

Ultrafast twin-peak rogue waves in a vector field

AVI KLEIN,¹ SHIR SHAHAL,¹ SARA MEIR,¹ HAMOOTAL DUADI,¹
KFIR SULIMANY,² OHAD LIB,² HADAR STEINBERG,² STANISLAV A.
KOLPAKOV,³  AND MOTI FRIDMAN^{1,*} 

¹Faculty of Engineering and the Institute of Nanotechnology and Advanced Materials, Bar-Ilan University, Ramat Gan 5290002, Israel

²The Racah Institute of Physics, the Hebrew University of Jerusalem, Israel

³School of Engineering and Applied Science, Aston University, Aston Triangle, Birmingham B4 7ET, UK
*mordechai.fridman@biu.ac.il

Abstract: Rogue waves, which were first described as an oceanographic phenomenon, constitute an important factor in the dynamics of many physical systems. Most of these systems were analyzed by scalar fields, while some of them, and specifically in optics, are described by a vector field. Thus, they differ from scalar systems in several crucial aspects. In this work, we study experimentally twin-peak rogue waves with a temporal imaging system capable of measuring the Stokes vectors as a function of time. We found that the two peaks in optical twin-peak rogue waves have orthogonal states of polarization and similar intensities. We observed this with two different systems, however, we do not have a theoretical explanation for this phenomena and could not explain it with current models.

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Rogue waves are ‘freak waves’ that arise from the combination of nonlinearity and dispersion in a noisy environment [1]. These waves have an important impact on the dynamic of numerous systems in oceanography [2–7], atmospheric science [8], Bose-Einstein condensation [9], and in finance [10]. Optical systems are great test-benches for investigating rare rogue waves due to the fast dynamics and short time-scales of optical waves. Indeed, optical rogue waves were measured in different systems [11–13]. We show that twin-peak rogue waves which are common in optics [14–16], have variations in the state of polarization along the rogue wave. This may explain why such rogue waves are rare in the ocean compared to single-peak and triple-peak rogue waves [2,17].

Scalar systems are governed by the nonlinear Schrodinger equation (NLSE), and analytically solutions of rogue waves are essential for predicting their dynamics. Scalar analytical solutions were developed by Peregrine [18] based on the work of Ma [19] and later by Akhmediev [20–22] and are known as rational solutions [23–31]. Other methods were introduced, such as the bi-linear method [32], and the Darboux transformation [27,33], which gave similar results. Another method is an effective potential technique [34], which is based on analyzing the statistical properties of soliton-soliton interactions [35–37] but lacks a complete derivation for the effective potential.

Extending these techniques to a vector field, where the nonlinearity couples between the components of the vector, leads to similar rational solutions in two dimensions [38–42] and in three dimensions [43,44]. Even when considering different coupling mechanisms, such as an energy exchange between the vector components, the results are similar to the rational solutions [45–48]. However, none of these models predict rogue waves with two peaks where each peak has different state of polarization.

Optics is a great test-bench for studying rogue waves and other rare events thanks to the fast dynamics of optical waves which allows to measure large number of rare events in short time-scales. However, due to the fast dynamics it is challenging to measure the dynamics of the structure of each rogue wave [14–16]. We developed time-lenses which measure the intensity,

phase and state of polarization dynamics in rogue waves with sub-picosecond resolution [49–53]. This allows us to measure the structure of rogue waves as a function of time and to test the different models [38–48].

We started by generating twin-peak rogue waves in a pulsed fiber laser. The laser scheme is presented in Fig. 1(a), and the peak distribution of its output is presented in (b) showing thick tail that deviates from the exponential distribution expected by stochastic models, indicating rogue waves formation [14]. We focused on twin-peak rogue waves and measured the state of polarization by a full-Stokes temporal imaging system, shown in (c) [49–53]. The oscilloscope trigger was set to catch rogue waves and we recorded over 400,000 events. In each measurement we checked if there are two peaks which are measured by all three time-lenses, due to the width of the time-lens it was not probable to measure triple-peak rogue waves [54]. Since the timing on the rogue waves is unknown and due to thermal and acoustical fluctuations, in most cases, the rogue waves were measured by only one or two time-lenses. We found over 7000 cases of twin-peak rogue waves which were measured by all three time-lenses and calculated their state of polarization by the same algorithm done in Ref. [49]. The MATLAB code is shown in the supplementary materials together with some of the data (see Code 1 [55], Dataset 1 [56], and Dataset 2 [57]).

