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Information Technology

Acknowledgements

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FIT5226

Multi-agent System & Collective Behaviour

Wk 6: Classical Game Theory (Intro)

Bernd Meyer, August 2024

Classical (Economic) Game theory

John von Neumann & Oskar Morgenstern (1944) *Theory of Games and Economic Behavior*

John von Neumann

Oskar Morgenstern

John Nash

Game Theory

History

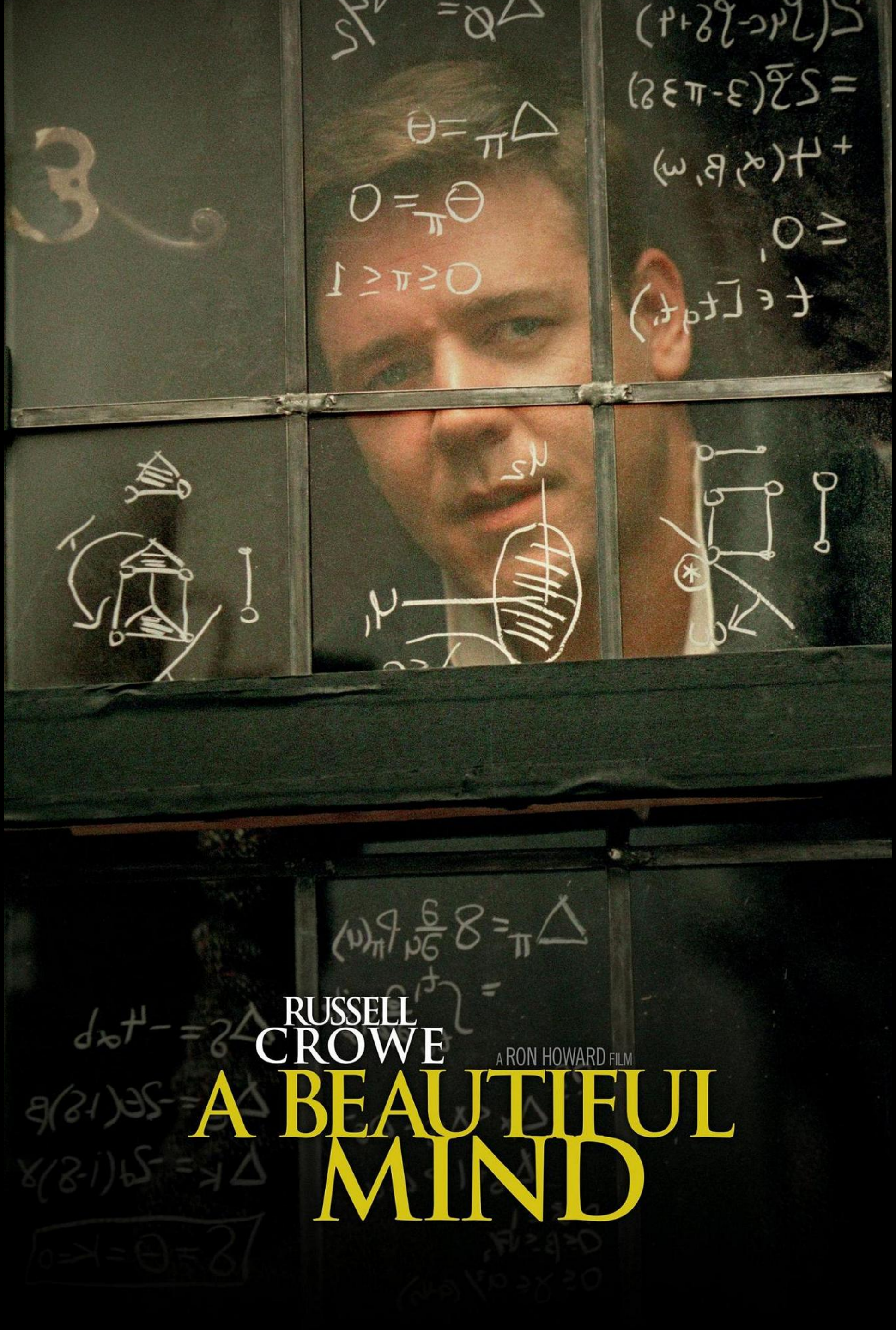
1921-1927: First approaches (E. Borel)

1928: J. von Neumann: “Zur Theorie der Gesellschaftsspiele”
in Mathematische Annalen Vol. 100

1944: J. von Neumann and O. Morgenstern:
“Theory of Games and Economical Behaviour”,
Princeton University Press
The defining Volume

1950: PhD John Nash (27 pages!)

