REVIEW ARTICLE

The Integrated Nested Laplace Approximation applied to spatial point process models

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ARTICLE HISTORY
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ABSTRACT
This template is for authors who are preparing a manuscript for a Taylor & Francis journal using the LaTeX document preparation system and the `interact` class file, which is available via selected journals' home pages on the Taylor & Francis website.

KEYWORDS

1. Introduction

spatial prediction is a high-dimensional problem
many dependent latent variables but joint dist not necessarily needed
INLA fast approximation for marginals useful for mapping [2]

2. Log-Gaussian Cox Process

Poisson process intensity $\lambda(\mathbf{s})$ events per unit area
Model $\log \lambda(\mathbf{s}) = Z(\mathbf{s})$, $Z(\mathbf{s})$ spatial Gaussian process
Random continuous function: $Z(\mathbf{s})$ a Gaussian random variable, Mean μ Matern covariance function
Poisson process log-likelihood: $\ell(\lambda) = C - \int \lambda(\mathbf{s})d\mathbf{s} + \sum \log(\lambda(\mathbf{s_i}))$
(Need to add covariate to notation)

3. INLA

Integrated Nested Laplace Approximation
Bayesian Hierarchical models, many latent Gaussian variables, few parameters
Laplace approximation in general: $\int \exp[h(x)]\mathrm{d}x$, Taylor expansion of $h(x)$
Example from @rinla
- $\mathbf{y} = (y_1, \ldots, y_n)'$ independent Gaussian observations
- $y_i \sim \mathrm{N}(\theta, \sigma^2)$

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- $\theta \sim \mathrm{N}(\mu_0, \sigma_0^2)$
- $\psi = 1/\sigma^2$, $\psi \sim \text{Gamma}(a, b)$
- The posterior distribution of ψ:

$$p(\psi|\mathbf{y}) \propto \frac{p(\mathbf{y}|\theta,\psi)p(\theta)p(\psi)}{p(\theta|\psi,\mathbf{y})}$$

- Laplace approximation:

$$\tilde{p}(\psi|\mathbf{y}) \propto \frac{p(\mathbf{y}|\theta,\psi)p(\theta)p(\psi)}{\tilde{p}_G(\theta^*|\psi,\mathbf{y})}$$

Repeat for θ, will depend on ψ
Provides marginal posteror for one entry at a time of a vector $\boldsymbol{\theta}$

4. The SPDE Approach

GP has dense covariance matrix
SPDE result approximates GP with CAR and sparse covariance matrix [1]

$$(\kappa^2 - \Delta)^{\alpha/2}(\tau Z(\mathbf{s})) = W(\mathbf{s})$$

Choose nodes $\mathbf{s}_i$ to model $Z(\mathbf{s}_i)$, build a triangular mesh, autoregressive model on the nodes
Change of basis: barycentric coordinates allow sparse matrices and simple linear inerpolation

5. Going Off the Grid

Log-likelihood:

$$\ell(Z) = C - \int \exp\left[Z(\mathbf{s})\right] d\mathbf{s} + \sum Z(\mathbf{s_i})$$

[3]:

$$\ell(Z) \approx C - \sum_i \tilde{\alpha}_i \exp\left[\sum_j z_j \phi_j(\tilde{\mathbf{s}}_i)\right] + \sum_i \sum_j z_j \phi_j(\mathbf{s_i})$$

(Poisson distribution)

6. Variable Sampling Effort

Observed a thinned process
Thinning process can be known or unknown

Scale SPDE node integration weights by thinning probabilities when known
Incorporate log-linear model for thinning probability when unknown [4]

7. Illustrations

Examples with data, maybe `bei` dataset, Victorville, or a simulation study

8. Conclusion

References

[1] F. Lindgren, H. Rue, and J. Lindström, *An explicit link between gaussian fields and gaussian markov random fields: the stochastic partial differential equation approach*, Journal of the Royal Statistical Society: Series B (Statistical Methodology) 73 (2011), pp. 423–498.

[2] H. Rue, S. Martino, and N. Chopin, *Approximate bayesian inference for latent gaussian models by using integrated nested laplace approximations*, Journal of the royal statistical society: Series b (statistical methodology) 71 (2009), pp. 319–392.

[3] D. Simpson, J.B. Illian, F. Lindgren, S.H. Sørbye, and H. Rue, *Going off grid: Computationally efficient inference for log-gaussian cox processes*, Biometrika 103 (2016), pp. 49–70.

[4] Y. Yuan, F.E. Bachl, F. Lindgren, D.L. Borchers, J.B. Illian, S.T. Buckland, H. Rue, T. Gerrodette, *et al.*, *Point process models for spatio-temporal distance sampling data from a large-scale survey of blue whales*, The Annals of Applied Statistics 11 (2017), pp. 2270–2297.