

Kea Francis

ME 363 Project Two

07/02/2018

- a) 2nd order undamped step input for $t \geq 0$

Handwritten equations on a piece of paper:

$$a) \quad m\ddot{x} + kx = \hat{F}t^2 \mathcal{H}(t-t_1)$$
$$\left[\ddot{x} + \frac{k}{m}x = \frac{\hat{F}}{m}t^2 \mathcal{H}(t-t_1) \right]$$

- b) $x_1(0 \leq t \leq t_1) = \text{free response} + \text{forced step response}$

The total system response is a combination of the free and forced response. I found the free and forced responses separately then added them together.

Handwritten derivation on lined paper:

b) $0 \leq t \leq t_1$

Forced Step Response

$$x_{\text{forced}}(t) = \frac{\hat{F}}{\omega_n^2} \left[1 - e^{-\zeta \omega_n t} \left(\cos(\omega_d t) + \zeta \frac{\omega_n}{\omega_d} \sin(\omega_d t) \right) \right] \mathcal{H}(t-t_1)$$
$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

Free Response

Homogeneous:

$$e^{-\zeta \omega_n t} \left(x_0 \cos(\omega_n t) - \frac{x_p(0)}{\omega_n} \cos(\omega_n t) + \frac{\dot{x}_0}{\omega_n} \sin(\omega_n t) - \frac{\dot{x}_p(0)}{\omega_n} \sin(\omega_n t) + \zeta x_0 \sin(\omega_n t) - \zeta x_p(0) \sin(\omega_n t) \right)$$

↓

$$x_{\text{free}}(t) = e^{-\zeta \omega_n t} \left(x_0 \cos(\omega_n t) + \frac{x_0}{\omega_n} \sin(\omega_n t) + \zeta x_0 \sin(\omega_n t) \right)$$
$$x_{\text{Tot}}(t) = \left[e^{-\zeta \omega_n t} \left(x_0 \cos(\omega_n t) + \frac{x_0}{\omega_n} \sin(\omega_n t) + \zeta x_0 \sin(\omega_n t) \right) + \frac{\hat{F}}{\omega_n^2} \left[1 - e^{-\zeta \omega_n t} \left(\cos(\omega_n \sqrt{1-\zeta^2} t) + \zeta \frac{\omega_n}{\omega_n \sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2} t) \right) \right] \mathcal{H}(t-t_1) \right]$$

- c)

$$c) \mathcal{L}[\ddot{x} + \frac{k}{m}x] = \mathcal{L}[\frac{\hat{F}}{m} t^2 \mathcal{H}(t_0 - t_1)]$$

$$s^2 X(s) - s \overset{\nearrow 0}{x(0)} - \overset{\nearrow 0}{\dot{x}(0)} + \frac{k}{m} X(s) =$$

$$\frac{\hat{F}}{m} \frac{2}{s^3} - \frac{1}{s}$$

$$\rightarrow X(s) (s^2 + \frac{k}{m}) = \frac{\hat{F}}{m} \frac{2}{s^3} - \frac{1}{s}$$

$$\rightarrow X(s) = \frac{2\hat{F}}{ms^4(s^2 + \frac{k}{m})} = \frac{2\hat{F}}{s^4(ms^2 + k)}$$

$$\frac{C_1}{(ms^2 + k)} + \frac{C_2}{s^4} + \frac{C_3}{s^3} + \frac{C_4}{s^2} + \frac{C_5}{s}$$

$$C_1 = \lim_{(ms^2 + k) \rightarrow 0} [(ms^2 + k) X(s)] = \frac{2\hat{F}}{s^4} \Big|_{s = \sqrt{-\frac{k}{m}}} = \frac{2\hat{F}}{-\frac{k^2}{m^2}}$$

$$C_2 = \lim_{s^4 \rightarrow 0} [s^4 X(s)] = \frac{2\hat{F}}{ms^2 + k} \Big|_{s=0} = \frac{2\hat{F}}{k}$$

d)

$$C. C_3 = \lim_{s^3 \rightarrow 0} \left[\frac{d}{ds} (s^3 X(s)) \right] = -\frac{2\hat{F}m}{K^2}$$