Original Interest: Economy and Warfare --- Strategy selection


RUSSELL
CROWE
A RON HOWARD FILM
A BEAUTIFUL
MIND

Non-cooperative Game Theory

Strategic interaction between rational, self-interested agents

non-cooperative means

the individual is the basic actor,

individuals pursue their own interests

cooperative or coalition game theory investigates how agents form coalitions and teams

Biological and Ecological Games

Competition for food

- **When beetles of the same size compete, they get equal shares of the food**
- **When a large beetle competes with a small beetle, the large beetle gets the majority of the food.**
- **But being large is not only positive: large beetles experience less of a fitness benefit from a given quantity of food, since some of it is diverted into maintaining their expensive metabolism**

Prisoners' Dilema

An Example of a <u>cooperative</u>, <u>non-zero sum game</u>

Situation: The police arrests two suspects, but has only little evidence. Both suspects are imprisoned in isolation and offered a "deal". If either acts as witness against the other one he is offered a reduced sentence.

- If both remain silent (cooperate with each other), they gain a little.
- If both talk, both are punished harder as if both remain silent.
- If A "talks" (=defects) and B remains silent (=cooperates with A), A gains most at the expense of B.

	Prisoner B stays silent (*cooperates*)	**Prisoner B betrays (*defects*)**
Prisoner A stays silent (*cooperates*)	Each serves 1 year	Prisoner A: 3 years Prisoner B: goes free
Prisoner A betrays (*defects*)	Prisoner A: goes free Prisoner B: 3 years	Each serves 2 years

Dilemma: If either player reacts "fully rationally" he/she would always defect (talk). However, the "irrational" hope is that the other one will also cooperate.

Such games become more even more interesting if they are iterated, so that A/B can gain information about B/A's strategy. (Without it being known when the game ends!)

A Biological Prisoner's Dilemma

Self-interested Agents: Rationality

Each Agent

- has their own, independent concept of how good an outcome is
- and chooses their actions according to these target states
- this does *not* necessarily means that
  the agent is selfish or acts against others
- agents **independently maximise their own 'utility'**
  ("rationality")

Example: friends and enemies

Alice choses her destination for the evening
She can go to to club, the movies, or stay home and watch netflix

her **utility** for these *by herself* is:

club: 100, movies: 50; home: 50

Alice dislikes Bob but likes Carol.
If she goes out, she might bump into either

Bob is at the club 60% of the time, and at the movies otherwise
Carol is at the movies 75% of the time, and at the club otherwise

If Alice runs into Bob at the movies or the club, her utility is only 10
If Alice sees Carol, her utility increases by a factor 1.5

$A = c$

	$B = c$ (60%)	$B = m$ (40%)
$C = c$ (25%)	15	150
$C = m$ (75%)	10	100

$A = m$

	$B = c$	$B = m$
$C = c$	50	10
$C = m$	75	15

Alice's **expected** utility for home (h): 50

Alice's expected utility for club (c):
$0.25(0.6 \cdot 15 + 0.4 \cdot 150) + 0.75(0.6 \cdot 10 + 0.4 \cdot 100) = 51.75.$
Alice's expected utility for movies (m):
$0.25(0.6 \cdot 50 + 0.4 \cdot 10) + 0.75(0.6(75) + 0.4(15)) = 46.75.$

A prefers the club (though Bob is likely to be there and Carol is not), she prefers staying home to the movies (though Carol almost always is there).

Utility Theory

Is it sensible (sufficient) to describe an agent's preferences as a utility (numerical) function?

Start from the comparison of only two outcomes by an agent

O1 ~ O2:	no preference
O1 >= O2:	weak preference "O1 is at least as desirable as O2"
O1 > O2:	strict preference

Lottery

Define a lottery that assigns probabilities to random outcomes

$$L = [p_1 : o_1, p_2 : o_2, \ \dots, p_n : o_n]$$

We can now consider a whole lottery as an outcome

Axioms 1/2

1. Completeness: a preference is defined between any two outcomes
2. Transitivity: $o_1 \succ o_2 \wedge o_2 \succ o_3 \implies o_1 \succ o_3$
3. Monotonicity:

   $o_1 \succ o_2 \wedge p > q \implies [p : o_1,\ 1 - p : o_2] \succ [q : o_1,\ 1 - q : o_2]$
4. Decomposability

   Let $p(L_i, o_j)$ denote the outcome assigned to o_j in lottery L_i

   $\forall j : p(L_1, o_j) = p(L_2, o_j)\,. \implies .L_1 \sim L_2$

Axioms 2/2

5. Substitutability:

$$o_1 \sim o_2 \implies$$
$$[p : o_1, p_3 : o_3, \ \dots \ , p_k : o_k] \sim [p : o_2, p_3 : o_3, \ \dots \ , p_k : o_k]$$

6. Continuity:

$$o_1 \succ o_2 \wedge o_2 \succ o_3 \implies \exists p : o_2 \sim [p : o_1, \ 1 - p : o_3]$$

Utility Theorem

von Neuman & Morgenstern (1944):

if a preference relation satisfies the above axioms, there exists a function $u : O \rightarrow [0,1]$ such that

1. $u(o_1) \geq u(o_2) \Leftrightarrow o_1 \succ o_2$
2. the agent prefers outcomes that maximise the expected value of u

