Bound Solitons Induced by Noise-Mediated Casimir-Like Interactions

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Abstract

We study experimentally and theoretically the steady states of optical solitons that aggregate into bunches in a nonlinear multimode interference saturable absorber based mode-locked fiber laser. By modifying the laser gain via its pumping current, we demonstrate a variety of steady states. We model the steady states using our recently proposed noise-mediated Casimir-like pulse interaction mechanism and obtain a quantitative agreement with our experimental results. We present a steady state phase diagram and demonstrate transitions between noise-mediated and coherently bound states without using additional modulators, asserting the noise-mediated interaction as a dominant mechanism in the complex soliton structures. The control of the complex soliton structures via the pumping current opens the door for the use of ultrafast lasers in telecommunication.

1 Introduction

Dissipative solitons are localized waves in open systems far from equilibrium, whose existence result from a balance of dissipative and dispersive effects. They appear in numerous physical areas including reaction-diffusion systems, neurological and ecological sciences, fluid dynamics, and photonics [1–7]. Photonics is a great platform to observe the nonlinear complex behavior of dissipative solitons [8–14]. In particular, collective nonlinear phenomena such as the aggregation of solitons into bunches have created great interest [15, 15–28].

Multi-soliton patterns exhibit an extensive pallet of short and long-range pulse interactions. Short-range interactions take place when pulse tails overlap [21, 29–32]. However, interactions over separations of orders of magnitude beyond the width of individual solitons must be mediated by the the gain [19, 33], or by acoustic response [17, 34, 35]. Furthermore, non-soliton components of optical pulses, such as dispersive-wave pedestals give rise to interactions with an intermediate range significantly longer than the soliton width, but much shorter than the cavity length [18, 36]. Recently, the manipulation of complex soliton states has been achieved utilizing optical modulation components [35, 37, 38]. A major challenge in the study of multi-soliton patterns is the distillation of an effective interaction model, which would determine their steady states.

Recently, we introduced a long-range interaction mechanism arising from the effect of gain depletion in the presence of noisy quasi-CW light in a mode-locked laser [33, 39]. The interaction results from the reduction of optical fluctuations due to gain depletion, following the passage of a pulse through the gain medium. Suppression of fluctuations decrease the temporal jitter of subsequent pulses and in this way bias it and sets the pulse motion. This noise-mediated interaction (NMI) mechanism shares some intriguing properties with the Casimir effect in quantum electrodynamics, where macroscopic objects experience an effective interaction due to suppression of electromagnetic field fluctuations [40]. In both the NMI and the Casimir effect, distant objects inhibit microscopic fluctuations in extended electromagnetic modes. The consequent breaking of spatiotemporal homogeneity gives rise to the weak interactions among these objects.

Optical solitons in anomalous-dispersion mode-locked lasers are often accompanied by weak and broad dispaers-wave pedestals [41]. When solitons are well-separated, the NMI is generally attractive [33], but when two solitons are close enough that their pedestals overlap, the interaction can become repulsive; consequently NMI can lead to the formation of bound states of solitons whose steady-state separation is comparable with the pedestal width. The overall phases of solitons in such bound states are not coherently locked, so that they can be viewed as *loosely bound.*

The propagation of dispersive waves in a mode-locked laser is highly sensitive to the overall net gain, which means that the pedestals can be controlled by the amplifier pump. Here, we use this control knob to experimentally observe and manipulate the steady state of a two-pulse waveform in an all-fiber laser passively mode-locked by a nonlinear multimode interference-based saturable absorber [42]. The NMI theory makes precise predictions about the dependence of the pulse separation on the pedestal power and width, which are in quantitative agreement with the experimental measurements, demonstrating that NMI is the dominant mechanism in long-range pulse interactions in mode-locked fiber lasers of this type.

Even though, as explained, the pedestal overlap induces effective inter-pulse repulsion, repulsion is strong enough to generate only for a large enough pedestal power. Our experiment confirm this theoretical prediction, and demonstrate a transition to a state with *tightly bound* solitons with locked phases, below the pedestal power threshold. The controllability of the pulse separation and the ability to switch from tightly- and loosely-bound soliton regime, constitute a significant step toward applications of multipulse laser waveforms.

