

```
[> restart
```

```
[> with(IterativeMaps): with(ImageTools):
```

```
[> `0r` := (1-x)^(r+1): `1r` := (r+1)*x*(1-x)^r: `2r` := (1/2)*r*(r+1)*x^2*(1-x)^(r-1): `3r` := (1/6)*(r-1)*r*(r+1)*x^3*(1-x)^(r-2): `4r` := (1/6)*(r-1)*r*(r+1)*x^(r-2)*(1-x)^3: `5r` := (1/2)*r*(r+1)*x^(r-1)*(1-x)^2: `6r` := (r+1)*x^r*(1-x): `7r` := x^(r+1): `0k` := (1-x)^(k+1): `1k` := (k+1)*x*(1-x)^k: `2k` := (1/2)*k*(k+1)*x^2*(1-x)^(k-1): `3k` := (1/6)*(k-1)*k*(k+1)*x^3*(1-x)^(k-2): `4k` := (1/6)*(k-1)*k*(k+1)*x^(k-2)*(1-x)^3: `5k` := (1/2)*k*(k+1)*x^(k-1)*(1-x)^2: `6k` := (k+1)*x^k*(1-x): `7k` := x^(k+1):
```

```
[>
```

```
[> l:=.2:m:=4:n:=20:s:=20:
```

```
[> `firstr` := `0r` + `1r` + `6r`:  
`secondr` := `1r` + `2r` + `3r` + `6r` + `7r`:  
`firstk` := `0k` + `1k` + `6k`:  
`secondk` := `1k` + `2k` + `3k` + `6k` + `7k`:
```

```
[> unassign('k');
```

```
[> bif:=Bifurcation([x],[firstr],[l],m,n,xmin=-.01,xmax=1.01):  
ColouringProcedures:-HueToRGB(bif):
```

```
Pbif:=plot('k'=m..n,p=-.01..1.01,axes=box,size=[600,600],  
background=bif,labelfont=[times,s],font=[time,s-6]):
```

```
F:=proc(k,x0)
```

```
if not k::numeric and x0::numeric then
```

```
return 'procname'(args);
```

```
end if;
```

```
LyapunovExponent(unapply(eval(firstr,r=`k`),x),x0,  
max_iter=2^17,epsilon=1e-4);
```

```
end proc:
```

```
Plyap:=plot(F(k,l),'k'=m..n,adaptive=false,numpoints=100,size=[600,200],  
labelfont=[times,s],font=[time,s-6]):
```

```
#plots:-display(Array([[Pbif],[Plyap]]),aligncolumns);
```

```
print(Pbif);print(Plyap);
```

```
bif:=Bifurcation([x],[secondr],[l],m,n,xmin=-.01,xmax=1.01):
```

```
ColouringProcedures:-HueToRGB(bif):
```

```
Pbif:=plot('k'=m..n,p=-.01..1.01,axes=box,size=[600,600],
```

```
background=bif,labelfont=[times,s],font=[time,s-6]):
```

```
F:=proc(k,x0)
```

```
if not k::numeric and x0::numeric then
```

```
return 'procname'(args);
```

```
end if;
```

```
LyapunovExponent(unapply(eval(secondr,r=`k`),x),x0,  
max_iter=2^17,epsilon=1e-4);
```

```
end proc:
```

```
Plyap:=plot(F(k,l),'k'=m..n,adaptive=false,numpoints=100,size=[600,200],  
labelfont=[times,s],font=[time,s-6]):
```

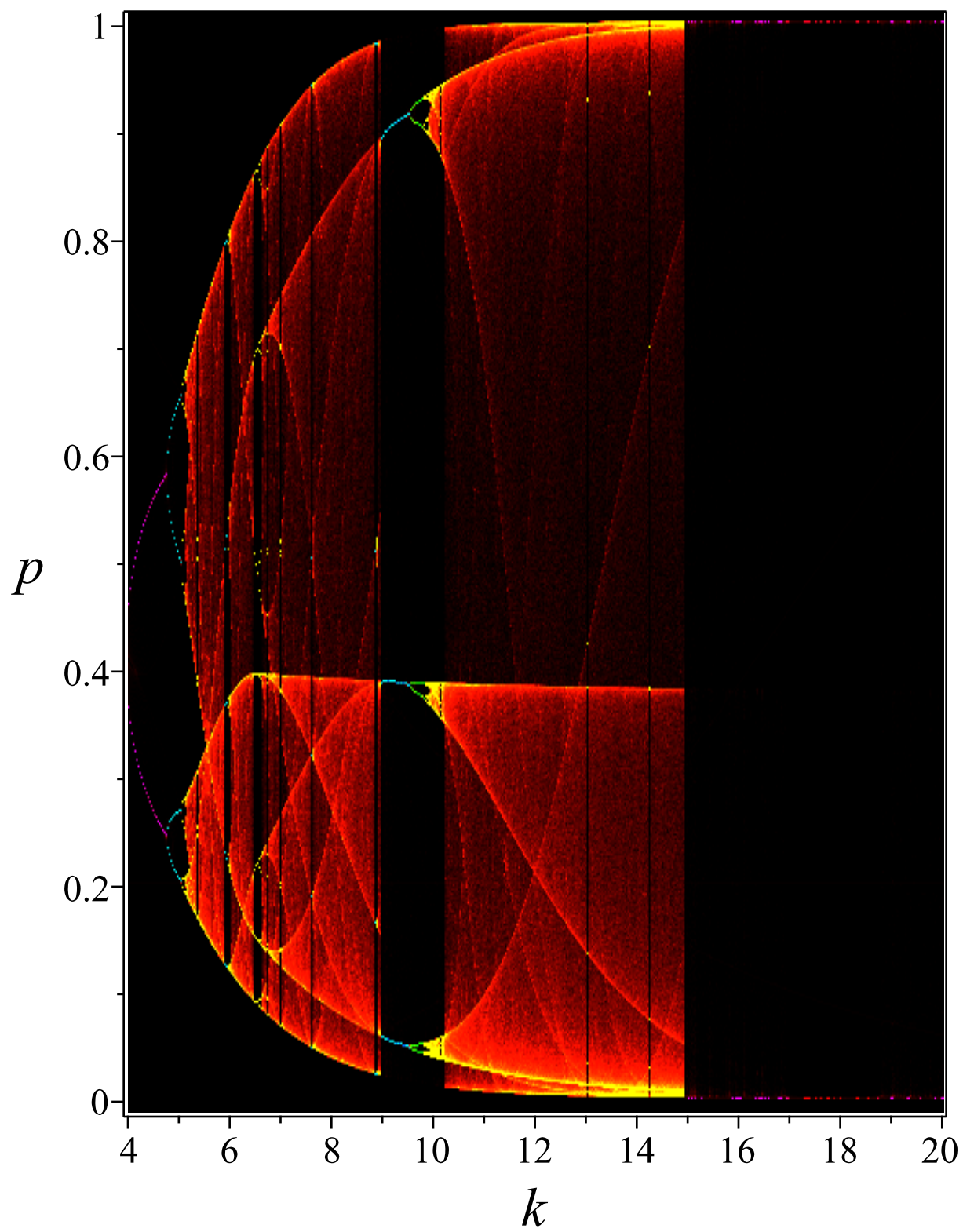
```
print(Pbif);print(Plyap);
```

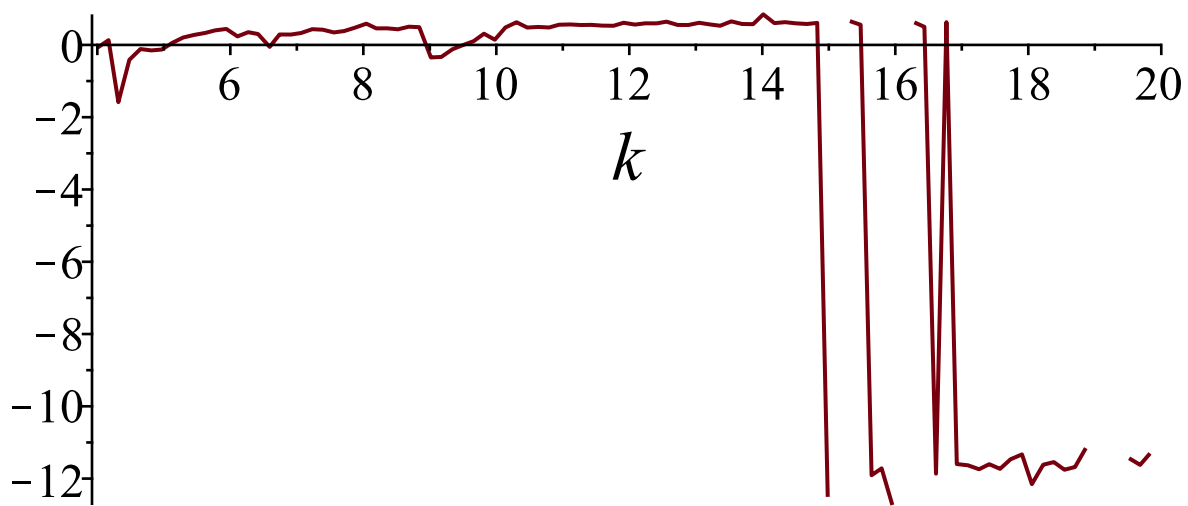
```
for k from 4 to 20 do:
```

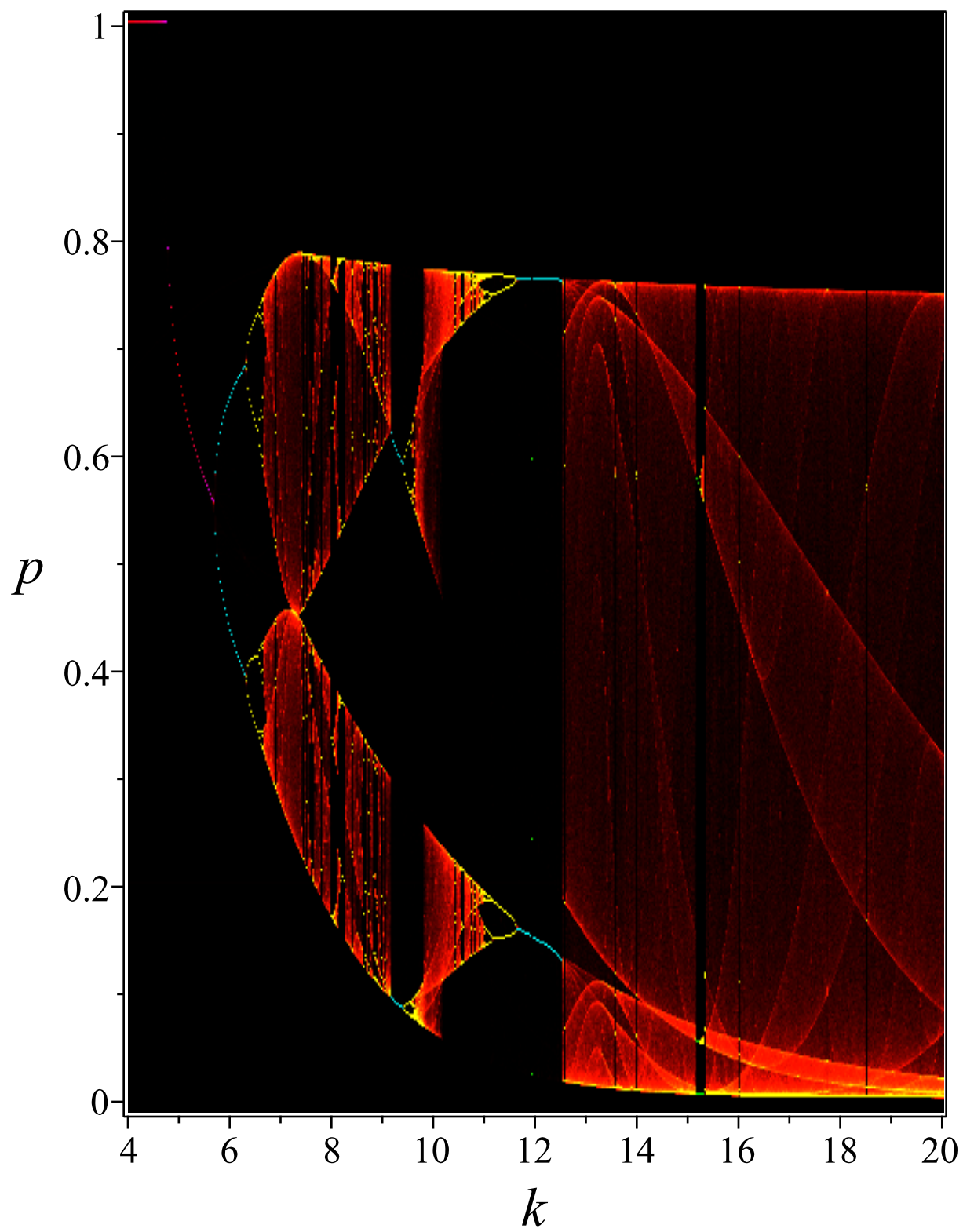
```
expr:=`firstk`*r+`secondk`*(1-r):
```

