

# HITS program for Maximum Score Estimation

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## Description

Hyperplanes Intersection Tabu Search (HITS) is a program that calculates the Maximum Score Estimator (MS) of the binary choice model (Manski, 1975; 1985). It is a variant of Tabu Search (TS) method (Glover, 1989) especially modified for MS. Specifically, it uses the gaussian elimination to locate the trial points of hyperplanes intersections (Pinkse, 1993). It uses the concept of “Neighbourhood” which is popular in trajectory-based methods of combinatorial optimization in order to search nearby solutions of the current solution and avoid complete enumeration. It also uses a “Tabu List” as a short-term memory of prohibited neighbouring solutions to which the search is prevented from moving to. The program is coded in Microsoft Visual Studio 2017 with Visual C++.

## Input

The basic input of the HITS program is 4 text files that contain the required information. Assume that we have the following dataset in a file called X.txt:

1	11.077	2	0	6	4.1	16.81	1
2	11.835	2	0	6	4.4	19.36	1
3	11.225	2	0	4	4.6	21.16	1
4	11.835	2	0	6	4.4	19.36	1
5	11.835	2	0	6	4.4	19.36	1
...							
2335	11.835	2	0	6	5.6	31.36	1
2336	11.352	2	0	4	4.7	22.09	1
2337	10.510	0	0	6	3.0	9.00	1
2338	11.222	0	0	17	5.4	29.16	1
2339	10.964	0	0	17	3.9	15.21	1

Also, assume the response variable saved in file y.txt as follows:

1	-1
2	-1
3	-1
4	-1
5	-1
...	
2335	1
2336	-1
2337	1
2338	1
2339	1

Then, the auxiliary text files to complete the estimation problem formulation have the following form:

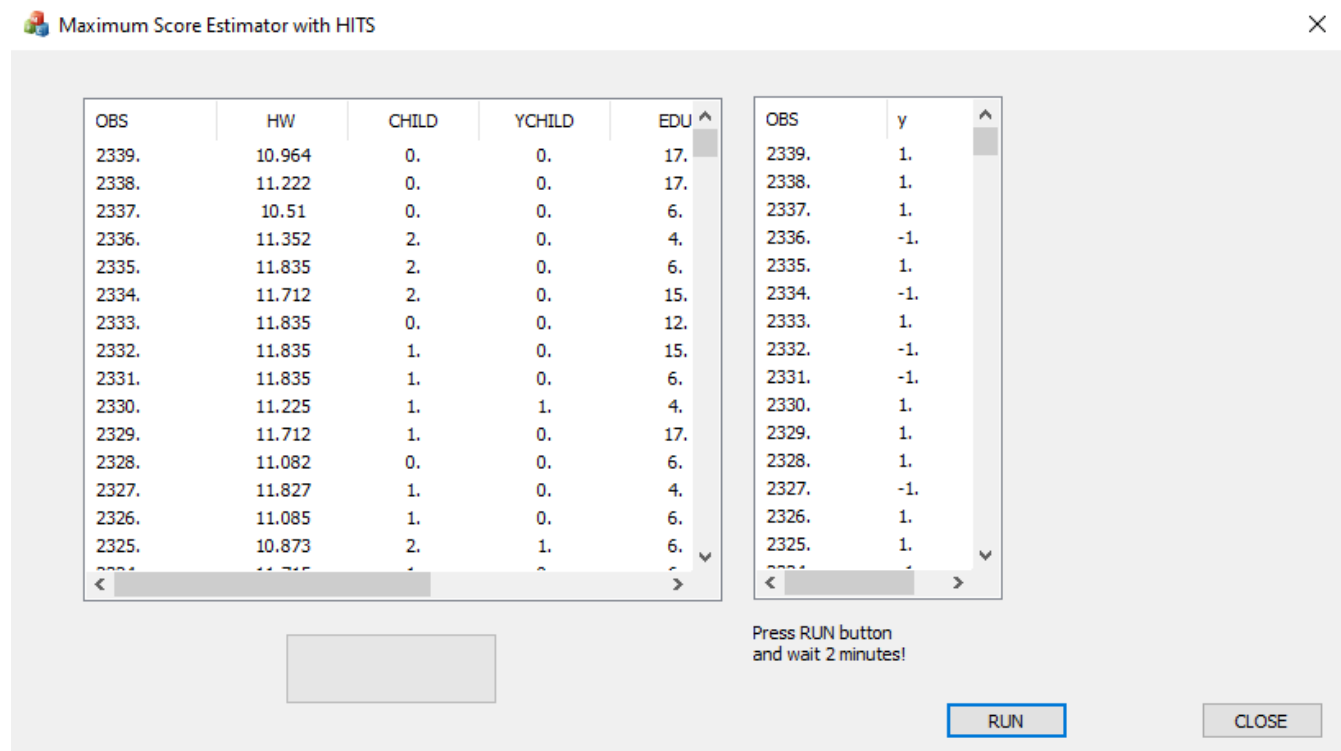
First the names.txt file looks like:

```
OBS HW CHILD YCHILD EDU AGE AGE2 INTCPT
```