2 Results

2.1 Mode-Locked Fiber Laser

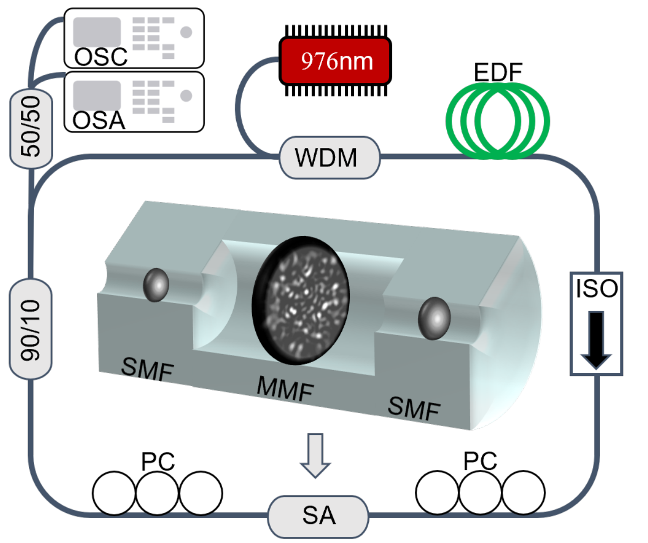

Figure 1: **Experimental setup.** SA: saturable absorber, SMF: single mode fiber, MMF: multimode fiber, EDF: erbium-doped fiber, WDM: wavelength-division multiplexer, ISO: optical isolator, PC: polarization controller, OSA: optical spectrum analyzer, OSC: oscilloscope.

The laser setup is an all-fiber integrated ring cavity operated in the anomalous dispersion regime. The cavity length is 14m, including a 2.7m erbium-doped fiber gain medium, as depicted in Figure 1. The erbium-doped fiber is core pumped by a 976 nm diode laser through a wavelength-division multiplexer. A polarization-independent optical isolator ensures unidirectional lasing and two fiber polarization controllers tune the overall low cavity birefringence. Finally, a 90/10 coupler provides the laser output.

The mode-locking operation is passively achieved by employing a nonlinear multimode interference-based saturable absorber [42–52]. In this scheme, light occupying a single spatial mode is coupled to a multimode fiber (MMF), whose output facet is spliced to a single mode fiber as in Figure 1. The light transmittance at the splicing point depends on the interference of the excited MMF modes. At high power, the interference is modified by Kerr nonlinearity making the transmittance at the splicing point power-dependent. In our setup, we utilize a 1.5 m long segment of graded-index multimode fiber (OM1; 62.5 μm core diameter, NA = 0.22) adopted as an effective saturable absorber of a passively mode-locked laser. This approach enables us to modify the absorber properties, such as linear loss, saturated loss, and saturation power, by adjusting the MMF position and the polarization controller. As a result, a wider range of soliton configurations can be achieved [47]. These solution structures were analyzed at the laser output with an optical spectrum analyzer (Yokagawa AQ6374), and a fast photodiode connected to a real-time oscilloscope (Tektronix MSO70804C).

Here, we produce stationary patterns of soliton pairs by varying the pump power beyond the threshold for stable single-pulse operation. The laser output spectrum is presented Figure 2(a) as a function of the pumping current . The central wavelength is 1561 nm, and the full width at half maximum bandwidth is ∼3.8 nm. At pump currents below 120 mA ??, we observe spectral fringes indicating that the solitons interact coherently, so that their phases are locked, i.e the solitons tightly bound; at higher pumping currents there are no interference fringes, so that the pulse phases are not coherently locked, which means that the solitons are loosely bound.

Narrow sidebands appear at the primary Gordon-Kelly resonances [53, 54], 1553 nm and 1567 nm, and their power grows with the pumping current. Figure 2(c) presents two spectral cross-sections of Figure 2(a), at pumping currents of 123 mA (blue) and 120 mA (red), exhibiting the Kelly sidebands and the interference fringes in the latter.