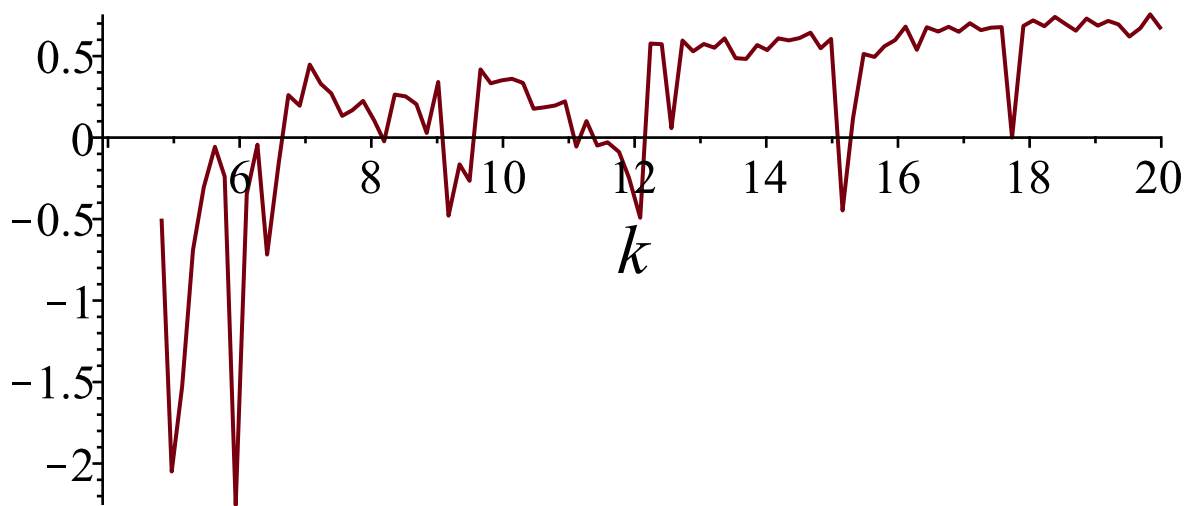
```
bif:=Bifurcation([x],[expr],[l],-.01,1.01,xmin=-.01,xmax=1.01):
```

```
ColouringProcedures:-HueToRGB(bif):
Pbif:=plot(theta=-.01..1.01,p=-.01..1.01,axes=box,size=[600,600],
background=bif,labelfont=[times,s],font=[time,s-6]):
F:=proc(theta,x0)
  if not theta::numeric and x0::numeric then
    return 'procname'(args);
  end if;
  LyapunovExponent(unapply(eval(expr,r=theta),x),x0,
    max_iter=2^17,epsilon=1e-4);
end proc:
Plyap:=plot(F(theta,1),theta=-.01..1.01,adaptive=false,numpoints=
100,size=[600,200],labelfont=[times,s],font=[time,s-6]):
print(firstk);print(secondk);print(Pbif);print(Plyap);
end do:
```

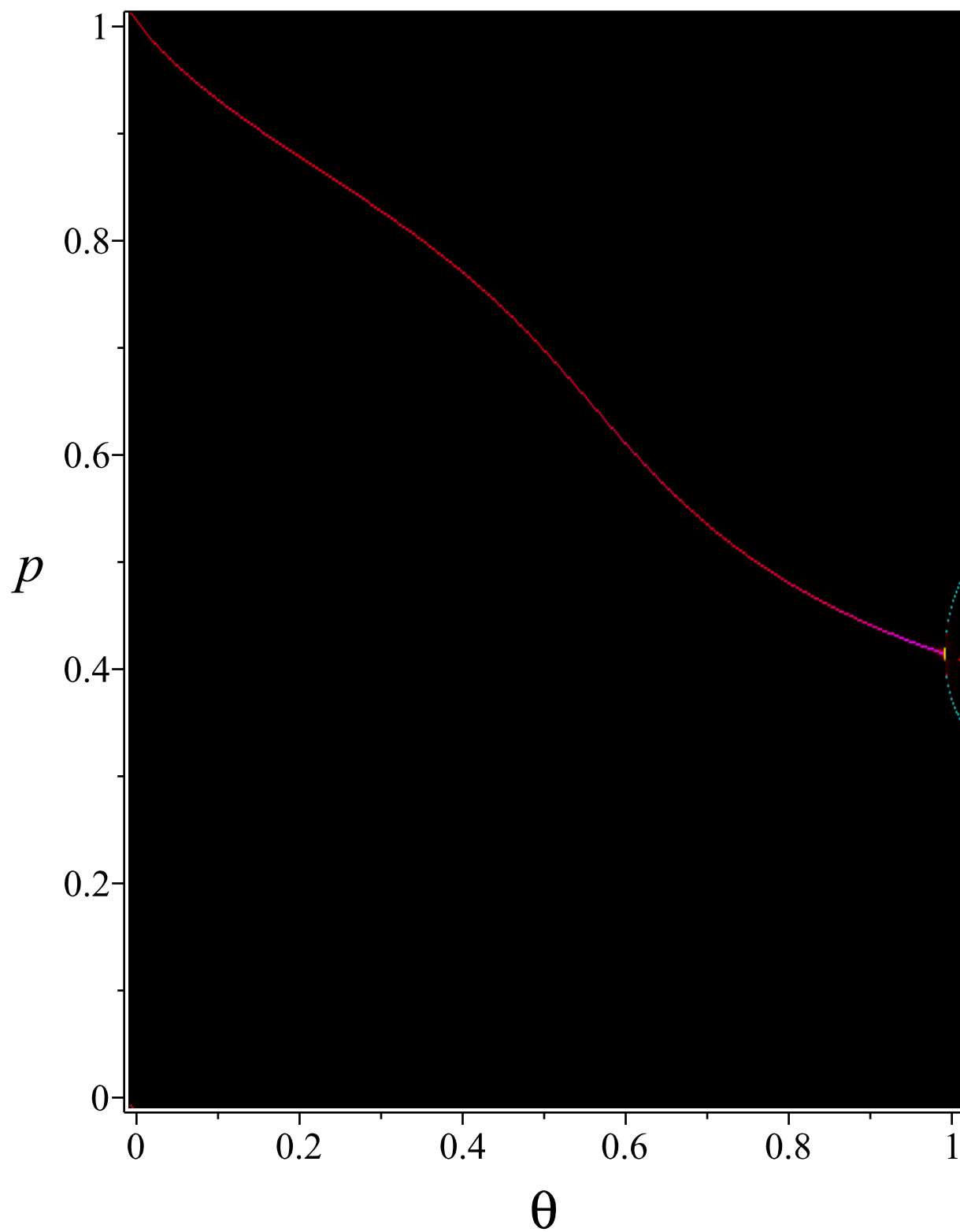


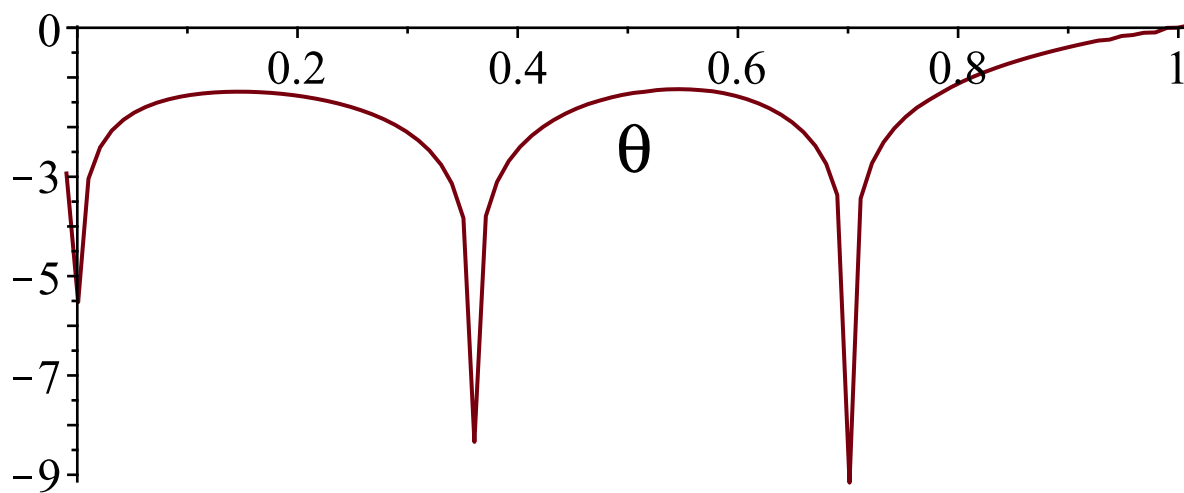




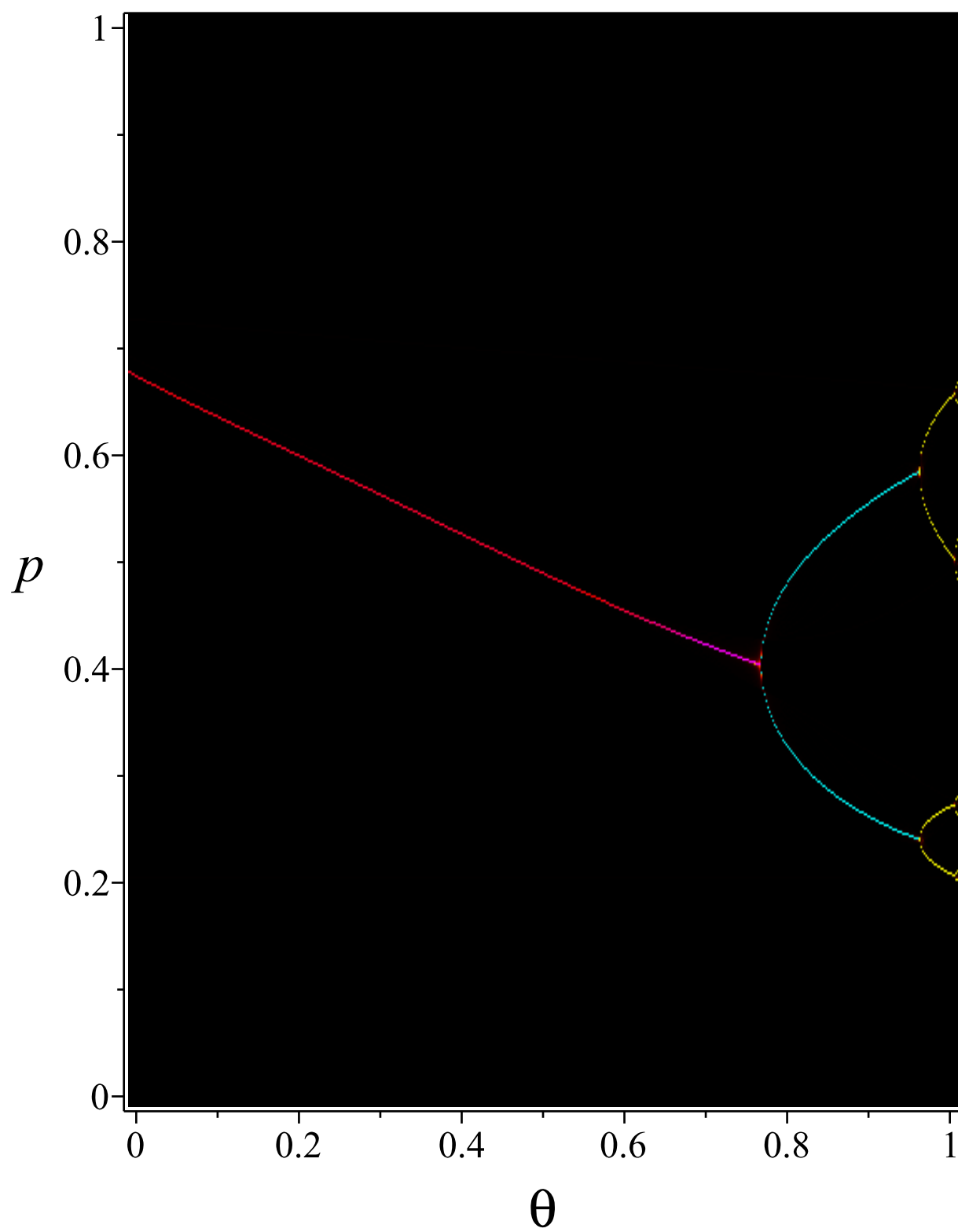


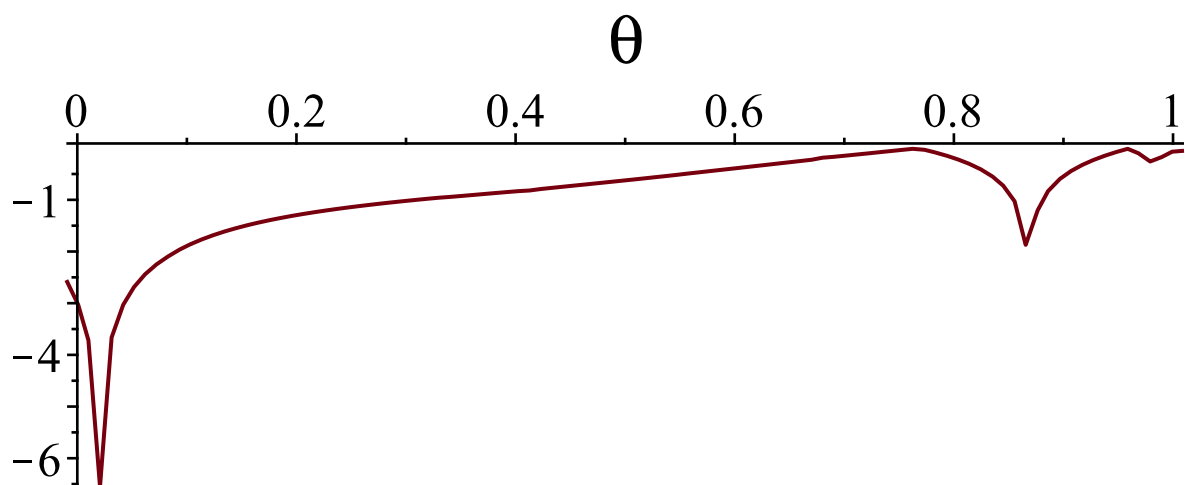
$$\frac{(1-x)^5 + 5x(1-x)^4 + 5x^4(1-x)}{5x(1-x)^4 + 10x^2(1-x)^3 + 10x^3(1-x)^2 + 5x^4(1-x) + x^5}$$



