Lab - Week 9

Classical Game Theory (Nash Equilibrium)

The purpose of this lab is to give you practice with determining outcomes of normal form games for pure strategies.

1. Dominated Strategies

Tasks

3.1 Define formally what a dominated strategy is.

3.2 Explain why a dominated strategy is an important concept.

3.3 Find out what the difference is between strictly dominated and weakly dominated. Explain!

3.4 We have defined the game *Morra* in the lecture. Which strategies are dominated in *Morra*? For this, recall that the payoff matrix for (our version of *Morra*) is:

$$P = \left(\begin{array}{c|c|c|c} 0 & 2 & -3 & 0 \\ \hline -2 & 0 & 0 & 3 \\ \hline 3 & 0 & 0 & -4 \\ \hline 0 & -3 & 4 & 0 \end{array}\right);$$

3.5 For each of the bi-matrix games below, name the dominated strategies. Which of the dominated strategies are weakly dominated, which strictly?

$$A = \left(\begin{array}{c|c} 2 & 1 \\ \hline 1 & 1 \end{array}\right); \quad B = \left(\begin{array}{c|c} 1 & 1 \\ \hline 1 & 3 \end{array}\right);$$

$$A = \left(\begin{array}{c|c|c|c} 2 & 1 & 3 & 17 \\ \hline 27 & 3 & 1 & 1 \\ \hline 4 & 6 & 7 & 18 \end{array}\right); \quad B = \left(\begin{array}{c|c|c|c} 11 & 9 & 10 & 22 \\ \hline 0 & 1 & 1 & 0 \\ \hline 2 & 10 & 12 & 0 \end{array}\right)$$

$$A = \left(\begin{array}{c|c|c} 3 & 3 & 2 \\ \hline 2 & 1 & 3 \end{array}\right); \quad B = \left(\begin{array}{c|c|c} 2 & 1 & 3 \\ \hline 2 & 3 & 2 \end{array}\right);$$

3.6 [optional - *at home unless time permits*] Write a (Python) program to automatically determine the pareto optimal outcomes for a bi-matrix game.

2. Nash Equilibrium

Nash Equilibrium in Pure Strategies

A pure Nash Equilibrium contains only one strategy of each player.

Tasks

4.1 Does the game in bi-matrix notation below have a pure Nash equilibrium? Give the equilibrium strategy profile or show why there is no pure Nash equilibrium.

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 4 & 2 \\ 3 & 6 \end{pmatrix};$$

4.2 Does the game from 4.1 have a mixed Nash equilibrium? Give the equilibrium point or give reasons why mixed Nash does not exist.

4.3 Define a two-player zero sum game that has a pure Nash equilibrium.