Figure 2(b) presents time domain traces of the cavity waveform intensity as a function of the pumping current, where two maxima are observed at high enough pump currents, above 121 mA. At lower pump currents the soliton separation is too small to be resolved in the oscilloscope, but the approximate conservation of total energy in the soliton waveform shows that the number of solitons has not changed. Indeed, the time-domain cross sections at pumping currents of 123 mA (blue) and 120 mA (red) presented in Figure 2(d), imply that both waveforms have equal total energy, proportional to the total area under the curves, as expected from the clamping of the soliton energy in anomalous-dispersion passively mode-locked lasers [55–57]. At 120 mA in particular, the separation time is estimated from the spectral interference pattern at a few ps, far below the resolution of the 8 GHz bandwidth of the oscilloscope.

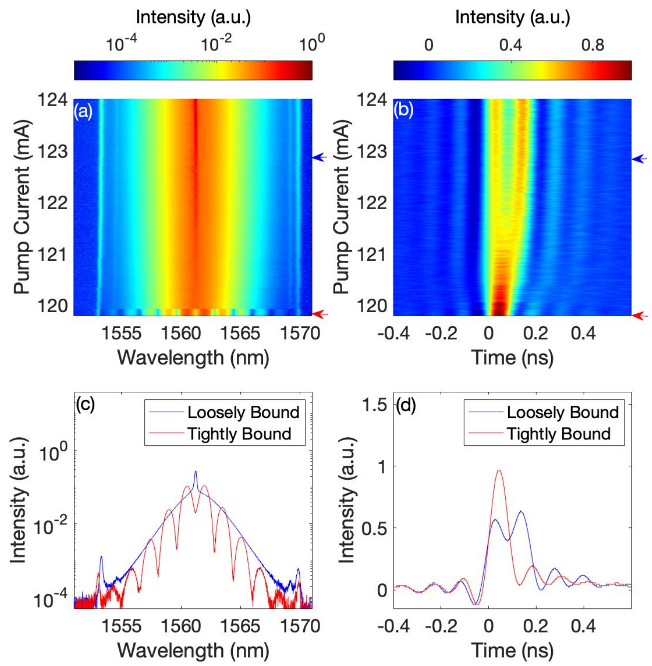

Figure 2: **Raw data.** (a) Optical spectrum analyzer traces versus pumping current. (b) Oscilloscope traces verses pumping current. (c) Optical spectrum analyzer traces at pumping current of 123 mA (blue) and 120 mA (red) demonstrate the two soliton binding regimes; Loosely bound (LB) pairs with fluctuating relative phase and Tightly bound (TB) pairs with coherently locked phases, and associated spectral. (d) Oscilloscope traces at pumping currents of 123 mA (blue) and 120 mA (red) demonstrate that the total energy in the soliton waveform remains nearly constant even when the inter-soliton separation is too small to be resolved by the oscilloscope.

2.2 Noise-Mediated Soliton Interaction

We begin this section by describing the NMI and calculating its steady states, highlighting the results from [33]. We then discuss cases where no loosely bound steady states exist, in which NMI causes attraction between the pulses for any separation. Considering the regimes where small separation is obtained, we explain the transition from a steady state caused by NMI to a steady state caused by a coherent interaction.

The NMI is a byproduct of the pulse timing jitter caused by nonlinear interaction between the pulses and a weak quasi-CW light that runs in the cavity in addition to the pulses. The instantaneous power of the quasi-CW light is determined by the time dependent gain profile.