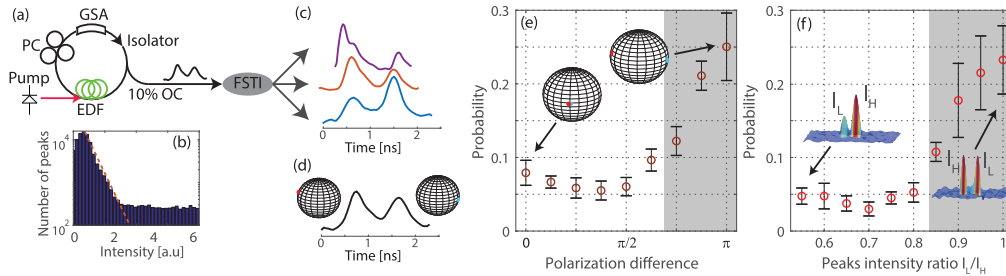


Fig. 1. (a) Rogue wave measurement scheme with a full Stokes temporal imaging. GSA - graphene saturable absorber, OC - output coupler, EDF - erbium doped fiber, PC - polarization controller, FSTI - Full Stokes temporal imaging. (b) Histogram of the peak intensities showing a large tail that deviates from the exponential distribution expected by stochastic models (red dashed curve). (c) The magnified output of three time-lenses, each projects on a different state of polarization. (d) Polarization measurement of the two magnified peaks in the twin-peak rogue wave, showing that the two peaks have orthogonal states of polarization. (e) Probability distribution for the relative state of polarization of the two peaks, as $S_1 \cdot S_2$ where S_1 and S_2 are the Stokes vectors of the two peaks, respectively. Typical states of polarization of the two peaks are presented when the two peaks have similar states of polarization and when the two peaks have orthogonal states of polarization. (f) Probability distribution for the peaks intensity ratio in twin-peak rogue waves. The x-axis denote the intensity ratio between the peaks. Typical twin-peak rogue waves with relative intensity ratio of 0.5 and 1 are presented at the insets where I_H denotes the intensity of the higher peak and I_L denotes the intensity of the lower peak.

The distribution of their state of polarization is presented in Fig. 1(e). The experimental results reveal that the probability for twin-peak rogue waves with states of polarization which are at least $3\pi/4$ apart, is 60%. This is emphasized by the grayed area in the figure. We also evaluated the intensity ratio between the higher peak and the lower peak. The intensity of the higher peak is denoted by I_H and the intensity of the lower peak is denoted by I_L . The probability distribution of the intensity ratio, $P(I_L/I_H)$ is shown in Fig. 1(f). The experimental results reveal that the probability for twin-peak rogue waves with similar intensity, which is emphasized by the grayed area, is over 80%. This indicates that the two peaks in optical twin-peak rogue waves have similar intensity and orthogonal state of polarization. These results were verified by building a second

ultrashort laser with a polarization rotator as a saturable absorber instead of the Graphene and observing the same results.

We numerically simulated different models of rogue waves in a vector field [38–48]. While some models predicted twin-peak rogue waves, they predicted the same twin-peaks in both states of polarization. To explain our results, we are looking for a model showing a single peak in each state of polarization with different timing leading to twin-peak rogue waves were the two peaks have orthogonal states of polarization.

To conclude, we measured the state of polarization of twin-peak rogue waves and observed that the two peaks in such waves tend to have peaks with orthogonal states of polarization and similar intensities. The results do not agree with any of the current models of rogue waves. The results are also relevant to other physical systems and can lead to new types of rogue waves where each peak in the rogue waves has a different state of polarization.

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Disclosures

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