STAT 532 Assignment 6 -2015
Due: Wednesday, Oct. 14th (by 4:00 pm)

Show all work **neatly** and **in order** for full credit. If plots are included they should be nicely scaled, supplement your discussion, and be presented in order (with the problem). In the body of the report, only include computer code that is explicitly asked for and output that is necessary to completely answer a question. Other well organized code and output can be included in the appendix so that I can check your work and provide comments if needed. I do not want to have to search through pages to find your answer, plots, or proof you did something. Homework assignments are a time to practice synthesizing information into reports as you will likely have to do for future jobs.

I expect you to *not* refer to any solutions you might find or homework's from previous students who have taken the class.

1. Suppose you are willing to assume your data come from a Cauchy distribution with unknown center (θ) and known scale (equal to 1), so the pdf (up to proportionality constant) is $p(y_i|\theta) \propto 1/(1+(y_i-\theta)^2)$. Assume you use a Normal prior distribution with a mean of 20 and a variance of 5 (based on previous studies and historical observations.) When you collect data you end up with $(23, 24, 25, 26.5, 27.5)$. Use the Metropolis algorithm to sample from the posterior distribution.

   (a) (12 pts) Use a Uniform jumping distribution centered at the current value. Explain/show how you decided on the tuning parameter. Display the posterior distribution, calculate a 99% posterior interval and the posterior probability that θ is greater than 30. Include the code for your sampler (but not for making plots, etc).

   (b) (6 pts) Use a Normal jumping distribution centered at the current value. Explain/show how you decided on the tuning parameter. You do not need to include your code for this one (assuming it is very similar to part (a)).

   (c) (2 pts) Discuss/compare the two algorithms (constructed from different jumping distributions). Was one easier to tune? Are the results similar? Which do you prefer and why? Etc.

2. (18 pts) Use the Metropolis-Hastings algorithm to obtain draws of y from a Poisson($\lambda = 8$) distribution using a Negative Binomial proposal distribution with mean equal to the current value of y. Provide plots to convince me your algorithm is working (remember you are plotting a discrete distribution!). Also, show me how you tuned the algorithm and how you assessed convergence. Include the code for your sampling, but not for making plots, etc.

3. (4 pts) Explain to a researcher in another discipline why we often rely on MCMC methods and other methods to sample from distributions to do Bayesian data analysis. Your answer should also make it clear that MCMC is not synonymous with Bayesian Statistics.

4. Write JAGS code for obtaining draws from the posterior distribution when you start with using the Normal with known variance ($\sigma^2 = 2$) for the likelihood piece and a conjugate Normal distribution for the prior. Set hyperparameter values for the prior and obtain draws from the posterior distribution using JAGS. You only have one piece of data and it is $y = 12$.

   (a) (2 pts) What is the analytical posterior distribution? (You can use previous results from class). Display the likelihood, the prior, and the posterior distribution on the same plot.

   (b) (1 pts) Include your JAGS model code here.

   (c) (1 pts) Provide a sample path plot

   (d) (2 pts) Provide a histogram of all posterior draws with a curve of analytical posterior density function overlaid on it.

   (e) (4 pts) Compare results using **three** additional sets of hyperparameters. Make them different enough that it is interesting. Nicely display your results graphically and discuss differences (keeping your sample size in mind).

   (f) (1.5 pts) What does *JAGS* stand for? What does *BUGS* stand for? What does *Stan* stand for?

5. Use your project article to answer the following questions. If the authors of your paper used the analytical posterior (rather than sampling from it using a computational algorithm), see me for additional questions.

   (a) (4 pts) Briefly explain what computational methods were used by the authors of the article to summarize or approximate the posterior distributions of interest. If it is not something we have covered in class yet, is it discussed elsewhere in the text book?

   (b) (3 pts) Briefly explain what methods the authors used to check convergence of the computational algorithms they used to approximate the posterior distribution.