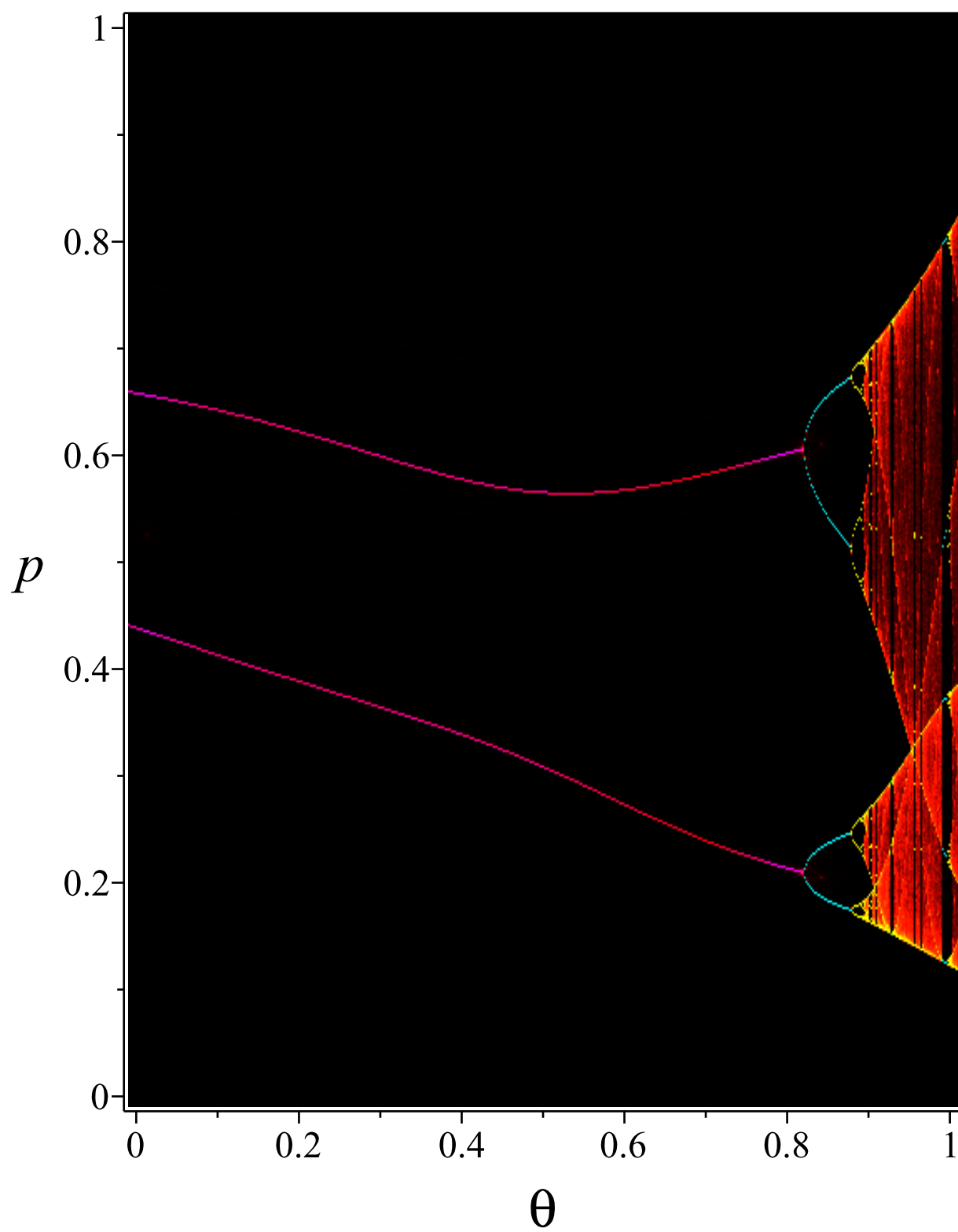


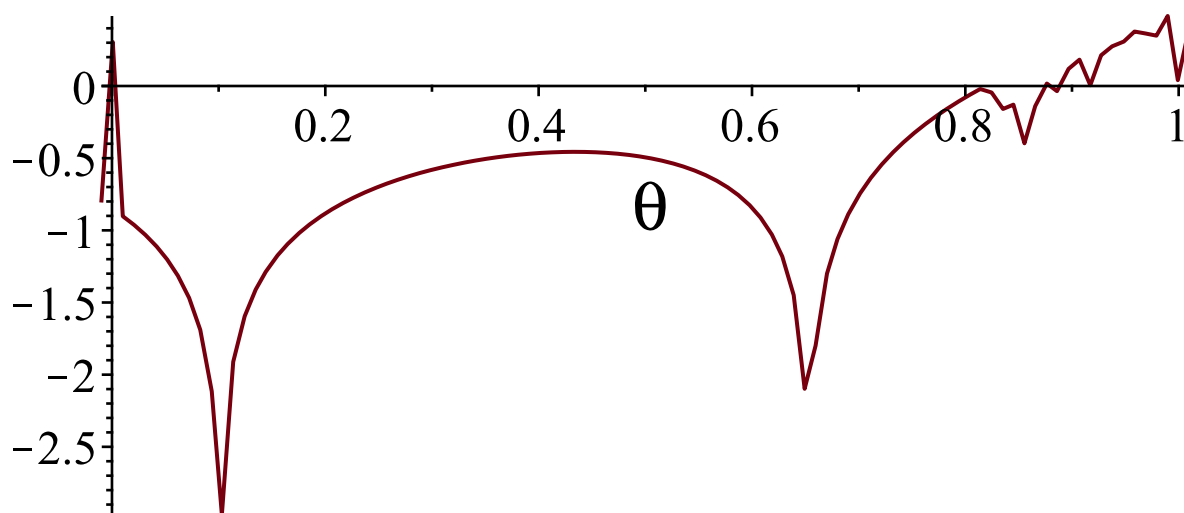
$$\frac{(1-x)^6 + 6x(1-x)^5 + 6x^5(1-x)}{6x(1-x)^5 + 15x^2(1-x)^4 + 20x^3(1-x)^3 + 6x^5(1-x) + x^6}$$



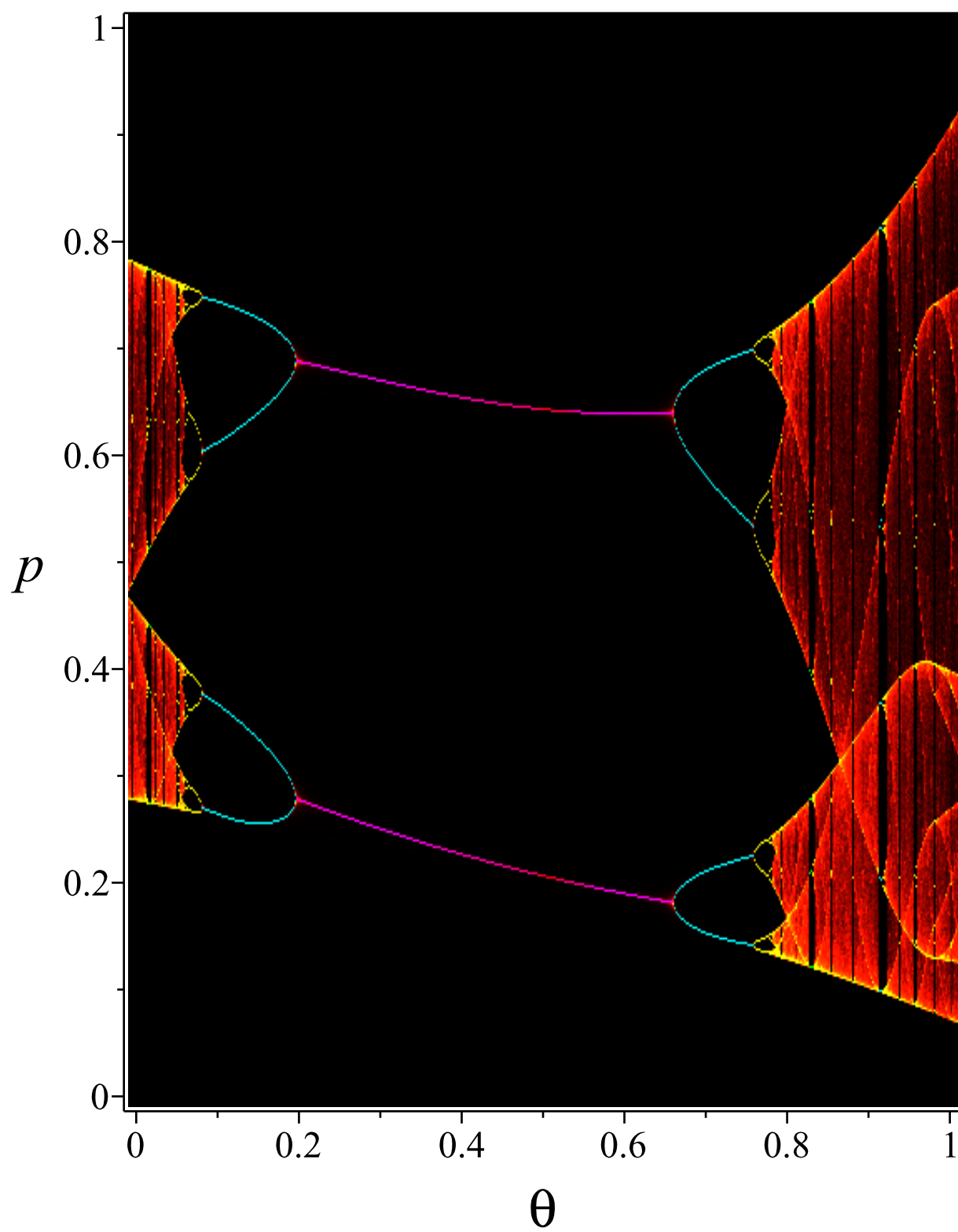


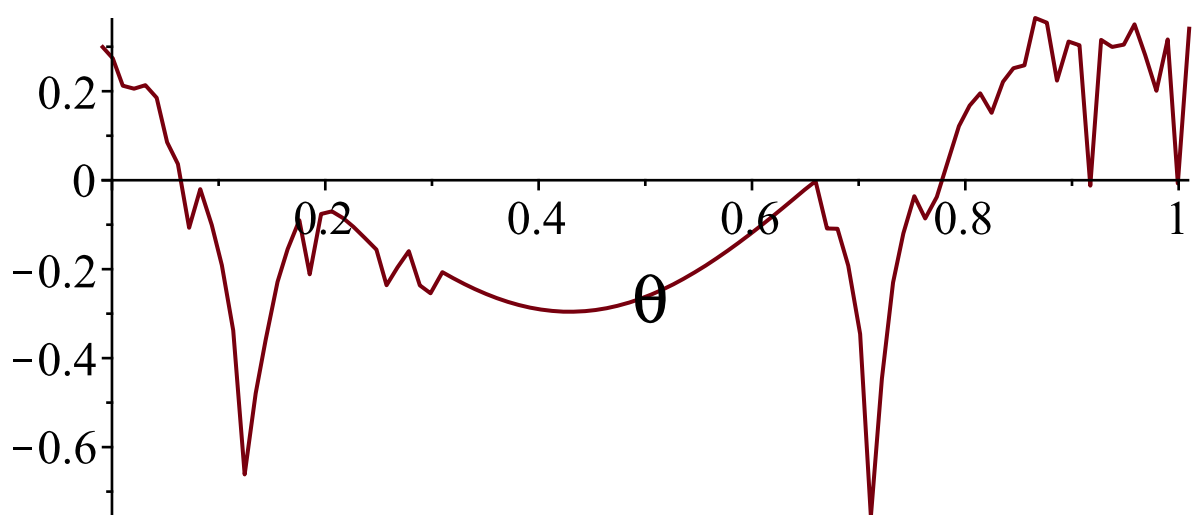
$$\frac{(1-x)^7 + 7x(1-x)^6 + 7x^6(1-x)}{7x(1-x)^6 + 21x^2(1-x)^5 + 35x^3(1-x)^4 + 7x^6(1-x) + x^7}$$



