods, *eg* internet [½]
- competitors with more sophisticated rating techniques may have selected against the insurer, leaving it with the poorer risks [½]
- recent increase in insurer's premium rates leading to loss of new business and reduced persistency [½]
- insurer may have just lost one or more big clients who previously introduced high volumes of business, *eg*: [½]
  - loss of preferred insurer status for national broker network [½]
  - end of arrangement with chain of garage outlets [½]
- company may have inefficient business processing systems and thus higher expense loadings within the premium basis than competitors [½]
- changes to own or other insurers' NCD scales may have resulted in reduction in customer loyalty [½]
- if insurer sells mainly through brokers:
  - predominance of direct writers into motor market with generally lower premiums reduces insurer's competitiveness [½]
  - reduction of commission rates to brokers could have resulted in reduced sales. [½]

Other areas for concern include:

- higher per policy fixed expenses due to lower volumes, leading to lower profits or possible losses [½]
- new business costs more to attract than the cost of a renewal, especially for telephone sales (for broker sales it is much less of an issue). Hence the costs of regaining previous market share increase the more current business declines. [1]

Possible remedies to stop decline are to:

- develop marketing plan to stop decrease or regain market share [½]
  - increase commission rates to key brokers (increases premium rates) [½]
  - redesign NCD scale to be more attractive to policyholders [½]
  - introduce new sales channel, *eg* direct sales if currently dependent on brokers [½]
  - advertising campaign to improve public awareness of insurer [½]
- reduce premium rates, but this will reduce profitability and return on shareholders capital. [½]

The report could also include a section on trends. This could cover:

- the company's own experience [½]
- wider market experience and what other insurers are doing. [½]

[Maximum 13]

(ii) ***Main observations***

- reduction in premiums, more for non-comp (17%) than for comp (11%) [½]
- reduction in vehicles covered of 10% is evenly spread across comp/non-comp [½]
- average premium falls marginally for comp (1%) but by 8% for non-comp [½]
- claim frequency up slightly for comp, but down 5% for non-comp [½]
- reduction in frequency for non-comp mitigates (to some extent anyway) the fall in average premium [½]
  - can't tell whether reduction is due to premium cuts, reflecting improving experience generally, or changing mix of business [1]
- Year 8 claim ratio improves from 31/12 of Year 8 to 31/12 of Year 9 suggesting possible over reserving at the end of Year 8. [½]
- The Year 9 claim ratio as at 31/12 of Year 9 could be similarly overestimated.[½]
- Assuming the same degree of prudence in reserving, the Year 9 claims ratio suggests an improvement for comp business (72% to 70.1%) but a deterioration in non-comp business (62.3% to 64.5%). [1]

[Maximum 5]

(iii) ***Points of clarification***

- What is the aim of the revised premium rates? [½]
- Is there a deliberate decision not to defend market share? [½]
- Has company made efforts or been successful at increasing the other non-motor business which makes up 20% of its portfolio? [½]
- How are competitors likely to react to our premium rate changes? Will they change their premium rates? If so, by how much? Critically, how will this affect our likely future volumes? [1]
- What are the driving forces behind the trends in experience? [½]

[Total 3]

(iv) ***Further details***

- more detailed analysis of motor experience split by rating groups [1]
- details of the rating structure compared to those of our competitors [½]
- how large claims have been allowed for [½]
- any unusual events causing accumulations, *eg* bad weather or a pileup [½]
- how we have allowed for reinsurance protection, cost and expected recoveries [1]
- consider quality of management, sales staff and underwriters and whether they have changed during the year [1½]
- how fixed expenses are to be split between the motor business and the rest of the portfolio [1]
- expected (or desired) future levels of business – needed for expense allocation [½]
- the appropriate level of loadings – *eg* for fixed and variable expenses, commission, investment income, contingencies [1]
- level of profit (or return on capital) that is required in the future [½]
- future level of inflation to be allowed for [½]
- period during which new premium rates will be in force [½]

[Total 9]

Part 6 – Questions

The split of the questions is described in the introduction to the Q&A Bank Part 1.

Note that the split between development questions and exam-style questions is somewhat subjective. For example, there have been past exam questions that test knowledge of the Core Reading, and so are similar to what we've included here as development questions. The exam-style questions involve more application and a wider range of ideas and are typically the more challenging questions in the exam.

1 Development questions

Question 6.1

The loss ratio experience of portfolios, A, B, C and D, over the last 5 years has been:

	A	*B*	C	*D*
1	77%	67%	81%	50%
2	23%	53%	76%	76%
3	63%	107%	75%	104%
4	87%	43%	69%	30%
5	75%	55%	49%	40%

(i) Calculate the mean and standard deviation of the observed loss ratios for each portfolio. On the evidence of these statistics, state, with reasons, which portfolios should have the largest and smallest risk premiums for each of the following stop loss covers in turn:

(a) all losses over 70%

(b) all losses over 100%

(c) losses in the layer 25% excess 90% [22]

(ii) Calculate the burning cost pure premium for each of the portfolios and each of the stop loss covers. Comment on your answers. [9]

(iii) Comment on the suitability of the above methods, and discuss how, in practice, you would rate a stop loss treaty. [15]

[Total 46]

2 Exam-style questions

Question 6.2

List the modules within a catastrophe model, and for each, state the primary data source used as its input. [5]

Question 6.3

A reinsurance company transacts only individual excess of loss reinsurance. The reinsurer calculates the reinsurance premium to be paid for each company in the following year as:

> Total claims paid by the reinsurer over the last five years for that individual company

divided by

> Total premiums written by the insurer over the last five years

multiplied by

> The expected level of premiums written by the insurer in the following year.

Discuss the advantages and disadvantages to the reinsurer of this method of calculating the premium. [8]

Question 6.4

You are the actuary of a large general insurance company that underwrites a broad and stable portfolio of risks. The company is in the process of reviewing its reinsurance strategy for the commercial property account. Historically, the company protected the commercial property portfolio using a ten-line surplus reinsurance treaty. The following underwriting statistics and large claim data for the commercial property account are available as at 31 December of Year 5.

Underwriting Statistics - Amounts in £000s

Underwriting year	Gross written premium	Net written premium	Gross incurred claims	Gross paid claims	Net incurred claims	Net paid claims
1	120,456	96,726	81,308	79,237	62,872	61,117
2	124,218	95,648	85,338	81,592	58,536	55,419
3	130,987	102,956	93,263	83,745	66,201	60,781
4	131,421	98,566	97,514	84,006	61,406	52,784
5	128,976	99,956	98,667	57,981	63,472	37,925

Individual claims incurred above £400,000 were as follows:

Under-writing year	Year of event	Gross paid £	Gross outstanding £	Date of reporting of claim	Date of payment if made	No of lines on surplus
1	1	674,321	0	4/5/Yr 1	30/11/Yr 2	1
1	1	1,452,908	0	30/7/Yr 1	22/11/Yr 1	5
1	1	1,265,943	0	7/11/Yr 1	4/6 Yr 2	3
3	3	812,364	0	1/4/Yr 3	8/12/Yr 3	1
3	3	0	984,213	6/7/Yr 3		4
3	3	6,374,306	0	10/12/Yr 3	6/5/Yr 5	9
4	4	0	542,164	21/5/Yr 4		3
4	4	0	794,002	5/11/Yr 4		4
4	4	0	803,000	10/4/Yr 5		9
5	5	0	850,000	17/8/Yr 5		1

The company is considering changing its reinsurance programme to a 20% quota share plus a risk excess of loss insurance, on the net account, for £4.5 million excess of £0.5 million with two free reinstatements. The premium for the risk excess of loss would be 2% of the gross premium income for the underwriting year.

(i) Calculate the pro-forma underwriting statistics as at 31st December of Year 5 for each of the underwriting years 1 – 5 assuming that the company had used the new reinsurance arrangements. [9]

(ii) Assuming an inflation rate of 5% per annum, and that the case estimates of the company include a 10% surplus, estimate what you would expect to be the burning cost for the excess of loss contract for underwriting year 6 based upon the company's historical data. State any assumptions that you make. [11]

(iii) Your general manager has asked for your advice in assessing the proposed changes to the reinsurance programme. Outline the matters that you would address in your analysis. (You should calculate any figures you would propose to include in your analysis.) [22]

[Total 42]

Part 6 – Solutions

1 Solutions to development questions

Solution 6.1

(i) ***Risk premiums***

The means and (sample) standard deviations are as follows:

A:	mean 65%	s.d. 25%
B:	mean 65%	s.d. 25%
C:	mean 70%	s.d. 12.5%
D:	mean 60%	s.d. 30%.

[½ for each figure; total 4]

Looking at these statistics, A and B are equally risky, so should have the same stop loss risk premium. [½]

C has a higher mean loss ratio, but a lower variability. D has the reverse. [½]

When deciding on the risk premiums for stop loss cover, we need to consider the excess pure premium and *not* just the probability that the limit will be breached. [1]

(a) *XS 70%*

C will have the highest probability of making a recovery under this contract. (For a symmetrical distribution the chance of exceeding the mean is 50%. For one skew to the right, as is more likely here, the probability tends to be somewhat under 50%.) [1]

However, for C to exceed 95%, for example, the result would have to be more than two standard deviations above the mean. The chance of this for the normal distribution is only about 2½%; for other distributions the probability may be of a similar order (possibly a little larger). [1½]

A or B make a recovery when the result is 0.2 standard deviations above the mean. This is a little less likely than C making a recovery. However, 90% is only 1 s.d. above the mean, and 2 s.d. above is 115%. This shows that the chance of a large recovery is greater for A or B than for C. The premium for A or B seems likely to be bigger than that for C (although this "evidence" is not totally conclusive). [2]

D makes a recovery when the result is 0.33 standard deviations above the mean. This is a little less likely than A or B making a recovery. However, 90% is only 1 s.d. above the mean, and 2 s.d. above is 120%. This shows that the chance of a large recovery is somewhat greater for D than for A or B. [1½]

It seems clear that C will have the smallest premium, because it is much less variable.[1]

The overall result between D and A/B seems to us to be a close call and the result of the calculations may well depend on the assumed distribution of the loss ratio. [1]

(b) *XS 100%*

D will have the greatest chance of any recovery and also the greatest chance of a large recovery. [1]

C will have the least chance of any recovery and also the least chance of a large recovery. [1]

Hence D should have the largest premium, A and B the next and C has the smallest. [1]

(c) *25% XS 90%*

This time we have, in terms of standard deviations above the mean, the layer:

A or B :	+1.0	to +2.0	
C :	+1.6	to +3.6	
D :	+1.0	to +1.8.	[½ for each figure, total 3]

Whenever the upper limit of this layer is breached (likeliest for D) a full recovery of 25% is made. So it is clear that the premium order must be the same as in (b), *ie* D should have the largest premium, A and B the next and C has the smallest. [2]

Using a statistical package we get the following illustrative pure premiums by fitting distributions of the stated form with the same moments as the empirical distributions:

(a)	*logNormal*	*A,B 7.64%*	*C 4.94%*	*D 7.76%*
	Gamma	*A,B 7.80%*	*C 4.97%*	*D 8.00%*
(b)	*logNormal*	*A,B 1.80%*	*C 0.12%*	*D 2.56%*
	Gamma	*A,B 1.55%*	*C 0.08%*	*D 2.25% (tail is thinner)*
(c)	*logNormal*	*A,B 2.09%*	*C 0.48%*	*D 2.21%*
	Gamma	*A,B 2.14%*	*C 0.42%*	*D 2.37%*

[Total 22]

(ii) ***Burning cost***

The burning costs pure premiums are just the average past recoveries, if the cover had been in place throughout. These are as follows:

(a)	A: 5.8%	B: 7.4%	C: 4.4%	D: 8.0%	
(b)	A: 0.0%	B: 1.4%	C: 0.0%	D: 0.8%	
(c)	A: 0.0%	B: 3.4%	C: 0.0%	D: 2.8%	[½ each]

These are dramatically different from the results in part (i). Now:

- B always has a much larger premium than A.
- D is not the largest for (b) or (c), but B is.
- A and C each have no premium at all for (b) and (c). [1 each]

[Total 9]

(iii) ***Rating a stop loss treaty***

Clearly neither (i) nor (ii), by itself, provides a suitable way to rate a stop loss treaty.

In particular, (i) penalises A relative to B, although past experience suggests that when B makes a loss at all it makes a relatively big loss, which is exactly what unlimited or high layer stop loss cover is protecting against. [1]

In (ii) it seems inappropriate to have a zero pure premium for cover (b) or (c) to A or C, despite there (presumably) being some chance that a recovery would be made in the future. [1]

Now what would we do in practice, given full information?

The ideal would be to decide, somehow, what the prospective distribution of the loss ratio for any company is. We could then calculate the pure premium for any layer of cover directly from that distribution. Whether it is an analytical or empirical one, we could do this given a suitable computer package. [1]

A view of the loss ratio can be derived from historical experience, suitably adjusted, as follows:

- Gather triangulated premium, paid claims and incurred claims data. [1]
- Obtain rate change information for the period covered. [½]
- Estimate the premium income to be written and rate changes for the period of cover being priced. [1]
- The rate changes will be used to put the premium on-level. [½]
- Obtain information on changes in mix of business, and policy terms and conditions, over the period. [½]
- Adjust claims data for inflation, historic and future, in order to put the claims on-level. [1]

Use an actuarial method (*eg* chain-ladder) to project the claims and premiums to ultimate, and calculate the loss ratio for each past year. [1]

We could average the set of resulting adjusted loss ratios (say) to give an estimate of the expected loss ratio for the period of cover. If we consider them as a set of observations from a statistical distribution, we could use their mean and standard deviation to parameterise a distribution. [1]

The log-normal distribution is commonly used for this purpose. [½]

As an alternative to the above, the losses may be modelled separately, for example, by splitting out attritional, large and catastrophe losses (or attritional and large/catastrophe). [1]

Then:

- The catastrophe losses could come from a vendor model. [½]
- The large losses could be modelled using a frequency / severity approach. [½]
- The attritional losses could be analysed as above. [½]

This may produce a better model of the overall volatility. [½]

When pricing a stop loss important considerations include:

- meeting risk transfer criteria [½]
- the particular terms of the stop loss in question [½]
- any inuring reinsurance. [½]

Once we have the final risk premium, we need to add the normal loadings, *ie*:

- expenses [½]
- commission (if applicable), particularly profit commission or other loss-sensitive features [1]
- retrocession [½]
- profit and contingencies and/or cost of capital [½]
- brokerage [½]
- investment income (as a deduction). [½]

Finally, a check with competitors' rates would be advisable. [½]

[Maximum 15]

2 Solutions to exam-style questions

Solution 6.2

The modules are:

- event module
- hazard module
- inventory (or exposure) module
- vulnerability module
- financial analysis module. [½ each]

The inventory and financial analysis modules both rely primarily on data input by the user (an insurer or reinsurer) of the models. [1]

The event and hazard modules are based on seismological and meteorological assessment. [1]

The vulnerability module relies on engineering assessment. [½]

[Total 5]

Solution 6.3

Advantages to the reinsurer

This method is simple, cheap and requires no underwriting expertise. [½]

Provided the insurer stays with the reinsurer, the reinsurer's profit is stable (with a time lag) since the insurer pays back the claim amount eventually. The reinsurer is taking little risk over the long term. [1]

All risks are charged premiums that reflect their past experience. [½]

Disadvantages to the reinsurer

There is no graduation of large claims. [½]

So if the insurer has a number of large claims that didn't quite result in recoveries, it could select against the reinsurer. Similarly if the insurer has unusually good experience the premium is inadequate. [1]

Since the reinsurer is not taking any risk, then the insurer is not actually benefiting from reinsurance cover. The only benefit is the opportunity to select against the reinsurer. [1]

Also, since the reinsurer is not taking a risk, there is no opportunity for risk profit. [½]

The data is based on claims paid. It should be based on claims incurred. There is no allowance for outstanding claims, IBNR, reopened claims or partial payments. [1]

There is no loading to cover the reinsurer's expenses, profit, *etc*. [½]

There should be an adjustment for claims inflation and inflation of the excess point or else the premium will probably be inadequate. [1]

The premium should be based on actual written premium not expected. Therefore an adjustment premium should be paid. Otherwise, this gives the insurer the opportunity to select against the reinsurer by mis-estimating the premium. [1]

There is no allowance for any changes that make the past data irrelevant. For example, the insurer's target market, underwriting and policy conditions. [1]

[Maximum 8]

Solution 6.4

> ***Comment***
>
> *This question is from a past fellowship examination, so is longer and more difficult than you would normally find in a Subject ST8 examination.*

(i) ***Calculation of figures***

> ***Comment***
>
> *There is a lot of number crunching to do in this part and a fair bit of time pressure given the marks available. Note that of the nine marks available here, only four marks are for the actual figures, the other five are method marks. Hence, as with all numerical exam questions, it is crucial that you give a good clear explanation or example of what you are doing. Do not fall into the trap of producing a lot of numbers, which if wrong, are completely meaningless and impossible for someone else to follow.*

UY	GWP	NWP	GCI	GCP	NCI	NCP
1	120,456	93,956	81,308	79,237	63,832	62,175
2	124,218	96,890	85,338	81,592	68,270	65,274
3	130,987	102,170	93,263	83,745	69,673	62,346
4	131,421	102,508	97,514	84,006	77,734	67,205
5	128,976	100,601	98,667	57,981	78,754	46,385

[4]

Explanation of figures above

All gross figures unchanged. [½]

Net written premium (NWP) = (1 – 0.2 – 0.02) × GWP (*ie* reduced for QS and XLS premiums). [1]

Net claims incurred (NCI):

- Reduce GCI by 20% for QS [½]
- Reduce by the XLS recoveries as follows:
  - reduce by 20% for QS [½]
  - adjust for recoveries up to £4.5*m* in excess of £0.5*m*. [½]

Eg NCI(Year 5) = 80% × 98,667 – (80% × 850 – 500) = 78,754 [½]

Net claims paid (NCP): methodology as for NCI. However the claims paid relate to claims paid to date for that underwriting year irrespective of the calendar year in which the claim was settled. [1]

Eg for Year 1, all XOL claims have been paid, so all are deducted. For Year 4, none have been paid, so none have been deducted. [½]

[Total 9]

(ii) ***Burning cost***

> ***Comment***
>
> *Do not worry if you struggled on this question. Spend a little time now working through the solution below to check you understand the method used.*
>
> *The important thing in a question like this one is to make sure that your method is clearly explained. Then just work step-by-step through the calculations. If you are running short of time, a brief description of the remaining steps is likely to gain some further marks, even if you do not have time to do the number crunching.*

Assumptions

- Business is written evenly over the year, therefore adjust the data by six months to reflect unexpired risks.
- Risk incidence is spread evenly over the year.
- Reporting delays are negligible, so ignore.
- Hence, the average UY claim occurs at the end of the UY (*ie* 31/12).
- Equal weights can be given to each year's data because we expect the exposure to be relatively constant. [½ each]

Method

1. Increase past claims for inflation to 31 December Year 6 values. [½]
2. Reduce outstanding amounts for outstanding reserve release. [½]
3. Net down by 20% for QS. [½]
4. Burning cost is total past claims divided by exposure. [½]

The figures show the risks in the same order as in the question.

Gross incurred	Gross inc net of over reserving	Inflated gross inc net of over reserving	Inc net of over reserving & QS	Cost to XLS
674,321	674,321	888,750	711,000	211,000
1,452,908	1,452,908	1,892,797	1,514,238	1,014,238
1,265,943	1,265,943	1,627,342	1,301,874	801,874
812,364	812,364	975,503	780,403	280,403
984,213	885,792	1,050,123	840,099	340,099
6,374,306	6,374,306	7,400,032	5,920,025	4,500,000
542,164	487,948	554,265	443,412	0
794,002	714,602	793,711	634,969	134,969
803,000	722,700	786,152	628,921	128,921
850,000	765,000	817,949	654,359	154,359
			Total:	**7,565,862**

[6]

Hence, the cost per year of exposure based on the 4½ years' data is £1,681,303.. [1]

Example: top risk

Original gross claim of 674,321 inflated at 5% *pa* for 5 years 7 months and 27 days gives 888,750.

There is no 10% of reserves to release in this case.

80% × 888,750 = 711,000

Less the retention of 500,000 gives the cost to the XLS of 211,000 [Maximum 11]

(iii) ***Issues to address in analysis***

> ***Comment***
>
> *This part of the question asks for figures but gives little direction as to what to do. We are going to be very pushed for time. If placed in this position it is going to make sense to make suggestions for which we can do some relatively simple number crunching.*
>
> *Read through the solution to this question and try to follow what is going on.*
>
> *One way to generate ideas here is to think about the reasons for reinsurance and whether we can check how these packages meet them.*
>
> *We should also minimise the use of reinsurance to the extent that it passes profits to reinsurers. Hence, consider the costs of the treaties against the recoveries made. We have the data to do this retrospectively.*

The current surplus arrangement

Year	***RP***	***RR***	***INCR***	***IGCR***	
1	23,730	18,404	65%	67%	
2	28,570	26,738	61%	68%	
3	28,031	26,652	64%	70%	
4	32,855	35,619	61%	73%	
5	29,020	33,680	61%	73%	[5]

RP Reinsurance premium, the difference between gross and net premium from the table in the question.

RR Reinsurance recoveries, the difference between net and gross claims incurred.

Gross claims incurred are taken from the table, but allowing for the 10% loading for the claims not yet settled, *eg* for Year 5, $57{,}981 + 0.9 \times (98{,}667 - 57{,}981) = 94{,}598$.

Net claims incurred are also from the table but allowing for reinsurance recoveries and the 10% loading on those not yet settled.

eg for Year 5, $37{,}925 + 0.9 \times (63{,}472 - 37{,}925) = 60{,}917$.

RR for Year 5 is the difference between these two.

IGCR = Gross claim ratio. The ratio of gross incurred to gross written premiums.

INCR = Insurer's net claims ratio. The ratio of net claims incurred to net written premiums.

XOL Arrangement

Year	*RP*	*RR*	*INCR*
1	2,409	1,215	50%
2	2,484	0	0%
3	2,620	4,859	185%
4	2,628	150	6%
5	2,580	112	4%

[3]

RP is 2% of gross written premium

RR is found by taking 80% of gross claim, then calculating the XOL recovery. For those not yet paid the claim is reduced by 10%, *eg* for Year 3:

$$\left(0.8\times 812.364-500\right)+4{,}500+\left(984.213\times 0.9\times 0.8-500\right)=4{,}859$$

Quota share

Year	*RP*	*RR*	*INCR*
1	24,091	16,220	67%
2	24,844	16,993	68%
3	26,197	18,462	70%
4	26,284	19,233	73%
5	25,795	18,920	73%

[2]

RP is 20% of the gross written premium.

