Sets

I set of instance types.

P set of possible computing cloud providers.

$PI_p \subseteq I$ set of instances that belong to provider.

L a set of levels the workflow is divided into.

G a set of task groups, tasks in a groups have the same computational cost and input/output size.

$L_l^L \subseteq G$ a set of task groups belonging to a level l.

Params

$n_p^{Pmax} \geq 0, \in \mathbb{Z}$ upper limit of number of instances allowed by a cloud provider p.

$n_i^{Imax} > 0, \in \mathbb{Z}$ upper limit of number of instances allowed by a cloud provider of instance i.

$p_{i_1}^I \geq 0$ a fee (in US dollars) for running the instance of type i for one hour.

$ccu_{i_1}^I \geq 0$ performance of instance of type i in CloudHarmony Compute Units (CCU).

$p^R \geq 0$ price per task for a queuing service, such as Amazon SQS.

$p_{i_1}^{Iout} \geq 0$ price in dollars per MiB for non-local data transfers.

$p_{i_1}^{Iin} \geq 0$ price in dollars per MiB for non-local data transfers.

$p_{i_1}^{Sout} \geq 0$ price in dollars per MiB for non-local data transfers.

$p_{i_1}^{Sin} \geq 0$ price in dollars per MiB for non-local data transfers.

$l_{i,s} \in 0, 1\ , \geq 0$ matrix showing which transfers are local (0) and which non-local (1).

$r_{i,s} \geq 0$ data transfer rates between a given storage site s and instance i in MiB per second.

$t_g^x \geq 0$ execution time in hours of a single task in a group g on a machine with the processor performance of 1 CloudHarmony Compute Unit (CCU).

$A_g^{tot} > 0, \in \mathbb{Z}$ number of tasks in a group g.

$d_g^{in} \geq 0$ data size for input of a task in group g.

$d_g^{out} \geq 0$ data size for output of a task in group g.

$t^D > 0, \in \mathbb{Z}$ total time allowed for completing workflow (deadline).

$t^{net}_{t,i,s} = \frac{d^{in}_t + d^{out}_t}{r_{i,s} \cdot 3600}$ *transfer time* in hours, i.e. time for data transfer between instances of type i and storage site s for a task in task group g.

$t^u_{t,i,s} = (\frac{t^x_t}{ccu^I_i},\ t^{net}_{t,i,storage})$ time in hours for processing a task in group g on instance of type i using storage site s.

$c^T_{t,i,s} = (d^{out}_t \cdot (p^{Iout}_i + p^{Sin}_s) + d^{in}_t \cdot (p^{Sout}_s + p^{Iin}_i)) \cdot l_{i,s}$ a cost of data transfer between an instance of type i and a storage site s when processing task in group g.

Variables

$N_{t,i,idx} \in 0, 1$ 1 iff (if and only if) instance of type i with index $k \in_i$ is launched to process task group g, otherwise 0 (binary).

$H_{t,i,idx} \in \mathbb{Z}, \geq 0, \leq t^D$ for how many hours the instance of index k is launched (integer).

$T_{t,i,idx} \in \mathbb{Z}, \geq 0, \leq A^{tot}_t$ how many tasks of g are processed on that instance (integer).

$D_l \in \mathbb{Z}, \geq 1, \leq t^D$ actual computation time for level l (real).

$D^t_l \geq 0, \leq t^D$ maximal number of hours (deadline) that instances are allowed to run at level l (integer).

Objectives

$$\underset{TotalCost}{\text{minimize}} \sum_{\substack{t \in G \\ i \in I \\ idx \in \{x \mid\ x \in [0,\ n^{Imax}_i - 1]\}}} p^I_i \cdot H_{t,i,idx} + (p^R + c^T_{t,i,storage}) \cdot T_{t,i,idx} \tag{1}$$

Objectives have the following meaning:

- 1 is total cost of executing workflow

Constraints

$$\sum_{l \in L} D_l^t \leq t^D \tag{2}$$

$$\forall_{l \in L} D_l \leq D_l^t + 1 \tag{3}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} H_{t,i,idx} \geq N_{t,i,idx} \tag{4}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} H_{t,i,idx} \leq t^D \cdot N_{t,i,idx} \tag{5}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} T_{t,i,idx} \geq N_{t,i,idx} \tag{6}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} T_{t,i,idx} \leq A_t^{tot} \cdot N_{t,i,idx} \tag{7}$$

$$\forall_{\substack{l \in L \\ t \in L_l^L \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} H_{t,i,idx} \leq D_l \tag{8}$$

$$\forall_{\substack{l \in L \\ t \in L_l^L \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} T_{t,i,idx} \cdot t_{t,i,storage}^u \leq D_l^t \tag{9}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} H_{t,i,idx} \geq T_{t,i,idx} \cdot t_{t,i,storage}^u \tag{10}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} H_{t,i,idx} \leq T_{t,i,idx} \cdot t_{t,i,storage}^u + 1 \tag{11}$$

$$\forall_{t \in G} \sum_{\substack{i \in I \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} T_{t,i,idx} = A_t^{tot} \tag{12}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [1, \ n_i^{Imax} - 1]\}}} H_{t,i,idx} \leq H_{t,i,idx-1} \tag{13}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [1, \ n_i^{Imax} - 1]\}}} N_{t,i,idx} \leq N_{t,i,idx-1} \tag{14}$$

$$\forall_{\substack{t \in G \\ i \in I \\ idx \in \{x| \ x \in [1, \ n_i^{Imax} - 1]\}}} T_{t,i,idx} \leq T_{t,i,idx-1} \tag{15}$$

$$\forall_{\substack{l \in L \\ p \in P}} \sum_{\substack{i \in PI_p \\ t \in L_l^L \\ idx \in \{x| \ x \in [0, \ n_i^{Imax} - 1]\}}} N_{t,i,idx} \leq n_p^{Pmax} \tag{16}$$

Constraints have the following meaning:

- 2 ensures that workflow finishes in given deadline
- 4 and 5 ensure that H may be allocated only iif N is 1
- 6 and 7 ensure that T may be allocated only iif N is 1
- 8 enforces level deadline on instances runtime
- 9 enforces level finishes work in D^t
- 10 adjust H respectively to T
- 12 ensures that all tasks are processed
- 13 to 15 reject symmetric solutions
- 16 enforces providers' instance limits