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ME 363 Project Two

07/02/2018

a) 2^{nd} order undamped step input for $t \geq 0$

a) $m\ddot{x} + kx = \hat{F} t^2 \mu(t - t_1)$

$$\boxed{\ddot{x} + \frac{k}{m} x = \frac{\hat{F}}{m} t^2 \mu(t - t_1)}$$

b) $x_1(0 \leq t \leq t_1) = free\ response + forced\ step\ response$

The total system response is a combination of the free and forced response. I found the free and forced responses separately than added them together.

b) $0 \leq t \leq t_1$

Forced Step Response

$$X_{forced}(t) = \frac{\hat{F}}{\omega_n^2}\left[1 - e^{-\zeta\omega_n t}\left(\cos(\omega_d t) + \zeta\frac{\omega_n}{\omega_d}\sin(\omega_d t)\right)\right]\mu(t - t_1)$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

Free Response

homogeneous:

$$e^{-\zeta\omega_n t}\left(X_0\cos(\omega_n t) - X_p(0)\cos(\omega_n t) + \frac{\dot{X}_0}{\omega_n}\sin(\omega_n t) - \frac{\dot{X}_p(0)}{\omega_n}\sin(\omega_n t) + \zeta X_0\sin(\omega_n t) - \zeta X_p(0)\sin(\omega_n t)\right)$$

$\downarrow$

$$X_{free}(t) = e^{-\zeta\omega_n t}\left(X_0\cos(\omega_n t) + \frac{\dot{X}_0}{\omega_n}\sin(\omega_n t) + \zeta X_0\sin(\omega_n t)\right)$$

$$\boxed{X_{Tot}(t) = \left[e^{-\zeta\omega_n t}\left(X_0\cos(\omega_n t) + \frac{\dot{X}_0}{\omega_n}\sin(\omega_n t) + \zeta X_0\sin(\omega_n t)\right)\right] + \frac{\hat{F}}{\omega_n^2}\left[1 - e^{-\zeta\omega_n t}\left(\cos(\omega_n\sqrt{1-\zeta^2}\, t) + \zeta\frac{\omega_n}{\omega_n\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\, t)\right)\right]\mu(t - t_1)}$$

c)

c) $\mathcal{L}\left[\ddot{x} + \frac{K}{m}x\right] = \mathcal{L}\left[\hat{F}/m\, t^2\, \mu(t_0 - t_1)\right]$

$$s^2 X(s) - s\,\cancelto{\emptyset}{x(0)} - \cancelto{\emptyset}{\dot{x}(0)} + \frac{K}{m}X(s) =$$

$$\frac{\hat{F}}{m}\ \frac{2}{s^3}\ \frac{1}{s}$$

$$\rightarrow\ X(s)\left(s^2 + \frac{K}{m}\right) = \frac{\hat{F}}{m}\ \frac{2}{s^3}\ \frac{1}{s}$$

$$\rightarrow\ X(s) = \frac{2\hat{F}}{ms^4\left(s^2 + \frac{K}{m}\right)} = \frac{2\hat{F}}{s^4(ms^2 + K)}$$

$$\frac{C_1}{(ms^2 + K)} + \frac{C_2}{s^4} + \frac{C_3}{s^3} + \frac{C_4}{s^2} + \frac{C_5}{s}$$

$$C_1 = \lim_{(ms^2+K)\to\emptyset}\left[(ms^2+K)X(s)\right] = \frac{2\hat{F}}{s^4}\Bigg|_{s=\sqrt{-\frac{K}{m}}} = \frac{2\hat{F}}{-\frac{K^2}{m^2}}$$

$$C_2 = \lim_{s^4\to\emptyset}\left[s^4 X(s)\right] = \frac{2\hat{F}}{ms^2+K}\Bigg|_{s=0} = \frac{2\hat{F}}{K}$$

d)

c. $C_3 = \lim\limits_{s^3 \to 0}\left[\frac{d}{ds}\left(s^3 X(s)\right)\right] = \frac{2\hat{F}m}{K^2}$