The reinsurance recoveries are equal to 20% of incurred claims, but those not yet settled are reduced by 10% to allow for the loading,

eg for Year 5 $=0.2\times\left(\left(98{,}667-57{,}981\right)\times 0.9+57{,}981\right)$

Quota share plus XOL together

Year	*RP*	*RR*	*INCR*
1	26,500	17,435	68%
2	27,328	16,993	70%
3	28,817	23,321	68%
4	28,913	19,382	75%
5	28,375	19,032	75%

[2]

Conclusions

Smoothing

Both the current surplus programme and the proposed QS/XLS package smooth the net claims experience. Surplus (ranging from 61–65%) has historically been more effective than the QS/XLS (68–75%). [1½]

Hence, any effects on statutory solvency could be predicted with greater confidence under the surplus cover. [1]

Solvency protection

The surplus cover gives unlimited cover where an EML is breached whereas the XSL recovery is capped at £4.5*m*. That could prove to be inadequate for some larger risks. [1]

Cost versus benefit

The net CI ratios show that the surplus treaty has been the most cost effective, with losses greater than the reinsurance premiums for the last two years (*ie* 108% and 116%). [1½]

This may give concern about the security of the reinsurer in the longer term if this performance is widespread across the reinsurer's other business. [1]

Moreover, it is unreasonable that the reinsurer will continue to write loss-making business. The reinsurer will increase their rates if this persists. [1]

Whilst the immediate benefit to the insurer is good, we should try to assess the likely availability of this cover or premiums for it in the future. [1]

The reinsurer was lucky in that nine lines of the surplus treaty were used for the largest claim. Ignoring that claim from the analysis, or if that claim did not occur, then the results would be different. [1]

Further, more detailed analysis is required, rather than just looking at loss ratios for the portfolio. [1]

[Maximum 22]

Part 7 – Revision Questions

This part contains 100 marks of questions testing the material from the whole course. You may like to try these questions under exam conditions as a mock exam.

Question 7.1

Explain what is mean by the term *burning cost.* [2]

Question 7.2

Explain the difference between *product costing* and *product pricing*. [2]

Question 7.3

The number of claims per year on an individual policy is negative binomial with parameters $k = 2$ and $p = 0.3$. The individual claim sizes are lognormally distributed with parameters $\mu = 5$ and $\sigma = 2$. You may assume that claim frequency and claim severity are independent. Find the mean and variance of the risk premium. [3]

Question 7.4

You are the pricing actuary for a large proprietary general insurer that writes personal lines business.

You have recently performed a full premium rating exercise for your comprehensive private motor business.

(i) List the main risk factors that are likely to have been used for this exercise. [5]

You are considering introducing a compulsory excess on its comprehensive private motor business for all new business.

(ii) Explain how you would calculate the percentage change in the premium rates from the previous calculation, in order to allow for the excess. [8]

[Total 13]

Question 7.5

Explain the following terms:

(a) estimated maximum loss

(b) expected maximum loss

(c) probable maximum loss

(d) possible maximum loss. [4]

Question 7.6

(i) List the main factors that, in practice, will determine the way that the expenses of an insurance company are allocated between classes. [5]

(ii) Suggest possible reasons why expense loadings in an insurer's current premium rates may prove to be inadequate. [3]
[Total 8]

Question 7.7

Describe the advantages and disadvantages of using a proprietary catastrophe model for pricing a small general insurance company's private motor insurance. [12]

Question 7.8

List the reasons why a general insurance company might want to use reinsurance. [9]

Question 7.9

You are given the following information for a class of general insurance business:

- The number of claims is Poisson.
- The severity distribution is lognormal with parameters $\mu = 10$ and $\sigma = 1.9$.
- Full credibility is defined as having a 95% probability of being within plus or minus 1% of the true aggregate loss.

Calculate the minimum number of expected claims that will be given full credibility. [7]

Question 7.10

Describe the main reasons for monitoring the business written by a general insurance company. [9]

Question 7.11

A colleague wants to fit some generalised linear models to some UK household data but has realised that he will need to group the postcode factor before he can include this in the model. The data he is using is at full postcode level. He has heard that spatial smoothing techniques might help but he does not really understand what these are.

(i) Explain briefly how spatial smoothing methods work. [2]

(ii) Describe the two main forms of spatial smoothing, explaining the types of claim for which each might be used. [13]
[Total 15]

Question 7.12

The random variable S has a compound Poisson distribution with Poisson parameter 45. The individual claim amounts are exponentially distributed with exponential parameter $\beta = 0.002$.

(i) Derive a formula involving $m!$ for the mth moment about zero of a random variable with an exponential distribution with parameter β. [3]

(ii) A normal distribution is used to approximate the aggregate claim distribution.

(a) Estimate the aggregate claim amount that will be exceeded with probability one in twenty.

(b) Estimate the aggregate claim amount that will be exceeded with probability one in a hundred. [5]

(iii) A translated gamma distribution is used to approximate the aggregate claim distribution.

(a) Estimate the parameters of the approximation that would be used.

(b) Find the corresponding figures for the 95% and 99% points of the translated gamma approximation.

(c) Comment on your answers. [8]
[Total 16]

Part 7 – Revision Solutions

Solution 7.1

The actual cost of claims paid or incurred during a past period of years expressed as an annual rate per unit of exposure. [1]

This is sometimes used (after adjustment for inflation, incurred but not reported (IBNR) and so on) as a method of calculating premiums for certain types of risks or monitoring experience, for example, motor fleets and non-proportional reinsurance. [1]

[Total 2]

Solution 7.2

Product costing is the calculation of the theoretical office premium to be charged for a particular class of business. [1]

Product pricing is the determination of the actual office premium. This will take account of current market conditions. [1]

[Total 2]

Solution 7.3

Denoting N as the number of claims, X as the individual claim size and PP as the pure risk premium for the individual risk:

$$E(N) = \frac{kq}{p} = \frac{2 \times 0.7}{0.3} = 4.667, \quad [½]$$

$$\text{var}(N) = \frac{kq}{p^2} = \frac{2 \times 0.7}{0.3^2} = 15.556, \quad [½]$$

$$E(X) = e^{\mu + \frac{1}{2}\sigma^2} = e^{5 + \frac{1}{2} \times 2^2} = 1,096.63 \text{ and} \quad [½]$$

$$\text{var}(X) = \left(e^{2\mu + \sigma^2}\right)\left(e^{\sigma^2} - 1\right) = 8,028.54^2. \quad [½]$$

So:

$$E(PP) = E(N)E(X) = 5,118, \text{ and} \quad [½]$$

$$\text{var}(PP) = E(N)\text{var}(X) + \text{var}(N)E^2(X) = 17,875^2. \quad [½]$$

[Total 3]

Solution 7.4

(i) ***Likely risk factors for private motor***

- the ability of the driver
- how long on risk (usually a year)
- the number of miles driven
- how much time the car is used
- how many passengers carried
- the density of the traffic where the car is driven
- the time of day the car is driven
- the speed at which the vehicle is usually driven and its general level of performance
- the ease with which the vehicle can be damaged and the cost of repairing it
- where the car is left when not used (the theft risk)
- size and weight of the vehicle
- fire risk. [½ each, maximum 5]

(ii) ***To calculate the percentage change in premiums***

Data needed:

- claims over the last, say five years, from the ground up
- exposure over the same period
- claim handling expenses and their inflation. [½ each]

First assess how much the risk premium changes as a result of introducing the excess. [½]

For every, say 100 policies, we can work out our total expected claims cost assuming no excess (*ie* from the ground up), by multiplying the expected frequency by severity. [½]

We then work out the frequency and average severity of claims that are beneath the proposed excess, and the frequency and average severity of claims that are above it. [1]

The new total claims cost will reduce by:

excess amount $\times$ number of claims expected above the excess [½]

plus

avg. severity of claims beneath the excess $\times$ expected no. of claims beneath the excess [½]

The percentage reduction in risk premium can therefore be calculated by comparison with the risk premium prior to the introduction of the excess. [½]

However, the office premium rate may not decrease by this percentage. [½]

For example, we may expect expenses to be lower due to the lower number of small claims. We would therefore adjust the office premium to allow for this change in expected expenses. [1]

We may also expect policyholders' attitudes to claiming to be different – some may not now bother claiming if they know there's an excess (*eg* on claims that only just breach the excess level), particularly if there is an NCD system in use. [½]

On the other hand, we may get people exaggerating claims in order to "recover" the excess. [½]

We may also need to change the allowance for reinsurance, if any. [½]

The profit loading may also change – in particular because on average we will now have smaller premiums. [1]
[Maximum 8]

Solution 7.5

The estimated maximum loss is the largest loss that is reasonably expected to arise from a single event in respect of an insured property. [½]

This may well be less than either the market value or the replacement value of the insured property and is used as an exposure measure in rating certain classes of business. [1]

The expected maximum loss is the same as the estimated maximum loss. [½]

The term "probable maximum loss" represents an attempt to quantify exposure, used in rating or to judge requirements for outwards reinsurance. [½]

It may be used as another term for estimated maximum loss, depending on the class of business. [½]

The term "possible maximum loss" implies the consideration of more remote scenarios than those for probable or estimated maximum loss and therefore carries a higher value. [1]

[Total 4]

Solution 7.6

(i) ***Expense allocation***

Main factors determining expense allocation between classes:

- results of latest expense analysis [½]
- basis used, *eg* ongoing or wind-up [½]
- management plans and perspectives [½]
- purpose of the allocation, *eg* setting premium rates or declaring profitability. [½]

For rate-setting:

- market is crucial, *ie* how competitive is class, are cross-subsidies possible? [½]
- what are the marginal costs? [½]
- what are the commission rates? [½]
- size and importance of class to the insurer [½]
- expectations for future competitive position and volume. [½]

When analysing profitability:

- ensuring the results are consistent with shareholders expectations and that the allocation is reasonable for internal and external analysts. [½]

[Maximum 5]

(ii) ***Inadequate expense loadings***

- The number of policies sold could be lower than planned leading to higher fixed expenses per policy than assumed when rating. [½]
- Higher than assumed inflation of costs, *eg* salary inflation. [½]
- Higher than expected legal and professional fees, possibly related to more unusual or complicated claims. [½]
- Higher levels (more expensive) new business, rather than renewal business, which is less expensive to process. [½]
- Heavy expenses from an unexpected change in circumstances, *eg* costs of new statutory requirements. [½]
- Unexpected one-off payments such as a special tax. [½]
- Poor management control of expenses. [½]
- The expense allocation may not have been appropriate. [½]

[Maximum 3]

Solution 7.7

The catastrophe model may be useful for adding a loading to premium rates to allow for the small possibility of a catastrophic event affecting many insured vehicles … [1]

… such as a flood (river or coastal), hailstorm or hurricane. [1]

The model may also assist with the following areas:

- reinsurance loadings [½]
- capital allocation and the RoC loading within premium rates. [½]

Both of these are particularly important areas for a small company. [½]

However, for a small company, the cost of such a model may be prohibitively expensive. [½]

In addition, catastrophic events for motor insurance are *so* rare that using a complex model may be "over the top". It may instead be better to use a very simple approach such as using margins elsewhere in the rating exercise. [1]

In particular, a small company will undoubtedly have an extensive reinsurance programme … [½]

… including catastrophe excess of loss reinsurance, which would cover claims from these events anyway. [½]

In this case, it would be sufficient to add a loading for the cost of reinsurance into the premium rates instead of using a catastrophe model … [1]

… although we would still need to consider the need for premiums to be competitive, particularly as the company is small. [½]

Using a catastrophe model has generic disadvantages too:

- the model will be partly subjective, as it has significant elements of judgement [½]
- the science behind the models is still relatively immature [½]
- due to the infrequent nature of the perils being modelled, it is difficult to know how "right" it is [½]
- the perils modelled are changing themselves, for example as a result of climate change [½]
- the accuracy of the models can be distorted, for example by:
  - changes in mix of business, *eg* by location [½]
  - changes in population trends [½]
  - changes in insurance terms and conditions [½]
- the models use approximations, which can be difficult to quantify [½]
- the models can be based on sparse data [½]
- different models often give different results [½]
- models won't allow for everything, for example they cannot compensate easily for inaccurate or missing data [½]
- it can be difficult to interpret and communicate the results. [½]

[Maximum 12]

Solution 7.8

Reinsurance may be used by an insurer:

- to limit exposure to risk (or spread risk) in respect of: [½]
  - single risks [¼]
  - aggregations of single risks [¼]
  - accumulations [¼]
  - multi-class losses [¼]
- to obtain additional business through reciprocity [½]
- to avoid single large losses in respect of: [½]
  - single large claims [¼]
  - catastrophes [¼]
- to smooth its results [½]
- to increase profitability by: [½]
  - increasing the stability of results (and so the ability to plan) [½]
  - taking advantage of cheap reinsurance [½]
- to enable it to declare profits from outstanding liabilities more quickly [½]
- to improve the solvency margin and hence reduce the risk of insolvency [½]
- to increase the capacity to accept risk, either: [½]
  - singly, *ie* to enable it to write larger risks, or [¼]
  - cumulatively, *ie* to enable it to write more business [¼]
- to obtain financial assistance to help with: [½]
  - new business strain [¼]
  - bolstering free assets [¼]
  - merger / acquisition [¼]
  - any other short-term cashflow needs [¼]
- to get technical assistance on: [½]
  - new risks [¼]
  - unusual risks [¼]
  - risks in new territories [¼]
- as a supervisory condition. [½]

[Maximum 9]

Solution 7.9

The standard for full credibility for the aggregate loss is $n_S = n_N + n_X$, where $n_N = \dfrac{y^2}{k^2}$ and $n_X = n_N \left(CV_X\right)^2$.

$\Phi(y) = \dfrac{1+P}{2} = \dfrac{1+0.95}{2} = 0.975$, so that $y = 1.960$. [1]

Hence, $n_N = \dfrac{y^2}{k^2} = \left(\dfrac{1.960}{0.01}\right)^2 = 38{,}416$. [1]

$n_X = n_N \left(CV_X\right)^2 = n_N \left(\dfrac{\sigma_X}{\mu_X}\right)^2$ where: [1]

$\mu_s = e^{\mu + \frac{1}{2}\sigma^2} = 133{,}920$ and $\sigma_s^2 = \left(e^{\mu+\frac{1}{2}\sigma^2}\right)^2 \left(e^{\sigma^2} - 1\right) = 803{,}143^2$ [2]

$n_X = 38{,}416\left(\dfrac{803{,}143}{133{,}920}\right)^2 = 1{,}381{,}679$ [1]

So $n_S = 38{,}416 + 1{,}381{,}679 = 1{,}420{,}095$. [1]

[Total 7]

Solution 7.10

Assessing performance against the organisation's goals [½]

The company will have certain overall goals (*eg* profit), and will put together more detailed targets designed to achieve these goals. [½]

Monitoring will enable the company to see whether these targets are going to be met, and hence the likelihood of achieving the overall goals. [½]

If expectations are not being met, decisions can be made in order to put the business back on target (*eg* re-pricing products). [½]

Managing risk [½]

Monitoring written business allows the company to assess how much risk is inherent in the portfolio (for example, accumulations). [½]

The amount of risk will be a factor in determining how much capital the company should hold … [½]

… and what its reinsurance purchasing strategy should be. [½]

Gaining market intelligence [½]

Monitoring written business can provide useful information about competitors' strategies. It can also allow the company to compare itself with the market and assess the position of the underwriting cycle. [1]

Satisfying regulators [½]

Market regulators may require periodic monitoring and reporting of written business.[½]

Influencing the market [½]

A company may be able to influence the market by publishing the results of its monitoring exercises. [½]

Assist with reserving [½]

The outputs of any monitoring exercise can be used for other purposes such as an input in to the reserving process, for example when setting *a priori* loss ratios within the BF method. [½]

Part of the actuarial control cycle [½]

Another reason for monitoring would be to validate assumptions in a model. [½]

By comparing actual experience with that expected, we can assess the need to adjust the assumptions, for example, in our premium rates. [½]
[Maximum 9]

Solution 7.11

(i) ***How spatial smoothing methods work***

Spatial smoothing methods take the fitted (predicted) values for each postcode and then adjust the relativities to take into account the values for neighbouring postcodes. [1]

These techniques are useful for factors like postcode, where there is a very large number of levels (postcodes), each with a low level of credibility. [½]

Indeed, for many individual postcodes, there would be no exposure and/or claims in the data but we still need to calculate a rate for each (we would not wish to rate these at zero cost), and so we can use spatial smoothing methods to help with this. [1]
[Maximum 2]

(ii) ***Two forms of spatial smoothing***

Distance-based smoothing [½]

Distance-based smoothing incorporates information about nearby postcodes based on the distance between the postcodes. [½]

The further away a postcode is from the one we are interested in, the less weight that will be given to its experience. [1]

This is true regardless of whether or not there is any reason to believe that nearby postcodes have similar claims experience. [1]

For example, if there is a boundary such as a river between two postcodes, this will be ignored by this method. [1]

Also, because urban postcode areas tend to be geographically smaller than rural ones, there will be a greater number of other postcodes that are physically close and so there may be more influence on a single postcode than we would like. [1]

For these reasons, distance-based smoothing is often more appropriate to use for weather-based perils (which take no notice of boundaries or whether a postcode is urban or rural) rather than perils like theft. [1]

Distance-based methods can also include dimensions other than distance – for example, the density of the population in an area – so that individual postcodes are more influenced by other postcodes that have similar characteristics as them. [1]

Adjacency-based smoothing [½]

Adjacency-based smoothing incorporates information about directly neighbouring (adjacent) postcodes only. [½]

Each postcode is therefore influenced by its direct neighbours. Each of these is influenced by their direct neighbours *etc*, so there is a ripple effect. [1]

We can incorporate distributional assumptions or prior knowledge of the claims processes into this method, although this may make the calculations complex. [1]

Boundaries between postcodes (*eg* rivers) can be reflected within the definitions of which postcodes neighbour each other. [½]

This is useful if the claims experience on one side of the boundary is not expected to be correlated in any way with that on the other side of the boundary. [½]

Urban and rural differences are often handled more appropriately using this method because each postcode is more likely to be influenced by postcodes that are similar in terms of risk than when distance-based smoothing is used. [1]

Adjacency-based smoothing methods can be used for non-weather-related claim types because the boundaries can be reflected and because of the urban/rural issue. [1]

Examples of non-weather-related claim types are theft or malicious damage. [½]
[Maximum 13]

Solution 7.12

(i) ***mth moment***

The m th noncentral moment of the $Exp(\beta)$ distribution is:

$$E(X^m) = \int_0^\infty x^m \beta e^{-\beta x} dx \qquad [1]$$

Since the total probability for the $Gamma(\alpha,\lambda)$ distribution is 1, we know that:

$$\int_0^\infty \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} dx = 1$$

Rearranging this gives:

$$\int_0^\infty x^{\alpha-1} e^{-\lambda x} dx = \frac{\Gamma(\alpha)}{\lambda^\alpha} \qquad [1]$$

Using this relationship with $\alpha = m+1$ and $\lambda = \beta$, we find that:

$$E(X^m) = \beta \int_0^\infty x^m e^{-\beta x} dx = \beta \frac{\Gamma(m+1)}{\beta^{m+1}} = \frac{m!}{\beta^m} \qquad [1]$$

[Total 3]

(ii) ***Normal distribution***

The mean and variance of the aggregate claim amounts are:

$$E(S) = \lambda m_1 = 45 \times \frac{1}{0.002} = 22,500 \qquad [1]$$

and $$\text{var}(S) = \lambda m_2 = 45 \times \frac{2}{0.002^2} = 22.5\text{m} = (4,743)^2 \qquad [1]$$

If we approximate the aggregate claim amounts with a normal distribution with the same mean and variance, we have that S is approximately $N[22,500,(4,743)^2]$. [1]

So $\dfrac{S-22,500}{4,743}$ has an approximately $N(0,1)$ distribution.

The upper 95% and 99% points of the $N(0,1)$ distribution are 1.6449 and 2.3263

So the critical amounts using a normal approximation are:

(a) 95% point $= 22,500 + 1.6449 \times 4,743 = 30,300$ [1]

(b) 99% point $= 22,500 + 2.3263 \times 4,743 = 33,500$ [1]

[Total 5]

(iii)(a) ***Parameters***

The parameters α, λ and k for the translated gamma distribution can be found by equating the mean, variance and skewness (using the result in part (i)) of the aggregate claim amounts:

$$E(S) = \frac{\alpha}{\lambda} + k = 22,500 \qquad [1]$$

$$\text{var}(S) = \frac{\alpha}{\lambda^2} = 22.5\text{m} \qquad [1]$$

$$\text{skew}(S) = \frac{2\alpha}{\lambda^3} = 45 \times \frac{6}{0.002^3} = 33,750\text{m} = (3,232)^3 \qquad [1]$$

Solving the last two equations, we find that:

$$\lambda = 2 \times \frac{22.5\text{m}}{33,750\text{m}} = 0.001333 \Rightarrow \alpha = 22.5\text{m} \times \lambda^2 = 40 \qquad [1]$$

From the first equation, we then get:

$$k = 22,500 - \frac{40}{0.001333} = -7,500 \qquad [1]$$

(iii)(b) ***95% and 99% points***

The critical amounts can be found using the result that $2\lambda(X-k)$ has a $\chi^2_{2\alpha}$ distribution.

From the *Tables*, the upper 95% and 99% points of the χ^2_{80} distribution are 101.9 and 112.3.

So the critical amounts using a translated gamma approximation are:

$$95\% \text{ point} = -7{,}500 + \frac{101.9}{2 \times 0.001333} = 30{,}710$$ [1]

and $$99\% \text{ point} = -7{,}500 + \frac{112.3}{2 \times 0.001333} = 34{,}610$$ [1]

(iii)(c) ***Comment***

The 95% point is similar using both approximations. However, the 99% is higher using the translated gamma distribution. This is because the aggregate claims distribution is positively skewed, with coefficient of skewness $3{,}232^3 / 4{,}743^3 = 0.32$. The translated gamma distribution, which is also a skewed distribution, can give a better approximation in the upper tail than the normal distribution, which is symmetrical. [1]

[Total 8]

Subject ST8: Assignment X1

2013 Examinations

Time allowed: 2½ hours, plus 15 minutes reading time

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Subject ST8: Assignment X1

2013 Examinations

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Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	**Total**	
_ 4	_ 4	_ 5	_ 4	__ 10	__ 14	_ 8	__ 12	__ 10	_ 9	__ 80	=_____%

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Notes on marker's section

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Question X1.1

List commonly-used rating factors for household property (buildings and contents) insurance. [4]

Question X1.2

(i) Explain the difference between facultative and treaty reinsurance. [2]

(ii) State the disadvantages of facultative reinsurance. [2]
[Total 4]

Question X1.3

(i) Explain why premiums may be used as a measure of exposure. [2]

(ii) Suggest reasons why they are not always appropriate. [3]
[Total 5]

Question X1.4

Define the following general insurance terms:

(a) Uberrima fides

(b) Professional indemnity insurance

(c) Nil claim. [4]

Question X1.5

You are sitting in the staff canteen of a general insurance company that writes all classes of business. You can't help yourself listening to the conversation of the student actuaries sitting at the table next to yours. The comments you think you hear are:

Comment A: "Motor claims are reported and settled quickly."

Comment B: "Late reported claims are bigger on average than claims reported quickly."

Comment C: "Claims from employers' liability do not occur very often, but when they do, they end up in court with large payments to the injured employee."

Discuss the points of clarification you would raise, assuming you were invited to join in with the conversation. [10]

Question X1.6

(i) Define the terms exposure measure, risk factor and rating factor. [4]

(ii) State an appropriate exposure measure and list the risk factors and rating factors for each of the following classes of business:

(a) personal motor

(b) marine hull. [10]

[Total 14]

Question X1.7

A general insurance company sells public liability business on a losses-occurring basis. It is considering moving to a claims-made basis to differentiate its product from those of its competitors.

