FIT5226

Multi-agent System & Collective Behaviour

Wk 2: Macroscopic Deterministic Models, Contagion Processes

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This week…

- Macroscopic / Mesoscopic treatment via system dynamics
- Discrete versus continuous modelling (time and state)
- Markov Chains as a main tool for discrete models
- The importance of noise

Macroscopic Approaches

In the simplest approach we model only macroscopic properties.

- treat all agents as equal and exchangeable
- divide the population into characteristic subgroups
- only keep track of subgroup counts or population ratios

to simplify things further we often make common assumptions

- infinite population limit (this enables us to work with ratios as reals)
- well-mixed population

  - sometimes we want spatially resolved systems
    (often: grid-world)

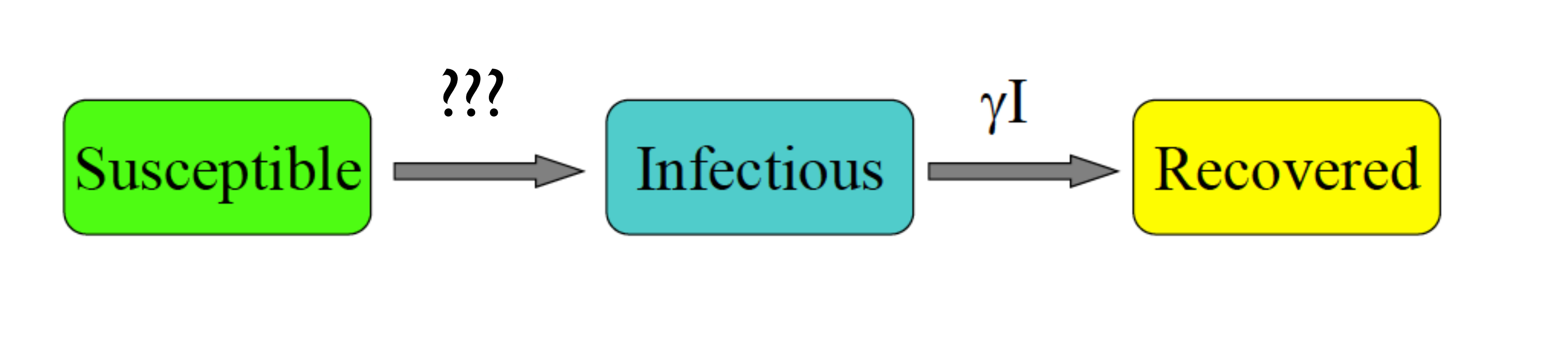
Deterministic Approaches

to simplify things even further we can look at a simplified process:
the deterministic development of the average.
(we will revise this later)

This gives rise to System Dynamics models

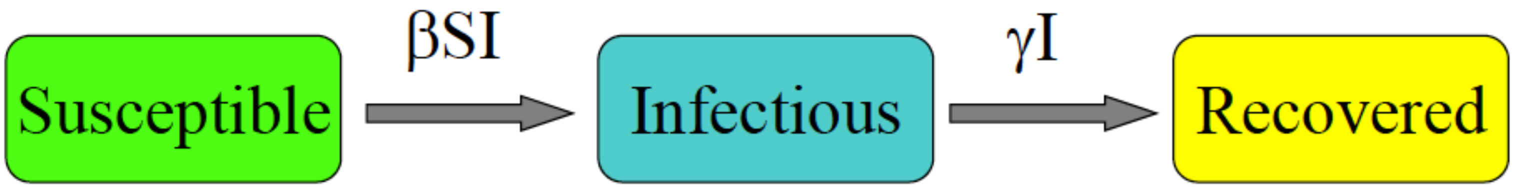
- Differential equation models
- expressed as "Box and Arrow" diagrams

Macroscopic Contagion Models SIR example



from Ecology of Disease, John Latto (UCSB)