Second the params.txt file looks like:

```
n = 2339
p = 7
b0 = -1
Nreps = 10
d = 1e300
//ntenure = 50
//niterovermax = 50
//niterover = 200
```

The user is welcome to double click the executable application DisplayGrid-MFC. The following screen is displayed with the estimation data already preloaded from the text files:



The observations of the dataset are presented in reverse order, counting from the last one (No. 2339) to the first one (No. 1, truncated in the Figure).

The required parameters for HITS program are five:

1. “n”: the number of rows in the loaded dataset or the number of observations in the regression.

2. **"p"**: the number of columns in the loaded dataset or the number of independent variables in the regression.
3. **"b0"**: is set to either 1 or -1 and is the coefficient of the first independent variable. This is done for identification and because the MS is identified up-to-scale. Multiplying the regression coefficients by any positive constant number provides the same objective function value. Prior knowledge can be used to select 1 or -1, or in general both values can be tried, and the regression will be run twice.
4. **"Nreps"**: the number of randomized independent runs of the tabu search algorithm. This is done to avoid getting trapped in local maxima using a single run.
5. **"d"**: the upper bound for the absolute value of each regression coefficient. It can be set to an arbitrary large value to allow unconstrained maximization. Alternatively, it can be set to a stricter value (say 1e5) to put bound constraints on the regression coefficients. The parameter d applied to each regression coefficient.

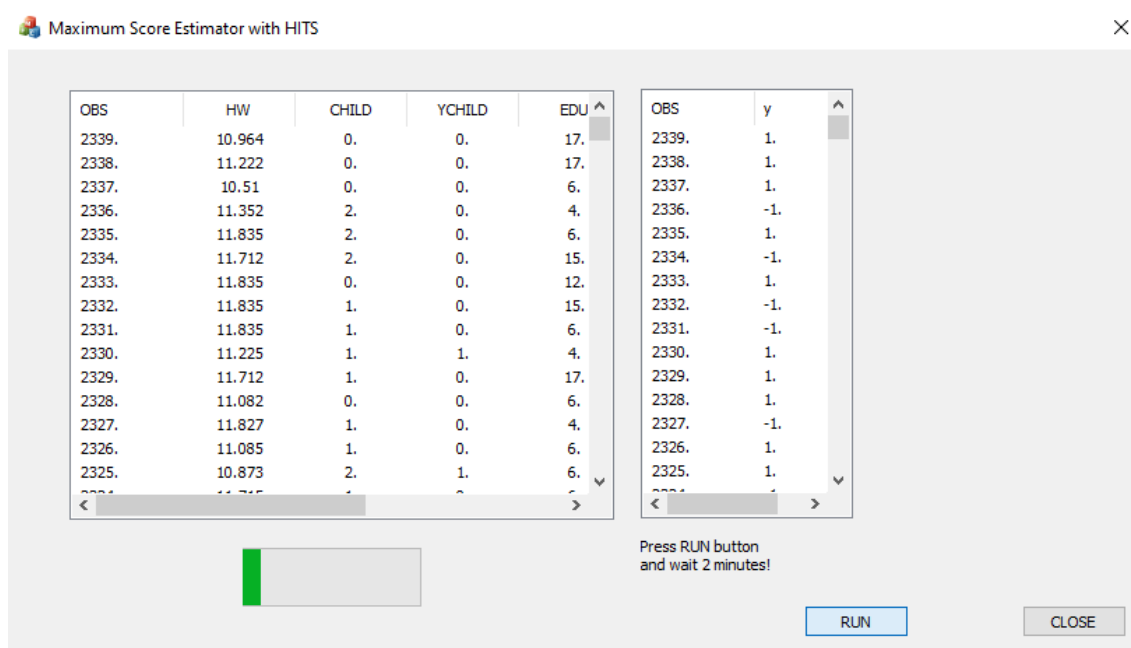
The selection of parameters depends on the size and nature of the problem. In general, the solution time increases almost proportionally with the Nreps. A rule of thumb is to keep the Nreps between 3 and 20. For an extremely large estimation problem (with  $N > 20,000$  and  $p > 10$ ) the user can specify  $Nreps=1$  and see what the program HITS provides as a solution of a single heavy run. The HITS software has currently be compiled with limits:  $Tmax=50,000$ ,  $pmax=20$ . Nevertheless, the user can set different limits and recompile the program for larger estimation problems.

