How to do partial elimination with BLAS and LAPACK

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1 Problem statement

Lets assume that we have equation system of size n. We want to partially eliminate first m variables from the system, so that $k = n - m$ variables may still be not eliminated. What we need to do is to calculate Shur complement of the equation system.

$$\begin{bmatrix} A_{m\times m} & B_{m\times k} \\ C_{k\times m} & D_{k\times k} \end{bmatrix}_{n\times n} x = \begin{bmatrix} E_{m\times 1} \\ F_{m\times 1} \end{bmatrix}_{n\times 1} \longrightarrow \begin{bmatrix} \boldsymbol{I}_{m\times m} & B'_{m\times k} \\ \mathbf{0}_{k\times m} & D'_{k\times k} \end{bmatrix}_{n\times n} x = \begin{bmatrix} E'_{m\times 1} \\ F'_{m\times 1} \end{bmatrix}_{n\times 1} \tag{1}$$

2 Algorithm

$$B' = A^{-1} \times B \tag{2}$$
$$E' = A^{-1} \times E \tag{3}$$
$$D' = D - C \times B' \tag{4}$$
$$F' = F - C \times E' \tag{5}$$

3 BLAS/LAPACK implementation

Let's assume that

- matrices have continuous columns (like in Foratran), which is not standard behavior in C,

- **G** is LHS matrix of the system that is composed of submatrices **A** to **D**,
- **H** is RHS vector of the system that is composed of subvectors **E** and **F**,
- **A** to **F** are pointers to the begining of the submatrices,
- **n**, **m** and **k** are defined as in introduction,
- we check if operation succeeded after operations that may fail (e.g. LU decomposition),
- the operation will be in-place and will modify input matrix.

Algorithm

1. Do the LU decomposition for submatrix A:
   `dgetrf(m, m, A, n, ipiv, status)`
   The row pivot vector returned in `ipiv` will be needed in next two steps.
2. $B = (PLU)^{-1} \times B$:
   `dgetrs(NoTrans, m, k, A, n, ipiv, B, n, status)`
3. $E = (PLU)^{-1} \times E$:
   `dgetrs(NoTrans, m, 1, A, n, ipiv, E, n, status)`
4. $D = D - C \times B$:
   `dgemm(NoTrans, NoTrans, k, k, m, -1.0, C, n, B, n, 1.0, D, n)`
5. $F = F - C \times E$:
   `dgemv(NoTrans, k, m, -1.0, C, n, E, 1, 1.0, F, 1)`
6. $A = \boldsymbol{I}_{m \times m}$ – A is identity matrix
7. $C = \mathbf{0}_{k \times m}$ – C is zero matrix

Steps 6 and 7 may be omited if you only need Schur complement (matrices D and F).