(i) Define the terms "losses-occurring basis" and "claims-made basis". [2]

(ii) Discuss this course of action from the point of view of the insurer. [6]

[Total 8]

Question X1.8

You are an actuary working for a large proprietary general insurance company that writes a wide variety of business in a developed country. One of the directors has suggested that the company should not renew any of its existing reinsurance programmes nor take out any further reinsurance.

(i) List the types of reinsurance treaty that the company may already have. [2]

(ii) Discuss the possible reasons for the director's suggestion and its disadvantages. [10]
[Total 12]

Question X1.9

You are the actuary for a general insurance company. The finance director has suggested that in order to save money on reinsurance, the company should increase its retention levels.

Describe briefly the factors you would take into account in assessing this suggestion. [10]

Question X1.10

You are an actuary working for a large general insurance company that writes a wide range of personal lines products. The company is looking to move into the pet insurance market writing a product that covers domestic cats and dogs. You have been asked by the personal lines director, who knows very little about this product, to write a report on the product features of pet insurance, and the considerations to be made in designing the product.

Outline the points you would make in your report. [9]

End of paper

Subject ST8: Assignment X2

2013 Examinations

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If you are submitting by **post**:

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- post your script to: **First Floor, McTimoney House, 1 Kimber Road, Abingdon, Oxfordshire, OX14 1BZ**
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Subject ST8: Assignment X2
2013 Examinations

Please complete the following information:	
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Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	**Total**	
_/3	_/3	_/6	_/8	_/8	_/6	_/9	_/10	_/14	_/13	_/80	=_____%

Grade: A B C D E **Marker's initials:** ________

Please grade your Assignment X1 marker by ticking the appropriate box.

[ ] **Excellent** – the marker's comments were thorough and very helpful
[ ] **Good** – the marker's comments were generally helpful
[ ] **Acceptable** – please explain below how the marker could have been more helpful
[ ] **Poor** – the marker's comments were generally unhelpful; please give details below

Please give any additional comments here (especially if you rate the marker less than good):

Note: Giving feedback on your marker helps us to improve the quality of marking.

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[ ]	Numbered all pages of your script (excluding this cover sheet)?
[ ]	Written the total number of pages (excluding the cover sheet) in the space above?
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[ ]	Rated your Assignment X1 marker?

Feedback from marker

Notes on marker's section

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or when you submit your next script.

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Question X2.1

Explain what is meant by the *insurance cycle* and give a brief description of how it arises. [3]

Question X2.2

Suggest reasons why two different modellers might select different variables even though their models are based on the same past experience. [3]

Question X2.3

List the regulatory restrictions that may be imposed on the actions of a general insurer. [6]

Question X2.4

A medium-sized general insurance company has been selling private motor business via brokers for many years. The company has recently started selling direct to the public, via the telephone. The telephone operation uses a smaller number of rating factors than the broker product and the rates offered are generally slightly cheaper. You have been asked to set up a management information system to record the movement statistics for the whole operation. Explain with reasons the main statistics that you would calculate. [8]

Question X2.5

You are an actuarial consultant who has been asked to review the premium rating calculations for a large household contents book of business.

Suggest appropriate checks that should be undertaken to ensure that the risk premiums have been calculated correctly. [8]

Question X2.6

Discuss the uncertainties underlying any estimates that might be made of an insurer's claims liabilities. [6]

Question X2.7

(i) Explain why there are usually many questions for an individual to answer on a private motor proposal form. [7]

(ii) Explain how a company might reduce the amount of data it collects for each private motor applicant, and yet still be able to write profitable insurance business. [2]
[Total 9]

Question X2.8

A general insurance company that writes only motor business exclusively through brokers has experienced a gradual but increasing reduction in its market share over the past five years. The company has decided to use the internet with a view to preventing further reduction and achieving 50% of its sales through this medium within the next five years. Discuss the possible effect this strategy may have in the following areas of the company's business:

- the cost structure
- broker arrangements
- calculation of premiums
- business mix and volumes
- reinsurance arrangements. [10]

Question X2.9

Two years ago, a new general insurance company entered the travel insurance market. Its strategy was to focus on growth, and over the two years it has grown rapidly.

The insurer is about to do a re-pricing exercise based on its own internal data.

(i) Explain why past experience might not necessarily be a good guide to the future. [7]

(ii) Discuss the areas of uncertainty arising from using this data. [7]
[Total 14]

Question X2.10

A general insurance company writes commercial property insurance in a small country. Up until now, the sale of alcohol in this country has been strictly controlled by the State. As a result, all of the country's pubs and bars were State-owned, with any risks being borne by the State (*ie* they were self-insured).

Regulation is about to change, and so private individuals (known as landlords) will be allowed to run pubs and bars. Your company is considering insuring these landlords.

(i) Describe the particular risks to the insurer of writing this type of business. [9]

(ii) Outline the restrictions and exclusions the insurer might place on the cover. [4]
[Total 13]

End of paper

Subject ST8: Assignment X3

2013 Examinations

Time allowed: 2½ hours, plus 15 minutes reading time

Instructions to the candidate

1. *Please note that we only accept the current version of assignments for marking, ie you can only submit this assignment in the sessions leading to the 2013 exams.*
2. *Attempt all of the questions, leaving space in the margin and beginning your answer to each question on a new page.*
3. *Write in black ink using a medium-sized nib because we will be unable to mark illegible scripts.*
4. ***Leave at least 2cm margin on all borders.***
5. *Attempt the questions as far as possible under exam conditions.*
6. *You should aim to submit this script for marking by the recommended submission date. The recommended and deadline dates for submission of this assignment are listed in the Study Guide for the 2013 exams, on the summary page at the back of this pack and on our website at **www.ActEd.co.uk**.*

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At the end of the assignment

If your script is being marked by ActEd, please follow the instructions on the reverse of this page.

In addition to this paper, you should have available actuarial tables and an electronic calculator.

Submission for marking

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If you are submitting by **email**:

- complete the cover sheet, including the checklist
- scan your script (and Marking Voucher if applicable) to a pdf document, then email it to: **ActEdMarking@bpp.com**.

Please note the following:

- Please title the email to ensure that the subject and assignment are clear *eg* "ST8 Assignment X3 No. 12345", inserting your ActEd Student Number for 12345.
- The assignment should be scanned the **right way up** (so that it can be read normally without rotation) and as a single document. We cannot accept individual files for each page.
- Please set the resolution so that the script is legible and the resulting PDF is less than 3 MB in size. **The file size cannot exceed 4 MB**.
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Subject ST8: Assignment X3

2013 Examinations

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Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	**Total**	
$\frac{}{4}$	$\frac{}{7}$	$\frac{}{17}$	$\frac{}{6}$	$\frac{}{13}$	$\frac{}{12}$	$\frac{}{21}$	$\frac{}{80}$	=_____%

Grade: A B C D E **Marker's initials:** ________

Please grade your Assignment X2 marker by ticking the appropriate box.

[ ] **Excellent** – the marker's comments were thorough and very helpful

[ ] **Good** – the marker's comments were generally helpful

[ ] **Acceptable** – please explain below how the marker could have been more helpful

[ ] **Poor** – the marker's comments were generally unhelpful; please give details below

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Feedback from marker

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Question X3.1

A general insurer wishes to introduce a new set of premium rates. Explain why the date the insurer chooses to introduce the rates is likely to be some time after the date of the latest available base statistics. [4]

Question X3.2

An insurance company operating in the land of "Claysoil" is worried about the claim costs on its household account, caused by subsidence. The insurer is considering excluding subsidence claims.

(i) List the advantages and disadvantages of this course of action. [4]

(ii) Explain briefly how the insurer could calculate the revised premium rate, assuming that this peril is excluded. [3]
[Total 7]

Question X3.3

A general insurer has announced a loss of £3*m* on its private motor business for the year to 30 September 2012. The last revision of premium rates took effect from 1 October 2011, and the next premium rate revision is planned for 1 January 2013.

A director has observed that the company wrote £60*m* of premiums in the year to 30 September 2012 and that premium rates should therefore be increased by 5% to remove the £3*m* loss.

(i) State the main reasons why this suggestion is probably inappropriate. [9]

(ii) List any further information you would require if you were asked to assess the rates to be charged from 1 January 2013. [8]
[Total 17]

Question X3.4

You are an actuary working for a large general insurer. The Farmers Union (FU) has approached your company asking you to insure its members' tractors. Each member will have to take out its own policy but the FU thinks that it can act as a broker between the farmers and your company to help reduce costs.

You have been asked to provide the managing director of your company with a report explaining the main issues to the company of providing this new insurance. Outline the content of the report. [6]

Question X3.5

You are an actuary working for a general insurance company that writes household contents insurance in a country where the market for this type of business is small, but developing. Premiums for this business are currently calculated based on a small number of rating factors.

It is proposed that the insurer introduces a new rating factor, namely the age of policyholder, on this business.

(i) Explain, using examples, the rationale behind this proposal. [5]

(ii) Describe the difficulties that the insurer may face before implementing this proposal. [8]

[Total 13]

Question X3.6

The following information applies to a class of business for which the numbers of claims follow a Poisson process. All policies are annual.

(a) Expected claim size is £1,000 at 1/7/2012.

(b) Excess fixed in monetary amounts at £300.

(c) Inflation of claims and expenses expected to be 1% per month.

(d) Policy covers first two losses only.

(e) Expected claim frequency is 25%.

(f) The policyholder pays a reinstatement premium of half the original premium on the date the first loss is settled.

(g) Acquisition expenses are 25% of premium.

(h) Per policy expenses are initially £1 per month throughout the life of the policy.

(i) Contribution to fixed expenses and profit should be 15% of gross premium.

(j) Assumed rate of investment return is 1% per month.

(k) There are no lapses or cancellations.

(l) Policies are not expected to be renewed.

(m) All premiums (net of acquisition costs) are received at the date of policy inception.

(n) Assume all claims and per policy expenses are paid 9 months after inception.

(o) Acquisition expenses and the contribution to fixed expenses and profit should be assumed to occur at policy inception.

Stating any further assumptions or approximations that you make, calculate an office premium to apply for all new policies commencing in 2013. [12]

Question X3.7

The number of claims, N, arising from a particular group of policies has a negative binomial distribution with parameters $k = 3$ and $p = 0.9$. Individual claim amounts, X, have the following distribution:

$$P(X = 500) = 0.5$$
$$P(X = 1{,}000) = 0.25$$
$$P(X = 2{,}000) = 0.25$$

The aggregate claim amount is denoted by S.

(i) Show that the relationship

$$P(N = n) = \left(a + \frac{b}{n}\right) P(N = n-1)$$

holds for $n = 1, 2, \ldots$, and determine the values of a and b. [2]

(ii) Calculate $P(S \le 2{,}000)$ using

(a) the recursive method
(b) a normal approximation
(c) a translated gamma approximation.

You should ignore any continuity corrections. [17]

(iii) Comment on your answers to (ii). [2]
[Total 21]

You may use without proof the following result for a compound negative binomial random variable, S:

$$Skew(S) = \frac{3kq^2}{p^2} m_1 m_2 + \frac{2kq^3}{p^3} m_1^3 + \frac{kq}{p} m_3$$

where $m_k = E(X^k)$, the k^{th} non-central moment of the claim amount distribution.

End of paper

Subject ST8: Assignment X4

2013 Examinations

Time allowed: 3 hours, plus 15 minutes reading time

Instructions to the candidate

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At the end of the assignment

If your script is being marked by ActEd, please follow the instructions on the reverse of this page.

In addition to this paper, you should have available actuarial tables and an electronic calculator.

Submission for marking

There are three methods for you to submit your script, namely by *email*, by *fax* or by *post*.

If you are submitting by **email**:

- complete the cover sheet, including the checklist
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Please note the following:

- Please title the email to ensure that the subject and assignment are clear *eg* "ST8 Assignment X4 No. 12345", inserting your ActEd Student Number for 12345.
- The assignment should be scanned the **right way up** (so that it can be read normally without rotation) and as a single document. We cannot accept individual files for each page.
- Please set the resolution so that the script is legible and the resulting PDF is less than 3 MB in size. **The file size cannot exceed 4 MB**.
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Subject ST8: Assignment X4
2013 Examinations

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Name:

Address:

ActEd Student Number (see Note below):

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Number of following pages: _______

Please put a tick in this box if you have solutions and a cross if you do not: ☐

Please tick here if you are allowed extra time or other special conditions in the Profession's exams: ☐

Time to do assignment
(see Note below):
_____ hrs _____ mins

Under exam conditions
(delete as applicable): yes / nearly / no

Note: If you spend more than 3 hours on the assignment, you should indicate on the assignment how much you completed within this time so that the marker can provide useful feedback on your chances of success in the exam.

Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	**Total**	
__ / 16	_ / 8	__ / 17	__ / 10	__ / 18	__ / 28	_ / 3	___ / 100	=_____%

Grade: A B C D E **Marker's initials:** ________

Please grade your Assignment X3 marker by ticking the appropriate box.

[ ] **Excellent** – the marker's comments were thorough and very helpful
[ ] **Good** – the marker's comments were generally helpful
[ ] **Acceptable** – please explain below how the marker could have been more helpful
[ ] **Poor** – the marker's comments were generally unhelpful; please give details below

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[ ]	Rated your Assignment X3 marker?

Feedback from marker

Notes on marker's section

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Question X4.1

A colleague is doing a pricing exercise, using GLMs, for a motor book of business and has come up with an initial model containing many potential rating factors.

He is unsure whether to keep one particular factor in the model and he has asked for your advice. This factor has five levels: A, B, C, D, Unknown.

The initial model contains 50,000 observations and has 80 parameters fitted. The scaled deviance for this model is 392.45.

Your colleague has fitted a model that excludes the factor in question and the scaled deviance has now increased to 401.97.

(i) Carry out a statistical test to decide whether or not this factor is statistically significant. Explain the rationale behind the test and state your conclusion clearly. [6]

(ii) Discuss the further considerations you would take into account when deciding whether or not to keep this factor in the model. [10]

[Total 16]

Question X4.2

A generalised linear model has independent normally-distributed responses Y_i, $i = 1, 2, \ldots, n$, and uses the identity link function.

(i) Show that the normal distribution is a member of the exponential family. [4]

(ii) Obtain the variance function for this model. [1]

(iii) Evaluate the expression:

$$d_i\left(y_i, \hat{\mu}_i\right) = 2\int_{\hat{\mu}_i}^{y_i} \frac{y_i - t}{V(t)} dt$$

and hence obtain an expression for the deviance of this GLM in terms of the observed responses y_i and fitted values $\hat{\mu}_i$. [3]

[Total 8]

Question X4.3

You have been working in an actuarial pricing department for the last year, and have done some work on generalised linear models (GLMs).

You have run a simple multiplicative generalised linear model using some household contents theft data. The tables below show the initial results from the frequency model and the exposures split by rating cell.

Relativities		
	Intercept	0.08
Area	Country	0.5
	Town	1.0
	City	1.5
No. of bedrooms	1-3	0.5
	4+	1.0
Age of policyholder	Young	2.0
	Old	1.0

Area	No. of bedrooms	Age of policyholder	Policy years
Country	1-3	Young	10,200
Town	1-3	Young	12,000
City	1-3	Young	17,500
Country	4+	Young	50
Town	4+	Young	200
City	4+	Young	50
Country	1-3	Old	6,700
Town	1-3	Old	12,000
City	1-3	Old	8,000
Country	4+	Old	18,250
Town	4+	Old	25,000
City	4+	Old	50

(i) Calculate the predicted claim frequency for each combination of the factor levels. [3]

(ii) Calculate one-way tables of frequency for these three factors. [5]

(iii) Comment on how well the one-way table results predict the GLM results. [9]
[Total 17]

Question X4.4

State and explain four distinct methods of factor simplification that can be used in a generalised linear model. [10]

Question X4.5

You are a pricing actuary who works for an insurance company that writes personal lines motor business.

(i) List the risk factors affecting the collision claims experience that relate to:

(a) the driver

(b) the environment

(c) the vehicle. [5]

(ii) Explain why these factors are not usually used directly for rating. [4]

(iii) Discuss, with examples, the factors that affect how effective a proxy rating factor might be. [9]
[Total 18]

Question X4.6

You are an actuary working for a small general insurance company that sells annual travel insurance direct to the public.

The policy includes £500 of cover for holiday cancellation beyond the control of the policyholder. Sales of policies have fallen in recent years, and evidence suggests that this is because the level of cover for this peril is too low. You have been asked to review the limit and the resulting new premium rate.

(i) List the main rating factors for annual travel insurance. [5]

You have obtained the following data from a travel association that shows costs of holidays sold in their affiliated travel agents over the last three months:

Range of individual holiday cost	*Total value of holidays sold*	*Number of holidays sold*
Under £500	£1.0*m*	3,000
£500 – £999	£5.0*m*	6,500
£1,000 – £1,499	£7.0*m*	5,500
£1,500 – £1,999	£9.0*m*	5,000
£,2000 – £2,999	£6.0*m*	2,500
£3,000 – £3,999	£3.5*m*	1,000
£4,000 upwards	£6.0*m*	1,000

You are thinking of increasing the limit of cover to either £1,000, £2,000, £3,000 or £4,000.

(ii) Using the data in the table, construct a table of ILFs for use in your analysis. [7]

(iii) List the assumptions you will need to make in order to use the factors you have calculated to estimate the increase in losses at each higher limit. [7]

(iv) Describe the other factors that may influence your choice of the new limit. [9]
[Total 28]

Question X4.7

Show that the Poisson distribution belongs to the exponential family. [3]

End of paper

Subject ST8: Assignment X5

2013 Examinations

Time allowed: 3 hours, plus 15 minutes reading time

Instructions to the candidate

1. *Please note that we only accept the current version of assignments for marking, ie you can only submit this assignment in the sessions leading to the 2013 exams.*
2. *Attempt all of the questions, leaving space in the margin and beginning your answer to each question on a new page.*
3. *Write in black ink using a medium-sized nib because we will be unable to mark illegible scripts.*
4. ***Leave at least 2cm margin on all borders.***
5. *Attempt the questions as far as possible under exam conditions.*
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At the end of the assignment

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Submission for marking

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If you are submitting by **email**:

- complete the cover sheet, including the checklist
- scan your script (and Marking Voucher if applicable) to a pdf document, then email it to: **ActEdMarking@bpp.com**.

Please note the following:

- Please title the email to ensure that the subject and assignment are clear *eg* "ST8 Assignment X5 No. 12345", inserting your ActEd Student Number for 12345.
- The assignment should be scanned the **right way up** (so that it can be read normally without rotation) and as a single document. We cannot accept individual files for each page.
- Please set the resolution so that the script is legible and the resulting PDF is less than 3 MB in size. **The file size cannot exceed 4 MB**.
- Before emailing to ActEd, please check that your scanned assignment includes all pages and conforms to the above.

If you are submitting by **fax**:

- only write on one side of the paper when completing the assignment
- complete the cover sheet, including the checklist
- fax your script (including cover sheet and Marking Voucher if applicable) to **0844 583 4501**.

In addition:

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- An email will be sent by the end of the next working day to confirm that we have processed your script. Please do not phone to check progress before then. If the fax was sent without error then it's very unlikely that there will be a problem.

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- please staple the cover sheet (and Marking Voucher if applicable) to the front of your assignment
- please do not staple more than one assignment together.

Subject ST8: Assignment X5

2013 Examinations

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Name: **Address:** **ActEd Student Number** (see Note below): [ ][ ][ ][ ][ ] **Note:** Your ActEd Student Number is printed on all personal correspondence from ActEd. Quoting this number will help us to process your scripts quickly. If you do not complete this box, your script may be delayed. If you do not know your ActEd Student Number, please email ActEd@bpp.com. **Your ActEd Student Number is not the same as your Faculty/Institute Actuarial Reference Number or ARN.**	**Number of following pages: _______** **Please put a tick in this box if you have solutions and a cross if you do not:** [ ] **Please tick here if you are allowed extra time or other special conditions in the Profession's exams:** [ ] Time to do assignment (see Note below): _____ hrs _____ mins Under exam conditions (delete as applicable): yes / nearly / no **Note:** If you spend more than 3 hours on the assignment, you should indicate on the assignment how much you completed within this time so that the marker can provide useful feedback on your chances of success in the exam.

Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	**Total**	
_ / 6	_ / 4	_ / 4	__ / 14	__ / 12	__ / 15	__ / 19	_ / 9	__ / 17	___ / 100	=_____%

Grade: A B C D E **Marker's initials:** ________

Please grade your Assignment X4 marker by ticking the appropriate box.

[ ] **Excellent** – the marker's comments were thorough and very helpful

[ ] **Good** – the marker's comments were generally helpful

[ ] **Acceptable** – please explain below how the marker could have been more helpful

[ ] **Poor** – the marker's comments were generally unhelpful; please give details below

Please give any additional comments here (especially if you rate the marker less than good):

Note: Giving feedback on your marker helps us to improve the quality of marking.

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Feedback from marker

Notes on marker's section

The marker's main objective is to give you advice on how to improve your answers. The marker will also assess your script quantitatively and qualitatively. The percentage score gives you a quantitative assessment. The grade is a qualitative assessment of how your script might be classified in the exam. The grades are as follows:

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or when you submit your next script.

This page has been left blank in case you wish to submit your script by fax.

Question X5.1

You are a general insurance actuary in charge of setting the premium rates for a certain class of business. You have decided to use the Bühlmann-Straub model to estimate the risk premium, and have collected the following data:

The volume measures for the business written in the last two years are $V_1 = 50$ and $V_2 = 100$.

Total losses for the previous two years have been $S_1 = 5,000$ and $S_2 = 2,500$.

You believe that $\theta_i \sim N(100, 525)$ and that $X_i | \theta_i$ is normally distributed with mean θ_i and variance $\sigma^2(\theta_i) / V_i = 300 / V_i$.

Calculate the Bühlmann-Straub risk premium for the total losses in year three, assuming that the volume measure for year three will be $V_3 = 25$. [6]

Question X5.2

A general insurance portfolio consists of N independent, identically distributed risks. The claim frequency per unit of exposure is a random variable F with $E(F) = 0.06$ and $\text{var}(F) = 0.10$. The standard for full credibility requires that the observed claim frequency lies within 2% of the expected population claim frequency 95% of the time.

Calculate the number of risks required for full credibility. [4]

Question X5.3

List the desirable qualities for a complement of credibility. [4]

Question X5.4

For a particular class of business, the number of claims is thought to follow a Poisson distribution. For full credibility, the insurer has always required the number of claims to be within 3% of the true mean, with a probability of 95%.

(i) Calculate the number of claims required for full credibility. [2]

An actuary who has recently joined the insurer is concerned that it is not sufficient to consider only the claim frequency when calculating credibility. He suggests that the claim severity should also be taken into account and estimates that, for this class of business, the individual claim sizes have the following distribution:

$$f(x) = c(1600 - 2x) \text{ for } 0 \le x \le 800 \quad \text{where } c \text{ is a constant.}$$

(ii) Calculate the revised number of claims required for full credibility, stating any assumptions you make. [7]

The new actuary is now beginning to doubt the assumption that the number of claims for this class actually does follow a Poisson distribution. He would now like to test how the numbers for full credibility would change under a different distributional assumption.

(iii) Calculate the revised number of claims required for full credibility, assuming that the number of claims has a binomial distribution with parameters $n = 2000$ and $p = 0.2$. [4]

(iv) Comment on the relative sizes of your answers to parts (ii) and (iii). [1]
[Total 14]

Question X5.5

You are the actuary of a general insurance company that writes many classes of insurance business through a broker network. A new director has recently joined the Board of Directors. Before his appointment, the director had little experience of the general insurance market, but did have some accounting experience.