$C_4 = \lim\limits_{s^2 \to 0}\left[\frac{d}{ds}\left(s^2 X(s)\right)\right] = \frac{2\hat{F}}{s^2(ms^2+K)}\Bigg|_{s=0} = \frac{2\hat{F}m^2}{K^3}$

$$2\hat{F} = \left(ms^4\left(s^2+\frac{K}{m}\right)\right)\left[\frac{C_1}{ms^2+K}+\frac{C_2}{s^4}+\frac{C_3}{s^3}+\frac{C_4}{s^2}+\frac{C_5}{s}\right]$$

$$= C_1 s^4 + C_2 ms^2 + K + C_4 ms^4 + Ks^2$$

$$+ C_5(ms^5 + Ks^3)$$

$$= C_1 s^4 + C_2 ms^2 + C_2 K + C_4 ms^4 + C_4 Ks^2 + C_5 ms^5 + C_5 ks^3$$

$$0 = C_5 ms^5 + C_5 ks^3$$

$$C_5 = \emptyset$$

$$X(s) = \frac{-\frac{2\hat{F}m^2}{k^2}}{(ms^2+k)} + \frac{2\hat{F}}{Ks^4} - \frac{2\hat{F}m}{k^2 s^3} + \frac{2\hat{F}m^2}{K^3 s^2} + \emptyset$$

e) $x_2(t \geq t_1) = free\ response + forced\ step\ response$

Since the system needed to be designed to abruptly stop when t = t1 I used two equations. The first equation accounted for the moment it changed to t1 and the second equation accounted for the moment just after it exceeds t1. The second equation just amounted to zero so that the system would know to stop.

d) for $t = t_1 = \pi/\omega_n$

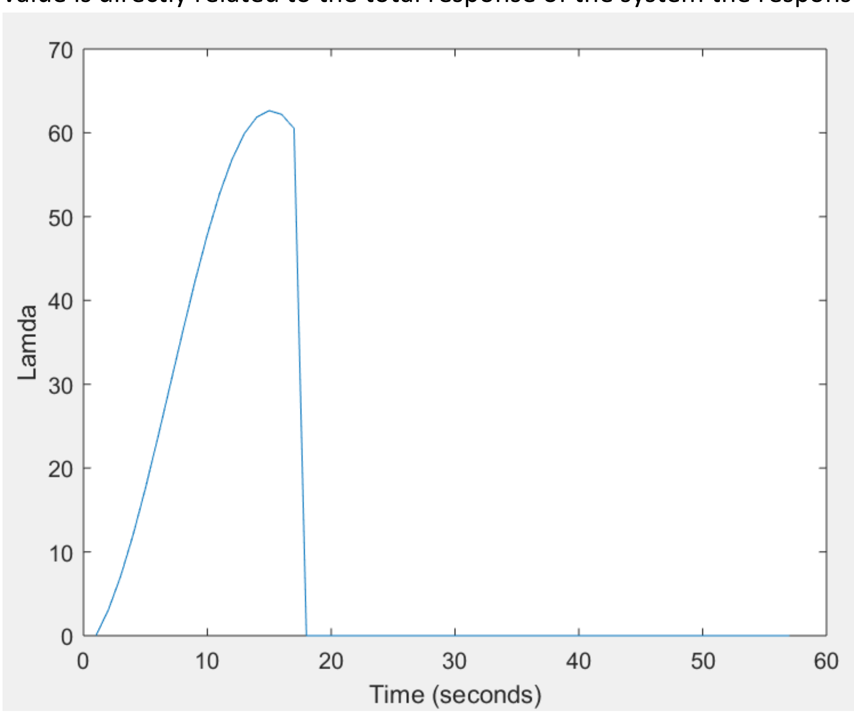
$$X_{TOT}(t) = \left[e^{-\xi\pi}\left(X_0\cos(\pi) + \frac{\dot{X}_0}{\omega_n}\sin(\pi) + \xi X_0\sin(\pi)\right)\right]$$
$$+\left[\frac{\hat{F}}{\omega_n^2} - \frac{\hat{F}}{\omega_n^2}e^{-\xi\omega_n t}\left(\cos\left(\sqrt{1-\xi^2}\,\pi\right) + \xi\frac{1}{\sqrt{1-\xi^2}}\sin\left(\sqrt{1-\xi^2}\,\pi\right)\right)\right]\mu(t-t_1)$$

