



Stat 534 Project: Extrapolation from Poisson Process Intensity Surface Models

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- Goals
 - Estimate inhomogeneous intensity surface from events in a subregion
 - Infer intensity across entire region
- Applications
 - Mapping where endangered species are located
 - Mapping geomagnetic anomalies prior to an unexploded ordnance (UXO) remediation

Maximum Likelihood Intensity Surface Fitting

- General point processes
 - The theory is not too complicated but the computation is very difficult
- Poisson processes
 - Doable with numerical methods
 - Log-likelihood of Poisson with intensity $\lambda(\mathbf{s})$ on region D (note typos in Diggle (2013))

$$\begin{aligned}\ell(\lambda) &= \{-\mu + n \log(\mu) - \log(n!)\} + \sum_{i=1}^n \{\log(\lambda(\mathbf{s}_i)) - \log(\mu)\} \\ &= \sum_{i=1}^n \log(\lambda(\mathbf{s}_i)) - \int_D \lambda(\mathbf{s}) d\mathbf{s} - \log(n!)\end{aligned}$$

where $\mu = \int_D \lambda(\mathbf{s}) d\mathbf{s}$

Poisson Process Log-Linear Model

- Assuming events are independent (conditional on the intensity function),

$$\log(\lambda(\mathbf{s})) = \mathbf{x}(\mathbf{s})^T \boldsymbol{\beta}$$

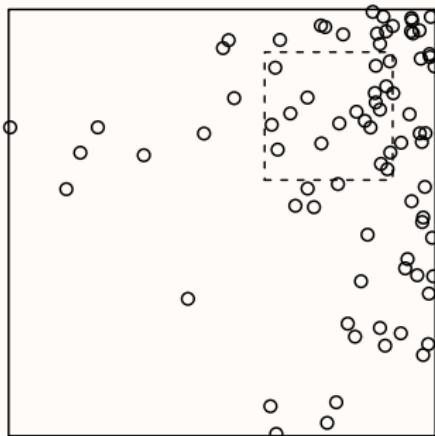
where $\mathbf{x}(\mathbf{s})^T$ is a row of predictors at location \mathbf{s}

- Predictors can include covariates, but they must be known across the whole region
- Berman and Turner (1992) use dummy points and quadrature to set up an approximation as a weighted Poisson regression
- Their method is implemented in spatstat's `ppm`, with `glm` from base R or `gam` from `mgcv` as the back-end, using the `quasi` family

Simple Example

- True model
 - Poisson process on the unit square
 - $\log(\lambda(x, y)) = 5x + 2y$
- But we don't observe $0.6 < x < 0.9, 0.6 < y < 0.9$

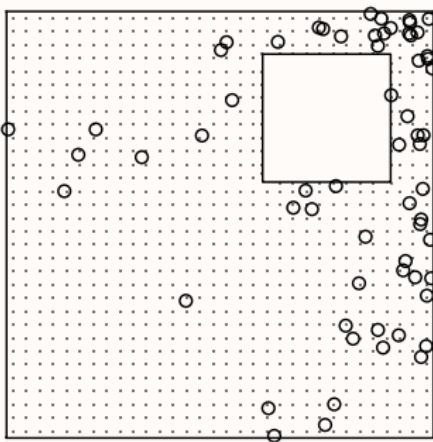
Event Locations



Estimated Model

- Fit the model $\log(\lambda(x, y)) = \beta_0 + \beta_1 x + \beta_2 y$

Data and Dummy Points

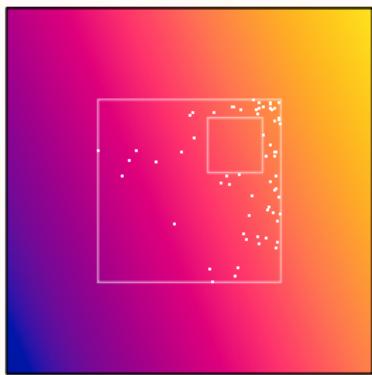


	Estimate	S.E.
$\hat{\beta}_0$	0.20	0.56
$\hat{\beta}_1$	4.54	0.59
$\hat{\beta}_2$	2.00	0.44