SIR model mathematically



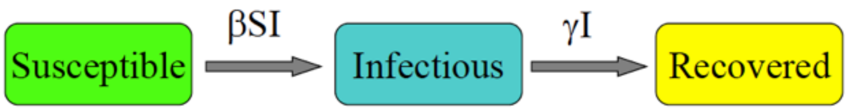
$$\frac{dS}{dt} = -\beta S I \qquad \frac{dI}{dt} = \beta S I - \gamma I \qquad \frac{dR}{dt} = \gamma I$$

from Ecology of Disease, John Latto (UCSB)

translating into a DE model means we are assuming:

- infinite population
- well-mixed population

Assumptions of the SIR model

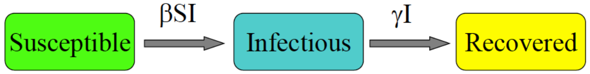


Assumptions of simple SIR model

- Population size is fixed
- No age structure
- Incubation period of disease is instantaneous
- Random mixing between susceptibles and infecteds
- Immunity is lifelong

from Ecology of Disease, John Latto (UCSB)

Peak Point of an Infectious Disease



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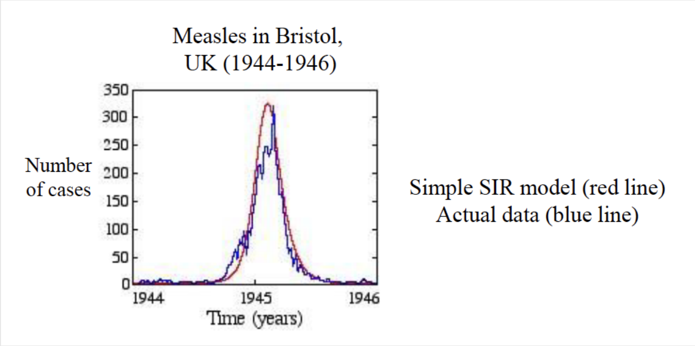


