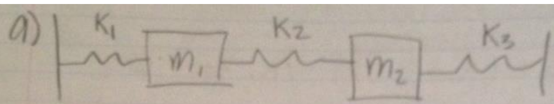
Kea Francis

ME 363 Dr. Boulet

Project Four

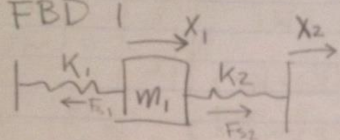
a)

a)



$\dot{X}_2 > \dot{X}_1 \Rightarrow \dot{X}_2 = 2 \quad \dot{X}_1 = 1$

FBD 1



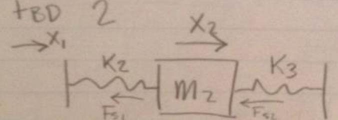
$F_{s1} = K_1 X_1$

$F_{s2} = K_2 (X_2 - X_1)$

$\Sigma F = m_1 \ddot{X}_1$

$-F_{s1} + F_{s2} = m_1 \ddot{X}_1$

FBD 2



$F_{s1} = K_2 (X_2 - X_1)$

$F_{s2} = K_3 X_2$

$\Sigma F = m_2 \ddot{X}_2$

$-F_{s1} - F_{s2} = m_2 \ddot{X}_2$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} \ldots$$

$$m_1 \ddot{X}_1 + K_1 X_1 - K_2 (X_2 - X_1) = \phi$$

$$m_2 \ddot{X}_2 + K_2 (X_2 - X_1) + K_3 X_2 = \phi$$

$$\Downarrow$$

$$m_1 \ddot{X}_1 + (k_1 + k_2) X_1 - K_2 X_2 = \phi$$

$$m_2 \ddot{X}_2 + (K_2 + k_3) X_2 - K_2 X_1 = \phi$$

$$\ldots + \begin{bmatrix} (K_1 + K_2) & -K_2 \\ -K_2 & (K_2 + K_3) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b)

b) $\omega_n = \sqrt{k/m}$

```
wn1 = sqrt(ksum1/m1);
wn2 = sqrt(ksum2/m2);
wn = [wn1;wn2];
```

```
wn =

    6.7082
    2.5820
```

The value of mass one and the sum of the spring values for the mass one diagram were used in calculation for the natural frequency of mass one. The same method was applied to find the natural frequency of mass two.

c)

c) $$(-\omega_n^2[m]+[k])\{u^n\} = \emptyset$$

$$\{U^n\} = \begin{Bmatrix} U_{1n} \\ U_{2n} \end{Bmatrix}$$

$$\lambda_1 = \frac{m_1 \omega_{n1}^2}{K_1} \qquad \lambda_2 = \frac{m_2 \omega_{n2}^2}{K_2}$$

$$-\lambda \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} + \begin{bmatrix} (K_1+K_2) & -K_2 \\ -K_2 & (K_2+K_3) \end{bmatrix} = U^n$$

```
lamda = (mass*wn.^2)./spring;
u = (-lamda*mass)+spring;
```

```
lamda =

    1.5000    3.0000
   -0.6667    0.4000
```

This portion of the MATLAB code results in the 2x2 matrix for the system’s mode shapes. Mass and spring are the 2x2 matrices for all the mass and spring values.

d) Since there is no input force the system will operate using the free response.

d) $t = \emptyset$

$$V_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad \omega_n = \begin{bmatrix} \omega_{n1} \\ \omega_{n2} \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ equilibrium

$$A = U^{-1} X_0$$

$$B = \begin{bmatrix} \omega_{n1} & 0 \\ 0 & \omega_{n2} \end{bmatrix}^{-1} u^{-1} V_0$$

$$X_{m1}(t) = u^1 (A_1 \cos(\omega_1 t) + B_1 \sin(\omega_1 t))$$

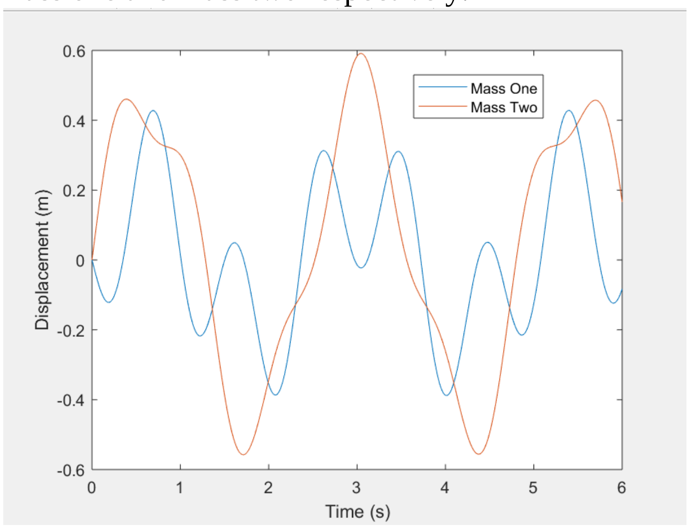
$$X_{m2}(t) = u^2 (A_2 \cos(\omega_2 t) + B_2 \sin(\omega_2 t))$$

$$X_{tot} = X_{m1} + X_{m2}$$

```
u =

   57.0000   21.0000
  -28.6667   48.8000

>> u(:,1)

ans =

   57.0000
  -28.6667

>> u(:,2)

ans =

   21.0000
   48.8000
```

xtot is the total displacement of the system while xm1 and xm2 are the displacements for mass one and mass two respectively.

e)



The displacements begin at zero for both masses. Mass two has a positive slope greater than the negative slope of mass one. These slopes represent the velocities of each mass.

MATLAB Code

```
clear
clc
close
a = 2;
b = 3;
c = 9;

m1 = a; %kg
m2 = b; %kg
k1 = 10*c; %N/m
k2 = 10*b; %N/m
k3 = 10*a; %N/m
x1_dot = -1; %m/s
x2_dot = 2; %m/s

ksum1 = (k1-k2)+k2;
ksum2 = (k2+k3)-k2;

wn1 = sqrt(ksum1/m1);
wn2 = sqrt(ksum2/m2);
wn = [wn1;wn2];

mass = [m1 0;0 m2];
spring = [(k1-k2) k2;-k2 (k2+k3)];

xo = [0;0];
vo = [-1;2];

lamda = (mass*wn.^2)./spring;
u = (-lamda*mass)+spring;
A = (u(1)^-1)*xo;
B = ([wn(1) 0;0 wn(2)]^-1)*(u^-1)*vo;

i = 1;
for t = 0:0.005:3
 x1 =u(:,1).*((A(1).*cos(wn(1).*t))+(B(1).*sin(wn(1).*t)));
 x2 = u(:,2).*((A(2).*cos(wn(2).*t))+(B(2).*sin(wn(2).*t)));
 xtot = x1+x2;
 xm1(i) = xtot(1);
 xm2(i) = xtot(2);
i = i +1;
end
```

```
y = [0:0.005:3];
plot(y,xm1)
hold on
plot(y,xm2)
xlabel('Time (s)')
ylabel('Displacement (m)')
legend('Mass One','Mass Two')
```