# How to do partial elimination with BLAS/LAPACK

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### 1 Problem statement

Lets assume that we have equation system of size n. We want to partially eliminate first m variables from the system, so that k = n - m variables may still be not eliminated. What we need to do is to calculate Shur complement of the equation system.

$$\begin{bmatrix} A_{m \times m} & B_{m \times k} \\ C_{k \times m} & D_{k \times k} \end{bmatrix}_{n \times n} x = \begin{bmatrix} E_{m \times 1} \\ F_{m \times 1} \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{I}_{m \times m} & B'_{m \times k} \\ \mathbf{0}_{k \times m} & D'_{k \times k} \end{bmatrix} x = \begin{bmatrix} E'_{m \times 1} \\ F'_{m \times 1} \end{bmatrix}$$
(1)

### 2 Algorithm

$$B' = A^{-1} \times B \tag{2}$$

$$E' = A^{-1} \times E \tag{3}$$

$$D' = D - C \times B' \tag{4}$$

$$F' = F - C \times E' \tag{5}$$

## 3 BLAS/LAPACK implementation

Let's assume that

• matrices use Foratran (column-wise) ordering, not the C one (row-wise),

- **G** is LHS matrix of the system that is composed of submatrices **A** to **D**,
- ullet H is RHS vector of the system that is composed of subvectors  ${f E}$  and  ${f F}$ ,
- A to F are pointers to the beginning of the submatrices,
- $\bullet$  n, m and k are defined as in introduction,
- we check if operation succeeded after operations that may fail (e.g. LU decomposition),
- the operation will be in-place and will modify input matrix.

#### Algorithm

- Do the LU decomposition for submatrix A: dgetrf(m, m, A, n, ipiv, status)
  The row pivot vector returned in ipiv will be needed in next two steps.
- 2.  $B = (PLU)^{-1} \times B$ : dgetrs(NoTrans, m, k, A, n, ipiv, B, n, status)
- 3.  $E=(PLU)^{-1}\times E$ : dgetrs(NoTrans, m, 1, A, n, ipiv, E, n, status)
- 4.  $D = D C \times B$ : dgemm(NoTrans, NoTrans, k, k, m, -1.0, C, n, B, n, 1.0, D, n)
- 5.  $F = F C \times E$ : dgemv(NoTrans, k, m, -1.0, C, n, E, 1, 1.0, F, 1)
- 6.  $A = I_{m \times m} A$  is identity matrix
- 7.  $C = \mathbf{0}_{k \times m} C$  is zero matrix