Last, there are 3 secondary parameters, that are hard coded like:  $ntenure=50$ ,  $niterover=200$ ,  $niterovermax=50$ . (In the code the variable  $niterovermax$  is called  $niterovernotimproved$ ). If the user wishes, they can change these default values and recompile. These values work well for standard problems with  $N < 20,000$  and  $p < 10$ .

## Running

During the run the user can see:

1. The progress bar filling in (proportionally to: run i over Nreps, the total number of runs).



2. The log file: DemoExample\_Cpp\_tabusearch\_opt\_log\_iter200.txt with the trajectory of the search.

```
DemoExample_Cpp_tabusearch_opt_log_iter200 - Notepad
File Edit Format View Help
this is Discrete tabu search algorithm
for maximum score estimation
=====1=====
Initial score is 0.000000
Initial attributes: 452 819 1122 1318 1368 1891
MaxScore = 0.557931 dzmax = 0.557931 Max Score Ever = 0.557931
MaxScore = 0.667379 dzmax = 0.109448 Max Score Ever = 0.667379
MaxScore = 0.667379 dzmax = 0.000000 Max Score Ever = 0.667379
MaxScore = 0.682343 dzmax = 0.014964 Max Score Ever = 0.682343
MaxScore = 0.687046 dzmax = 0.004703 Max Score Ever = 0.687046
MaxScore = 0.686618 dzmax = -0.000428 Max Score Ever = 0.687046
MaxScore = 0.690038 dzmax = 0.003420 Max Score Ever = 0.690038
MaxScore = 0.690466 dzmax = 0.000428 Max Score Ever = 0.690466
MaxScore = 0.690894 dzmax = 0.000428 Max Score Ever = 0.690894
MaxScore = 0.691321 dzmax = 0.000428 Max Score Ever = 0.691321
MaxScore = 0.690894 dzmax = -0.000428 Max Score Ever = 0.691321
MaxScore = 0.691321 dzmax = 0.000428 Max Score Ever = 0.691321
MaxScore = 0.691321 dzmax = 0.000000 Max Score Ever = 0.691321
MaxScore = 0.691321 dzmax = 0.000000 Max Score Ever = 0.691321
MaxScore = 0.691749 dzmax = 0.000428 Max Score Ever = 0.691749
MaxScore = 0.691749 dzmax = 0.000000 Max Score Ever = 0.691749
MaxScore = 0.691749 dzmax = 0.000000 Max Score Ever = 0.691749
MaxScore = 0.692176 dzmax = 0.000428 Max Score Ever = 0.692176
MaxScore = 0.692176 dzmax = 0.000000 Max Score Ever = 0.692176
MaxScore = 0.692176 dzmax = 0.000000 Max Score Ever = 0.692176
MaxScore = 0.692176 dzmax = 0.000000 Max Score Ever = 0.692176
MaxScore = 0.692176 dzmax = 0.000000 Max Score Ever = 0.692176
MaxScore = 0.692176 dzmax = 0.000000 Max Score Ever = 0.692176
Ln 1, Col 1 100% Windows (CRLF) UTF-8
```

