Stat 534 Project: Extrapolation from Poisson Process Intensity Surface Models

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- Goals
  - Estimate inhomogeneous intensity surface from events in a subregion
  - Infer intensity across entire region
- Applications
  - Mapping where endangered species are located
  - Mapping geomagnetic anomalies prior to an unexploded ordnance (UXO) remediation

Maximum Likelihood Intensity Surface Fitting

- General point processes
  - The theory is not too complicated but the computation is very difficult
- Poisson processes
  - Doable with numerical methods
  - Log-likelihood of Poisson with intensity $\lambda(\mathbf{s})$ on region D (note typos in Diggle (2013))

$$\ell(\lambda) = \{-\mu + n\log(\mu) - \log(n!)\} + \sum_{i=1}^{n} \{\log(\lambda(\mathbf{s}_i)) - \log(\mu)\}$$

$$= \sum_{i=1}^{n} \log(\lambda(\mathbf{s}_i)) - \int_D \lambda(\mathbf{s})d\mathbf{s} - \log(n!).$$

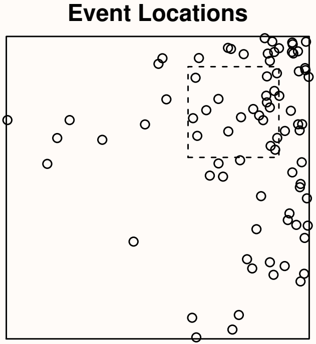
where $\mu = \int_D \lambda(\mathbf{s})d\mathbf{s}$

Poisson Process Log-Linear Model

- Assuming events are independent (conditional on the intensity function),
$$\log(\lambda(\mathbf{s})) = \mathbf{x}(\mathbf{s})^T \beta$$
where $\mathbf{x}(\mathbf{s})^T$ is a row of predictors at location $\mathbf{s}$
- Predictors can include covariates, but they must be known across the whole region
- Berman and Turner (1992) use dummy points and quadrature to set up an approximation as a weighted Poisson regression
- Their method is implemented in `spatstat`'s `ppm`, with `glm` from base R or `gam` from `mgcv` as the back-end, using the `quasi` family

Simple Example

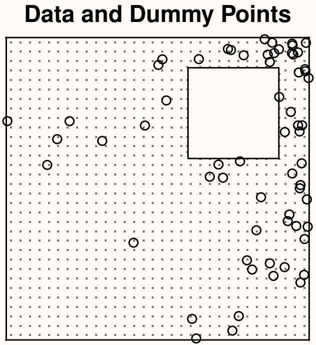
- True model
  - Poisson process on the unit square
  - $\log(\lambda(x, y)) = 5x + 2y$
- But we don't observe $0.6 < x < 0.9, 0.6 < y < 0.9$



Estimated Model

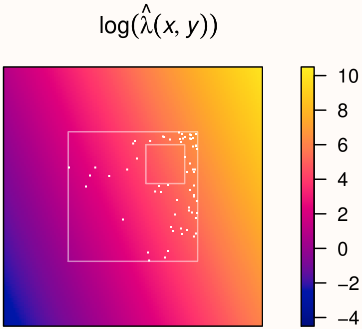
- Fit the model $\log(\lambda(x, y)) = \beta_0 + \beta_1 x + \beta_2 y$

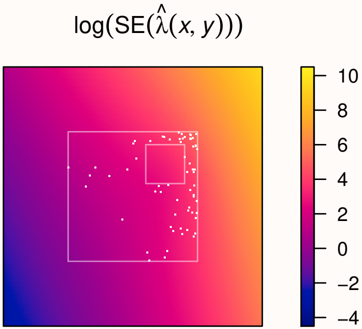
	Estimate	S.E.
$\widehat{\beta}_0$	0.20	0.56
$\widehat{\beta}_1$	4.54	0.59
$\widehat{\beta}_2$	2.00	0.44



