Subject 6.456: Adaptive Array Processing

Problem Set No. 1

Fall 2021

Issued: Thursdsay, September 9, 2021 **Due: October 12, 2021**

Submit your homework assignment in a pdf document via the course website.
A scanned copy of handwritten or typed work and printed figures is fine.

Problem 1.1. You have an N-element array along a 1-dimensional "wire" with uniform spacing between array elements. The speed of propagation is constant throughout the environment.

1) You can add additional array elements and you can make one of two changes. The first is that you can keep the total aperture of the array the same and decrease the uniform spacing between each element. The second possible change is that you can keep the spacing between elements that same and increase the total aperture of the array. If you want to decrease the mainlobe "beamwidth" of the array (i.e., you want to be able to resolve signals that are more closely spaced in spatial frequency (k)), which change will accomplish this? If you want to increase the maximum temporal frequency of a signal for which there is no spatial aliasing, which change will accomplish this? Explain your reasoning in each case.

2) Assume that when the speed of propagation equals c_0, the maximum temporal frequency of a signal for which there will be no spatial aliasing is ω_0. Assume also that when the speed of propagation equals c_1, the maximum temporal frequency of a signal for which there will be no spatial aliasing is ω_1, and that $c_1 > c_0$. Which of the following is true? Explain your reasoning.

   - $\omega_1 > \omega_0$
   - $\omega_1 = \omega_0$
   - $\omega_1 < \omega_0$

3) The signals propagating along the array are restricted to a finite band of wavenumbers. When the spacing between array elements is d meters, the normalized wavenumbers of the propagating signals are limited to the region greater than $-\pi/4$ radians/array element spacing and less than $\pi/2$ radians/array element spacing. What is the maximum spacing allowed between uniformly spaced array elements such that there will be no spatial aliasing (that is, the wavenumber of any received signal can be unambiguously determined)?

Problem 1.2. You have a 7-element standard linear array (spacing between elements, $d = \lambda/2$) oriented along the z-axis. The elements are number $n = 1, \ldots, 7$ from one end to another starting at the bottom element. You use a uniform weighting with the array steered to broadside (i.e., $\theta' = 90$ degrees). Plot the beampattern for the array as a function of both θ and ψ_z for the following cases:

1) All elements are fully functional.

2) The first element fails.

3) The third element fails.

4) The third, fourth, and sixth elements fail.

5) The second, forth, and sixth elements fail.

In all cases, normalize the beampattern so that the peak response equals one. Compare and explain the cause for the differences, if any, between the mainlobe width and peak sidelobe height for the following cases. If there is not difference between the mainlobe width and peak sidelobe height, explain why not.

a) Cases 1 and 2.

b) Cases 1 and 3.

c) Cases 1 and 5. If one case shows aliasing and the other does not, explain why this occurs.

d) Cases 4 and 5. If one case shows aliasing and the other does not, explain why this occurs.

Problem 1.3. *Van Trees:* Problem 2.4.6 You may assume that you are in free space and the sound speed is a constant value c.

Problem 1.4. *Van Trees:* Problem 2.7.1 (Note: In part (a) the aperture is continuous. In parts (b) and (c), plot the respective beampatterns (magnitude). In part (c), the equivalent array should also have a triangular weighting. Plot the beampatterns out to include the 3rd sidelobe away from the mainlobe on each side in each part.)

Problem 1.5. **Field Data Processing Assignment:** The goal of this assignment is giving you the opportunity to process some real data from a field experiment and interpret the results. Include an annotated copy of the matlab code that you used to process the data (put in comments to say what each section of the code is doing). The code can be written fairly compactly. My code to implement the processing for this problem is about 30 lines long.

Download the matlab file `pset1_prob6_JD176.mat` from the course website location where you found this problem set. This file contains received acoustic data and signal parameters from an experiment that took place in the ocean off the coast of the Hawaiian island of Kauai this summer. The receiving array was nominally vertical and had 24 sensors. The first element of the array is at the bottom of the array and the twenty fourth element is at the top of the array. The array was approximately 15 meters below the sea surface. The transmitter was approximately 3 km from the receiver and was 45 meters below the sea surface. Denote the elevation angle describing the direction to a source by θ and let it be measured from the positive z-axis.

The received signal in this file has already be preprocessed by modulating it down to baseband. That is, it has already been moved from begin centered at the carrier frequency (Fc) to being centered at zero. Therefore, signals that appear at a frequency f_o in this data propagated at a frequency of f_o + Fc. The frequency band of the transmitted signal was from approximately 9 kHz to 17 kHz.

The signal has also already undergone temporal matched filtering with the baseband transmitted pulse so that the signal that you see here is an estimate of the arrival structure (impulse response) of the environment between the transmitter and each sensor in the array.

The variables included in the file are:

- `bbrxsig` : a 3 dimensional matrix of time samples of the preprocessed receive signal from the sensors, each column is the time series at one sensor. The matrix contains 250 time series corresponding to receptions at different times. Each of the 250 segments of data is separated by approximately one tenth of a second.

- `Fs` : the sampling rate of the array data (Hz)

- `Fc` : the carrier frequency of the transmitted signal (Hz)

- `d` : the distance between adjacent sensors (m)
- `c` : the propagation speed of the medium (m/s)

Your objective is to generate a two-dimensional plot of the magnitude squared of the output of the beamformer for each of the 250 segments of data as a function of delay and angle. For each segment of data, you will need to take the Fourier Transform of the baseband signal as received at each sensor, implement the beamformer in the frequency domain to get an output frequency coefficient for each angle, then inverse Fourier Transform these to get an output time series as a function of angle. In this case, "time" corresponds to delay in the impulse response of the environment.

Generate your output using the pcolor command in matlab. Plot your outputs in dB. That is `20 * log10` of the absolute value of the output time series. Set the color scale (use the `caxis` commend) to cover from the peak of the output to 30 dB below this value. Label the x-axis as angle in degrees and the y-axis as delay in milliseconds. Use the `jet` colormap with 100 levels and set the shading to 'flat' (the command `shading('flat')` will do this). Use the `colorbar` command to generate a colorbar along the right side of the plot. An example of an output plot is in the file `prob6_figure.pdf` in the directory with the rest of the problem set material.

Implement your beamformer using both conventional uniform array weights and one set of weights of your choice. Choose the weights so that the peak sidelobe level of the beampattern is at least 30 dB below the peak of the mainlobe. Note on the assignment what weights you choose and briefly describe why. Briefly describe the differences that you observe in the outputs for the two different sets of array weights.

Are any of the "arrivals" in the impulse response that you observe likely to be aliased from angles other than where you observe them? If so, which ones and why? If not, why not?

You will hand in three figures from each of your two beamfomers. Use the data from segments 16, 31, and 34 to generate the figures that you hand in. The remainder of the data is provided to give you more data on which to practice and allow you to observe the variablity in and rate of change of the environment. For example, if you focus on the arrivals between about 70 and 110 degrees elevation angle and 6 and 16 milliseconds delay over the 250 data segments (approximately 25 seconds worth of data), you will see a very structured pattern of fluctuations.