3. The info file: DemoExample\_Cpp\_tabusearch\_opt\_info\_iter200 .txt with advanced debug information.

```
DemoExample_Cpp_tabusearch_opt_info_iter200 - Notepad
File Edit Format View Help
this is Discrete tabu search algorithm
for maximum score estimation
=====1=====
Initial score is 0.000000
Initial attributes: 452 819 1122 1318 1368 1891
14004 neighbourhood moves checked for the current steps
840 better domains than current solution
out of bounds domains rejected
Elapsed for iteration 1 is 152.000000 ms
MaxScore = 0.557931 dzmax = 0.557931 Max Score Ever = 0.557931
Attributes are 1005 819 1122 1318 1368 1891
14004 neighbourhood moves checked for the current steps
3512 better domains than current solution
out of bounds domains rejected
Elapsed for iteration 2 is 531.000000 ms
MaxScore = 0.667379 dzmax = 0.109448 Max Score Ever = 0.667379
Attributes are 819 1005 1122 642 1368 1891
14004 neighbourhood moves checked for the current steps
0 better domains than current solution
out of bounds domains rejected
Elapsed for iteration 3 is 706.000000 ms
MaxScore = 0.667379 dzmax = 0.000000 Max Score Ever = 0.667379
Attributes are 642 166 1005 1122 1368 1891
14004 neighbourhood moves checked for the current steps
677 better domains than current solution
out of bounds domains rejected
Elapsed for iteration 4 is 1040.000000 ms
MaxScore = 0.682343 dzmax = 0.014964 Max Score Ever = 0.682343
Ln 1, Col 1 100% Windows (CRLF) UTF-8
```

## Output

The basic output of HITS is a text file with name DemoExample\_Cpp\_tabusearch\_opt\_log\_iter200.txt that contains all the information:

```
this is Discrete tabu search algorithm
for maximum score estimation
=====1=====
Initial score is 0.000000
Initial attributes: 452 819 1122 1318 1368 1891
MaxScore = 0.557931 dzmax = 0.557931 Max Score Ever = 0.557931
MaxScore = 0.667379 dzmax = 0.109448 Max Score Ever = 0.667379
MaxScore = 0.667379 dzmax = 0.000000 Max Score Ever = 0.667379
MaxScore = 0.682343 dzmax = 0.014964 Max Score Ever = 0.682343
MaxScore = 0.687046 dzmax = 0.004703 Max Score Ever = 0.687046
...
MaxScore = 0.692604 dzmax = 0.000000 Max Score Ever = 0.693031
MaxScore = 0.692176 dzmax = -0.000428 Max Score Ever = 0.693031
MaxScore = 0.692604 dzmax = 0.000428 Max Score Ever = 0.693031
MaxScore = 0.692604 dzmax = 0.000000 Max Score Ever = 0.693031
MaxScore = 0.691749 dzmax = -0.000855 Max Score Ever = 0.693031
Argmax is -1.443236 -0.043792 0.598771 11.926891 -1.653683 -9.622923
Max Score Ever is 0.693031
MaxScore at Argmax is 0.693031
coefficient of variable 0 b(0) is -1.000000
attributes are 71 317 1291 1651 892 2016
ntenure= 50 niterover = 200 niternotimproved = 50
Time of computation was 76.170000 seconds
=====2=====
```

Then moving on to independent run no. 2 in the same file

```
this is Discrete tabu search algorithm
for maximum score estimation
...
=====2=====
Initial score is 0.000000
Initial attributes: 407 1201 1661 1746 1924 2009
MaxScore = 0.647713 dzmax = 0.647713 Max Score Ever = 0.647713
MaxScore = 0.669517 dzmax = 0.021804 Max Score Ever = 0.669517
MaxScore = 0.669089 dzmax = -0.000428 Max Score Ever = 0.669517
MaxScore = 0.681915 dzmax = 0.012826 Max Score Ever = 0.681915
MaxScore = 0.684481 dzmax = 0.002565 Max Score Ever = 0.684481
...
MaxScore = 0.693886 dzmax = -0.000428 Max Score Ever = 0.694314
MaxScore = 0.693886 dzmax = 0.000000 Max Score Ever = 0.694314
```

```

MaxScore = 0.694314 dzmax = 0.000428 Max Score Ever = 0.694314
MaxScore = 0.694314 dzmax = 0.000000 Max Score Ever = 0.694314
MaxScore = 0.694314 dzmax = 0.000000 Max Score Ever = 0.694314
Argmax is -0.764674 -1.187283 0.670576 7.147826 -1.077536 -1.618442
Max Score Ever is 0.694314
MaxScore at Argmax is 0.694314
coefficient of variable 0 b(0) is -1.000000
attributes are 237 918 1094 1328 1360 2016
ntenure= 50 niterover = 200 niternotimproved = 50
Time of computation was 76.980000 seconds
=====3=====