We assume that the time-dependent gain profile is depleted by the pulses. In our model, the pulse consists of an incoherent sum of a soliton and a low amplitude pedestal centered around the pulses, caused by Kelly-Gordon resonances [41,53] pedestal. The temporal width of the solitons is much smaller than any other relevant timescale, leading to a sharp drop in the gain profile. The pedestal waveform of the nth pulse, normalized by the pulse energy and governed by the Kelly resonant, is $\tilde{E}_{ped}/(2\tilde{w}t_R)e^{-\frac{|t-t_n|}{\tilde{w}t_R}}$ where $\tilde{E}_{ped}$ and $\tilde{w}$ are the pedestal energy and width normalized by the pulse energy and the cavity round trip time (t_R), respectively. These variables are the only unitless quantities determining the normalized stable separation time. The gain depletion due to the pedestals smoothens the gain profile. We a assume a constant recovery rate of the gain in the medium between pulses (Figure 3):

$$\frac{\partial g}{\partial t} = \frac{\partial g_u - g}{\partial t_{rec}} \tag{1}$$

where g_u is the unsaturated gain and t_{rec} is the recovery time; these parameters are determined by the pumping regime. Given the depletion resulting from the solitons as well as the pedestals, and the linear recovery, a saw-tooth tooth temporal gain profile shape is obtained as seen in Figure 3.

Accordingly, the quasi-CW power also exhibits a smoothed saw-tooth profile. The interaction between the quasi-CW light and the pulses causes diffusion of the pulse position which redetermines the quasi-CW power profile in the cavity. This diffusion induces a drift velocity of the pulses, similar to a Brownian motion in an inhomogeneous environment that depends on the particles' position.

The diffusion constant D_n of the nth pulse is proportional to the local intensity of the quasi-CW, in turn determined by the local net gain as:

$$D_n \propto \left(\frac{1}{\sqrt{l - g_n^-}} + \frac{1}{\sqrt{l - g_n^+}}\right) \tag{2}$$

where l is the total small signal loss and g_n^-, g_n^+ are the gain coefficients before and after the nth pulse, respectively. The variations in g are small compared with the mean gain $\hat{g}$, but may induce large changes in the net loss. Denoting the timing of the nth pulse by t_n, its diffusion constant by D_n, and the slow time coordinate or propagation distance by z, then:

$$\frac{\partial \langle t_n \rangle}{\partial z} = \frac{1}{2}\frac{\partial D_n}{\partial t_n} \tag{3}$$

where the brackets indicate noise-averaging.

Analyzing the drift equation, we find an effective two-pulse potential that determines the steady states. The steady states separation time, governed by the potential minimum, increases with the pedestal energy. The minimum of the effective potential, calculated assuming $\tilde{w} \ll 1$, leads to a stable separation time of [33]:

$$\tilde{t}_s = \tilde{w}\ln\left(\frac{\tilde{E}_{ped}}{\tilde{w}}\right) \tag{4}$$

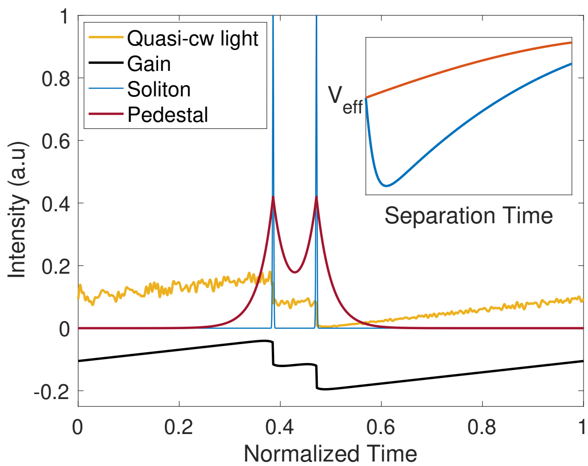

Figure 3: **Illustration of the NMI mechanism.** Passage of soliton pulses through the gain medium depletes the instantaneous gain. A pair of pulses yields a smoothed saw-tooth gain profile. The quasi-CW light light that runs in the cavity is determined by the gain profile, and thus also exhibits a smoothed saw-tooth profile. The NMI is a by-product of the pulse timing jitter caused by nonlinear interaction between the pulses and a quasi-CW light. The inset shows the effective potential for the noise-mediated interaction of the loosely bound (blue) and tightly bound (red) states.

where $\tilde{t}_s$ is the stable separation time normalized by the cavity round trip time.