Example Games: pure competition

- Players have exactly opposing interests
- Only two players (otherwise, what does exactly opposing mean?)
- Let u_i be the utility for Player i for a combination of actions ("action profile") a

$$\exists c \forall a \in A : u_1(a) + u_2(a) = c$$

Two-person Zero-sum games

Two-person Zero-sum games are the best understood form of games
These games are a special case of pure competition

Two players take turns in making a choice from a set of possibilities

Three possible outcomes:

- draw (no one wins anything)
- player A wins $x which are paid by player B
- player B wins $y which are paid by player A

Matching Pennies

Each player shows one side of a coin.
To win, one player must match, the other mismatch

	H	T
H	1	-1
T	-1	1

Rock-Paper-Scissors

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

Rock-Paper-Scissors

2023 RPS TOURNAMENT OF CHAMPIONS

for Charity (Cancer Council)

Morra

Each Player hides one or two coins.

- each player guesses how many coins the opponent has hidden.
- There is a win if and only if one player guesses correctly, the other one incorrectly.
- In case of a win an amount equal to the total number of hidden coins are paid to the winner.

Morra Payoff Matrix

<u>Strategies</u> for both players:

(1/1) Hide one coin, guess "one coin"
(1/2) Hide one coin, guess "two coins"
(2/1) Hide two coins, guess "one coin"
(2/2) Hide two coins, guess "two coins"

	(1/1)	(1/2)	(2/1)	(2/2)
(1/1)	0	2	–3	0
(1/2)	–2	0	0	3
(2/1)	3	0	0	–4
(2/2)	0	–3	4	0

As this is zero-sum, a single matrix is enough to denote the outcomes (instead of a bi-matrix notation)

We speak of the "column player" and the "row player".

- the pay-off is given for the row player (by convention)
- the row player tries to maximize the outcome
- column player tries to minimize the outcome

Defining Games

Finite, n-person game: $\langle N, A, u \rangle$:

- N is a finite set of n players, indexed by i
- $A = A_1 \times \ldots \times A_n$, where A_i is the action set for player i
  - $a \in A$ is an action profile, and so A is the space of action profiles
- $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

Writing a 2-player game as a matrix:

- row player is player 1, column player is player 2
- rows are actions $a \in A_1$, columns are $a' \in A_2$
- cells are outcomes, written as a tuple of utility values for each player

GT - RL Terminology

Game Theory	**Reinforcement Learning**
Player	Agent
Action	Action
Payoff	Reward
Strategy	Policy
Action profile	Joint action

GT utility/payoff vs RL Reward

- much like the reward in RL, payoff in GT
  - is a scalar value,
  - indicates how well agent is performing
  - shares the underlying assumptions of utility theory

- However, differences are that RL reward
  - is typically <u>cumulative</u> over time
  - depends on action <u>and environment state</u>
    - <u>in RL players' actions influence the environment state</u>
  - <u>can be delayed</u>
    - eg. investing before cashing in later
  - <u>in GT the reward is instantaneous</u>

More Example Games

Should you send your packets using correctly-implemented **TCP** (which has a "backoff" mechanism) or use a defective implementation (which doesn't)?

Consider this situation as a two-player game:

- both use a correct implementation: both get 1 ms delay
- one correct, one defective: 4 ms delay for correct, 0 ms for defective
- both defective: both get a 3 ms delay.

TCP Backoff Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

Consider this situation as a two-player game:

- both use a correct implementation: both get 1 ms delay
- one correct, one defective: 4 ms delay for correct, 0 ms for defective
- both defective: both get a 3 ms delay.

<u>Note</u>: the bimatrix notation gives the direct payoff, so both players try to maximise

General From: Prisoners' dilemma

	C	D
C	a, a	b, c
D	c, b	d, d

- any such game with $c > a > d > b$

Traffic Game

	L	R
L	*1,1*	*0,0*
R	*0,0*	*1,1*

- which side of the road do you drive on?

Cooperation Games

	C	D
C	a, a	b, c
D	c, b	d, d

- players have aligned interests,
- no conflict
- $\forall a \in A, \forall i, j, u_i(a) = u_j(a)$

- this is still <u>not</u> cooperative game theory

Battle of the Sexes

	B	F
B	*2, 1*	*0, 0*
F	*0, 0*	*1, 2*

- combines elements of co-operation and of competition

Take home lessons

- Game Theory provides a suitable tool to analyse the interactions of multiple agents
- The basis of game theory is the assumption of rational behaviour
- “Rational” is defined by certain assumptions about reasoning and behaviour of the players, including this of utility theory (these are not based in psychology but logical assumption)
- Classical game theory only analyses outcomes of games, it does not talk about how players “learn” to play a game
- Part of the GT toolbox is a “zoo” of paradigmatic games that encapsulate typical scenarios (for example, pure competition, coordination, mixes of these, conflicts between individual advantage and good group outcomes etc).
- Two-person zero-sum games model the simplest form of direct and pure competition