Problem Description:

Problem T2.1 (100 Points) We know by some "magical" calculation

$$I = \int_{-1}^{1} e^{(x^5)} dx \approx 2.0949681713212$$

The integrand looks like

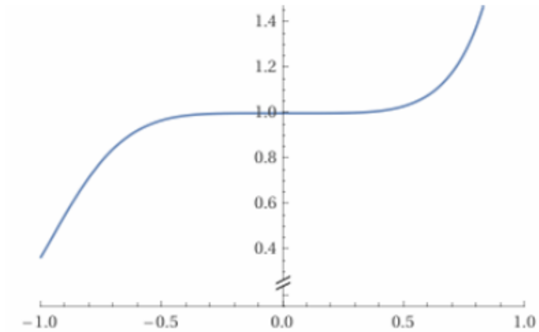

Figure 1. The integrand for this problem.

For this test, you write programs with the following methods with associated given parameters to compute the integral. You need sufficient details in implementing these algorithms.

Method	Number of Mesh Points	Your Integral	Your Error	Credits to Give
Midpoint	$N = 100$			20
Simpson 1/3	$N = 101$			20
Simpson 3/8	$N = 101$			20
Gaussian Quadrature	$N = 5$			20
Monte Carlo	$N = 1000$			20

Algorithm Description:

To approximate the integral I= e^(x^5) dx, the boundary is set to [-1,1]. the Midpoint Method divides the interval into N subintervals, evaluates the function at the midpoints, and sums the results, multiplying by the subinterval width. Simpson's 1/3 Rule applies quadratic interpolation, weighting the function evaluations at the endpoints and intermediate points, and calculating a weighted sum. Simpson's 3/8 Rule uses cubic interpolation with different weights for each set of points. Gaussian Quadrature evaluates the function at predefined roots of the Legendre polynomial, with corresponding weights, to achieve high accuracy with fewer points. Monte Carlo Integration randomly samples points within the interval, evaluates the function at those points, averages the results, and scales by the interval length.

Results:

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Midpoint: Integral = 2.0947725685, Error = 0.0001956028
Simpson 1/3: Integral = 2.0949693709, Error = 0.0000011996
Simpson 3/8: Integral = 2.0820747975, Error = 0.0128933738
Gaussian Quadrature: Integral = 2.0922416734, Error = 0.0027264979
Monte Carlo: Integral = 2.0926809027, Error = 0.0022872686
```