In the case where $\tilde{E}_{ped} < \tilde{w}$, the minimum of the potential appears at zero separation time (inset of Figure 3) such that the noise-mediated interaction causes attraction at all separations.

In the case where the separation time is not much longer then the soliton duration, the NMI can be weak compared to the coherent interaction which decreases exponentially with the separation time. Therefore, there is a threshold for a phase transition between the NMI induced steady state, i.e loosely bound, and the coherent interaction induced steady state, i.e tightly bound. This phase transition, found empirically at $\tilde{t}_s = 0.5 \cdot 10^{-3}$, is 35 times longer than the pulse duration as will be shown in the next section.

2.3 Soliton Steady States Analysis

We begin this section by applying the theory of NMI to explain the steady states of soliton pairs that we observe experimentally. We then present the steady states in a phase-space diagram. The phase diagram allows us to propose an approach to observe the phase transition between loosely and tightly bound steady states by adjusting the pumping current.

We extract from Figure 1(a-b) the measured separation times between the pulses, and the Kelly sidebands energy, which corresponds to the pedestal energy (See supplementary information, section 1). In Figure 4(a) we present the separation time of the soliton pair, normalized by the cavity round trip time, versus the pedestal energy, normalized by the pulse energy. The dots and triangles correspond to data measured at the loosely bound regime (LB), and at the tightly bound regime (TB) which is identified by the interference fringes in the measured power spectrum. Figure 4(a) shows three different data sets obtained by adjusting the polarization controller to modify the SA behavior. The curves in Figure 4(a) correspond to the prediction of our NMI model, showing a good agreement with the experimental results. We assumed pedestal widths of 77ps, 56ps and 46ps (corresponding to normalized values of $0.80{\cdot}10^{-3}$, $1.10{\cdot}10^{-3}$, $0.66{\cdot}10^{-3}$) for the experiment 1 (yellow), experiment 2 (red), experiment 3 (blue) data sets, respectively. While we could not measure the pedestal width directly using the oscilloscope, we estimate its lower bound by the measured width of the Lorentzian shaped Kelly sidebends [41]. We obtain 12.2±0.14ps, 9.8ps ±0.3ps, 10.4 ±0.14ps, (corresponding to normalized values of $0.17 \cdot 10^{-3}$, $0.14 \cdot 10^{-3}$, $0.15 \cdot 10^{-3}$) for experiment 1,2,3, respectively. We note we obtain the lower bounds without compensate for the spreading of the spectrum due to the $50pm$ resolution of the optical spectrum analyzer, hence the actual pedestal width should be closer to the value we found by the fitting.

We further propose an approach to observe the phase transition between loosely and tightly bound steady states by adjusting the pumping current. By decreasing the pumping current, we decrease the normalized pedestal energy (see supplementary information Figure 2(a) for more details). According to Equation 4, by deceasing the pedestal energy, we continuously decrease the separation time.

By decreasing the pumping current, two scenarios may occur. The first scenario is that the separation time decreases down to the scale of the soliton duration. In this scenario, a phase transition occurs and the separation is locked by the coherent interaction, as explained in the last section. This phase transition occurs between loosely and tightly bound states, presented in the experiment 1 dataset in Figure 4 (b). We note that to observe the transition we use the pumping current as the only knob.

The second scenario that may occur by decreasing the pumping current is that the soliton pair state in the cavity becomes unstable before we the threshold for the phase transition between the loosely and tightly bound state. This case is presented in experiment 2 and 3 data sets in Figure 4 (b). This scenario is measured in different configuration of the polarization controllers, adjusted to maximize the pedestal energy (higher than 10^{-3}). The phase transition can not be observed by reducing the pumping current since the annihilation of one of the soliton occurs before the solitons are close enough to bound