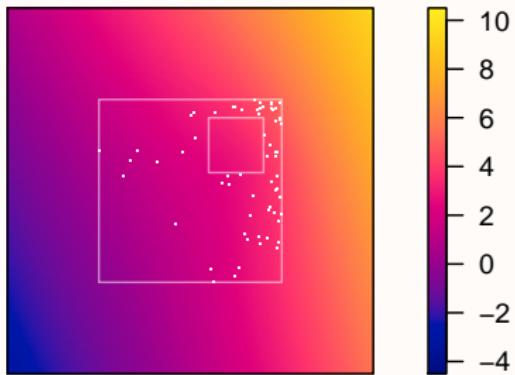
Extrapolate the Surface

- Use the predict method
- Specify a new window $-0.5 < x < 1.5, -0.5 < y < 1.5$

$\log(\hat{\lambda}(x, y))$

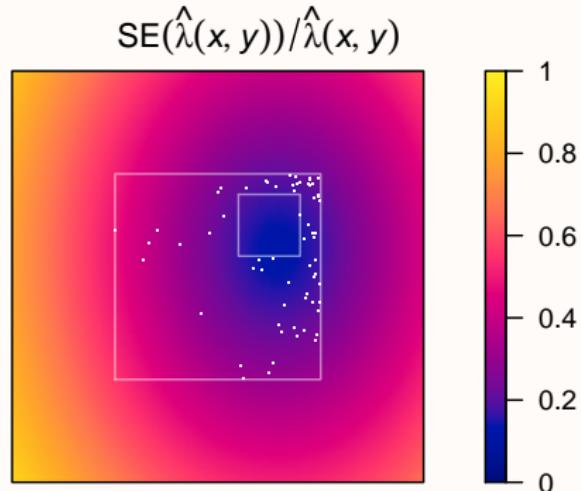


$\log(SE(\hat{\lambda}(x, y)))$



Where is the Uncertainty?

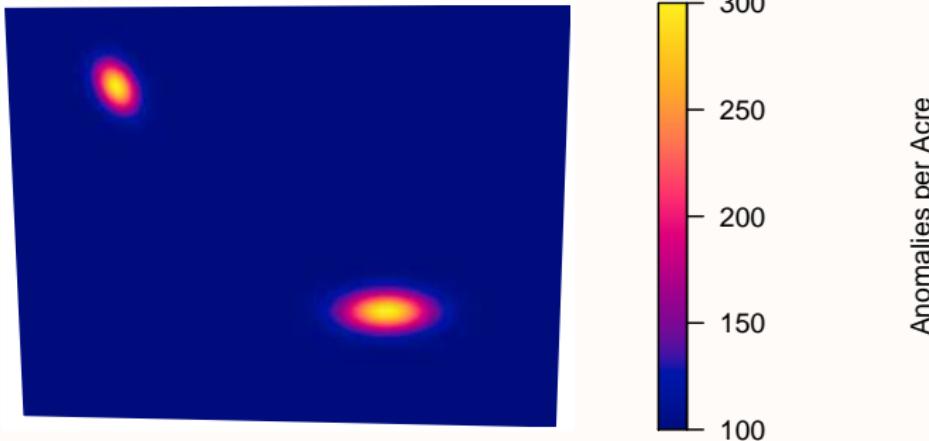
- Relative standard error is lowest where the highest intensity was observed



Simple UXO Site

- 952.38 acre region (roughly 7,625 ft by 5,709 ft)
- High density of geomagnetic anomalies around targets
- Low density of background anomalies