The director has stated that, because claim costs are a much higher percentage than expense costs of the premium paid by the customer, expense analyses are not important when setting premium rates. As a consequence, in allocating expenses for premium rating he suggests that the following method be used:

Take the total expenses from the statutory returns by accounting class, and divide by the corresponding premium. Load this as a percentage of premiums for each accounting class.

(i) State the reasons why an expense analysis should be carried out. [4]

(ii) Discuss the director's proposed method and suggest a better alternative. [8]
[Total 12]

Question X5.6

(i) Define *strike rate*. [1]

A large general insurer, which sells business via a variety of distribution channels, has noticed a steady decrease in its strike rate over the last few months.

(ii) Outline possible reasons why this might have occurred, and suggest actions that the general insurer could take in order to improve its strike rate. [14]
[Total 15]

Question X5.7

Explain the considerations that should be taken into account when using credibility theory in practice. [19]

Question X5.8

You are the actuary in charge of pricing a household property book. You have decided to use a credibility approach to future claim frequency, which you then multiply by your historic average claim size, to obtain the pure risk premium P on a policy. *ie*:

$P = \bar{S} \times \left[Z\bar{X} + (1-Z)M \right]$, with $0 \le Z \le 1$ and where:

- Z is the credibility factor, calculated using the square root rule
- $\bar{X}$ is the average past claim frequency experience on policies in the portfolio
- M is a benchmark average claim frequency experience obtained from industry statistics
- $\bar{S}$ is the average past claim size on policies in the portfolio.

Your standard for full credibility is 1,082 claims and you have 900 observed claims.

(i) Describe the disadvantages of using this method for calculating Z. [6]

(ii) You decide to change your method of calculating Z to a Bayesian approach, but you are concerned that the change in method will cause your premium rates to seem inconsistent with previous years. Calculate the Bayesian credibility parameter k that will leave the credibility factor Z unchanged. [3]
[Total 9]

Question X5.9

You work for a general insurer that sells only commercial property insurance.

(i) Explain why you might wish to monitor the insurer's premium rate changes. [2]

(ii) Suggest, with reasons, a suitable definition of *premium rate* for monitoring the profitability of the commercial property book. [2]

(iii) Discuss four ways in which the rate changes could be calculated. [13]
[Total 17]

End of paper

Subject ST8: Assignment X6

2013 Examinations

Time allowed: 3 hours, plus 15 minutes reading time

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Subject ST8: Assignment X6
2013 Examinations

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Address:

ActEd Student Number (see Note below):

Note: Your ActEd Student Number is printed on all personal correspondence from ActEd. Quoting this number will help us to process your scripts quickly. If you do not complete this box, your script may be delayed. If you do not know your ActEd Student Number, please email ActEd@bpp.com. **Your ActEd Student Number is not the same as your Faculty/Institute Actuarial Reference Number or ARN.**

Number of following pages: _______

Please put a tick in this box if you have solutions and a cross if you do not: ☐

Please tick here if you are allowed extra time or other special conditions in the Profession's exams: ☐

Time to do assignment (see Note below): _____ hrs _____ mins

Under exam conditions (delete as applicable): yes / nearly / no

Note: If you spend more than 3 hours on the assignment, you should indicate on the assignment how much you completed within this time so that the marker can provide useful feedback on your chances of success in the exam.

Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	**Total**	
_ / 5	_ / 2	_ / 5	_ / 8	_ / 7	__ / 26	_ / 4	__ / 14	__ / 29	___ / 100	=_____%

Grade: A B C D E **Marker's initials:** ________

Please grade your Assignment X5 marker by ticking the appropriate box.

[ ] **Excellent** – the marker's comments were thorough and very helpful
[ ] **Good** – the marker's comments were generally helpful
[ ] **Acceptable** – please explain below how the marker could have been more helpful
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Feedback from marker

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Question X6.1

State the four main factors that would determine the level of ceding commission paid on a motor quota-share reinsurance policy. [5]

Question X6.2

Define the term *stochastic event set* and explain briefly how it is used within a catastrophe model. [2]

Question X6.3

List the perils that may be modelled within a catastrophe model. [5]

Question X6.4

An insurer expects 50 claims per year from a particular class of insurance that it writes. The distribution of individual claim amounts (£x) is given by:

$$f(x) = 55{,}000 / (50{,}000 + x)^2 \qquad \text{for } 0 < x < 500{,}000 \text{ and}$$
$$f(x) = 0 \qquad \text{for } x > 500{,}000.$$

Stating any assumptions you make, calculate the premium that a reinsurer would charge for an individual excess of loss contract of £200,000 in excess of £100,000, given that the reinsurer requires an expenses and contingency margin of 35% of the premium. Outline the circumstances under which the insurer will choose not to use the reinsurance. [8]

Question X6.5

Explain the term *demand surge* and how it affects catastrophe models. [7]

Question X6.6

It is February 2013. You are a pricing actuary, working for a large global reinsurer in the London Market. Your company writes a wide range of reinsurance contracts and is one of the lead players in many lines of business.

One of the underwriters has passed you the details of a stop-loss treaty, which was due to renew on 1 January 2013 for a two-year period, but which has yet to be agreed. The treaty provides unlimited cover for a portfolio of private cars, attaching at an incurred loss ratio of 40%. Bodily injury claims are excluded from the treaty. There is also a profit commission payable.

Amongst the pile of correspondence is a note from the broker that is placing the business, addressed to the underwriter:

John,

Why are you not happy to accept the renewal of this treaty at the expiring terms? The lead underwriter is happy with it and you were happy to follow him last time. Not only that, but the experience has once again been good – there have been no losses to the treaty at all.

Regards, Julie.

You sift further through the information supplied and find the following experience statistics:

Year	Incurred Loss Ratio
2007	30%
2008	35%
2009	30%
2010	30%
2011	35%
2012	38%

Inflation of car parts and associated labour costs has been 3% per annum over the period, and the figures were calculated by the ceding company.

The underwriter has asked you to prepare some discussion points for a forthcoming meeting with the broker.

(i) Outline how premium rates for this stop-loss policy would be calculated. [9]

(ii) List, with reasons, the factors other than price that the underwriter should consider before writing the business. [10]

(iii) Outline the contents of a reply for the underwriter to send to the broker in advance of the meeting, which should include possible non-price changes to the deal to make it more attractive to the reinsurer. [7]
[Total 26]

Question X6.7

A general insurance company is assessing its need for property natural catastrophe reinsurance. It has been suggested that the best approach is to use a stochastic model to estimate the likelihood of catastrophic claims and also the expected cost should a claim arise. Describe briefly how this would work. [4]

Question X6.8

(i) State the two main approaches for assessing the risk premium for non-proportional reinsurance using a cedant's loss experience, and outline the steps involved in each approach. [12]

(ii) State the three main factors that the choice of approach would depend on. [2]
[Total 14]

Question X6.9

You have been asked to build a catastrophe model to assist general insurers in their management of their flood risk for their domestic household insurance in a developed country.

(i) Explain why traditional rating methods, such as the burning cost approach, may be unsuitable for dealing with flood risk. [2]

(ii) Outline briefly the components of the model. For each module, state the data needed and any problems you envisage. [21]

(iii) List the uses that insurers could make of your model. [6]
[Total 29]

End of paper

For the session leading to the April 2013 exams – ST Subjects

Marking vouchers

Subjects	*Assignments*	*Mocks*
ST4, ST5, ST7, ST8	20 March 2013	26 March 2013
ST1, ST2, ST6, ST9	26 March 2013	3 April 2013

Series X Assignments

Subjects	*Assignment*	*Recommended submission date*	*Final deadline date*
ST1, ST2, ST4 – ST9	**X1**	**21 November 2012**	16 January 2013
ST1, ST2, ST4 – ST9	**X2**	**5 December 2012**	30 January 2013
ST1, ST2, ST4 – ST9	**X3**	**16 January 2013**	13 February 2013
ST1, ST2, ST4 – ST9	**X4**	**30 January 2013**	27 February 2013
ST1, ST2, ST4 – ST9	**X5**	**13 February 2013**	13 March 2013
ST4, ST5, ST7, ST8	**X6**	**20 February 2013**	20 March 2013
ST1, ST2, ST6, ST9		**27 February 2013**	26 March 2013

Mock Exams

Subjects	*Recommended submission date*	*Final deadline date*
ST4, ST5, ST7. ST8	**13 March 2013**	26 March 2013
ST1, ST2, ST6, ST9	**20 March 2013**	3 April 2013

We encourage you to work to the recommended submission dates where possible. Please remember that the turnaround of your script is likely to be quicker if you submit it well before the final deadline date.

For the session leading to the September/October 2013 exams – ST Subjects

Marking vouchers

Subjects	*Assignments*	*Mocks*
ST4, ST5, ST7, ST8	28 August 2013	4 September 2013
ST1, ST2, ST6, ST9	4 September 2013	11 September 2013

Series X Assignments

Subjects	*Assignment*	*Recommended submission date*	*Final deadline date*
ST1, ST2, ST4 – ST9	**X1**	**5 June 2013**	24 July 2013
ST1, ST2, ST4 – ST9	**X2**	**19 June 2013**	31 July 2013
ST1, ST2, ST4 – ST9	**X3**	**3 July 2013**	7 August 2013
ST1, ST2, ST4 – ST9	**X4**	**17 July 2013**	14 August 2013
ST1, ST2, ST4 – ST9	**X5**	**31 July 2013**	21 August 2013
ST4, ST5, ST7, ST8	**X6**	**14 August 2013**	28 August 2013
ST1, ST2, ST6, ST9		**21 August 2013**	4 September 2013

Mock Exams

Subjects	*Recommended submission date*	*Final deadline date*
ST4, ST5, ST7, ST8	**21 August 2013**	4 September 2013
ST1, ST2, ST6, ST9	**28 August 2013**	11 September 2013

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Assignment X1 Solutions

This document sets out the marking schedules for the questions in Assignment X1.

Notes to the marker or student are given in italics.

Comments about the question are made, where relevant, in boxes.

The text of the marking schedule is in ordinary type.

Mark allocations are [Right hand justified in square brackets]

A limited number of bonus marks are available, where markers feel that a good point is relevant and deserves credit.

Important note to students who are new to Subject ST8

A key skill for this exam is to be able, where the question demands it, to generate a wide range of relevant points. The ability to do this develops with increasing familiarity with the content of the course (and with practice). Schedules for the early assignments are likely, therefore, to include a number of ideas that you have not been able to generate, first time round. Don't be put off by this, but carefully study the marking schedules and learn from the points that you didn't make in your attempt.

Solution X1.1

Common rating factors include:

- sum insured
- number of rooms
- location
- the voluntary or compulsory use of excesses
- whether there is any business use of the property
- whether the policyholder owns or rents the property
- if the property is normally unoccupied during the day
- whether it is a house or flat or some other construction
- type and standard of construction
- age of the building
- type of locks and/or burglar alarms fitted
- whether smoke alarms have been fitted
- high risk contents
- previous claims experience
- family composition
- smoker / non-smoker
- type of cover ("new for old" or indemnity)
- age of policyholder.

[¼ each, maximum 4]

Solution X1.2

(i) ***Facultative vs treaty***

Under facultative reinsurance cover:

- each risk is negotiated individually with the reinsurer [½]
- there is no obligation for the ceding office to offer the business, neither is the reinsurer obliged to accept it [½]
- each case is therefore considered on its own merits, and the reinsurer is free to quote whatever terms and conditions it sees fit to impose for that risk. [½]

Under treaty reinsurance:

- there is an arrangement between the cedant and the reinsurer, whereby the cedant agrees to place certain defined business with the reinsurer [½]
- the reinsurer is obliged to accept the business. [½]

[Maximum 2]

(ii) ***Disadvantages of facultative reinsurance***

The main disadvantages of facultative reinsurance are:

- it is a time-consuming and costly exercise to place such risks individually [½]
- there is no certainty that the required cover will be available when needed [½]
- even if cover is available, the price and terms may be unacceptable [½]
- the direct writer may be unable to accept a large risk until it has been able to find the required reinsurance cover; this inhibits the insurer from accepting business automatically when it is offered, thus reducing its standing in the market. [½]

[Total 2]

Solution X1.3

(i) ***Why premiums may be used as a measure of exposure***

Premiums can generally be used as a measure of exposure because they are usually quite closely related to the insurer's assessment of the amount of risk on a policy. [1]

They are objectively measurable and practical for many purposes. [1]

(ii) ***Reasons why they are not always appropriate***

- not suitable as a measure of exposure for premium rating [½]
- premium bases change over time [½]
- market pressures may lead to distortion of premium bases [½]
- premiums may be set wrongly, *eg* if little experience of new type of risk [½]
- loadings applied to risk premium (for expenses, profit *etc*) change over time [½]
- premiums are subject to inflation [½]

[Total 3]

Solution X1.4

(a) ***Uberrima fides***

Latin for "utmost good faith". [½]

This honesty principle is assumed to be observed by the parties to an insurance, or reinsurance, contract. [½]

An alternative form is uberrimae fidei: "of the utmost good faith". [½]

(b) ***Professional indemnity insurance***

Professional indemnity insurance provides cover for professionals (*eg* actuaries, accountants) … [½]

… against liabilities they may incur due to negligence in the services and advice that they provide to their clients. [½]

(c) ***Nil claim***

A claim that results in no payment by the insurer, because, for example: [½]

- the claim is found not to be valid [½]
- the amount of the loss turns out to be no greater than the excess [½]
- the policyholder has reported a claim in order to comply with the conditions of the policy, but has elected to meet the cost in order to preserve any entitlement to no-claim discount. [½]

[Maximum 4]

Solution X1.5

> ***Comment***
>
> *The purpose of this question is to make you more aware of exactly what you are writing down. Avoid sloppy, inexact statements. The statements made by the three students in the question are not atypical of what students do sometimes write in their answers.*
>
> *The schedule contains a number of marginal points. Full marks can be obtained without giving all of these details.*

Comment A

Generally true. Most motor claims are reported and settled quickly, especially for property damage claims. [1]

But reporting delays do sometimes occur, for example: [½]

- an injury to a third party, such as a back problem, may get worse [½]
- a small loss may initially make the policyholder wonder whether to claim and lose NCD [½]
- an injured party may not claim until they have recovered. [½]

Settlement delays are not uncommon, for example: [½]

- where there is protracted debate on which party (and hence whose insurer) is liable [½]
- for bodily injury conditions to stabilise (and amount established). [½]

[Maximum 3]

Comment B

Again generally true, but depends on class. [1]

For example, in household buildings, late reporting might be mainly due to amounts being relatively small … [½]

… or large subsidence losses which may not be evident for some time. [½]

For employers' liability, for example, late reporting may also result from minor amounts (many asbestosis claims are for small amounts, since they are paid to old people) but other latent diseases can give large settlements compared with a typical early-reported injury. [1]

Other factors affecting the average claim size:

- impact of inflation [½]
- impact of claims settled at no cost [½]
- cautious reserving (which makes unsettled claims appear big). [½]

[Maximum 3]

Comment C

"Not very often", yes per employee, … [½]

… but an office may receive many thousands of claims each year! [½]

Claim frequency varies mainly by type of business. [½]

A large proportion of claims are for relatively minor injuries, so small payments. [1]

A few claims will be for more serious injuries … [½]

… and others for disease / illness (only injury is mentioned in the comment). [½]

Only a very small proportion go to court … [½]

… and some of these will establish that there is no liability for the insurer. [½]
[Maximum 4]

Solution X1.6

(i) ***Definitions***

An *exposure measure* is a basic unit used by an insurer to measure the amount of risk. It is usually stated over a given period. [1]

A risk factor is a factor that is expected to influence the intensity of risk. It is usually backed up by statistics. [1]

A rating factor is a factor that is used to determine the premium rate charged. [1]

Rating factors tend to be measurable risk factors or proxies for risk factors. They should be measurable, verifiable and objective. [1]
[Total 4]

(ii)(a) ***Personal motor***

The exposure measure would be vehicle years. [½]

The risk factors include:

- the number of miles driven
- the density of the traffic where the car is driven
- the ability of the driver
- the speed at which the vehicle is usually driven and its general level of performance
- the ease with which the vehicle can be damaged and the cost of repairing it
- the theft risk
- weight of the vehicle
- fire risk. [¼ each, maximuml 1½]

The rating factors include:

- type of cover (*eg* comprehensive)
- policy excess
- the occupation of the policyholder and other drivers
- whether there are additional drivers of the vehicle as well as the policyholder
- sex of main driver (if allowed as a rating factor)
- age of policyholder and other drivers
- whether or not driving is restricted to certain named drivers
- make and model of vehicle
- the extent of any modification to the engine or body
- location of policyholder (*eg* postal code)
- where the vehicle is kept overnight: on the road / on a driveway / in a garage *etc*
- whether or not the driver has any driving convictions
- past experience. [¼ each, maximum 3]

(ii)(b) ***Marine hull***

The exposure measure would be the insured value of the hull. [½]

The risk factors include:

- size of vessel
- type of vessel
- age of vessel
- condition and maintenance of vessel
- miles travelled
- routes travelled
- crew training / ability
- management ability. [¼ each, total 2]

The rating factors include:

- sum insured
- type of vessel
- commercial category (and whether missiles are covered)
- use of vessel / scope of voyages
- geographic region / destination
- the number and experience of the crew
- deductible
- past experience (loss history). [¼ each, total 2]

[Total 10]

Solution X1.7

(i) ***Definitions***

A *losses-occurring basis* is where the policy provides cover for losses occurring in the defined period no matter when they are reported. [1]

A *claims-made basis* is where the policy provides cover for all claims reported to an insurer within the policy period irrespective of when they occurred. [1]

(ii) ***Changing from a losses-occurring to a claims-made basis***

\+ There will be less uncertainty over future claims. [½]

\+ In particular, exposure to latent claims will be limited, since if claims are not reported within the period, then they will not be covered. [½]

\+ It should be easier to reserve for claims. [½]

+ Claims will be reported more quickly, since policyholders will have to meet a "deadline" if they want their claims to be accepted. [½]

+ The insurer will be able to determine its profits more quickly. [½]

+ There will be greater clarity as to which period of insurance cover each claim relates to. [½]

+ There will be less scope for expensive legal action between insurers to determine who is liable for any particular claim. [½]

– There is a risk that a customer takes out a policy knowing that a claim has already occurred, with a view to claiming for it. [½]

– This basis is out of line with the rest of the market, which may make the policy less marketable … [½]

– … and less well understood, which may lead to customer dissatisfaction. [½]

– Policyholders may face gaps in coverage if they move from this insurer to another insurer, making them reluctant to buy policies from this insurer in the first place. [½]

– The policy may not meet the needs of the customer, since policyholders may require cover for latent claims. [½]

– The claims that emerge will be from different periods of exposure. This may make it harder to analyse experience. [½]

– There may be an increased number of claims reported (as policyholders are more likely to report claims as soon as they become aware of them) which could increase claims handling costs. [½]

[Maximum 6]

Solution X1.8

(i) ***Types of reinsurance***

These could include:

- quota share
- surplus treaty
- risk excess of loss
- aggregate excess of loss
- catastrophe excess of loss
- stop loss
- financial reinsurance. [¼ each, with ¼ bonus if all mentioned]

(ii) ***Reasons***

The company is well diversified and so does not need reinsurance for that purpose. [½]

Also, if it is large it is probably well-capitalised, which means that it could rely on the cushion of free assets rather than reinsurance. [½]

A large company should have sufficient experience and so is less reliant on technical assistance from the broker and/or reinsurer. [½]

The company may have a particularly bad relationship with the existing reinsurance brokers. [½]

Much of the business written may not be volatile, leading to less need for reinsurance. In particular, there may be few large risks that could lead to large claims. [½]

Alternatively, the company may have taken the view that the benefit gained from the reinsurance was not sufficient. For example, few recoveries may have been made in recent years. [½]

In any case, it may be that the reinsurance is deemed expensive in the current climate. This could be due to a lack of capacity in the reinsurance market. [½]

The company may have decided that suitable reinsurance is not actually available in the market at the current time. [½]

Not having reinsurance would mean that profit would not be ceded. Shareholders may thus receive better returns. [½]

Also, the company could spend the money elsewhere, perhaps more fruitfully. [½]

There would also be the expense saving resulting from not having to organise the reinsurance purchase and manage ongoing reinsurance recoveries. [½]

Disadvantages

The company may not be able to meet catastrophe claims or claims resulting from an aggregation of risk. [½]

It is likely that the company covers liability business. Individual claims on this type of business can be very large. [½]

Even a large, well-financed company may not be able to cope with these sorts of events without reliance on reinsurance. [½]

For example, even if it has the finances to pay for unexpected claims, it will still need to protect its solvency margin for other purposes, *eg* expanding the business. [½]

If the company did cease its reinsurance arrangements, this may weaken any relationship with the broker. This would be particularly important if it later decided to re-enter the agreements, perhaps when reinsurance rates soften or when the need arises due to new ventures. [1]

Not having reinsurance would probably concern the regulators … [½]

…and would look out of line with competitors. [½]

Rating agencies would also be concerned. A downgraded credit rating could result in loss of credibility for the insurer and, ultimately, loss of business. [½]

Reinsurance may even be compulsory for some classes. [½]

The shareholders will want smooth dividends. This will be more difficult to achieve without reinsurance. [½]

[Maximum 10]

Solution X1.9

It would be important to take into account the existing reinsurance arrangements of the insurer. In particular:

- details of existing reinsurance arrangements (types of reinsurance, retention levels *etc*) [½]
- appropriateness of existing reinsurance arrangements. [½]

The type(s) of business sold should also be considered. In particular:

- the classes of business written [½]
- the size of the risks [½]
- the volatility of claims experience [½]
- the geographical spread of risks [½]
- possible accumulations of risk [½]
- the risk of catastrophes [½]
- the predictability of the business written, which will depend on: [½]
  - the extent of past claims experience, say 5–10 years [½]
  - the goodness of fit of statistical distributions to past claims experience. [½]

Features relating to the company will be important, in particular:

- the size of the company [½]
- the solvency margin / free assets [½]
- the company's appetite for risk. [½]

Features relating to the available reinsurance will also be important, in particular:

- alternatives to reinsurance [½]
- availability of reinsurance with higher retentions [½]
- cost of reinsurance (perceived value for money) [½]
- reinsurance premiums relative to gross premium income [½]
- the security of the available reinsurance. [½]

The reasons why reinsurance has been purchased at all should be taken into account:

- the limitation of exposure to risk / need to spread risks [½]
- the need to stabilise results [½]
- the effect on the solvency position [½]
- the need for financial support [½]
- the need for technical assistance. [½]

Finally, the impact on the business of the change should be considered:

- the effect on profitability of higher retention levels [½]
- the effect on business volumes [½]
- the effect on the investment freedom of the insurer [½]
- the possible reduction in administrative costs if there is less contact with reinsurers. [½]

[Maximum 10]

Solution X1.10

Consideration should be given to the benefit provided, any maximum levels of benefits, and any indexation of these benefit levels. [1]

Benefits might include:

