Question 1

(a) Total number of ways in which people can pick books $= \underbrace{n}_{\text{first person's choices}} \cdot \underbrace{(n-1)}_{\text{second "}} \cdot (n-2) \cdots 2 \cdot 1$

$= n!$

Number of cases when every person picks his/her own book. $= 1$

$\therefore$ $\boxed{\text{Probability} = \frac{1}{n!}}$

(b) Number of ways, the first m people pick the books $= 1$

Number of ways in which the next $(n-m)$ people pick books $= (n-m)!$

$\therefore$ $\boxed{\text{Probability} = \frac{(n-m)!}{n!}}$

(c) Number of ways the first m people pick the books $= m!$

Number of ways the next $(n-m)$ people pick the ~~ways =~~ books $= (n-m)!$

$\boxed{\text{Probability} = \frac{((n-m)!)(m!)}{n!}}$

(d) For any person, the probability of picking a clean book is $(1-p)$.

$\therefore$ If the first m people pick clean books.

$\therefore$ $\boxed{\text{Probability} = (1-p)^m}$

(e) ~~m people picking clean books.~~

(e) Probability of m people picking clean books $= (1-p)^m$

Probability of $(n-m)$ people picking unclean books $\cancel{(n-m)}^m = p^{(n-m)}$.

$\therefore$ $\boxed{\text{Probability} = {}^nC_m (1-p)^m \, p^{(n-m)}}$

~~Question 2~~

~~$|x_i - \mu| \le \sqrt{}$~~

Question 2

$$|x_i - \mu| = \sqrt{(x_i-\mu)^2} \le \sqrt{\sum_{i=1}^{N} (x_i-\mu)^2}$$

$$\sqrt{\sum_{i=1}^{n} (x_i-\mu)^2} = \frac{\sqrt{\sum_{i=1}^{N} (x_i-\mu)^2 \cdot \cancel{\ }} \cdot \sqrt{N-1}}{\sqrt{N-1}}$$

$$= \sigma\sqrt{N-1}$$

$\therefore$

$$|x_i - \mu| \le \sigma\sqrt{N-1} \quad \forall \; \cancel{i \in \mathbb{N}} \; i \in \{1, 2, \ldots N\}$$

Hence proved. :)

Comparison with Chebyshev's →

We know that Chebyshev's inequality is

$$\frac{\left|\{|x_i-\mu|>k\sigma\}\right|}{n} \leq \frac{1}{k^2}$$

& our inequality derived from question is

$$|x_i-\mu| \leq \sigma\sqrt{n-1} \quad \forall\, i \in \{1,2,\dots,n\}$$

Hence our inequality states that →

$$\frac{\left|\{|x_i-\mu| \leq \sigma\sqrt{n-1}\}\right|}{n} = 1$$

(exact 1)

$|A|$ = cardinality(A)

But from Chebyshev's, $\{\text{put } k=\sqrt{n-1}\}$

$$\frac{\left|\{|x_i-\mu| \leq \sigma\sqrt{n-1}\}\right|}{n} \geq 1-\frac{1}{\sqrt{n-1}^2} = \frac{n-2}{n-1}$$

Hence Chebyshev's gives a range of values for this $k=\sqrt{n-1}$. Hence our inequality is better as we get exact value

Also it shows the limit of use of Chebyshev, after $k \geq \sqrt{n-1}$, Chebyshev gives useless ranges when answers are exact.