STAT 532 Assignment 10 -Fall 2015
Due: November 24 (by 4:30 pm)

Show all work **neatly** and **in order** for full credit. Same instructions as previous homeworks...ask if you need clarification.

1. (10 pts) Go through the information in Chapter 7 about information criteria. See the other information posted under Chapter 7 materials in Content. The paper by Link and Barker (2006) is interesting and accessible. Go through the R-code in `SchoolsNormalHierarchical_DICexplore_F15.R` to hopefully help better understand DIC. Feel free to add to the code to make the investigation more complete for the future and find any mistakes in my code! Comment and or ask questions here to show me you did go through it and think about it.

2. A couple of cupcake fanatics are interested in whether there is evidence of a substantial difference in taste between a cupcake made by Liz Arnold compared to one made by Megan Higgs. They round up 28 friends and give them the trial (which they make sure is double-blind by hiring some extra helpers). It turns out that 21 of the people say Liz's cupcakes taste better than Megan's. They want to use this data to day something about the probability that a Liz-made cupcake is perceived to be tastier than a Megan-made cupcake.

   (a) (6 pts) Go ahead and use the beta-binomial model for inference. Do the analysis using four different Beta priors: Beta(0,0), Beta(1,1), Beta(2,2), and an informative prior of your choosing (you should actually use prior knowledge you have for this one). Justify your choice of informative prior. Graphically compare the posterior distributions, and write a short summary sentence for the results from at least one of the priors.

   (b) (4 pts) Now, let's say they define "substantial" to mean that the probability of choosing the real one to have a better taste is greater than or equal to 0.7. Therefore, using all four priors compare the following hypotheses using Bayes Factors: $M_1 : \theta \geq 0.7$ and $M_2 : \theta < 0.7$. Make a table showing results and write a brief summary sentence for each prior.

3. (4 pts) Explain to a non-stats grad student why priors matter for Bayes factors and why vague priors meant to represent a lack of information about the parameters pose serious problems if you are interested in Bayes factors. Using the following example to help: Assume data come from $N(\mu, 1)$ distribution and you will use prior distribution $\mu \sim N(\mu_0, \tau^2)$. You want to compare Model 1 where $\mu = 0$, to Model 2 where μ is unknown. What happens to the Bayes Factor comparing the two models as τ^2 gets large? Show work.

4. (4 pts) Briefly relate a Bayes Factor to a Likelihood Ratio for a stats grad student not taking this class.

5. Suppose you have data $y = (1, 3, 5, 7, 7)$ and you would like to assess evidence that they come from a Poisson versus a geometric distribution. Assume you do not favor one model *a priori*.

   (a) (3 pts) You specify the two distributions of interest as a Poisson(5) or a geometric(0.15), so the mean of the Poisson is 5 and the mean of the geometric is $(1 - 0.15)/(0.15) = 5.67$.

      i. What are the prior odds for the geometric model? Show work.

      ii. What are the posterior odds for the geometric model? Show work.

      iii. What is the Bayes Factor comparing the two models? Show work.

   (b) (6 pts) Now, suppose we want to compare models for unknown means. For the Poisson you decide to use $\lambda \sim Unif(0, 30)$, and for the geometric you use a uniform prior on the mean, which is $(1 - \pi)/\pi \sim Unif(0, 30)$.

      i. What are the prior odds for the geometric model? Show work.

      ii. What are the posterior odds for the geometric model? Show work.

      iii. What is the Bayes Factor comparing the two models? Show work.