Extrapolate the Surface

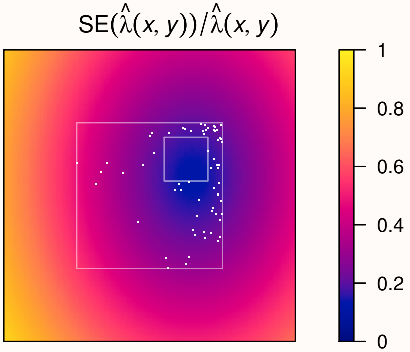
- Use the `predict` method
- Specify a new window $-0.5 < x < 1.5, -0.5 < y < 1.5$





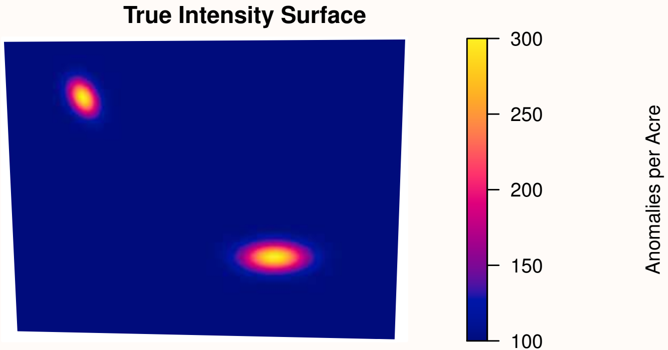
Where is the Uncertainty?

- Relative standard error is lowest where the highest intensity was observed



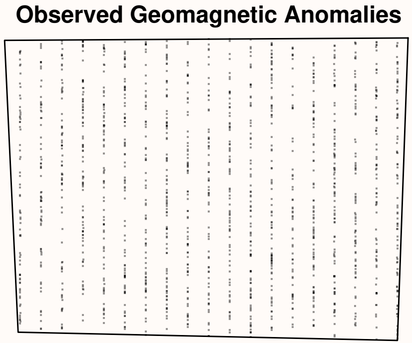
Simple UXO Site

- 952.38 acre region (roughly 7,625 ft by 5,709 ft)
- High density of geomagnetic anomalies around targets
- Low density of background anomalies



Observed Events

- Metal detectors record anomalies in six foot wide strips along parallel transects with 396 feet between centerlines
- Observed 14.7 acres, 1.5% of the site