$X_0(t_1) = 1$ for a step response
$\dot{X}_0(t_1) = \emptyset$

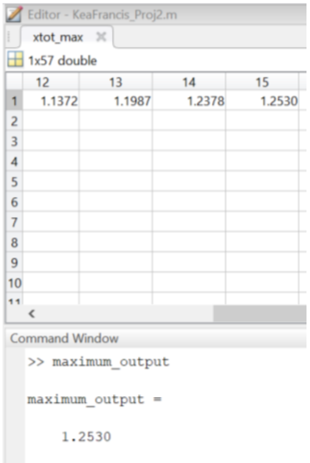
for $t > t_1$ abruptly stops

$X_{TOT} = \emptyset$

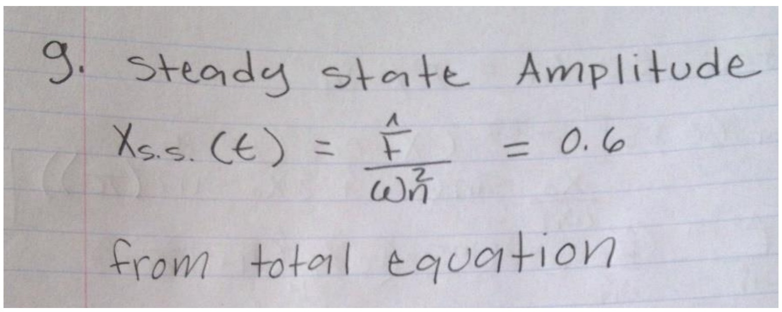
f) The curve looks believable. There is an abrupt stop after the time reaches t1 and continues to zero similar to a step function. Since the system does not have damping and since the lamda value is directly related to the total response of the system the response can oscillate freely.



g) The maximum value for the total response was 1.25 which occurred after 0.75 seconds.



h) To derive the steady state equation, I looked for the portion of the total response equation that wasn't transient.




```
format short
clear all
clc

m = 2; %kg
k = 10*3; %N/m
F_hat = 9; %N/s^2
%damp = 0; %damping

x0 = 0;
xdot = 0;

wn = sqrt(k/m); %rad/s
t = 0;
t1 = pi()/wn; % time that the disturbance abruptly stops
theta = wn*t; %

i = 1;

while theta < 11
```

```
damp = 0;
wd = wn*sqrt(1-damp^2);
phase = atand(sqrt(1-damp^2)/damp)+pi(); %phase angle

ss = F_hat/(wn^2); %steady state of response

if t > 0
xdot = 1; %check reasoning
xfree = exp(-damp*wn*t)*(x0*cos(wn*t))+((xdot/wn)*sin(wn*t))+(damp*x0*sin(wn*t));
xforce = (F_hat/wn^2)*(1-exp(-damp*wn*t)*(cos(wd*t)+(damp*(wn/wd))*sin(wd*t))); %forced
step response for second order system
xtot = xfree + xforce;
xtot_max(i) = xfree + xforce; %used to determine the maximum x(t)
lamda(i) = ((k^2)/(m*F_hat))*xtot;
end

if t > t1
    xtot = 0;
    lamda(i) = ((k^2)/(m*F_hat))*xtot;
end

i = i + 1;
t = t + 0.05;
theta = wn*t;
thet(i) = wn*t;
end

maximum_output = max(xtot_max);

plot(lamda)
xlabel('Time (seconds)')
ylabel('Lamda')
```