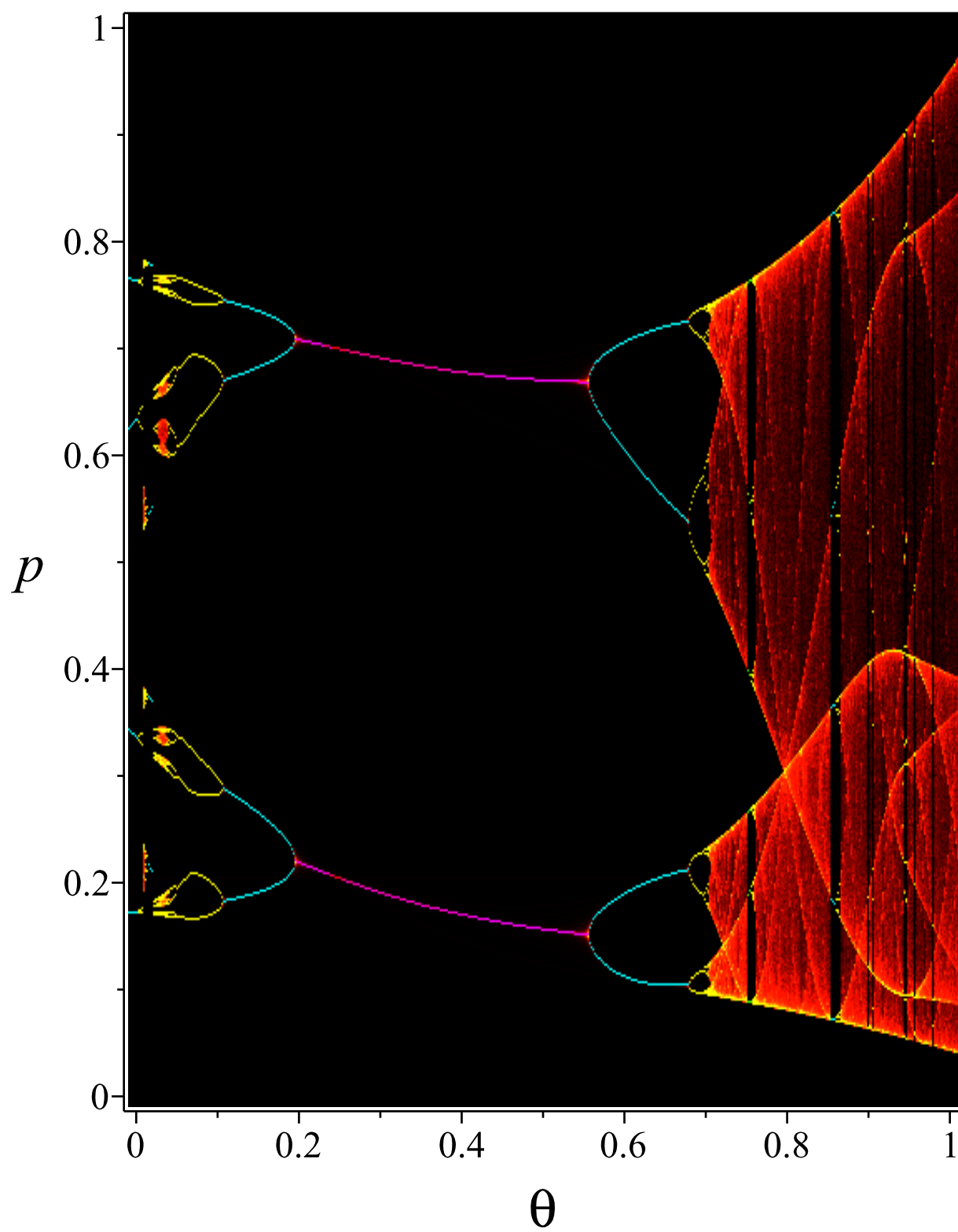


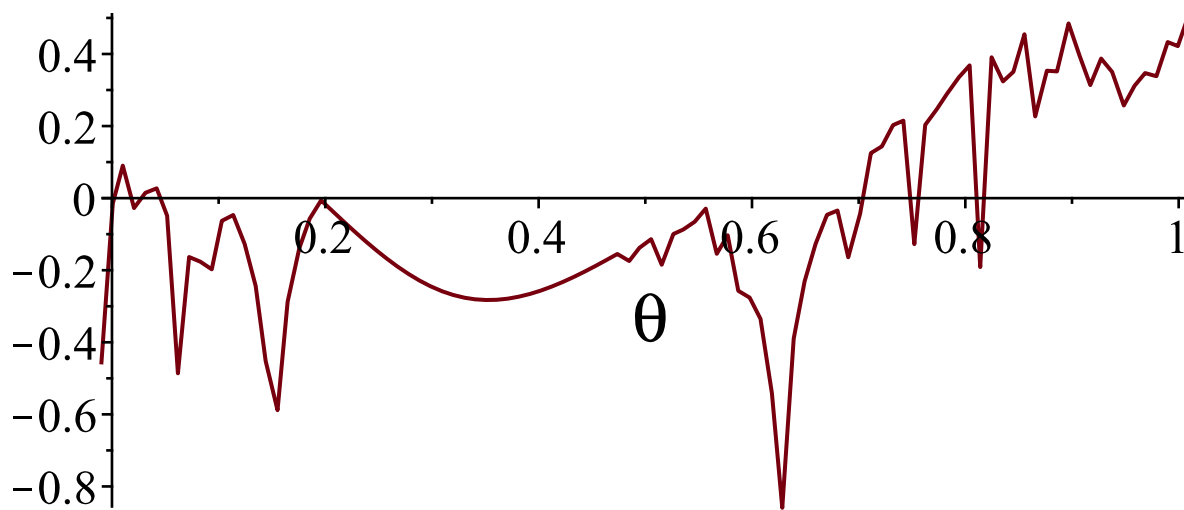
$$\frac{(1-x)^8 + 8x(1-x)^7 + 8x^7(1-x)}{8x(1-x)^7 + 28x^2(1-x)^6 + 56x^3(1-x)^5 + 8x^7(1-x) + x^8}$$



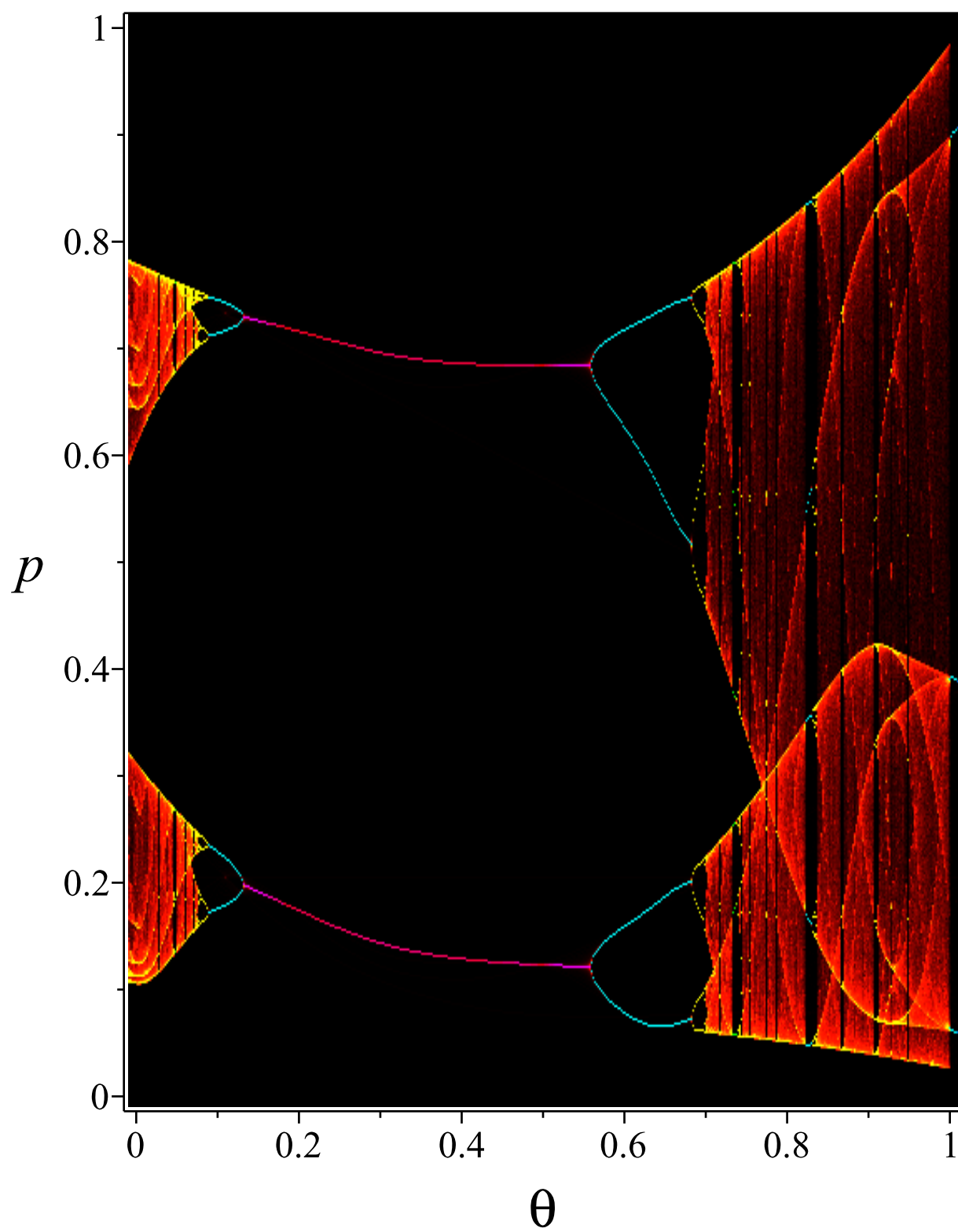


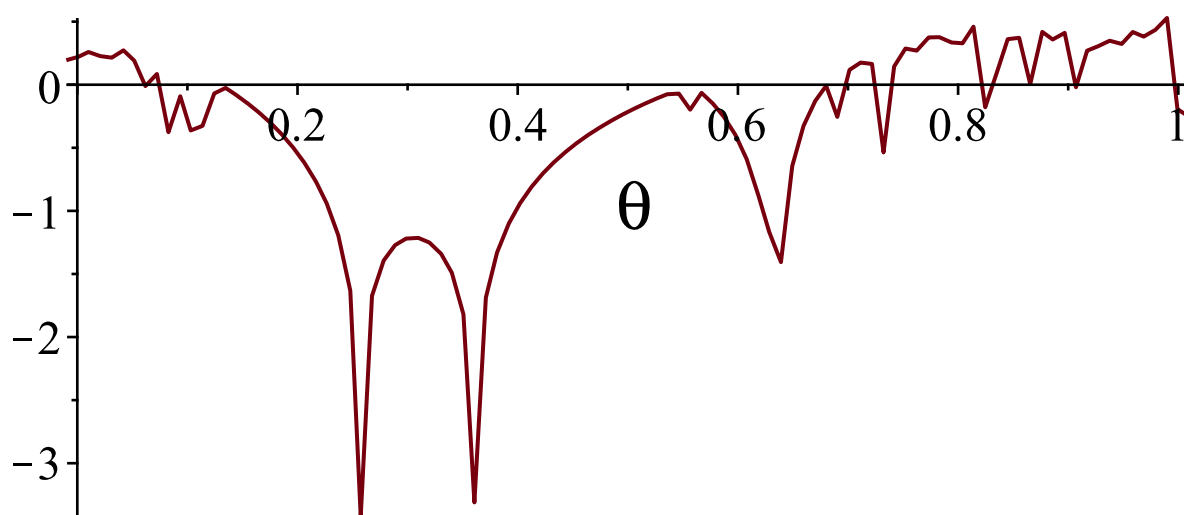
$$\frac{(1-x)^9 + 9x(1-x)^8 + 9x^8(1-x)}{9x(1-x)^8 + 36x^2(1-x)^7 + 84x^3(1-x)^6 + 9x^8(1-x) + x^9}$$



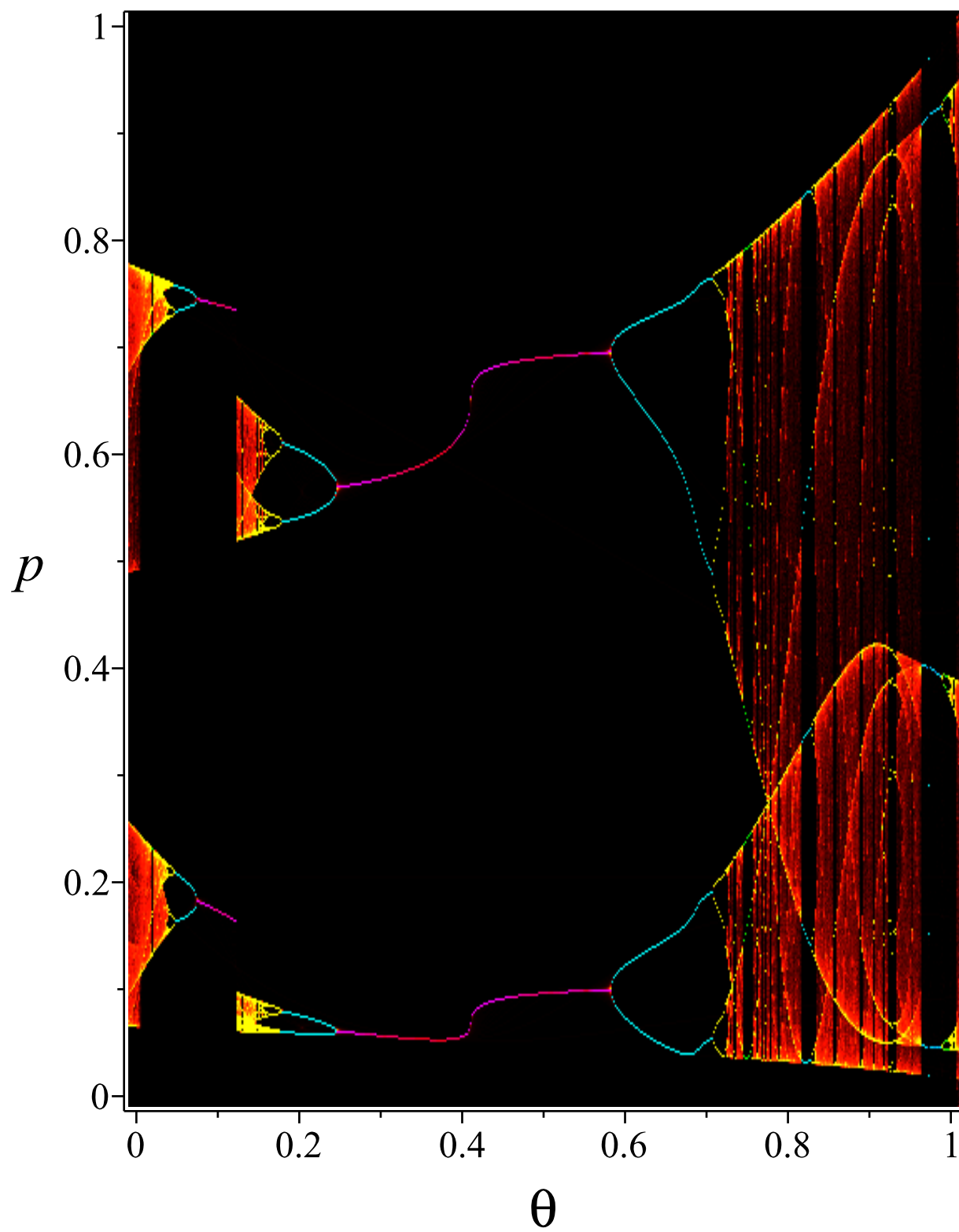


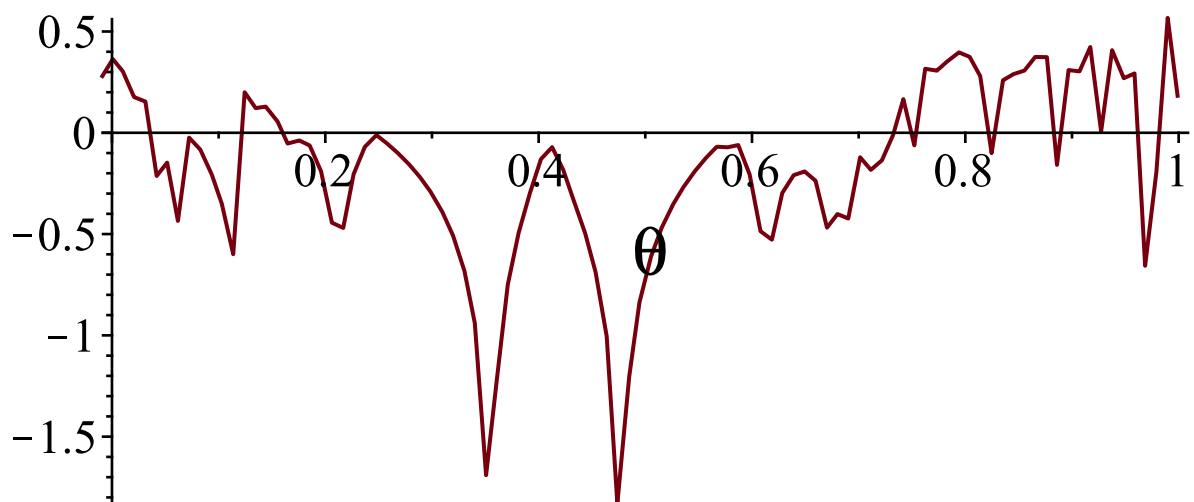
$$\frac{(1-x)^{10} + 10x(1-x)^9 + 10x^9(1-x)}{10x(1-x)^9 + 45x^2(1-x)^8 + 120x^3(1-x)^7 + 10x^9(1-x) + x^{10}}$$