```

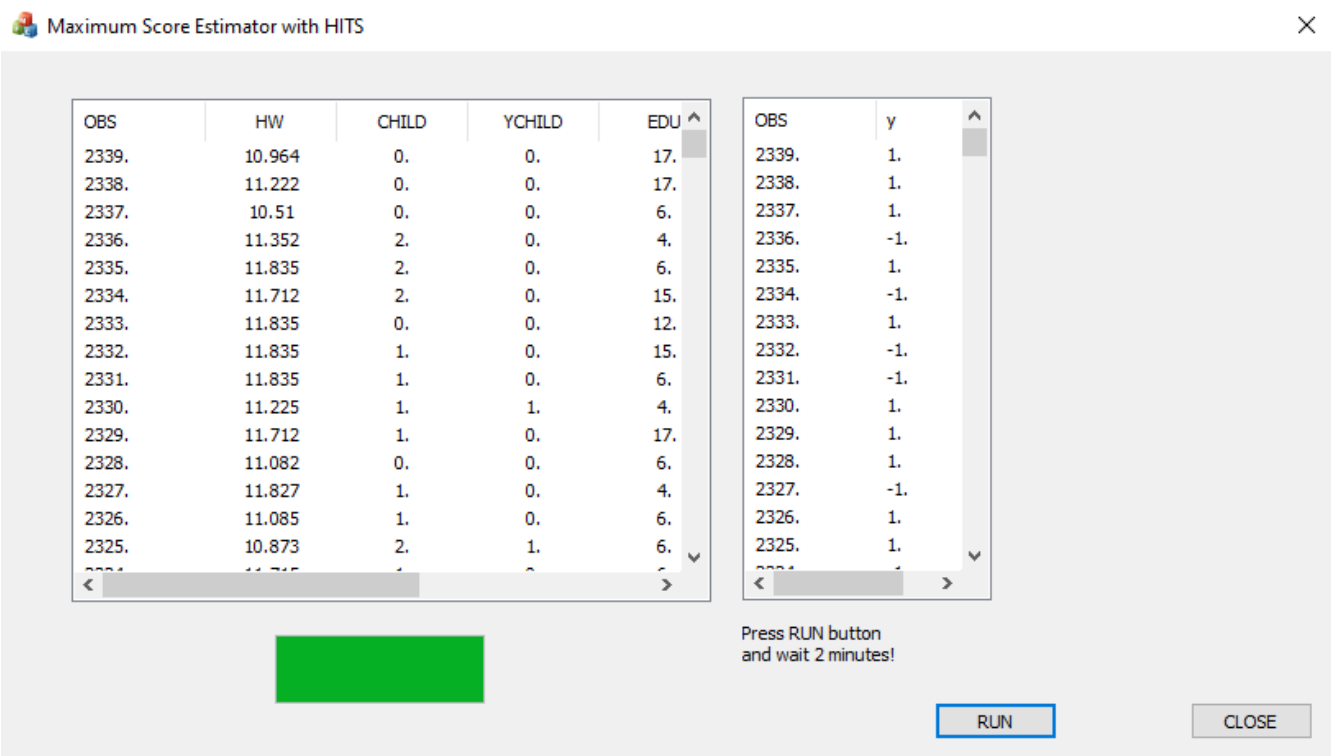
Up to finally the independent run no. 10 (Nreps was set to 10):

```

this is Discrete tabu search algorithm
for maximum score estimation
...
=====10=====
Initial score is 0.617785
Initial attributes: 20 638 645 1375 1616 2149
MaxScore = 0.640017 dzmax = 0.022232 Max Score Ever = 0.640017
MaxScore = 0.649423 dzmax = 0.009406 Max Score Ever = 0.649423
MaxScore = 0.657973 dzmax = 0.008551 Max Score Ever = 0.657973
MaxScore = 0.662676 dzmax = 0.004703 Max Score Ever = 0.662676
MaxScore = 0.667379 dzmax = 0.004703 Max Score Ever = 0.667379
...
MaxScore = 0.690894 dzmax = 0.000000 Max Score Ever = 0.690894
MaxScore = 0.690894 dzmax = 0.000000 Max Score Ever = 0.690894
MaxScore = 0.690894 dzmax = 0.000000 Max Score Ever = 0.690894
MaxScore = 0.689611 dzmax = -0.001283 Max Score Ever = 0.690894
MaxScore = 0.689611 dzmax = 0.000000 Max Score Ever = 0.690894
Argmax is -1.158853 0.147361 0.786379 8.194124 -1.107319 -5.667072
Max Score Ever is 0.690894
MaxScore at Argmax is 0.690894
coefficient of variable 0 b(0) is -1.000000
attributes are 9 337 616 1217 1846 2206
ntenure= 50 niterover = 200 niternotimproved = 50
Time of computation was 89.513000 seconds

```

The software at the time of inspection of results presents the following screen with the progress bar filled in 100%:



Then, the user can complete the analysis hitting CLOSE button and inspect the solution output files already discussed above.