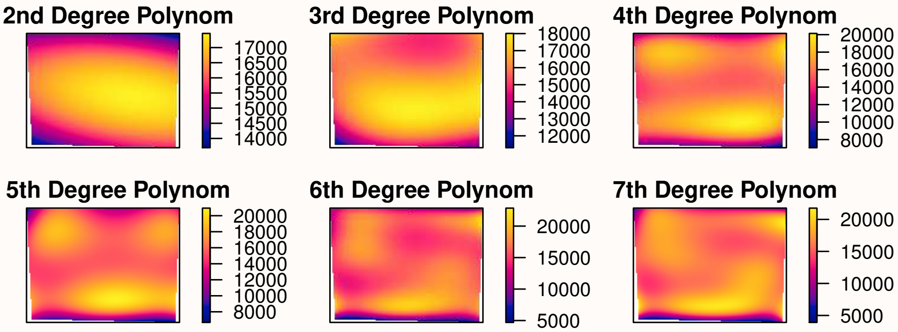
Trend Surface Models

- Polynomial models

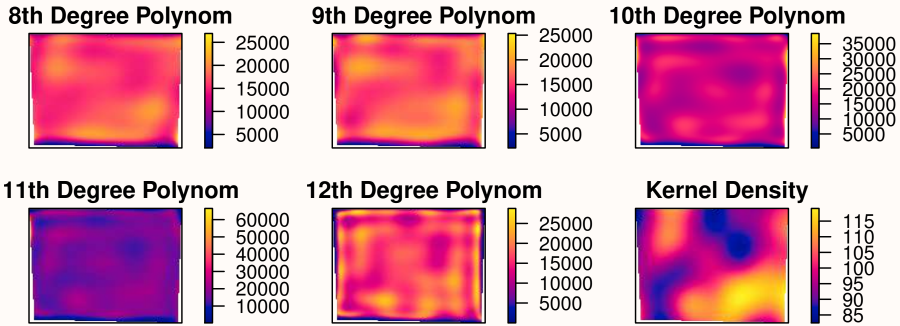
$$\log(\lambda(x,y)) = \sum_{i=0}^{p}\sum_{j=0}^{p-i}\beta_{ij}x^i y^j; \qquad p = 2,3,\ldots,12$$

- Can approximate complicated surfaces
- Expect two peaks, so even $p \geq 4$ could work well
- Rescaling x and y to mean 0 and variance 1 reduces numerical instability for large p

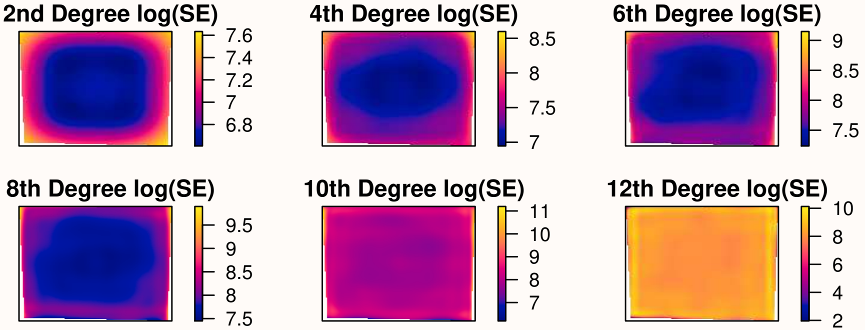
Extrapolated Surfaces



Extrapolated Surfaces

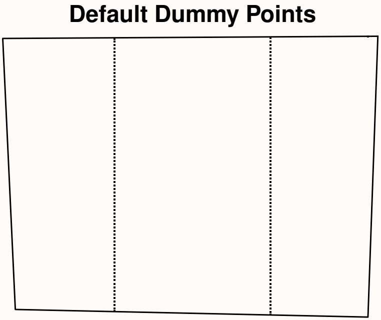


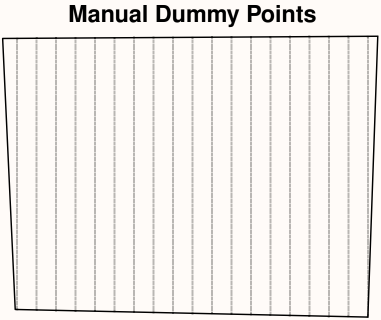
Standard Errors



Problems with Implementation

- By default, `ppm` places dummy points on a grid across a bounding box
  - Dummy points outside the actual window are discarded
  - Only 160 points on two transects are kept
- I place 128 evenly-spaced dummy points along each transect
  - 2,443 are used





Problems with Implementation

- For $p \geq 18$, `ppm` cannot compute SEs because the "Fisher information matrix is singular" — 190 coefficients
- The `plot` method gives an error about infinite values
- When the window isn't specified, the `predict` method's default grid misses all but two transects
- The predict method does not work with spline smoothers
- The magnitudes of the predictions are much too large

Conclusion

- Polynomial surfaces are flexible but require much faith in the model
- How to check these models?
- How much of the region must be observed?
- Can implement with existing R packages but not easily
- What scale are the prediction SEs on?

References I

Berman, Mark and Rolf Turner (1992). "Approximating point process likelihoods with GLIM". In: *Applied Statistics*, pp. 31–38.

Diggle, Peter J. (2013). *Statistical Analysis of Spatial and Spatio-Temporal Point Patterns*. 3rd ed. CRC Press.