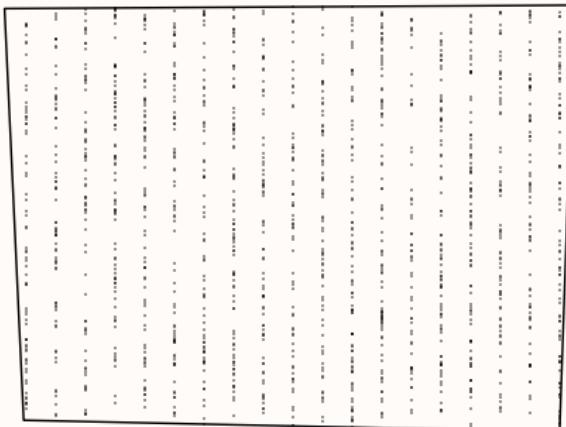
True Intensity Surface



Observed Events

- Metal detectors record anomalies in six foot wide strips along parallel transects with 396 feet between centerlines
- Observed 14.7 acres, 1.5% of the site

Observed Geomagnetic Anomalies



Trend Surface Models

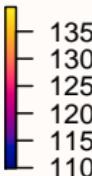
- Polynomial models

$$\log(\lambda(x, y)) = \sum_{i=0}^p \sum_{j=0}^{p-i} \beta_{ij} x^i y^j; \quad p = 2, 3, \dots, 12$$

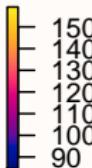
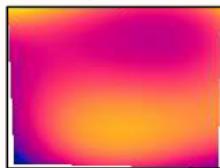
- Can approximate complicated surfaces
- Expect two peaks, so even $p \geq 4$ could work well
- Rescaling x and y to mean 0 and variance 1 reduces numerical instability for large p

Extrapolated Surfaces

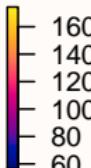
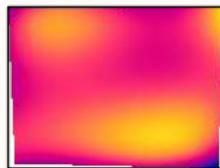
2nd Degree Polynom



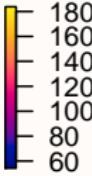
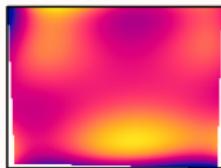
3rd Degree Polynom



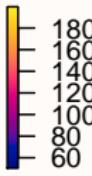
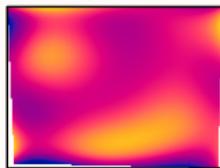
4th Degree Polynom



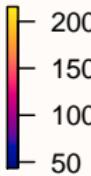
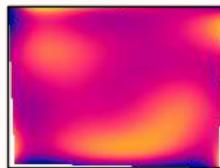
5th Degree Polynom



6th Degree Polynom

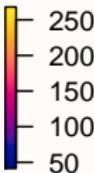
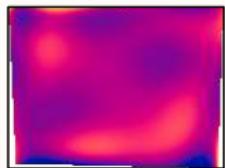


7th Degree Polynom

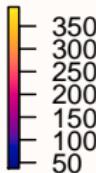
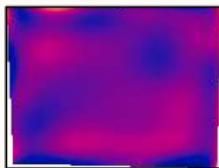


Extrapolated Surfaces

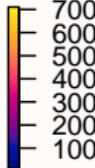
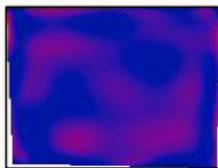
8th Degree Polynom



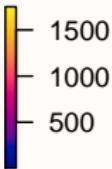
9th Degree Polynom



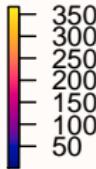
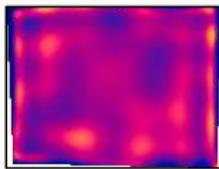
10th Degree Polynom



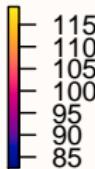
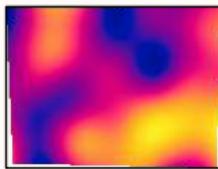
11th Degree Polynom