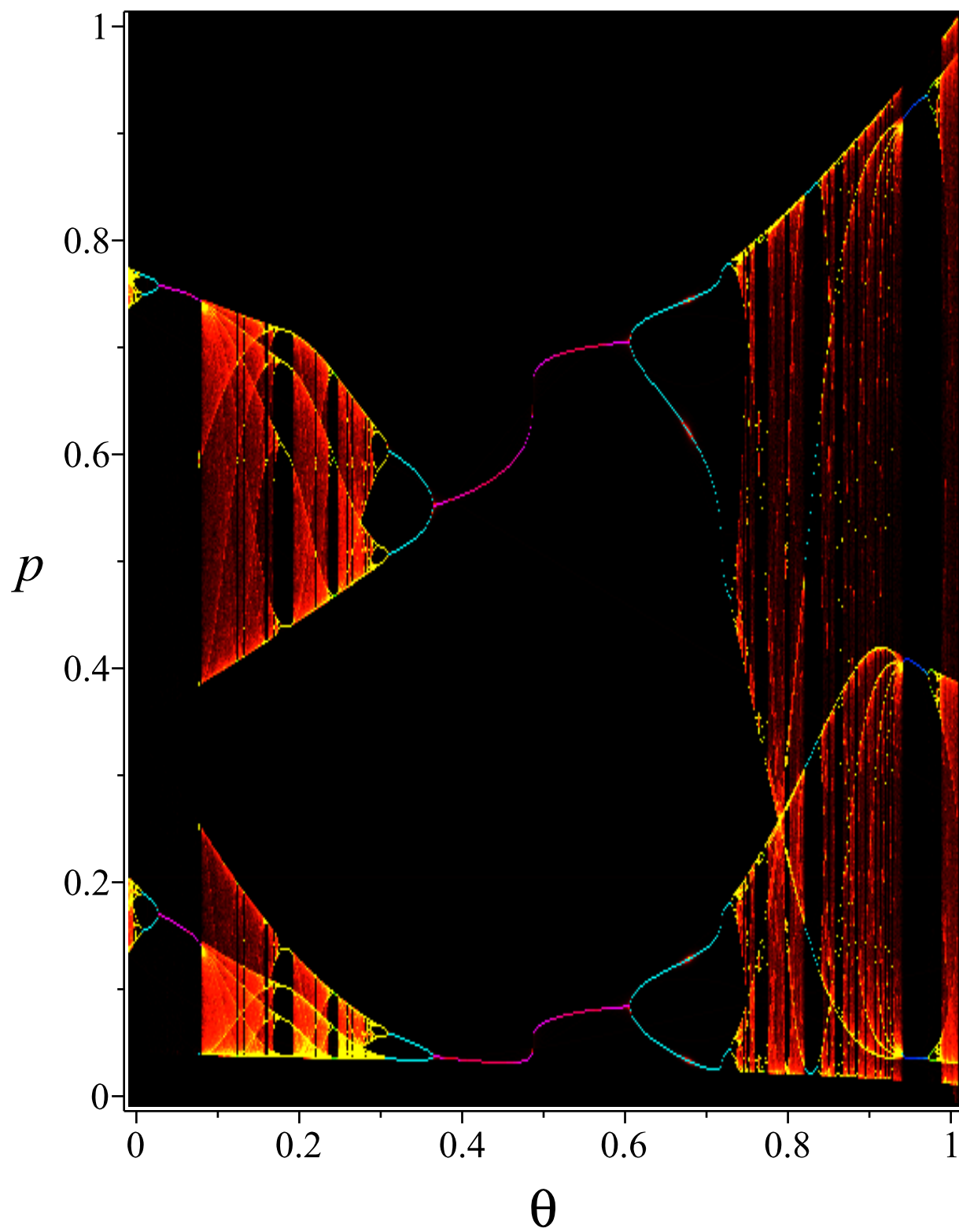


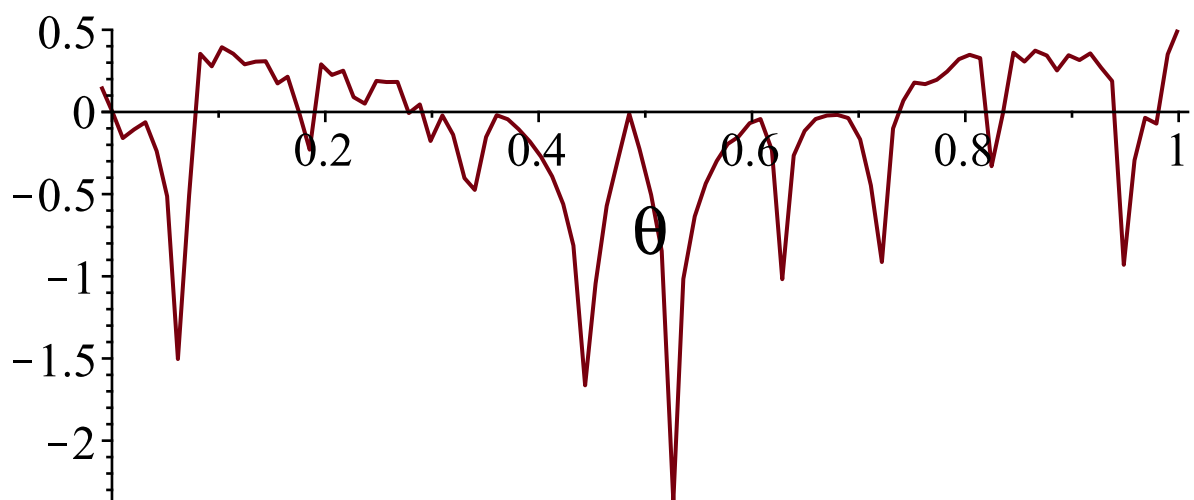
$$\frac{(1-x)^{11} + 11x(1-x)^{10} + 11x^{10}(1-x)}{11x(1-x)^{10} + 55x^2(1-x)^9 + 165x^3(1-x)^8 + 11x^{10}(1-x) + x^{11}}$$



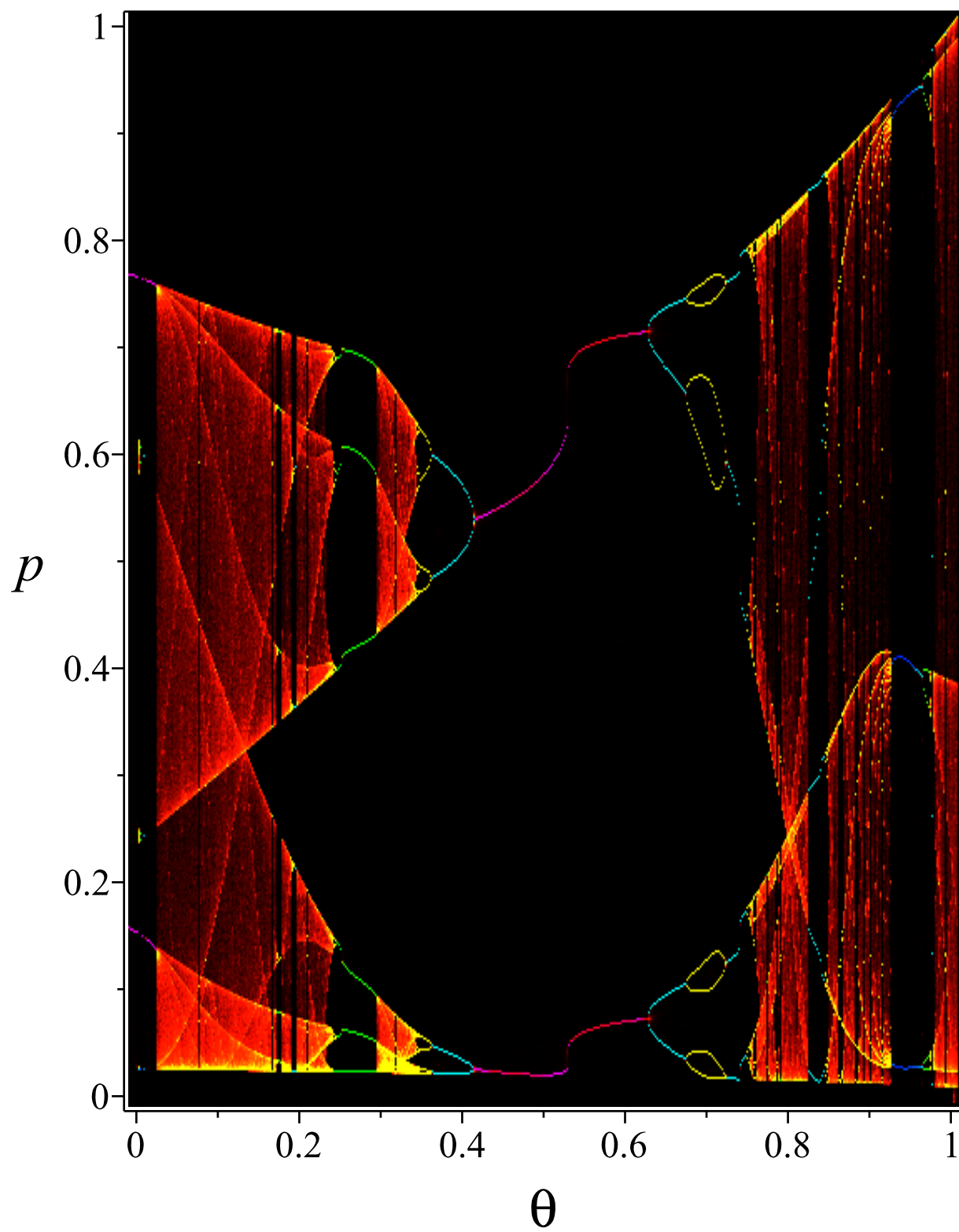


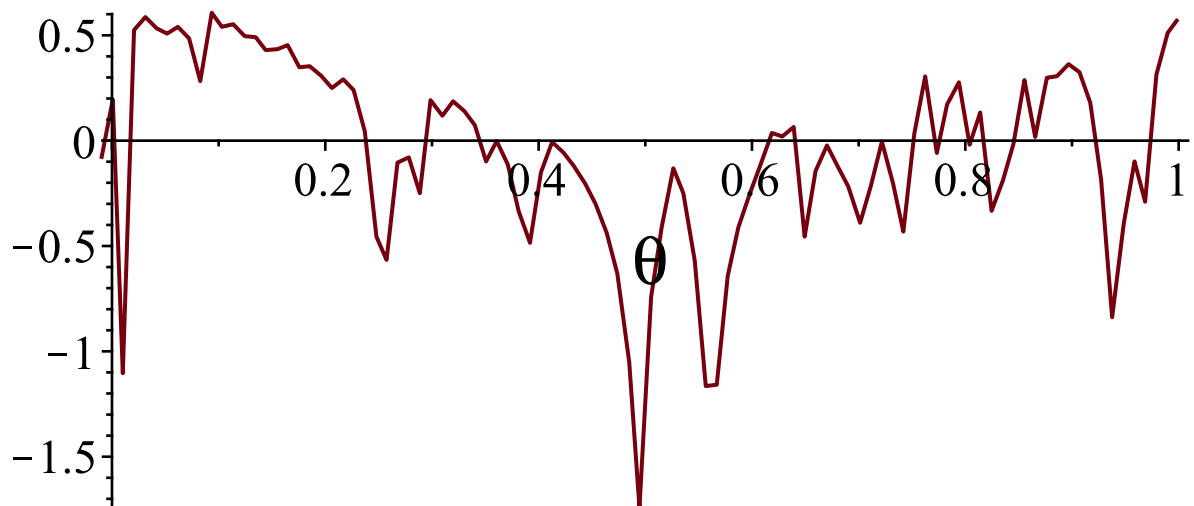
$$\frac{(1-x)^{12} + 12x(1-x)^{11} + 12x^{11}(1-x)}{12x(1-x)^{11} + 66x^2(1-x)^{10} + 220x^3(1-x)^9 + 12x^{11}(1-x) + x^{12}}$$