at the peak point I stops to increase and starts to decrease

$$\frac{di}{dt} = 0$$

$$\beta S i - \gamma i = 0$$

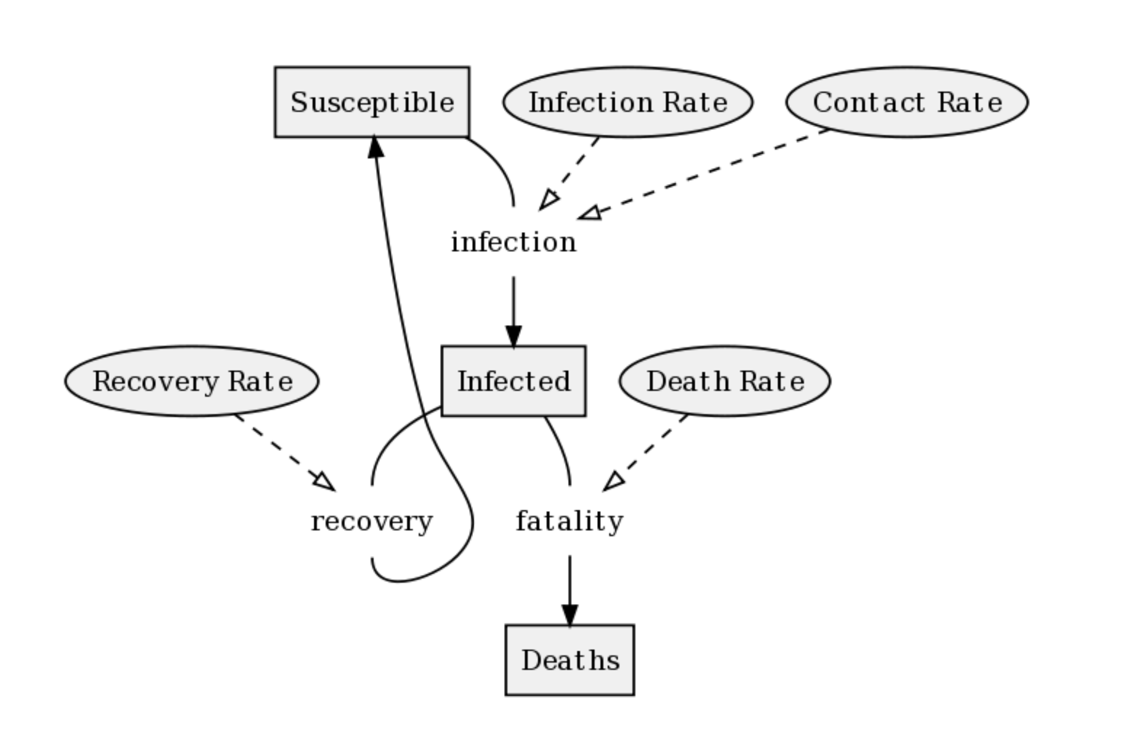
$$\beta S i = \gamma i$$

$$S = \gamma / \beta$$

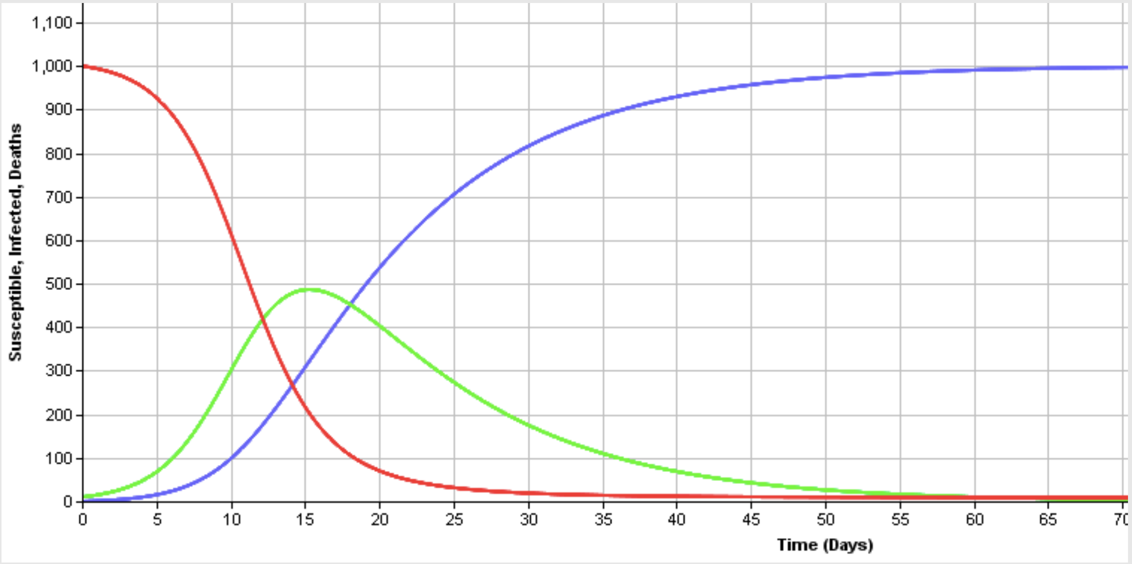
The system does not admit a general analytic solution!

The Influence of Mortality Rates

Some diseases may lead to mortality. How does this influence disease spread?



The Influence of Mortality Rates



Scenario 1:

Susceptible - Initial Value: 1000

Death Rate - Value: .1

Recovery Rate - Value: 0.001

Infection Rate - Value: 0.005

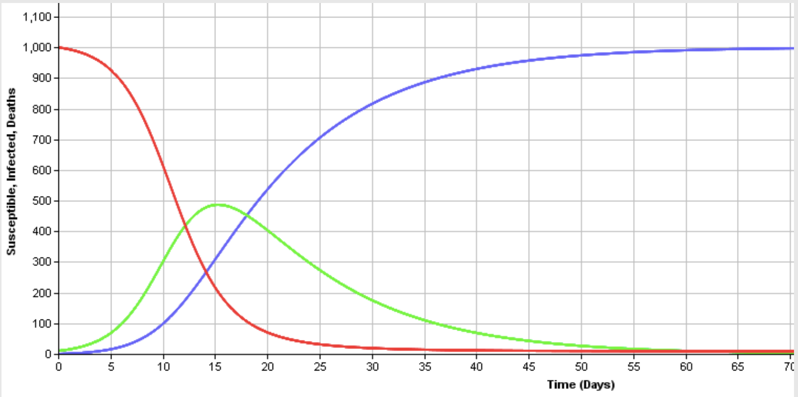
Contact Rate - Value: 0.1

red is susceptible

blue is deaths

green is infected

The Influence of Mortality Rates



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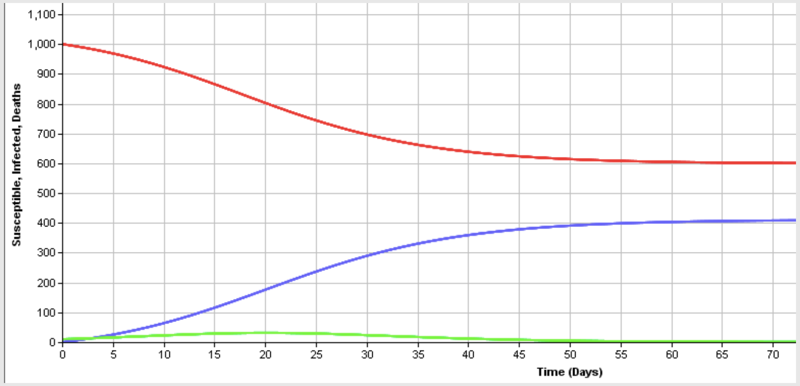
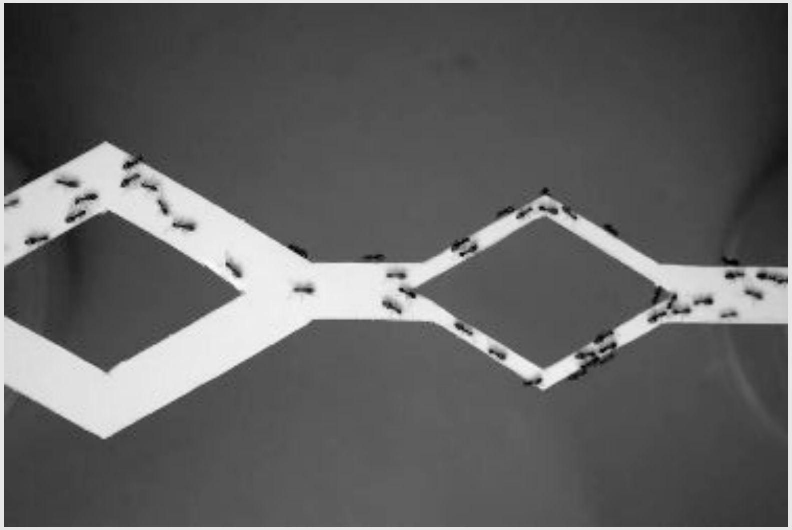


Scenario 2:
Death Rate - Value: .4
(only change)

Agents with a behavioural model
- a general decision model

Example: Ant Foraging by Mass Recruitment



- Pheromone-mediated trails
- Pheromone deposit modulated by trail quality
- Individuals follow trail probabilistically
- Pheromone evaporates over time

➡ *Self-limiting positive feedback loop*

Note that ants collectively perform a (very simple) form of reinforcement learning

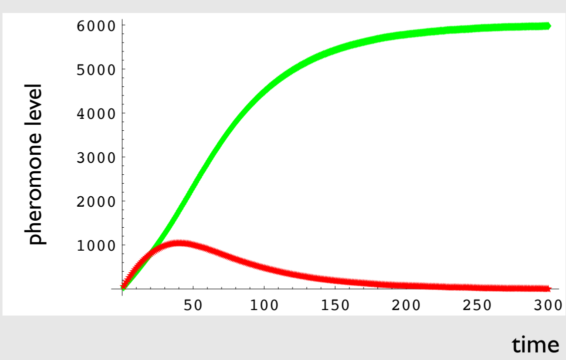
Deneubourg Standard Model

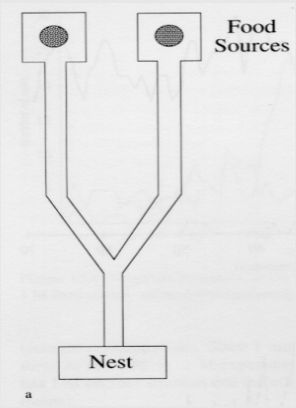
$$\frac{\partial C_i}{\partial t} = \frac{1}{l_i} \cdot \frac{f(C_i)}{f(C_1) + f(C_2)} - \rho C_i$$