- vets fees, which may be capped at a maximum amount per year, per illness or per treatment [½]
- hospital benefits, which may be capped at a maximum total amount, or a maximum stay in hospital [½]
- death benefit (for death by accident or illness up to a certain age), which should cover the value of the pet, up to a maximum value [½]
- loss benefit, which should cover the value of the pet, up to a maximum value [½]
- advertisement costs if the animal goes missing [½]
- kennel fees if the owner is hospitalised [½]
- quarantine costs [½]
- liability cover if the pet attacks a third party [½]
- liability cover if the pet damages property. [½]

There may be different bands of benefit. [½]

Bearing in mind the benefits provided, it would be necessary to consider the perils that are likely to be insured. [½]

The period of cover of the contract may be one year (so that it is an annually renewable product) or a single premium contract covering the pet over its lifetime. [1]

Terms and conditions would need to be clearly defined. [½]

The terms and conditions of the product may include:

- the level of excess (if any), which may be a fixed amount, or a percentage of the claim [½]
- exclusions, *eg*:
  - certain routine treatments and vaccinations [½]
  - any pre-existing conditions [½]
  - certain breeds [½]
  - certain uses of pet [½]
  - claims resulting from incidents outside of certain geographical limits [½]
  - war / nuclear risks [½]
- eligibility criteria, such as minimum and maximum age at entry [½]
- any cancellation terms under the policy. [½]

In determining benefit levels, terms and conditions, customer needs would need to be considered. [½]

It may be necessary to offer different products through different distribution channels. [½]

[Maximum 9]

Assignment X2 Solutions

Solution X2.1

In any given class of general insurance over a period of time, companies move from large profits to losses (or very small profits) and back again. The effect is much more pronounced for some classes of business than for others. [1]

When profits are large, existing companies seek to expand and new companies are attracted into the market. [½]

Increased competition drives down premium rates (or weakens underwriting controls), and profits are subsequently reduced, or losses are made. [½]

Faced with inadequate profits, some insurers contract or withdraw from the market. [½]

Premium rates can then increase for the remaining companies, who subsequently make good profits again. [½]

[Total 3]

Solution X2.2

- the modellers might select a different period of data on which to base their analysis [½]
- the modellers might select a different set of data to use to determine parameters (leaving other data for validation purposes) [½]
- the modellers might adjust the base data differently, for example in terms of data cleaning or reserve estimates for individual claims [½]
- if the modellers have access to the data in different formats, then they might have different rating factors available to them [½]
- the modellers might place a different weighting on the extent to which they base their selection on the results of statistical tests compared to the subjective judgement they apply [½]
- if both use subjective judgement, then their selections are likely to differ [½]
- the modellers might have different purposes for the investigation [½]

[Maximum 3]

Solution X2.3

Restrictions on the type and volume of business that a general insurer can write or classes for which the insurer is authorised. [½]

Restrictions on the territories in which the insurer can write business. [¼]

Limits or controls on the premium rates that can be charged. [½]

A requirement that the general insurer maintains a minimum level of solvency, measured in some prescribed manner. [½]

Restrictions on the types of assets or the amount of a particular asset that a general insurer can take into account for the purposes of demonstrating solvency (including matching requirements). [½]

A requirement to use prescribed bases for calculating premiums or for valuing the general insurer's assets and/or liabilities when demonstrating solvency. [½]

A requirement to take account of uncertainties and risks in the business when calculating the solvency requirement. [½]

Restrictions on which individuals can hold key roles in companies. [½]

Licensing of agents to sell insurance and requirements on the methodology of sale (including advertising restrictions). [½]

Legislation to protect policyholders should a general insurer fail. [½]

Restrictions on the information that may be used in underwriting and premium rating. [½]

A requirement to deposit assets to back outstanding claims reserves. [½]

A requirement to pay protection levies to consumer protection bodies. [½]

A requirement to provide detailed reports and accounts at prescribed intervals. [¼]

A requirement to purchase reinsurance. [¼]

A requirement to hold a claims equalisation and/or catastrophe reserve. [¼]

Limitations on contract terms and conditions, *eg* minimum required level of cover. [¼]

Limitation on ownership, *eg* can only own 49% of an Indian company. [¼]

Monopoly and merger restrictions. [¼]

Requirement to have an office in a location if writing business there. [¼]

Compulsory covers, *eg* requirement to offer terrorism cover in some countries. [¼]

A prescription to hold certain assets. [¼]

Data protection procedures. [¼]

Requirement to uphold customer treatment standards. [¼]
[Maximum 6]

Solution X2.4

The statistics would be split into two: those for the new product and those for the existing broker product. [½]

For the new product

We would need to calculate new business volumes, split by premium income and number of policies. This would allow the company to compare their actual experience with what was expected. [1]

This would be used, for example, to:

- test the competitiveness of the premium rates [½]
- judge the effectiveness of any advertising [½]
- monitor capital requirements (if this is an issue) [½]
- allow for expenses (*eg* spreading fixed costs) when setting new rates. [½]

The business volumes will be monitored across different rating groups, to judge where the premium rates are out of line with the market. [1]

This will aim to reduce the impact of selection. Selection is a particular risk in this case since we have fewer rating factors than the broker product. [1]

Once the business has been in force for a while, the company will want to measure renewal and cancellation rates. [1]

This will enable it to test the competitiveness of its premium rates and to judge the loyalty of its customers. It might also indicate other factors such as the perceived quality of service. [1]

For the existing product

New business volumes, possibly split by broker, might reveal if there are:

- policyholders who realise that the new direct product is cheaper, and get cheaper premiums by cutting out the broker [½]
- falls in sales from brokers who do not like the new telephone operation [½]
- general trends in the market away from broker sales. [½]

For both products, endorsement rates will be used as a check that all exposure data is accounted for in a claim analysis. Endorsement rates may also be useful for expense allocation and to plan staffing levels. [1]

[Maximum 8]

Solution X2.5

- check for completeness of the data, *eg* raw claims and policy data reconciles with the accounting information [½]
- high level checks of frequency and average cost over periods of time [½]
- compare data with that used in a previous review [½]
- check that claims have attached correctly to the policy data [½]
- check that data hasn't been corrupted in the cleaning process, *eg* using one-way tables by rating factors [½]
- check that the data has then fed through correctly into any rating software packages [½]
- reconcile any adjustments for IBNR and IBNER with figures produced by the claims reserving department [½]
- check that frequency and average cost models combine correctly, and that all claim causes are included [½]
- check that frequency, inflation or any other trends have been correctly projected to the expected claims payment dates for the new rating series [½]
- comparison of risk premium rates with a previous analysis [½]
- adjustments made to allow for new rating factors have been made on a consistent basis across each peril [½]

- check that appropriate allowance has been made for individual large claims and accumulations [½]
- obtain sample quotes for new business and renewals [½]
- as you are a consulting actuary, you are likely to have information regarding competitors' risk premiums and therefore can check against this [½]
- gross up risk premium for general level of expenses, *etc* and compare with office premiums of competitors [½]
- check that only household contents data included (*eg* where combined buildings and contents business) [½]
- check that recent and appropriate data has been used [½]
- check credibility of data in cells and homogeneity within cells [½]
- check suitability of model, including appropriate rating cells [½]
- check that the increases in risk premiums look reasonable by comparing with recent years' loss ratios [½]

[Maximum 8]

Solution X2.6

With a few exceptions, such as personal accident insurance, the size of payment to be made will not be known in advance, particularly if it may be subject to inflation. [1]

When considering the patterns arising in the dates of claims payments there may be several factors acting, for example:

- the policyholder may not even know the contingency giving rise to the claim has taken place, for instance a slowly developing disease, or
- the claim may have been notified to the broker but they have not yet passed it on to you the insurer. [1]

Even after full details of the claim have been reported to an insurer, different members of the claims staff could reach different but equally valid conclusions about the facts and so produce different estimates of the liability. [1]

There may be protracted discussions about whether or not the claim falls within the terms of the policy. [½]

During the period to settlement there may be long discussions with the reinsurer who may take a different view about the claim from the insurer. This can result in delays and lower than expected recoveries. [½]

Even if the insurer could estimate the liabilities arising from the current year's business, it will be very difficult for it to estimate the liabilities arising from future years. These will be made up from:

- claims from business which renews with the insurer – it will be difficult to predict how much of the business will renew at the end of each year, and
- claims from completely new business, which will be even harder to estimate. [1]

Business mix and volume may not be the same as in the past. [½]

Standards of underwriting or claims handling may change, impacting liabilities. [½]

Unexpected legislative changes may affect liabilities. [½]

There will be uncertainty over the extent and eventual value of any large losses or catastrophe claims. [½]

Latent claims may emerge. [½]

The chosen statistical model or its parameters may not be suitable. [½]

The assumptions underlying the statistical method / model may no longer be appropriate (for example the run-off triangle assumption for development). [½]

[Maximum 6]

Solution X2.7

(i) ***Why there are so many questions on motor proposal forms***

Reasons include:

- many different risk factors [½]
- risk factors difficult to measure and verify, so information is collected on various rating factors instead, to use as proxies for the risk factors [½]
- many different possible causes of claims [½]
- high variability in risk levels [½]
- insurers collect data for rating factors that they are not currently using. [½]
- market practice – potential customers seem willing to answer a large number of questions, particularly if they think this may entitle them to a cheaper price [1]
- the method of sale, *eg* internet or telephone, may enable many questions to be answered relatively quickly. [1]

The risks faced by an insurer are:

- how likely the policyholder is to claim, and [½]
- how big each claim will be, on average. [½]

There are many factors affecting each of these (*ie* many different risk factors), and there are many different causes of accidents or claims. [½]

In order to charge individuals premiums that reflect the risk, data needs to be collected to define the risk as accurately as possible. In practice, many of the risk factors may not be directly measurable or verifiable, and rating factors are instead used as proxies for the risk factors. [1]

For example, "how good a driver is" might be indicated by a combination of: age, number of years' driving experience, recent experience, accidents allowed or not, occupation, convictions. [1]

Information is also collected for marketing purposes. [½]
[Maximum 7]

(ii) ***How to reduce the amount of data***

The first possibility is to investigate claims experience carefully and decide which, if any, rating factors can be removed. Some of the rating factors may be proved to have negligible impact on the level of risk. [1]

This approach may still lead to problems if the company's rating structure is very different from the rest of the market, due to the risk of selection. [½]

Other possibilities are to:

- offer insurance to niche groups that can be defined by fewer rating factors [½]
- automatically decline to quote for anything but standard risks. [½]

[Maximum 2]

Solution X2.8

The cost structure

There will be one-off set-up costs, *eg* IT staff and equipment. [½]

Advertising costs may need to increase in the short term, but may fall in the longer term where less expensive means are available on the internet. [½]

Cheaper claims handling may become possible, *eg* due to greater use of internet or email. [½]

The overall long-term costs across the company should be lower as efficiency gains are achieved. [½]

Overall fees to brokers are likely to fall as less business is obtained through them. [½]

The cost structure will change as more of the expenses are fixed (maintaining an infrastructure) and less are variable (commission). [½]

Broker arrangements

Brokers may be unhappy about the introduction of the new sales channel. [½]

This may lead to further reduction in broker business. [½]

The number of brokers and their commission arrangements, together with the pricing strategy, may need to be changed. [½]

Capital efficiency may improve because the balance of premiums tied up with brokers (and hence not investible) is likely to reduce. [½]

Calculation of premiums

Set-up costs will need to be amortised over a suitable period in the premium calculation. [½]

Separate premium rates are likely to be appropriate for internet sales, *eg* to reflect the lower expenses. [½]

A more sophisticated premium calculation system may be possible for internet sales. [½]

Need to consider consistency between the broker and internet premium, but note that the internet rates will need to be competitive in the market. [½]

It may be possible to obtain policy information in more detail, which may allow the use of new rating factors. [½]

It should be easier to update rates more frequently to allow for new experience as it emerges. [½]

Business mix and volumes

The business mix will change, probably steering towards the more financially aware. [½]

It will probably worsen a little as underwriting will be less tight so we are more likely to accept a few bad risks. [½]

The age profile and geographical spread of customers is also likely to change. [½]

This will affect claims experience and persistency. [½]

Projected business volumes will hopefully improve overall, but are highly uncertain. [½]

Much depends upon the competitiveness of the premium rates, the level of advertising, design of the website and whether people actually want insurance to buy over the internet. [½]

Renewal rates may improve due to the ease of renewal via the internet (alternatively, they may deteriorate due to more customers shopping around). [½]

Reinsurance arrangements

Reinsurance arrangements are likely to be reviewed, *eg* to reflect changing business mix/volumes. [½]

Initially, there may be an increased need for obtain technical assistance from the reinsurer on the new sales channel. [½]

There may be an increase in large losses as bad risks may be filtered out less well over the internet. Hence we may require more Risk XL reinsurance. [½]

The business may grow sufficiently to let the company rely less on reinsurance. [½]
[Maximum 10]

Solution X2.9

(i) ***Why past experience may not be a good guide to the future***

Inflation can invalidate claims amount data. [½]

For travel insurance, there might be many different types of inflation, *eg*:

- price inflation (which will affect the cost of replacing lost items) *vs* medical inflation (which will affect medical costs from accident / bodily injury claims) [½]
- different rates of inflation in different countries. [½]

With many different types of inflation to consider, it may be difficult to analyse the underlying claims experience. [½]

Similarly, there are many possible trends that could affect both claim frequencies and claim amounts. [½]

For example:

- crime rates in different countries could affect the frequency of theft claims [½]
- popularity of different countries could affect the cost of holidaying there, which would affect the size of cancellation claims. [½]

Currency is also a significant risk for travel insurance. Exchange rates are liable to fluctuate significantly, which could invalidate past claim sizes, although an explicit adjustment could be made for this. [1]

Since the insurance company is relatively new, and growing rapidly, its claims management and payment procedures will not be established (and so may be quite volatile). This will make it difficult to assess claims run-off patterns from past data. [½]

Furthermore, underwriting and/or claims management procedures may change in the future. [½]

Alongside the re-price there may also be changes in the cover provided, *eg*:

- policy wordings [½]
- excess levels. [½]

Other recent / future changes that might invalidate the use of past experience include changes in:

- the distribution channel used [½]
- the mix of business [½]
- legislation. [½]

The level of competition may have changed leading to a change in the stage of the insurance cycle, which would invalidate certain items of experience (such as profit margins). [½]

Finally, the two year period to which the data applies may have been unusual, *eg* it may have been a severe recession. [½]

[Maximum 7]

(ii) ***Uncertainty arising from using this data***

Even though the insurance company is growing rapidly, there is unlikely to be enough internal data to accurately assess claim frequencies and severities. [½]

In particular, there will almost certainly be a lack of data on:

- unusual risks [½]
- low-frequency claim types. [½]

The insurer may have no experience of certain types of claims. For example, it may not have experienced a significant (catastrophic) event, such as a tsunami. [½]

There is also unlikely to be enough of a history of data to identify trends. [½]

The lack of data will prevent the insurer from holding any data back to use for validation purposes, which could result in the model "over-fitting" the data. [½]

Since the insurer has been focussing on growth, it may have been lax in its data handling – in particular its data entry and subsequent data manipulation. [½]

Also, this is a new product and so there could be teething problems associated with the IT systems used for recording data. [½]

As a result, the data may be:

- incomplete [¼]
- incorrect. [¼]

Since the insurer is taking data up to the current time, it will have IBNR claims to estimate, which is a further area of uncertainty. It could leave a period between the end of the exposure period and the analysis, but lack of data will almost certainly rule this option out. [1]

In addition, many claims may not be fully developed. This is fairly likely for travel insurance because verifying and settling claims in other countries can be difficult. [½]

Assumptions on volume and mix will also be particularly difficult to predict since the insurer will not have data from a stable period of time. [½]

Expenses are uncertain – particularly for a relatively new insurer that will have some relatively large set-up costs. [½]

Since the insurance company is growing, there will also be uncertainty over:

- commission, since the bigger the insurer gets, the more power it will be able to exert over distributors [½]
- staffing levels (administrators, underwriters, claims assessors *etc*) [½]
- per policy expense loadings. [½]

[Maximum 7]

Solution X2.10

(i) ***Particular risks***

There is likely to be a lack of claims data available with which to accurately price the product. Even if the State could provide statistics on claims this is unlikely to be sufficiently detailed or relevant for this purpose, particularly given this is a small country. [1]

Volumes of business will be uncertain. It will be difficult to predict how many people will own pubs, and hence require this type of insurance. Competition from other insurers will also be a very relevant factor here. [1]

It will be difficult to predict the mix of business – *eg* by type of pub or location – the insurer will be undertaking. [½]

The insurer will have little experience in knowing what underwriting and rating factors to ask for and use (and to what extent), which may leave it open to undesirable risks and anti-selection. [1]

As there have not been any similar products in the market it will be difficult to establish clear and appropriate policy conditions (including exclusions). This may result in payment of claims that are not intended (or may lead to low sales volumes). [1]

Claim sizes at any one time could be very variable – for example, some liability claims (*eg* as a result of a serious injury) could be large. [½]

Also, claims may be very volatile over time – for example, there may be the occasional serious fire leading to total damage to the property (and many liability claims). [½]

The volatility and variability of claims is made worse because the country is small, and so experience will be limited. [½]

There will be uncertainty over the public's attitude to claiming. For example, there may be an increasing number of members of the public suing the pubs for accidents occurring on the premises, leading to an increase in liability claims. [1]

Crime rates will be uncertain. This will, for example, affect claims for arson and theft. [½]

The attitude of landlords to claiming is uncertain. For example, fraudulent claims may be an issue (*eg* a new pub fails to make money, and so the landlord makes false or exaggerated insurance claims to make up the loss). [1]

The impact of legislation may be uncertain. The government is likely to enforce controls on these new pubs (*eg* restrictions on opening hours), which are likely to change as the situation develops. [1]

Uncertainty over economic conditions is another source of risk. This may affect sales volumes (and renewals), as people are less likely to want to run pubs (and afford insurance premiums) in times of economic depression. [½]

Possible accumulations of risk may arise, *eg* if the pubs insured are all in a similar location (it is a small country). [½]

Reinsurance may not be available or may not be offered at a suitable price (*eg* liability cover may be scarce or expensive). [½]

[Maximum 9]

(ii) ***Restrictions and exclusions***

The insurer might decide not to cover certain properties that lead to a risk that is unacceptable to the insurer. [½]

For example:

- buildings with flammable materials, *eg* pubs with thatched roofs [½]
- buildings in high-risk locations, such as on river banks (flooding risks). [½]

The following exclusions might also apply:

- any deliberate loss, such as arson initiated by the landlord
- situations where the policyholder has not taken reasonable steps to prevent the loss (*eg* a break-in where security measures have not been activated)
- liability which is the fault of a third party
- loss arising as a result of war, terrorism and similar risks
- loss arising from illegal or criminal activity
- loss or theft where the police have not been informed
- loss or theft of cash (and other items that would be difficult to verify)
- costs below a specified excess level or above a specified policy limit. [½ each]

[Maximum 4]

Assignment X3 Solutions

Solution X3.1

The delay may be due to:

- the length of time that can elapse before sufficient claims have been notified [1]
- delays in processing and analysing the claims experience [1]
- the time taken to assess and agree with management the new premium rates and premium structure to be charged [1]
- the time required to administer and implement the new rates [½]
- the time delay between the risk period and the payment of claims [½]
- the possible need to obtain approval from a regulatory body. [½]

[Maximum 4]

Solution X3.2

(i) ***Excluding subsidence claims***

Advantages

- Future claim frequency will fall, so total claim costs will be lower.
- The uncertainty as to future claim frequency and claim amount from subsidence claims will also be eliminated.
- Future claim handling expenses will be lower.
- Policyholders may not want the cover and may prefer lower premiums.

Disadvantages

- Policyholders will want the cover, so loss of new business.
- Policyholders expect cover, so possible ill-will and cost of arguing with policyholders later.
- May be better alternatives to the problem like higher premiums, excesses or use of reinsurance.
- Administration problems if different policy conditions for different groups of policyholders.

Markers, award bonus marks for any other valid points [½ per point, total 4]

(ii) ***Premium calculation***

The company would need to revise its claim frequencies and claim amounts, to give distributions excluding subsidence cover. [½]

The claim frequency would be estimated by looking at past claims, by peril, excluding subsidence claims from the analysis. [½]

The claim frequency to include in the rates would be a projection of trends, to give expected future claim frequency. Therefore, the projection of trends should exclude subsidence claims. [½]

The claim amount would be adjusted by looking at past experience of claim amounts, excluding subsidence claims, to give an average cost per "non-subsidence" claim. [½]

When inflating the claim amount the office would need to ignore subsidence claim inflation from its inflation assumption. [½]

The revised risk premium is calculated by adjusting the frequency and amount to allow for any policy excess. [½]

Other loadings would also be adjusted, such as expense loadings, profit and contingency margins. [1]

[Maximum 3]

Solution X3.3

Comment

This type of question requires a very broad, lateral thinking approach, perhaps combined with a checklist of all the key concepts for Subject ST8. There are many potentially important points that could be given for the first part of the question so you need to exercise some judgement when selecting the "main reasons". It is essential that you give enough clear explanation of each of your points. Your explanations should be clear and succinct.

Markers (including examiners) do not like:

(a) long waffly, rambling explanations which go on and on and somehow never seem to come to any clear and distinct point, going round and round in circles, repeating the same thing over and over again, which have lots of words to be read, which is even more of a problem if your writing is bad, but at the end of the day you are never too sure what it was that was being said in the first place.

(b) spurious statements where the marker is not clear whether you know how and why the given point might be relevant, eg "mix of business". The marker will not give you the benefit of the doubt, so it is up to you to convince the marker that you do know what you are talking about. Examiners describe this as a "scattergun" technique or "note-form". They don't like it.

(c) repetition of the same point in different words. This tends to occur because you know you need to write more but can't think what to write or because you don't really understand and so don't actually recognise that you are saying the same thing twice. Either way examiners assume that it means you have absolutely no idea about anything to do with general insurance.

(i) ***Main reasons why the suggestion is probably not appropriate***

The period of exposure (01/10/11 – 30/9/12) includes business written before 01/10/11 (the last date of premium revision). [½]

About half of the incurred claims may have arisen from premiums written prior to 01/10/11. [½]

So, a straight percentage increase based on a loss over this financial year may not be appropriate. [½]

The loss in 2011/12 might not have been representative of the expected future experience. [½]

2011/12 might have produced exceptionally bad claims experience from an unusual event (*eg* exceptionally icy or stormy winter). [½]

No allowance has been made for important trends:

- inflation of claims costs and expenses [½]
- trends in experience (*eg* increase in car thefts). [½]

Investment income and gains affect profit but have not been considered. [½]

No allowance appears to have been made for what the rest of the market is doing. [½]

Our profit / loss will depend on the number of policies sold, which is affected by our pricing policy compared with that of competitors. [½]

The loss might be due to an inappropriate rating structure. A flat 5% increase would not address this. [½]

The problem may be expense overrun. This could be addressed by being more efficient (*ie* reduce costs where possible). [½]

Policy conditions may have changed. [½]

The "loss" might not be as drastic as at first appears. It depends on the basis used to determine the loss with regard, for example, to:

- investment income
- reserving basis
- allocation of expenses between classes. [1½]

In particular, consider whether the loss is still there on a realistic reserving basis, allowing for accurate allocation of expenses and investment income. [1]

Even if the above points were ignored, the calculation of 5% (presumably 3÷60) is inappropriate because even if the rates had been 5% higher:

- we would have sold fewer policies (so fixed costs may be less well covered) [½]
- some of the increased premium would be paid out as commission [½]
- we would want to make some profit. [½]

[Maximum 9]

(ii) ***List of information required***

Company data for full claims and exposure experience analysis over a reasonable period. This should be suitably matured (say 9 months) and if the company is small then you would need to go back for a number of years. [1]

You need details of policies (*ie* risk factor data) and corresponding claims data (*ie* amounts and dates paid). Claims data should include best estimates of outstanding claims. [1]

This is the key idea. It is not advisable or necessary here, though, to list all the detailed data required to undertake this analysis, because there are lots of other relevant ideas you need to allocate time to.

