Q1 $X_1, X_2, X_3 \dots X_n \longrightarrow$ independent identically distributed random variables.

cdf $= F_X(x)$

pdf $= f_X(x) = F_X'(x)$.

$P(Y_1 \le y) = P(X_1 \le y, X_2 \le y \dots X_n \le y)$

As $Y_1 = \max\{X_1, X_2 \dots X_n\}$.

$\therefore$ if Y_1 is smaller than/equal to some value, then all X_i are also smaller than/equal to that value as all X_i are smaller than/equal to Y_1.

As $X_1, X_2 \dots X_n$ are independent.

$\therefore$ ~~$P(X_1 \le y)$ F~~

$\therefore P(X_1 \le y, X_2 \le y \dots X_n \le y) = P(X_1 \le y) . P(X_2 \le y) \dots P(X_n \le y)$

$\therefore P(Y_1 \le y) = P(X_1 \le y). P(X_2 \le y) \dots P(X_n \le y)$.

$P(X_i \le y) = F_X(y) \quad \forall\ i \in \{1, 2 \dots n\}$.

$\therefore P(Y_1 \le y) = [F_X(y)]^n$

$$\boxed{F_{Y_1}(y) = [F_X(y)]^n} \longrightarrow \text{cdf of } Y_1.$$

Pdf of $Y_1 = F_{Y_1}'(y) = f_{Y_1}(y) = n . [F_X(y)]^{n-1} . f_X(y)$

$$\therefore \boxed{f_{Y_1}(y) = n . f_X(y) . [F_X(y)]^{n-1}} \longrightarrow \text{pdf of } Y_1.$$

Similarly for Y_2.

$P(Y_2 > y) = P(X_1 > y, X_2 > y \dots X_n > y)$

As $Y_2 \le X_i \quad \forall\ i \in \{1, 2 \dots n\}$.

$\therefore$ if $y < Y_2 \Rightarrow y < X_i$

As $X_1, X_2 \dots X_n$ are independent.

$\therefore P(X_1 > y, X_2 > y \dots X_n > y) = P(X_1 > y). P(X_2 > y) \dots P(X_n > y)$

$$P(X_i > y) = 1 - P(X_i \le y)$$

$$P(X_i > y) = 1 - F_X(y) \quad \forall \quad i \in \{1, 2, \ldots n\}.$$

$$\therefore P(Y_2 > y) = [1 - F_X(y)]^n$$

$$1 - P(Y_2 \le y) = [1 - F_X(y)]^n$$

$$\therefore \boxed{F_{Y_2}(y) = 1 - [1 - F_X(y)]^n} \to \text{cdf of } Y_2.$$

$$F'_{Y_2}(y) = f_{Y_2}(y) = (-n)[1 - F_X(y)]^{n-1}(-f_X(y))$$

$$\boxed{f_{Y_2}(y) = n \cdot f_X(y) \cdot [1 - F_X(y)]^{n-1}} \to \text{pdf of } Y_2.$$