evaporation

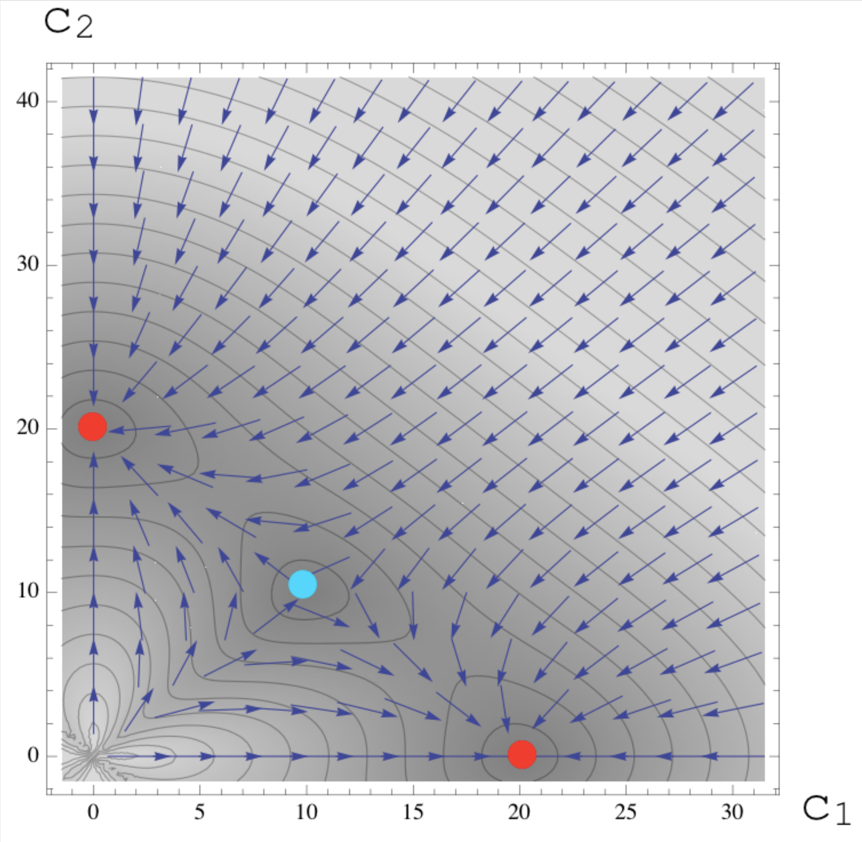
proportion of population choosing path *i*

amount of (modulated) deposit





Predictions of the Deneubourg model



$$\frac{\partial C_i}{\partial t} = \frac{1}{l_i} \cdot \frac{f(C_i)}{f(C_1) + f(C_2)} - \rho C_i$$

$$f(x) = (k + x)^{\mu}$$

Streamline plot for pheromone model

The only stable fix-points are where one of the branches carries no pheromone (and no traffic). Note that this depends on the parameters setting. It applies for many commonly used parameter ranges but not always.

A Common Binary Choice Model

$$\frac{\partial X_i}{\partial t} = f\left(X_i, X_j\right) - \rho X_i$$

with positive non-linear feedback $f(X_i, X_j)$
monotonically increasing in X_i-X_j

Similar models for human social decision making and opinion formation

- "Hits & Flops" dynamics in economic markets (Weisbuch and Stauffer, Physica A, 2000)
- Fashion Trends (Donangelo et al, Physica A, 2000)
- Adopting Innovations: The Bass Model (Capasso and Bakstein 2003)
- Recommender Systems (a la Amazon, iTunes, Google, etc)

Take home lessons

- One way to model the behaviour of large systems of interacting elements is to focus on the development of a few parameters for the whole system (often, a single one)
- Such a model can often be conveniently expressed by a system of (partial) differential equations.
- This allows us to use a vast toolbox of mathematical methods for differential equations to analyse the behaviour.
- However, in such a (O)DE model means we are only capturing the deterministic development of a parameter, ie. we are applying some form of statistical averaging between states of a system.