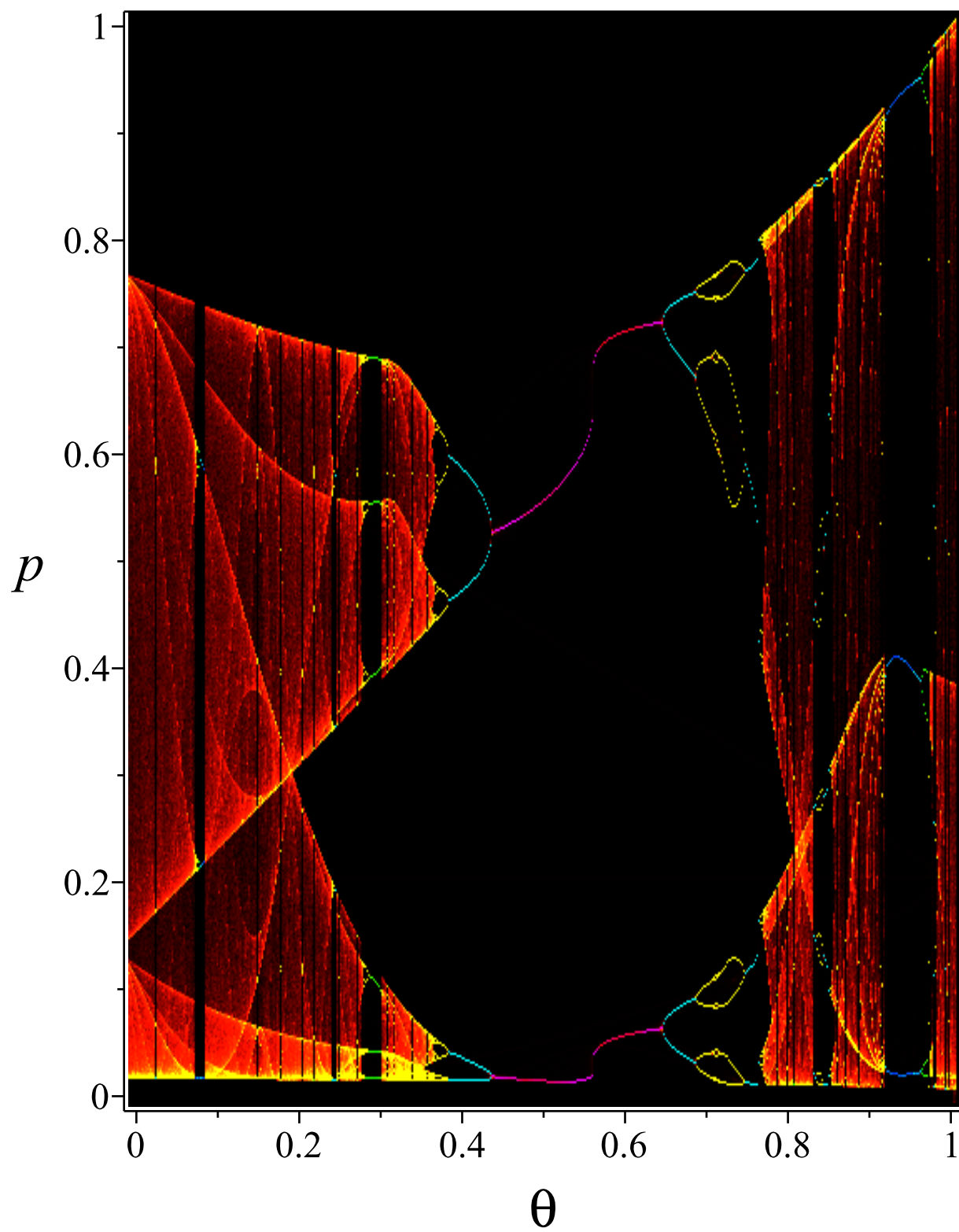
$$\theta = \frac{(1-x)^{13} + 13x(1-x)^{12} + 13x^{12}(1-x)}{13x(1-x)^{12} + 78x^2(1-x)^{11} + 286x^3(1-x)^{10} + 13x^{12}(1-x) + x^{13}}$$

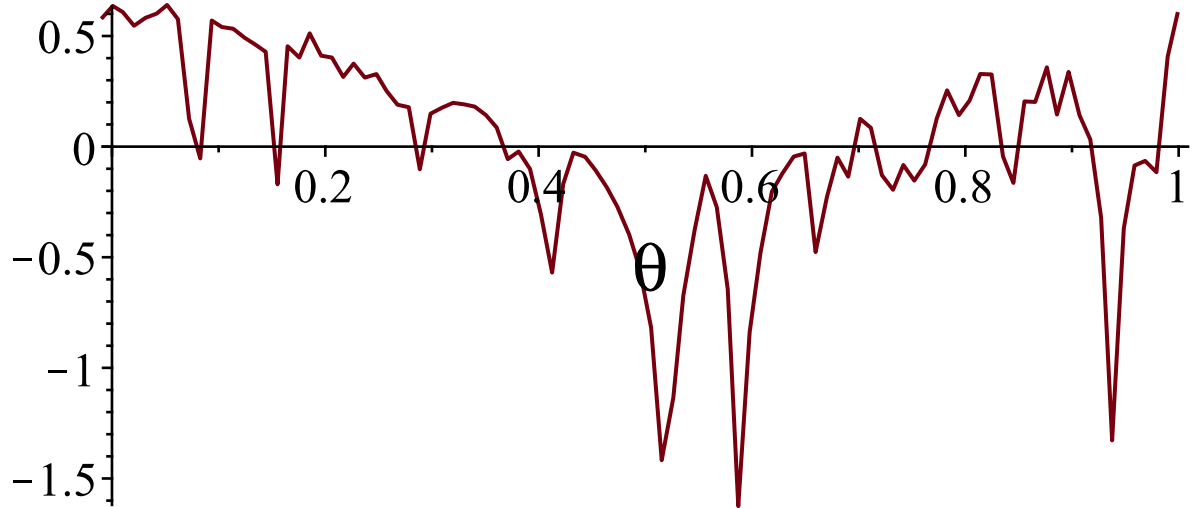




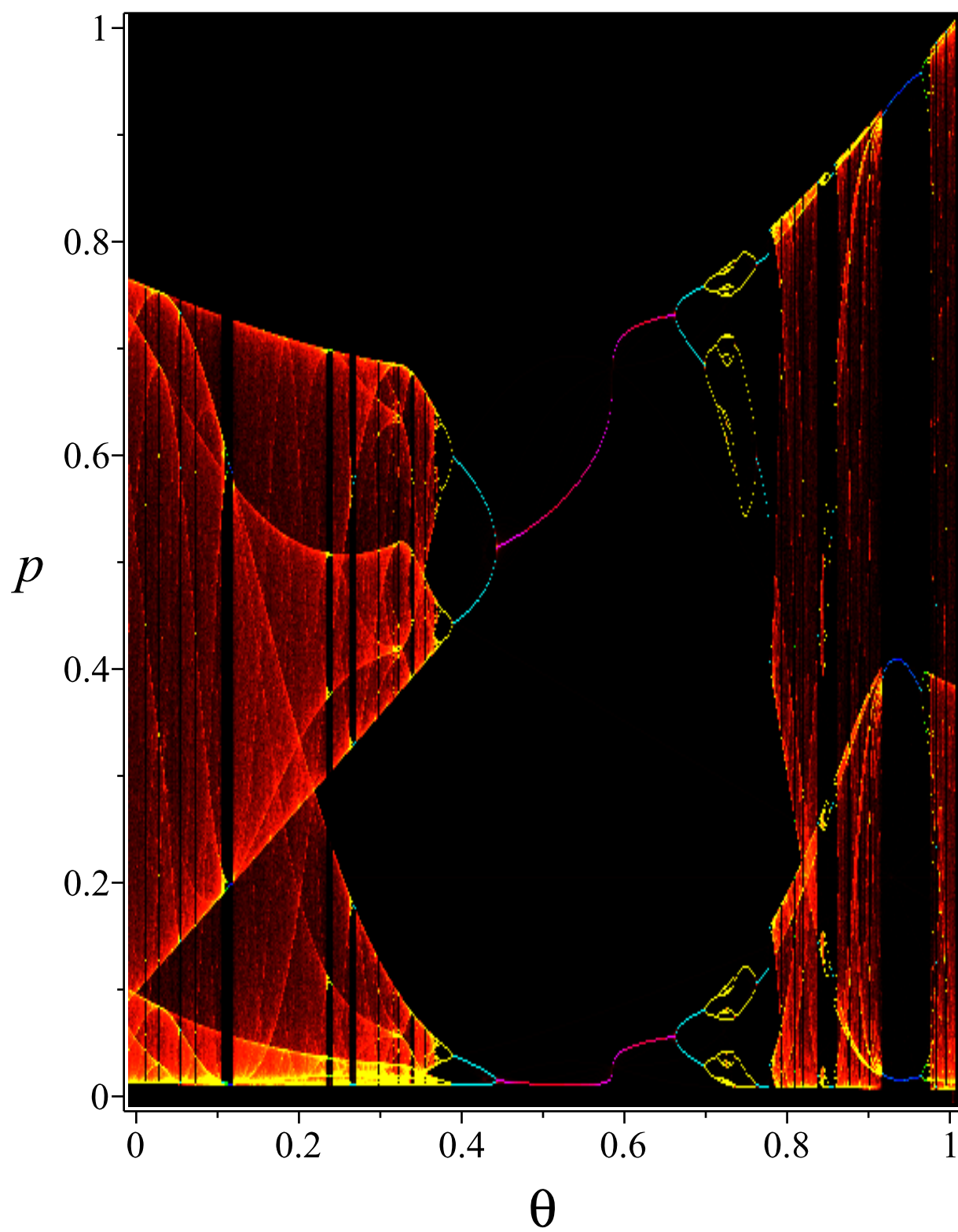
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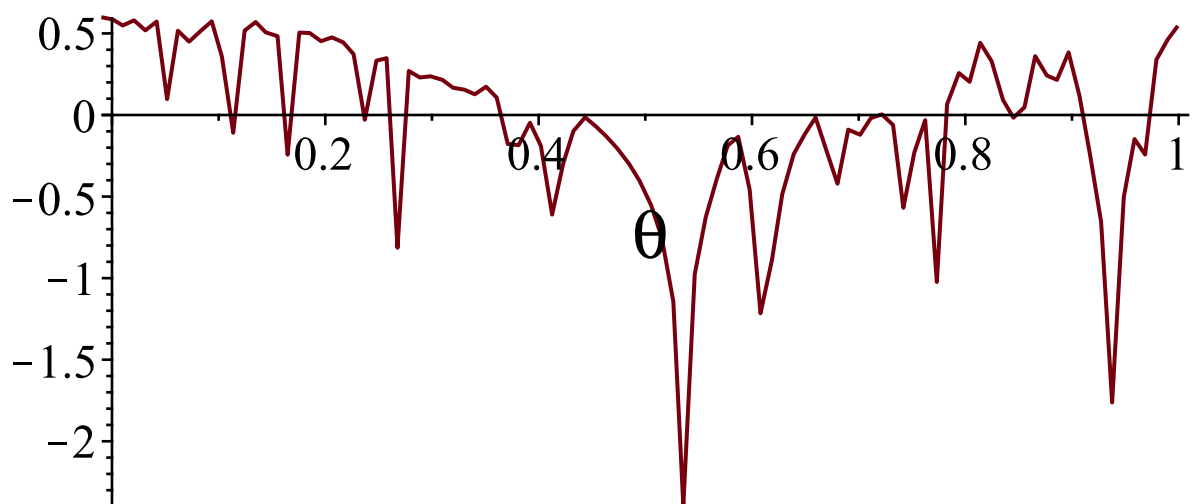
$$\frac{(1-x)^{14} + 14x(1-x)^{13} + 14x^{13}(1-x) + 14x(1-x)^{13} + 91x^2(1-x)^{12} + 364x^3(1-x)^{11} + 14x^{13}(1-x) + x^{14}}{2}$$



