# Question 4
# Handwritten Solution
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Let us define event $E_1 : \{ Q_1 < q_1 \}$
$E_2 : \{ Q_2 < q_2 \}$ and $E : \{ Q_1 Q_2 < q_1 q_2 \}$
Given : $Q_1$ and $Q_2$ and $q_1, q_2$ are non-negative
Hence for $Q_1 < q_1$ and $Q_2 < q_2$,
$Q_1 Q_2 < q_1 q_2$
So all values satisfying $Q_1 < q_1$ AND $Q_2 < q_2$ also satisfy $Q_1 Q_2 < q_1 q_2$
Now $\{ Q_1 < q_1 \text{ AND } Q_2 < q_2 \} \equiv E_1 \cap E_2$
and $\{ Q_1 Q_2 < q_1 q_2 \} \equiv E$
Hence all values belonging to $E_1 \cap E_2$ also belong to $E$
$\therefore \quad E_1 \cap E_2 \subseteq E$
$E_1 \cap E_2$ is subset of $E$.
* Now property of probability :
$A \subseteq B \implies P(A) \leq P(B)$
so $P(E_1 \cap E_2) \leq P(E)$
* Now as $Q_1, Q_2$ are independent, events $Q_1 < q_1$ and $Q_2 < q_2$ are also independent $\implies E_1, E_2$ are independent.
For any independent events $A, B$,
$$P(A \cap B) = P(A) \cdot P(B)$$

So, $P(E_1 \cap E_2) = P(E_1) . P(E_2)$

so $P(E) \geqslant P(E_1 \cap E_2) = P(E_1) . P(E_2)$

or $P(E) \geqslant P(E_1) . P(E_2)$

and given $\longrightarrow$ $P(E_1) \geqslant 1 - P_1$ and

$P(E_2) \geqslant 1 - P_2$

so $P(E_1) . P(E_2) \geqslant (1 - P_1)(1 - P_2)$

This inequality is okay as both $P(E_1), P(E_2) \geqslant 0$ are non-negative and $1 - P_1$ and $1 - P_2$ also $\geqslant 0$

Hence $P(E) \geqslant P(E_1) . P(E_2) \geqslant (1 - P_1)(1 - P_2)$

or $P(E) \geqslant (1 - P_1)(1 - P_2)$

$= 1 - P_1 - P_2 + P_1 P_2$

or $P(E) \geqslant 1 - P_1 - P_2 + P_1 P_2 \geqslant 1 - P_1 - P_2$

(as $P_1 P_2 \geqslant 0$)

so $P(E) \geqslant 1 - (P_1 + P_2)$

or $P\left( Q_1 Q_2 < q_1 q_2 \right) \geqslant 1 - (P_1 + P_2)$

Hence proved.