Also we need:

- details of how existing rates were derived
- details of large claims and periods of unusually severe conditions
- period new rates are to be in force
- inflation data and projection of future inflation
- details of changes in the risks (*eg* mix, policy conditions, underwriting or claims settlement)
- details of environmental factors that might affect claims (*eg* economic, legal or technical changes)
- variable per-policy handling expenses
- required per-policy contribution to fixed costs
- commission levels
- estimates of likely sales volumes (*eg* based on modelled past experience)
- expected investment rates
- profit loading and return on capital
- contingency loading
- market considerations (*eg* details of competitors' premium rates, the stage of the insurance cycle, the acceptability of the increase to the market and the level of brand loyalty)
- details of reinsurance arrangements.

[½ each, maximum 6]
[Total 8]

Solution X3.4

Main issues to the company

If the quote is accepted it will result in expanding to a new class of business and hopefully giving a new profitable stream of income. [½]

Need to decide who will be responsible for the administrative duties including the sales and claims administration. If the FU carries out a lot of the duties for the company, we need to decide how closely it should be monitored. The commission paid to the Union will have to reflect the work that they do. [1]

If the FU acts as a broker, this may not reduce costs as there will be duplication of administration. [½]

Need to ensure adequate systems are in place to cope with the new business. Systems currently used for other motor business can hopefully be used with little modification needed. [½]

Accounts and reserving processes must be set up or changed to cope with the new class. [½]

The expansion should fit in with the company's strategic plan. It needs to complement the existing portfolio and not lead to new excessive accumulations of risk. [½]

Consider whether the company has enough experience to be confident of quoting a premium rate. The risks should not be too great and rates can be revised once reliable data has been collected. [1]

Consider whether adequate reinsurance is in place. If not, see if it is available at an acceptable price. [½]

Staff should be adequately trained to be able to cope with the new business. [½]

The company may effectively have a monopoly over the tractor insurance market which means it could be very profitable. Can the business be retained over the long term? Can the connection with the FU be used to gain other business from farmers like household insurance? [1]

[Maximum 6]

Solution X3.5

(i) ***Rationale behind the proposal***

The age of policyholder is likely to be an indicator of future claims experience. [½]

For example:

- older people may be more likely to stay at home during the day, and this reduces the opportunity for theft claims [½]
- there may be lower moral hazard with older policyholders, *eg* they may be more careful not to leave windows open. [½]

The use of this additional rating factor should remove some residual heterogeneity. [½]

The age of the policyholder is objective, easily measurable and verifiable. [½]

The age of the policyholder may also be an indicator of future persistency. For example, younger policyholders may be more likely to shop around and so not renew policies (leading to higher per-policy expenses). [1]

Charging a premium according to this risk factor will encourage more of the lower risks (*ie* older policyholders) to take out policies with the insurer … [½]

… and similarly, fewer of the higher risks (*ie* younger policyholders) will take out the policy. [½]

This should lead to better experience overall, and hence cheaper premiums generally, or higher profitability for the insurer. [½]

This will be particularly important if other insurers are also rating by age of policyholder, in which case this insurer will be subject to anti-selection if it does not rate by age. [1]

If premiums get cheaper as a policyholder gets older, this should result in lower premiums on renewal, which will improve persistency levels. [½]

[Maximum 5]

(ii) ***Difficulties***

The insurer will need to calculate the extent to which the age of the policyholder will affect the premium charged. [½]

If data on the age of the policyholder has already been collected then the likely impact could be calculated from the insurer's own data, *eg* by calculating premium rates directly for each age or age-band (or by statistical methods). [½]

But if the age of the policyholder has not historically been collected, this will instead need to be estimated using external data, which may not be available … [½]

… or from a purely subjective view, which may not be accurate. [½]

In any case, the data available may not be sufficiently reliable or credible to be able to calculate an accurate premium. [½]

If the adjustment to the premium rates to allow for age is incorrect, this may result in anti-selection. [½]

Application procedures may need to be amended in order to collect and use the required information (*ie* ask for the date of birth of the policyholder) from now on. [½]

Procedures may need to be put in place to verify that the age of the policyholder is correct (*eg* random checks on a sample of claims or insist on sight of birth certificate on application). [½]

In households with more than one occupant, the age of the policyholder will only make a crude allowance for the ages of all the people in the household. [½]

Customers may object to being asked their age, and this may result in reduced sales. [½]

As the rating structure is currently fairly simple, the addition of one more rating factor may be seen to over-complicate the product. [½]

The insurer may be accused of discriminating against people of a certain age (by charging them a higher premium for cover). [½]

The rating factor may not be liked by sellers (*eg* if it is seen to make the product more difficult to sell), and some sellers may be less willing to sell this insurer's products. [½]

The insurer will need to consider:

- the need to smooth rates from one age (or age band) to the next [½]
- the need for a simple rating structure, *eg* using 10-year age bands. [½]

Rating manuals (or quotation software) will need to be redesigned, and sellers may need to be trained to explain the effect of policyholders' age on the premiums charged. [½]

The insurer may need regulatory approval to allow for the additional rating factor – *eg* it may need to provide statistical evidence that age of policyholder does have an impact on claims experience. [½]

The proposed change could make a significant difference to the premium paid by existing policyholders on renewal, which may adversely affect renewal rates. For example, a younger policyholder might see a significant premium increase when the new rating factor is introduced. [½]

The age of the policyholder might not actually make a significant difference to the predictability of claims – in which case it may not be worth adding it in as a rating factor. [½]

Alternative rating factors, which may reflect the risk better and/or be more acceptable, should be investigated. [½]

Reinsurance arrangements may need to be reviewed following the proposal. [½]
[Maximum 8]

Solution X3.6

> ### *Comment*
>
> *This question is not as bad as it first looks. A methodical, step by step approach should see you through.*
>
> *In the past the presentation of many students' answers has been poor. Many just give a page of algebra or numbers with very little explanation, which makes some of the scripts difficult to follow. This inevitably results in lost marks. Also, many students make assumptions without explicitly stating them. Avoid these tendencies!*

Claim cost

From (n), and assuming policies commence on average on 01/07/13 [½]

... average date of claim settlement is 01/04/14. [½]

From (a), (b) and (c), the amount of claim payment is:

$$1000 \times 1.01^{21} - 300 = £932.39. \quad [1]$$

Discounted to 01/07/13 at given rate of interest (1% per month), claim cost per claim is:

$$932.39 \div 1.01^{9} = £852.52. \quad [1]$$

Expenses per policy

Information on the per-policy expense is not very clear. Assume: £1 at 01/01/13, inflating at 1% per month. For contracts starting 01/07/13, the starting amount is increased by 6 months of inflation. So the present value of these expenses at inception is:

$$12 \times 1.01^{6} = 12.74 \quad [1]$$

The expenses are assumed to be paid after 9 months (3 months later than the expected average date), so there is another 3 months to discount at 1%, giving £12.37 [1]

Probabilities of claim

Probability of no claims = $P(N=0) = 0.25^0 \times e^{-0.25} =$ 0.77880 [1]

Probability of one claim = $P(N=1) = 0.25^1 \times e^{-0.25} =$ 0.19470 [1]

Probability of two or more = $P(N \geq 2) = 1 - 0.77880 - 0.19470 =$ 0.02650 [1]

Calculation of premium

$$\text{Premium} = \Pr(N=1) \times (\text{value at 1/7/13 of claim cost} - 1/2 \text{ premium})$$

$$+ \Pr(N \geq 2) \times (2 \times \text{value at 1/7/13 of claim cost} - 1/2 \text{ premium})$$

$$+ \text{ per policy expense as at 1/7/13}$$ [2]

$$+ \text{ acquisition costs}$$

$$+ \text{ profit loading}$$

To be precise, the reinstatement premium should be discounted for nine months at 1% per month. We have ignored this added complexity.

$$P = 0.1947 \times (852.52 - P/2) + 0.0265 \times (2 \times 852.52 - P/2) + 12.37 + 0.25P + 0.15P$$

$$P = 223.54 \div 0.7106$$

$$P = 314.58$$ [1]

Hence the required premium is £315. [1]

[Total 12]

Marks would be deducted for:

- *sloppy explanations*
- *not stating assumptions explicitly*
- *calculation errors*
- *erroneous logic.*

Markers would be particularly harsh on answers that are not close to £300.

Solution X3.7

(i) ***Finding a and b***

Since N has a negative binomial distribution with parameters $k=3$ and $q=0.1$:

$$P(N=n)=\binom{n+2}{n}(0.9)^3(0.1)^n$$

and:

$$\frac{P(N=n)}{P(N=n-1)}=0.1\times\binom{n+2}{n}\Big/\binom{n+1}{n-1}=0.1\frac{(n+2)(n+1)}{(n+1)n}=0.1+\frac{0.2}{n}.$$

Hence the relationship holds with $a=0.1$ and $b=0.2$. [2]

(ii)(a) ***Using the recursive method***

The recursive formula states that:

$$P(S=s)=\sum_{x=1}^{s}\left(a+\frac{bx}{s}\right)P\left(X=x\right)P\left(S=s-x\right),\quad s\geq 1$$

and:

$$P(S=0)=P(N=0)$$ [1]

So, working in units of £500, we have:

$$P(S=0)=P(N=0)=0.9^3=0.729$$ [1]

$$P(S=1)=0.1\left(1+\frac{2\times 1}{1}\right)P(X=1)P(S=0)=0.1\times 3\times 0.5\times 0.729$$

$$=0.10935$$

[1]

$$P(S=2)=0.1\left(1+\frac{2\times 1}{2}\right)P(X=1)P(S=1)+0.1\left(1+\frac{2\times 2}{2}\right)P(X=2)P(S=0)$$

$$=0.06561$$

[1]

$$P(S=3)=0.1\left(1+\frac{2\times 1}{3}\right)P(X=1)P(S=2)+0.1\left(1+\frac{2\times 2}{3}\right)P(X=2)P(S=1)$$

$$=0.01185$$

[1]

and:

$$P(S=4)=0.1\left(1+\frac{2\times 1}{4}\right)P(X=1)P(S=3)+0.1\left(1+\frac{2\times 2}{4}\right)P(X=2)P(S=2)$$
$$+0.1\left(1+\frac{2\times 4}{4}\right)P(X=4)P(S=0)=0.05884.$$

[1]

Hence the probability that the aggregate claim amount is less than or equal to £2,000 is:

$$P(S\le 4)=0.729+0.10935+0.06561+0.01185+0.05884=0.97465$$ [1]

(b) ***Using a normal approximation***

The mean and variance of S must first be calculated from the formulae:

$$E(S)=E(N)E(X)$$

and:

$$Var(S)=E(N)Var(X)+\left[E(X)\right]^2 Var(N)$$ [1]

From the formulae in the *Tables*:

$$E(N)=\frac{kq}{p}=\frac{3\times 0.1}{0.9}=\frac{1}{3}$$ [1]

and:

$$Var(N)=\frac{kq}{p^2}=\frac{10}{27}$$ [1]

Now, working in units of £1,000, we have:

$$E(X)=(0.5\times 0.5)+(1\times 0.25)+(2\times 0.25)=1$$

$$E(X^2)=(0.5^2\times 0.5)+(1^2\times 0.25)+(2^2\times 0.25)=1.375$$ [1]

and:

$$Var(X)=1.375-1^2=0.375$$ [1]

It follows that:

$$E(S)=\frac{1}{3}$$ [1]

and:

$$Var(S) = \left(\frac{1}{3} \times 0.375\right) + \left(\frac{10}{27} \times 1^2\right) = 0.495370 \qquad [1]$$

We can now estimate the required probability as follows:

$$P(S \le 2) \approx P\left(N(0,1) \le \frac{2 - \frac{1}{3}}{\sqrt{0.495370}}\right) \approx \Phi(2.37) = 0.99111 \qquad [1]$$

(c) ***Using a translated gamma approximation***

We shall assume that $S - k \sim Gamma(\alpha, \lambda)$ for some constants α, λ and k.

The values of α, λ and k have to be estimated using the method of moments. We already know the values of the first and second central moments of S. The value of the third may be found from the equation:

$$Skew(S) = \frac{3kq^2}{p^2} m_1 m_2 + \frac{2kq^3}{p^3} m_1^3 + \frac{kq}{p} m_3$$

where m_i denotes the i th non-central moment of X, $i = 1,2,3$. [1]

The third non-central moment of X is:

$$E(X^3) = (0.5^3 \times 0.5) + (1^3 \times 0.25) + (2^3 \times 0.25) = 2.3125 \qquad [1]$$

So:

$$Skew(S) = \left[9\left(\frac{0.1}{0.9}\right)^2 \times 1.375\right] + \left[6\left(\frac{0.1}{0.9}\right)^3\right] + \left[\frac{1}{3} \times 2.3125\right] = 0.931842. \qquad [1]$$

The values of α, λ and k can now be found from the equations:

$$E(S) = \frac{\alpha}{\lambda} + k = \frac{1}{3} \qquad Var(S) = \frac{\alpha}{\lambda^2} = 0.495370$$

$$Skew(S) = \frac{2\alpha}{\lambda^3} = 0.931842.$$

[1]

Solving these equations gives $\alpha = 0.55997$, $\lambda = 1.06321$ and $k = -0.19335$. This means that we should approximate the distribution of $S + 0.19335$ by $Gamma(0.55997,\ 1.06321)$. [1]

In order to estimate $P(S \le 2)$ using the *Tables*, we have to convert the Gamma distribution to a χ^2 distribution. Recall that, if $Y \sim Gamma(\alpha, \lambda)$, then $2\lambda Y \sim \chi^2_{2\alpha}$.

Hence $2 \times 1.06321(S + 0.19335) \sim \chi^2_{1.12}$ approximately, so we can estimate the probability using a χ^2_1 distribution:

$$\begin{aligned} P(S \le 2) &= P(S + 0.19335 \le 2.19335) \\ &= P\left(2.12642(S + 0.19335) \le 4.6640\right) \\ &\approx P\left(\chi^2_1 \le 4.6640\right) \end{aligned}$$

[1]

From the *Tables*, we see that:

$$P\left(\chi^2_1 \le 3.841\right) = 0.95 \text{ and } P\left(\chi^2_1 \le 5.024\right) = 0.975$$ [1]

Interpolating between these values gives:

$$P\left(\chi^2_1 \le 4.6640\right) \approx 0.95 + \left(\frac{4.6640 - 3.841}{5.024 - 3.841}\right) \times 0.025 = 0.9674$$ [1]

Note: To obtain an estimate for the probability using 1.12 df, we could interpolate between the probabilities calculated using 1 and 2 df. Using a similar method to the above gives $P(\chi^2_2 \le 4.6640) \approx 0.9261$. *Interpolating between degrees of freedom then gives* $P(\chi^2_{1.12} \le 4.6640) \approx 0.9624$. *However, estimating using 1 df is acceptable.*

[Maximum 17]

(iii) ***Comments***

The three methods used in (ii) gave the following results:

Method	$P(S \leq 2)$
Recursive formula (exact method)	0.9747
Normal approximation	0.9911
Translated gamma approximation	0.9674

The exact distribution of S is compound negative binomial, which is positively skewed.

The normal approximation has overestimated $P(S \leq 2)$. This can be explained by the fact that the normal distribution is symmetric and has a thinner right-hand tail than a positively skewed distribution. [1]

The translated gamma distribution is positively skewed and this approximation has slightly underestimated $P(S \leq 2)$. We can conclude that the approximate distribution must have a thicker right-hand tail than the exact distribution. However, the translated gamma approximation is the better of the two. [1]

Assignment X4 Solutions

Solution X4.1

(i) ***Statistical test***

Define Model 1 to be the initial model and Model 2 to be the reduced model.

These two models are nested, so we can use a χ^2 test to compare the changes in scaled deviance between the models. [1]

The scaled deviance for Model 1 = 392.45 (given).

The scaled deviance for Model 2 = 401.97 (given).

Degrees of freedom for Model 1 = 50,000 – 80 = 49,920. [½]

Degrees of freedom for Model 2 = 50,000 – 76 = 49,924 (since the reduced model has only 1 level for this factor instead of 5 so there are 4 fewer parameters fitted). [1]

We know that, under the null hypothesis, there is no difference between Model 1 and Model 2:

$$D_2^* - D_1^* \sim \chi^2_{df_2 - df_1}$$ [1]

The difference between the scaled deviances is 401.97 – 392.45 = 9.52. We should compare this with the χ^2_4 distribution. [½]

The upper 5% point of the χ^2_4 distribution is 9.488 (from page 169 of the *Tables*). [1]

Our test statistic of 9.52 exceeds this value. So, at the 5% significance level, we would reject the reduced Model 2 in favour of the initial Model 1. [1]

Therefore, based on our statistical test, and assuming a 5% significance level, we would keep this factor in the model. [1]
[Maximum 6]

(ii) ***Further considerations***

This factor is only just significant, based on the 5% statistical test, so we might want to conduct a more detailed analysis. [½]

We do not know what this factor is, although we know it has five levels. We would need to investigate whether these five levels are groupings of more detailed levels. If so then we could try including the original ungrouped factor in the model instead, to test for significance ... [1]

… although this might be difficult if the factor has been grouped due to there being insufficient data. [½]

We would need to look at the parameter values associated with each of the five levels to see if they are as we would expect, relative to each other. [½]

We could draw a graph of the values, to enable us to see this more clearly. [½]

For example, it could be that the relativity for the "Unknown" level is so different to the relativities for the other levels that it is this alone that is making the factor appear statistically significant. [1]

If this is the case then we may conclude that the factor is not really adding much in terms of predictive power for the future. [½]

Alternatively, we could group "Unknown" with one of the other levels and re-fit the model to see whether it is then statistically significant. [½]

We would want to fit an interaction between this factor and some measure of time, to enable us to see whether the pattern observed for the relativities is consistent over time. [1]

If the pattern is not consistent then we may reject this factor on the basis that it does not show a stable pattern and is therefore not useful for predicting the future. [1]

We should consider whether the factor is likely to be acceptable to policyholders … [½]

… and brokers, *eg* their systems may be unable to handle an extra rating factor. [½]

We could investigate whether this factor has been used in previous rating exercises for this book of business. If so, we might be more inclined to keep it in the model this time. [1]

We could also find out whether this factor is used by other insurers in the personal lines motor market. If we drop the factor from our model while other insurers continue to use it then we could suffer from anti-selection. [1]

If we haven't used this factor before then we need to consider the practicalities of how easily it could be incorporated into our rating algorithms. If it is likely to take a lot of IT time to build the relevant tables then this would be an argument for not using the factor. [1]

[Maximum 10]

Solution X4.2

(i) ***Normal distribution is a member of the exponential family***

If $Y \sim N\left(\mu,\sigma^2\right)$, then the pdf of Y is:

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]$$

To show that the normal distribution belongs to the exponential family, we have to show that its pdf can be written in the form given on Page 27 of the Tables, which is:

$$f_Y(y) = \exp\left[\frac{y\theta - b(\theta)}{a(\varphi)} + c(y,\varphi)\right]. \qquad [½]$$

The θ term must be a function of the mean μ and the $c(y,\varphi)$ term does not contain μ.

For the normal distribution, we have:

$$f_Y(y) = \exp\left[-\ln\left(\sqrt{2\pi}\sigma\right) - \frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]$$

$$= \exp\left[-\ln\left(\sqrt{2\pi}\sigma\right) - \frac{½y^2 - y\mu + ½\mu^2}{\sigma^2}\right]$$

$$= \exp\left[\frac{y\mu - ½\mu^2}{\sigma^2} - \ln\left(\sqrt{2\pi}\sigma\right) - \frac{½y^2}{\sigma^2}\right] . \qquad [1½]$$

This is in the required form if we define:

$$\varphi = \sigma \qquad [½]$$

$$c(y,\varphi) = -\ln\left(\sqrt{2\pi}\varphi\right) - \frac{½\, y^2}{\varphi^2}$$ [½]

$$a(\varphi) = \varphi^2$$ [½]

$$\theta = \mu$$ [½]

$$b(\theta) = \frac{1}{2}\theta^2 .$$ [½]

[Maximum 4]

(ii) ***Variance function***

The variance function is given by:

$$V(\mu) = b''(\theta) .$$ [½]

Here we have:

$$b'(\theta) = \theta \qquad \text{and} \qquad b''(\theta) = 1 .$$

So:

$$V(\mu) = 1 .$$ [½]

[Total 1]

(iii) ***Deviance***

Since $V(\mu)=1$, we have:

$$d_i(y_i,\hat{\mu}_i)=2\int_{\hat{\mu}_i}^{y_i}\frac{y_i-t}{V(t)}dt$$

$$=2\int_{\hat{\mu}_i}^{y_i}(y_i-t)\,dt$$

$$=2\Big[y_i\,t-½t^2\Big]_{\hat{\mu}_i}^{y_i}$$

$$=2\Big[\big(y_i^2-½\,y_i^2\big)-\big(y_i\,\hat{\mu}_i-½\,\hat{\mu}_i^2\big)\Big]$$

$$=y_i^2-2y_i\,\hat{\mu}_i+\hat{\mu}_i^2$$

$$=(y_i-\hat{\mu}_i)^2$$ [2]

The deviance is then:

$$D=\sum_{i=1}^{n}d_i(y_i,\hat{\mu}_i)=\sum_{i=1}^{n}(y_i-\hat{\mu}_i)^2$$ [1]

[Total 3]

Solution X4.3

(i) ***Predicted claim frequencies***

The predicted claim frequencies are shown in the last column of the table below.

They are calculated by multiplying together the relativities for the relevant level of each factor with the intercept term.

So, for example, the predicted frequency for the first row is $0.08 \times 0.5 \times 0.5 \times 2 = 0.04$.

Area	No. of bedrooms	Age of policyholder	Predicted frequency
Country	1-3	Young	0.04
Town	1-3	Young	0.08
City	1-3	Young	0.12
Country	4+	Young	0.08
Town	4+	Young	0.16
City	4+	Young	0.24
Country	1-3	Old	0.02
Town	1-3	Old	0.04
City	1-3	Old	0.06
Country	4+	Old	0.04
Town	4+	Old	0.08
City	4+	Old	0.12

[3]

(ii) ***One-way tables***

Policy years

First we need to calculate the total number of policy years (exposure) for each level of each rating factor. This is obtained by adding the exposure for each rating cell that relates to each level.

For example, take Area = Country. The total number of policy years is calculated as $10,200 + 50 + 6,700 + 18,250 = 35,200$. [2 for correct numbers]

Number of claims

Then we need to calculate the total number of claims for each level of each rating factor. We know the predicted claim frequency and the exposure for each cell so we can calculate the number of claims in each cell and then add these across each level of each rating factor.

For example, for Area = Country, the total number of claims is calculated as
$(10,200 \times 0.04) + (50 \times 0.08) + (6,700 \times 0.02) + (18,250 \times 0.04) = 1,276$.