$$C_4 = \lim_{s^2 \rightarrow 0} \left[\frac{d}{ds} (s^2 X(s)) \right] = \frac{2\hat{F}}{s^2(ms^2+K)} \Big|_{s=0} = -\frac{2\hat{F}m^2}{K^3}$$

$$2\hat{F} = (ms^4(s^2 + \frac{K}{m})) \left[\frac{C_1}{ms^2+K} + \frac{C_2}{s^4} + \frac{C_3}{s^3} + \frac{C_4}{s^2} + \frac{C_5}{s} \right]$$

$$= C_1 s^4 + C_2 ms^2 + K + C_4 ms^4 + Ks^2$$

$$+ C_5(ms^5 + Ks^3)$$

$$= C_1 s^4 + C_2 ms^2 + C_2 K + C_4 ms^4 + C_4 Ks^2 + C_5 ms^5 + C_5 Ks^3$$

$$0 = C_5 ms^5 + C_5 Ks^3$$

$$C_5 = 0$$

$$X(s) = \frac{2\hat{F}m^2}{K^2} + \frac{2\hat{F}}{Ks^4} - \frac{2\hat{F}m}{K^2s^3}$$

$$+ \frac{2\hat{F}m^2}{K^3s^2} + 0$$

e) $x_2(t \geq t_1) = \text{free response} + \text{forced step response}$

Since the system needed to be designed to abruptly stop when $t = t_1$ I used two equations. The first equation accounted for the moment it changed to t_1 and the second equation accounted for the moment just after it exceeds t_1 . The second equation just amounted to zero so that the system would know to stop.

d) for $t = t_1 = \pi/\omega_n$

$$X_{TOT}(t) = \left[e^{-\xi\pi} \left(X_0 \cos(\pi) + \frac{X_0}{\omega_n} \sin(\pi) + \xi X_0 \sin(\pi) \right) \right] + \left[\frac{\hat{F}}{\omega_n^2} - \frac{\hat{F}}{\omega_n^2} e^{-\xi\omega_n t} \left(\cos(\sqrt{1-\xi^2}\pi) + \xi \frac{1}{\sqrt{1-\xi^2}} \sin(\sqrt{1-\xi^2}\pi) \right) \right] \mathcal{U}(t-t_1)$$

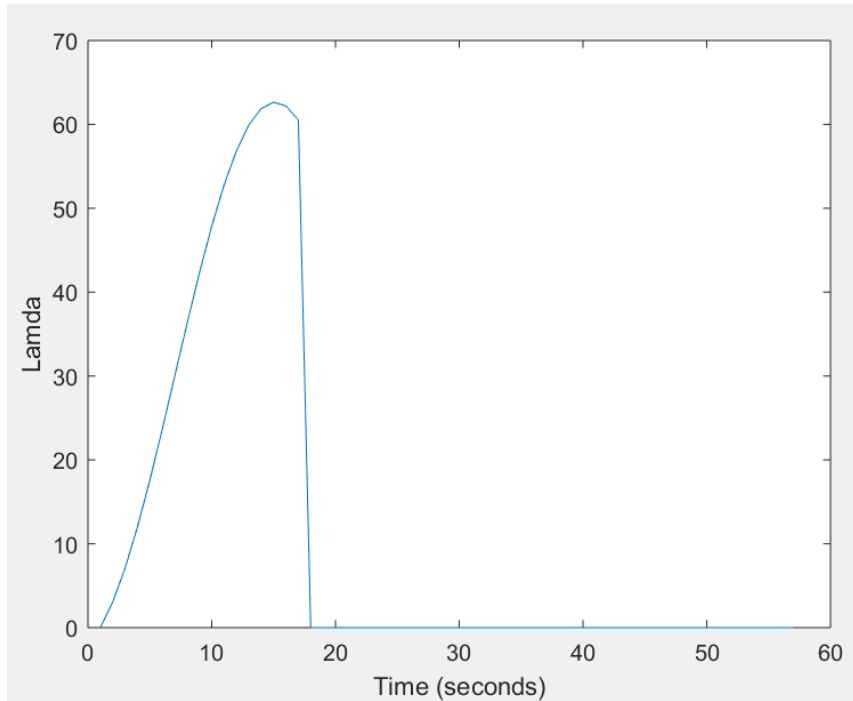
$$X_0(t_1) = 1 \text{ for a step response}$$