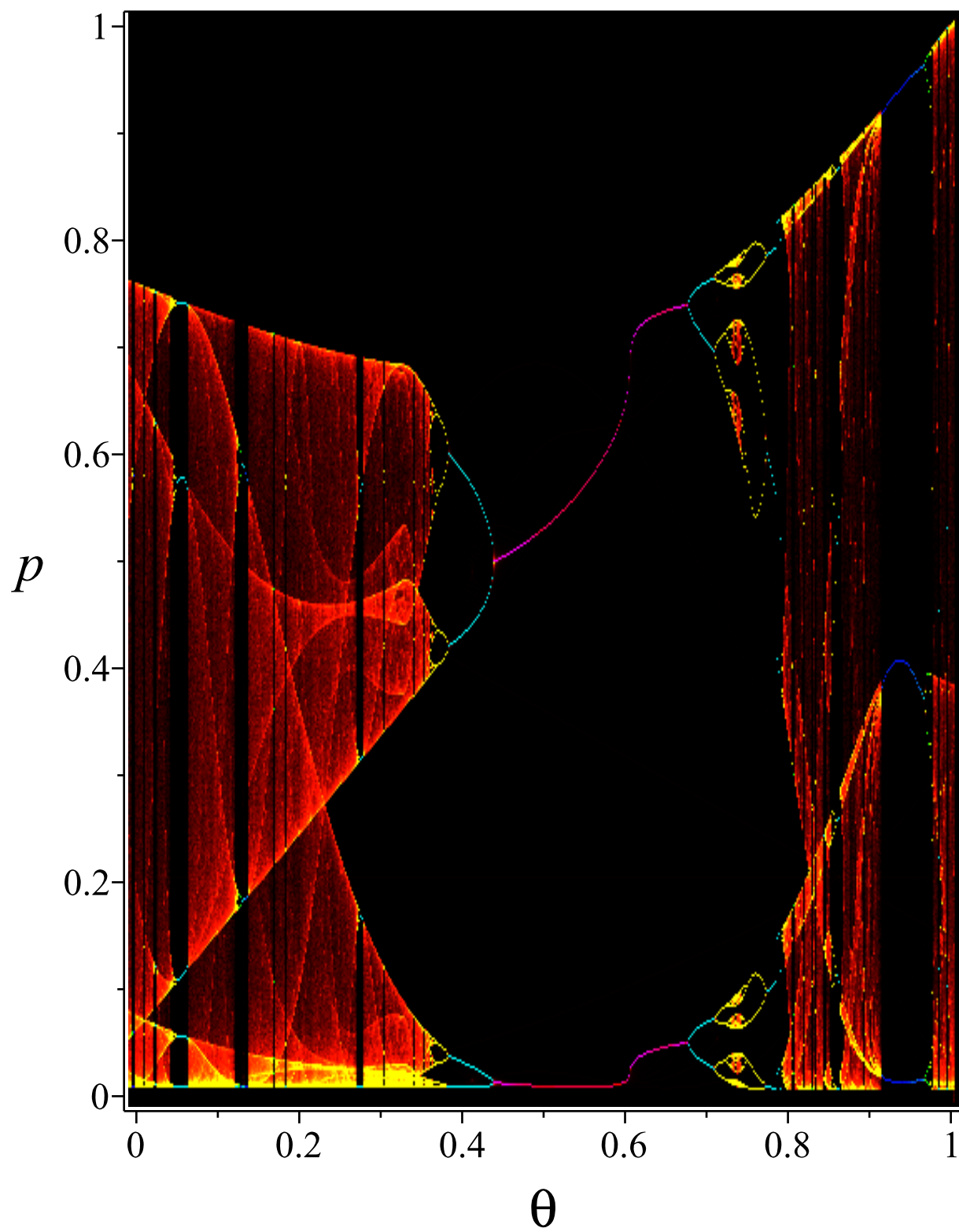


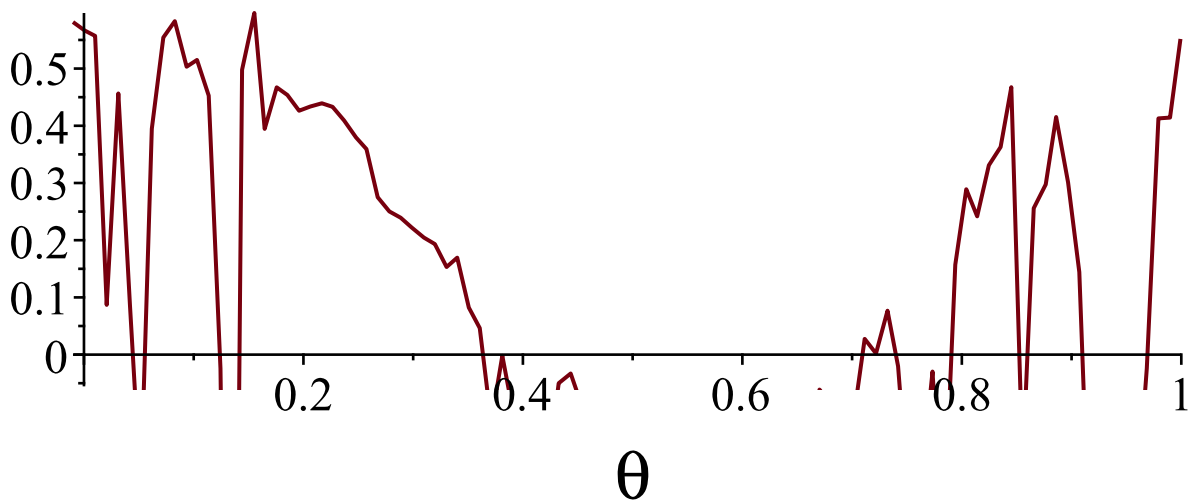
$$\frac{(1-x)^{15} + 15x(1-x)^{14} + 15x^{14}(1-x)}{15x(1-x)^{14} + 105x^2(1-x)^{13} + 455x^3(1-x)^{12} + 15x^{14}(1-x) + x^{15}}$$



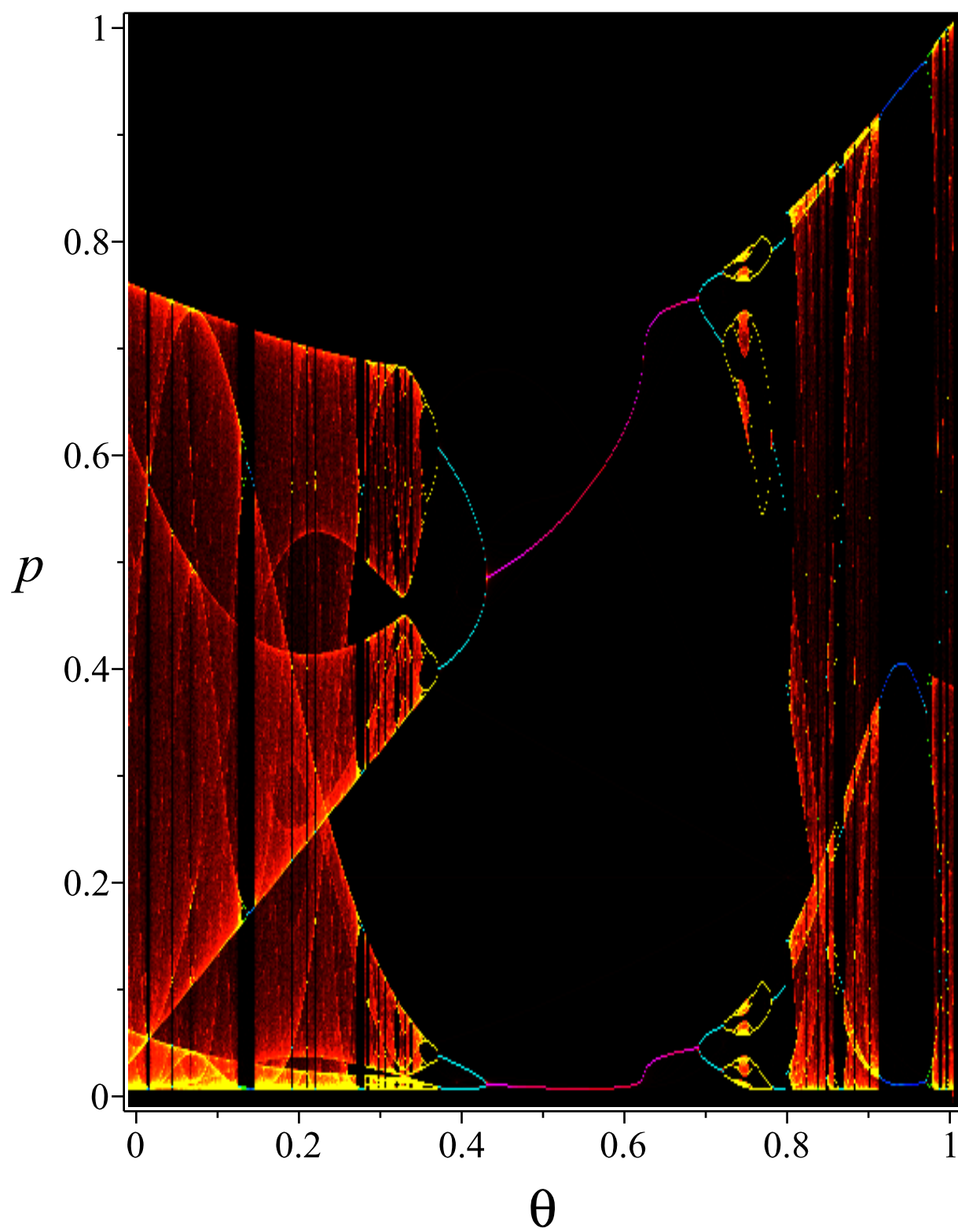


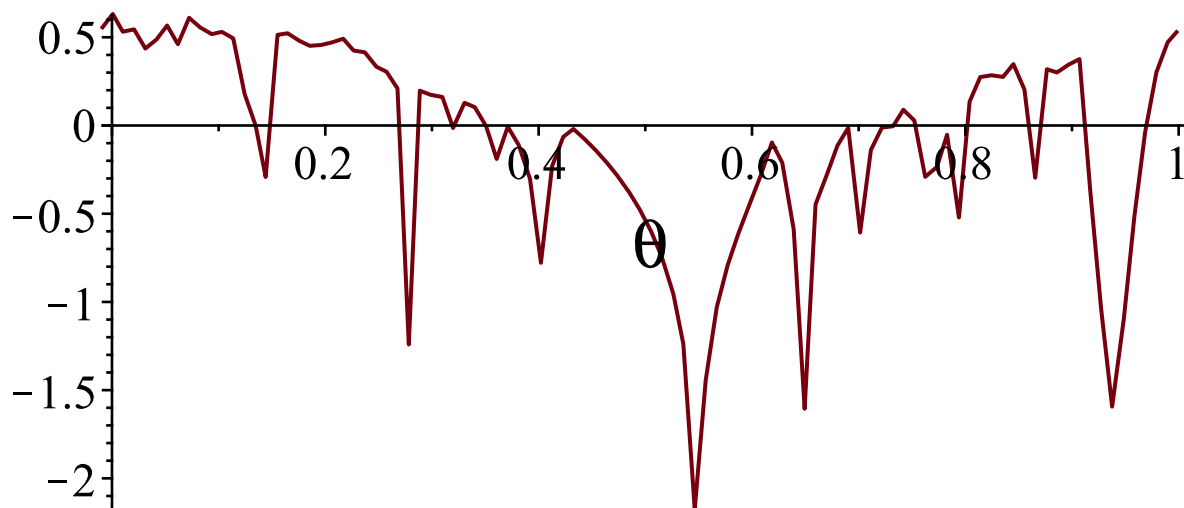
$$\frac{(1-x)^{16} + 16x(1-x)^{15} + 16x^{15}(1-x)}{16x(1-x)^{15} + 120x^2(1-x)^{14} + 560x^3(1-x)^{13} + 16x^{15}(1-x) + x^{16}}$$



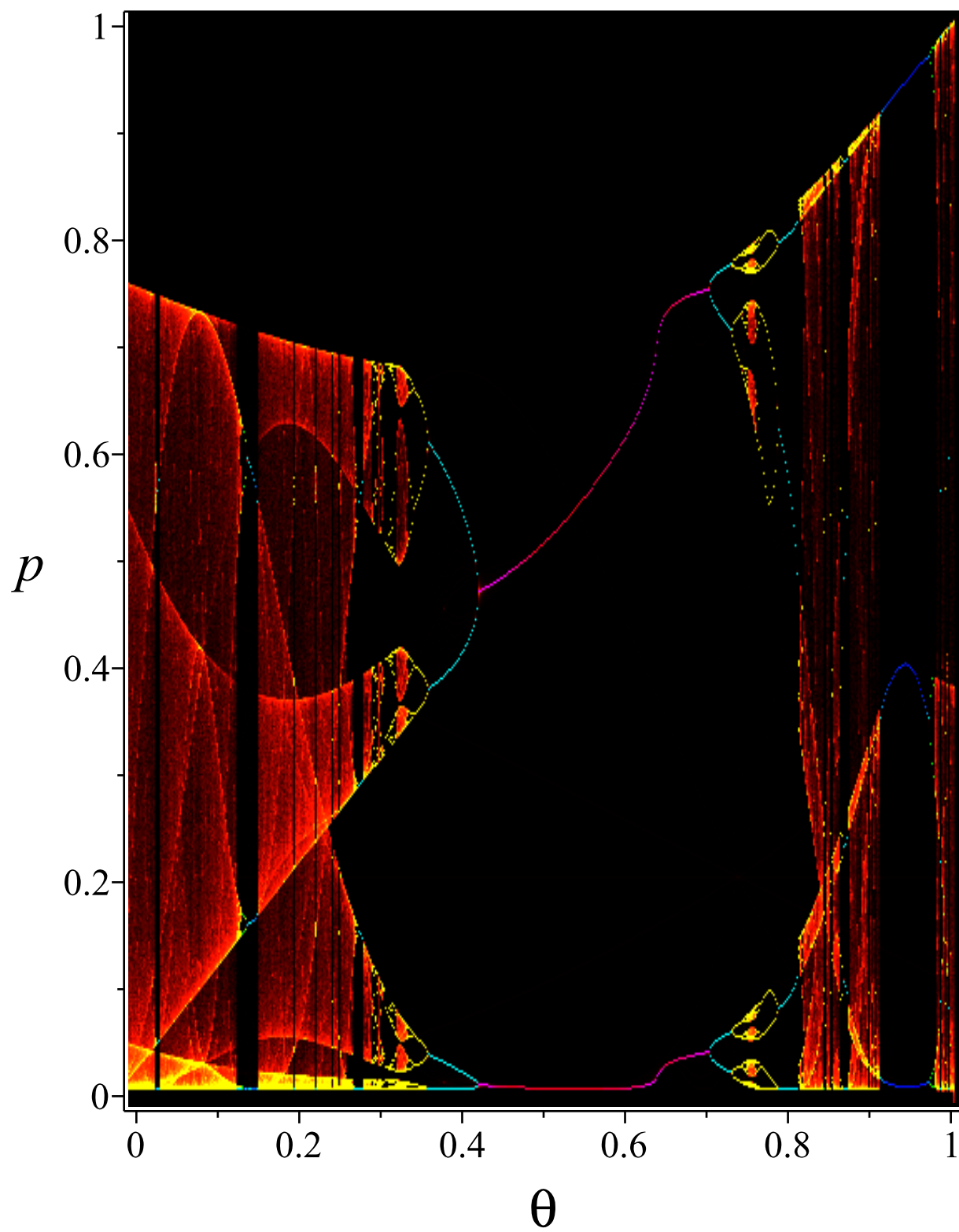


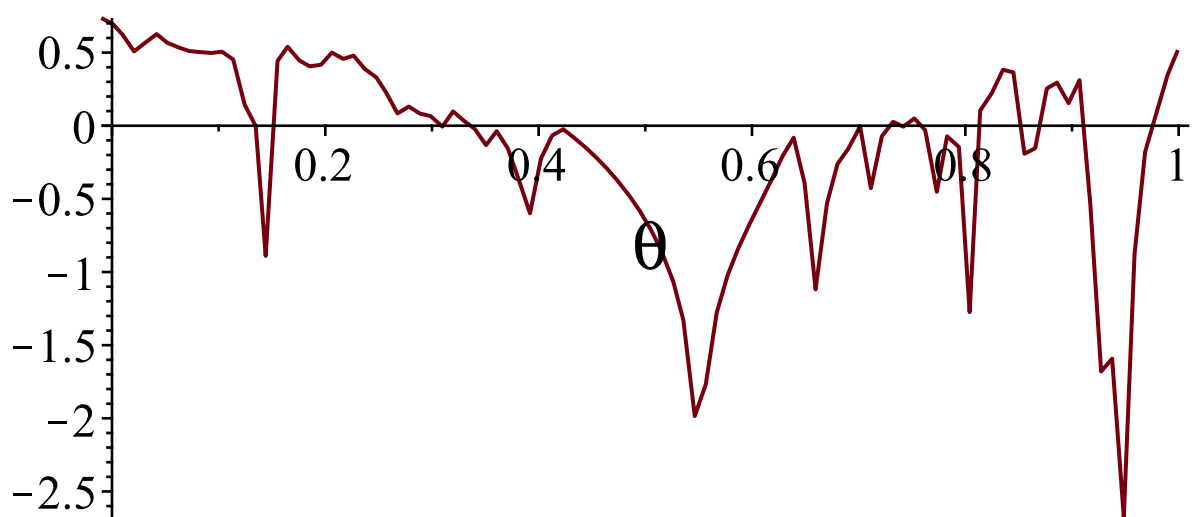
$$\frac{(1-x)^{17} + 17x(1-x)^{16} + 17x^{16}(1-x)}{17x(1-x)^{16} + 136x^2(1-x)^{15} + 680x^3(1-x)^{14} + 17x^{16}(1-x) + x^{17}}$$



