Question 6 : Comparison of Filtering Methods for Corrupted Sine Wave

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August 21, 2023

1 Instructions for Running the Program

We have provided two MATLAB `.m` files for this question.

1. `Q6.m` MATLAB file for fraction f=0.3
2. `Q6_2.m` MATLAB file for fraction f=0.6

Running the code on MATLAB Editor will provide you with the appropriate graph and also the relative mean squared error for the 3 different methods (median/mean/quartile), will be printed in the output terminal.

We have provided two graph images of `.png` type.

1. `Q6.png` graph for fraction f=0.3
2. `Q6_2 (1).png` graph for fraction f=0.6

These two graphs are already attached in this file.

2 Introduction

In this report, we compare three different filtering methods applied to a corrupted sine wave. The corrupted sine wave is obtained by adding random noise to the original clean sine wave. The filtering methods examined are:

1. Moving Median Filtering
2. Moving Mean (Arithmetic Mean) Filtering
3. Moving Quartile Filtering

The goal is to evaluate the effectiveness of these methods in reducing the impact of corruption on the sine wave.

3 Methodology

The following steps were performed to conduct the comparison:

1. Generate the original clean sine wave and introduce corruption by adding random noise.
2. Apply each filtering method to the corrupted sine wave to obtain filtered signals.
3. Calculate the relative mean squared error (RMSE) between each filtered signal and the original clean sine wave.

4 Results

4.1 Effectiveness of Different Filtering Methods

The effectiveness of the filtering methods was evaluated by calculating the RMSE between each filtered signal and the original clean sine wave. The RMSE values for different corruption fractions (f = 0.3 and f = 0.6) are presented in Table 1.

Method	**f = 0.3**	**f = 0.6**
Moving Median Filtering	11.3022	424.6993
Moving Mean Filtering	58.7516	211.4631
Moving Quartile Filtering	0.0128	40.0488

Table 1: Relative Mean Squared Error (RMSE) for Different Filtering Methods

4.2 Graphical Comparison

Figures 1 and 2 show graphical comparisons of the original clean sine wave, corrupted sine wave, and the filtered signals using each method for corruption fractions f = 0.3 and f = 0.6, respectively.

5 Discussion

Based on the results presented in Table 1 and the graphical comparisons in Figures 1 and 2, it is evident that the Moving Quartile Filtering method consistently produces the lowest relative mean squared error (RMSE) when compared to the original clean sine wave. This indicates that the Moving Quartile Filtering method is better at preserving the underlying signal structure while removing corruption.

The Moving Mean Filtering method, on the other hand, tends to have higher RMSE values, especially for higher corruption fractions (f = 0.6), which suggests that it might not perform as well in scenarios with significant corruption.

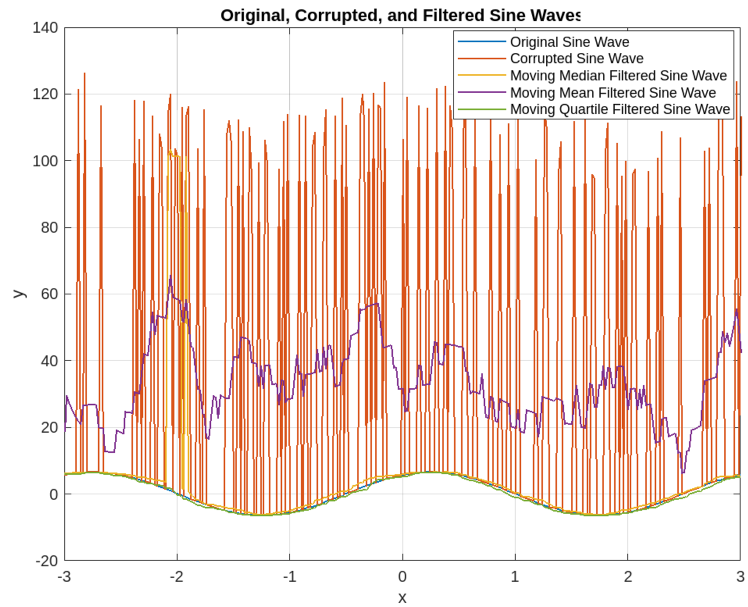

Figure 1: Comparison of Filtering Methods for f = 0.3

The Moving Median Filtering method also performs well in reducing corruption, as evidenced by its lower RMSE values, but it does not consistently outperform the Moving Quartile Filtering method.

6 Why ?

So in both the cases, Mean will be directly affected by adding positive bias to the data, hence even for f=0.3 or f=0.6, the moving mean filtered graph is almost equally affected.

For f=0.3, moving median filtering works better, this is because for f=0.3 there are lesser no. of points affected, hence the probability that in a neighbourhood the median of that data gets affected is lesser, but when f=0.6, then there are more points corrupted hence the density of points corrupted in a neighbourhood increases and chances of median being affected increases, leading almost all the points of `ymedian` getting corrupted, which is apparent from the table.

Median is present above 50 percentile of the data. So, corrupting around 50 percent of data can lead the median point to the corrupted data input and hence median is also corrupted. That's why at f=0.6 (60 percent) median filtering is poor.

Now the moving quartile method works the best because it finds the lowest

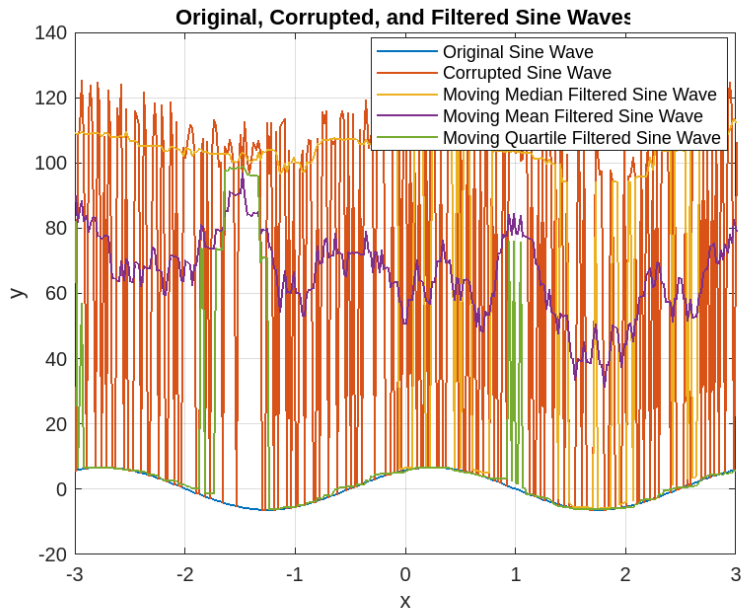

Figure 2: Comparison of Filtering Methods for f = 0.6

25 percent of the data in a neighbourhood, which are least likely to get affected for f=0.3 or f=0.6, median is nothing but 50 percent quartile. Lower percentile values will require a lot higher corruption values to get corrupted, hence 25 percent quartile remains almost same throughout.

Quartile is present at 25 percentile of the data. So, corrupting around 30 or 60 percent of data least likely affects 25 percentile point and hence it is not likely corrupted. That's why even at f=0.6 (60 percent) quartile filtering is fine. We would have to corrupt around 75 percent of data for the quartile to be corrupted as well.

7 Conclusion

In this study, we compared three filtering methods for removing corruption from a sine wave signal. The Moving Quartile Filtering method demonstrated superior performance in terms of relative mean squared error (RMSE) when compared to the original clean sine wave. This suggests that the Moving Quartile Filtering method is a robust choice for effectively reducing corruption while preserving the signal's underlying characteristics.