[2 for correct numbers]

Frequency

Then, for each level of each factor, we can calculate the one-way claim frequency by dividing the number of claims by the number of policy years. [1 for correct numbers]

The table below summarises the results.

Area	No. of bedrooms	Age of policyholder	Policy years	Total no. of claims	One-way frequency
Country			35,200	1,276	3.6%
Town			49,200	3,472	7.1%
City			25,600	2,598	10.1%
	1-3		66,400	4,562	6.9%
	4+		43,600	2,784	6.4%
		Young	40,000	3,516	8.8%
		Old	70,000	3,830	5.5%

[Total 5]

(iii) ***One-way results vs. GLM results***

The following table shows the one-way claim frequencies together with the GLM frequencies for each level of each factor:

Area	No. of bedrooms	Age of policyholder	One-way frequency	GLM frequency
Country			3.6%	4.0%
Town			7.1%	8.0%
City			10.1%	12.0%
	1-3		6.9%	4.0%
	4+		6.4%	8.0%
		Young	8.8%	16.0%
		Old	5.5%	8.0%

The GLM frequencies for each level of each factor are calculated as the relativity for that level multiplied by the intercept term (the base-level frequency). [1]

The one-way table significantly understates the claim frequency for young policyholders compared to the GLM. [½]

The exposure distribution shows that young policyholders tend to live in smaller houses (1-3 bedrooms), which are less risky, and this is masking their relatively poor claims experience. [1]

Conversely, the one-way frequency for houses with 1-3 bedrooms is overstated because it is more likely that the riskier younger policyholders will live there. [1]

The vast majority of the old policyholders live in the country or in a town, with only a few (11.5%) of them living in the higher-risk cities. The one-way frequency for old people therefore appears lower than it really is because it is being pulled down by the fact that they live primarily in low-risk areas. [1]

The one-way frequency for larger houses (4+ bedrooms) is also lower than the GLM value because, although large houses are twice as risky as small houses, it is almost exclusively the lower-risk old people who live in them. [1]

The one-way table frequencies for area are generally closer to the GLM values than for the other two factors. [½]

This is because the distribution of age by area is relatively similar to the distribution of age overall … [½]

… and the distribution of house size by area is relatively similar to the distribution of house size overall. [½]

The main exception when considering the area factor is the city. Over two-thirds (69%) of people living in the city are young (compared with only 36% of people in the data that are young). This will make the city appear even riskier when looking at one-way tables. [1]

Also, although 4+ bedroom houses make up nearly 40% of the total houses in the data, hardly any (0.2%) of these are located in the city. This will make the city appear less risky when looking at one-way tables. [1]

These two sets of correlations for the city work in different directions, with house size having the strongest effect. This is why the one-way frequency table for area understates the "true" frequency. [1]

[Maximum 9]

Solution X4.4

1. Group and summarise the data prior to loading into the GLM software [½]

This method can be used to remove redundant codes from the data … [½]

… especially when there is little exposure associated with these … [½]

… for example, data relating to unknown levels of a factor. [½]

However, this method requires some knowledge or experience of the pattern that would be expected from the GLM. [½]

2. Group data within the modelling package [½]

Two or more levels of a factor can be grouped together within the model, to create a custom factor. [½]

The grouped levels will then be assigned a single parameter value when the model is run. [½]

3. ***Fit a curve*** [½]

Instead of grouping levels of a factor, we can fit a polynomial curve to the factor. [½]

This could be a quadratic, a cubic or a higher order curve. [½]

The parameters associated with this factor in the model are then the parameters from the polynomial, excluding the constant term. [1]

A higher order of polynomial will provide a better fit to the data, but may be less useful as a predictor of future experience. A balance should be struck between parsimony and predictive accuracy. [1]

A fitted curve is sometimes referred to as a variate. [½]

4. Fit a piecewise curve [½]

Here, we would split the levels of a factor into different sections and fit either a custom factor or a curve to each section. [1]

Therefore, this is a combination of methods 2 and 3 above. [½]

At the boundary of each section, the join can be disjoint or constrained to being piecewise continuous, depending on which is more appropriate for the model. [1]

[Maximum 10]

Solution X4.5

(i) ***Collision risk factors***

The collision risk factors that relate to the driver are:

- level of skill
- number of miles typically driven
- attitude to risk. [½ each]

The collision risk factors that relate to the environment in which the car is typically driven are:

- density of traffic (may be affected by when as well as where the car is driven)
- presence of hazards (*eg* potholes or ice)
- the type of roads where the car is used (motorway, country lane *etc*)

[½ each]

The collision risk factors that relate to the vehicle are:

- the performance and speed capability of the car
- the repair costs if the vehicle is damaged
- safety features to protect passengers (*eg* airbags)
- safety features to improve car control (*eg* ABS, traction control)
- the size and weight of the car. [½ each]

[Maximum 5]

(ii) ***Why the factors are not used directly in rating***

Although the factors listed in part (i) help to define the genuine risk for an individual policyholder, many are not used as rating factors because:

- they may not be readily measurable [½]
- they may be subjective [½]
- they may change over time [½]
- they may be difficult to verify [½]
- they may be open to manipulation [½]
- they cannot be defined by the customer at the point of sale. [½]
- they may be unacceptable to brokers [½]
- they may be unacceptable to policyholders [½]
- they may be out of line with the market. [½]

[Maximum 4]

(iii) ***Effectiveness of a proxy rating factor***

A proxy rating factor will be more effective if it is a good direct measure of a genuine risk factor. [1]

For example, length of licence (number of years since passed test) might give a good direct indication of driving skill (although it doesn't take into account the number of miles driven during that time), while age of driver may only give a partial indication of the driver's attitude to risk. [1]

A rating factor will be more effective if it is a factual quantity that is known to the proposer, as opposed to something that is open to debate. [1]

For example, postcode is a well-defined fact that all policyholders are likely to know, whereas they might not remember exactly when they passed their driving test. [1]

If the rating factor has an obvious "direction" then the proposer might be tempted to under or overstate this in order to get a cheaper premium. This will make the rating factor less effective. [1]

For example, most prospective policyholders will know that a lower estimated annual mileage will lead to a lower quoted premium, and so they may deliberately underestimate this. [1]

If there is a lot of overlap between the information provided by different proxy rating factors then the effect of each one will be less significant. [1]

For example, for young drivers, there is a lot of overlap between their age, their NCD level and their driving experience. A 17 year old will only be able to have less than a year's experience and no earned NCD so there will be a lot of correlation between these factors. [1]

A rating factor will be less effective if there is no way of checking whether the policyholder is telling the truth about the factor in question. [1]

For example, "miles driven on motorways" may be difficult to check. [1]

Markers: give equivalent credit for all alternative, relevant and valid examples.

[Maximum 9]

Solution X4.6

(i) ***Rating factors***

As the specific holiday details are not known for annual policies, there are not many rating factors used, but they would normally include: [½]

- number of people covered, together with their age and sex [1]
- countries covered (normally grouped into UK only/US/worldwide) [1]
- reason for travel (pleasure or business) [½]
- type of holiday coverage, *eg* winter-sports cover or not [½]
- health (although existing medical conditions are usually excluded) [½]
- level of excess (if optional) [½]
- level of cover (if optional) [½]
- maximum holiday length (if optional). [½]

[Maximum 5]

(ii) ***ILF calculation***

We need to calculate the limited expected value (LEV) at the base of £500, and at each of the proposed limits (ignoring probability of a claim – see part (iv)).

At the base of £500, 3,000 holidays were sold with a total value of £1m. All of these would get refunded in full in the event of a claim. In addition, each of the 21,500 other holidays would expect to receive £500 as their claims would be capped at that level.

This gives:

$$LEV_{£500} = E\left[X \wedge 500\right] = \left(1m + 500 \times 21{,}500\right)/24{,}500 = 480 \quad [1]$$

Similarly for the other levels proposed:

$$LEV_{£1{,}000} = \left(1m + 5m + 1{,}000 \times 15{,}000\right)/24{,}500 = 857 \quad [1]$$

$$LEV_{£2{,}000} = \left(22m + 2{,}000 \times 4{,}500\right)/24{,}500 = 1{,}265 \quad [1]$$

$$LEV_{£3{,}000} = \left(28m + 3{,}000 \times 2{,}000\right)/24{,}500 = 1{,}388 \quad [1]$$

$$LEV_{£4{,}000} = \left(31.5m + 4{,}000 \times 1{,}000\right)/24{,}500 = 1{,}449 \quad [1]$$

Hence the ILFs are:

$$ILF_{£1,000} == 857/480 = 1.79$$ [½]

$$ILF_{£2,000} = 1,265/480 = 2.64$$ [½]

$$ILF_{£3,000} = 1,388/480 = 2.89$$ [½]

$$ILF_{£4,000} = 1,449/480 = 3.02$$ [½]

[Total 7]

(iii) ***Assumptions***

- There is no excess on claims. [½]
- Frequency is independent of the limit. [½]
- The distribution of claim size is independent of the number of claims and of the limit. [1]
- Sales from the affiliated travel agents are typical of those insured by us in terms of, *eg*: [½]
  - cost [½]
  - mix of business (*eg* type of holiday, destination) [½]
  - type of policyholder (*eg* young/old). [½]
- We have ignored the possibility of using different ILFs for different rating cells, *eg* business or leisure travel. [½]
- The three months were typical of sales over the year … [½]
- … which is unlikely to be the case due to seasonality. [½]
- We have ignored inflation since the data was produced. [½]
- We have ignored any other trends in holiday costs. [½]
- We have ignored the associated claims handling expenses at each size of loss, which implicitly assumes they are proportionately the same. [1]
- There is no allowance for the possible extra variability of claims at higher limits, which may, for example, result in higher expected costs. [1]
- The effect of a higher limit would not affect claim frequency or severity for other perils covered by the policy. [½]

[Maximum 7]

(iv) ***Other factors to consider***

We should check the evidence to ensure that the limit is the true reason for the falling sales volumes. [½]

The limit may be out of line with competition, so we should check the levels in the marketplace. [½]

We should quantify the demand for a rise in limits at various levels. This could be by undertaking some market research or by talking to travel agents. [1]

This would primarily be to get an idea of likely holiday costs in the future (since this is the key driver of the cancellation claim amount). [½]

We should assess the likely effect on renewals of existing business … [½]

… as some policyholders will not need/want an increase in the limit and might resent the extra premium payable at renewal. [½]

We may therefore wish to leave existing business at the old limit, or give the option of moving to the new limit and premium level. [½]

The expense of changing the limits needs to be allowed for perhaps by amortising the expense over a number of years' business. [½]

This expense arises due to the cost of:

- this investigation
- making appropriate IT changes
- changes to marketing literature
- changes to policy wording. [½ each, maximum 1]

We should anticipate any need to increase the limit again in the future due to inflation or further changes in competitors' limits, *etc*. This might mean that we err for a slightly higher limit at this stage to avoid further changes in the near future. [1]

We should ask our reinsurers for their opinions. We are a small company and will therefore probably have extensive reinsurance. [½]

As we are small, we should also consider the possible effects on volumes – solvency may be tight and rapid increases might imperil this. [½]

We should review our profit criteria and ensure that the new premium level is appropriate. [½]

We will need to undertake a full profit testing exercise to assess the balance of volumes, premiums and claims. [½]

We should investigate the effect that the change in limit would have on the total premium charged for the contract. For example, given the other perils covered, the change may not have that great an effect. [1]

We should compare the new premiums with those charged by other companies. [½]

We should check any legal or social constraints on the limits we use. For example, there may be pressure from travel agents, or from press articles, on suitable limits. [½]

We might want to consider implementing the change over time, so that any premium rate hikes are not too dramatic. [½]

[Maximum 9]

Solution X4.7

The probability function of the Poisson distribution (with mean λ) is:

$$P(Y=y)=\frac{\lambda^y e^{-\lambda}}{y!}.$$ [½]

Hence:

$$P(Y=k)=\exp\left[k\ln(\lambda)-\lambda-\ln(k!)\right].$$ [½]

Now replace y with y_i, and let $\theta_i=\ln(\lambda)$. [½]

Then:

$$P(Y=y_i)=\exp\left\{y_i\theta_i-\lambda-\ln(y_i!)\right\}$$ [½]

or equivalently:

$$P(Y=y_i)=\exp\left\{y_i\theta_i-e^{\theta_i}-\ln(y_i!)\right\}.$$

This is of the form:

$$\exp\left\{\frac{y_i\theta_i-b(\theta_i)}{a_i(\varphi)}+c(y_i,\varphi)\right\}$$

where:

$$a_i(\varphi)=1$$

$$b(\theta_i)=e^{\theta_i}$$ [½ mark for each]

$$c(y_i,\varphi)=-\ln(y_i!).$$

[Maximum 3]

Assignment X5 Solutions

Solution X5.1

From the information given in the question, we have:

Year	Volume	Total losses	Claims ratio
1	50	5,000	100
2	100	2,500	25
Total	150	7,500	50

[1]

and

$$\mu(\theta_i) = E(X_i|\theta_i) = \theta_i$$ [½]

$$\sigma^2(\theta_i) = V_i\left[\text{var}(X_i|\theta_i)\right] = V_i \times \frac{300}{V_i} = 300$$ [½]

$$\beta = E(\mu(\theta_i)) = E(\theta_i) = 100$$ [½]

$$\phi = E(\sigma^2(\theta_i)) = E(300) = 300$$ [½]

$$\lambda = \text{var}(\mu(\theta_i)) = \text{var}(\theta_i) = 525$$ [½]

So:

$$Z = \frac{V_i}{V_i + \phi/\lambda} = \frac{150}{150 + 300/525} = 0.996205$$ [1]

Now we take the observation statistic for year three as the long-run observed claims ratio $= 50$, and the ancillary statistic is β.

So the Bühlmann Straub credibility premium per unit of volume is:

$$\begin{aligned} C^{BE} &= Z \times X_3 + (1-Z) \times \beta \\ &= Z \times 50 + (1-Z) \times 100 \\ &= 50.189 \end{aligned}$$

[1]

So the risk premium for the total losses in year three is $50.189 \times 25 = 1,254.744$. [½]

[Total 6]

Solution X5.2

We are given that $P = 0.95$, so:

$$\Phi(y) = \frac{1+P}{2} = 0.975\text{, giving } y = 1.960\text{.} \qquad [1]$$

The mean and variance of F are not equal, so we know that claim frequency is not a Poisson distribution. [½]

Therefore we need to use the general formula to establish the number of *claims* required for full credibility: [½]

$$n_N \geq \frac{y^2}{k^2}\left(\frac{\sigma_N^2}{\mu_N}\right) = \frac{1.960^2}{0.02^2}\left(\frac{0.10}{0.06}\right) = 16{,}006.67 \qquad [1]$$

So the number of *risks* required for full credibility is $\dfrac{16{,}006.67}{0.06} = 266{,}777$ [1]

[Total 4]

Solution X5.3

Complement of credibility

The desirable qualities for a complement of credibility are:

- accuracy, *ie* low variance around next year's mean claim cost [½]
- unbiasedness as a predictor of next year's mean claim cost (*ie* the differences between the predictor and the subsequent loss costs should average out near zero) [½]
- independence from the base statistic [½]
- availability of data [½]
- ease of calculation [½]
- explainable relationship to the claim costs of the class or individual being rated [½]
- ease of explanation to managers and customers [½]
- low probability of error occurring in the calculation of the complement [½]
- time needed to calculate the complement is not excessively great. [½]

[Maximum 4]

Solution X5.4

(i) ***Number of claims for full credibility***

$$\Phi(y) = \frac{1+P}{2} = \frac{1.95}{2} = 0.975$$ [½]

So $y = 1.960$. [½]

$$n_N \geq \frac{y^2}{k^2} \text{ where } k = 0.03$$ [½]

so

$$n_N \geq \frac{1.960^2}{0.03^2} = 4268.44 = 4269 \text{ claims.}$$ [½]

[Total 2]

(ii) ***Revised number of claims for full credibility***

Assume

- we still want to be within 3% of the mean with a probability of 95% [½]
- the normal approximation applies. [½]

n_N is still 4268.44, from part (i). (Note that we use the unrounded figure here.) [½]

We need to find $n_X = n_N \left(CV_X\right)^2 = n_N \left(\frac{\sigma_X}{\mu_X}\right)^2$.

$$\int_0^{800} c(1600 - 2x)dx = 1$$

Integrating gives:

$$\left[1600cx - cx^2\right]_0^{800} = 1$$

so

$$c = 0.0000015625$$ [1]

Therefore,

$$\begin{aligned} E(X) = \mu_x &= \int_0^{800} 0.0000015625x(1600-2x)dx \\ &= \left[\frac{0.0025x^2}{2} - \frac{0.000003125x^3}{3}\right]_0^{800} \\ &= 800 - 533.33333 \\ &= 266.66667 \end{aligned}$$

[1]

and

$$\begin{aligned} E\left(X^2\right) &= \int_0^{800} 0.0000015625x^2(1600-2x)dx \\ &= \left[\frac{0.0025x^3}{3} - \frac{0.000003125x^4}{4}\right]_0^{800} \\ &= 426666.66667 - 320000 \\ &= 106666.66667 \end{aligned}$$

[1]

So

$$\begin{aligned} \operatorname{var}(X) &= 106666.66667 - 266.66667^2 \\ &= 35555.5538 \end{aligned}$$

[1]

$$n_X = 4268.44 \times \frac{35555.5538}{266.66667^2} = 2134.22$$ [1]

and

$$n_S = n_N + n_X = 4268.44 + 2134.22 = 6402.66 = 6403 \text{ claims.}$$ [1]

[Maximum 7]

(iii) ***Revised number of claims for full credibility (binomial)***

$E(X) = np = 2000 \times 0.2 = 400$. [½]

$\text{var}(X) = np(1-p) = 200 \times 0.2 \times 0.8 = 320$. [½]

Assuming the normal approximation still applies,

$$n_N = \left(\frac{y^2}{k^2}\right)\left(\frac{\sigma_N^2}{\mu_N}\right)$$ [½]

$$= \frac{1.960^2}{0.03^2} \times \frac{320}{400} = 3414.76$$ [½]

and

$$n_S = \left(\frac{y^2}{k^2}\right)\left(\frac{\sigma_N^2}{\mu_N} + \frac{\sigma_X^2}{\mu_X^2}\right)$$ [1]

$$= \frac{1.960^2}{0.03^2}\left(\frac{320}{400} + \frac{35555.5538}{266.66667^2}\right) = 5548.98 = 5549 \text{ claims.}$$ [1]

[Total 4]

(iv) ***Relative sizes of answers***

The number of claims required for full credibility using a binomial distribution is lower than that using a Poisson distribution ... [½]

... because the binomial distribution has a lower variance than the Poisson. [½]

For the binomial distribution, the variance will be lower than the mean, whereas for the Poisson the variance equals the mean. [½]

[Maximum 1]

Solution X5.5

(i) ***Reasons for doing an expense analysis***

Expenses can still account for a large proportion of the premium charged ... [½]

… especially if the business written suffers from low persistency rates. [½]

We will want to allocate all the insurer's expenses correctly between the different classes of business it writes and within rating groups for each class. [1]

If there are any cross-subsidies either between classes or within a class then an expense analysis will help us to understand these. [½]

This allocation of expenses will enable us to more accurately measure the past performance of each class. [½]

It will also help us to determine the appropriate level of expense allowance to be included in any future premium-rating exercise, after adjusting for inflation. [½]

An expenses analysis might indicate whether all the expenses are expected to be covered by future premiums – if not, then we need to have some idea of the amount not covered. [1]

An expenses analysis may help with financial planning. [½]
[Maximum 4]

(ii) ***Director's proposed method***

In formula terms, the director's suggestion is:

$$P = E \times P + C$$

where:

P is the premium to charge
E is the expense ratio for a particular class, derived from the statutory returns
C is the non-expense element of the premium (which includes the claims). [1]

This is a simple approach. [½]

However, it will lead to high expense loadings for high premium business and low expense loadings for low premium business … [½]

… which will lead to cross-subsidies between high and low premium business, and a consequent risk of anti-selection. [1]

The director's method also makes no allowance for:

- inflation of expenses [½]
- changes in volumes of business [½]
- one-off charges [½]
- other trends [½]

from the base period associated with the statutory returns to the period for which the new premiums are to apply.

In addition, this method makes no allowance for the different costs associated with the processing of new business, renewals, *etc* within an accounting class. [½]

Alternative method

Instead, we could use a formula along the lines of:

$$P = C \times (1 + ce) + (p \times P) + pp + (c \times P) + O$$

where:

P = premium to charge
C = claims cost
ce = claims expenses as a proportion of claims cost
p = per premium expenses as a proportion of premium
pp = per policy expenses in £
c = commission as a proportion of P
O = other items, eg profit, contingencies, investment income.

where ce, p, c and pp are after allowing for inflation and other trends from the base period of the analyses to the average date for which the new premiums are expected to apply.