12th Degree Polynom

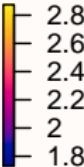
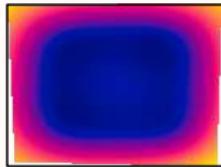


Kernel Density

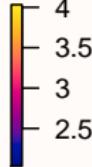
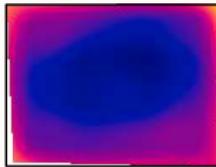


Standard Errors

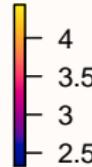
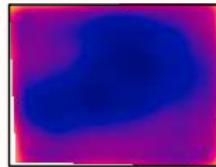
2nd Degree log(SE)



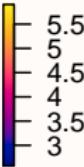
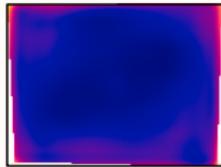
4th Degree log(SE)



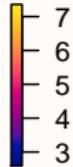
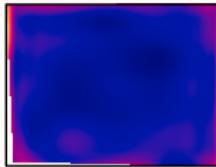
6th Degree log(SE)



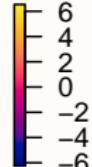
8th Degree log(SE)



10th Degree log(SE)



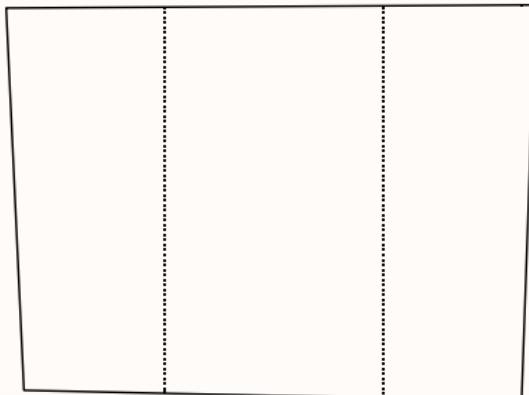
12th Degree log(SE)



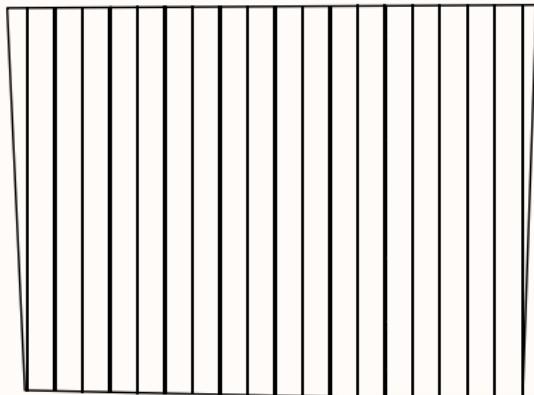
Problems with Implementation

- By default, `ppm` places dummy points on a grid across a bounding box
 - Only 160 points on two transects are kept
- I draw 500,000 points from a uniform distribution on the observed region
- Warton and Shepherd (2010) recommend using enough dummy points that the maximized log-likelihood converges

Default Dummy Points



Manual Dummy Points



Problems with Implementation

- For $p \geq 18$, `ppm` cannot compute SEs because the “Fisher information matrix is singular” — 190 coefficients
- The `plot` method gives an error about infinite values
- When the window isn’t specified, the `predict` method’s default grid misses all but two transects
- The `predict` method does not work with spline smoothers

Conclusion

- Polynomial surfaces are flexible but require much faith in the model
- How to check these models?
- How much of the region must be observed?
- Can implement with existing R packages but not easily

References |

- Berman, Mark and Rolf Turner (1992). "Approximating point process likelihoods with GLIM". In: *Applied Statistics*, pp. 31–38.
- Diggle, Peter J. (2013). *Statistical Analysis of Spatial and Spatio-Temporal Point Patterns*. 3rd ed. CRC Press.
- Warton, David I and Leah C Shepherd (2010). "Poisson point process models solve the “pseudo-absence problem” for presence-only data in ecology". In: *The Annals of Applied Statistics* 4.3, pp. 1383–1402.