Homework 4 - Stat 534
Due Friday, February 10, 2017

1. For $\lambda = 30$ generate 9 realizations of CSR on the unit square. For each realization, construct a kernel estimate of $\lambda(\mathbf{s})$. How do the estimated intensity functions compare to the constant intensity under CSR? What precautions does this exercise suggest with regard to interpreting estimates of intensity from a single realization (or data set)? The following R code will simultaneously produce the data and plots.

```
par(mfrow=c(3,3))
for(i in 1:9) plot(density(rpoispp(30)))
```

   Provide me with the images.

2. In class we looked at a heterogeneous Poisson process on the unit square with with intensity function
$$\lambda(x, y) = \exp(5x + 2y)$$
   (a) Simulate a realization of the process using the following R code.

   ```
   sim.dat<-rpoispp(function(x,y){exp(5*x + 2*y})
   ```

   Plot the results and comment.

   (b) Plot simulation envelopes for the K function (or some suitable modification of it) and comment.

   (c) Fit a trend model to your data using `ppm`. Provide me with the parameter estimates and associated standard errors.

   (d) Check the fit using `quadrat.test`. Use `method="MonteCarlo"` instead of the large sample chi-squared test. Plot the results Discuss.

   (e) Compare these results from this model with those to a model fit under an assumption of CSR. Summarize the results. Provide me the model comparison results (AIC comparisons are fine).

   (f) Plot a nonparametric estimate of the intensity function. Compare the fitted surface you got using `ppm` with the nonparametric (kernel density estimate) surface in some suitable way.

3. Recall the use of the `nncorr` statistic in the Finland Pines data set. The distribution of heights (the marks) was of interest. We saw that the nearest neighbor correlation between heights was -0.1839798. We questioned whether or not this was unusual. Carry out a randomization test to assess this. You can use the `rlabel` command to scramble the marks if you want. Provide me with a histogram of the randomization distribution and a p-value. Discuss **BRIEFLY** your results. Provide me with your R-code, also. (Note - you need to extract the correlation from the `nncorr` output. Here is how to do that:

```
nncorr(finpines.ht)["correlation"]
correlation
 -0.1839798
```

4. Let's derive a K function for something other than a CSR process. We will assume a Neyman-Scott process with the following properties.

   i The parent process is a homogeneous Poisson process with intensity λ.

   ii The number of offspring produced by each parent (N) is homogeneous Poisson with intensity μ.

   iii The position of each offspring is determined by a bivariate normal distribution with mean $(0, 0)$ (i.e. it is centered over the parent) and variance-covariance matrix $\sigma^2\mathbf{I}$. Note that this implies that the x and y coordinates are determined independently of one another with the same variance.

   Consider 2 offspring from the same parent located at (X_1, Y_1) and (X_2, Y_2).

   (a) What is the distribution of

   $$W = \frac{(X_1 - X_2)^2}{2\sigma^2} + \frac{(Y_1 - Y_2)^2}{2\sigma^2}?$$

   (b) Note that the Euclidean distance between the 2 points is

   $$H = \left[(X_1 - X_2)^2 + (Y_1 - Y_2)^2\right]^{1/2} = \left(2\sigma^2 W\right)^{1/2}.$$

   Derive the cdf of H.

   (c) Recall that we were told that the K function for Neyman-Scott processes with homogeneous Poisson parent processes and radially symmetric $f(h)$ is

   $$K(h) = \pi h^2 + \frac{E(N(N-1))}{\lambda E(N)^2} F(h).$$

   Use the results from above to find the K function for the described process.