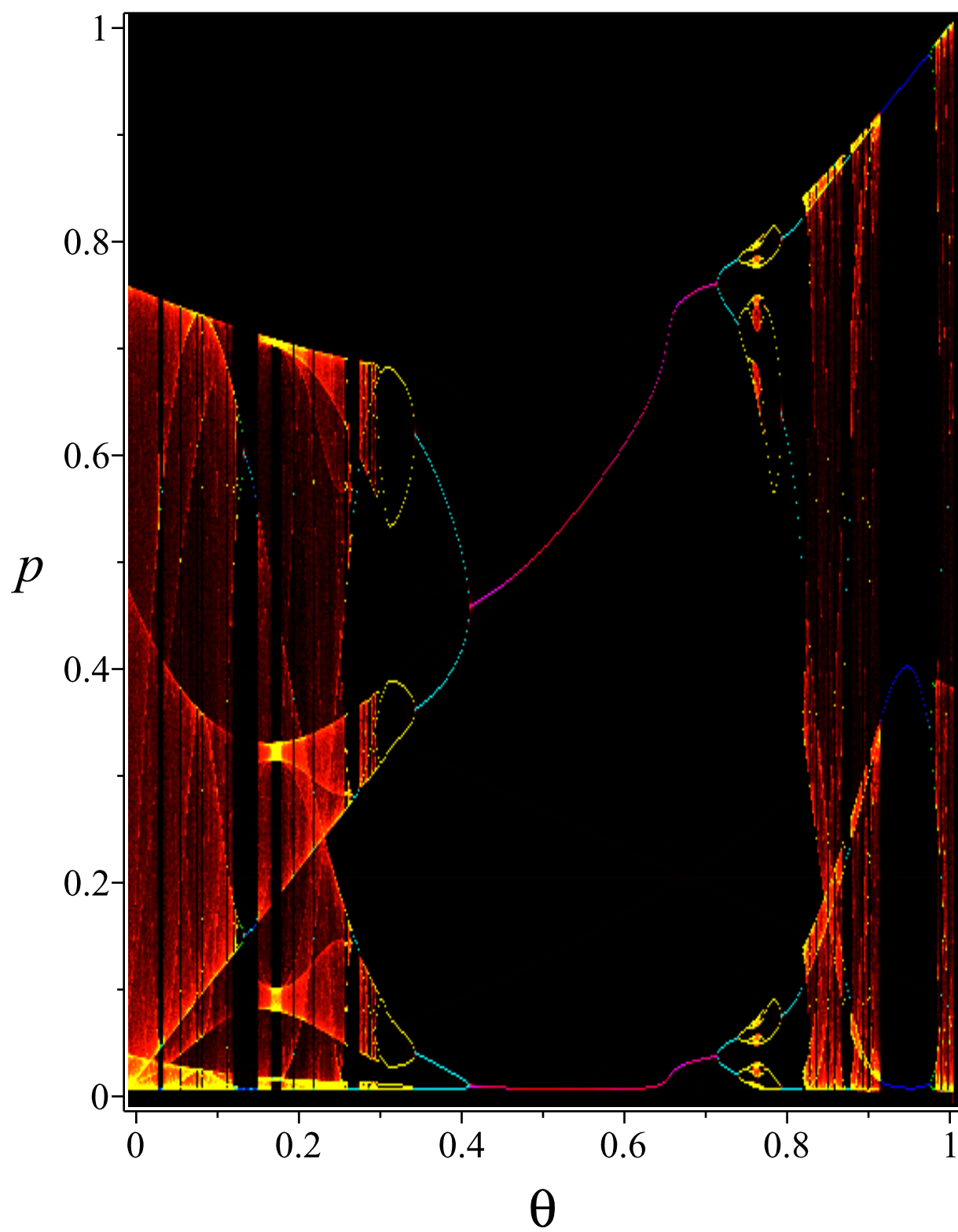


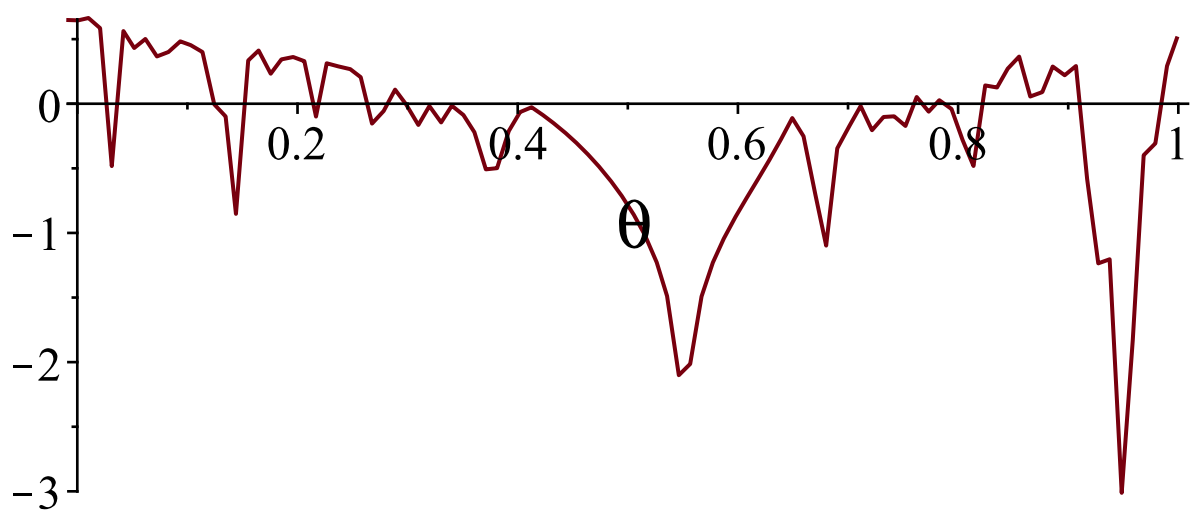
$$\theta = \frac{(1-x)^{18} + 18x(1-x)^{17} + 18x^{17}(1-x)}{18x(1-x)^{17} + 153x^2(1-x)^{16} + 816x^3(1-x)^{15} + 18x^{17}(1-x) + x^{18}}$$



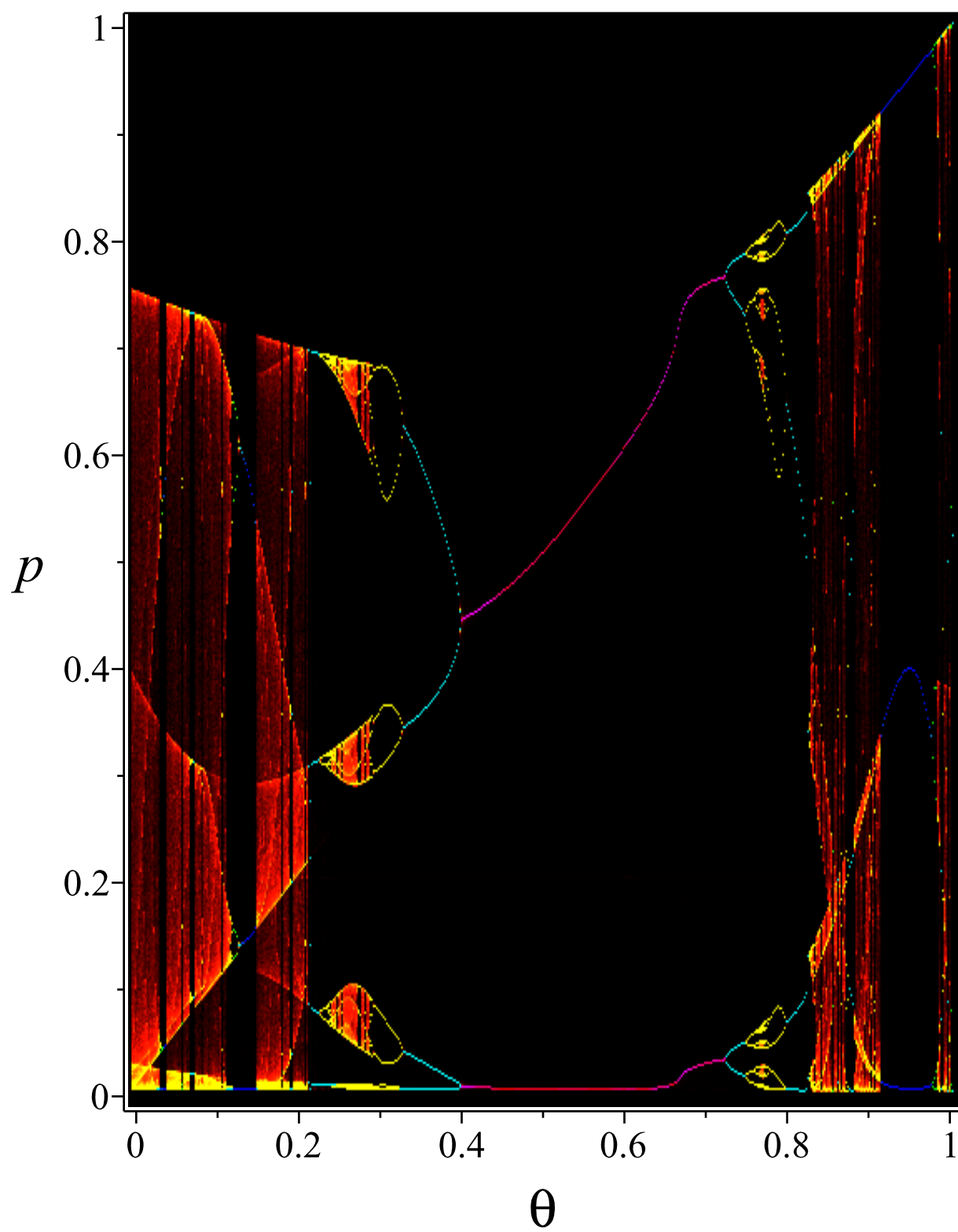


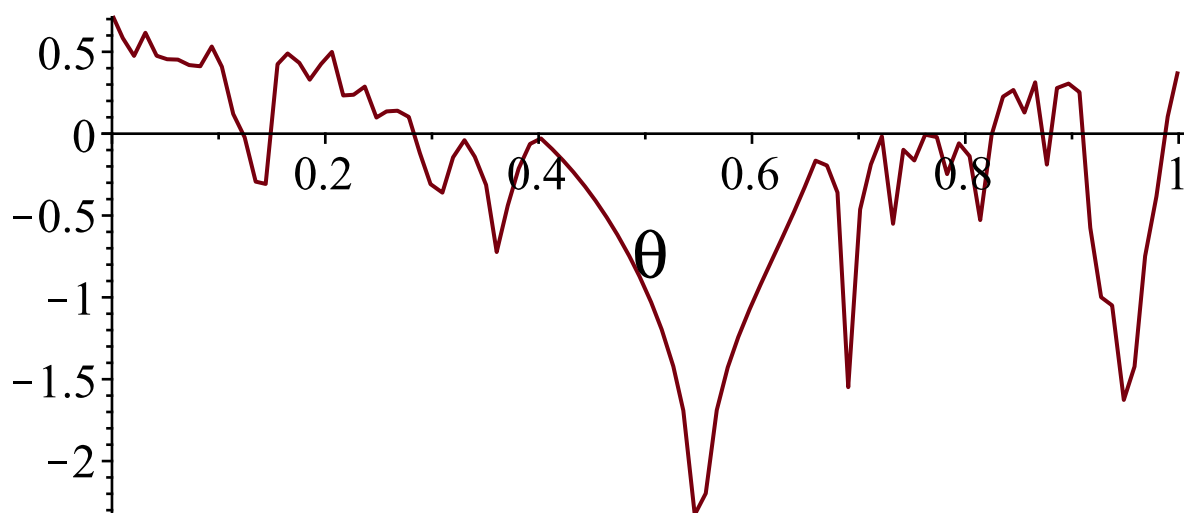
$$\frac{(1-x)^{19} + 19x(1-x)^{18} + 19x^{18}(1-x)}{19x(1-x)^{18} + 171x^2(1-x)^{17} + 969x^3(1-x)^{16} + 19x^{18}(1-x) + x^{19}}$$





$$\frac{(1-x)^{20} + 20x(1-x)^{19} + 20x^{19}(1-x)}{20x(1-x)^{19} + 190x^2(1-x)^{18} + 1140x^3(1-x)^{17} + 20x^{19}(1-x) + x^{20}}$$





$$\frac{(1-x)^{21} + 21x(1-x)^{20} + 21x^{20}(1-x)}{21x(1-x)^{20} + 210x^2(1-x)^{19} + 1330x^3(1-x)^{18} + 21x^{20}(1-x) + x^{21}}$$

