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Q5 By definition of conditional probability of event E given event F, is $\frac{P(E \cap F)}{P(F)}$

(a) $$P(C_i|Z_1) = \frac{P(C_i \cap Z_1)}{P(Z_1)}$$

we assume that contestant is equally likely to choose any door

$\therefore P(Z_1) = 1/3$

$\because$ C_i and Z_1 are indipedent events.

$\therefore P(C_i \cap Z_1) = P(C_i)P(Z_1) = \frac{1}{3} \cdot \frac{1}{3}$

$\Rightarrow P(C_i|Z_1) = 1/3$ for $i = 1,2,3$

(b) $$P(H_3|C_i, Z_1) = \frac{P(H_3 \cap C_i \cap Z_1)}{P(C_i \cap Z_1)}$$ ($\because$ $(C_i, Z_1) = \{C_i \cap Z_1\}$ ~~$\{C_i, Z_1\} = \{C_i \cap Z_1\}$~~)

Case I $i=1$

In this case $\because$ Car is behind door 1

$\therefore$ Host is equally likely to open door 2 or door 3.

now, we since host opens either door 2 or door 3.

$P(H_3 \cap C_1 \cap Z_1) + P(H_2 \cap C_1 \cap Z_1) = P(C_1 \cap Z_1)$ (H_3, H_2 are mutually exclusive)

$\therefore 2P(H_3 \cap C_1 \cap Z_1) = P(C_1 \cap Z_1)$

$\therefore P(H_3 \cap C_1 \cap Z_1) = \frac{P(C_1 \cap Z_1)}{2}$

$\Rightarrow P(H_3|C_1, Z_1) = 1/2$

Case II $i=2$

In this case, since Car is behind door 2, host will never open door 2.

$\therefore P(H_2 \cap C_2 \cap Z_1) = 0$

now as $P(H_3 \cap C_2 \cap Z_1) + P(H_2 \cap C_2 \cap Z_1) = P(C_2 \cap Z_1)$

$\therefore P(H_3 \cap C_2 \cap Z_1) = P(C_2 \cap Z_1)$

$\therefore P(H_3|C_2, Z_1) = 1$

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Q.5 **Case III** $i=3$

In this case since Car is behind door 3 $\therefore$ host will never open door 3.

$\therefore P(H_3 \cap C_3 \cap Z_1) = 0$

$\therefore P(H_3 | C_3, Z_1) = 0$

(c) $P(C_2 | H_3, Z_1) = \dfrac{P(C_2 \cap H_3 \cap Z_1)}{P(H_3 \cap Z_1)}$ $\left(\because \cancel{\{H_3, Z_1\}} = \{H_3 \cap Z_1\} \right)$ $(H_3, Z_1) = \{H_3 \cap Z_1\}$

now $\because C_1, C_2, C_3$ are mutually exclusive and exhaustive collection of events for car being behind any door.

$\therefore P(H_3 \cap Z_1) = P(H_3 \cap C_1 \cap Z_1) + P(H_3 \cap C_2 \cap Z_1) + P(H_3 \cap C_3 \cap Z_1)$

$= \dfrac{P(C_1 \cap Z_1)}{2} + P(C_2 \cap Z_1) + 0$ (from (b))

$= \dfrac{3}{2} P(C_2 \cap Z_1)$ $\left(\because P(C_1 \cap Z_1) = P(C_2 \cap Z_1) = 1/9 \right.$ from (a) $\left.\right)$

$\therefore P(C_2 | H_3, Z_1) = \dfrac{2}{3} \dfrac{P(C_2 \cap H_3 \cap Z_1)}{P(C_2 \cap Z_1)} = \dfrac{2}{3} \dfrac{P(C_2 \cap Z_1)}{P(C_2 \cap Z_1)} = 2/3$

(d) $P(C_1 | H_3, Z_1) = \dfrac{P(C_1 \cap H_3 \cap Z_1)}{P(H_3 \cap Z_1)} = \dfrac{P(C_1 \cap Z_1)}{2 P(H_3 \cap Z_1)} = \dfrac{1}{2} \cdot \dfrac{2}{3} \dfrac{P(C_1 \cap Z_1)}{P(C_2 \cap Z_1)} = \dfrac{1}{2} \cdot \dfrac{2}{3} \dfrac{P(C_2 \cap Z_1)}{P(C_2 \cap Z_1)} = 1/3$

(e) $\because P(C_1 | H_3, Z_1) < P(C_2 | H_3, Z_1)$

$\therefore$ There is more probability that the car is behind door 2 if contestant has chosen door 1 and host had opened door 3.

$\therefore$ The contestant must switch in order to increase his / her chances for wining.

NOTE:- This extra probability in case C_2 is due to host baisness, not to open the door with car $\therefore$ ~~sp~~ skiping some possibilities.

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Q5 (f) we start with calculating $P(H_3 \cap C_i \cap Z_1)$

$\because$ Host is equal likely to open door 2 or door 3, irrespective of where car is.

$$\therefore P(H_3 \cap C_i \cap Z_1) = P(H_2 \cap C_i \cap Z_1)$$

$\therefore$ as $P(H_3 \cap C_i \cap Z_1) + P(H_2 \cap C_i \cap Z_1) = P(C_i \cap Z_1)$

$$\therefore 2P(H_3 \cap C_i \cap Z_1) = P(C_i \cap Z_1)$$

$$\therefore P(H_3 \cap C_i \cap Z_1) = \frac{P(C_i \cap Z_1)}{2}$$

now we calculate,

$$P(C_2 | H_3, Z_1) = \frac{P(C_2 \cap H_3 \cap Z_1)}{P(H_3 \cap Z_1)}$$

$$= \frac{P(C_2 \cap Z_1)/2}{\sum_{i=1}^{3} P(H_3 \cap C_i \cap Z_1)} = \frac{P(C_2 \cap Z_1)/2}{3\,P(C_2 \cap Z_1)/2} \quad \left(\because P(C_i \cap Z_1) = 1/9,\ i = 1,2,3\right)$$

$$= 1/3$$

similarly,

$$P(C_1 | H_3, Z_1) = \frac{P(C_1 \cap H_3 \cap Z_1)}{P(H_3 \cap Z_1)} = \frac{P(C_1 \cap Z_1)/2}{3\,P(C_2 \cap Z_1)/2} = \frac{P(C_2 \cap Z_1)/2}{3P(C_2 \cap Z_1)/2} = 1/3$$

$$\because P(C_2 | H_3, Z_1) = P(C_1 | H_3, Z_1)$$

$\therefore$ There is equal probability that the car is behind door 2 or door 3 if contestant has chosen door 1 and host hand opened door 3.

$\therefore$ It is bad nor good to switch.

NOTE:- $P(C_2 | H_3, Z_1) + P(C_1 | H_3, Z_1) \neq 1$

Because $P(C_3 | H_3, Z_1) \neq 0$, i.e., the case when contestant loses if car is behind door 3.