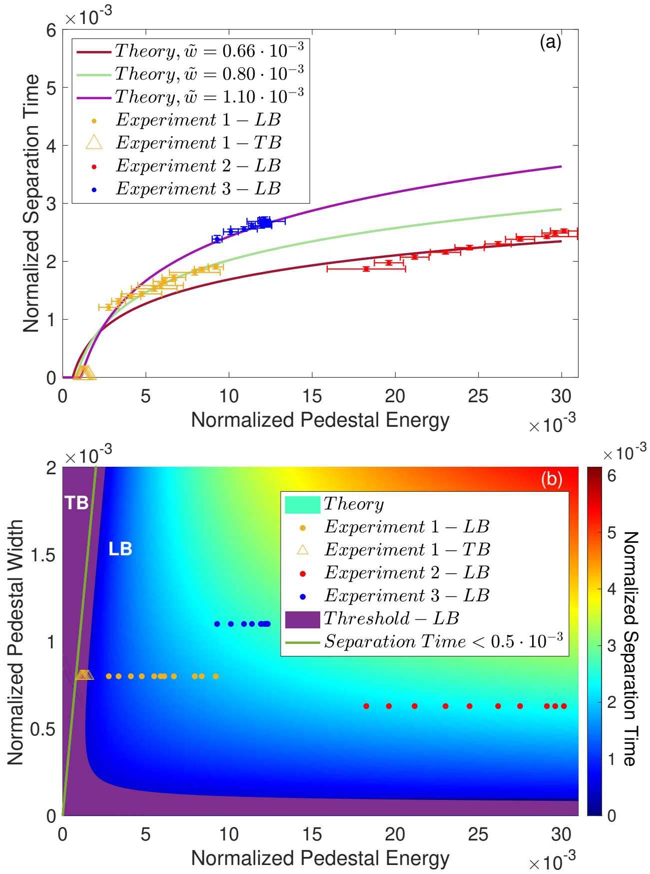

Figure 4: **Soliton steady states analysis.** (a) The measured separation time of the soliton pair, normalized by the cavity round trip time, versus the pedestal energy, normalized by the pulse energy. Three different data sets obtained by adjusting the polarization controller to modify the nonlinear interference in the MMF and thus change the SA properties. The NMI theory (solid line) agrees with the experimental data in the loosely bound (LB) regime and the tightly bound (TB) regime. (b) Theoretically predicted stable separation time for the two pulse loosely bound state plotted as a function of normalized pedestal width and normalized pedestal energy. The yellow line indicates the threshold for a loosely bound state at a non-zero separation time. The purple shaded area marks separation times smaller then $0.5 \cdot 10^{-3}$ in scale of the soliton duration, the NMI is weak comparing to the coherent interaction. Therefore, a phase transition is occur between loosely and tightly bound states. The experimental results are marked by dots (loosely bound) and triangles (tightly bound). The pedestal energy and width are normalized by the total pulse energy, and the cavity round trip time respectively. The dots correspond to the measured steady states presented in panel (a).

coherently.

To achieve these two scenarios in our cavity, the modification of the saturable absorber properties is essential. The staturable absorber properties determines the ratio between the pedestal energy and the soliton energy. Fortunately, our saturable absorber properties, determined by the nonlinear multimode interaction, are dependent of the polarization state of the light. Therefore by adjusting the polarization controller we can observe both scenarios in the cavity. In practice, in most cases we observe the second scenario while at extremely low pedestal energy, we observe the first scenario.

3 Conclusions

We observe and manipulate soliton steady states experimentally and model them with the noise-mediated interaction mechanism. We present a theoretical steady states phase diagram in which we map the areas of parameter space for the tightly and loosely bound states. For the loosely bound states, we calculate the stable separation time and obtain good agreement with the experimental results.

We introduce an approach to control the phase transition between loosely and tightly bound states by decreasing the pedestal energy, utilizing the pumping current with no additional modulators. We also present the ability to avoid this transition by a different configuration of the polarization controller.

We note that the NMI theory predicts the phase transition independently by reducing the pedestal energy or increasing its width, rather then the pumping current. Instead, an inline spectral filter could lead to the same effect on the pedestal energy to control the steady states. The obtained quantitative agreement with the experiment paves the way to the understanding of complex multi-solitons structures and their control. Thus, this system opens the door for the use of ultrafast lasers in telecommunication.

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