The final result is taken by inspecting the lines

Max Score Ever is **0.694314**

MaxScore at Argmax is 0.694314

For each of the runs and manually selecting the run with the highest Max Score Ever / MaxScore at Argmax.

In our case, this is run No. 2 (and run No. 8 in a tie, not shown in this document) with objective function value 0.694314.

The coefficients are taken from the run No. 2 as

Argmax is **-0.764674 -1.187283 0.670576 7.147826 -1.077536 -1.618442**

Max Score Ever is **0.694314**

MaxScore at Argmax is 0.694314

coefficient of variable 0 b(0) is -1.000000

The first coefficient is -1.0 (we set it like that in params.txt for this run). Then the remaining coefficients are **-0.764674 -1.187283 0.670576 7.147826 -1.077536 -1.618442** as they come in the columns of X.txt. Note that in X.txt the constant term is included in the last column.

The following information is used for debugging:

attributes are 237 918 1094 1328 1360 2016

n tenure= 50 niterover = 200 niternotimproved = 50

Time of computation was 76.980000 seconds

The computation time was 76.98 seconds for run no.2 only and the attributes 237 918 1094 1328 1360 2016 are the observations id's (numbers from 1 to 2339) that define the linear system  $p \times p$  that is equivalent to the final solution of HITS.

## Acknowledgements

I would like to thank Dr. **Alexandros Louka** and Professor **Yannis Biliadis** for their valuable help and the shared ideas for the development of HITS. The development of the HITS software was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the '2nd Call for H.F.R.I. Research Projects to support Post-Doctoral Researchers' (Project Number: 902).

## References

- C. F. Manski, Maximum score estimation of the stochastic utility model of choice, *Journal of Econometrics* 3 (1975) 205–228.
- C. F. Manski, Semiparametric analysis of discrete response: Asymptotic properties of the maximum score estimator, *Journal of econometrics* 27 (3) (1985) 313–333.
- C. A. Pinkse, On the computation of semiparametric estimates in limited dependent variable models, *Journal of Econometrics* 58 (1993) 185–205.
- Glover, F., 1989. Tabu search—part i. *ORSA Journal on Computing* 1, 190–206.

## Contents of the zip file

### Programs

**DisplayGrid-MFC.exe**: GUI program for HITS method for maximum score estimation.

### Input files

**X.txt**: input file regarding the design matrix X of the regression

**y.txt**: input file regarding the target y of the regression (values -1 or 1)

**names.txt** input file regarding the headers of names of columns of X

**params.txt**: input file regarding the options of the estimation problem

**3kp250.dat**: input file including a MOMDKP with 3 objective functions, 3 constraints and 250 items

**3kp500.dat**: input file including a MOMDKP with 3 objective functions, 3 constraints and 500 items

**4kp100.dat**: input file including a MOMDKP with 4 objective functions, 4 constraints and 100 items

### Output files

**DemoExample\_Cpp\_tabusearch\_opt\_log\_iter200.txt**: main output file of Martins regression (N=2339, p=7)

**DemoExample\_Cpp\_tabusearch\_opt\_info\_iter200.txt**: extra output file of Martins regression (N=2339, p=7)



**DemoExample\_Cpp\_tabusearch\_opt\_res\_iter200.txt:** same as the \_info\_ file above (a simple backup file).

#### **Reference files**

**X\_Martins.txt:** simply a backup of X.txt

**y\_Martins.txt:** simply a backup of y.txt

**X\_Martins\_Gams .txt:** simply a backup of X.txt with first two lines suitable for GAMS input.

**y\_Martins\_Gams .txt:** simply a backup of y.txt with first line suitable for GAMS input.

#### **Information file**

**manual\_HITS.pdf:** the manual of the program