$$\dot{X}_0(t_1) = 0$$

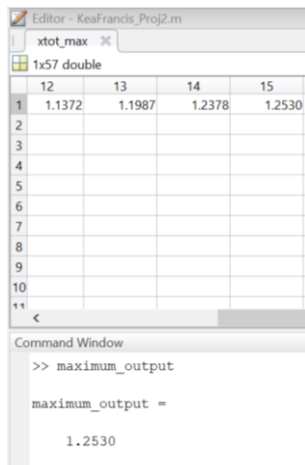
for $t > t_1$ abruptly stops

$$X_{TOT} = 0$$

- f) The curve looks believable. There is an abrupt stop after the time reaches t_1 and continues to zero similar to a step function. Since the system does not have damping and since the lamda value is directly related to the total response of the system the response can oscillate freely.



- g) The maximum value for the total response was 1.25 which occurred after 0.75 seconds.

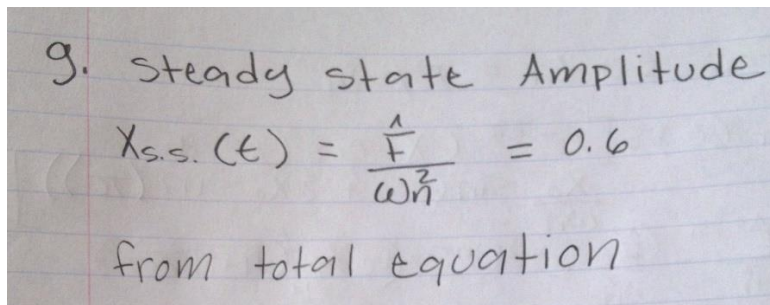


The image shows a MATLAB Editor window titled 'Editor - KeaFrancis_Proj2.m' with a variable 'xtot_max' of type 'double' and size '1x57'. Below it is a table with 4 columns (12, 13, 14, 15) and 11 rows (1 to 11). The values in the table are:

	12	13	14	15
1	1.1372	1.1987	1.2378	1.2530
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				

Below the table is the Command Window showing the command 'maximum_output' and the result 'maximum_output = 1.2530'.

- h) To derive the steady state equation, I looked for the portion of the total response equation that wasn't transient.



The image shows handwritten notes on lined paper. The text reads: '9. Steady state Amplitude', followed by the equation $X_{s.s.}(t) = \frac{\hat{F}}{\omega_n^2} = 0.6$, and then 'from total equation'.

format short

clear all

clc

m = 2; %kg

k = 10*3; %N/m

F_hat = 9; %N/s^2

%damp = 0; %damping

x0 = 0;

xdot = 0;

wn = sqrt(k/m); %rad/s

t = 0;

t1 = pi()/wn; % time that the disturbance abruptly stops

theta = wn*t; %

i = 1;

while theta < 11

```

damp = 0;
wd = wn*sqrt(1-damp^2);
phase = atand(sqrt(1-damp^2)/damp)+pi(); %phase angle

ss = F_hat/(wn^2); %steady state of response

if t > 0
    xdot = 1; %check reasoning
    xfree = exp(-damp*wn*t)*(x0*cos(wn*t))+((xdot/wn)*sin(wn*t))+ (damp*x0*sin(wn*t));
    xforce = (F_hat/wn^2)*(1-exp(-damp*wn*t)*(cos(wd*t)+(damp*(wn/wd))*sin(wd*t))); %forced
    %step response for second order system
    xtot = xfree + xforce;
    xtot_max(i) = xfree + xforce; %used to determine the maximum x(t)
    lamda(i) = ((k^2)/(m*F_hat))*xtot;
end

if t > t1
    xtot = 0;
    lamda(i) = ((k^2)/(m*F_hat))*xtot;
end

i = i + 1;
t = t + 0.05;
theta = wn*t;
thet(i) = wn*t;
end

maximum_output = max(xtot_max);

plot(lamda)
xlabel('Time (seconds)')
ylabel('Lamda')

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