Computational Physics
Homework 1

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September 7, 2014

1 Problem 1

Program forward and backward recursion to calculate the integral

$$I_n = \int_0^1 \frac{x^n}{x+5} dx \tag{1}$$

comparing single, double and quadruple precision with the ”exact” result.

1.1 Description of program

set up single, double and quadruple arrays to store the output. Set up single, double and quadruple initial conditions, define a function to calculate the ”exact” value. open a file for output.

create for loop which runs forwards in one program and backwards in the other and fill in the array in the correct order. In the case of forward recursion, fill in the output during this loop, for the backward recursion this requires another loop.

2 Problem 2

one kilometer rail is expanded by 1 meter, how much does it buckle?

2.1 diagram

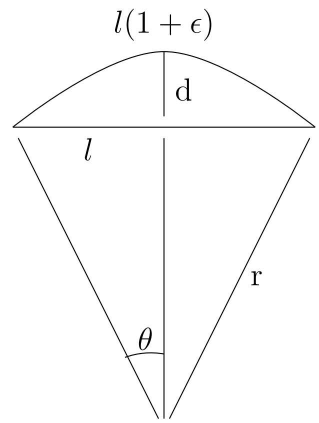

Figure 1: diagram defining the variabls used in the problem

2.2 calculation

We start with three equations which will allow us to find d, in terms of the known variables l and ϵ.

$$l(1+\epsilon) = 2r\theta \tag{2}$$
$$l = 2r\sin\theta \tag{3}$$
$$d = r - r\cos\theta = 2r\sin^2\left(\frac{\theta}{2}\right) \tag{4}$$

where we have used a trig identity to get rid of the numerically bad expression $1-\cos\theta$.

By using a trig identity on equation 3 we get

$$l = 4r\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \tag{5}$$

we can combine this with equation 4 to get

$$\frac{d}{l} = \frac{1}{2}\tan\left(\frac{\theta}{2}\right). \tag{6}$$

By comparing equations 2 and 3 we can get an expression for θ in terms of ϵ

$$\frac{\sin\theta}{\theta} = \frac{1}{1+\epsilon} \tag{7}$$

this equation cannot be solved analytically so we must expand the RHS.

$$\frac{\theta - \frac{\theta^3}{6} + \frac{\theta^5}{120}}{\theta} = \frac{1}{1+\epsilon} \tag{8}$$

i have expanded to 5th order in θ in order to get the required accuracy (I tried only going to 3rd order but the final answers differ in the 5th significant digit).

This equation simplifies to a quadratic equation in θ^2

$$\frac{1}{120}(\theta^2)^2 - \frac{1}{6}\theta^2 + \frac{\epsilon}{1+\epsilon} = 0 \tag{9}$$

This can be solved using the alternative quadratic formula

$$\theta^2 = \frac{\frac{2\epsilon}{1+\epsilon}}{\frac{1}{6} + \sqrt{\frac{1}{36} - \frac{\epsilon}{30(1+\epsilon)}}} \tag{10}$$

using $\epsilon = 0.001$ we have

$$\theta^2 = 0.005995803 \tag{11}$$

$$\Rightarrow \theta = 0.07743257 \tag{12}$$

we finally place this in equation 6 to get

$$d = 19.368m \tag{13}$$