[3 for development of appropriate formula]

This formula should then be applied separately for each product. [½]

A further refinement would be to load expenses according to numbers of claims rather than just overall claim cost. However, this would add greater complexity to the formula, which may not be justified. [1]

This formula addresses the main problems associated with the director's proposed method. [½]

[Maximum 8]

Solution X5.6

(i) ***Strike rate***

This is usually defined as the number of written policies divided by the number of quoted policies in a given period, possibly adjusted for declinatures. [1]

(ii) ***Reasons for a reducing strike rate and measures to improve***

We should first make sure that credibility of data is not an issue. We may find that the drop is not significant or is just a blip. [½]

There could have been a change in the mix of business, *eg* by type of business or distribution channel, towards business with lower strike rates. We should investigate the mix of business and analyses strike rates by these factors, and review our strategy if necessary. [1]

The sales staff (*eg* telephone staff) may be unmotivated to complete sales. This could be addressed by reviewing / offering incentives, bonuses and pay scales. [1]

In particular, brokers may not be getting enough commission and so do not have the motivation to complete a sale. A review of the commission levels and perhaps increasing the initial commission may improve the strike rate. The commission should target the more effective brokers. [1½]

The sales staff may not be competent in selling, and so not assess needs fully or offer alternative products. Training of sales staff should improve this situation. [1]

There may be greater competition, *eg* the launch of a similar product by a competitor. Increasing the marketing spend or redesigning the product may help here. [1]

Premiums may be uncompetitive. A full pricing review and comparison with competitors will quantify the effect of this. [1]

There could be a delay in reporting of take-up rates, so we don't yet know how many quotes were actually taken up. Limiting the quotes validity period to, say, a month would help to reduce this problem. [1]

It may be that renewal quotes are included in the calculated strike rate. In this case, customers could be leaving at renewal due to poor customer service. [½]

There could have been IT issues. For example, if quotes are given over the internet, there may have been a problem with the reporting of take-up or quote numbers, perhaps due to time lags or some other practical reason. It would be wise to ask the IT department to investigate whether this is the case. [1]

The system may make it very difficult for customers to renew, for example by taking a long time to process or by the system "hanging". [½]

Internet quote engines may not be well-designed (*eg* by asking too many questions, or questions that the policyholder is reluctant to answer). As a result, prospective policyholders are not inclined to take up the quote. A redesign might help, perhaps reducing or rewording the questions, or by looking at how our engines differ from those of our competitors. [1]

A recent marketing campaign may have stopped, or competitors may have increased their marketing spend. We can check this with our marketing department. [½]

It may be that the product is not suited to the prospective policyholders' needs. Better up-front marketing of the product would ensure that quotes are only requested by policyholders who deem it suitable first. [1]

It may be that the company is targeting the wrong customers, for example, those likely to heavily shop around for quotes. We should analyse the quotes given to ensure that the target market is in line with our strategy. [1]

If we quote via aggregator (price comparison) websites, we may find that our prices are, for some reason, uncompetitive. We should review what gets shown on these sites to make sure we're happy with where we are "placed". [1]

If we have only recently started to quote via aggregators, then we should expect a drop in strike rates because aggregator strike rates are generally lower than those for other channels. [½]

A poor strike rate isn't *necessarily* a problem as long as the profit per policy makes up for the drop in business. [½]

Markers: Give marks for any other reasonable explanations with suggested actions to address. [Maximum 14]

Solution X5.7

Choice of model

From a practical perspective, it is important to choose a model that is simple and easy to use since it will be used very frequently. [½]

This will help to minimise the chance of error in calculating the credibility estimate… [½]

… and make it easier to explain the choice of premium to managers, underwriters and customers. [½]

Since the premium charged is explicitly affected by experience rating, and this is understood by policyholders, the insurer must take care that the policyholder perceives the model to be fair … [½]

… otherwise there is a risk of losing business. [½]

In practice therefore, insurers should consider imposing a predetermined "swing" to the experience-rating model … [½]

… in other words, there should be a maximum limit imposed on the impact that a single claim can have on the premium. [½]

Large insureds should be rated entirely on their own experience. [½]

The choice of model should be a good fit to the business being modelled, *ie* the resulting estimate should have a low variability around the true underlying figure. [1]

The estimate should also be unbiased. [½]

So, an insurer should aim to minimise model error … [½]

… however there should be a balance between accuracy and simplicity. [½]

If the model is a simplified version of a theoretical credibility model, it should have a similar level of accuracy to the theoretical credibility model … [½]

… since this means that the model is more likely to have been simplified correctly. [½]

Model error will depend on any error in the base statistic as well as any error in the complement. [½]

So both figures should be as accurate as possible. [½]

The choice of ancillary data should be appropriate to the business being modelled. [½]

So there should be an explainable relationship between the choice of complement and the class or individual being rated. [½]

The complement should be independent from the base statistic. [½]

Data considerations

Consideration should be given to the level of grouping to be applied to the data. [½]

Too many subgroups will reduce the credibility of each data cell, whereas not enough grouping will introduce heterogeneity into each cell and make accurate pricing more difficult. [1]

As much reliable data should be collected as possible. [½]

Ways to increase the amount of data available for the base statistic include, for example, collecting:

- more years of historic data [½]
- data from different locations, or country-wide data. [½]

However, a balance should be struck between the volume of data available and the appropriateness of this data. [½]

The cost of collecting data should also be taken into account. [½]

Consider giving more weight to more stable data, *eg* claim frequency instead of claim amounts. [½]

Partial premiums (for each claim type) may be used and the final premium could give more weight to the more stable parts of the premium. [½]

Possible sources of ancillary data include national / regional / industry data. [½]

The ancillary data should be adjusted to ensure that the credibility complement is suitable. [½]

Where this is problematic, the percentage change in complement may be used. [½]

Or the credibility complement may be based on prior year's analysis. [½]

This essentially takes more years' data into account. [½]

However, there is the risk in this case that the insurer is slow to react to adverse changes in claims experience. [1]

Application of judgement

Consider the extent to which an individual risk should be charged for its own experience, especially if it has recently experienced a large claim. [½]

If the individual risk is not to be fully charged for its own claims, consider over how many risks should the surplus be spread, and which specific risks would be chosen. [½]

Consider patterns in recent frequency and severity in order to apply inflation and other trends to a risk premium based on past experience. [½]

Judgement should be applied where the premium calculated using credibility theory is significantly different from that suggested by normal underwriting. [½]

The results of each approach should be carefully examined, and checks done to ensure that key features have not been missed in any of the methods. [½]
[Maximum 19]

Solution X5.8

(i) ***Disadvantages of the square root rule***

The distribution of the X_j's (*ie* the average claims experience on the *j*-th risk) is unspecified. [½]

There is therefore no reason why the premium P is a more accurate estimator of future claims experience than M. [1]

If the standard for full credibility is n_F observations, such that P_F is the probability of your estimated premium falling within $\pm k\%$ of the true underlying value, then Z is a function of n_F and therefore a function of P_F and k. However, the model does not specify the optimal values of P_F and k. [2]

The model assumes that the estimate of future claims experience M based on external data is appropriate to the portfolio being considered. [1]

The model assumes that the observed claims experience $\bar{X}$ is also an appropriate approximation to the expected claims experience. [1]

For example, no allowance has been made in $\bar{X}$ or $\bar{S}$ for inflation, unusual claims experience, any changes in business mix / terms and conditions *etc* which may mean that past experience is not a good estimator of the future. [1]

[Maximum 6]

(ii) ***Bayesian credibility parameter k***

$$Z_B = \frac{n}{n+k}$$ [½]

and

$$Z_C = \left(\frac{n}{n_F}\right)^{1/2}$$ [½]

Where n is the number of claims and n_F is the number required for full credibility.

We require $\frac{n}{n+k} = \left(\frac{n}{n_F}\right)^{1/2}$, which gives $k = n_F\left(\frac{n}{n_F}\right)^{1/2}\left(1-\left(\frac{n}{n_F}\right)^{1/2}\right) = n_F Z(1-Z)$.[1]

For $n_F = 1{,}082$ and $n = 900$, this gives $Z = \left(\frac{900}{1{,}082}\right)^{1/2} = 0.91203$ and $k = 86.813$. [1]

[Total 3]

Solution X5.9

(i) ***Reasons for monitoring rate changes***

- To assess performance against goals, *eg* profitability targets.
- To manage risk, *eg* identifying variability of past profitability helps in deciding how much capital is needed to support the business.
- To gain market intelligence, *eg* assess the underwriting cycle.
- To satisfy regulators (may require premium rate monitoring)
- To help with reserving, *eg* by giving (pure) rate indices for use in the Bornhuetter-Ferguson method.

[½ mark for each, maximum 2]

(ii) ***Definition of premium rate***

We could use premium per unit of risk-adjusted exposure. [1]

For commercial property business, the exposure would be well-defined in terms of either sum insured or expected maximum loss. [½]

By adjusting for risk, we can allow for the very heterogeneous features of each commercial property. [½]

The premium would be net of commission. [½]
[Maximum 2]

Markers: justified alternatives are acceptable.

(iii) ***Ways to calculate rate changes***

Direct calculation of the premium rate [½]

Here, we calculate the premium rate for every policy, then compare the results from year to year. [½]

The advantages of this method are that:

- many factors affecting the expected loss can be taken into account [½]
- the absolute premium rate is calculated in addition to the rate change. [½]

For commercial property risks, we probably have the premium rate data already. [½]

However, quantifying the effect of softer factors, such as subtle changes in terms and conditions, may be difficult. [½]

The method takes time and is data-hungry. [½]

Price a standard risk [½]

This is a similar method but the premium rate is calculated for a specified "standard" risk or sample of risks which reflect the business mix of the portfolio as a whole. [½]

This method is simpler, quicker and less data-onerous than calculating the expected loss for every risk written. [½]

However, the method requires the additional assumption that the rate change for the standard risk is equal to the rate change for the entire portfolio. [½]

The method may be unsuitable for heterogeneous business such as commercial property risks, as there is rarely a "standard" risk. [½]

Measure rate changes on individual renewals [½]

For a heterogeneous book, it can be difficult to calculate the expected loss for a particular policy because the number of similar policies does not provide a credible basis for calculation. We address this problem by considering only the renewing policies and estimating the *change* in expected losses for these policies without determining the expected losses themselves. [1]

The aim of this method is to express the premium rate at t_2 as a proportion of the premium rate at t_1 for every renewed policy. If a rate change is calculated in this way then we do not need to know the absolute level of the premium rate. [1]

The model allows for the effect on premium rates of factors that are proportional to the expected loss – for example, limit / attachment points, the coinsurance share and the exposure measure. [1]

The model can be easily extended to allow for other factors – for example, claim inflation in excess of exposure change, change in rating factors and change in policy duration. [1]

The rate change for a group of renewed policies can be expressed as:

$$Rate\,Change_{t_1 \to t_2} = \frac{\sum Prem_{t_2}}{\sum As\text{-}if\ Prem_{t_1}} - 1 \qquad [1]$$

The "as-if" premium is the premium that would be charged for the renewal at time t_2 but on the rates applicable at time t_1. [½]

The main disadvantage with this method is that the impact of new and lost business written at different premium rates is ignored. [½]

It can also be difficult to quantify some of the soft factors (see above). [½]

Underwriter's view of the rate change [½]

This method involves recording how the underwriters perceive premium rates to be changing. [½]

The main advantage of this method is that it can allow for more of the soft factors mentioned above that would otherwise be unquantifiable. [½]

This is particularly useful for a commercial property account, where much of the pricing is performed by underwriters anyway. [½]

The main disadvantage is that it is very subjective, and therefore difficult to ensure consistency between underwriters and over different time periods. [½]

It is also difficult to verify in detail analytically. [½]

There may also be confusion around pure rate changes and hence mis-pricing. [½]
[Maximum 13]

Assignment X6 Solutions

Solution X6.1

- The profitability of the business ceded, *ie* the loss ratio. [1]
- Competitive pressures, such as how keen the reinsurer is to obtain motor quota share business. [1]
- The level of other cashflows, *ie* the reinsurer's expenses, profit and contingency loadings, investment income, brokerage, retrocession costs. [1½]
- The level of the cedant's acquisition expenses and claims handling expenses. [1]
- Loss sensitive factors, such as sliding scale or profit commission. [½]

[Total 5]

Solution X6.2

A stochastic event set is a set of possible future events … [½]

… created using past experience and a scientific understanding of the underlying causes of the natural hazards. [½]

It will include events that have never been observed historically. [½]

The model calculates the effect of these events on the insured portfolio, providing a detailed understanding and representation of the actual locations insured. [1]

[Maximum 2]

Solution X6.3

Perils include:

- hurricanes
- earthquakes
- tornadoes
- hailstorms
- winter storms
- river floods
- coastal floods
- disease

- warfare
- nuclear disasters
- wildfire (uncontrolled wilderness fires)
- terrorism. [½ each, maximum 5]

Solution X6.4

Average cost per claim can be calculated as follows (in £000s):

$$\int_{100}^{300} \frac{55(x-100)}{(50+x)^2}\,dx + \int_{300}^{500} \frac{11{,}000}{(50+x)^2}\,dx$$

$$= 55\left[\log_e(x+50) + \frac{150}{(x+50)}\right]_{100}^{300} - 55\left[\frac{200}{(x+50)}\right]_{300}^{500}$$

[2]

which is equal to

= 15.173 + 11.429 = 26.60 [1]

The expected number of claims is 50 and a contingency/expenses margin of 35% of the premium is required. The reinsurance premium is therefore given by:

$$(50\times 26{,}602)/0.65 = £2{,}046{,}000$$ [1]

Reasonableness check:

The proportion of claims falling in the band might be about 30% and the average payment from the reinsurer might be about £100,000, giving £30,000 as the cost per claim to the reinsurer. Hence answer looks OK.

The assumptions used here are:

- claim inflation can be ignored
- *reinsurer* also expects 50 claims from the portfolio and agrees the claim amount distribution
- no further profit loading is required (assumed to be covered by the contingency loading)
- this figure is before any profit commission arrangements.

[½ mark each]

Circumstances in which reinsurance would not be used:

- the 35% being paid towards expenses and contingencies is considered high in relation to the risks covered. That is, if the variance of the claim numbers and individual claim amounts is low, then the reinsurance would not be used. (Variance can be calculated in a similar way to the premium, if you have a few spare minutes!)
- if the company is financially strong, *ie* has large free reserves in excess of the required minimum margin
- if the company has other sources of finance, *eg* a parent company
- this business is relatively insignificant (*ie* a very large insurer)
- the company is not risk averse. [½ mark each]

[Maximum 8]

Solution X6.5

Demand surge reflects the basic economic reality of reduced supply and increased demand following a natural catastrophe. [1]

It is the temporary increase in repair / mitigation costs above the standard level of costs, resulting from the secondary impacts of the natural catastrophe itself. [1]

This increase is typically caused by:

- shortage of building materials due to, for example, damage to timber yards rendering available materials unusable [½]
- increased demand for building materials to repair / replace damaged properties [½]
- shortage of skilled labour due to, for example, people evacuating the area [½]
- increased demand for skilled labour to repair / rebuild properties. [½]

In practice, the labour shortage is often short-lived, as wages would increase, attracting new labour supplies. [½]

However, we may need to bring in labour from further afield which could be expensive *eg* involve paying accommodation costs. If the work takes longer than expected due to lack of materials, this will further increase the accommodation costs. [1]

When using catastrophe models, it is important for actuaries to understand the level of severity adjustments already included in the model and then decide on what other adjustments are appropriate, given the use to which the modelled output will be put. [1]

It can be difficult to decide on the adjustments for demand surge. [½]

The level of demand surge will depend on, for example:

- the timing of the catastrophe [½]
- existing supply and demand [½]
- location (which can affect the availability of labour, for example). [½]

[Maximum 7]

Solution X6.6

(i) ***Premium calculation***

We would start by looking at last year's pricing analysis, if any. At the very least we would see how the data provided this year differs from that provided in previous years, to assess any under-reserving or anomalies. [1]

We would hope to collect other relevant data, including:

- triangulations of paid claims (in order to allow for the timing of claims), incurred claims and premiums [½]
- slip terms [½]
- methodology used to calculate outstanding reserves [½]
- external data for use as benchmarks (*eg* loss ratios, development patterns, similar business written for other cedants). [1]

For motor property damage, however, we would not expect to see large delays in claims reporting or settlement, so we might be happy to assume that all years are fully run-off, maybe with the exception of the latest year. [½]

We would then adjust all the data to allow for:

- IBNR, if any
- claims inflation – using the 3% *pa* provided
- premium rate changes
- risk and exposure changes
- one-off events, large claims (unlikely for property damage), catastrophes and trends. [½ each]

We would then fit a model to the loss ratio, checking the goodness of fit and adjusting if necessary. Suitable claims distributions might include the log-normal distribution, for example. If the data is available, we might try to model frequency and severity separately. [1]

We would then calculate the expected loss to the layer using a stochastic approach for the forthcoming year of exposure, and allowing the possibility of the profit commission applying. [1]

Finally, we would add on the other loadings for contingencies, profit, normal commissions, brokerage and expenses. [1]

We would also make a small deduction for investment returns, which would be based on the claims payment pattern expected. [½]

[Maximum 9]

(ii) ***Important factors other than price***

Terms and conditions detailed on the slip, in order to assess other costs or possible exposure problems. [½]

Lack of underwriting control. Motor insurance is particularly vulnerable to competitive forces and short-term losses are often deemed to be "acceptable", whilst insurers build market share. [1]

The current position in the insurance cycle – particularly important for motor, to assess the likelihood of further rate reductions due to competitive pressures. [1]

Quality of the cedant's underwriting expertise and profit experience. A visit to the cedant may help here. [1]

Other similar business written by the reinsurer – do we want more exposure to this class? [½]

Cost of capital needed to back this business – particularly as the downside is unlimited. [½]

Opinions from the other reinsurers, particularly the lead underwriter. [½]

Relationship with the broker – effects on other business she passes us. [½]

Relationship with the cedant – other business we participate on. [½]

Availability of retrocession if required, if we wanted to limit our exposure. [½]

Standing in the market of other following reinsurers, if any, and their line sizes. Have they reduced their lines? Are they reputable reinsurers? [½]

Who is the lead underwriter and what is his standing in the market? [½]

The quality, quantity and credibility of the data we have been given – poor data often hides poor experience. [½]

The possibility of writing other business through this broker or with this cedant. [½]

Reason for such a late finalisation of the treaty – possibly due to lack of interest in the market? [½]

Reason for two-year term? – will lock in to current reinsurance rates, and opens up exposure to low original rates even further. [½]

Experience so far for 2013 – data should be available now and can supplement our analysis. [½]

Downside (say feasible maximum loss) in relation to upside (premium less expenses and profit commission). [½]
[Maximum 10]

(iii) ***Reply to underwriter***

The following points should be made within the reply. Note that many of the points are covered in (ii), but you need to be selective about those you use for the reply to the broker, as there is a (profitable?) relationship at stake.

Yes, the underlying experience has so far been good … [½]

… but the price depends on the expected cost to the layer, not the actual historic cost to the layer. [½]

The expected cost will depend on the predicted variability of the claims experience, which historically has been reasonably high … [½]

… and is showing signs of worsening in recent years. [½]

Additionally, we ought to allow for changes in profitability over the period. [½]

There is clearly some concern in the market as the deal isn't placed yet. [½]

Can we visit the cedant to get an idea of their competency? [½]

Whether it is possible to limit the downside, for example: [½]

- add a loss participation clause
- reduce our share
- revise the commissions, *eg* profit commission
- reduce the term to one year
- raise the attachment point
- put in an upper limit. [½ each]

[Total 7]

Solution X6.7

This approach would normally begin with the output from a proprietary catastrophe model. [½]

This usually comes in the form of files (AEPs and OEPs) showing the distribution of events. [½]

An occurrence exceedance probability (OEP) file considers the probability that the largest individual event loss in a year exceeds a particular threshold. [½]

An aggregate exceedance probability (AEP) file considers the probability that the aggregate losses from all loss events in a year exceeds a particular threshold. [½]

These files can be used in the stochastic frequency-severity model to simulate catastrophe loss experience in an annual period. [½]

The model could be run several thousand times for each of a number of different retention limits. [½]

For each retention limit, we can then derive the distribution of the annual reinsurance recoveries, along with the expected annual recoveries and the variance. [1]

By varying the retention limit the model could be used to find a suitable level. [½]

[Maximum 4]

Solution X6.8

(i) ***Two approaches***

Burning cost approach [½]

- Apply trends to the fully-ground-up historical loss data, which should include the paid and case reserve positions for each loss at regular points in time. [½]
- Apply the reinsurance contract terms to each trended loss to give trended losses to the layer, allowing for stability clauses if necessary. [1]
- Aggregate these amounts by year of loss or underwriting year to produce triangulations of paid and incurred loss development for losses to the layer. [1]
- Develop the triangles to ultimate using appropriate development patterns. [½]
- Supplement the data with benchmarks if necessary. [½]
- Allow for any exposure changes during the historical period covered by the loss data, for example by adjusting for rate changes. [½]
- We could, for example, take the average of each year's exposure-adjusted losses to the layer to arrive at an estimate of the loss cost for the year being priced. Alternatively, we could divide each year's losses to the layer by the exposure to give a loss rate for each year, then take the average and apply to the exposure for the year being priced. [1]
- Examine individual years' exposure-adjusted losses to the layer in case there are any remaining trends. [½]
- Allow for any changes in basis of cover. [½]

Frequency / severity model [½]

- Apply trends to the individual losses as per the burning cost method. [½]
- Allow for any material expected development on open claims (IBNER), for example by using development triangles. [½]
- Use the large loss count development to determine IBNR, in order to estimate the ultimate large loss count for each historic year. [½]
- Adjust for exposure as in the burning cost calculation. [½]
- Fit distributions to frequency and severity. [½]
- Combine the two distributions to produce a stochastic model for large losses, and hence model the reinsurance recoveries. [1]

For both methods, allow for practical issues such as:

- the appropriateness of the census point
- the choice of inflation rates
- treatment of shock losses
- discounting
- changes in the mix of business. [½ each]

[Maximum 12]

(ii) ***Factors affecting choice of approach***

- the approach to loading for profit – *ie* does it require a volatility measure or a distribution of outcomes? [1]
- the volume of loss data [½]
- time / resource constraints. [½]

[Total 2]

Solution X6.9

(i) ***Main problem with traditional methods***

Traditional rating approaches work well for a high frequency, low severity risk. [½]

They are much less appropriate for flood risk because:

- floods are a low frequency, high severity event [½]
- the observed losses may not be reflective of the true underlying risks as the period over which losses have been observed may be much lower than the return period of the losses under consideration … [½]
- … or the loss in question may not have occurred [½]
- for example, it is not appropriate to use 10 years' worth of data to estimate a flood event that is deemed to occur once every 50 years. [½]

[Maximum 2]

(ii) ***Model components***

Event module [½]

This will contain the "event set" with each flood event defined by its physical parameters, primarily: [½]

- location [½]
- expected frequency and severity [½]
- type of flood, *ie* sea or river. [½]

The frequencies and severities will need to take into account trends in sea and river levels and in the weather. [½]

Although this data can come from industry bodies, it may be difficult to interpret. [½]

In any case, floods can be infrequent and are unpredictable. This makes obtaining reliable frequency and severities difficult. [1]

The effect of climatic change (*eg* global warming) can be difficult to quantify and needs considerable expertise. [½]

There will always be population trends to consider – for example, any trends in the location of buildings. [½]

Hazard module [½]

This module will describe the consequence of each flood event. [½]

For example:

- the depth of floodwater by area [½]
- the duration of flooding [½]
- the speed of water flow [½]
- the area covered by each flood. [½]

There is a need to obtain and interpret survey information on coastal defences, *eg* location, height and quality. [1]

This requires considerable expertise and time. [½]

Flood maps and terrain models could be used, if available. [½]

We should take into account any impending government initiatives, although again the results may be difficult to quantify. [½]

Inventory module [½]

This will consist of a detailed exposure database of insured properties. [½]

The key details held for each property will be:

- type of cover (buildings or contents, new for old or indemnity) [1]
- type of property [¼]
- location [¼]
- construction [¼]
- number of floors [¼]
- sum insured [¼]
- age [¼]
- occupancy [¼]
- elevation [¼]
- previous flooding [¼]
- proximity to rivers/coasts/lakes. [¼]

One problem here is that the policyholder may not know about:

- a property's proximity to rivers and its vulnerability to flooding [½]
- any previous flood experience to that property (or may give false information, giving rise to moral hazard) [½]
- height of property above sea level or river level [½]

and he or she may be resistant to answering these types of questions … [½]

…although the data sources for the Hazard module should help to mitigate this problem. [½]

Vulnerability module [½]

This module will describe the degree of loss to each property resulting from exposure to flood (perhaps expressed as a percentage of sum insured). [½]

There will be considerable uncertainty involved in assessing the exact effects of flood on individual properties. [½]

Financial analysis module [½]

This module translates the total ground-up loss into an insured loss. [½]

In order to do this, it will need policy conditions for each policy, for example:

- limits
- flood excess
- coverage terms and conditions. [1]

Reinsurance details will also be needed so that net claims can be assessed. [½]
[Maximum 21]

(iii) ***Uses for flood catastrophe model***

The prime use is to monitor the aggregate insured losses for risk tolerance purposes. [1]

Capital assessment and allocation, both internally and externally. [1]

Reinsurance planning and assessment. [½]

Operational risk assessment. [½]

To assist in the disaster recovery planning process. [½]

Pricing – exposure rating for larger risks, property-specific pricing, and overall catastrophe loadings for the whole portolio. [1]

To help set underwriting guidelines. [½]

To help set appropriate reserves. [½]

To help in the design of insurance-linked derivatives, *eg* catastrophe bonds. [½]

Financial planning. [½]
[Maximum 6]