REVIEW ARTICLE

The Integrated Nested Laplace Approximation applied to spatial point process models

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ARTICLE HISTORY
Compiled November 12, 2019

ABSTRACT
This template is for authors who are preparing a manuscript for a Taylor & Francis journal using the LaTeX document preparation system and the `interact` class file, which is available via selected journals' home pages on the Taylor & Francis website.

KEYWORDS

Delete Me Later - Manuscript Organization from JAS Author Instructions

- **Introduction:** Please introduce one or more current statistical research methods that have been widely used by other discipline(s) in the Introduction section or divide the discussion into two or more subsections within this Introduction section.
- **Methodology or Approach:** Present the method(s) and relevant applications being reviewed in this or more sections.
- **Application of your Methodology:** Present some applications of the reviewed methods to some recent data in this or more sections. Also note: Simulation studies can be presented as a sub-section in this section or in a new standalone section.
- **Conclusion and Discussion:** In this conclusion and discussion section, provide conclusions of your review and provide anticipated future direction(s) in this area.

1. Introduction

spatial prediction is a high-dimensional problem

many dependent latent variables but joint dist not necessarily needed

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1.1. Log-Gaussian Cox Process

Poisson process intensity $\lambda(\mathbf{s})$ events per unit area

Model $\log \lambda(\mathbf{s}) = Z(\mathbf{s})$, $Z(\mathbf{s})$ spatial Gaussian process

Random continuous function: $Z(\mathbf{s})$ a Gaussian random variable, Mean μ Matern covariance function

Poisson process log-likelihood: $\ell(\lambda) = C - \int \lambda(\mathbf{s})d\mathbf{s} + \sum \log(\lambda(\mathbf{s_i}))$

(Need to add covariate to notation)

1.2. INLA

INLA fast approximation for marginals useful for mapping [**?**]

Integrated Nested Laplace Approximation

Bayesian Hierarchical models, many latent Gaussian variables, few parameters

Laplace approximation in general: $\int \exp[h(x)]\mathrm{d}x$, Taylor expansion of $h(x)$

Example from @rinla

- $\mathbf{y} = (y_1, \ldots, y_n)'$ independent Gaussian observations
- $y_i \sim \mathrm{N}(\theta, \sigma^2)$
- $\theta \sim \mathrm{N}(\mu_0, \sigma_0^2)$
- $\psi = 1/\sigma^2$, $\psi \sim \mathrm{Gamma}(a, b)$
- The posterior distribution of ψ:

$$p(\psi|\mathbf{y}) \propto \frac{p(\mathbf{y}|\theta,\psi)p(\theta)p(\psi)}{p(\theta|\psi,\mathbf{y})}$$

- Laplace approximation:

$$\tilde{p}(\psi|\mathbf{y}) \propto \frac{p(\mathbf{y}|\theta,\psi)p(\theta)p(\psi)}{\tilde{p}_G(\theta^*|\psi,\mathbf{y})}$$

Repeat for θ, will depend on ψ

Provides marginal posteror for one entry at a time of a vector $\boldsymbol{\theta}$

2. Methodology

2.1. The SPDE Approach

GP has dense covariance matrix

SPDE result approximates GP with CAR and sparse covariance matrix [**?**]

$$(\kappa^2 - \Delta)^{\alpha/2}(\tau Z(\mathbf{s})) = W(\mathbf{s})$$

Choose nodes $\mathbf{s}_i$ to model $Z(\mathbf{s}_i)$, build a triangular mesh, autoregressive model on the nodes

Change of basis: barycentric coordinates allow sparse matrices and simple linear inerpolation

2.2. Going Off the Grid

Log-likelihood:

$$\ell(Z) = C - \int \exp\left[Z(\mathbf{s})\right] d\mathbf{s} + \sum Z(\mathbf{s_i})$$

[**?**]:

$$\ell(Z) \approx C - \sum_i \tilde{\alpha}_i \exp\left[\sum_j z_j \phi_j(\tilde{\mathbf{s}}_i)\right] + \sum_i \sum_j z_j \phi_j(\mathbf{s_i})$$

(Poisson distribution)

2.3. Variable Sampling Effort – Delete this subsection?

Observed a thinned process
Thinning process can be known or unknown
Scale SPDE node integration weights by thinning probabilities when known
Incorporate log-linear model for thinning probability when unknown [**?**]

3. Applications

3.1. Simulation Study

3.2. Data Application

Examples with data, maybe `bei` dataset or Victorville

4. Conclusion and Discussion

References